

Enhancing Undergraduate Education in Applied Mathematics through Multidisciplinary and Global Problem solving



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Innovative Pedagogical Practices, Curricular Reforms and Teaching Resources in Applied Mathematics Education July 10, 2018





About Me (Padhu)

























Problem VS Exercise

Padmanabhan Seshaiyer George Mason University How many of the 16 movies can you find?





Padmanabhan Seshaiyer George Mason University How many of the 16 movies can you find?



What is "e"?









Student Engagement Model





Students as Consumers



Students as Producers







Students as peer-reviewers

Padmanabhan Seshaiyer

Life-long Skills



Teach less, Learn More

- Teacher-directed
- Direct Instruction
- Knowledge
- Content
- Basic Skills
- Facts and Principles
- Theory
- Curriculum
- Time-slotted
- One-size-fits-all
- Competitive
- Classroom
- Text-based
- Summative Tests
- Learning for School

- Learner-Centered
- Interactive exchange
- Skills
- Process
- Applied Skills
- Questions and Problems
- Practice
- Projects
- On-demand
- Personalized
- Collaborative
- Global Community
- Web-based
- Formative evaluations
- Learning for Life

How Many?

- How many Piano Tuners are in Chicago?
- Population = 3,000,000

- Number of families in Chicago = 750,000
- Number of Pianos in Chicago = 150,000
- Number of Pianos Repaired in a year = 1000
- Number of Piano Tuners in Chicago = 150

How Many?

 How many kernels of popcorn would it take to fill this room?



How Many?

 How many kernels of popcorn would it take to fill this room?



Fermi Problems

- How many piano tuners are there in Chicago?
- How many people in the world are talking on their cell phones in any given minute?
- How many kernels of popcorn would it take to fill this room?
- How many new passenger cars are sold each year in the USA?
- How many pennies would need to be stacked to reach the height of Mount Everest?







Imagine looking under the Microscope





Petri Dish

Bacteria

Patch of Bacteria in^{Ga} the Petri-dish



Find the **AREA** of the bacterial colony.









Can we connect the George Mason University Petri-Dish Area finding with "Find the integral"?



Spectrophotometer



Absorbance = $-\log_{10} \frac{I_t}{I_0}$

Developing a bacteria growth model from experimental data

Measurements of bacterial density at pH 6.25. The units of "Population Density" are those of absorbance as measured by a spectrophotometer.

Time (min)	Population Density
0	0.022
16	0.036
32	0.060
48	0.101
64	0.169
80	0.266

Cells divide in half (fission) Each cell takes the same time.

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What is the change?

t Time	P(t) Population density over time	P(t+1) – P(t) Population Change over unit time
0	0.022	
1	0.036	0.014
2	0.060	0.024
3	0.101	0.041
4	0.169	0.068
5	0.266	0.097

Bacteria takes longer than 16 minutes to divide. That is more than 1 time step to mature and divide.

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Some Cell Biology

- Bacteria cells reproduce by dividing into two cells
- Each cell takes about the same time to mature and divide



t	P(t+1)-P(t)
0	0
1	0.014
2	0.024
3	0.041
4	0.068
5	0.097

Some assumptions!

- A certain fraction of the cells divide each time step.
- Each cell takes about the same time to mature and divide

	•	
P(t+1) -	P(t) =	c P(t)

	P(t+1)-P(t)
0	0
1	0.014
2	0.024
3	0.041
4	0.068
5	0.097

Visualizing Population Change with respect to Population Density



Fitting a linear model!



Fitting an exponential model!

$$P(t+1) - P(t) = \frac{2}{3} P(t), P(0) = 0.022$$
$$P(t+1) = \frac{5}{3} P(t)$$



Fitting an exponential model!

RECURSIVE
$$\begin{cases} P(t+1) - P(t) = \frac{2}{3} P(t) \\ P(0) = 0.022 \end{cases}$$

EXPLICIT
$$P(t) = 0.022 \left(\frac{5}{3}\right)^{t}$$

How did we do? $P(t) = 0.022 \left(\frac{5}{3}\right)^{t}$

Time	Population density over time	Computed Population density over unit time
0	0.022	0.022
1	0.036	0.037
2	0.060	0.061
3	0.101	0.102
4	0.169	0.170
5	0.266	0.283

Wait...there is more!

Measurements of bacterial density at pH 6.25. The units of "Population Density" are those of absorbance as measured by a spectrophotometer.

Time (min)	Time index t	Population Density	\uparrow		
0	0	0.022	1.2 -		
16	1	0.036	- 1 shity		
32	2	0.060	р и 0.8-		
48	3	0.101	- a.o grilati.		
64	4	0.169	d 0.4-		
80	5	0.266	0.2 -		_ ×
96	6	0.360	o 📜 🧧	× ×	× 4
112	7	0.510		-	
128	8	0.704			
144	9	0.827			
160	10	0.928			



Carrying Capacity

- Maximum population size that the environment can sustain indefinitely.
- When the population reaches its environment limits, its growth slows down.



Incorporating Logistic Growth

- P(t) = Population size in time period t
- M = Carrying Capacity

 $\frac{P(t)}{M} =$ fraction of carrying capacity that is used

 $1 - \frac{P(t)}{M}$ = unused fraction of carrying capacity

$$P(t+1) - P(t) = r P(t) \left(1 - \frac{P(t)}{M}\right)$$
Modeling Population

Let y(t) be a homogeneous population density and y_0 be the initial population.



Predator-Prey Problem



$$\frac{dY}{dt} = \alpha_1 Y - \alpha_2 XY$$
$$\frac{dX}{dt} = -\alpha_3 X + \alpha_4 XY$$
$$Y(0) = Y_0$$
$$X(0) = X_0$$

About those "Models"

All models are wrong, but

some are more wrong than others.

C.W. Clark and M. Mangel (2000) Dynamic State Variable Models in Ecology.

Solution Methodology



Multidisciplinary Research

- As concluded by the National Research Council: Undergraduate education will not change in a permanent way through the efforts of "Lone Rangers." Change requires ongoing interaction among communities of people and institutions that will reinforce and drive reform.
- Research that happens across traditional STEM disciplines and at the edges of traditional disciplines.
- Here is the problem,

find the Mathematics to solve it!





Social Dynamics Modeling













Cyclic Migration Without Regulation





Posters on the Hill

$\begin{array}{c} 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.04 \\ 0.02 \\ 0.05 \\ 0.$

Parameter Estimation











Make solar energy economical



Manage the nitrogen cycle



Advance health informatics



Provide energy from fusion



Provide access to clean water



Engineer better medicines



Restore and improve urban infrastructure

Develop carbon

sequestration

methods



Reverse-engineer the brain



Prevent nuclear terror



Secure cyberspace



Enhance virtual reality



Advance personalized learning



Engineer the tools of scientific discovery





GRAND CHALLENGES FOR ENGINEERING

India's Water and Sanitation Crisis





163 M 210 M

people lack access to safe water

people lack access to improved sanitation



Disease Dynamics Modeling Identifying the threshold or tipping point (Kermack and McKendrick, 1927, 1932, 1933)

- Individuals are found in three stages
 - -Susceptible
 - -Infected
 - -Recovered





SIR Dynamics



Graphical User Interface



Graphical User Interface (data)



2-patch SIWR Model



$$\frac{dS_1}{dt} = \mu N_1 - b_{w_1} W S_1 - b_{I_1} S_1 I_1 - \mu S_1$$

$$\frac{dI_1}{dt} = b_{w_1} W S_1 + b_{I_1} S_1 I_1 - \gamma I_1 - \mu I_1$$

$$\frac{dR_1}{dt} = \gamma I_1 - \mu R_1$$

$$\frac{dS_2}{dt} = \mu N_2 - b_{w_2} W S_2 - b_{I_2} S_2 I_2 - \mu S_2$$

$$\frac{dI_2}{dt} = b_{w_2} W S_2 + b_{I_2} S_2 I_2 - \gamma I_2 - \mu I_2$$

$$\frac{dR_2}{dt} = \gamma I_2 - \mu R_2$$

Padmanabhan Seshaiyer Mathematical Models^{George Mason University} for Zika Transmission



Mathematical Models for Fighting Zika Virus, Carrie Manore and Mac Hyman, SIAM News, May 2016

Mathematical Models George Mason University for Zika Transmission



Mathematical modeling, analysis and simulation of the spread of Zika with influence of sexual transmission and preventive measures, Letters in Biomathematics, Vol 4, pp 148-166, 2017.





Global Challenge

Recruitment of Gangs





Age of Gang Members by Area Type, 2011 100 90 80 70 60 Percent 50 40 30 20 10 0 Larger Cities Suburban Counties Smaller Cities **Rural Counties** Adult (18 and Over) Juvenile (Under 18)

Modeling Gangs in Puerto-Rico



Governing Differential Equations

Youth Population

(1)
$$\frac{dS_{y}}{dt} = -\left[\beta_{yy}a_{1}\left(\frac{S_{y}G_{y}}{N_{y}}\right) + \beta_{yA}a_{1}\left(\frac{S_{y}G_{A}}{N_{A}}\right)\right] + f\rho_{y}R_{y} + \phi_{y}G_{y}$$

(2)
$$\frac{dG_{y}}{dt} = \beta_{yy}a_{1}\left(\frac{S_{y}G_{y}}{N_{y}}\right) + \beta_{yA}a_{1}\left(\frac{S_{y}G_{A}}{N_{A}}\right) + (1-f)\rho_{y}R_{y} - \phi_{y}G_{y} - \alpha_{y}G_{y}$$

(3)
$$\frac{dR_{y}}{dt} = \alpha_{y}G_{y} - \rho_{y}R_{y}$$

Adult Population

$$(4) \quad \frac{dS_A}{dt} = -\left[\beta_{AA}a_2\left(\frac{S_AG_A}{N_A}\right) + \beta_{Ay}a_2\left(\frac{S_AG_y}{N_y}\right)\right] + f\rho_A R_A + \phi_A G_A$$

$$(5) \quad \frac{dG_A}{dt} = \beta_{AA}a_2\left(\frac{S_AG_A}{N_A}\right) + \beta_{Ay}a_2\left(\frac{S_AG_y}{N_y}\right) + (1-f)\rho_A R_A - \phi_A G_A - \alpha_A G_A$$

$$(6) \quad \frac{dR_A}{dt} = \alpha_A G_A - \rho_A R_A$$



Partnerships for Enhanced Engagement in Research (PEER) Science



Computational Mathematics, Modeling and Analysis of Biological, Bio-inspired and Engineering Systems

PI: Burton Mwamila

Vice Chancellor, The Nelson Mandela African Institute of Science and Technology U.S. Partner: Padmanabhan Seshaiyer (Professor, George Mason University)

http://sites.nationalacademies.org/PGA/dsc/peerscience/PGA_084056



Global Problem Solving

- Location: Tanzania
- Problem: Poaching





Why is this a rich STEM Problem?

- Dynamics and Mechanics
- Search Algorithms
- Sensors and Electronics
- Control Systems and Feedback
- Communications
- Swarming
- Mapping Algorithms
- Machine Learning
- Applications: Anti-poaching, Remote Sensing, Agriculture, Transporting materials, Oil-gas-mineral exploration, Search & Rescue, Surveillance and many more!
 - STEM Education



Modeling using Drones

Agriculture



credit: diydrone.com

Search and Rescue



credit: gizmag.com

Surveillance





credit: Far-drone-hull-repair.rcquadcopterkit.net



Drones and Mechanics



 $\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \sum \frac{\mathbf{F}}{m} = \frac{1}{m} f_T \mathbf{R} \begin{bmatrix} 0 \\ 0 \\ \frac{4}{\sum_{i=1}^4 \gamma_i^2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \frac{1}{m} f_D \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$ $\begin{bmatrix} \dot{\omega}_{\phi} \\ \dot{\omega}_{\phi} \\ \dot{\omega}_{\phi} \\ \dot{\omega}_{\phi} \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}^{-1} \begin{bmatrix} \tau_{\phi} - (I_{zz} - I_{yy})\omega_{\theta}\omega_{\psi} \\ \tau_{\theta} - (I_{xx} - I_{zz})\omega_{\psi}\omega_{\phi} \\ \tau_{\psi} - (I_{yy} - I_{xx})\omega_{\phi}\omega_{\theta} \end{bmatrix}$

credit: norunway.com







PD Control = $K_{p}e(t) + K_{d}\frac{d}{dt}e(t) \rightarrow \gamma = [\gamma_{1}, \gamma_{2}, \gamma_{3}, \gamma_{4},]$

Drones and Target Detection





Target Detection

$$d_{a_i}^t = \begin{cases} 0 & \text{if } x_\tau \neq a_i \text{ at time } t \\ 1 & \text{if } x_\tau = a_i \text{ at time } t \end{cases}$$

Measurement Error

- β = missed detection (missing the poacher)
- α = false alarm (detecting something that's not there; a ranger. a cat, etc)

$$Bel(x_{\tau}) = Pr(x_{\tau} = a_k | D^t) = \frac{Pr(d_k^t | x_{\tau} = a_k, D^{t-1}) Pr(x_{\tau} = a_k | D^{t-1})}{Pr(d_k^t | D^{t-1})}$$

Theorem

For a uniform distribution, the belief function:

$$Pr(x_{\tau} \in A | D^{t} = \mathbf{0}) = \frac{t\beta\delta + (1-\alpha)(|A|-t)\delta}{t\beta\delta + (1-\alpha)(|A|-t\delta)}$$

converges to the prior belief, δ .

	0.5		Belief Fur	nction alpha=0.1,	beta=0.2		
	0.45 -	-					-
	0.4 -						-
	0.35	(-
Function, B	0.3 -	1					-
	0.25						-
Belief	0.2 -						-
	0.15						-
	0.1						-
	0.05 -						-
	0	5	10	15	20	25	30
				Gridsize			

Bees, Coffee and Maria



$$\frac{dC}{dt} = C\left[r_1 - \frac{r_1}{k_1}C + c_1\left(\frac{s_2B}{1 + \tau_2s_2B}\right) - q_1\left(\frac{s_1B}{1 + \tau_1s_1C}\right)\right]$$

$$\frac{dB}{dt} = B\left[r_2 - \frac{r_2}{k_2}B + c_2\left(\frac{s_1C}{1 + \tau_1s_1C}\right)\right]$$

В

С

$ au_1$	Bee Feeding Time
τ_2	Bee Pollination Time
s ₁	Coffee Attraction Rate (Pollination)
s ₂	Bees Search Rate (Food)
k_1, k_2	Carrying Capacity





Design Thinking in Higher Education DICE: Design Thinking, Innovation, Creativity & Entrepreneurship







Project PROGRESS: Promoting Renewable energy research On the Grid to create a Responsible and Engaged STEM workforce in Solar Sustainability across the Commonwealth

Padmanabhan Seshaiyer STEM Accelerator Program



Modeling of real-world applications



- 1. Modeling, Analysis and Computation of Fluid Structure Interaction Models for Biological Systems, S. Minerva Venuti and P. Seshaiyer (Mentor), SIAM Undergraduate Research Online, Vol. 3, pp 1-17 (2010).
- 2. Transforming Practice Through Undergraduate Researchers, P. Seshaiyer, Council on undergraduate research Quarterly, Fall 2012.

University, Community College, K-12 STEM Collaboration



Sarah M. Venuti (Undergraduate Student) Avis Foster (Undergraduate Student) Courtney Chancellor (Undergraduate Student) Stephanie Alley (Undergraduate Student) Kris Kappmeyer (K-12 Teacher) Kurt Litsch (High School Student) Alicia Hamar (High School Student) Archis Bhandarkar (High School Student) Rohan Banerjee (High School Student) James Nong (Graduate Student) Andrew Samuelson (Graduate Student)





Transforming Practice Through Undergraduate Researchers, P. Seshaiyer, Council on undergraduate research Quarterly, Fall 2012.

How does new research/innovations evolve?







Padmanabhan Seshaiyer



	Scientific Experimental Method		Engineering Design Method
•	Pose a question	•	Define a problem
•	Research the question	•	Research the problem
•	Construct an answer, explanation, or hypothesis to be tested	•	Design, plan, and build a prototype or solution to be tested
•	Test the hypothesis through experiments that attempt to disprove it	•	Test the prototype or solution to see if it solves the problem
•	Analyze the results and draw a conclusion about the answer	•	Analyze the results and improve the solution to the problem
•	Communicate the results and compare with the others' results	•	Communicate the results and implement or market the solution as a product or service
•	Repeat the process with more refined questions or with new questions that arose in the process	•	Repeat the process with refined or new ideas for better solutions, or with new problems that arose in the process
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What type of transformative research and training can one do?

- Modify Key Assumptions
- Build Realistic Geometry
- Optimize Mathematical Techniques
- Enhance Mathematical Software
- Match Experimental Data
- Refine Mathematical Model
- Perform Parameter Estimation Studies

Contact

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