



Improving Students' Understanding of Numerical Analysis & Mathematical Statistics through 3D Printing

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Design creative and tangible projects that can be open ended.

1. Numerical Analysis
2. Mathematical Statistics

Play is the highest form of research.

- Albert Einstein



Numerical Analysis

Students Applied Math Majors and Computer Science majors

Prerequisites Calculus I & II, Linear Algebra, Intro to Programming

- Topics**
1. solving linear systems,
 2. curve fitting (splines)
 3. numerical integration and differentiation
 4. root finding methods
 5. solving optimization problems
 6. MATLAB

Idea Many of the topics in a typical numerical analysis course are used by 3D printing design tools



The Problem



In 1915 the Coca-Cola Bottling Association put out a call glass companies across the U.S. to create a “distinctive package” to protect their product from “copycat” brands. In particular, they asked the glass companies to design

“bottle so distinct that you would recognize if by feel in the dark or lying broken on the ground.”

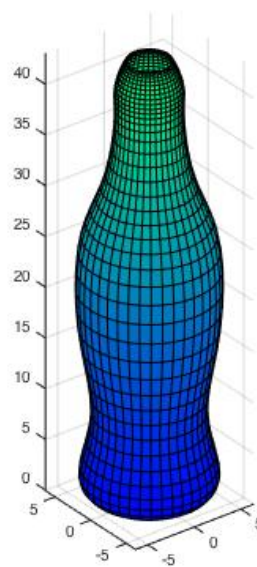
<http://www.coca-colacompany.com/stories/the-story-of-the-coca-cola-bottle>



Assignment: Bottle Design

Topic: Curve Fitting

Design an attractive bottle in MATLAB. Design the bottle's profile in 2D using parametric splines. To obtain the 3D model of the bottle, rotate the curve around the z-axis to create the bottle shape in 3D.





MATLAB

1. Pick Points

```
x = [0 3 6 5 6 6 4 3 3 2.5 2];  
y = [0 0 2 8 16 25 31 37 40];
```

2. Fit *parametric splines* $S_x(t)$ and $S_y(t)$

```
t = 1:length(x);  
tvals = linspace(t(1), t(end), 500);  
xvals = spline(t, x, tvals);  
yvals = spline(t, y, tvals);  
  
for i = 1:length(tvals)/5  
plot(xvals(1:5*i), yvals(1:5*i), 'LineWidth', 2)  
drawnow  
end
```

3. Update your selection of points to get a better fit.





4. Rotate the silhouette around the y -axis using the following code.

```
theta = 0:pi/20:2*pi;  
r = xvals;  
z = yvals;  
xx = bsxfun(@times,r',cos(theta));  
yy = bsxfun(@times,r',sin(theta));  
zz = repmat(z',1,length(theta));  
figure, surf(xx,yy,zz)
```

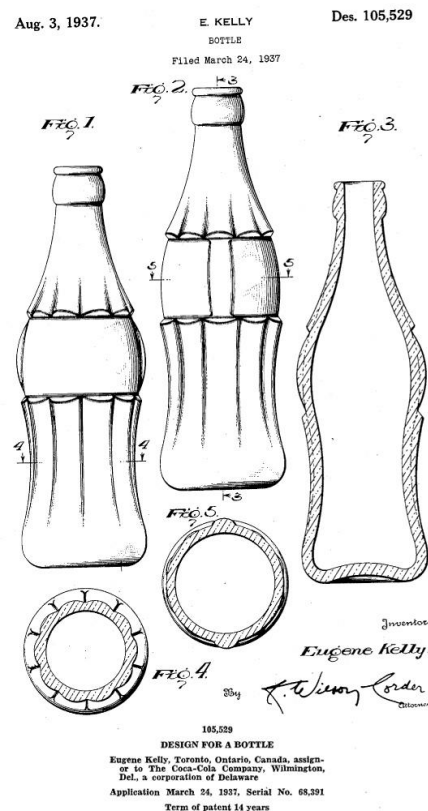


Assignment: Adjust Design to Meet Specifications

Topic: Numerical Integration

Calculate the volume of your bottle and then rescale your bottle so that it holds 6.5 ounces of fluid like the original coke bottle did. If the wall of your bottle has to be 2mm thick, estimate how much material is needed to create your bottle?

Save your bottle as an STL file and print it.



Recall from calculus that if a curve is given by parametric equations

$$x(t) = S_x(t) , \quad y(t) = S_y(t)$$

then the volume enclosed your contour curve is

$$V = \pi \int_{t_1}^{t_n} [S_x(t)]^2 S'_y(t) dt = \pi \sum_{i=1}^{n-1} \int_{t_i}^{t_{i+1}} [S_x(t)]^2 S'_y(t) dt$$



MATLAB

```

sx=spline(t,x);
xvals=ppval(sx,tvals);
sy=spline(t,y);
yvals=ppval(sy,tvals);
coefs2=sx.coefs;
coefs1=sy.coefs;

V=0;
for i=1:length(t)-1

fV=@(z) (coefs2(i,4)+coefs2(i,3).*(z-t(i))+coefs2(i,2).*...
(z-t(i)).^2+coefs2(i,1).*(z-t(i)).^3).^2.*(coefs1(i,3)+...
2*coefs1(i,2).*(z-t(i))+3*coefs1(i,1).*(z-t(i)).^2);

V=V+quad(fV,t(i),t(i+1));
end
V=V*pi;
```





First Print

Students 3D computer models were printed by converted to stl files via `stlwrite(filename, varargin)`





Assignment: Optimal Redesign

Topic: Univariate Minimization

Determine the minimal the amount of material needed to create a bottle that holds 6.5 ounces of liquid with a wall thickness of 2 mm. Is the optimized bottle unique? Use what you know about the bottle that uses the least amount of resources to redesign your bottle to use fewer materials than your original design.

I attempted to print the redesigned bottle in bamboo-fill ...



Assignment: Exhibition

Topic: Writing Across the Curriculum

Prepare a short statement to be displayed with your bottles that describes the inspiration for your designs as well as the key features you decided to keep in your redesign. Be sure to include the amount of materials needed for the original design as well as the amount you saved in the redesign.



 Assignment: Message in a Bottle**Topic:** Writing Across the Curriculum

Write a reflection on the project. Some questions you may address are: What were some of the challenges of the bottle design project? What did you learn about the roles mathematics and scientific computing play in design? What is the feasibility of optimal design and how does it limit (or expand) one's creativity? Are 3D computer models sufficient or is there an added benefit to generating a tangible version of your bottle via the 3D printer?



Pros and Cons

Pros

1. Open ended project
2. Concrete utilization methods learned in class
3. Creativity / Personalization
4. Memorable demonstration of how mathematics plays a role in design
5. Natural Conversation about Error
6. Incentives students to engage in class

Cons

1. Too Many Bottles!
2. Students didn't get to interact enough with the printer
3. Unmotivated students
4. Unrealistic Designs
5. Some bottles didn't hold water
6. Coding was too easy or too hard



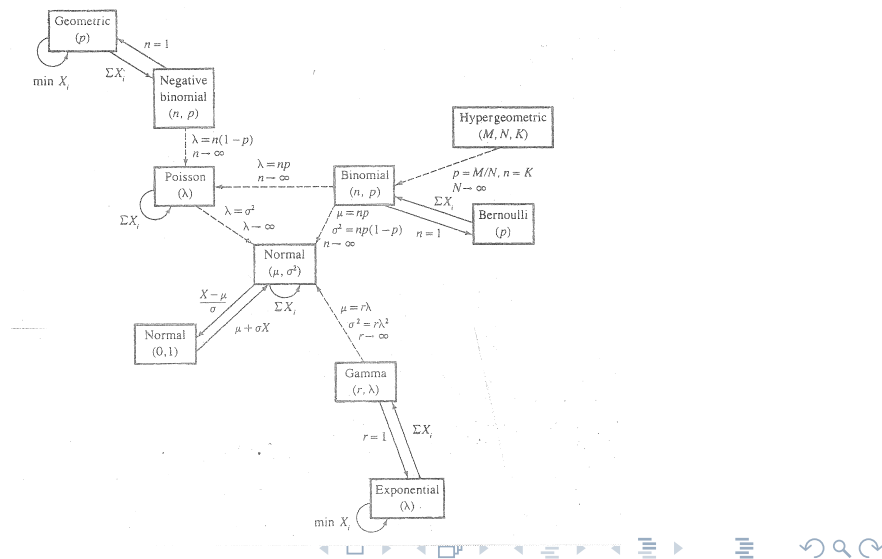
Mathematical Statistics: Probability Theory

Students Applied Math Majors and Computer Science majors

Prerequisites Calculus I &II, Multivariable Calculus

- Topics**
1. Probability
 2. Discrete and Continuous Random Variables
 3. Distributions

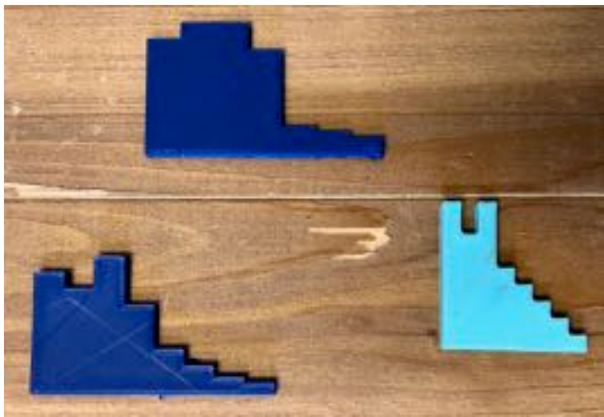
Idea Random samples and understanding the relationships between the distributions





Random Sample Chips

Distribute chips and have students group together by *similar* looking distributions.



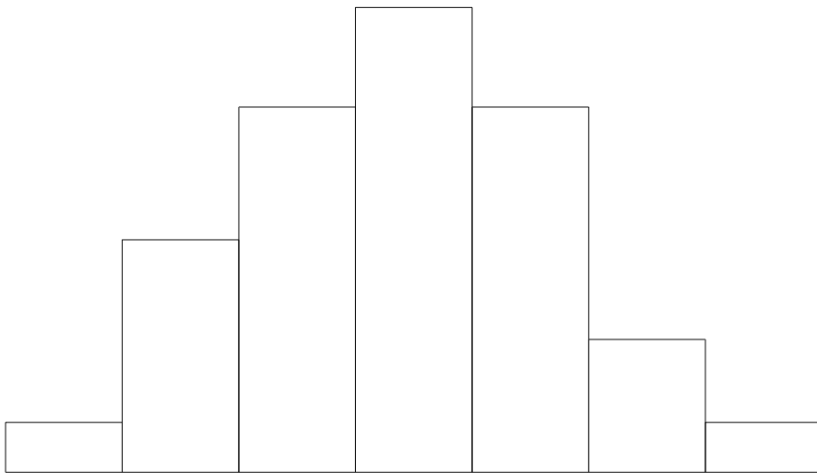
1. What are the chips representing?
2. What assumptions are being made?
3. Are the random samples from discrete or continuous distributions?
4. Are there subgroups within your group? (e.g. Geometric and Negative Binomial)

How to make Random Sample Chips

1. Create a random sample in R and plot its histogram.

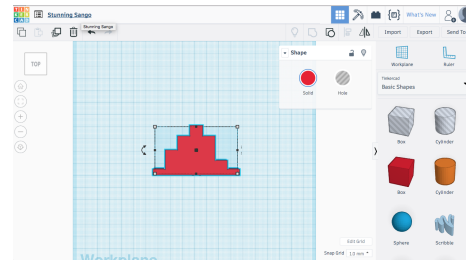
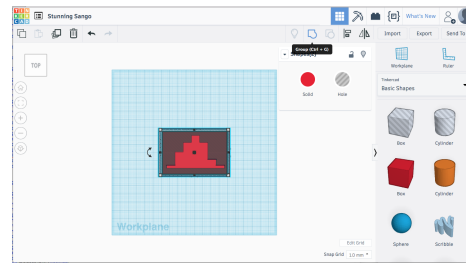
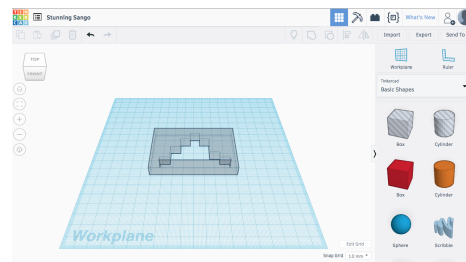
```
hist(rbinom(100,30,0.5), axes = FALSE, xlab='', ylab='', main='')
```

2. Export the graph as an .svg file.



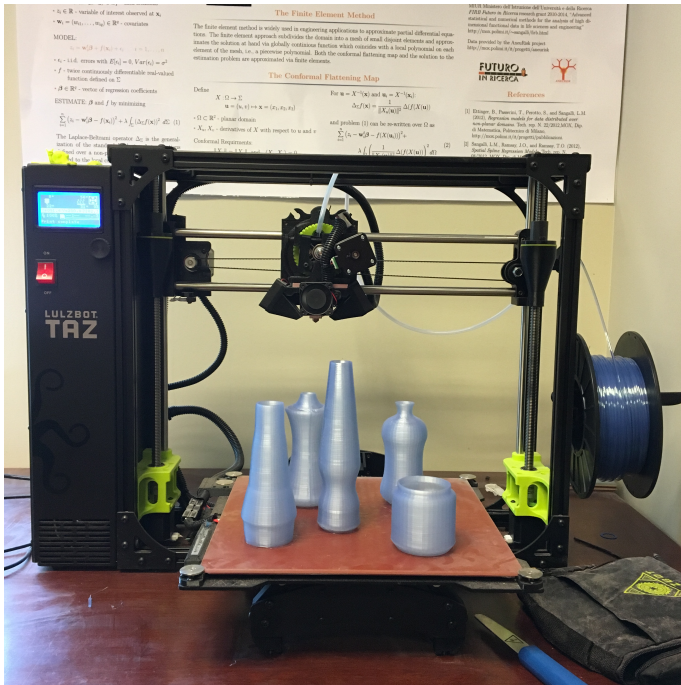
How to make the chips

3. Load it into Tinkercad and select to view it as hole.
4. Intersect the object with box.
5. Group the objects and export the final product as an .stl file





Thank You



EMORY

Fund for Innovative Teaching
(FIT) Grant Proposal for
Academic Year 2016-17

Questions?



