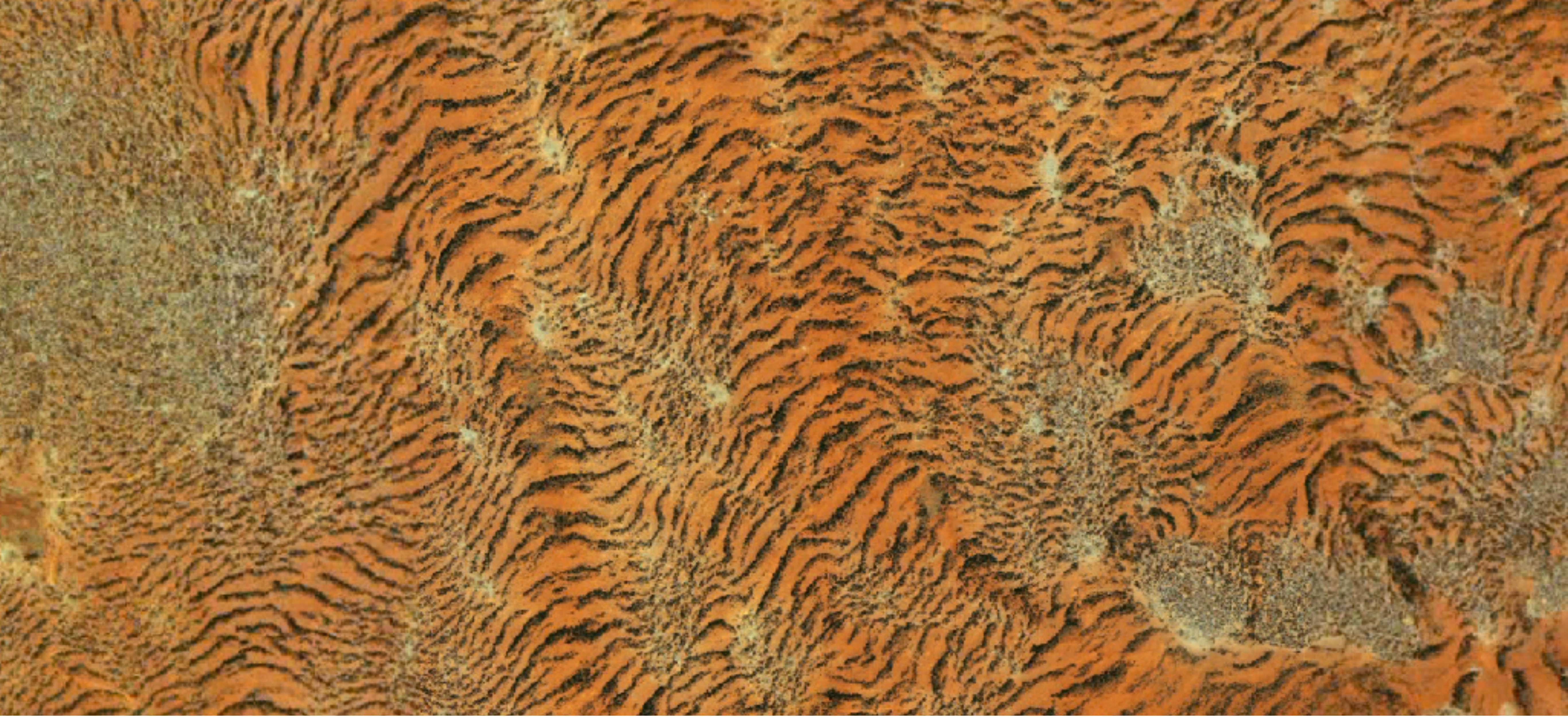


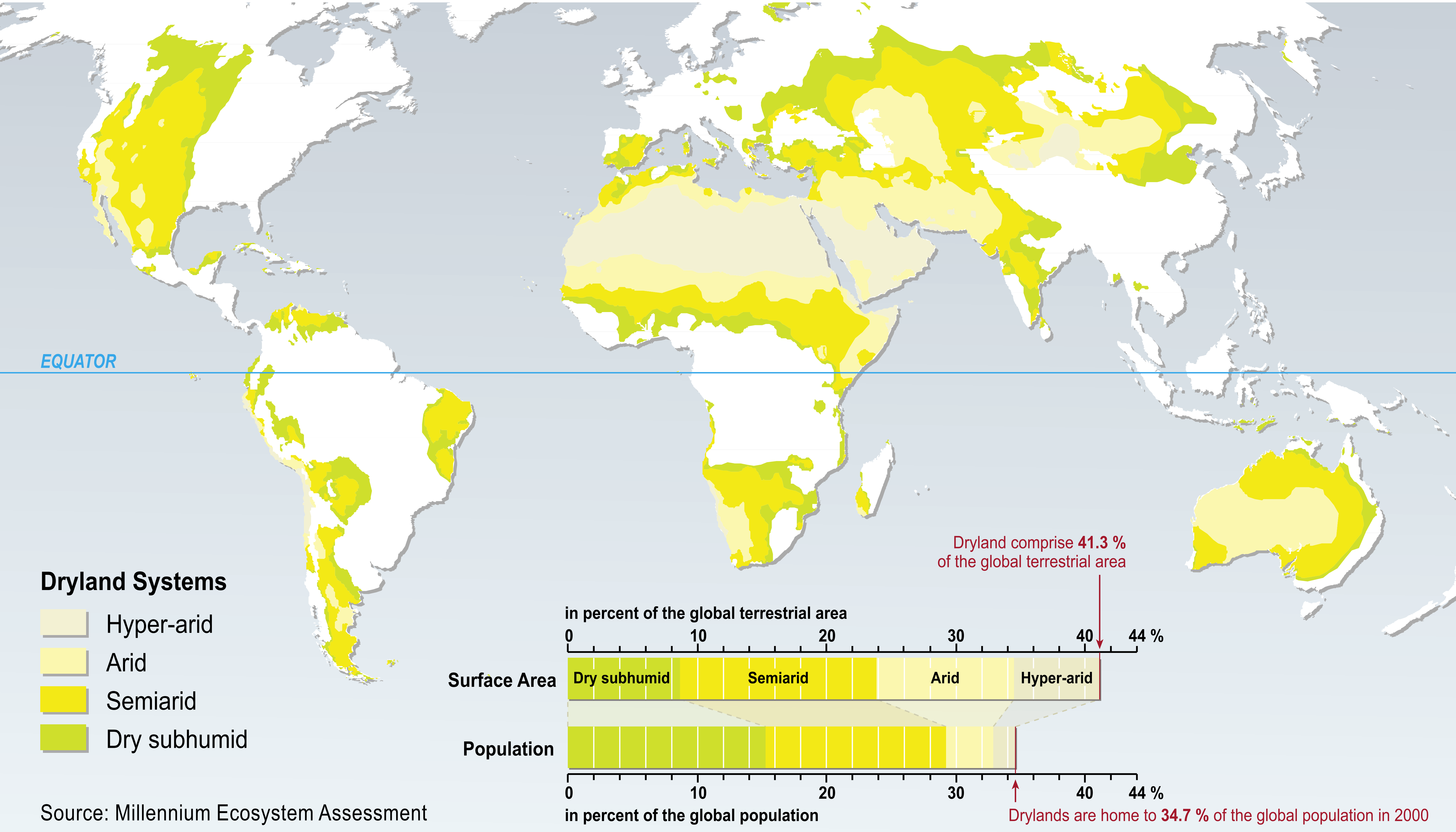
Pattern Formation in the Drylands:

Self Organization in Semi-Arid Ecosystems

Mary Silber
Committee on Computational and Applied Mathematics
+ Dept. of Statistics, University of Chicago

Drylands: water-controlled ecosystems

with infrequent, discrete, and largely unpredictable water inputs.

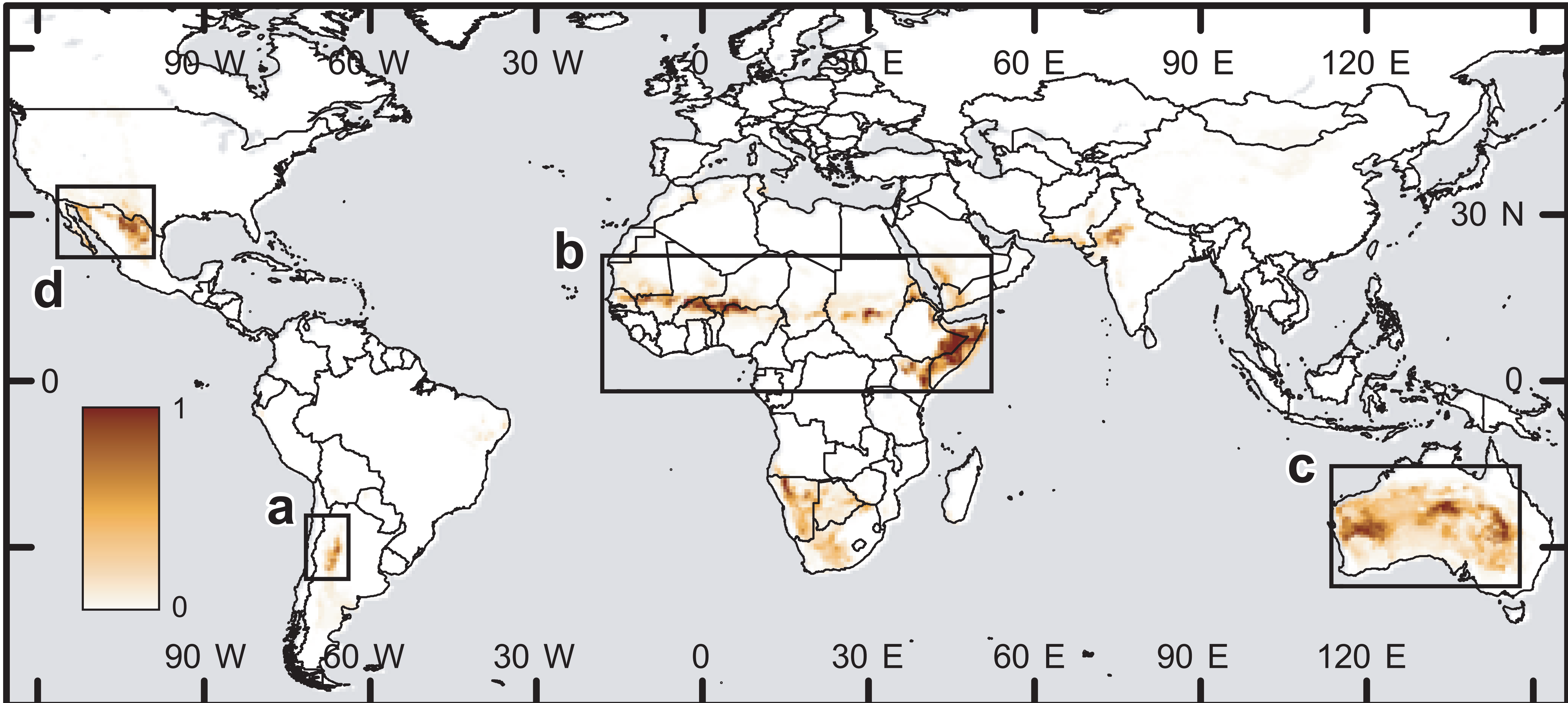


Millennium Ecosystem Assessment (2005)

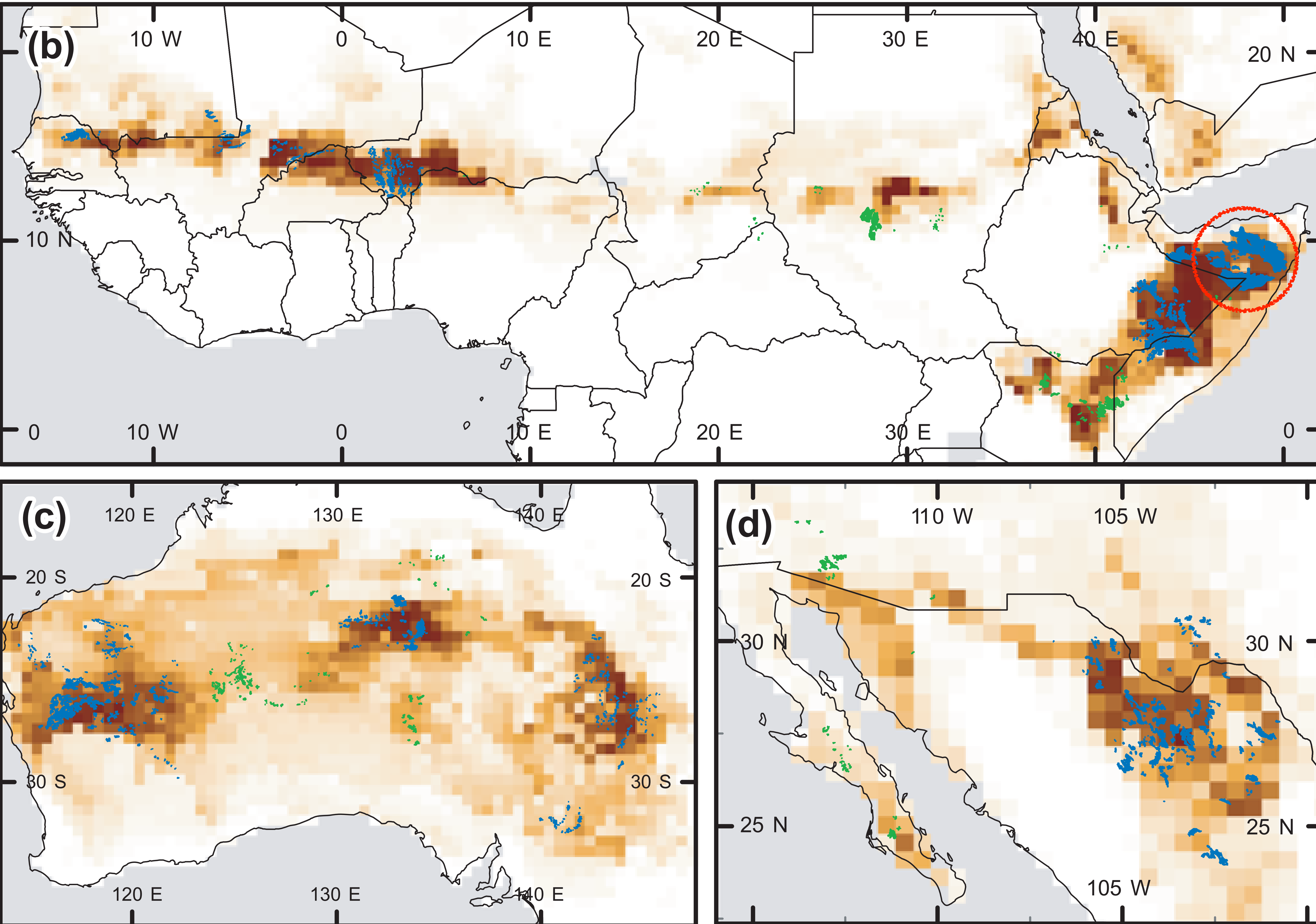
Patterned Drylands: Dry, Hot, Flat,…

The global biogeography of semi-arid periodic vegetation patterns (2008)

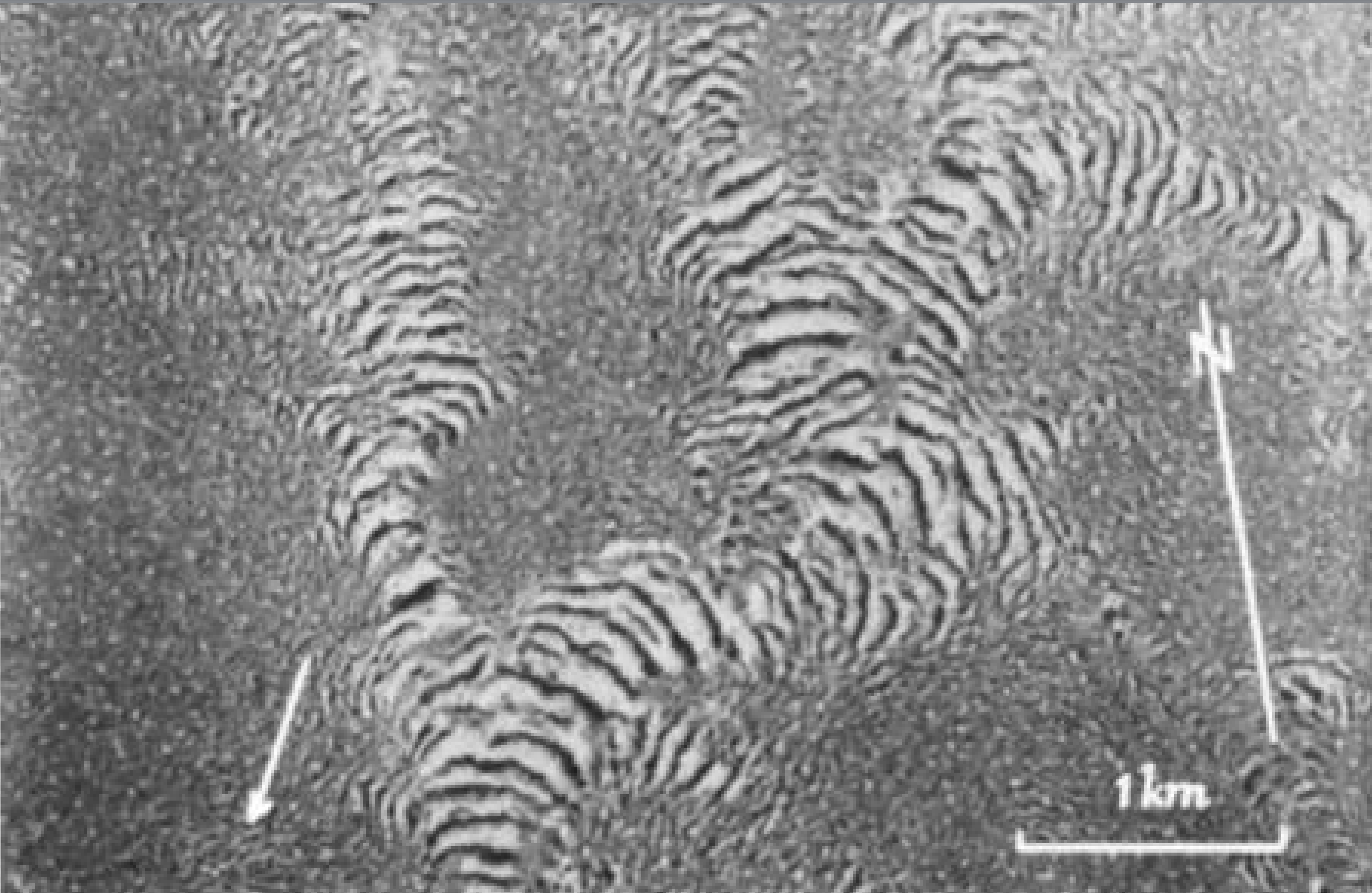
Vincent Deblauwe[1*], Nicolas Barbier[2], Pierre Couteron[3], Olivier Lejeune[4] and Jan Bogaert[1]



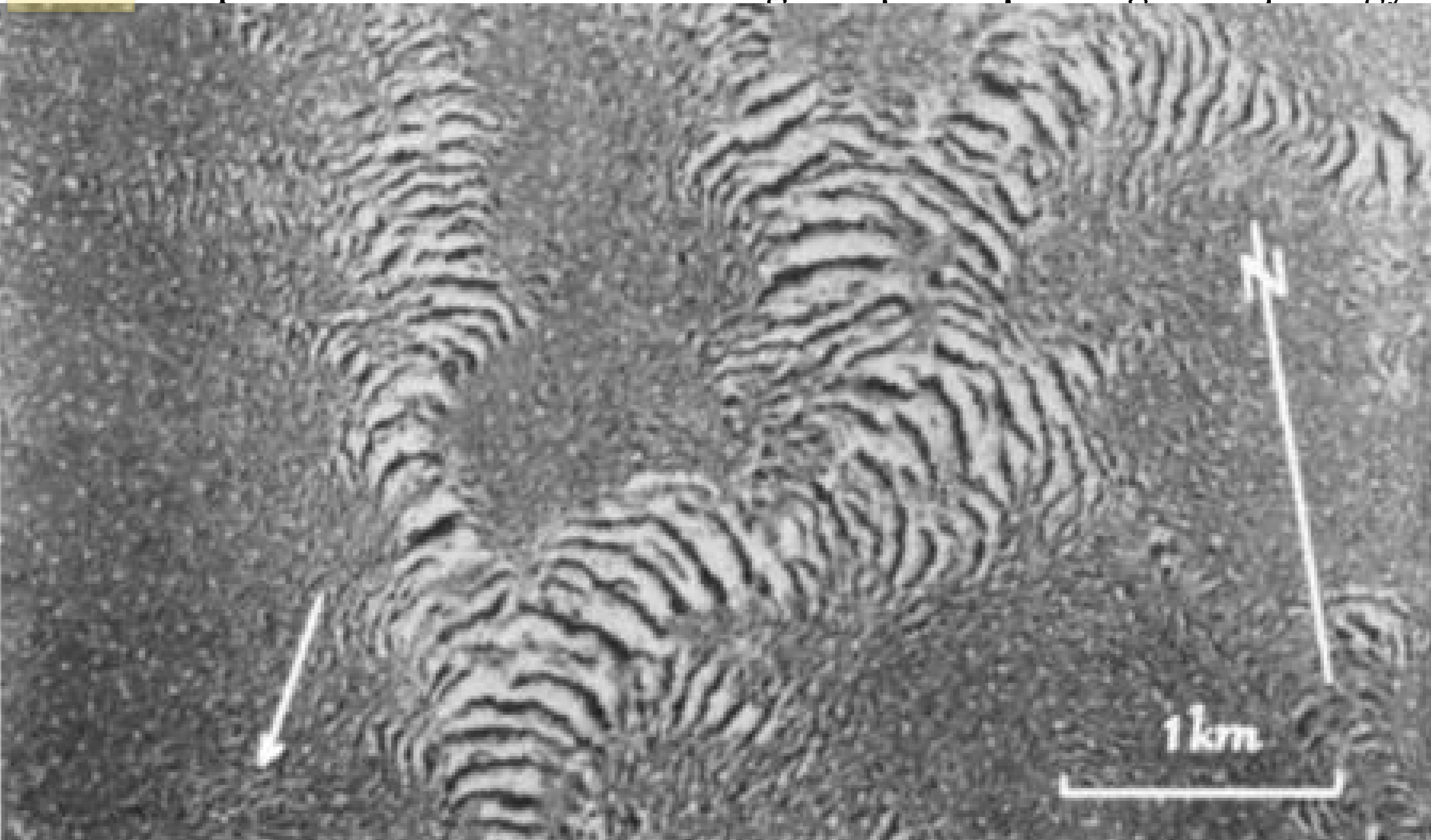
Deblauwe, et al. (2008)



Vegetation Patterns in the Semi-Desert Plains of British Somaliland
Author(s): W. A. Macfadyen
Source: *The Geographical Journal*, Vol. 116, No. 4/6 (Oct. - Dec., 1950), pp. 199-211



as I believe, must be investigated by physics and mathematics; and the whole matter must be studied on air photographs, since on the ground it proved difficult to recognize the patterns at all. While the superficial deposits are of importance, the underlying solid geology seems to have no particular significance except in its influence in moulding the pre-requisite geomorphology.



Slide from Karna Gowda, a modern day explorer

British Somaliland, circa 1950

Ethiopia, circa 2015

	"Wetlands" *(Pattern Formation in Fluids)*	"Drylands" *(Pattern Formation in the Environment)*
Equations	Navier-Stokes+BCs	models exist, but not validated due to lack of experiments
Parameters	often excellent specs	Some inferred at order of magnitude level; some constrained to match phenomena; some models have a lot
Time-scales	seconds - "PhD-scale"	decades-centuries
Spatial-scales	cm scale - "table-top"	10m-"landscape scale"
Symmetries	excellent approximation in controlled experiments	opportunity presented by heterogeneities?
Mechanisms	well developed and validated understanding of pattern formation mechanisms	generic mechanisms invoked

“Wetlands” (Pattern Formation in Fluids)

example of pattern formation via spontaneous symmetry-breaking instability

Experiments were performed at frequencies between 20 – 150 Hz.

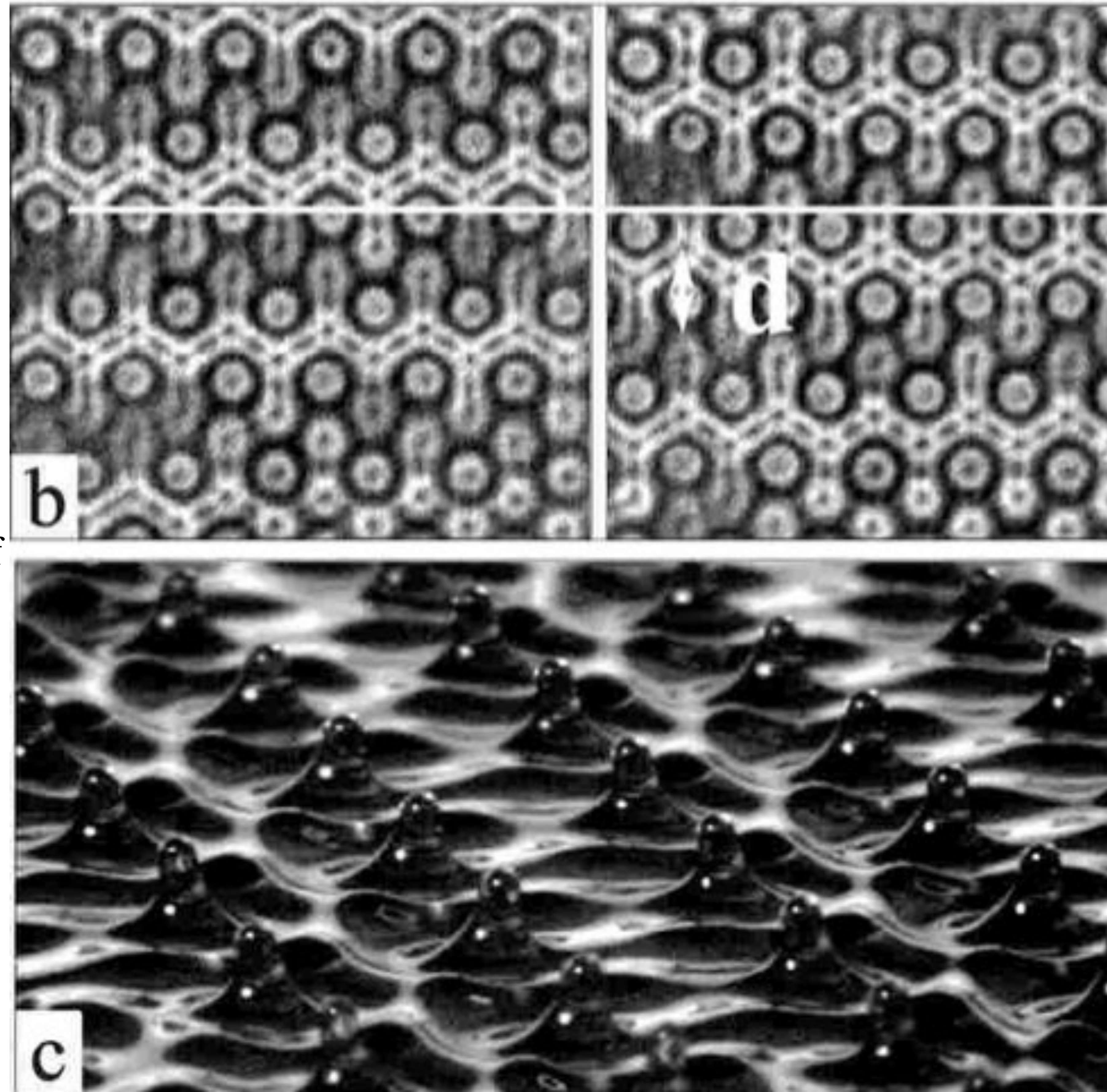
1-cm-thick, black-anodized aluminum plate of 14.4 cm diameter supported fluid, machined to 10 micron flatness.

Silicone oil density of 0.95 g/cm, surface tension of 21.5 dyne/cm.

Newtonian fluid viscosity range of 1 – 100 cS, highly temperature dependent.

Stable fluid temperature of 300.05°C was used.

Resultant viscosity variations less than 0.04 cS.



Arbell & Fineberg 2002

	"Wetlands" *(Pattern Formation in Fluids)*	**"Drylands"** *(Pattern Formation in the Environment)*
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Why study dryland patterns?

- Patterns are so Earthy and beautiful.
- Challenging applied direction for a “mature field” of pattern formation.
- Occur in ecosystems vulnerable to desertification, meant to feed a third of the world population! Is there useful information in the patterns? Any “early warning signs”?

Early Warning Signs Proposals
(some examples)

Changes in pattern morphology (on flat terrain)

Band wavelength coarsening (on sloped terrain)

(e.g. Doelman et al.; Sherrat et al.)

Disturbance recovery via front propagation

(e.g. Meron et al.)

Changes in patch size distribution

(e.g. Kefi et al.)

Early Warning Signs Proposals
(some examples)

Changes in pattern morphology (on flat terrain)

I. Vegetation Patterns in Mathematical Models

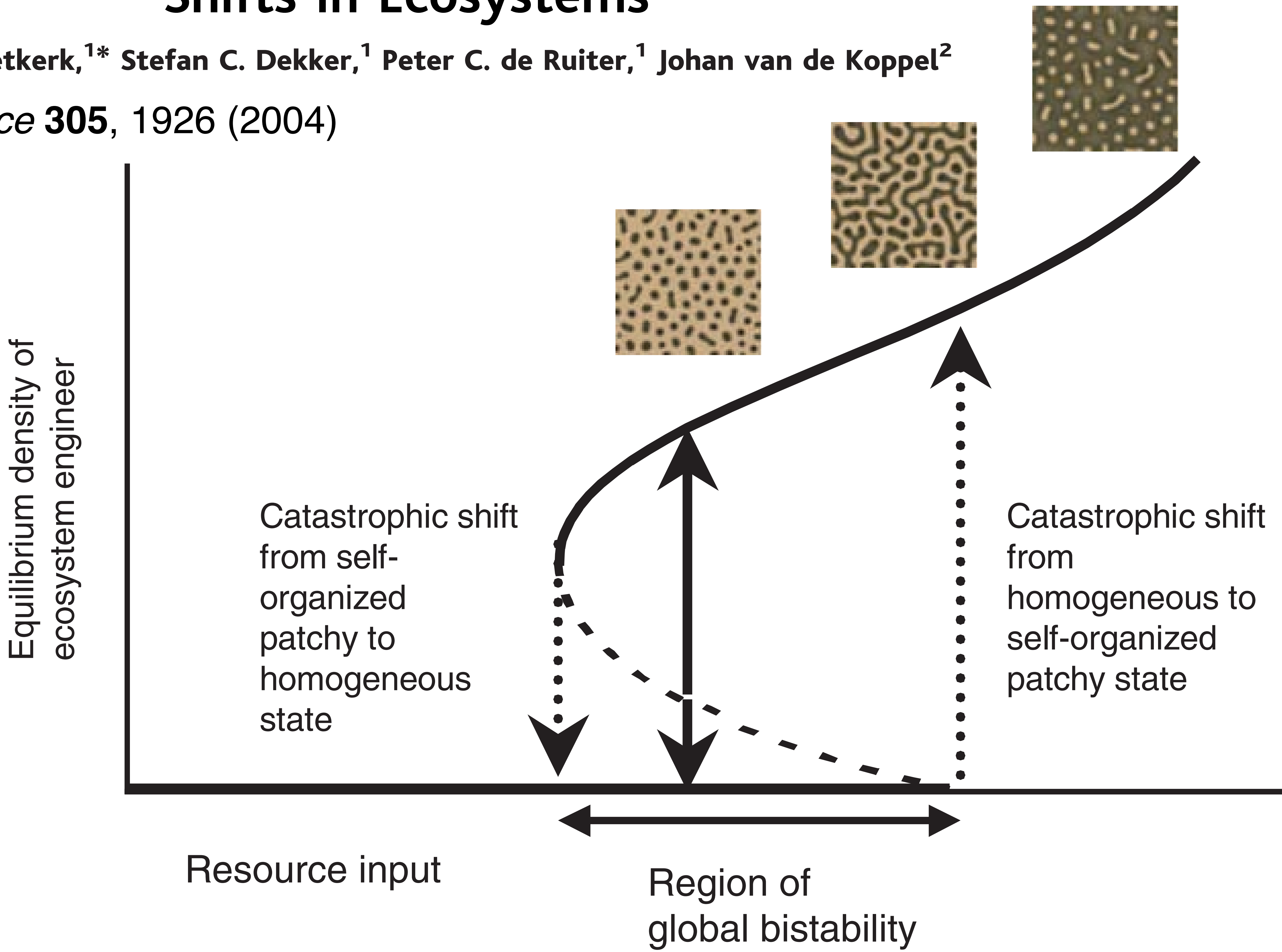
Band wavelength coarsening (on sloped terrain)

II. Vegetation Patterns in the Horn of Africa

Self-Organized Patchiness and Catastrophic Shifts in Ecosystems

Max Rietkerk,[1*] Stefan C. Dekker,[1] Peter C. de Ruiter,[1] Johan van de Koppel[2]

Science **305**, 1926 (2004)

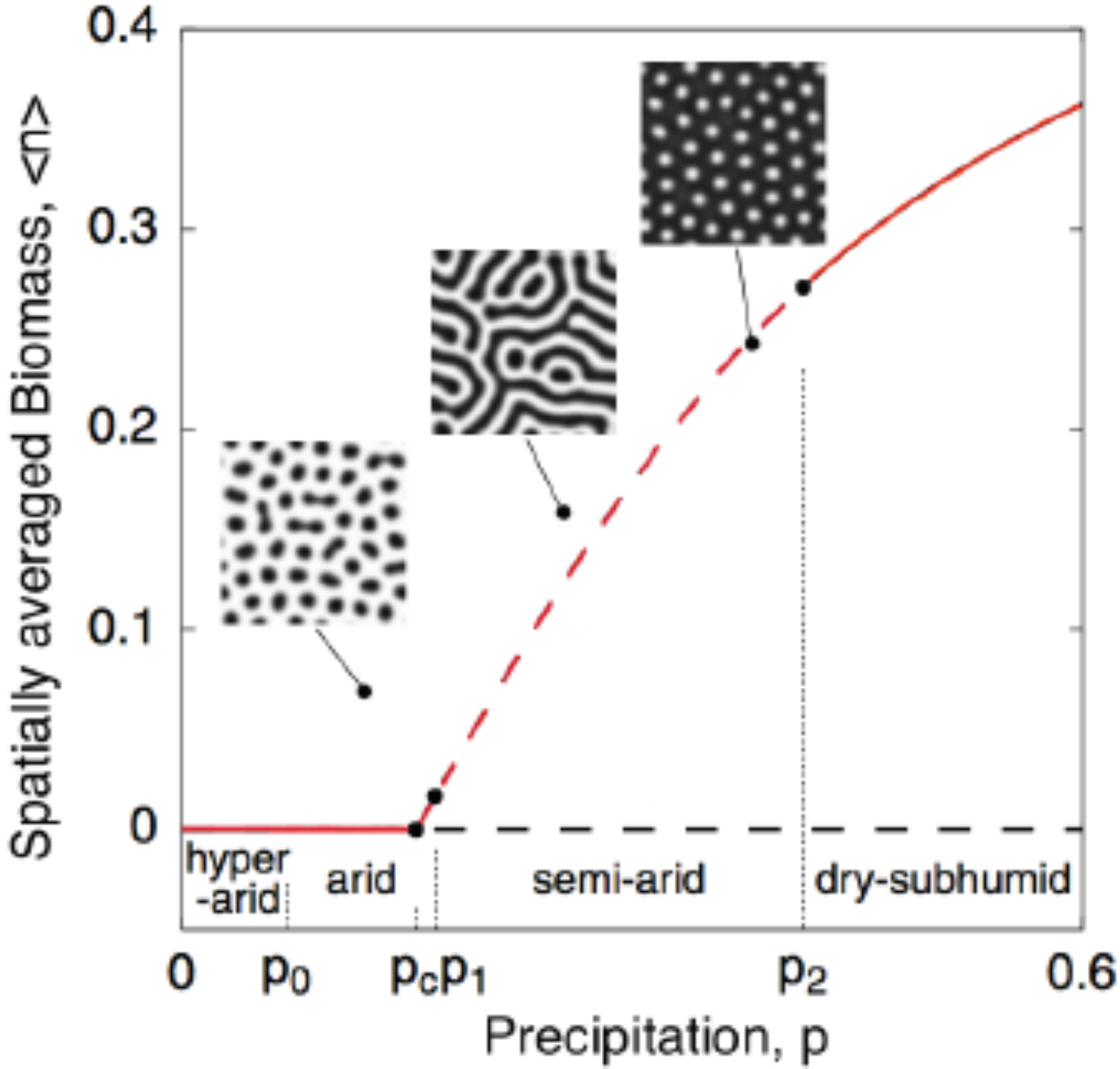


Vegetation Pattern Models: Turing mechanism

Diversity of Vegetation Patterns and Desertification

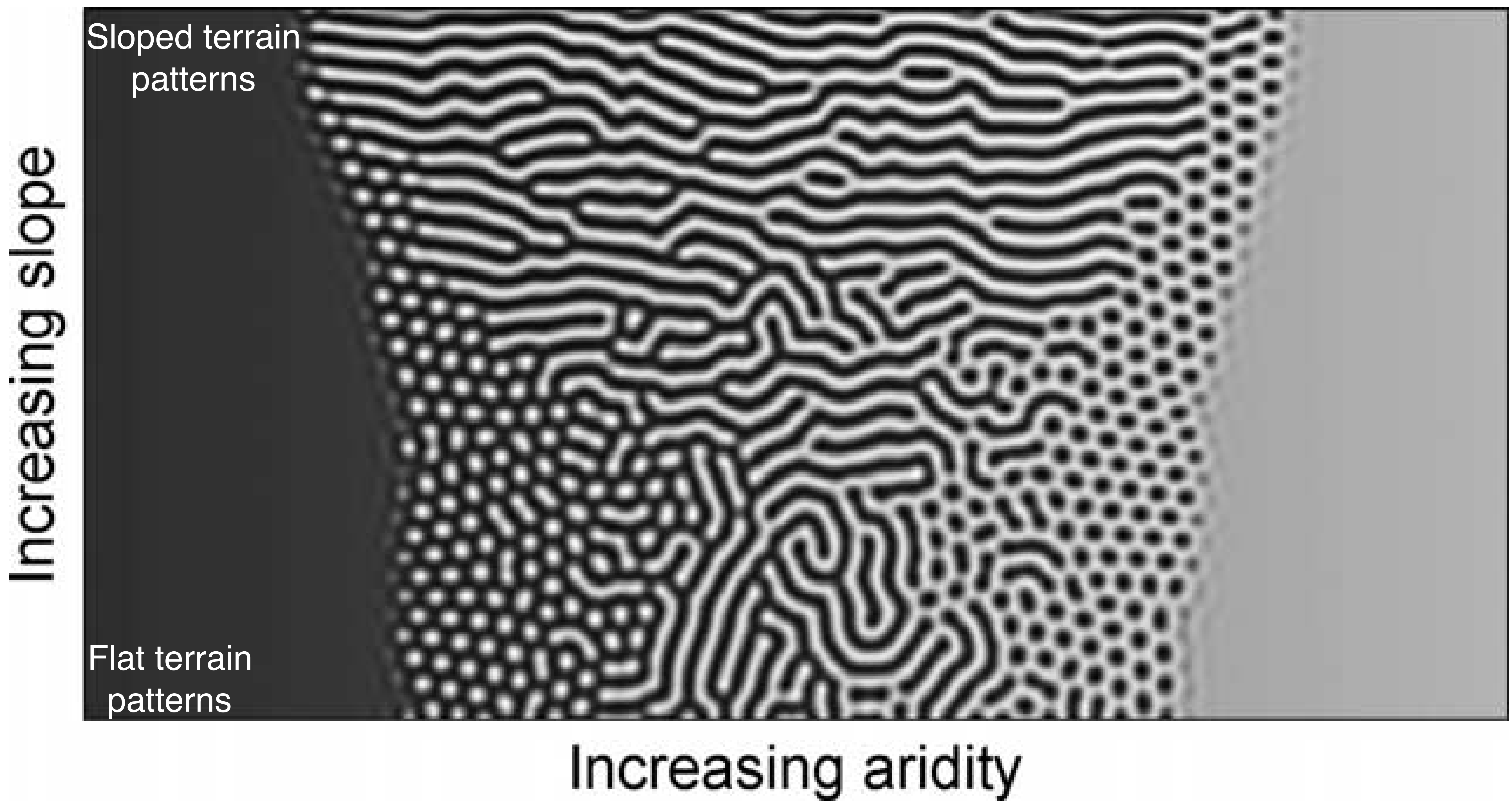
J. von Hardenberg,[1,4] E. Meron,[1,3] M. Shachak,[2] and Y. Zarmi[1,3]

Phys. Rev. Lett. (2001)

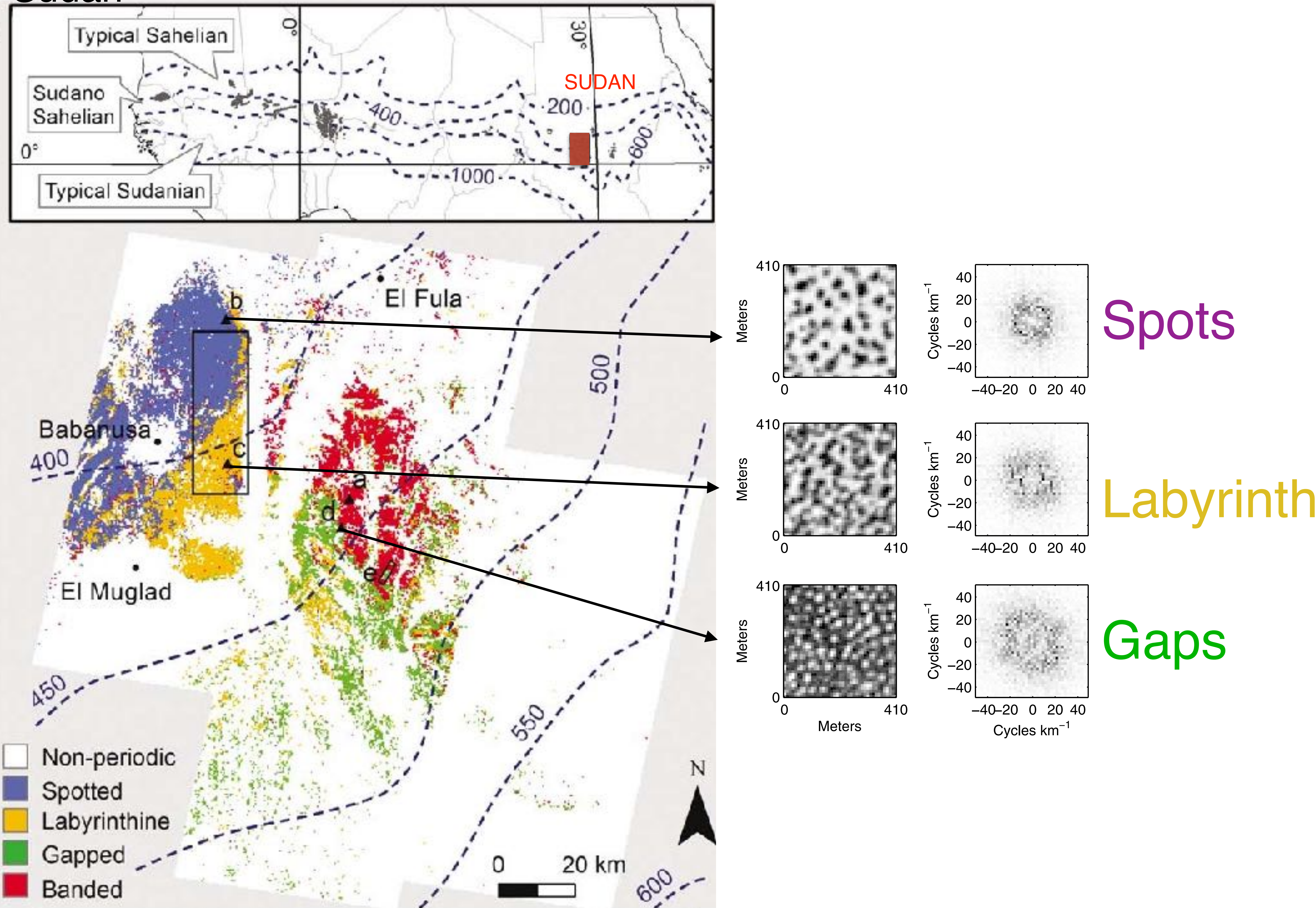


Environmental modulation of self-organized periodic vegetation patterns in Sudan

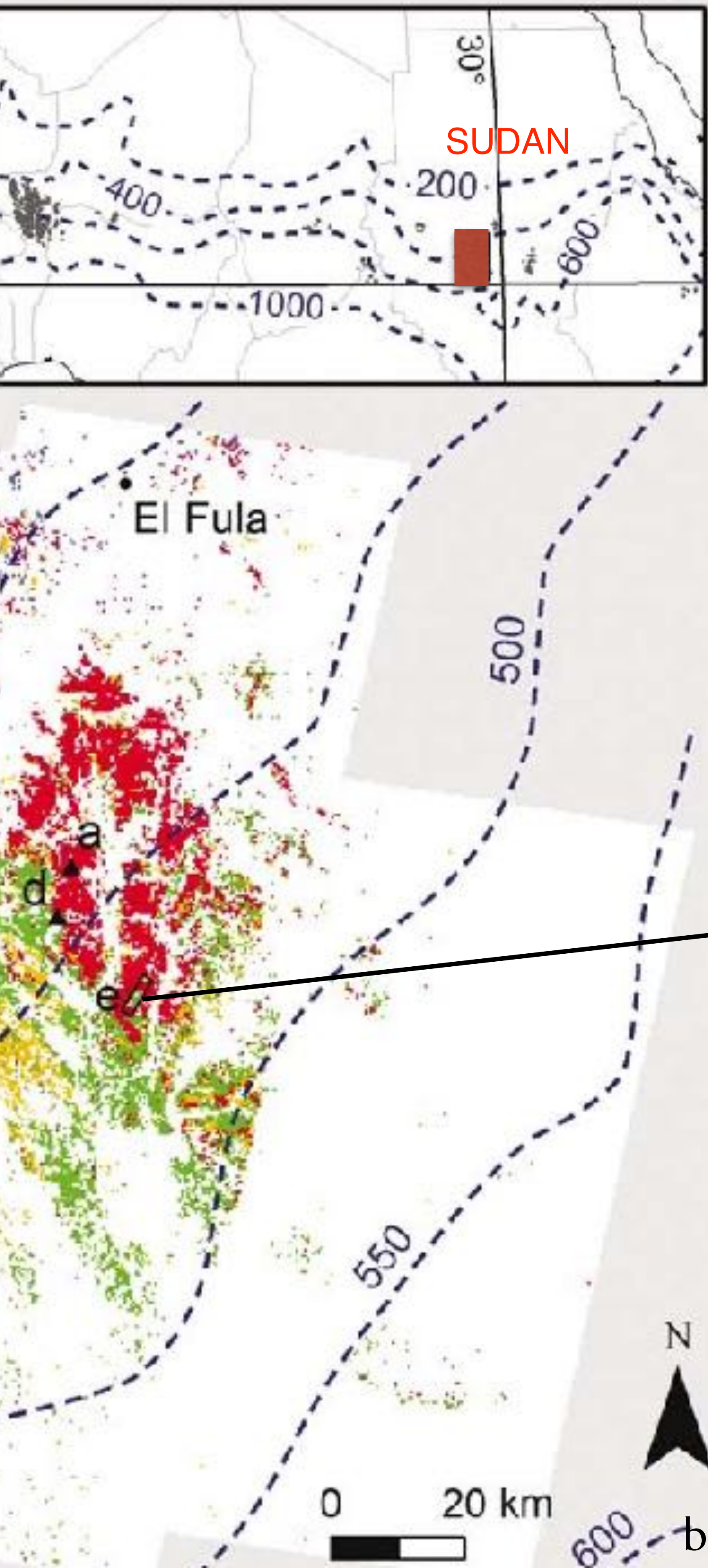
Vincent Deblauwe, Pierre Couteron, Olivier Lejeune, Jan Bogaert and Nicolas Barbier
Ecography 34: 990–1001, 2011

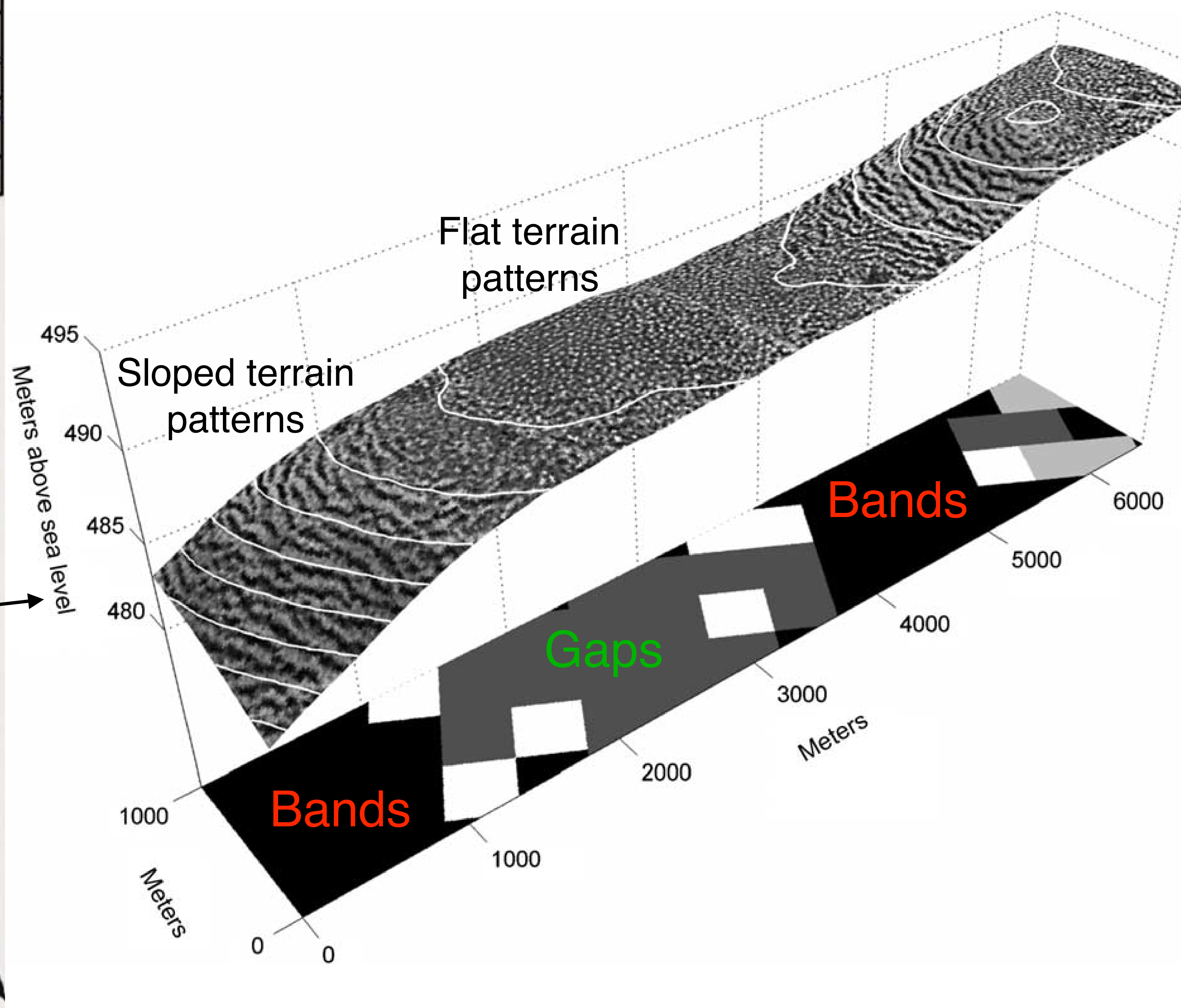


Deblauwe et al. (2011)
Sudan



Deblauwe et al. (2011)
Sudan





bands (black), gaps (dark gray), labyrinths (light gray) and non-periodic (white).

Vegetation Pattern Models: Turing mechanism

Diversity of Vegetation Patterns and Desertification

J. von Hardenberg,[1,4] E. Meron,[1,3] M. Shachak,[2] and Y. Zarmi[1,3]

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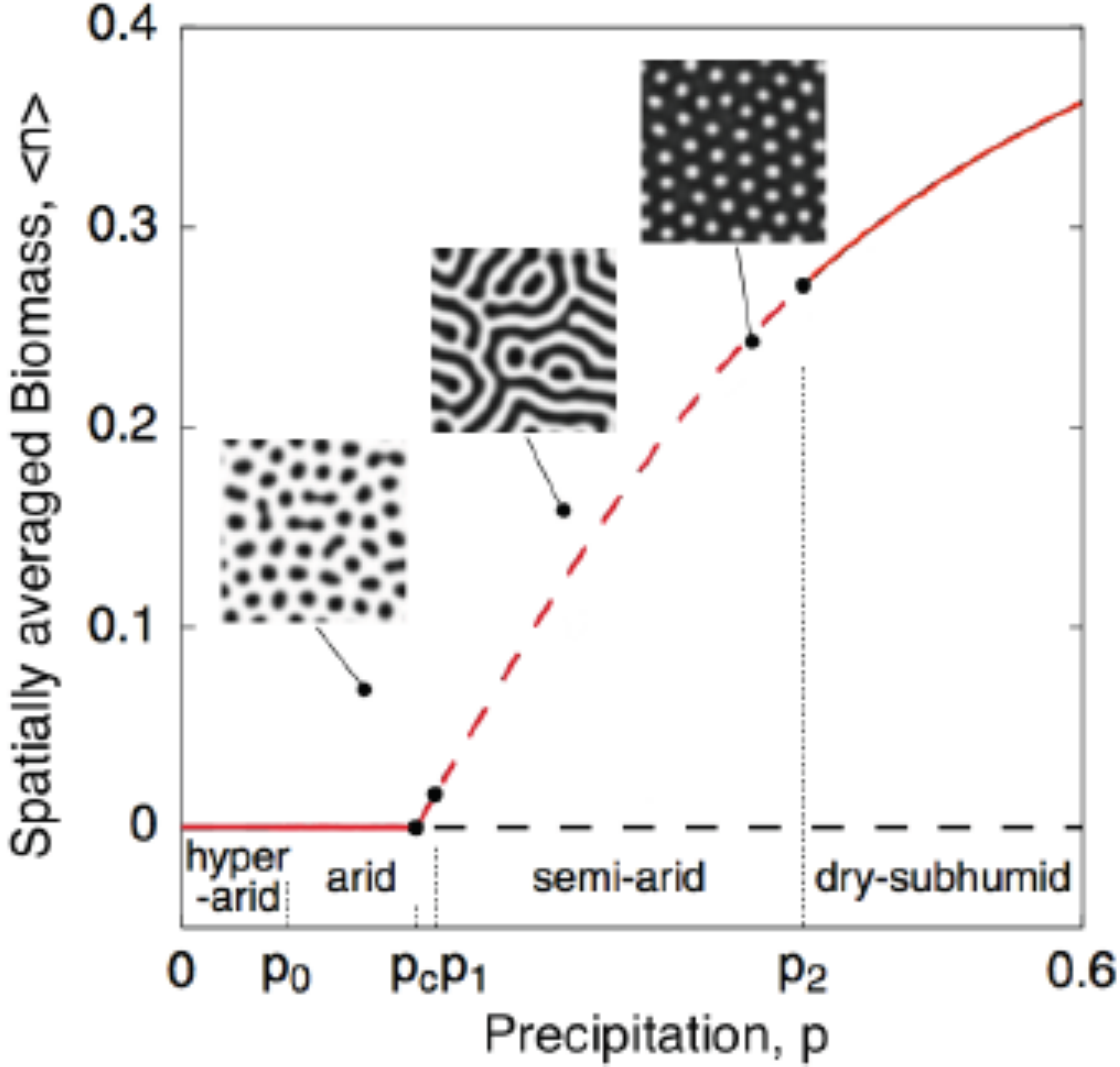
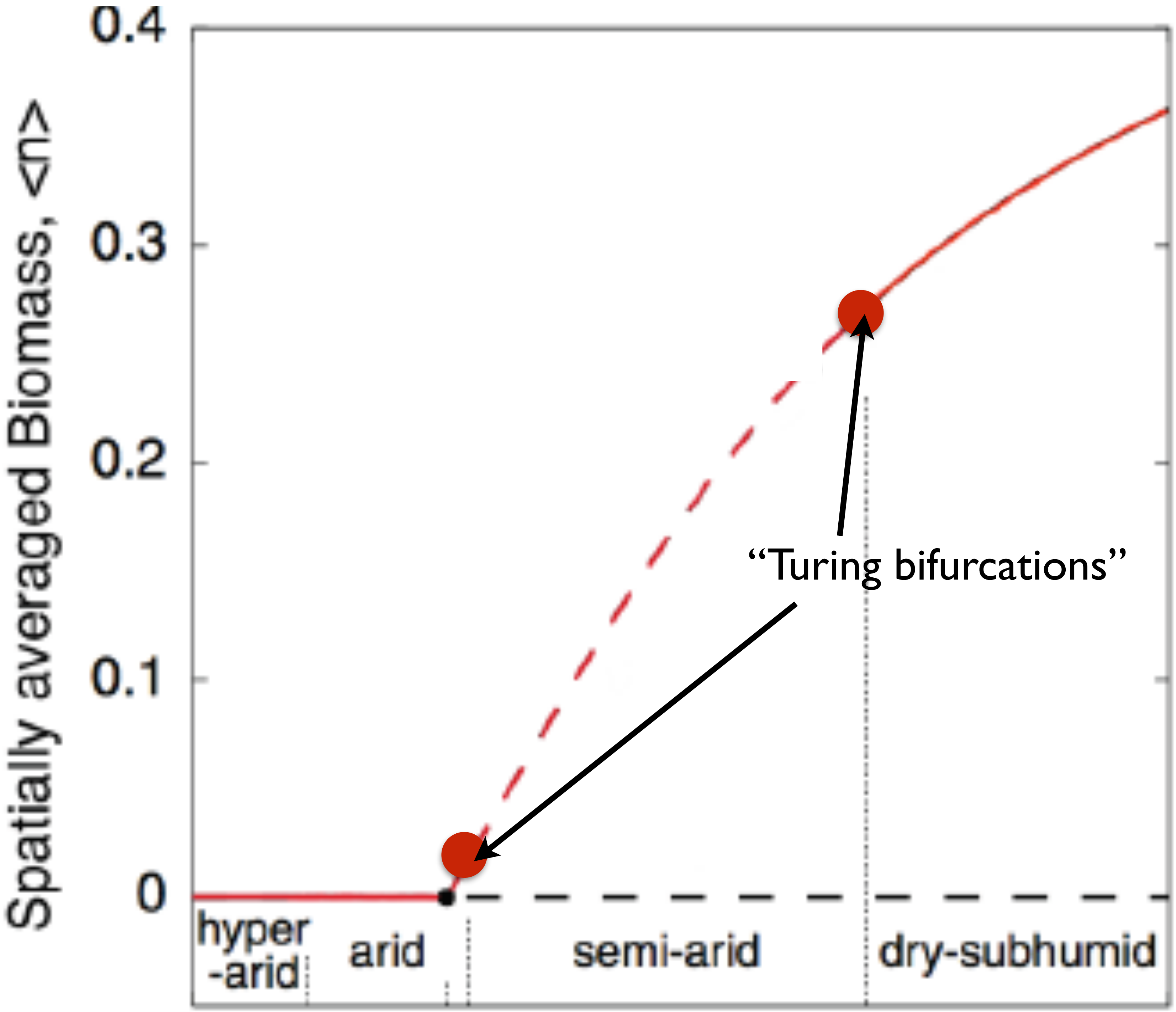
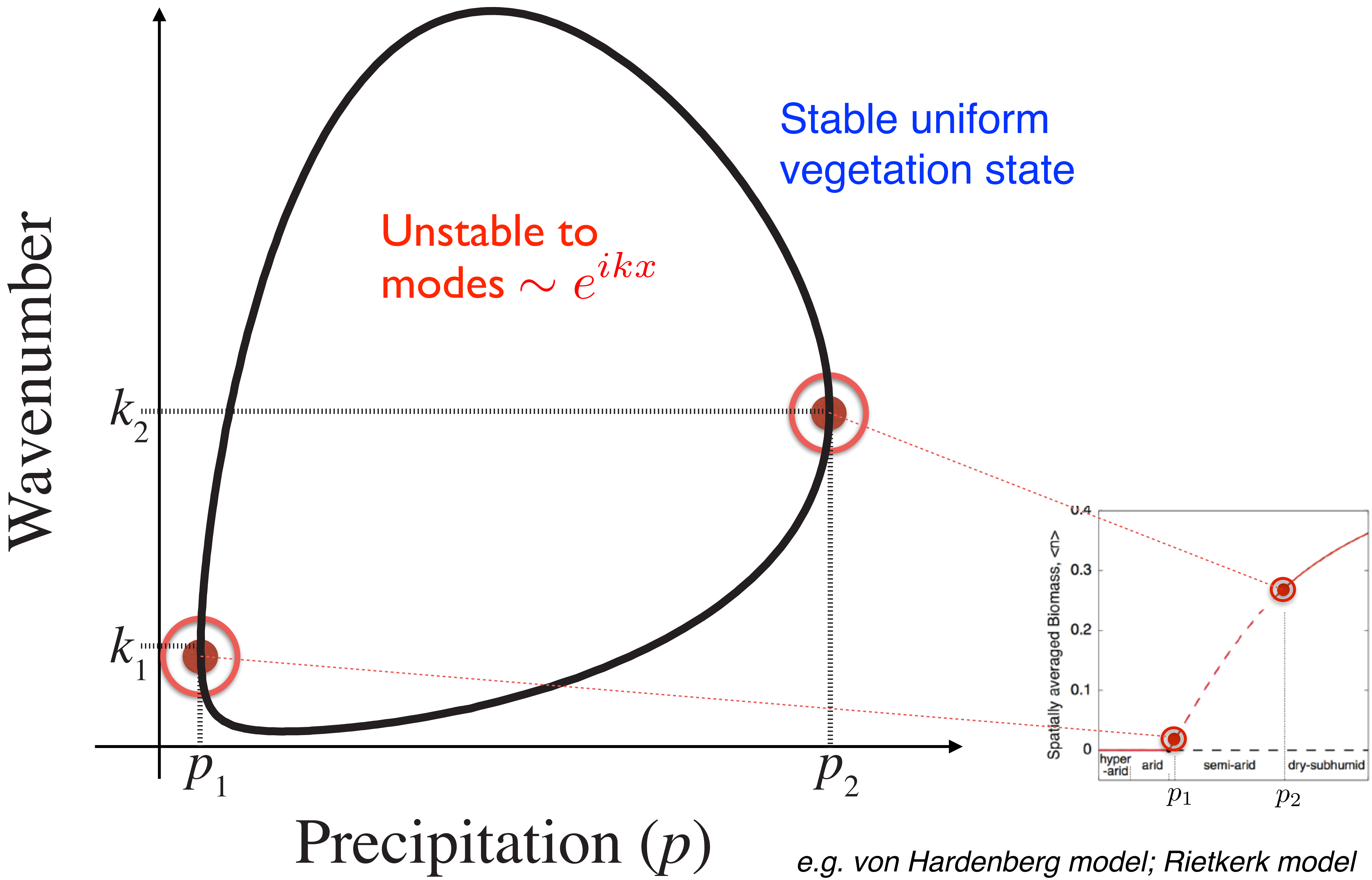




$$\text{uniform} \leftrightarrow \text{patterns} \leftrightarrow \text{uniform}$$

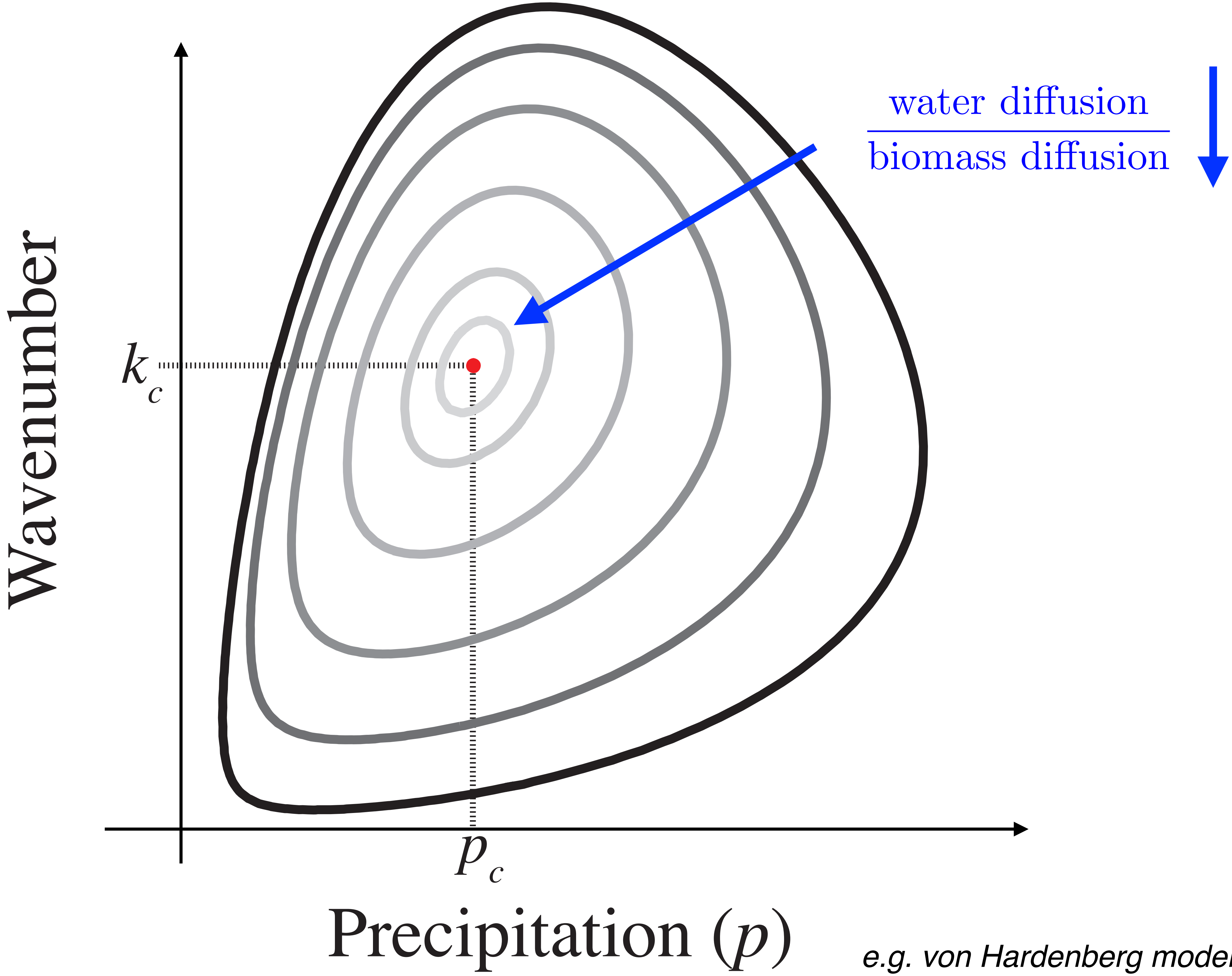
Bubble of linear instability to Fourier mode perturbations: "Turing Bubble"



e.g. von Hardenberg model; Rietkerk model

“Degenerate Turing Bubble”

(work with K. Gowda and H. Riecke, PRE 2014)



e.g. von Hardenberg model; Rietkerk model

"Degenerate Turing Bubble"

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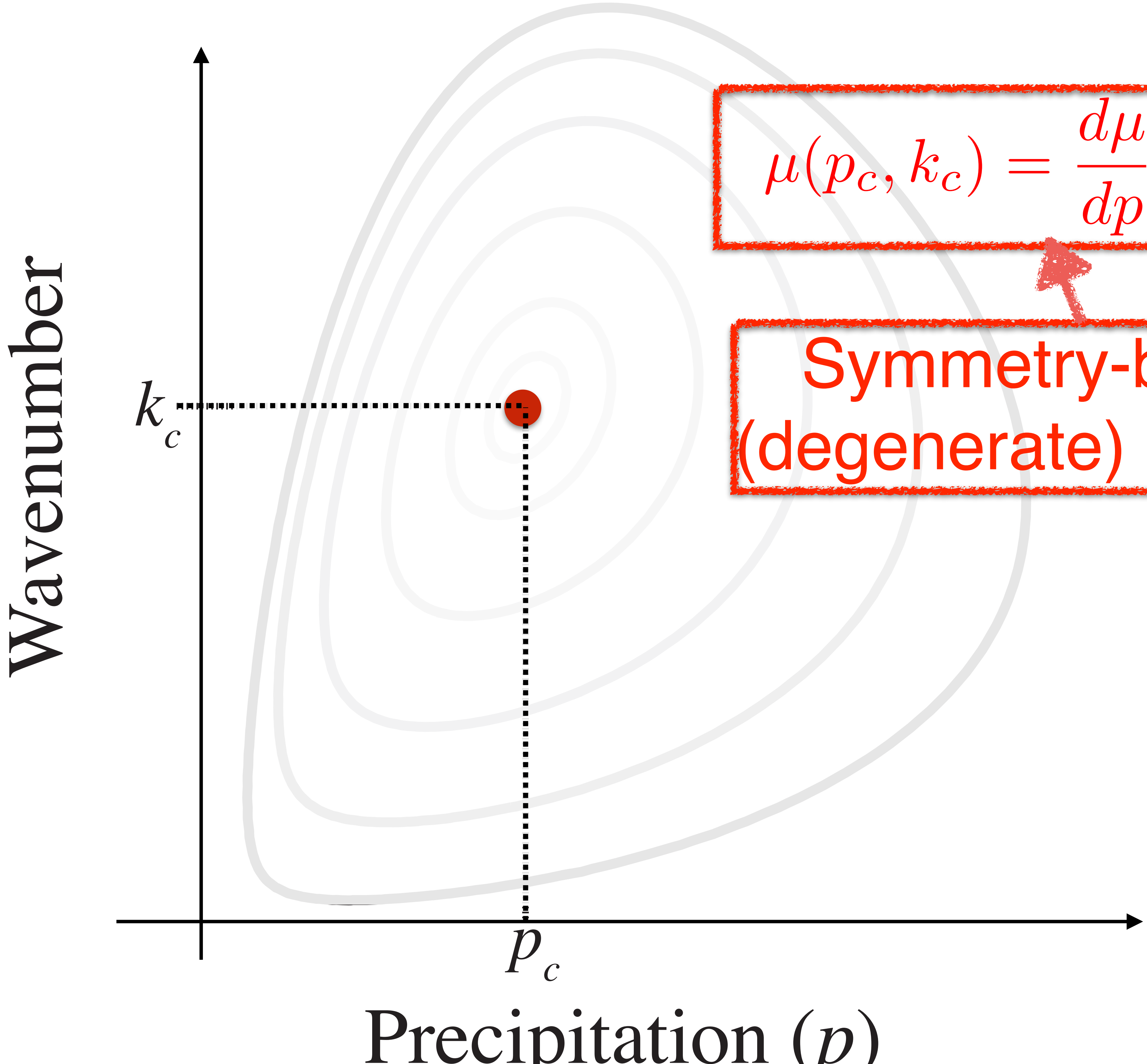
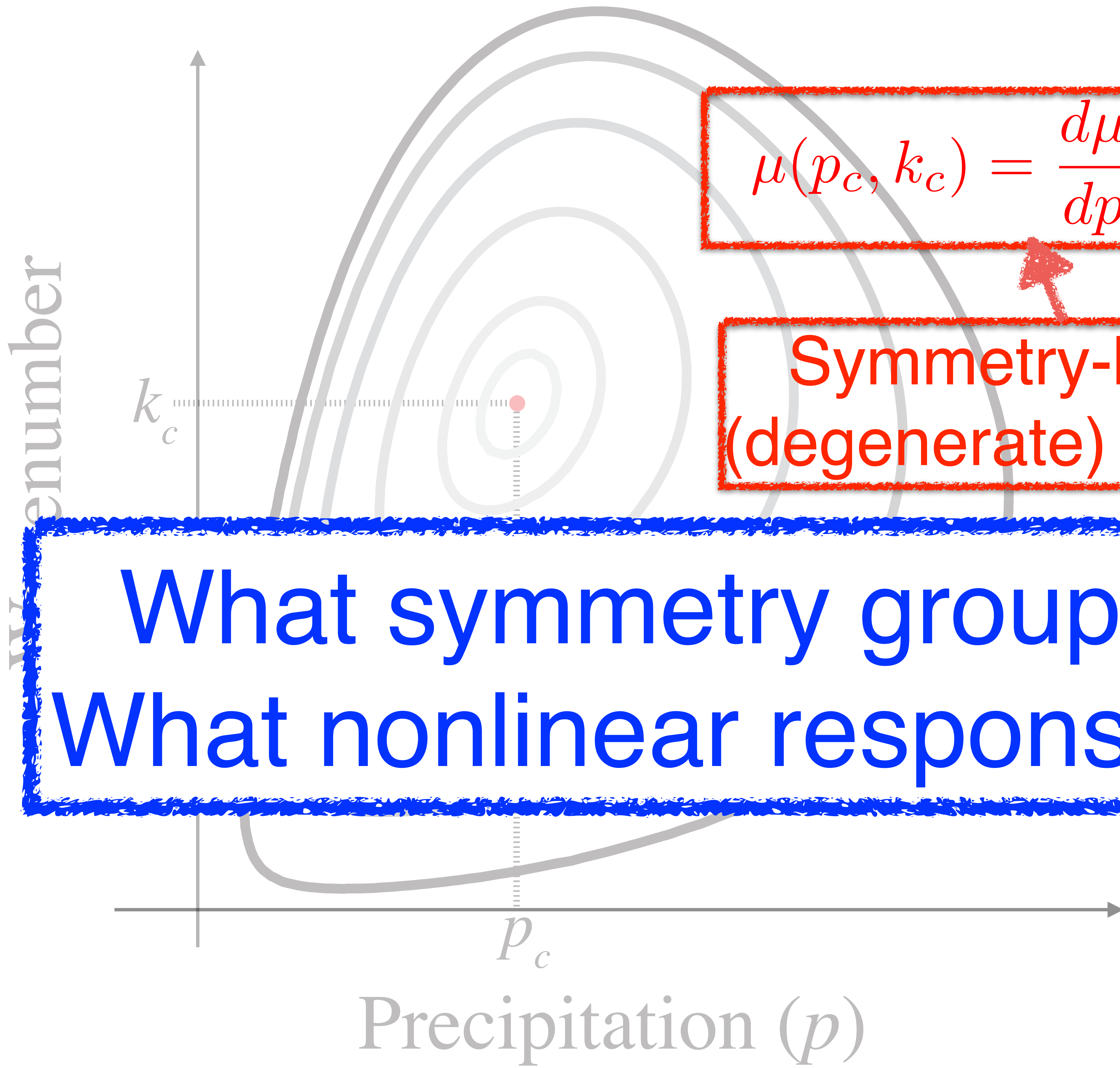


$$\mu(p_c, k_c) = \frac{d\mu}{dp}(p_c, k_c) = 0$$

Symmetry-breaking (degenerate) bifurcation

“Degenerate Turing Bubble”

(work with K. Gowda and H. Riecke, PRE 2014)

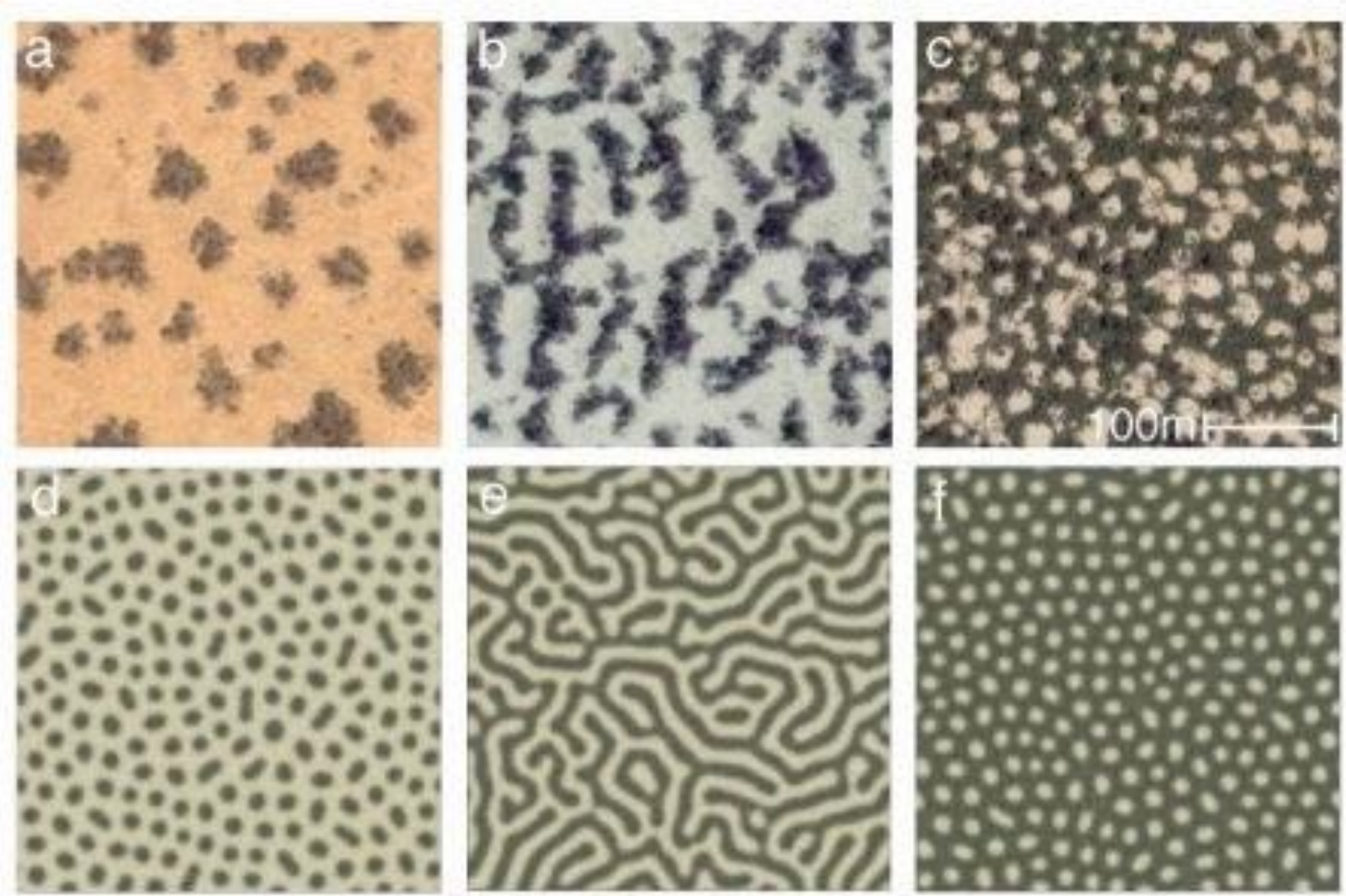


$$\mu(p_c, k_c) = \frac{d\mu}{dp}(p_c, k_c) = 0$$

Symmetry-breaking (degenerate) bifurcation

What symmetry group?
What nonlinear response?

“Spots” “Labyrinths” “Gaps”

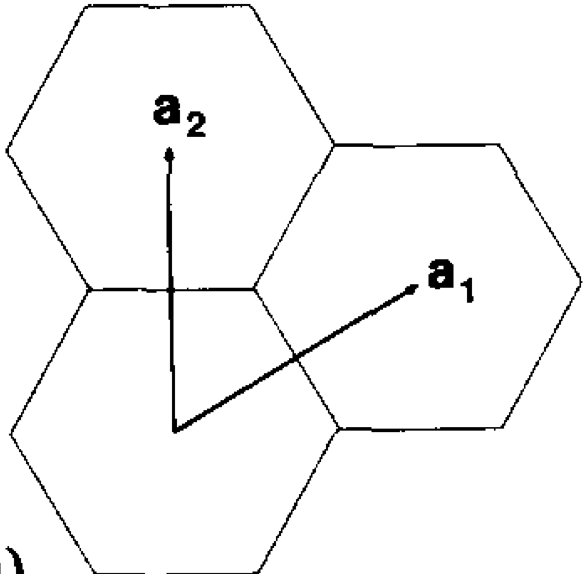


“Up-hexagons” Distorted “Stripes” “Down-hexagons”

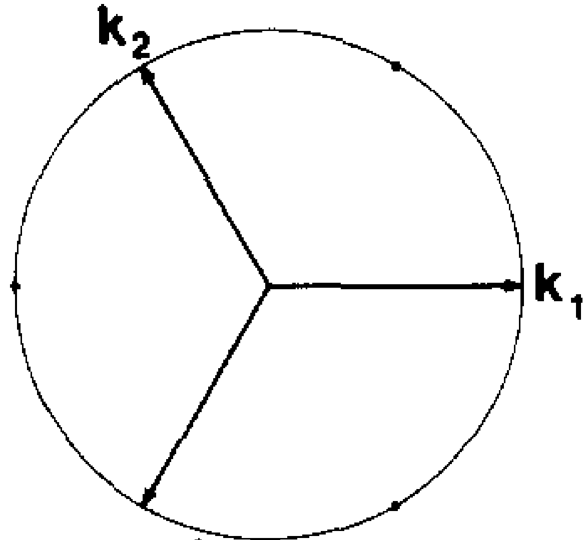
SYMMETRIES AND PATTERN SELECTION IN RAYLEIGH–BÉNARD CONVECTION

M. GOLUBITSKY, J.W. SWIFT and E. KNOBLOCH

Physica 10D (1984)



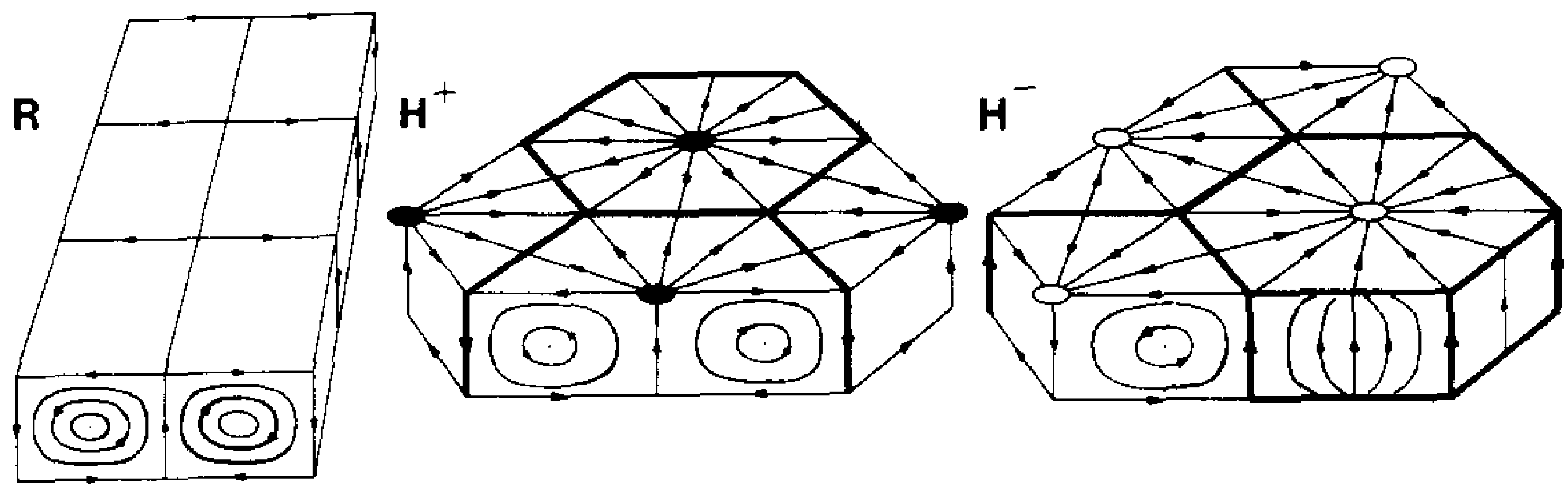
(a)



(c) $k_3 = -k_1 - k_2$

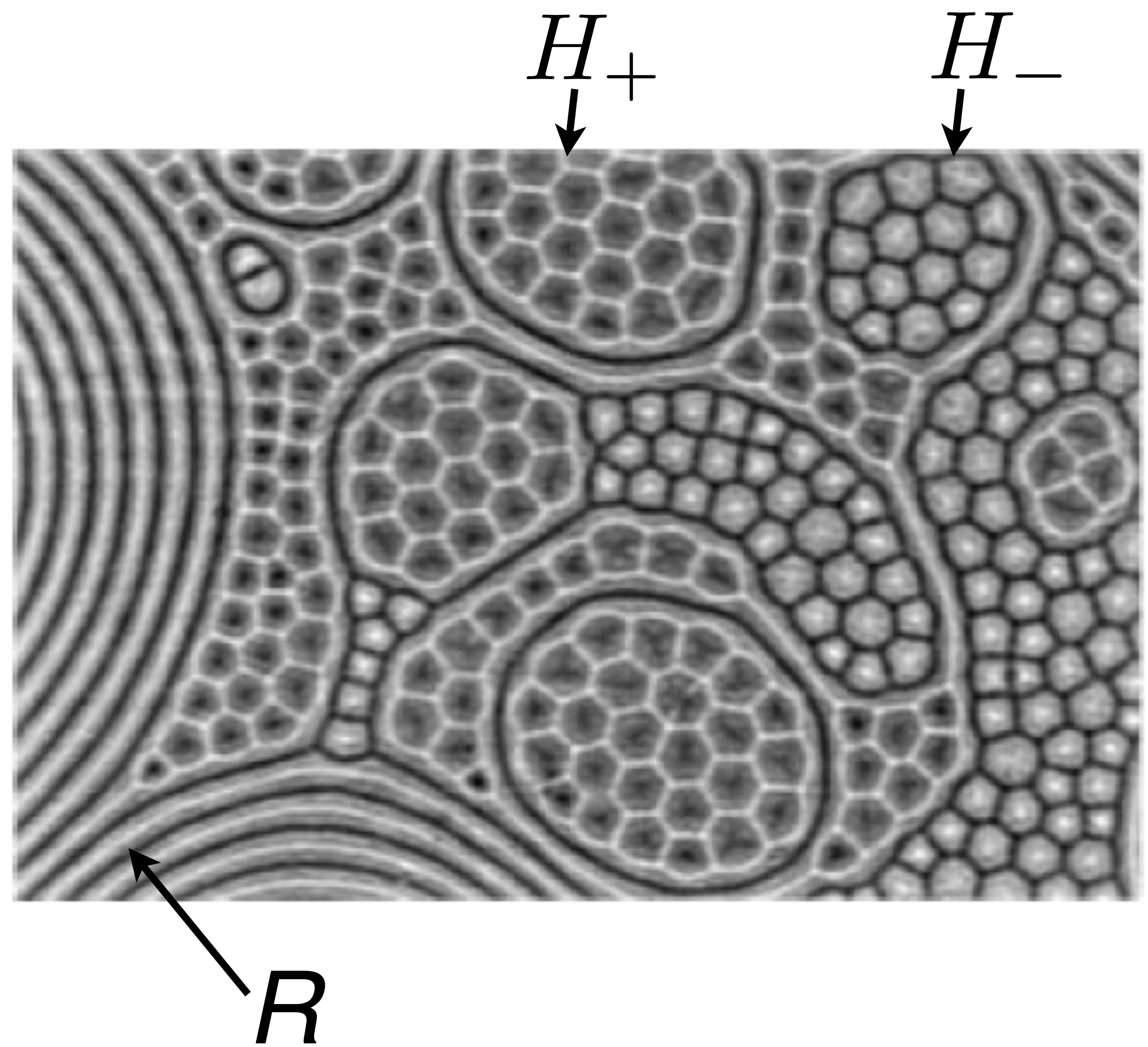
$$\Gamma_n = T^2 + D_6(0)$$

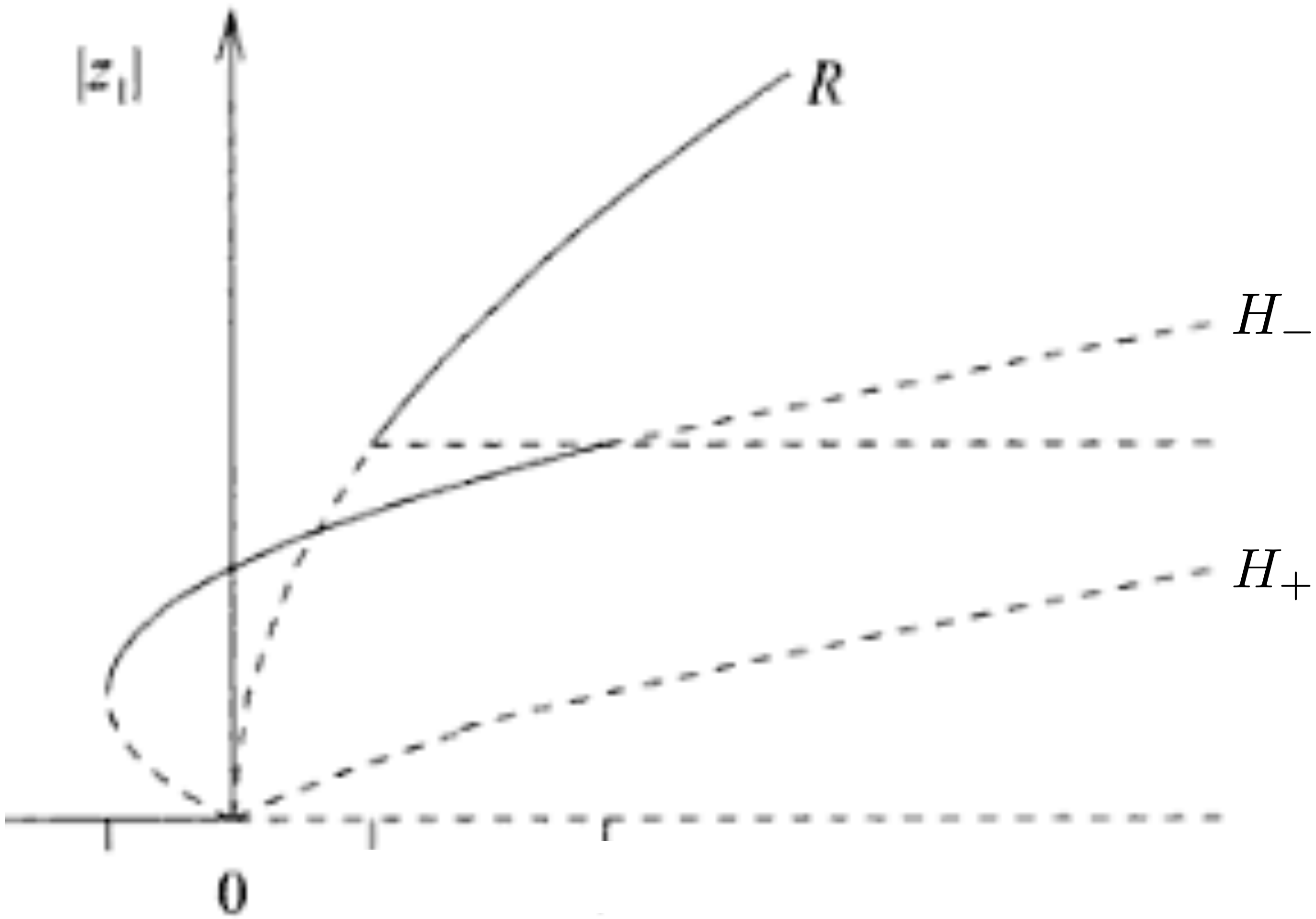
$$S^1 + \mathbb{Z}_2^2(R) \qquad D_6(H^+, H^-)$$



Assenheimer & Steinberg PRL (1996)

Fig. from R.B.Hoyle "Pattern Formation:…"(2006)



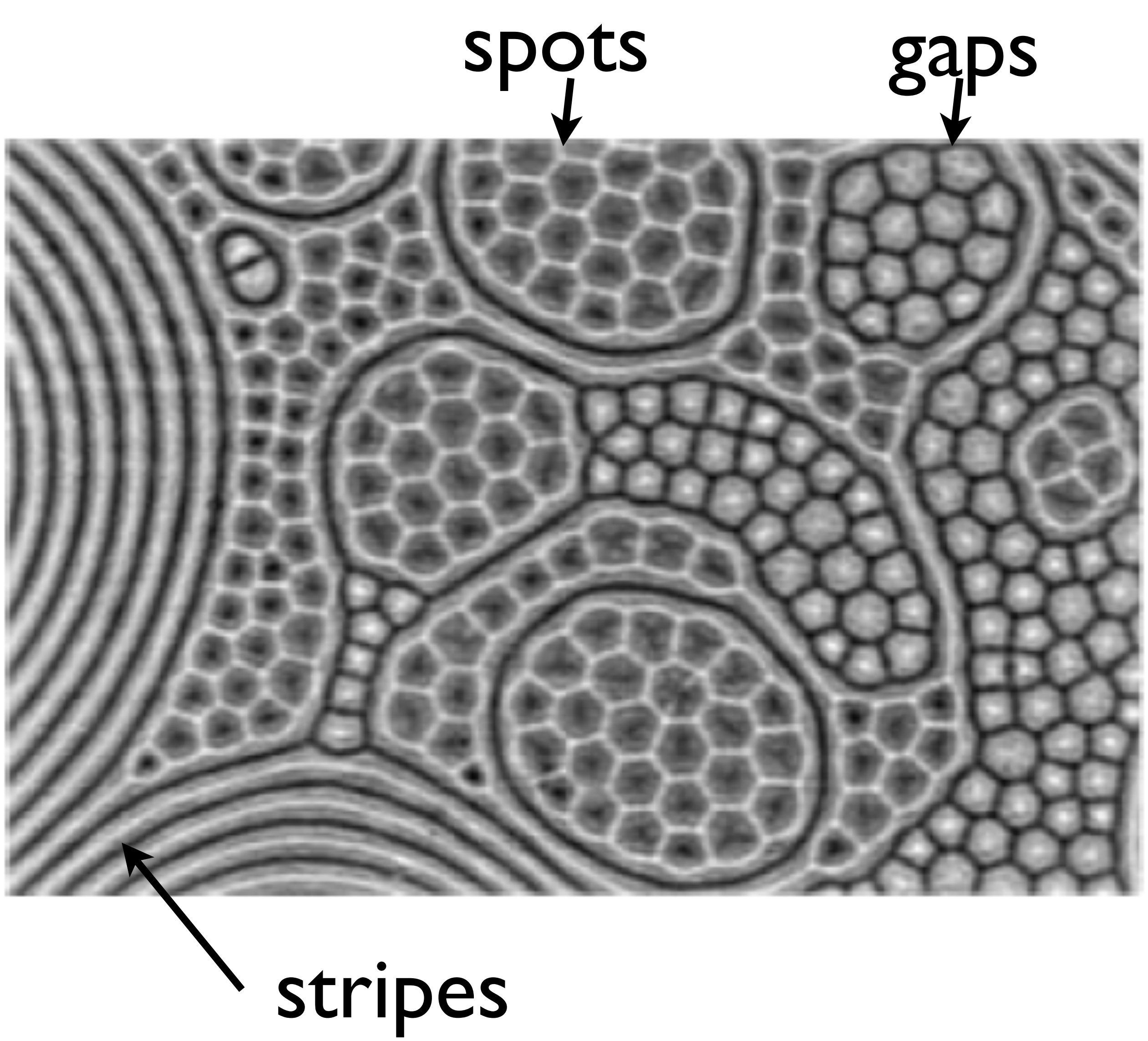


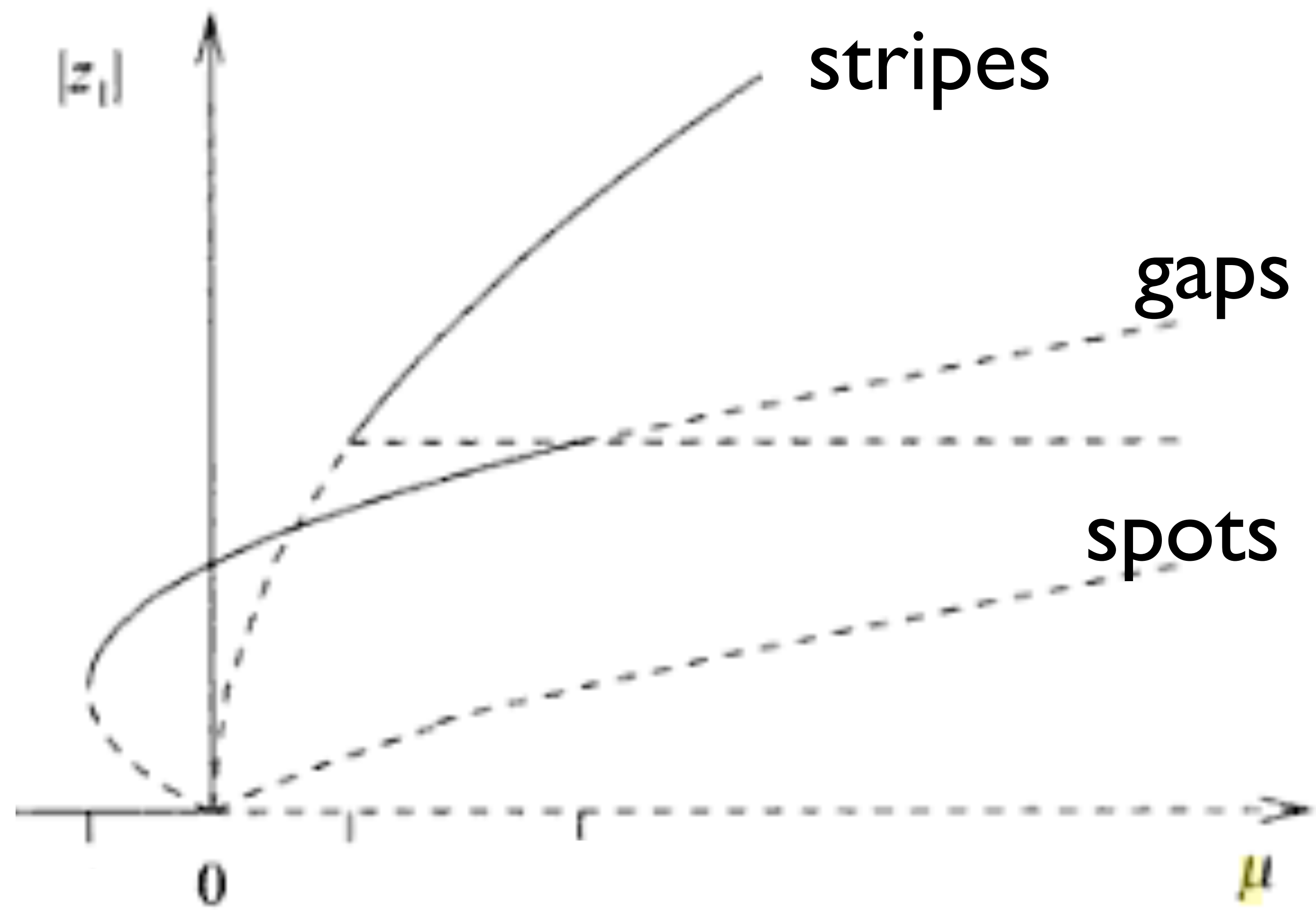
$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

$$\dot{z}_2 = \mu z_2 + a\bar{z}_1\bar{z}_3 - b|z_2|^2 z_2 - c\left(|z_1|^2 + |z_3|^2\right) z_2 + \ldots$$

$$\dot{z}_3 = \mu z_3 + a\bar{z}_1\bar{z}_2 - b|z_3|^2 z_3 - c\left(|z_1|^2 + |z_2|^2\right) z_3 + \ldots$$

Assenheimer & Steinberg PRL (1996)





$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

$$\dot{z}_2 = \mu z_2 + a\bar{z}_1\bar{z}_3 - b|z_2|^2 z_2 - c\left(|z_1|^2 + |z_3|^2\right) z_2 + \ldots$$

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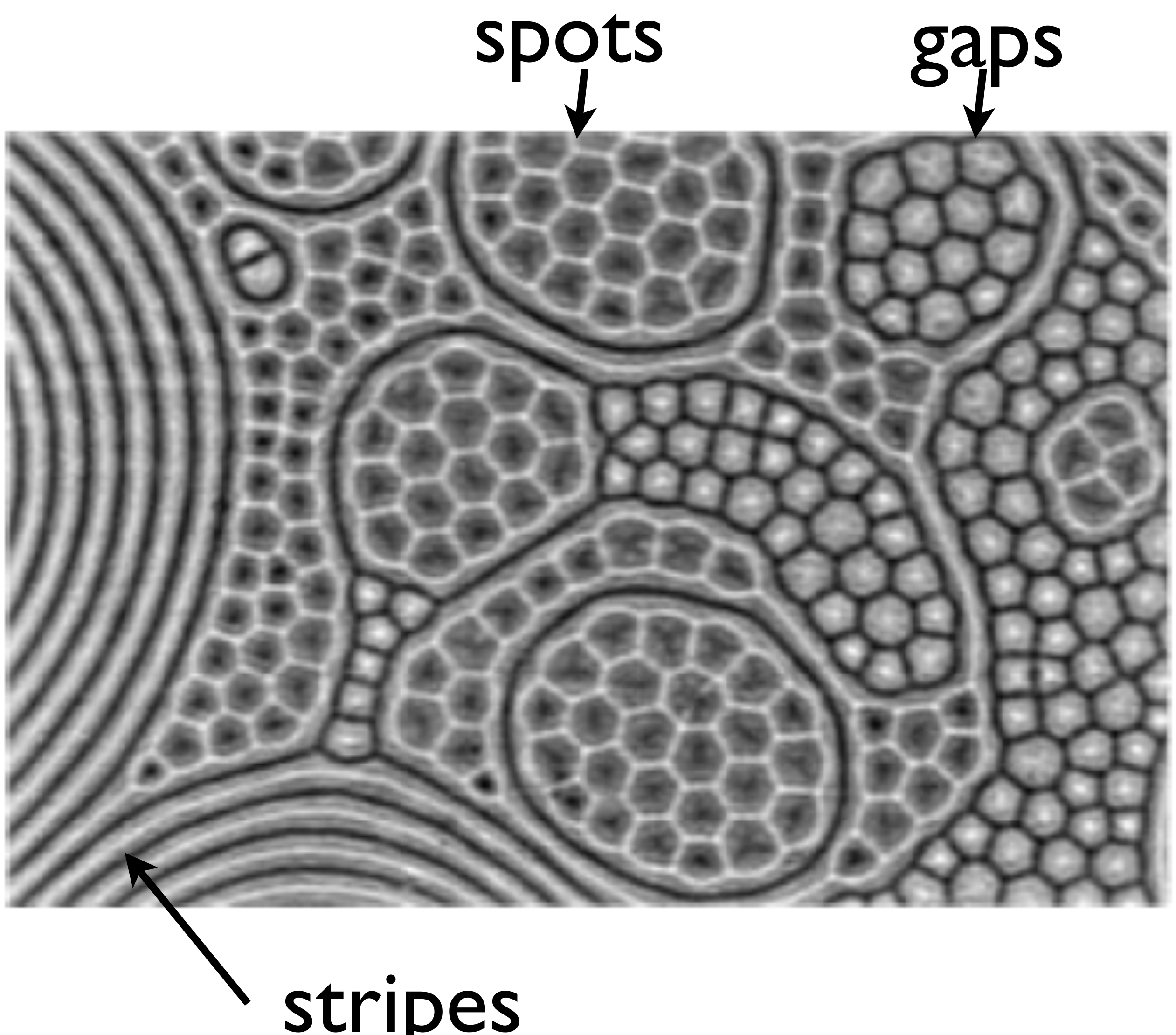
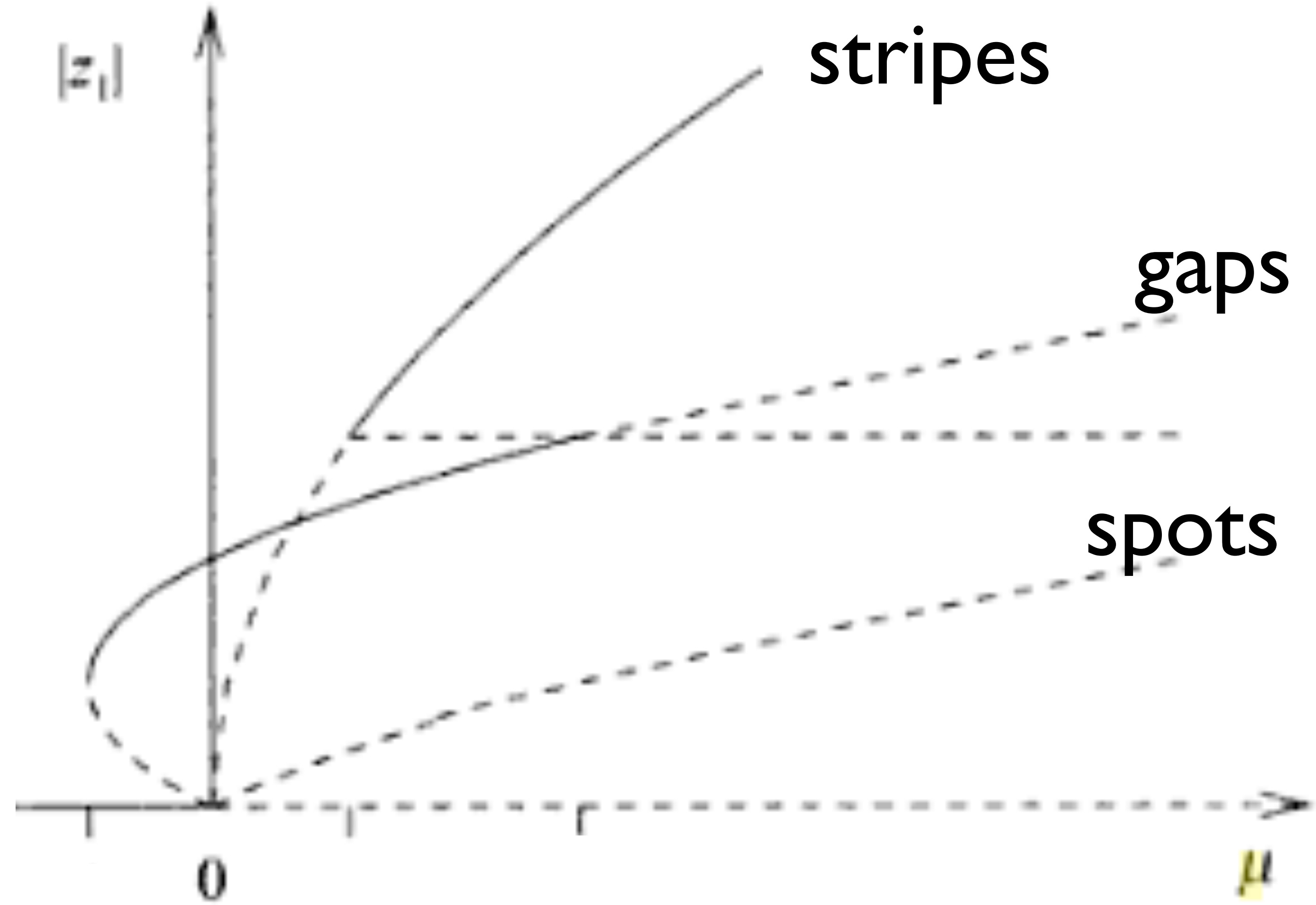
Assenheimer & Steinberg PRL (1996)





$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

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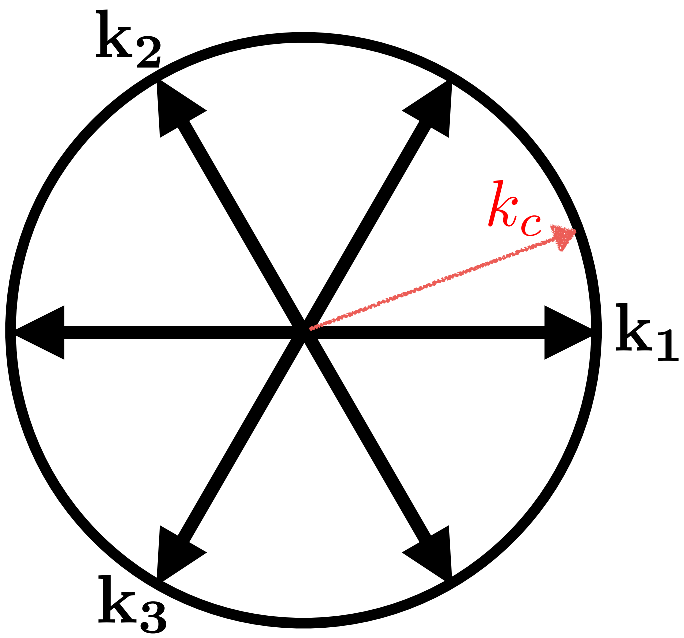
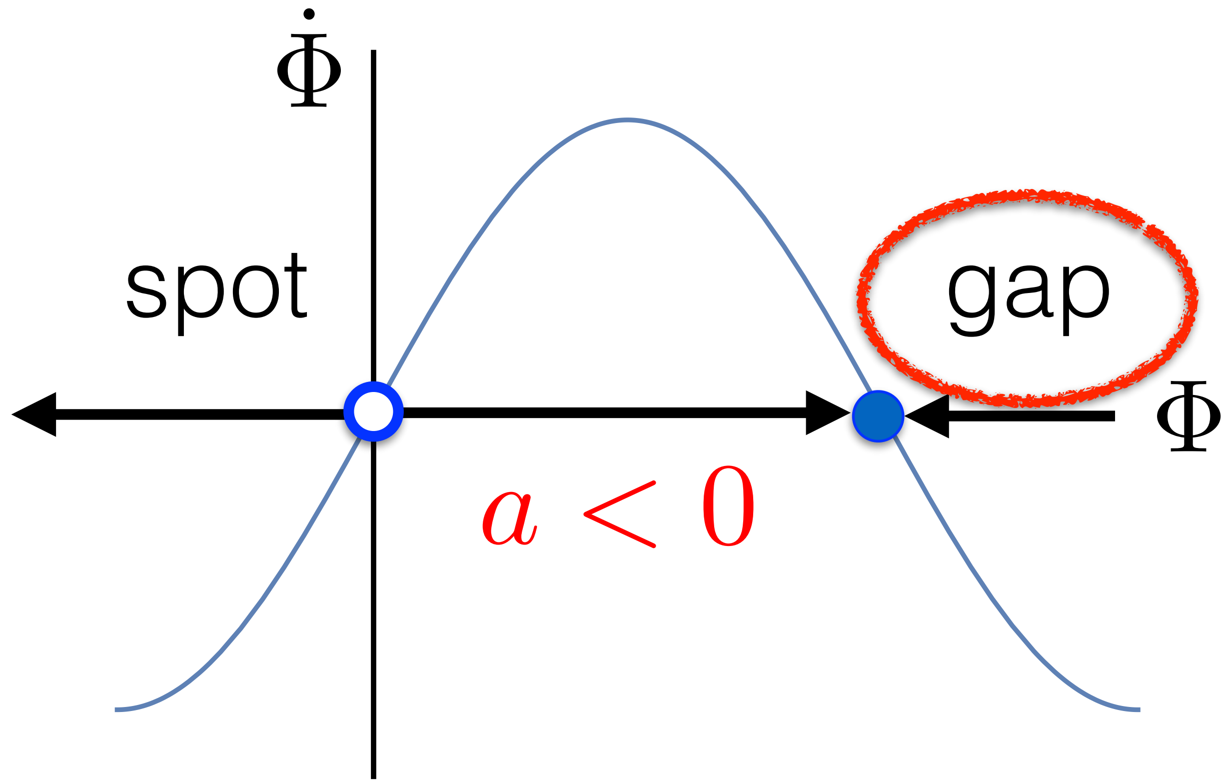
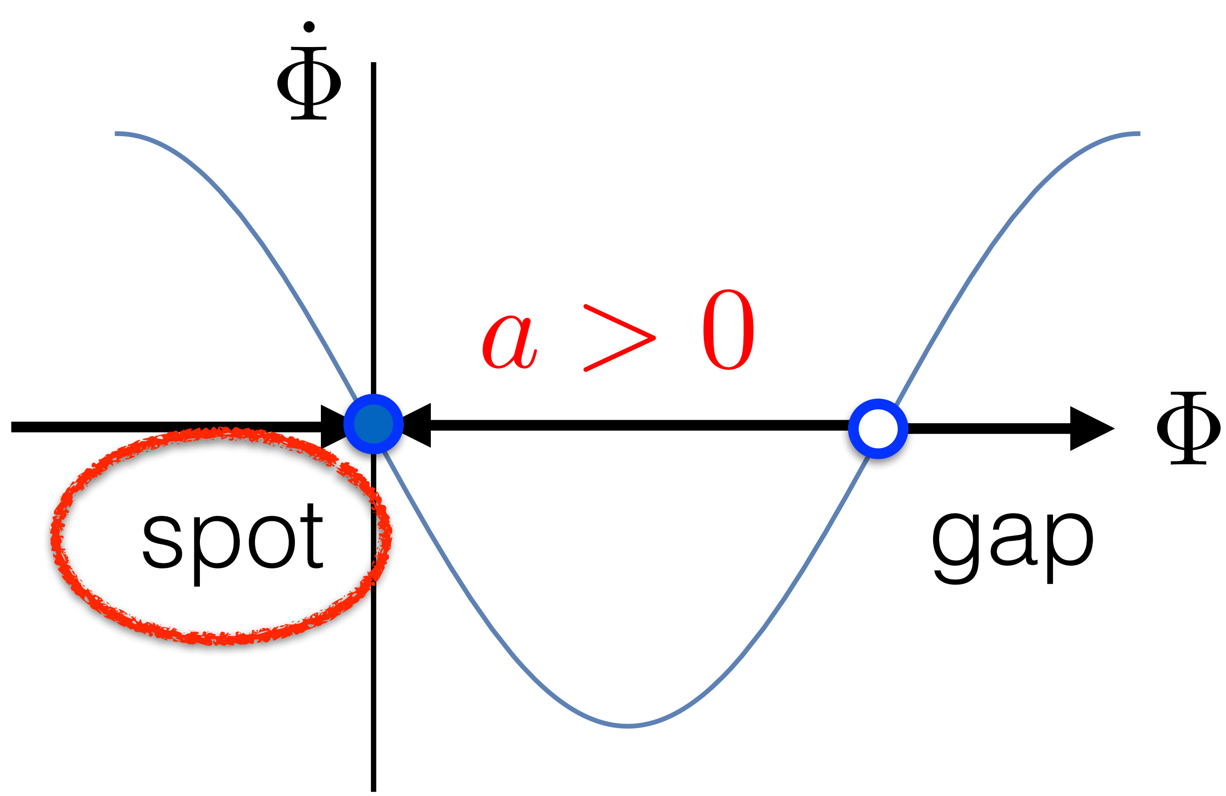
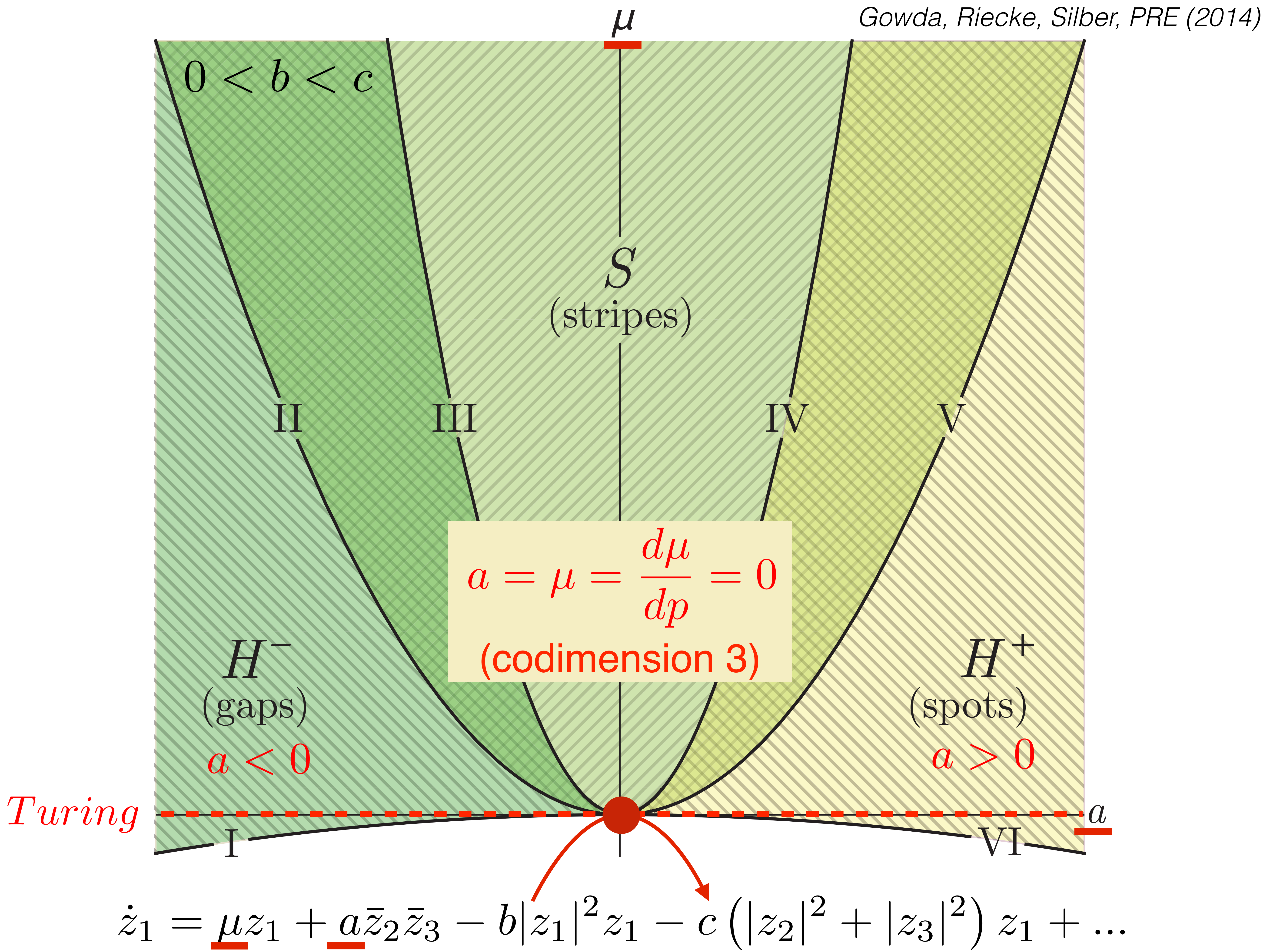
$$\dot{z}_j = \lambda z_j + a\bar{z}_{j+1}\bar{z}_{j-1} + \cdots$$ } near Turing bifurcation pt. $(\lambda = 0)$



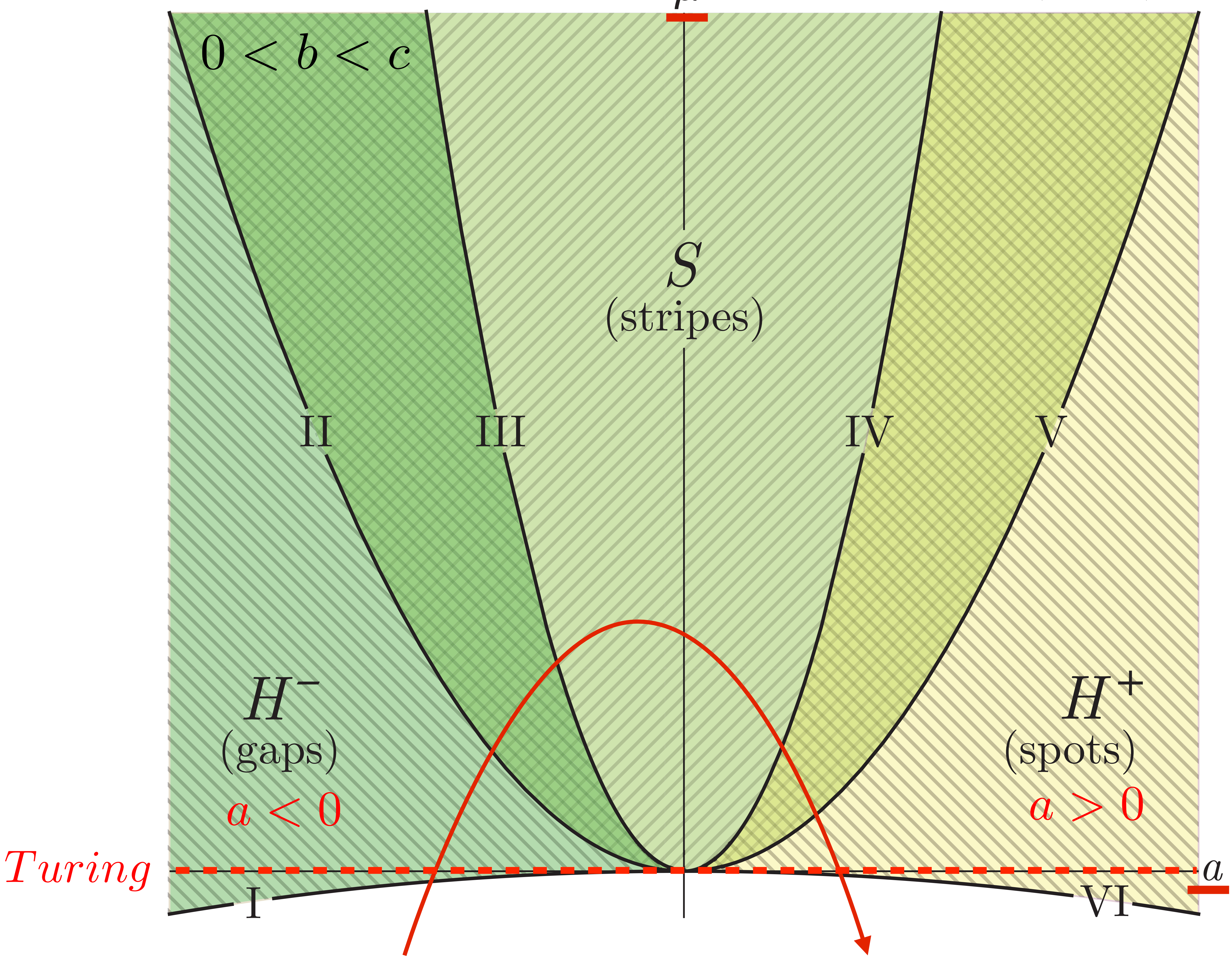






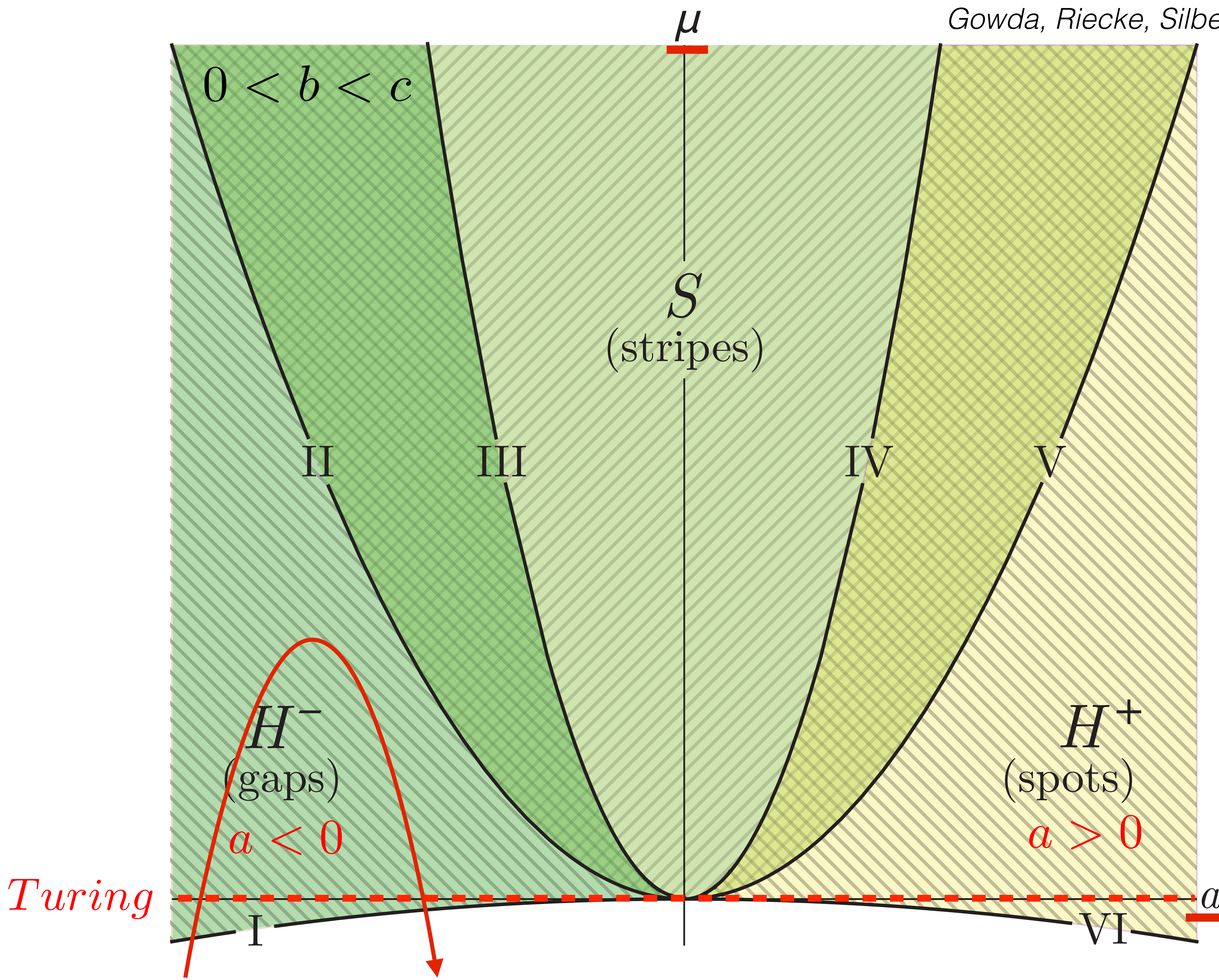
Gowda, Riecke, Silber, PRE (2014)
μ
$0 < b < c$
S
(stripes)
II
III
IV
V
$a = \mu = \frac{d\mu}{dp} = 0$
(codimension 3)
H^-
(gaps)
$a < 0$
H^+
(spots)
$a > 0$
Turing
a
I
VI
$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$

Gowda, Riecke, Silber, PRE (2014)



$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

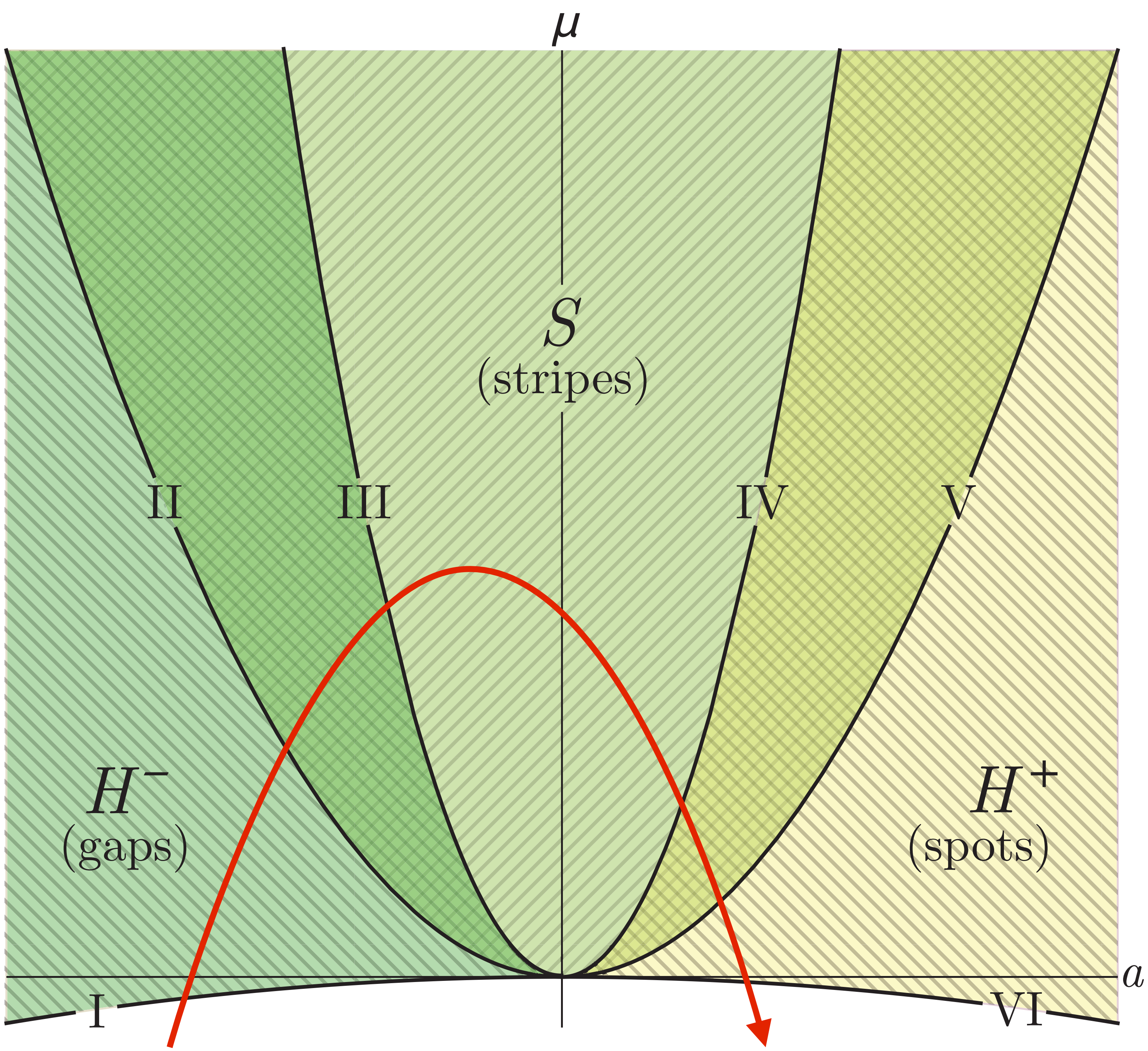
Gowda, Riecke, Silber, PRE (2014)

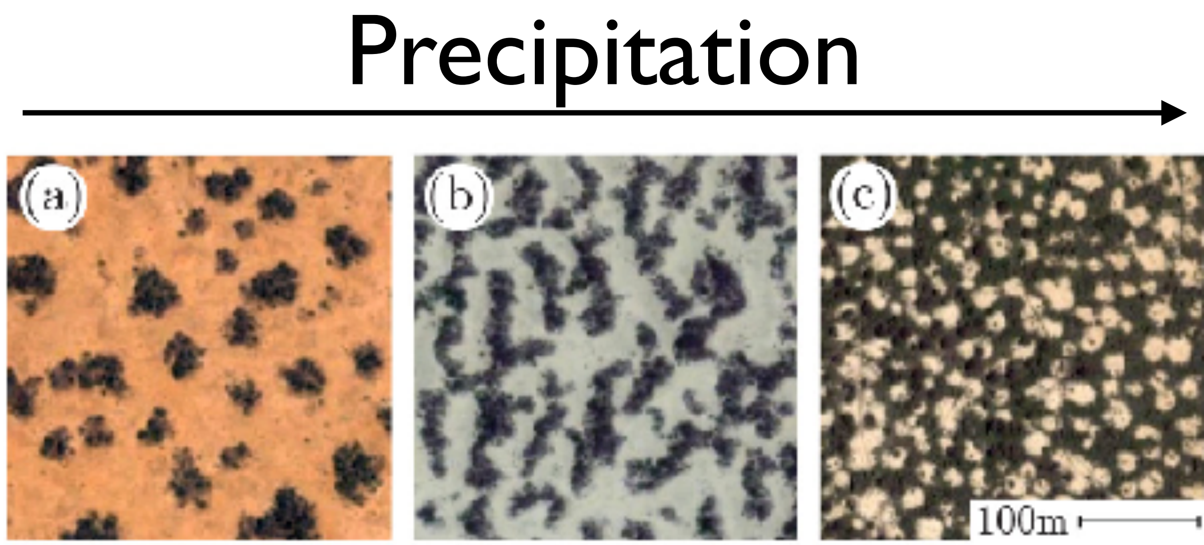


$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

Proposal:

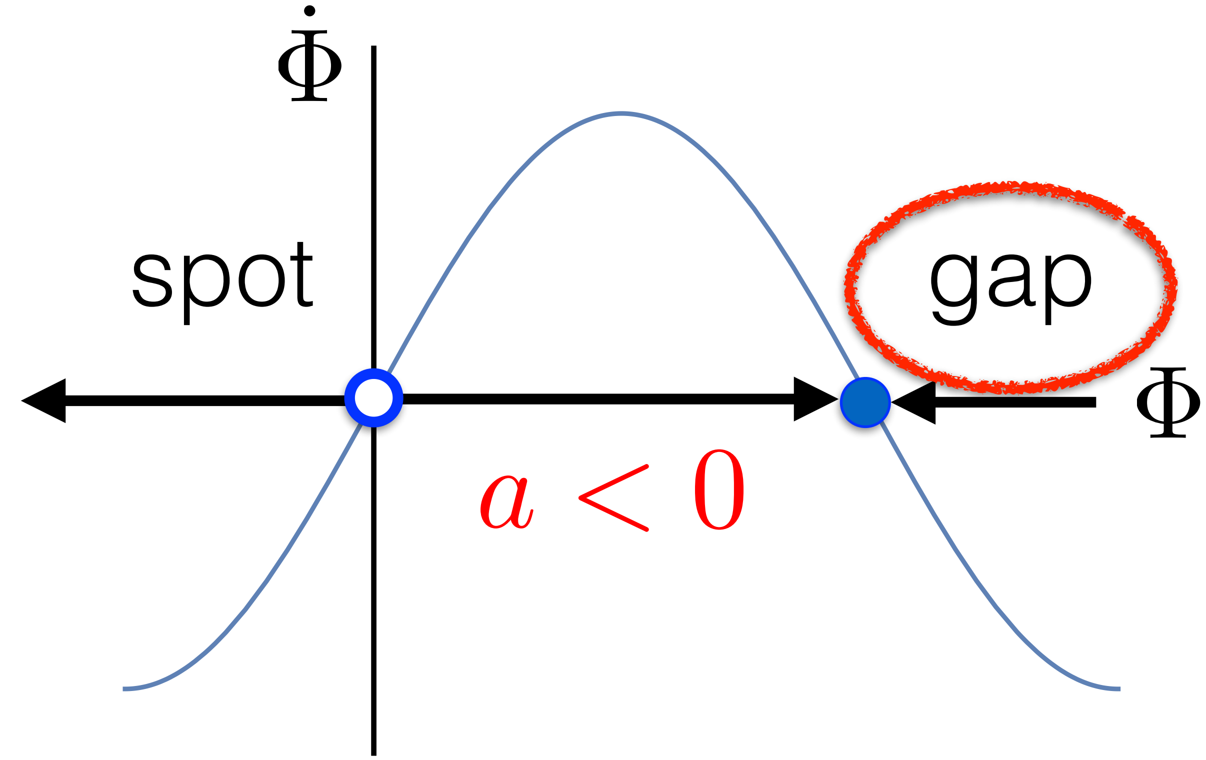
Quadratic coefficient changing sign from negative to positive, with decreasing precipitation serves as a proxy for the "standard sequence":

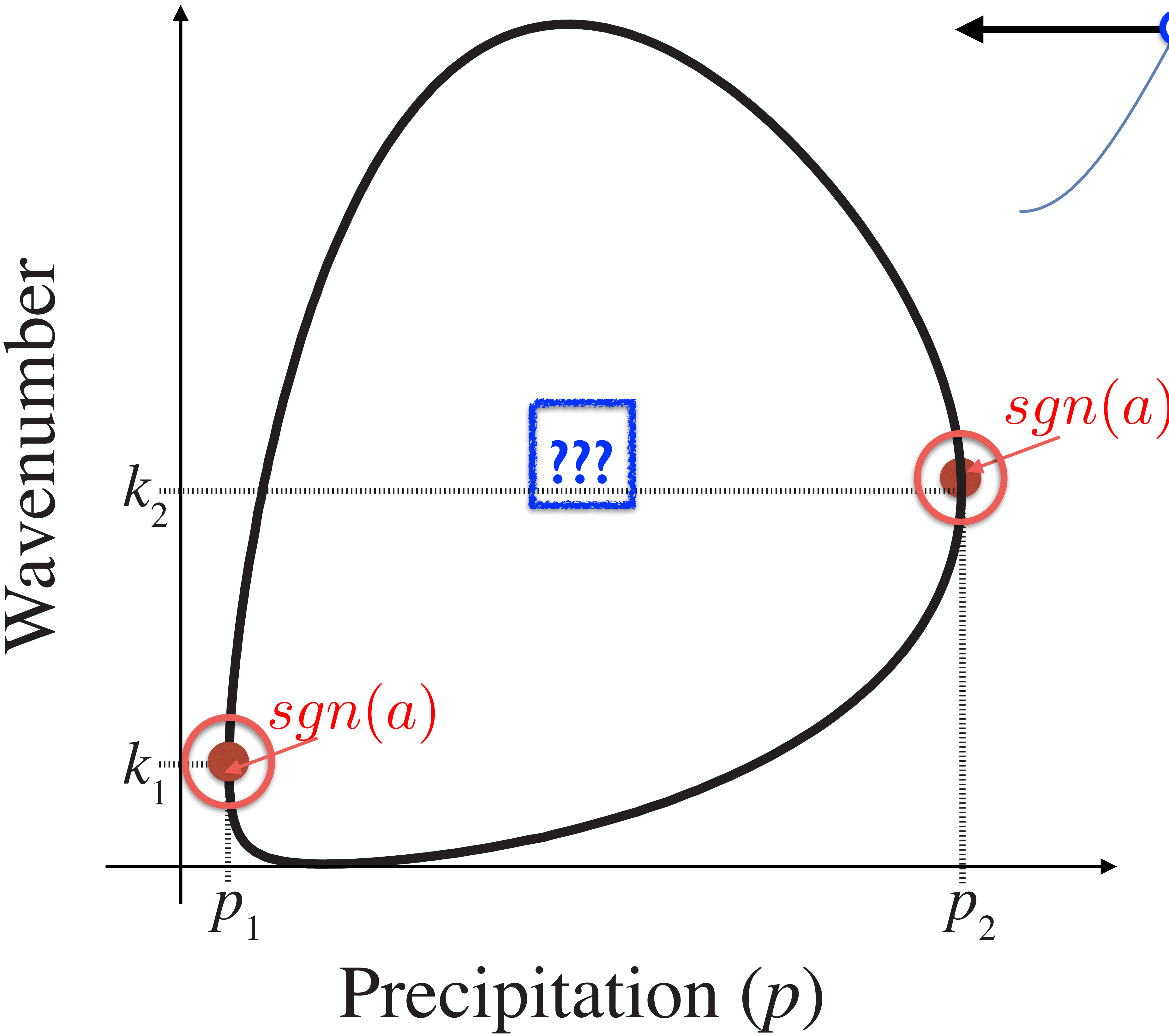




Therein lies the ecology?

$$\dot{z}_j = \lambda z_j + a\bar{z}_{j+1}\bar{z}_{j-1} + \cdots$$





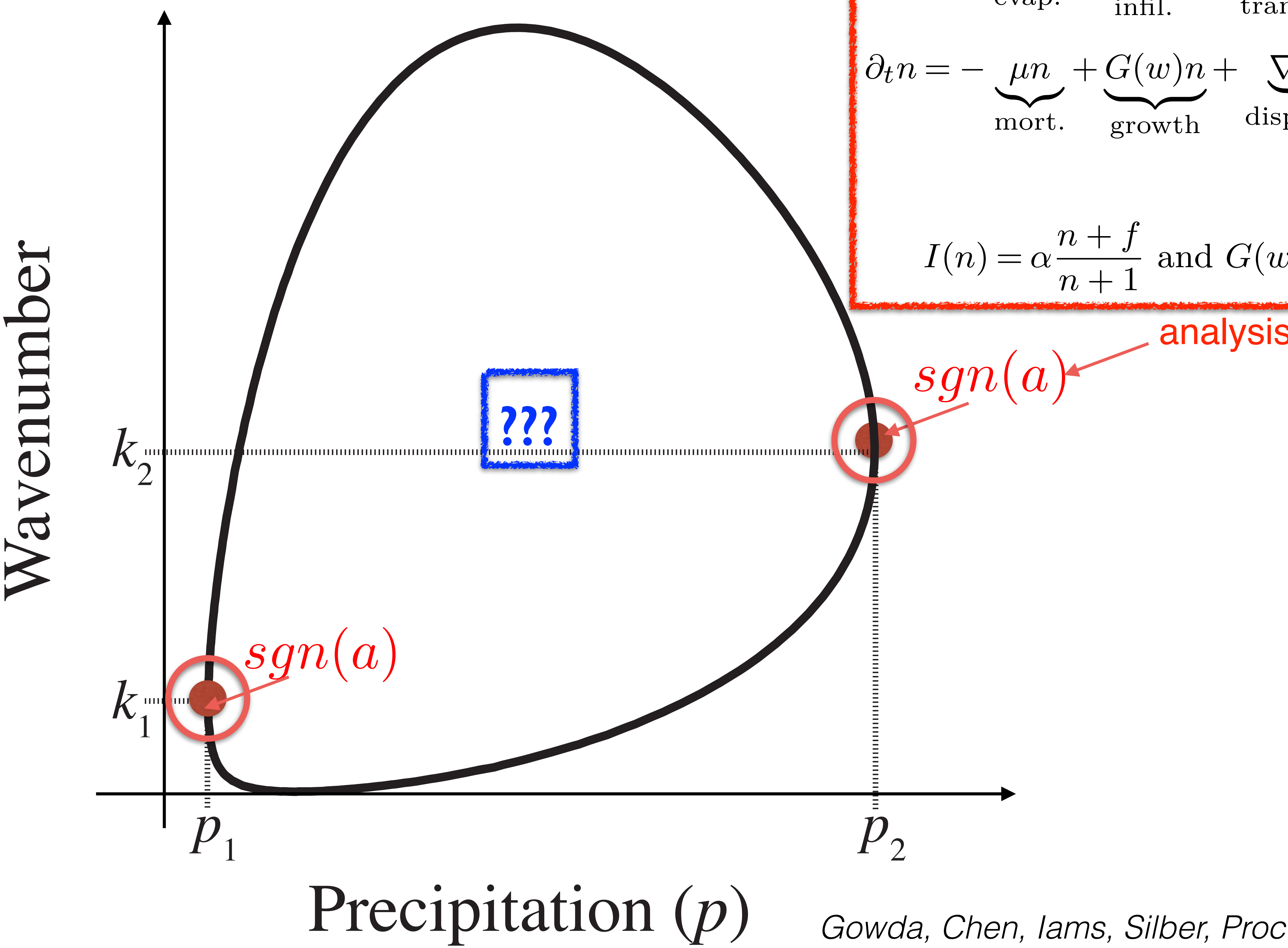
$$\dot{z}_j = \lambda z_j + a\bar{z}_{j+1}\bar{z}_{j-1} + \cdots$$

$$\partial_t h = \underbrace{p}_{\text{precip.}} - \underbrace{I(n)h}_{\text{infil.}} + \underbrace{D_h \nabla^2 h}_{\text{diffusion}},$$

$$\partial_t w = -\underbrace{\nu w}_{\text{evap.}} + \underbrace{I(n)h}_{\text{infil.}} - \underbrace{\gamma G(w) n}_{\text{transp.}} + \underbrace{D_w \nabla^2 w}_{\text{diffusion}},$$

$$\partial_t n = -\underbrace{\mu n}_{\text{mort.}} + \underbrace{G(w)n}_{\text{growth}} + \underbrace{\nabla^2 n}_{\text{dispersal}},$$

$$I(n) = \alpha \frac{n+f}{n+1} \text{ and } G(w) = \frac{w}{w+1}.$$



Gowda, Chen, Iams, Silber, Proc. R. Soc. A (2016)

$$\dot{z}_j = \lambda z_j + a\bar{z}_{j+1}\bar{z}_{j-1} + \cdots$$

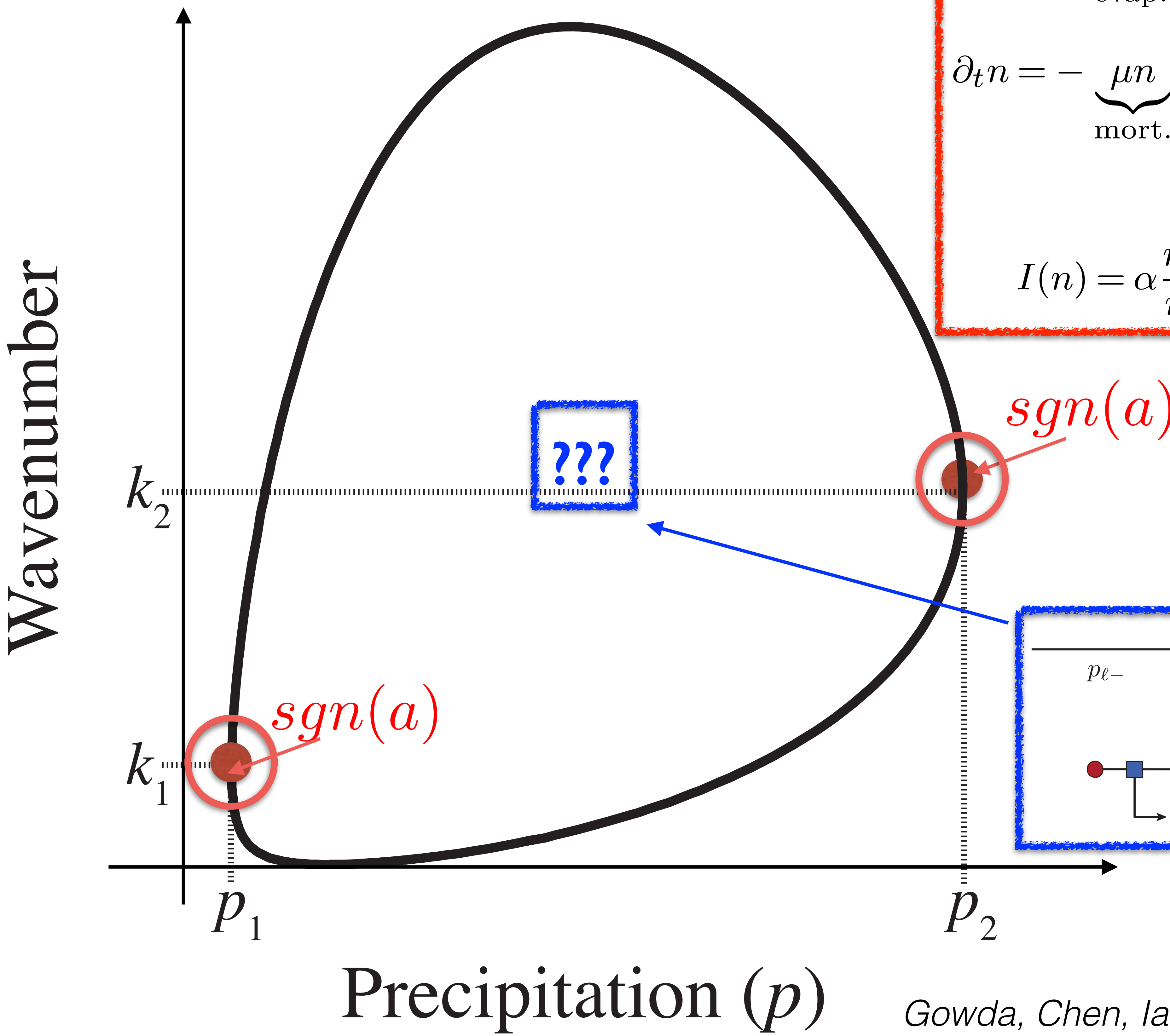
$$\partial_t h = \underbrace{p}_{\text{precip.}} - \underbrace{I(n)h}_{\text{infil.}} + \underbrace{D_h \nabla^2 h}_{\text{diffusion}},$$

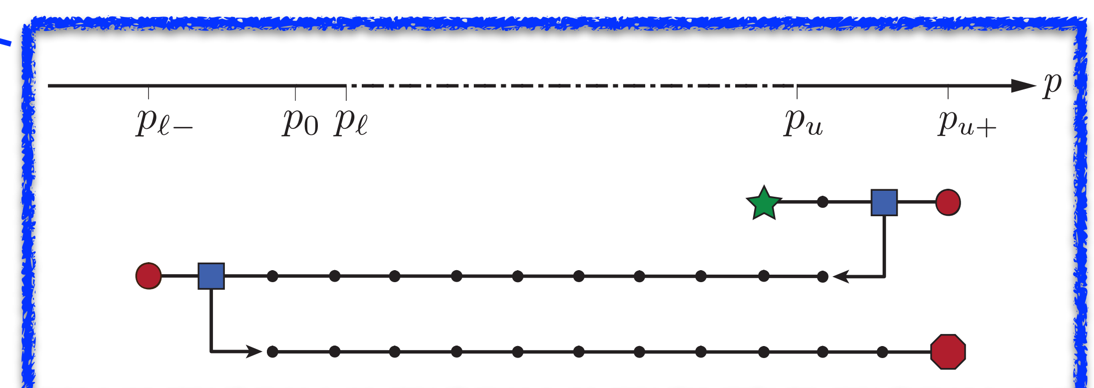
$$\partial_t w = -\underbrace{\nu w}_{\text{evap.}} + \underbrace{I(n)h}_{\text{infil.}} - \underbrace{\gamma G(w) n}_{\text{transp.}} + \underbrace{D_w \nabla^2 w}_{\text{diffusion}},$$

$$\partial_t n = -\underbrace{\mu n}_{\text{mort.}} + \underbrace{G(w) n}_{\text{growth}} + \underbrace{\nabla^2 n}_{\text{dispersal}},$$

$$I(n) = \alpha \frac{n+f}{n+1} \text{ and } G(w) = \frac{w}{w+1}.$$

analysis





simulation

Gowda, Chen, Iams, Silber, Proc. R. Soc. A (2016)

Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat.* **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1}}_{growth} b + \underbrace{\nabla^2 b}_{dispersal} ,$$

$$\partial_t h = \underbrace{p}_{precip.} - \underbrace{\alpha \frac{b+f}{b+1}}_{infil.} h + \underbrace{D_h \nabla^2 h}_{diffusion} .$$

$$D_h \gg 1$$

$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1}}_{infil.} h - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1}}_{transp.} b + \underbrace{D_w \nabla^2 w}_{diffusion} ,$$

biomass density (*b*)

surface water (*h*)

soil water (*w*)

Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat.* **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1} b}_{growth} + \underbrace{\nabla^2 b}_{dispersal},$$

p = bifurcation parameter

$$\partial_t h = \underbrace{p}_{precip.} - \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} + \underbrace{D_h \nabla^2 h}_{diffusion}.$$

$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1} b}_{transp.} + \underbrace{D_w \nabla^2 w}_{diffusion},$$

biomass density (*b*)

surface water (*h*)

soil water (*w*)

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Growth rate

$$G(w) = \frac{w}{w+1}$$

biomass density (b)

surface water (h)

soil water (w)

Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat.* **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1} b}_{growth} + \underbrace{\nabla^2 b}_{dispersal} ,$$

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$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1} b}_{transp.} + \underbrace{D_w \nabla^2 w}_{diffusion} ,$$

Infiltration rate

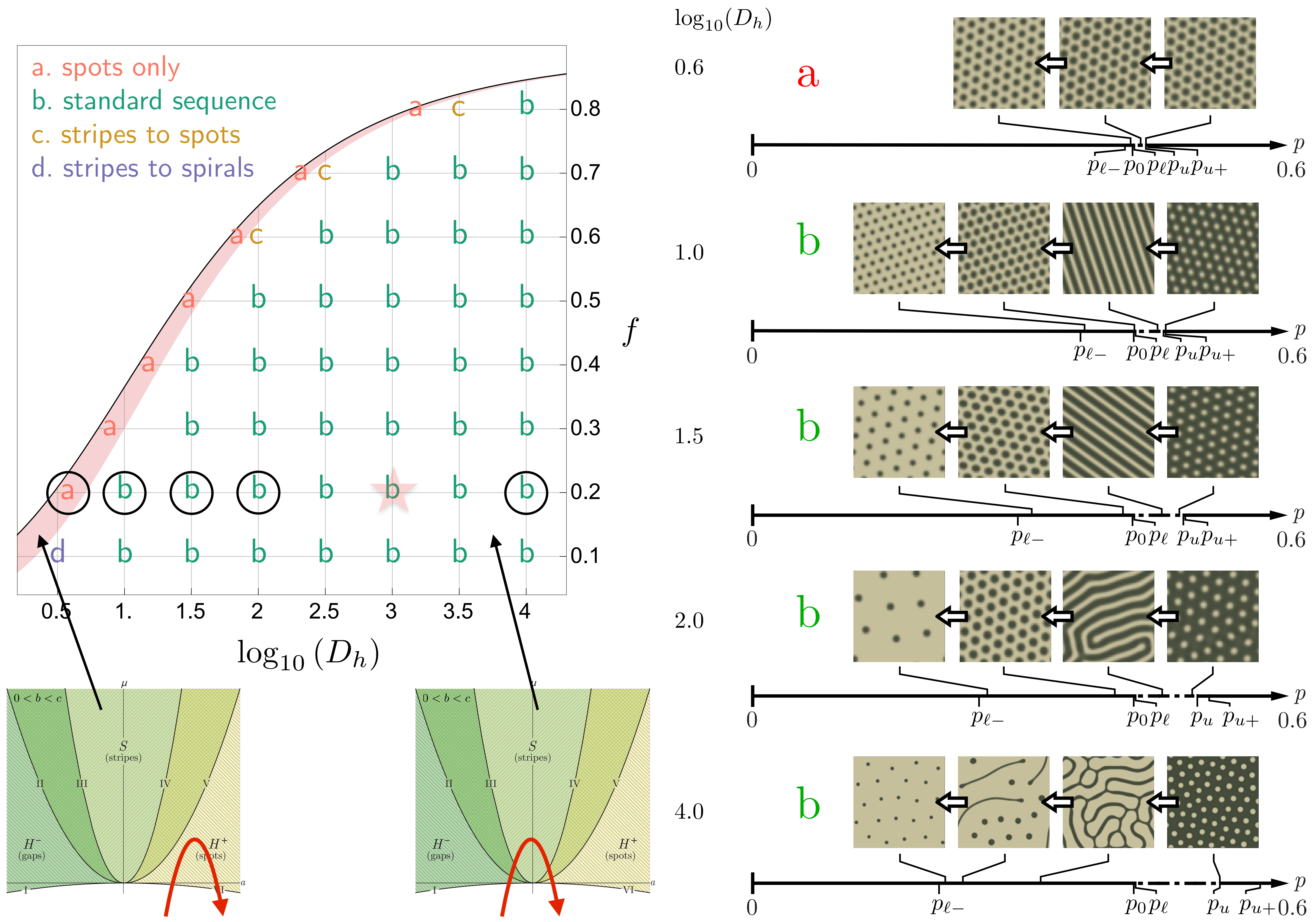
$$I(b) = \frac{b+f}{b+1}$$

biomass density (*b*)

surface water (*h*)

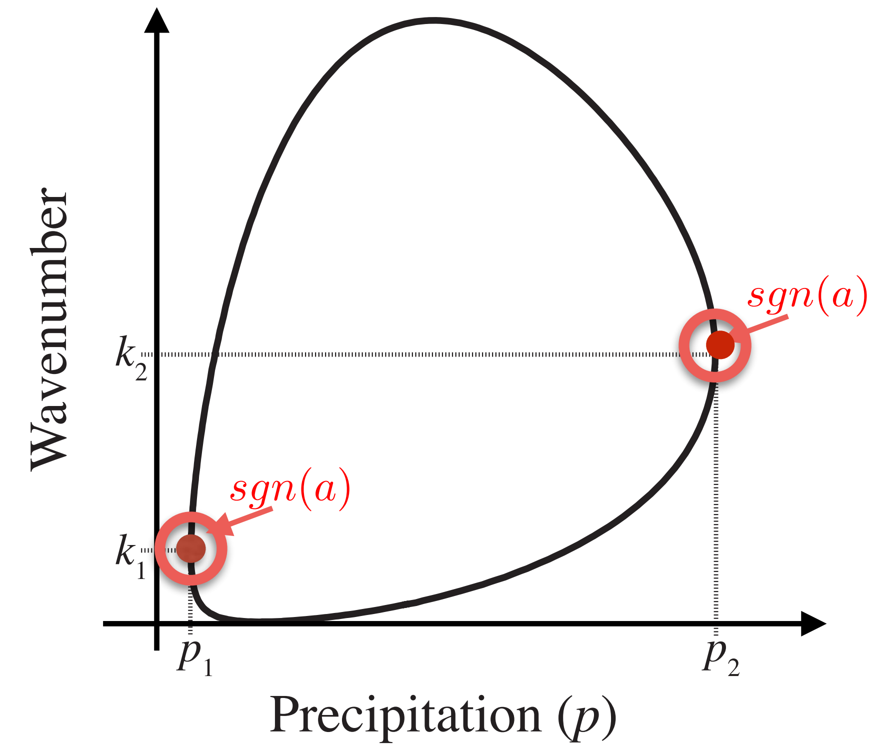
soil water (*w*)

Gowda, Chen, Iams, Silber, Proc. R. Soc. A (2016)

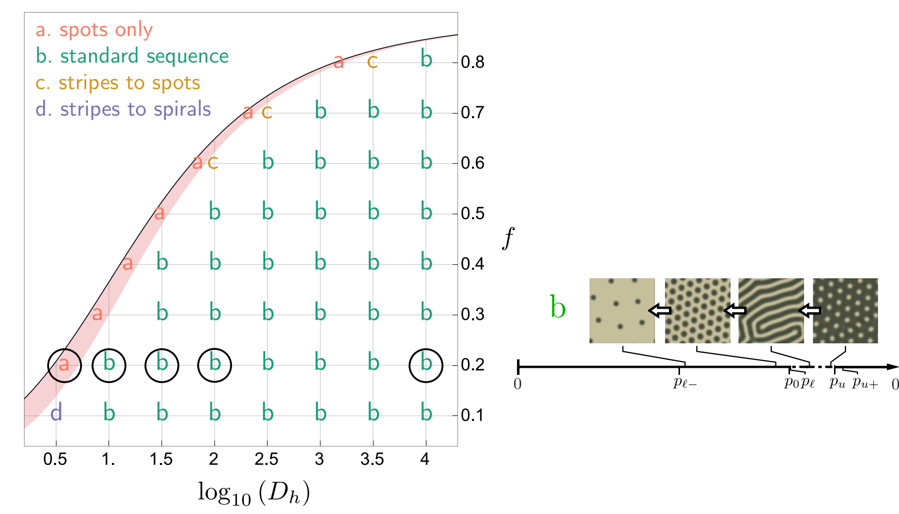


Part I Summary

“Degenerate Turing Bubble” suggests “nonlinear proxy” for “standard sequence”



Numerical simulations to test proxy’s skill



Analytic expression for proxy derived from model provides ecological model insights

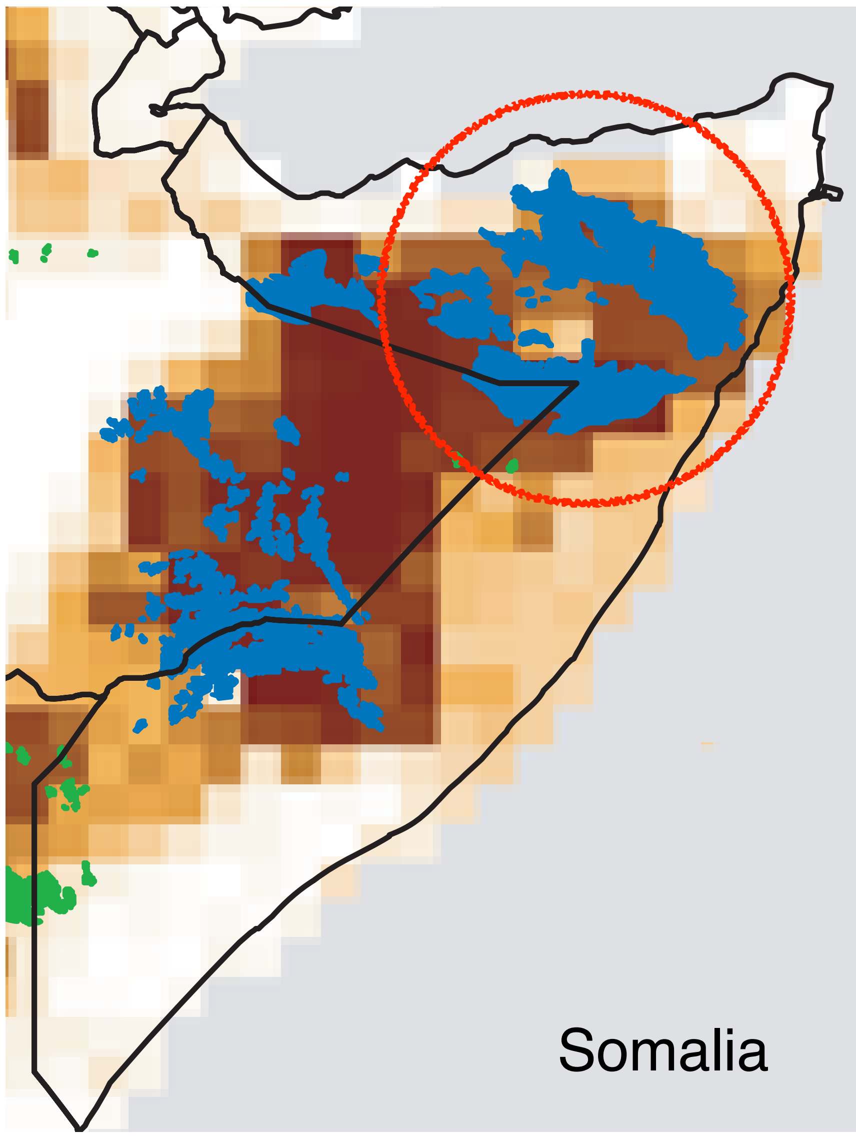
$$a_\ell = C_\ell G'(w_0) + \mathcal{O}(\epsilon)$$

$$a_u = C_u I''(b_0) + \mathcal{O}(\sqrt{\epsilon})$$

$$\epsilon \sim 1/D_h, \; C_\ell, \; C_u > 0$$

II. *Vegetation Patterns in the Horn of Africa*

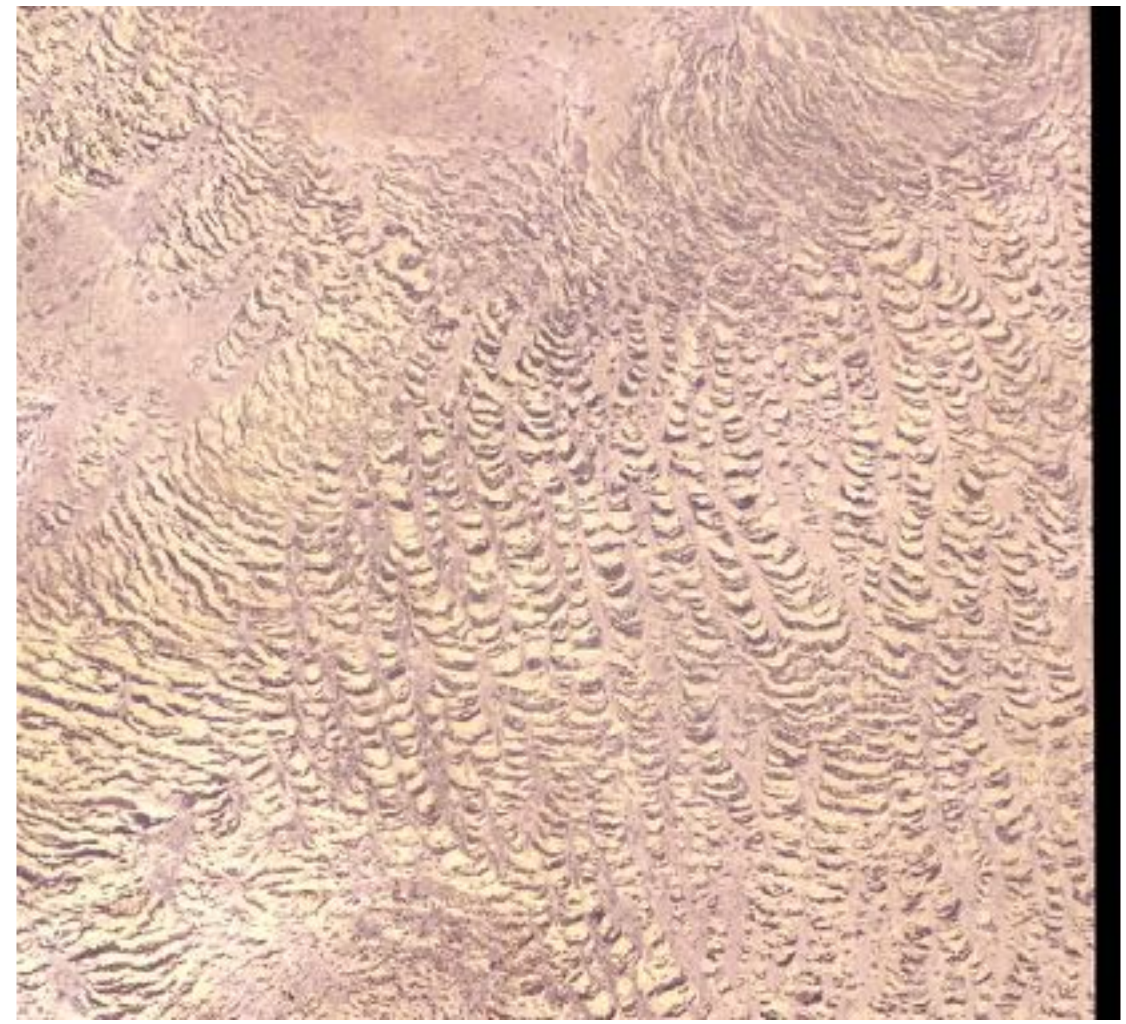
with Karna Gowda (Northwestern) and Sarah Iams (Harvard)



Deblauwe, et al. (2008)

Macfadyen (1950)

We compare aerial imagery from the 50s, 60s, and modern satellites.



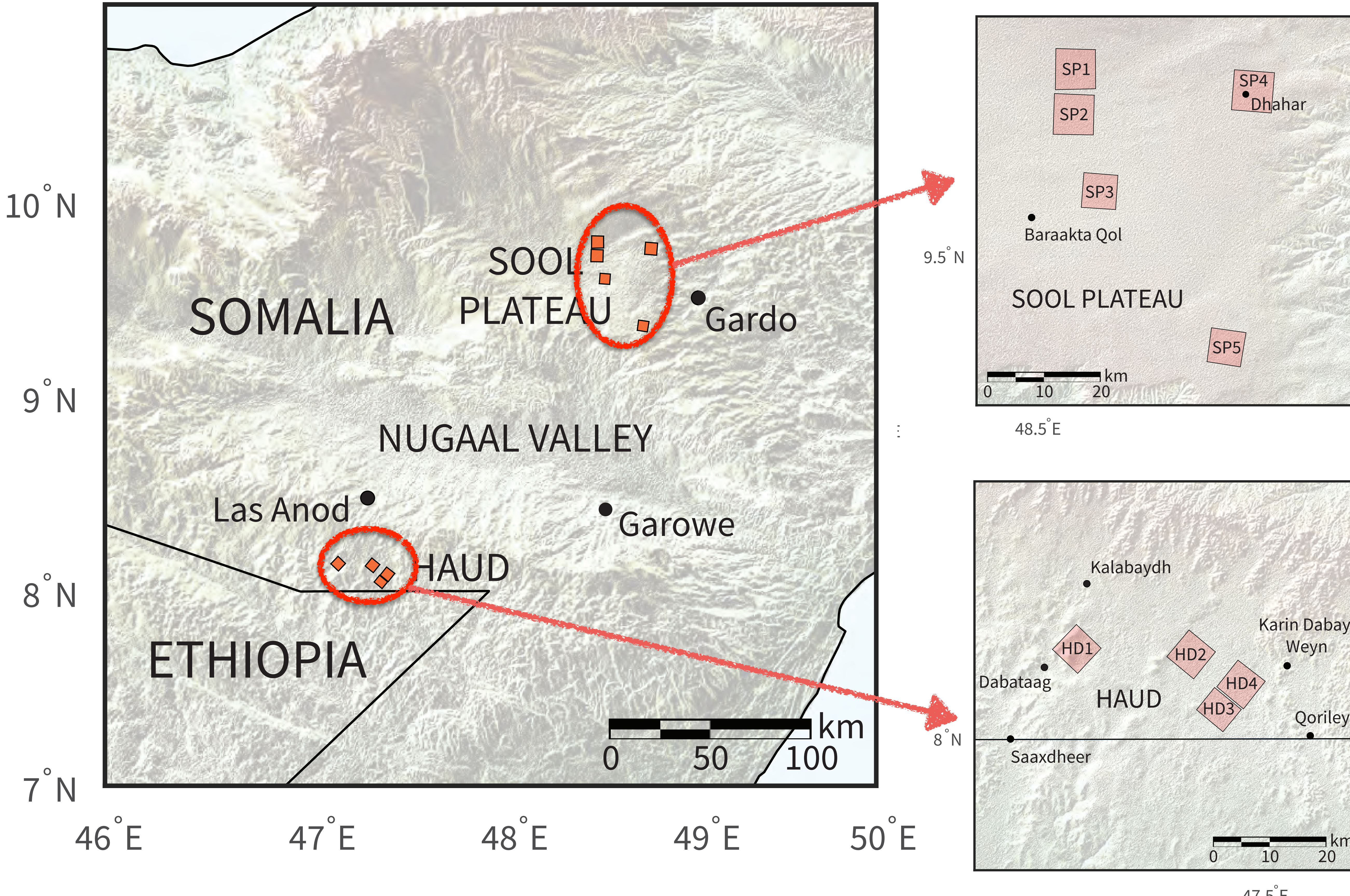
Aerial survey photography taken over banded regions in Somalia (1952) +spy satellite photos (1967)

- ~2 m/pixel resolution
- 1 channel (grayscale)
- Aligned via control points (ArcGIS)

Modern satellite imagery (2004-2016) thanks to DigitalGlobe Foundation

- 0.5-2.4 m/pixel
- 4-8 channels
- Can compute vegetation indices (NDVI, SAVI)

2 regions, 9 photographs, $\sim 50\ km^2$ each



Gowda, Iams, Silber (2017)

Related Aerial/Satellite Image Studies

(some of our inspiration!)

Niger:

Valentin & d'Herbès (1999)
Wu, Thurow & Whisenant (2000)
Barbier, Couteron, Lejoly, Deblauwe, Lejeune (2006)

Sudan:

Deblauwe, Couteron, Lejeune, Bogaert, & Barbier (2011)

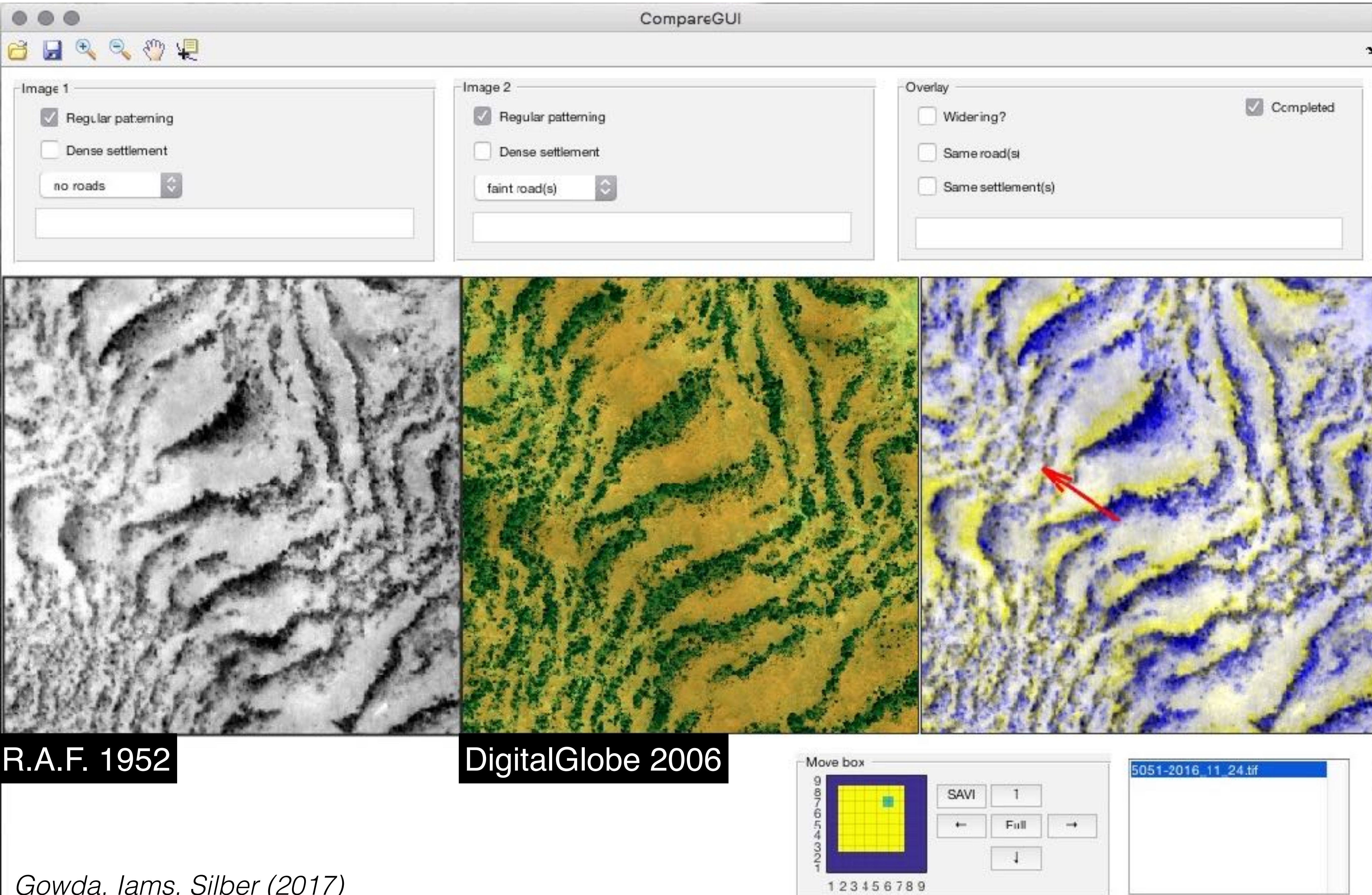
Australia, Morocco, **Somalia**, Texas:

Deblauwe, Couteron, Bogaert, & Barbier (2012)

Texas:

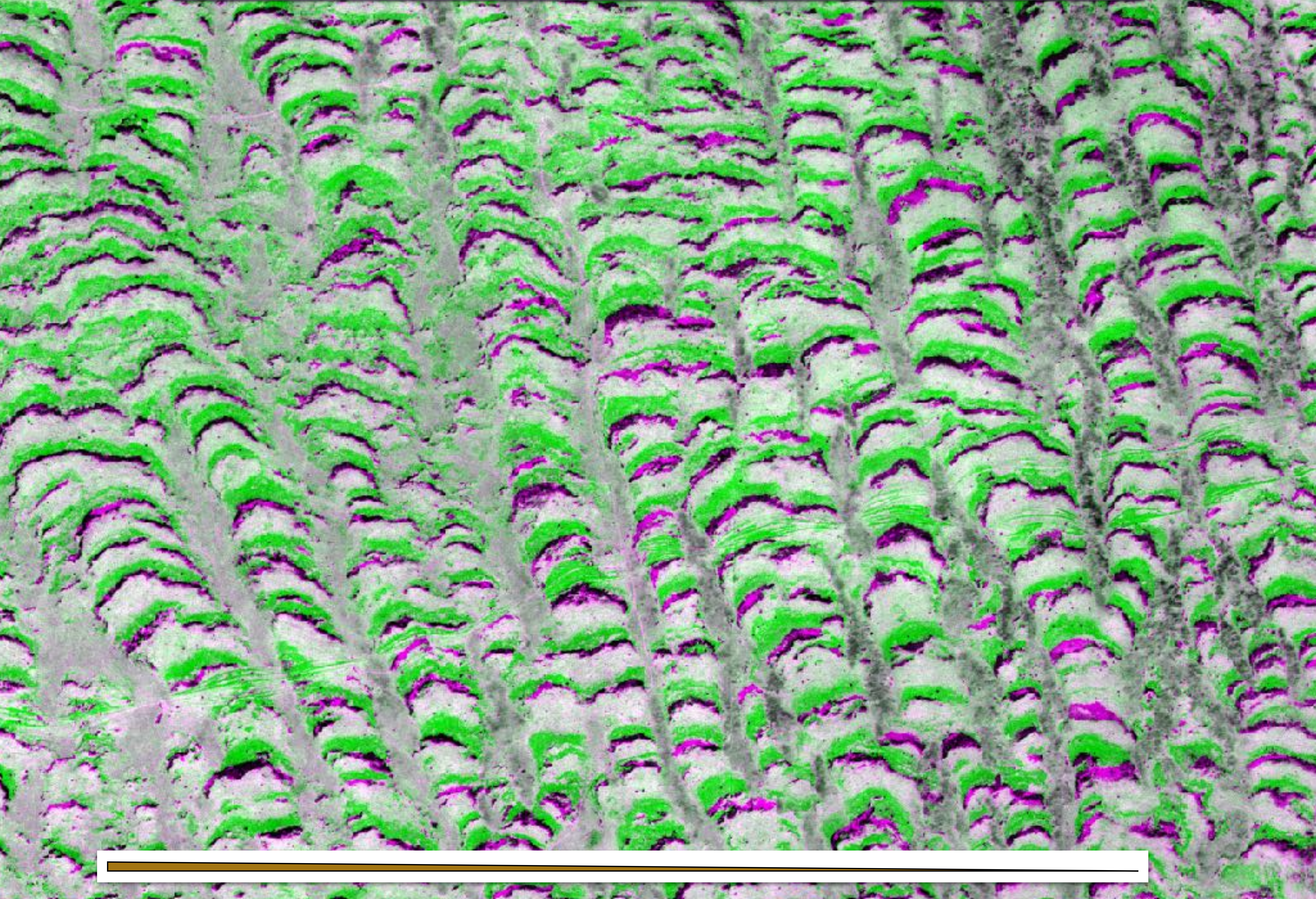
Penny, Daniels, Thompson (2013)

Karna Gowda's GUI



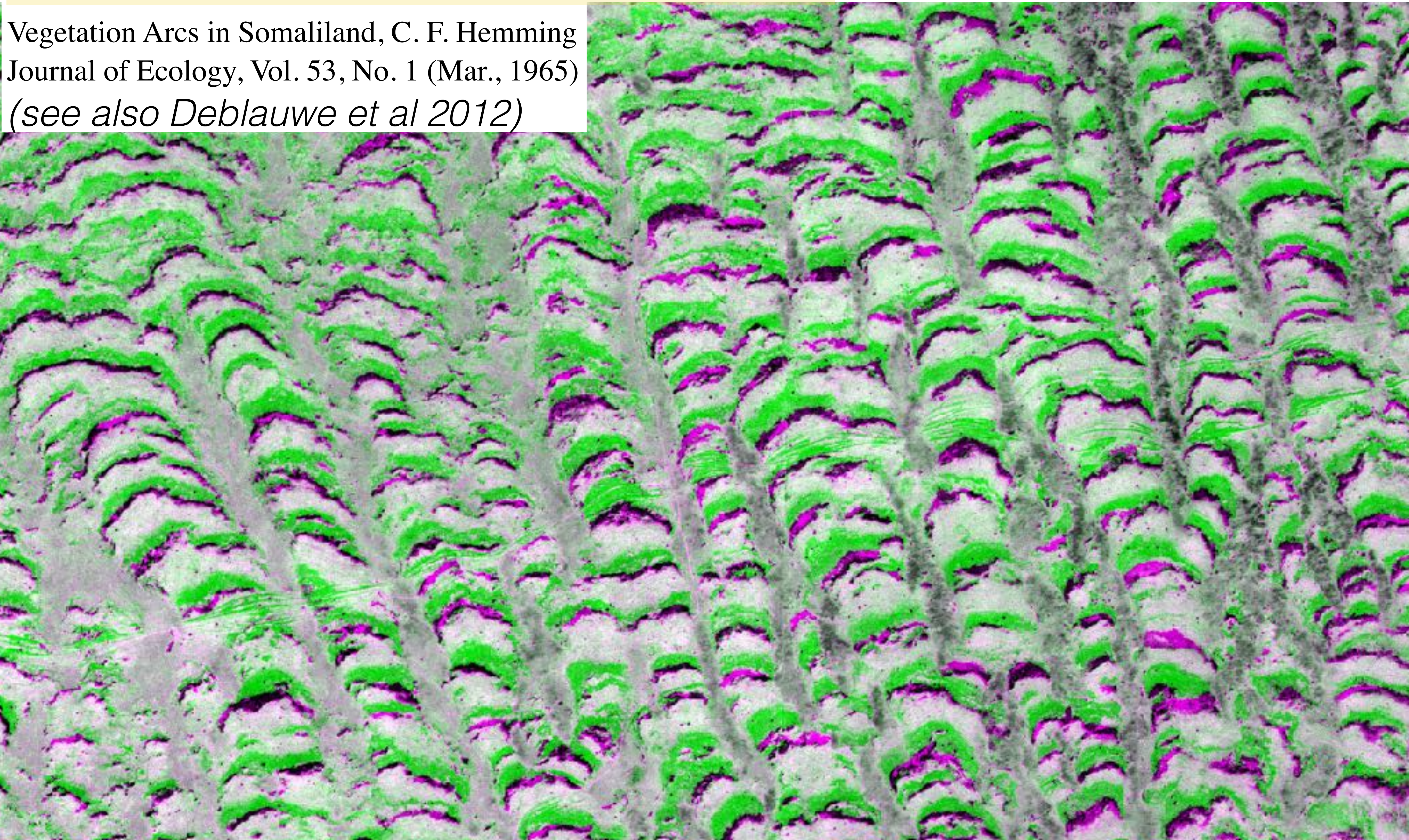
Gowda, Iams, Silber (2017)

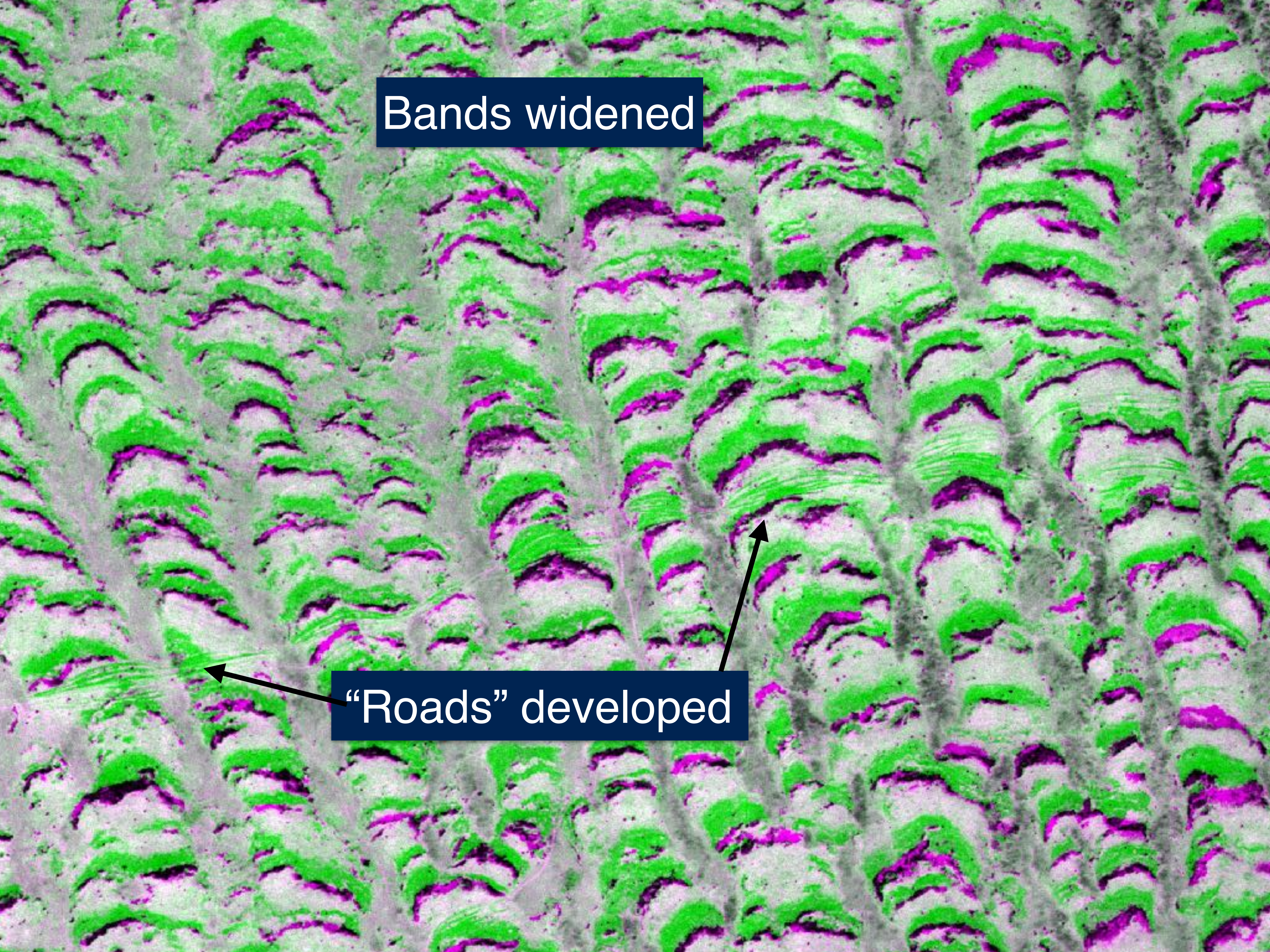
Vegetation's Slow March Through the Desert



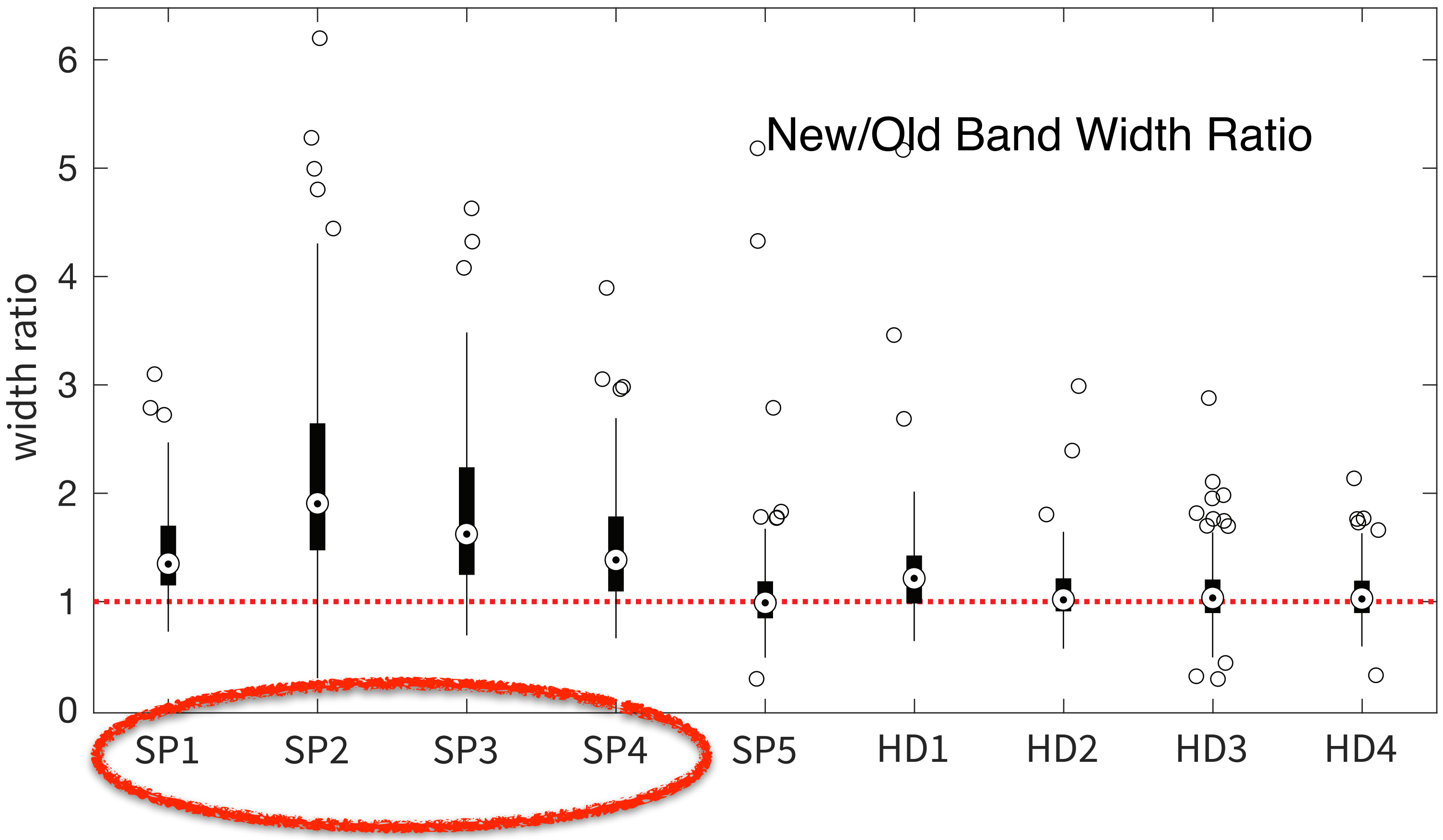
The rate of migration of vegetation arcs is of interest. As far as is known no measurements have been made, but having looked at many advancing upper edges it is estimated that up-flow colonization may occur at an average rate of 6–12 in. (15–30 cm) per year. The small arc surveyed in detail was about 60 ft (18 m) wide and might therefore take between 60 and 120 years to move one arc's width and abandon any trees now living

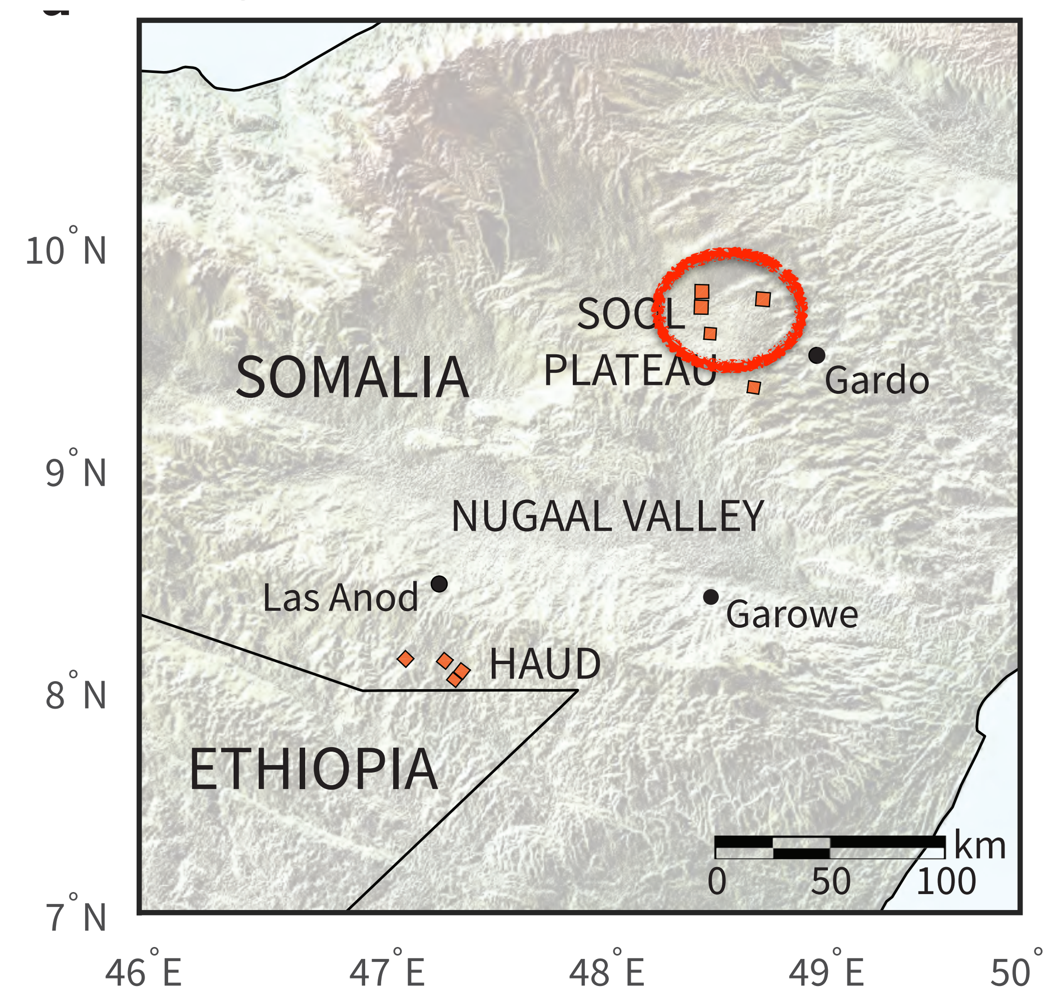
Vegetation Arcs in Somaliland, C. F. Hemming
Journal of Ecology, Vol. 53, No. 1 (Mar., 1965)
(see also Deblauwe et al 2012)

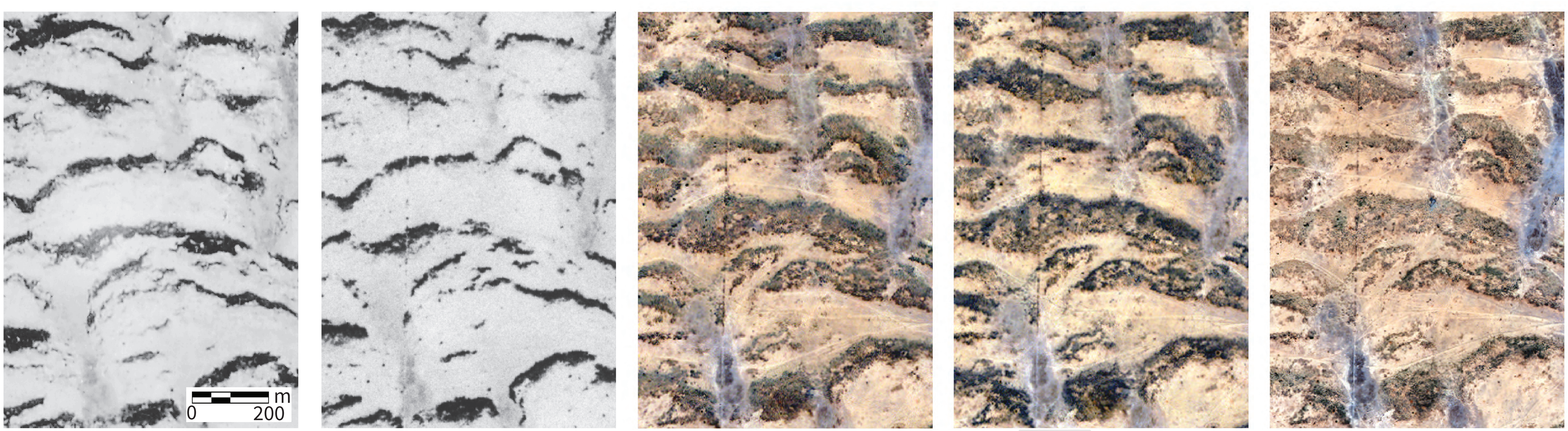



Bands widened
“Roads” developed

Vegetation Band Widening?

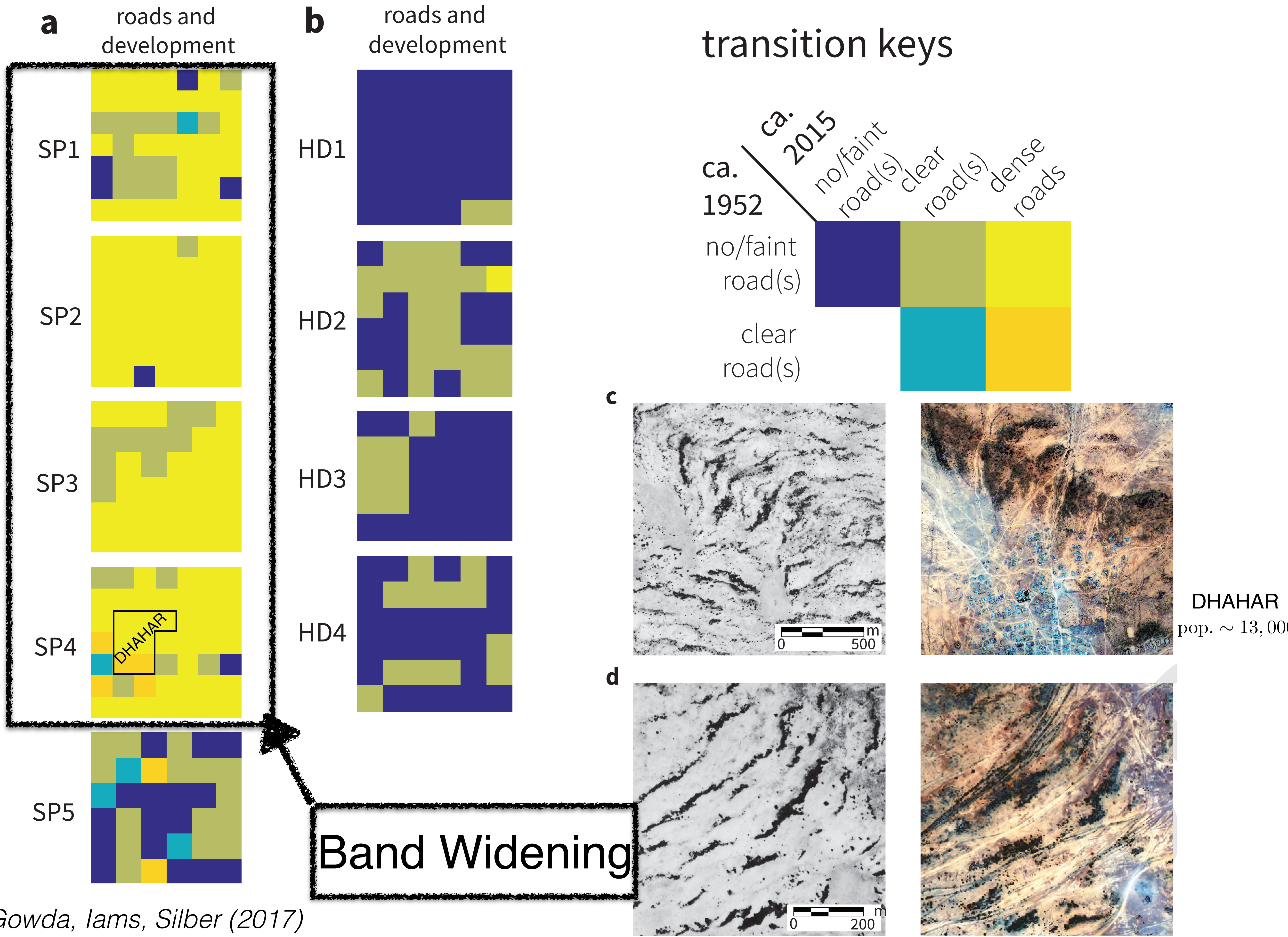




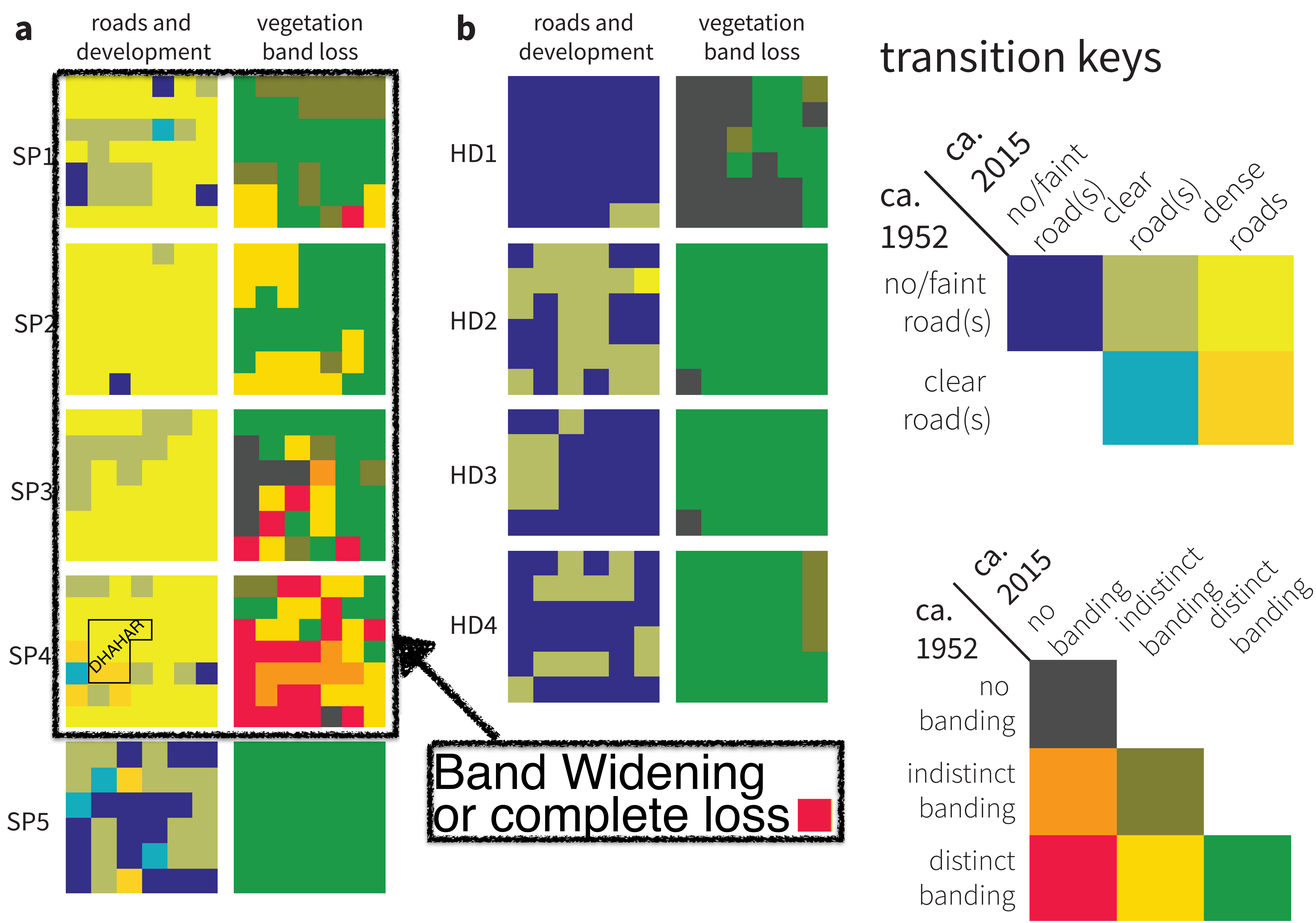


Feb. 1952 Dec. 1967 Jun. 2004 Mar. 2006 Dec. 2011

Gowda, Iams, Silber (2017)



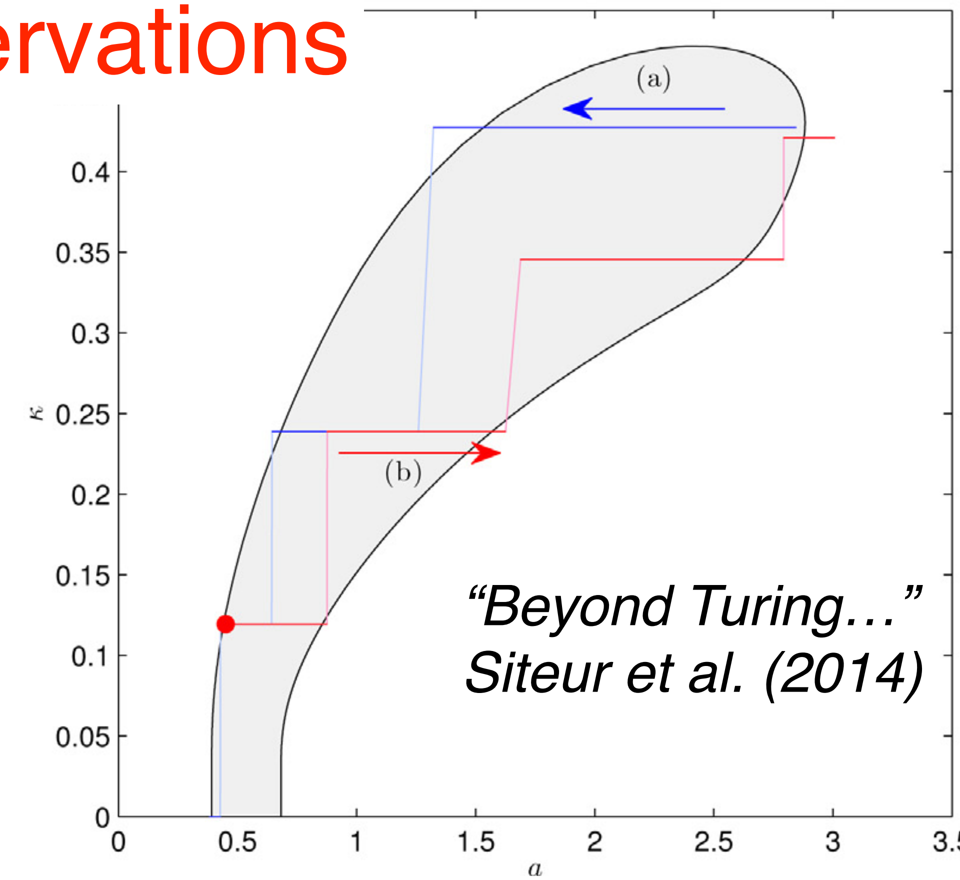
Gowda, Iams, Silber (2017)

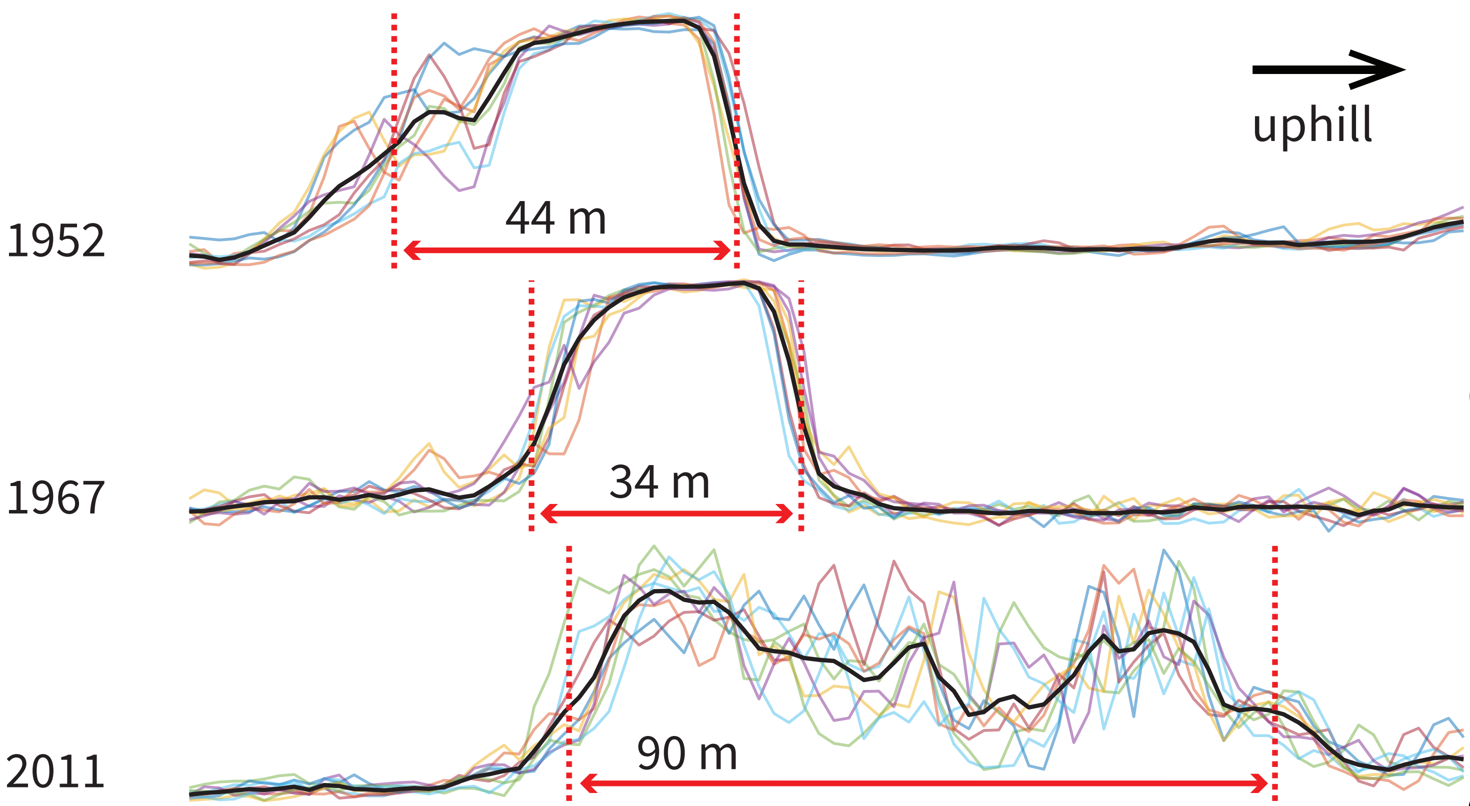


Gowda, Iams, Silber (2017)

Summary Related to Observations

Wavelength change (coarsening) *not observed & hard to observe* - bands may be quite resilient. *(See Doelman and co., Sherratt and co.)*





Gowda, Iams, Silber (2017)

Changes to band-interband ratio, migration rate, vegetation "pulse profile": easier to monitor on modern satellite timescale of decades?

Sool Plateau: band-interband ratio *increased* in regions of *increased* human impacts (based on "road proxy") — *not* what we expected…

Due to change in vegetation composition? Or some form of human impacts not captured by models?

Desertification

"land degradation in arid, semi-arid and dry sub-humid regions resulting from various factors, including climatic variations and human activities." (United Nations)

We used roads/tracks as a convenient proxy for human activity; "easy" to detect in the satellite images.

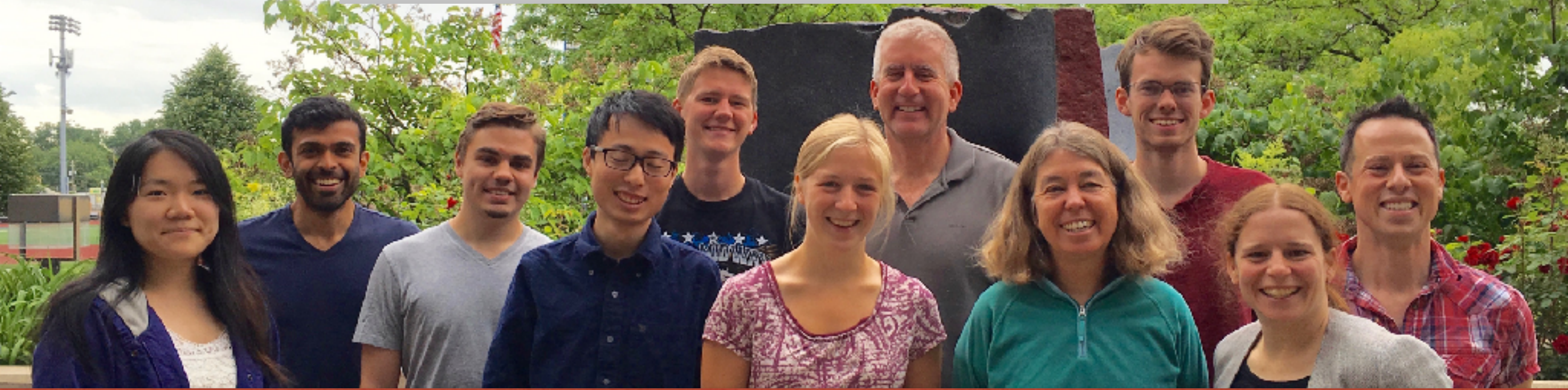
And they may be more than just a proxy - they may be directly influencing the ecohydrology, as described in Hemming (1966).

Vegetation arcs are found in areas without any incised drainage pattern, though they may adjoin such areas. This indicates that the rain water is absorbed either where it falls, or where it arrives after non-erosive sheet-flow.

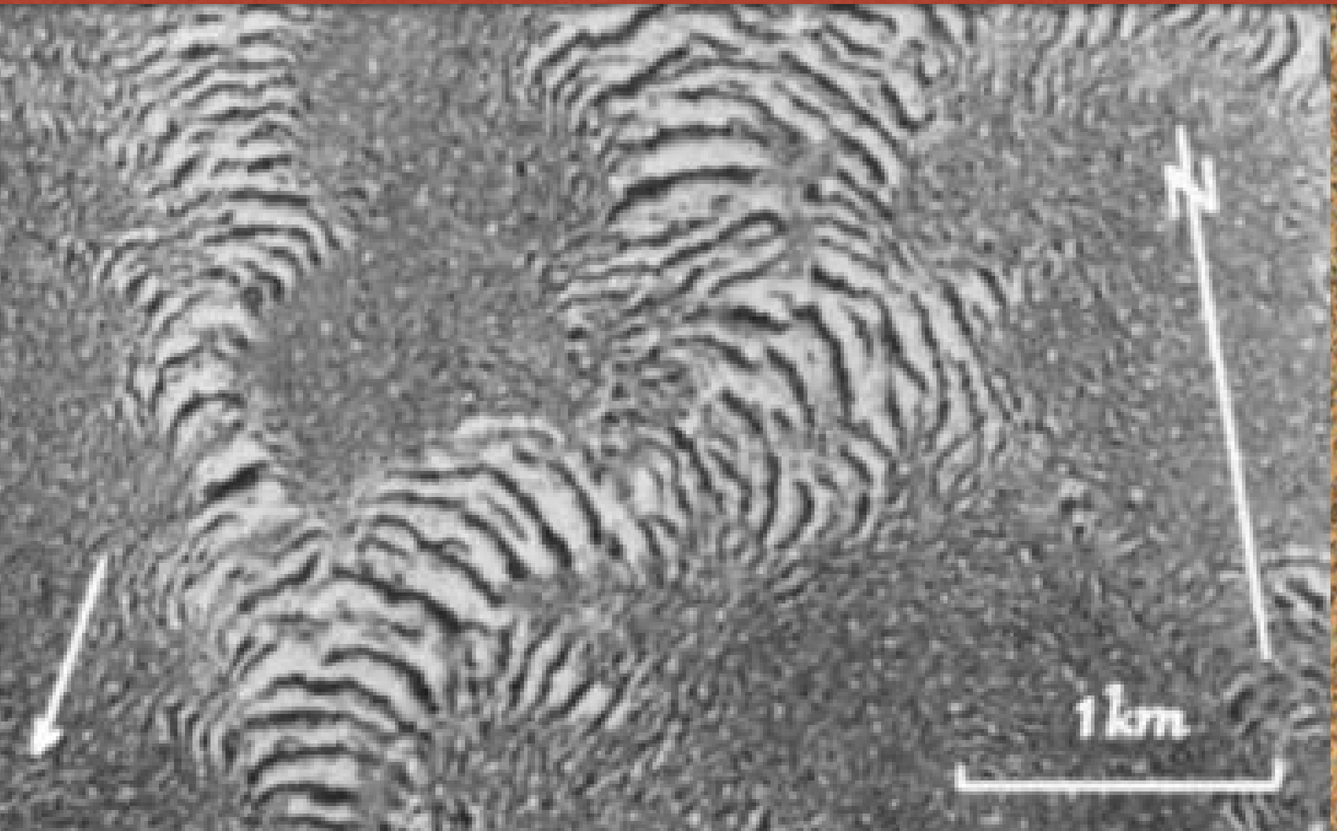
Hemming, Journal of Ecology (1965)

	"Wetlands" (Pattern Formation in Fluids)	"Drylands" (Pattern Formation in the Environment)
Equations	Navier-Stokes+BCs	models exist, but not validated due to lack of experiments
Parameters	often excellent specs	Some inferred at order of magnitude level; some constrained to match phenomena; some models have a lot
Time-scales	seconds - "PhD-scale"	decades-centuries
Spatial-scales	cm scale - "table-top"	10m-"landscape scale"
Symmetries	excellent approximation in controlled experiments	opportunity presented by heterogeneities?
Mechanisms	well developed and validated understanding of pattern formation mechanisms	generic mechanisms invoked

Topographic Influences on Patterns

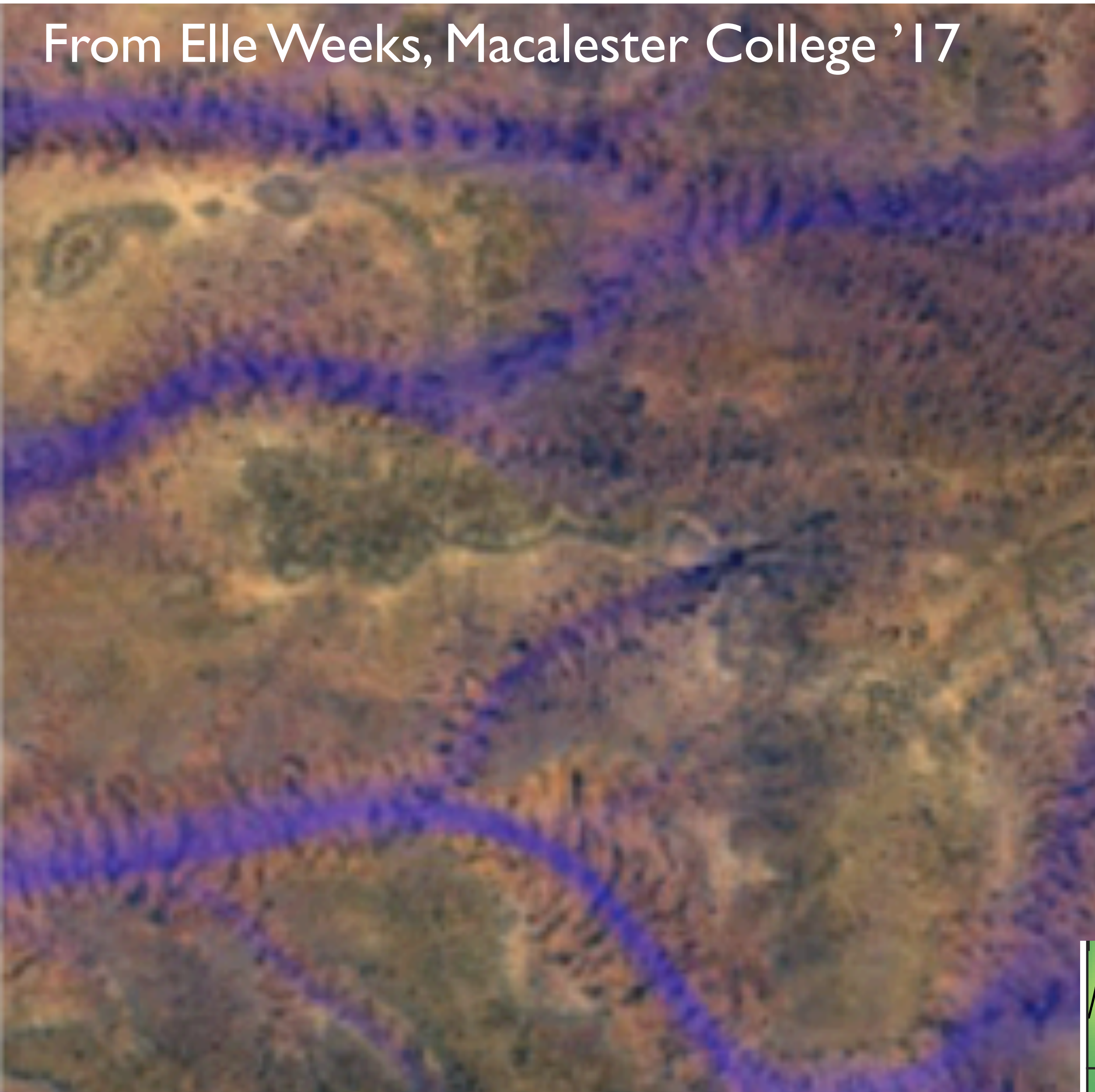


2016 Summer Undergraduate Project @ Macalester College
Macalester (Jake Ramthun, Elle Weeks, Prof. Chad Topaz)
+ Harvey Mudd (Jordan Haack, Dan Schmidt, Gavin Zhang, Prof. Andy Bernoff)
+Sarah Iams (Harvard)+Karna Gowda & Yuxin Chen (Northwestern)

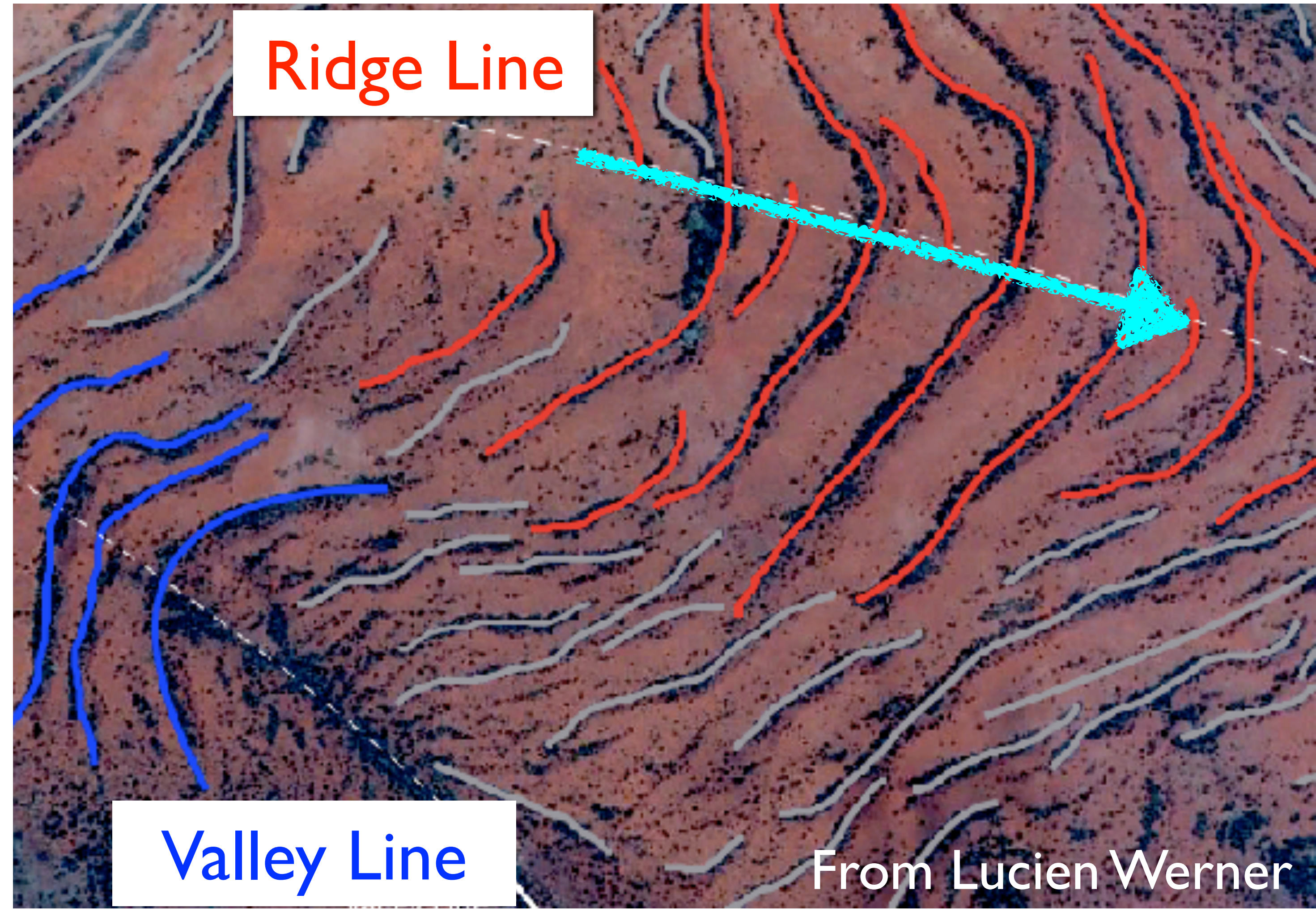


Topographic Influences on Patterns

"Channels"

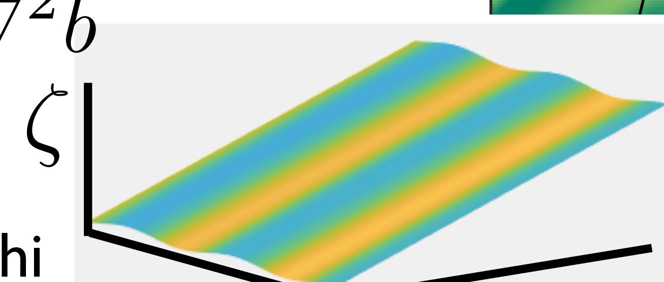


"Arcs"

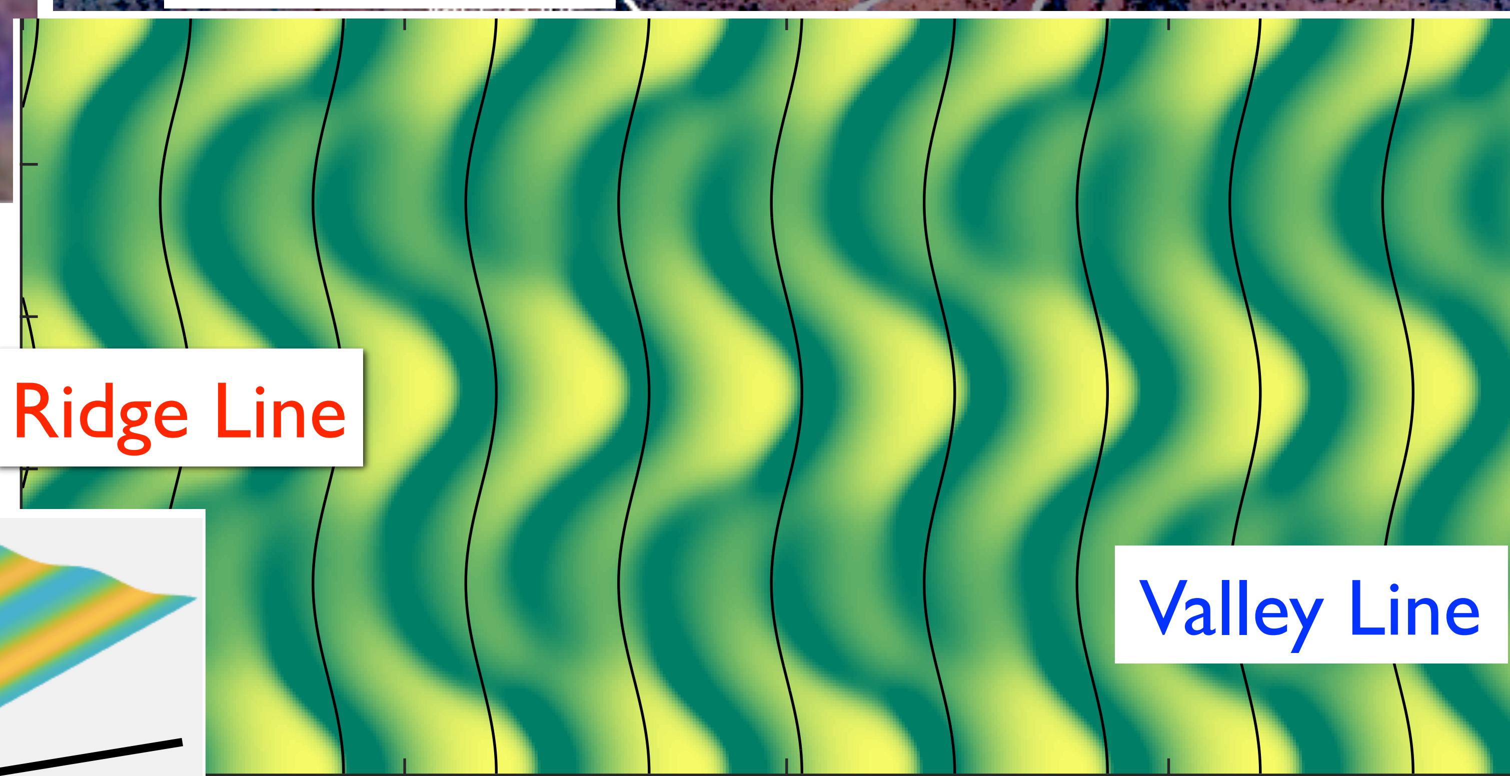


$$\frac{\partial w}{\partial t} = p - w - wb^2 + \nabla \cdot (w \nabla \zeta)$$

$$\frac{\partial b}{\partial t} = -mb + wb^2 + \nabla^2 b$$



From Punit Gandhi



Why study dryland patterns?

- Patterns are so Earthy and beautiful.
- Challenging applied direction for a “mature field” of pattern formation.
- Occur in ecosystems vulnerable to desertification, meant to feed a third of the world population! Is there useful information in the patterns? Any “early warning signs”?

Thanks to

Karna Gowda, Sarah Iams
Yuxin Chen, Hermann Riecke
Chad Topaz, Jake Ramthun, Elle Weeks
Andy Bernoff, Jordan Haack, Dan Schmidt
Punit Gandhi, Lucien Werner