Component-Based Model Reduction for Industrial Scale Problems

Dr. David Knezevic, CTO

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1. Context
2. SCRBE
3. Coupled SCRBE/FE for Nonlinear

Context

In many industries, engineers need to model large, complex systems:

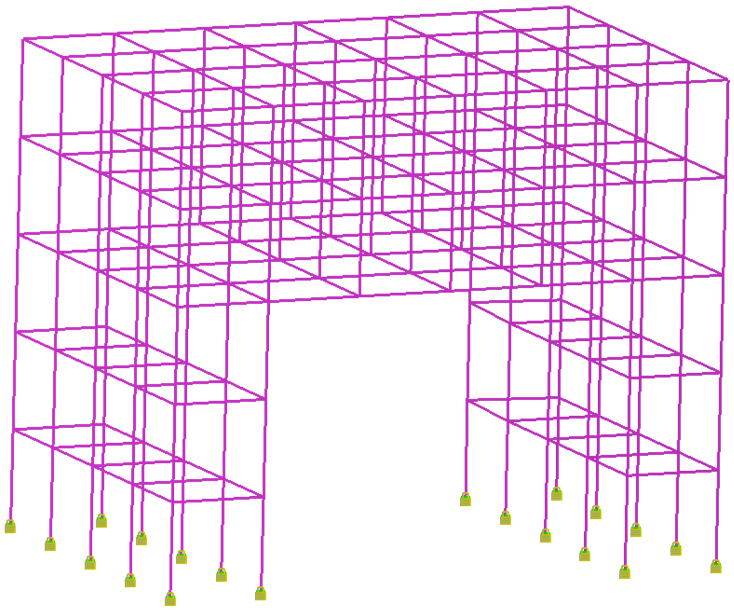
- Ships
- Oil platforms
- Mining machinery
- Wind turbines
- Aircraft
- …

Detailed modeling of large systems with FE is considered impractical in most circumstances

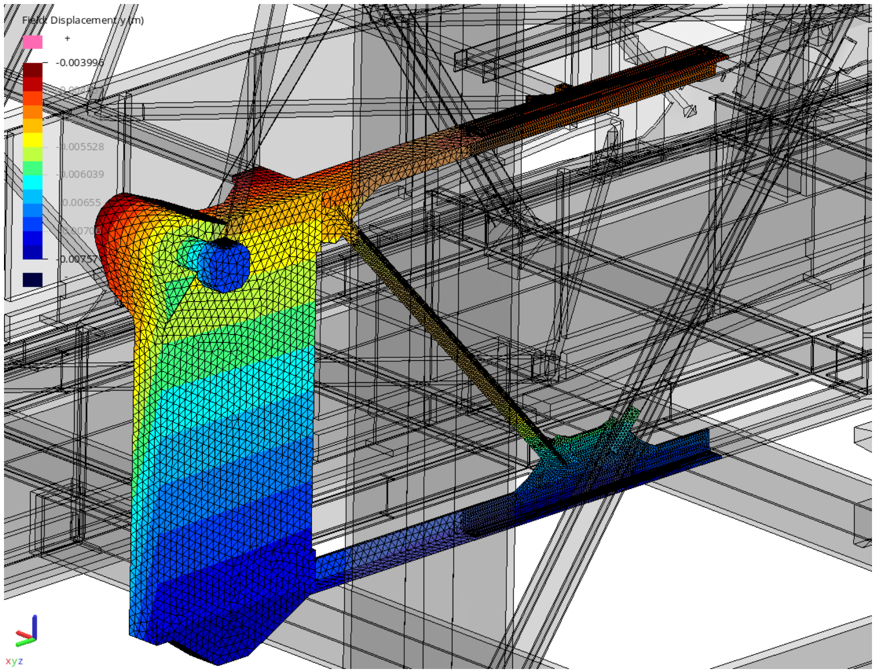
Context

Two traditional solutions for this:

Low fidelity models:
e.g. beam-element representations, or very coarse meshes



Localized detailed models:
e.g. detailed 3D model of a subregion, with appropriate BCs



Context

These approaches are widely used, but they have major drawbacks:

Low fidelity models:

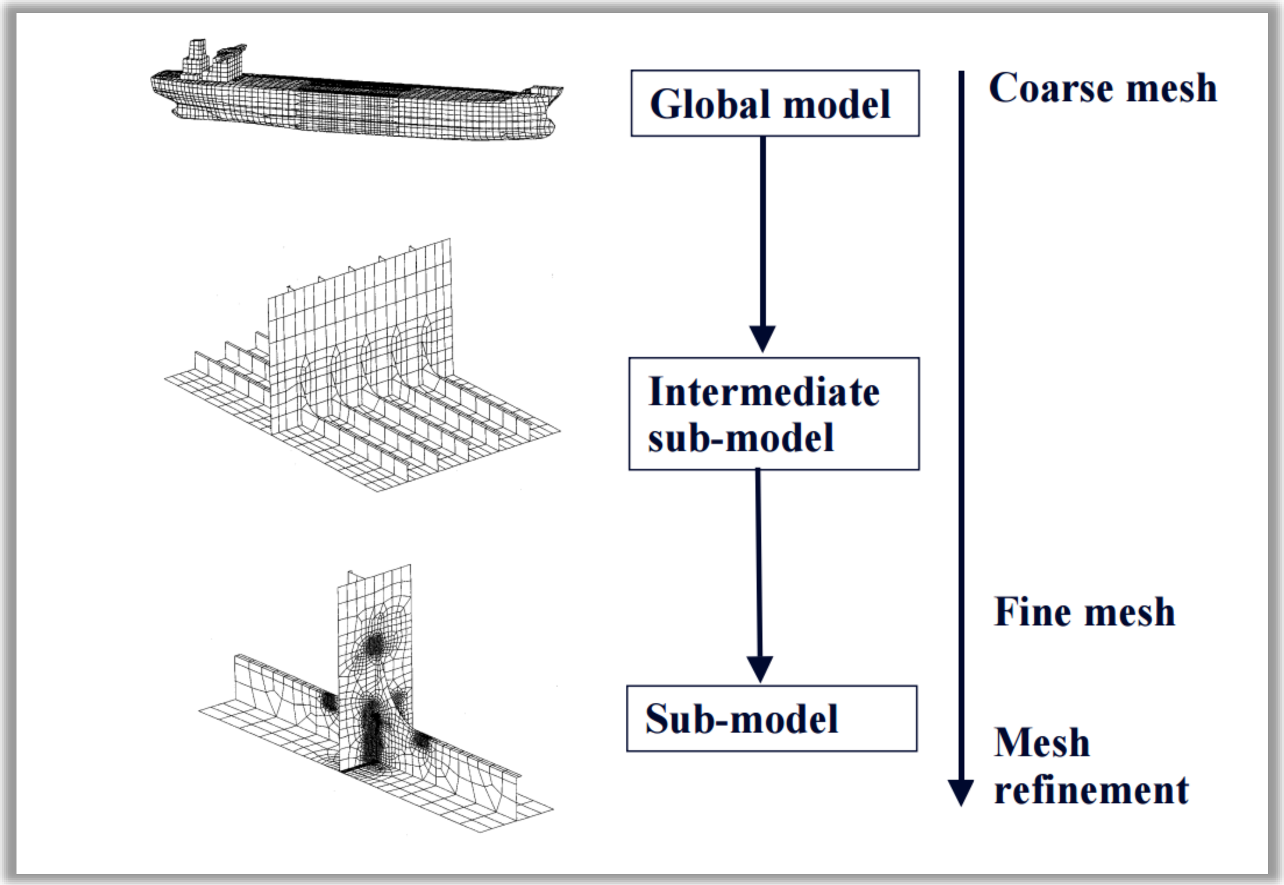
- Provide limited engineering insight
- Cannot represent the full range of systems (e.g. beam models are only good for “beam-like” structures)

Localized detailed models:

- Local models of course cannot capture global effects
- Difficult to impose load cases, have to infer BC data
- Isolating local model can introduce extra assumptions

Context: Standard Approach for Hulls

Standard workflow: DNV-RP-C206



The standard workflow involves juggling many models. **Slow and error prone!**

Akselos's Mission

Akselos

Provide computational tools that enable efficient, detailed, reconfigurable analysis of large systems.

We achieve this via:

- Component-based parametrized model reduction (SCRBE)
- Cloud-based simulation back-end (GCE)
- GUI for assembling models, visualizing results
- HPC software stack (libMesh, PETSc)
- The full range of element types (beams, shells, solids)
- Full FE solver suite for linear and nonlinear analysis
- Coupled SCRBE/FE solver for fast "locally nonlinear" solves

(Try this out for free at community.akselos.com)

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SCRBE: Static Condensation Reduced Basis Element

(Huynh, Knezevic, Patera, “A Static Condensation Reduced Basis Element Method: Approximation and A Posteriori Error Estimation", M2AN, 2012.)

Several related approaches in the literature:

- Maday, Ronquist: RB Element Method, Lagrange multipliers to “glue” non-conforming approximations
- Chen, Hesthaven, Maday: Seamless Reduced Basis Element Method, DG formulation to avoid Lagrange multipliers
- Nguyen: Multiscale Reduced Basis method, similar to MsFEM but uses RB for cell problems
- Iapichino, Quarteroni, Rozza: Reduced Basis Hybrid method, couples components via coarse grid and Lagrange multipliers
- Heinkenschloss et al.: Balanced truncation model reduction on subdomains coupled to FE
- Craig-Bampton, Component Mode Synthesis: Widely used in industry for non-parametrized eigenvalue or dynamic problems

SCRBE: Static Condensation + RB

Core idea: Consider a system with two components, Ω_1 and Ω_2, connected on port P:

$$\begin{bmatrix} A_{P,P} & A_{P,\Omega_1} & A_{P,\Omega_2} \\ A^T_{P,\Omega_1} & A_{\Omega_1,\Omega_1} & 0 \\ A^T_{P,\Omega_2} & 0 & A_{\Omega_2,\Omega_2} \end{bmatrix} \begin{bmatrix} \mathbb{U} \\ u_{\Omega_1} \\ u_{\Omega_2} \end{bmatrix} = \begin{bmatrix} f_P \\ f_{\Omega_1} \\ f_{\Omega_2} \end{bmatrix}$$

Solve for the non-port DOFs (SCRBE does this via RB method):

$$A_{\Omega_i,\Omega_i} u_{\Omega_i} = f_{\Omega_i} - A^T_{P,\Omega_i} \mathbb{U}$$

Substitute into the port DOF rows to get a system that involves port DOFs only:

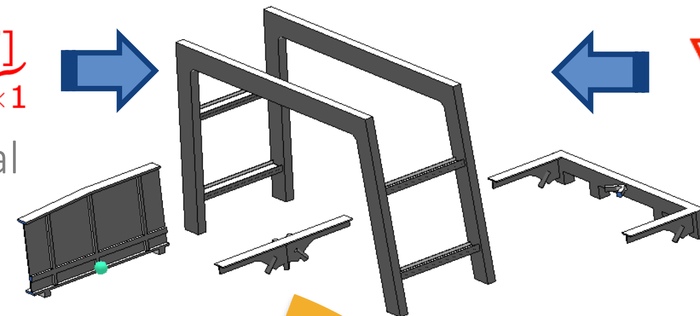
$$\mathbb{A}(\mu)\mathbb{U}(\mu) = \mathbb{F}(\mu)$$

SCRBE

Create/update components (similar to **parameterized** substructuring)

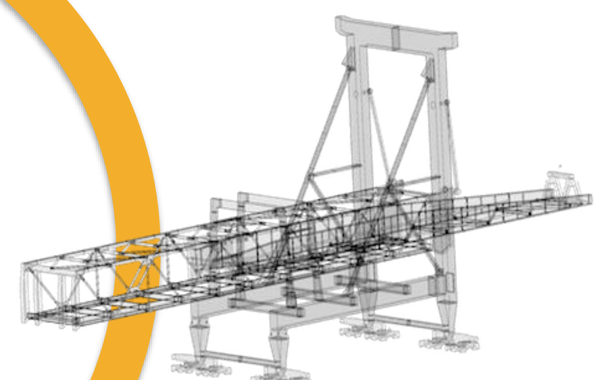
$$\underbrace{[\mathbf{A}]}_{\mathcal{N}\times\mathcal{N}}\ \underbrace{[\mathbf{x}]}_{\mathcal{N}\times 1} = \underbrace{[\mathbf{f}]}_{\mathcal{N}\times 1}$$

Computational Approx.



$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F} = 0$$

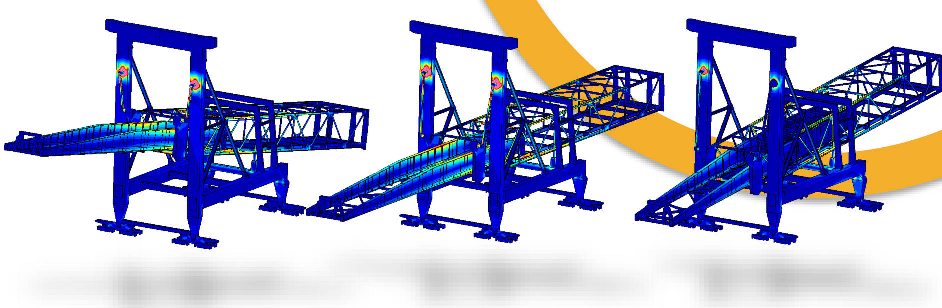
Physics



Assemble model

Set parameters and solve

(Orders of magnitude speedup compared to FE)



Key features of SCRBE:

- Provides major speedup compared to FE for linear PDEs, e.g. >1000x for large-scale problems
- Component-based approach enables very large systems, e.g. >100m FE DOFs
- Efficiently handles systems with many parameters, e.g. >1000
- Permits “topological changes”: add/remove/replace components
- Enables a posteriori error analysis based on residual-based RB error analysis

SCRBE: Port Modes

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Classical substructuring typically uses “all” DOFs on a port

Craig-Bampton, CMS (typically used for modal/dynamic) uses truncated port representations

We advocate truncated port representations in SCRBE:

- Retains good accuracy in practice (typically <1% error wrt FE, sufficient for engineering purposes)
- Provides larger speedup, reduces data footprint

Several port truncation methods for SCRBE have been published: pairwise methods, empirical, “optimal modes”

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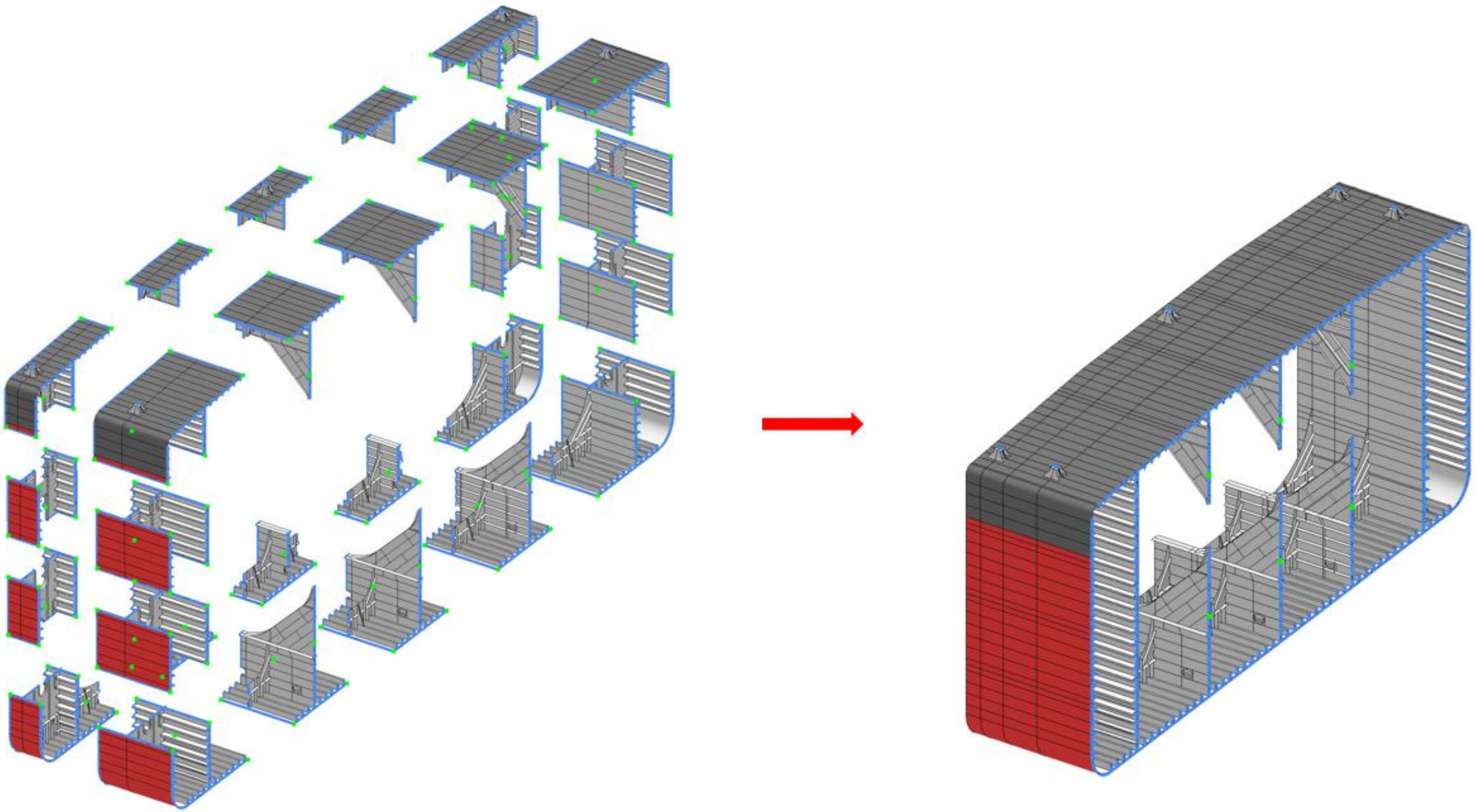
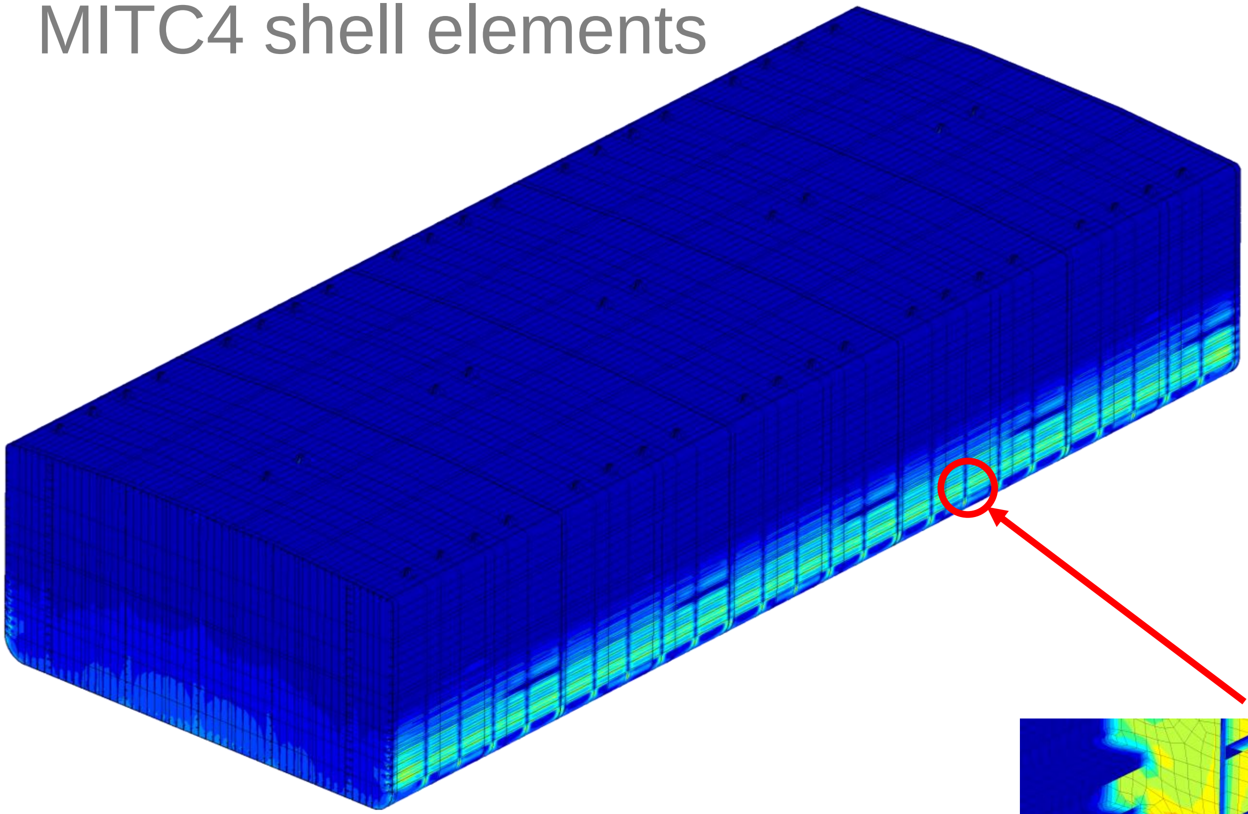
Example: SCRBE model of "Aframax" (type of tanker) ship hull

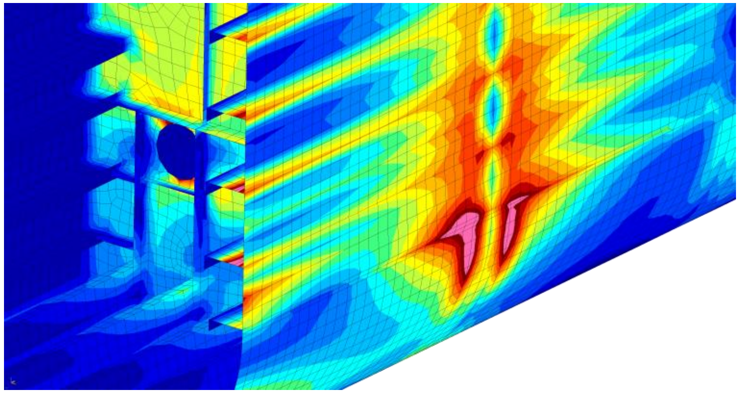
Ship Hull



Ship Hull

- Cargo segment of hull, ~17 million FE DOFs
- MITC4 shell elements





- SCRBE solve time: ~2 seconds
- Accuracy validated for smaller cases: <1% L_2 error wrt FE

Ship Hull

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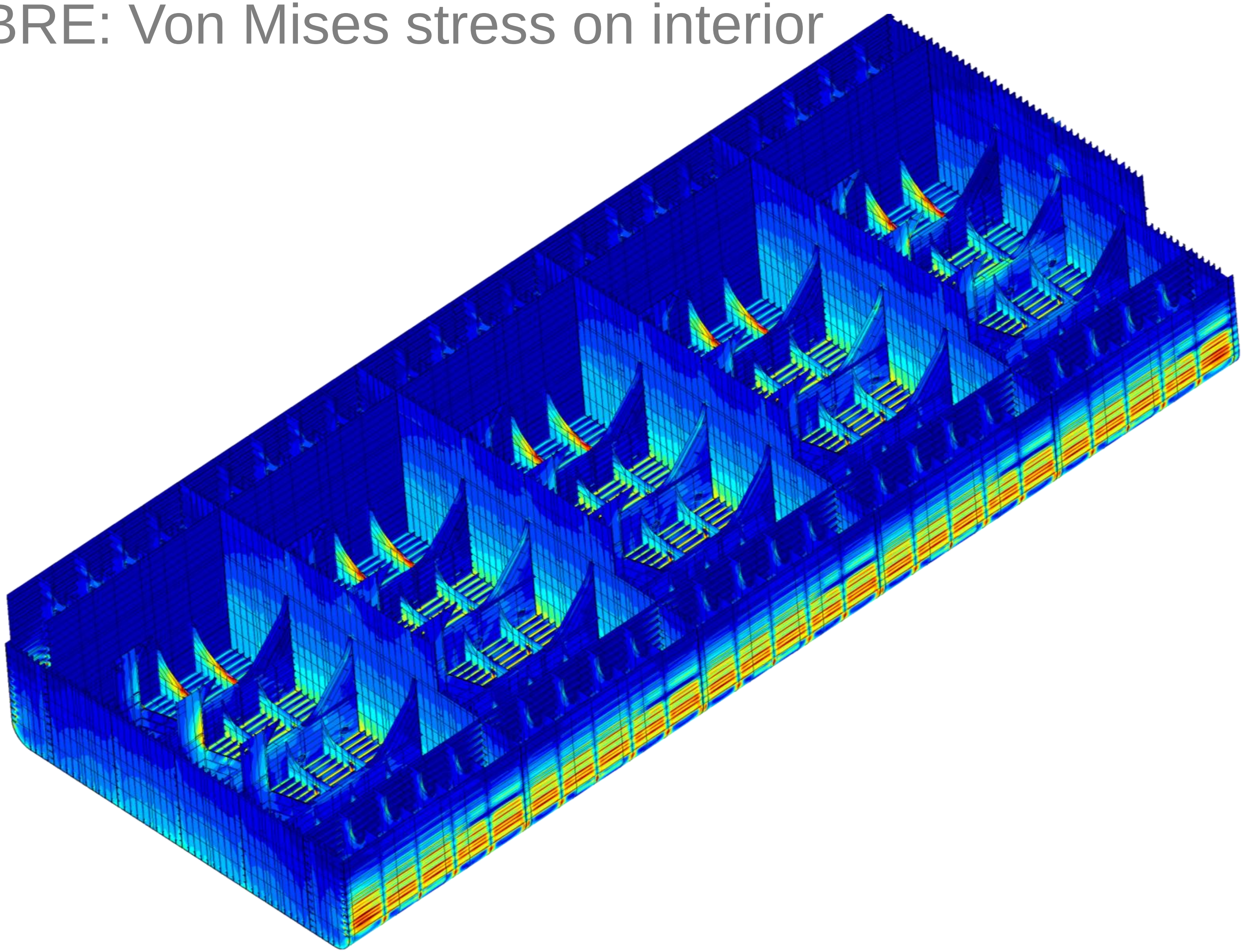
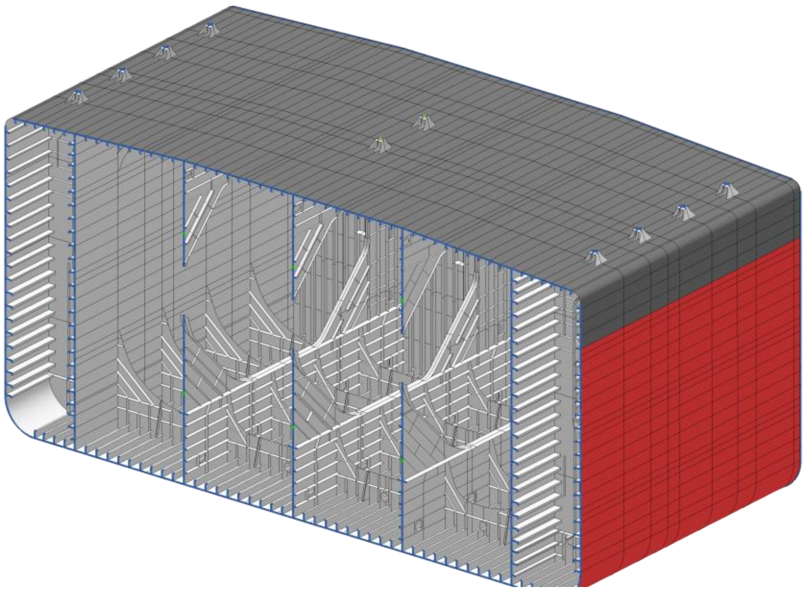
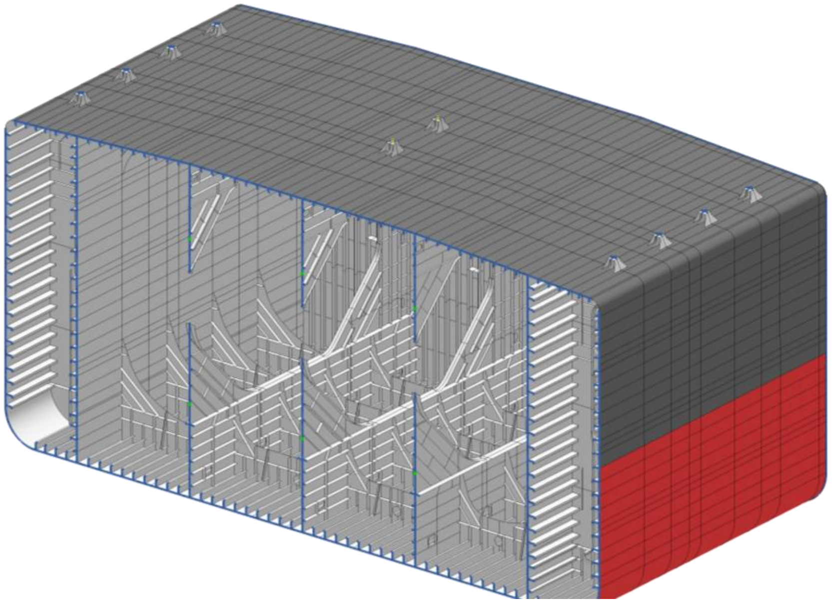
SCBRE: Von Mises stress on interior



Impose different load cases by configuring loads on each component

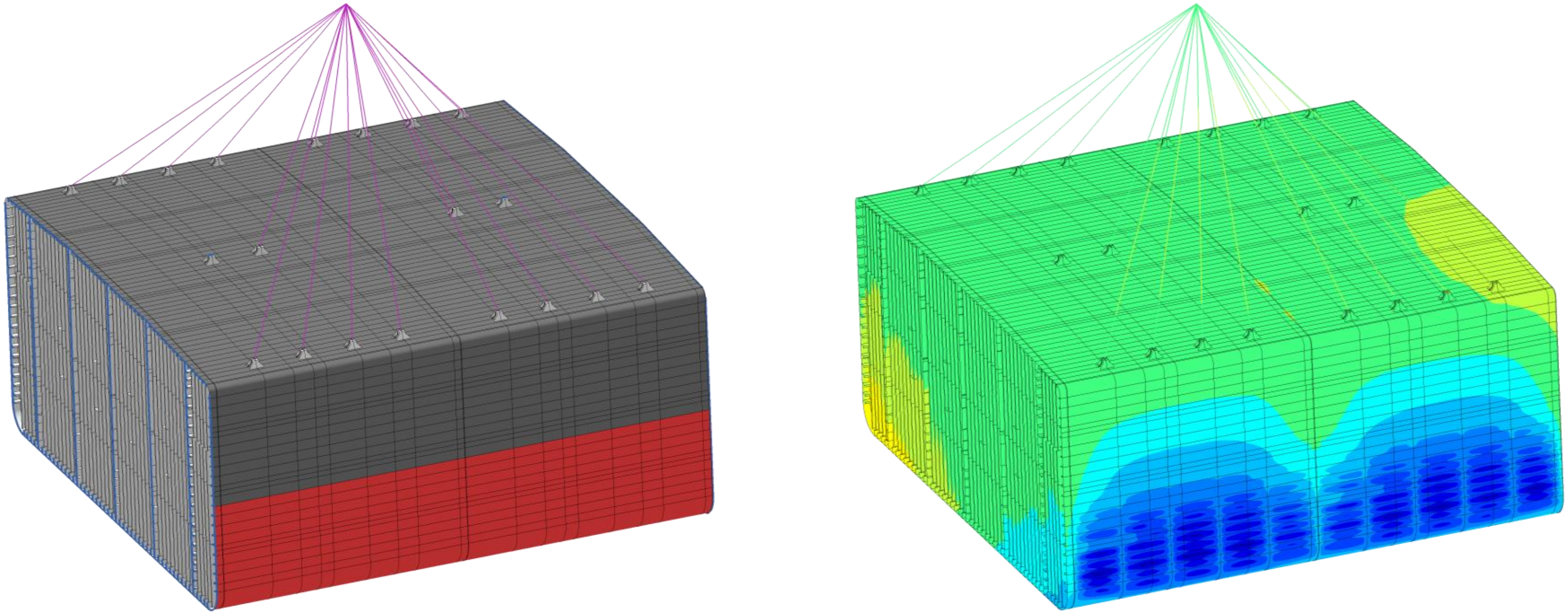


Heavy-ship condition



Light-ship condition

Typically beam elements connected to masses are used to provide a (coarse) model of top-side structure



Ship Hull

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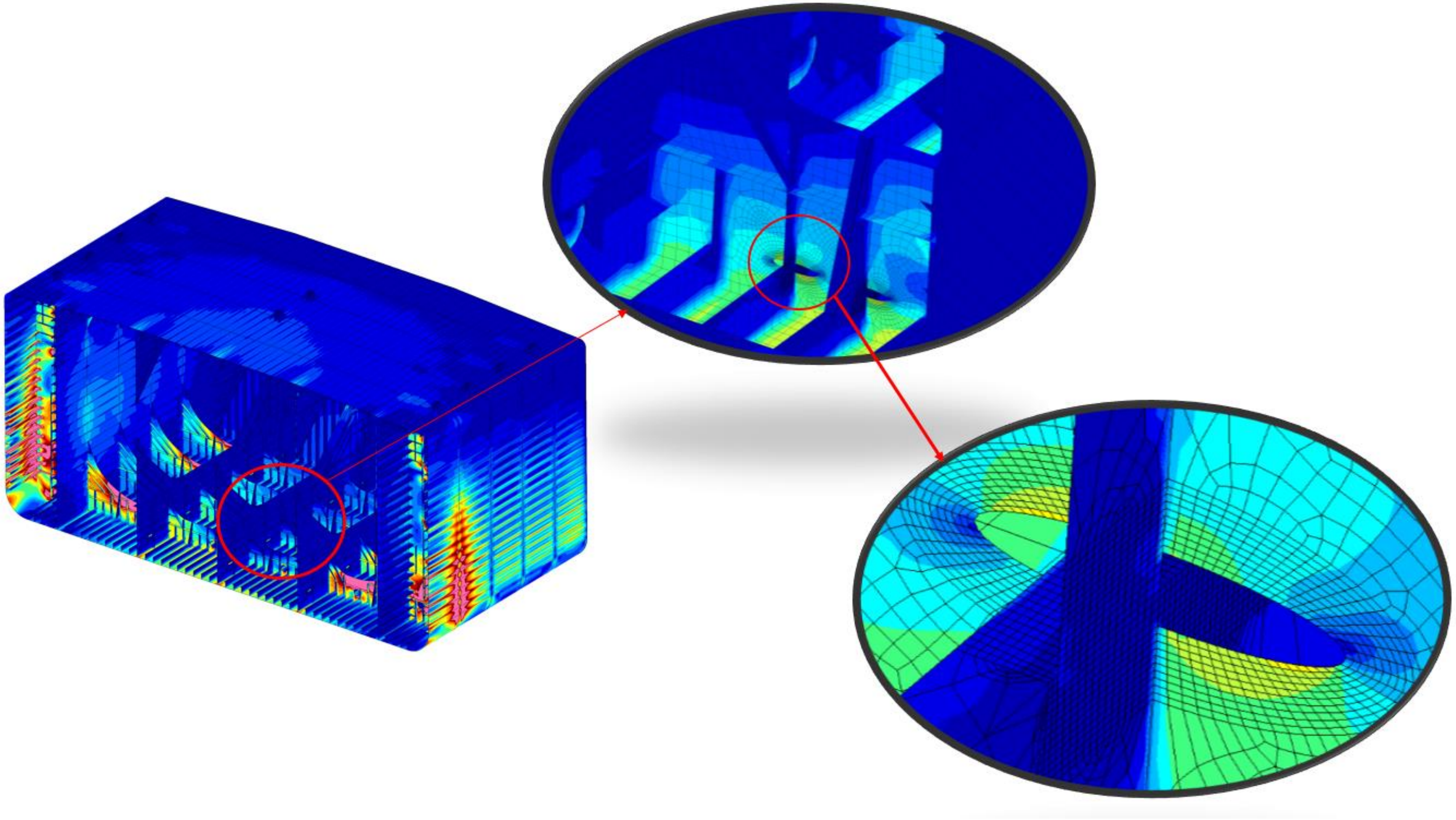
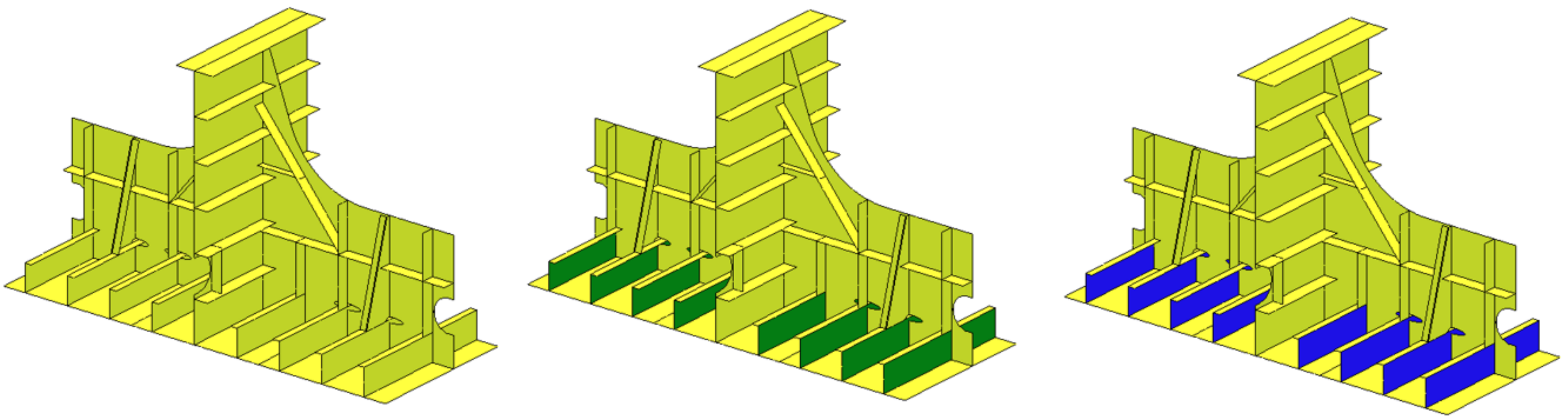
"Online" solve cost is independent of number of FE DOFs, hence we can easily include highly detailed meshing



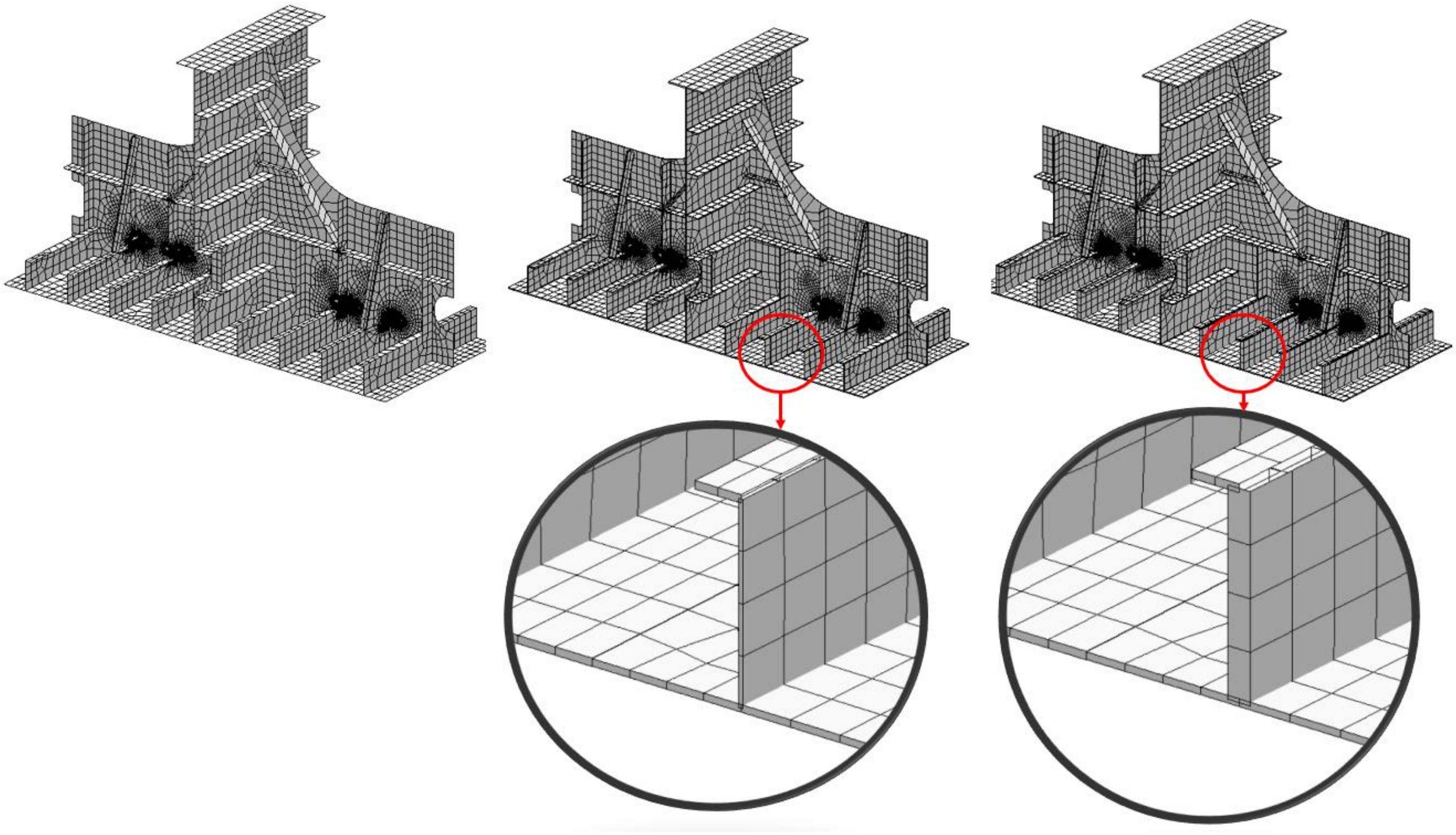
Ship Hull

Young's modulus, Poisson ratio, density are parametrized (different colors correspond to different grades of steel)

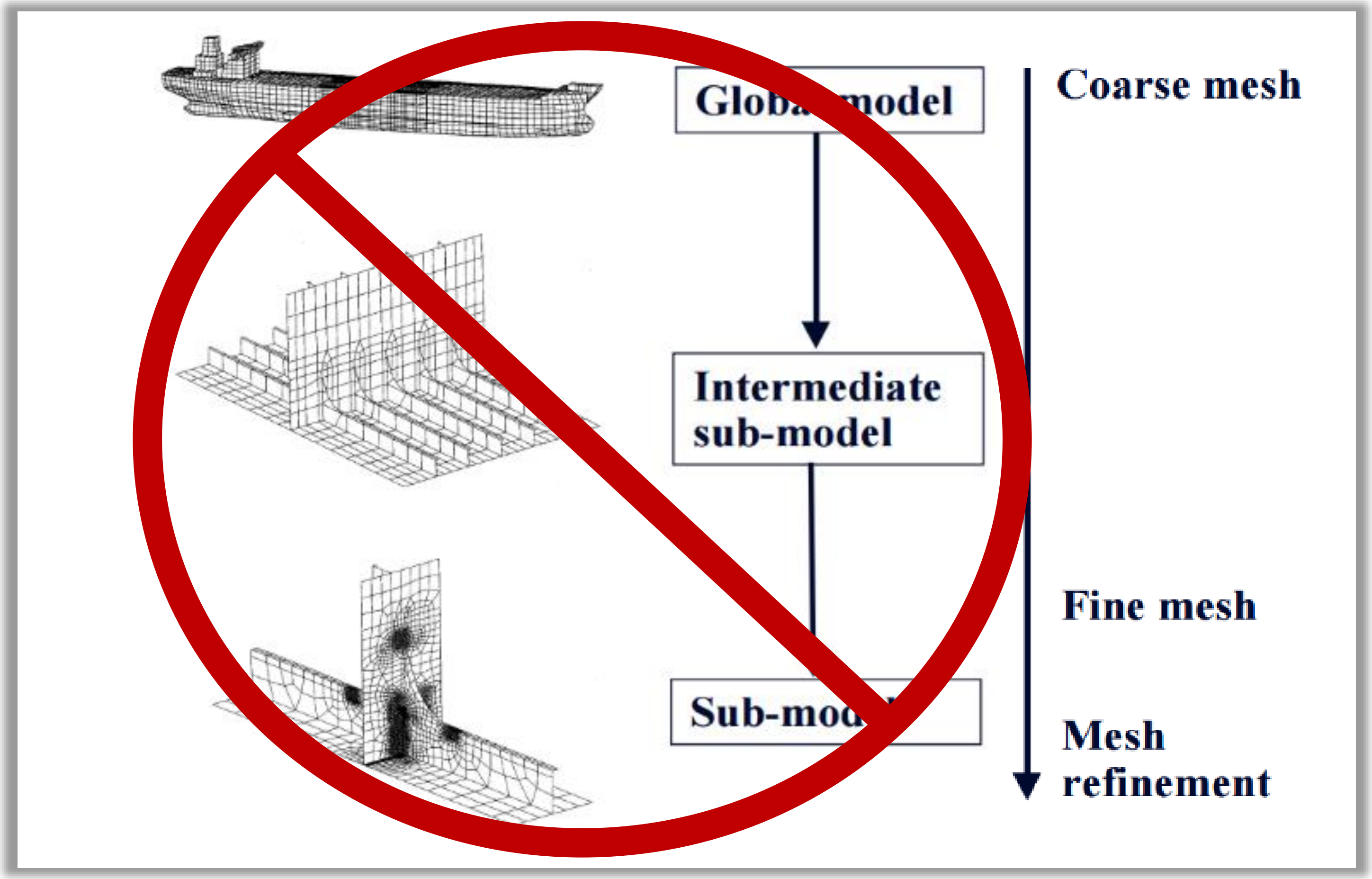


Ship Hull

Shell thickness can be parametrized too (a "geometric parameter" that doesn't require EIM!)

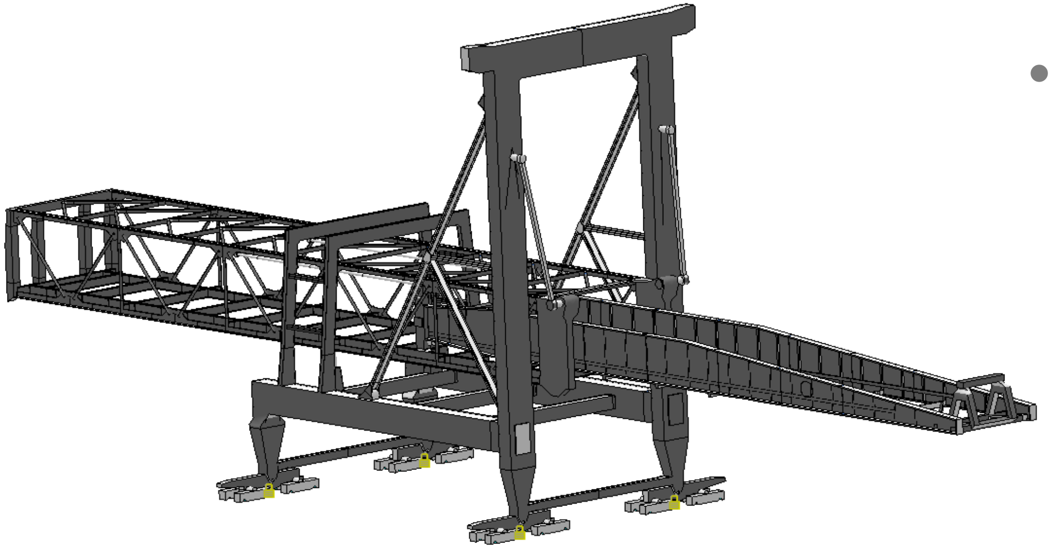


SCRBE: A Better Workflow for Hulls!



SCRBE enables a global parametrized hull model with a fine mesh everywhere

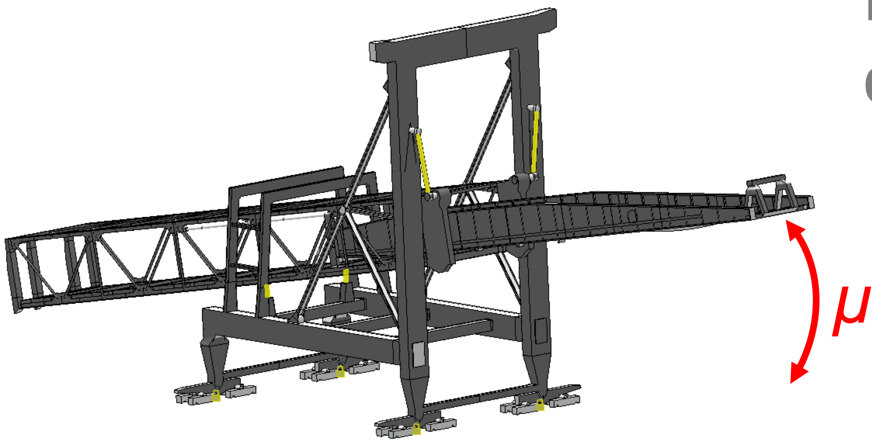
Shiploader

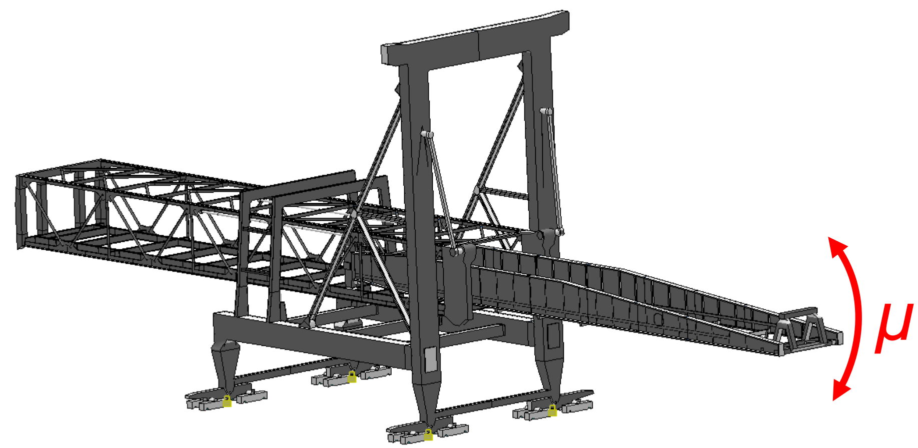


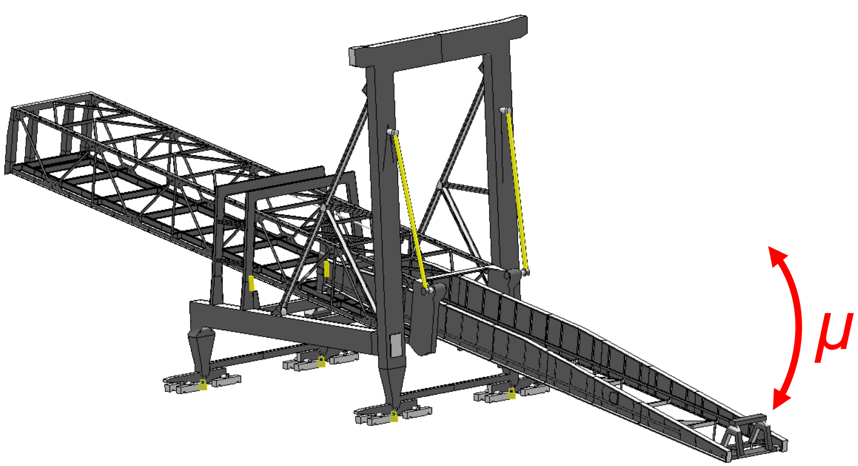
- P2 TET mesh
- 20,001,330 FE DOFs
- 166 components
- 287 connections
- SCRBE solve time: ~1s
- Accuracy validated for smaller cases: $<1\%$ L_2 error wrt FE

Shiploader: Angle Parameter

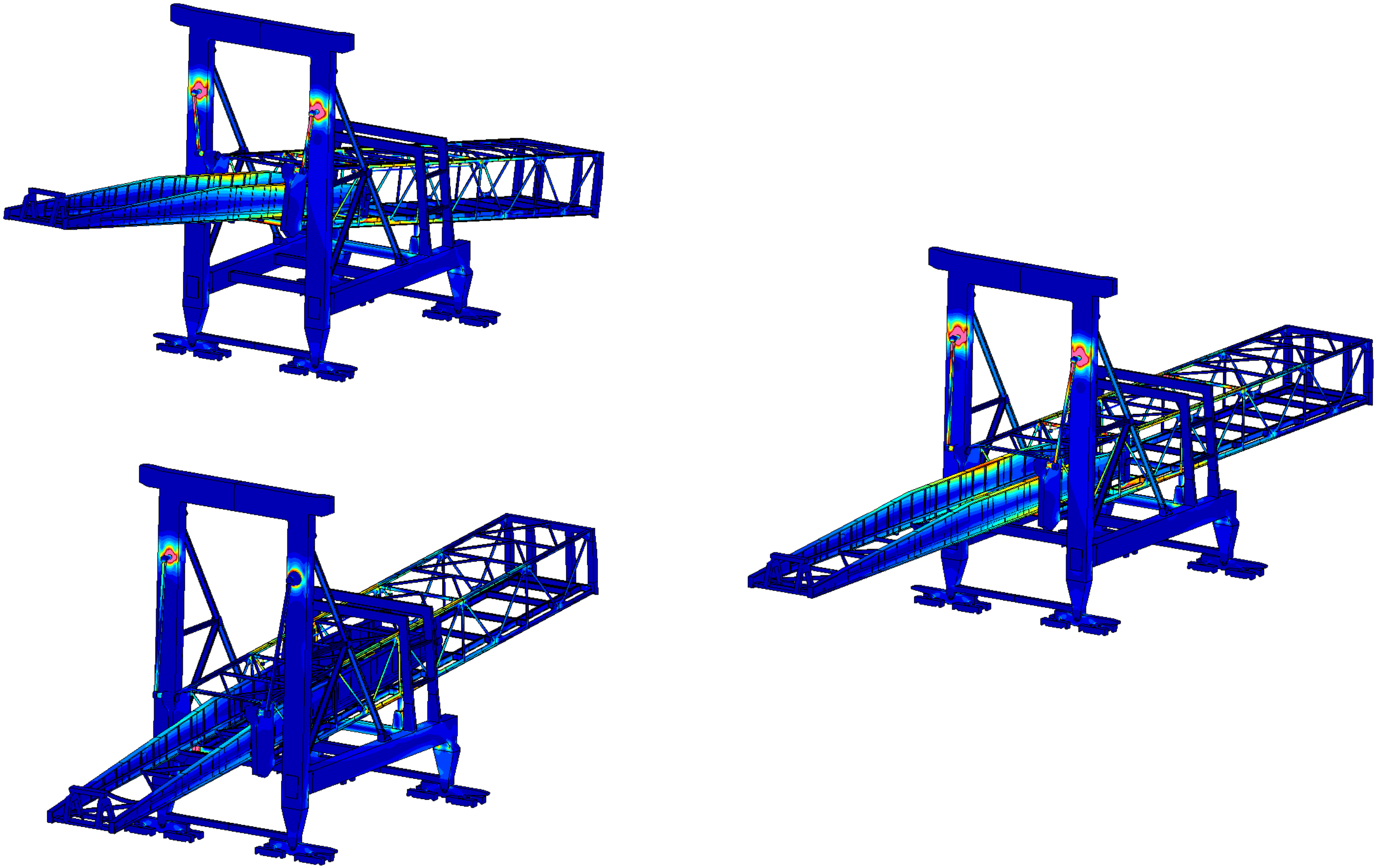
EIM within components
enables geometric parameters





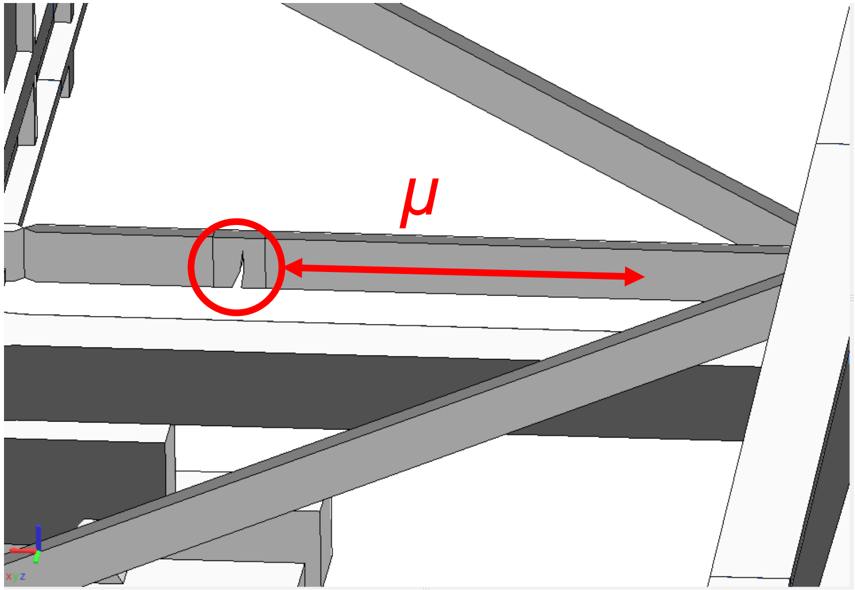


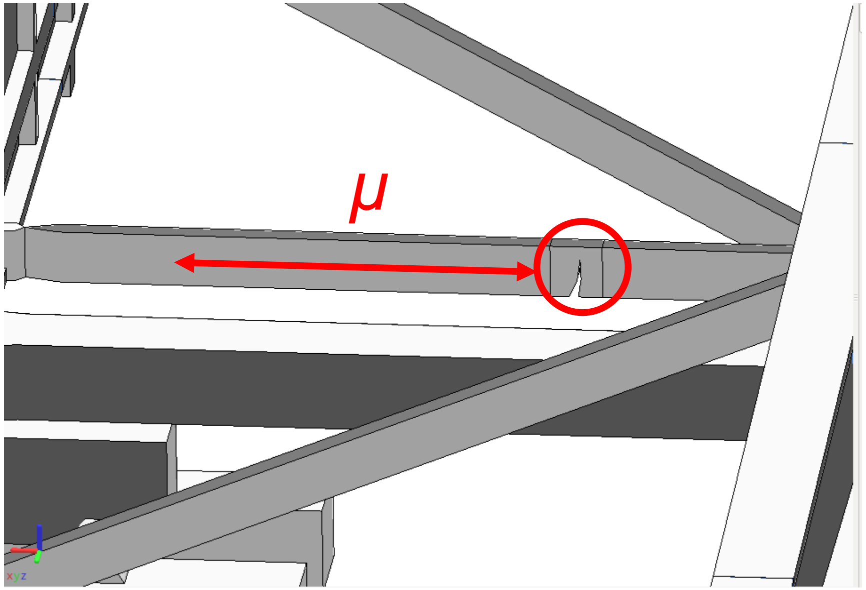
Shiploader: Angle Parameter



Shiploader: “Crack Location” Parameter

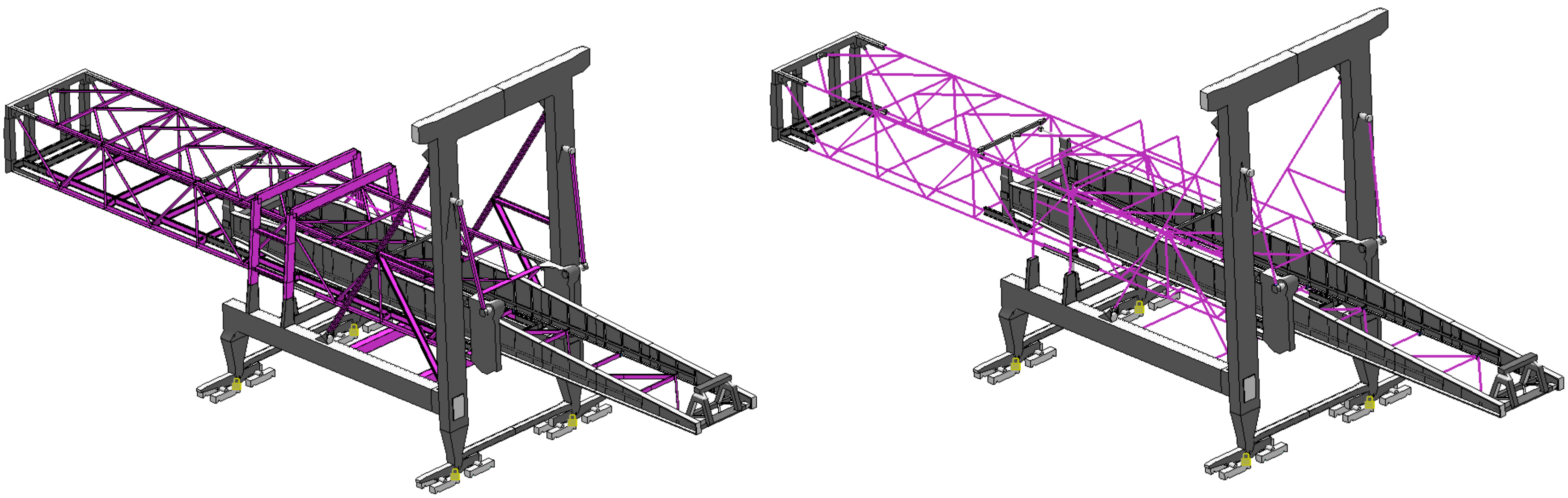
EIM within components
enables geometric parameters





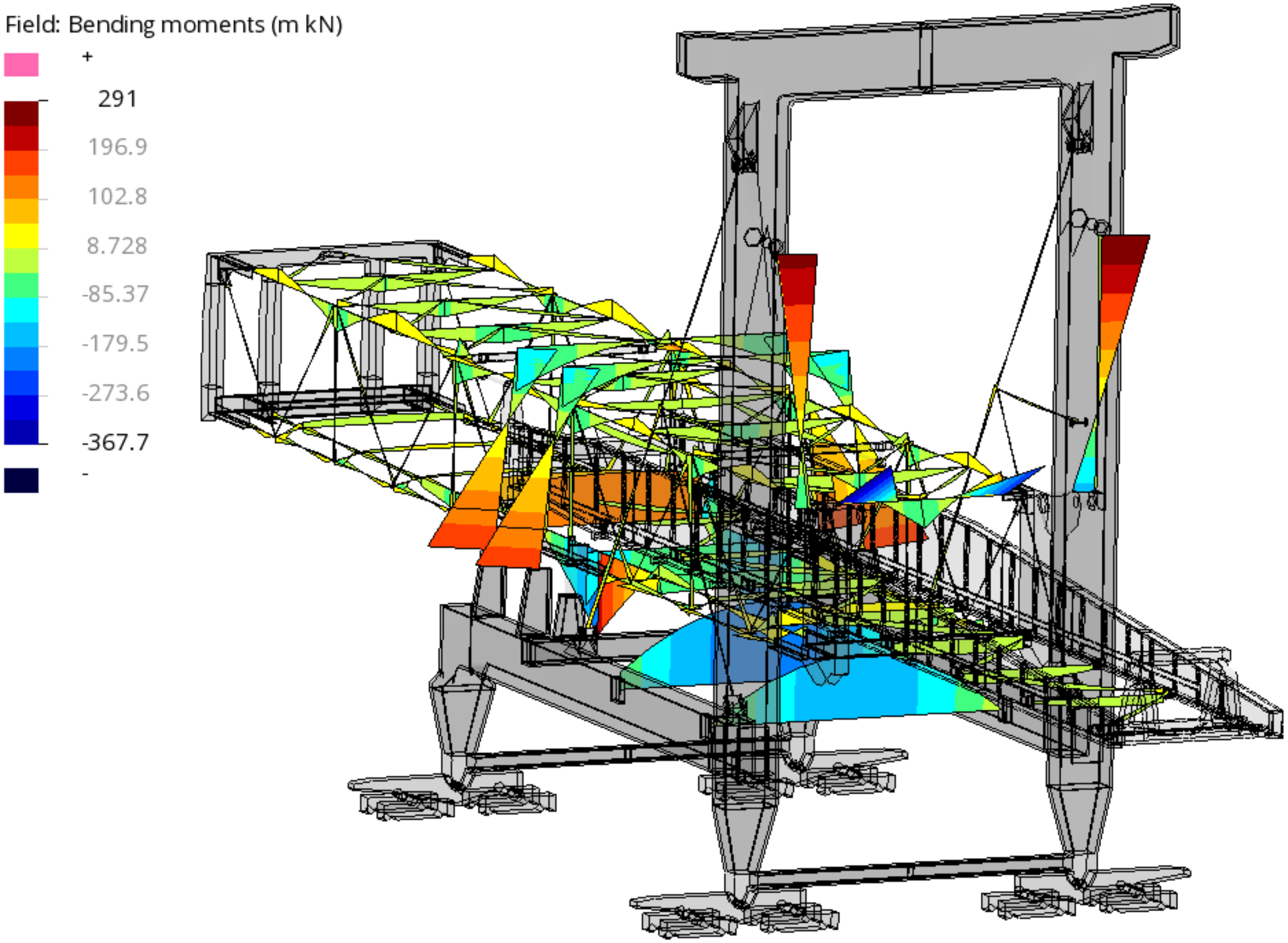
Shiploader: Hybrid 1D/3D

We can connect 3D components to 1D beams on ports, hence use beam elements in appropriate regions



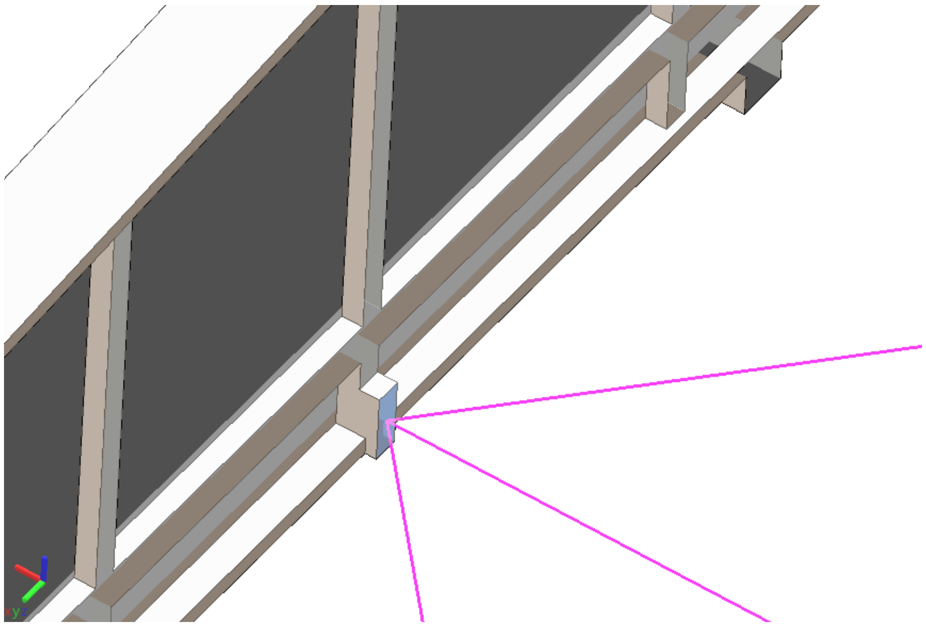
Shiploader: Hybrid 1D/3D

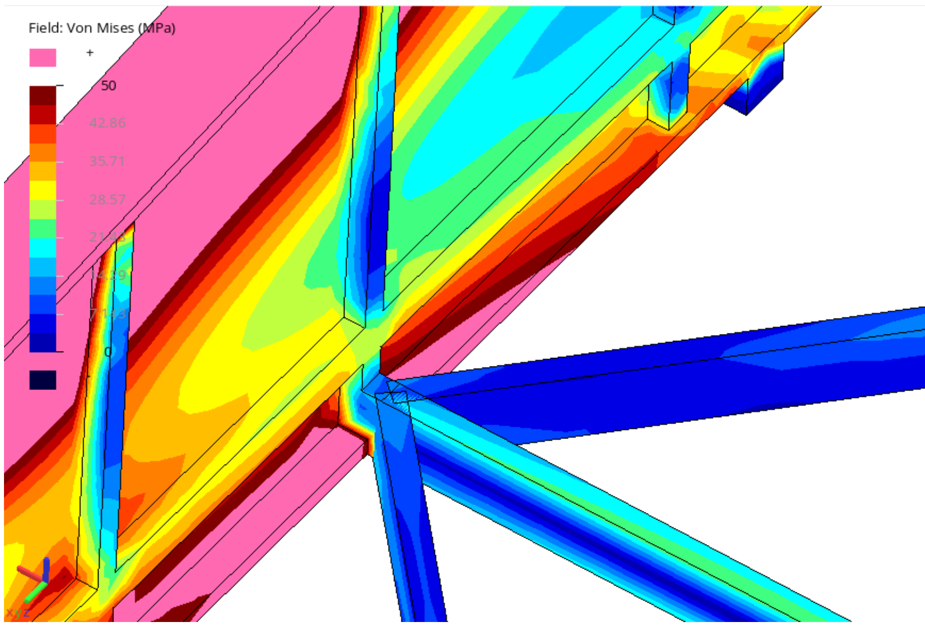
Moment diagrams on beam elements



Shiploader: Hybrid 1D/3D

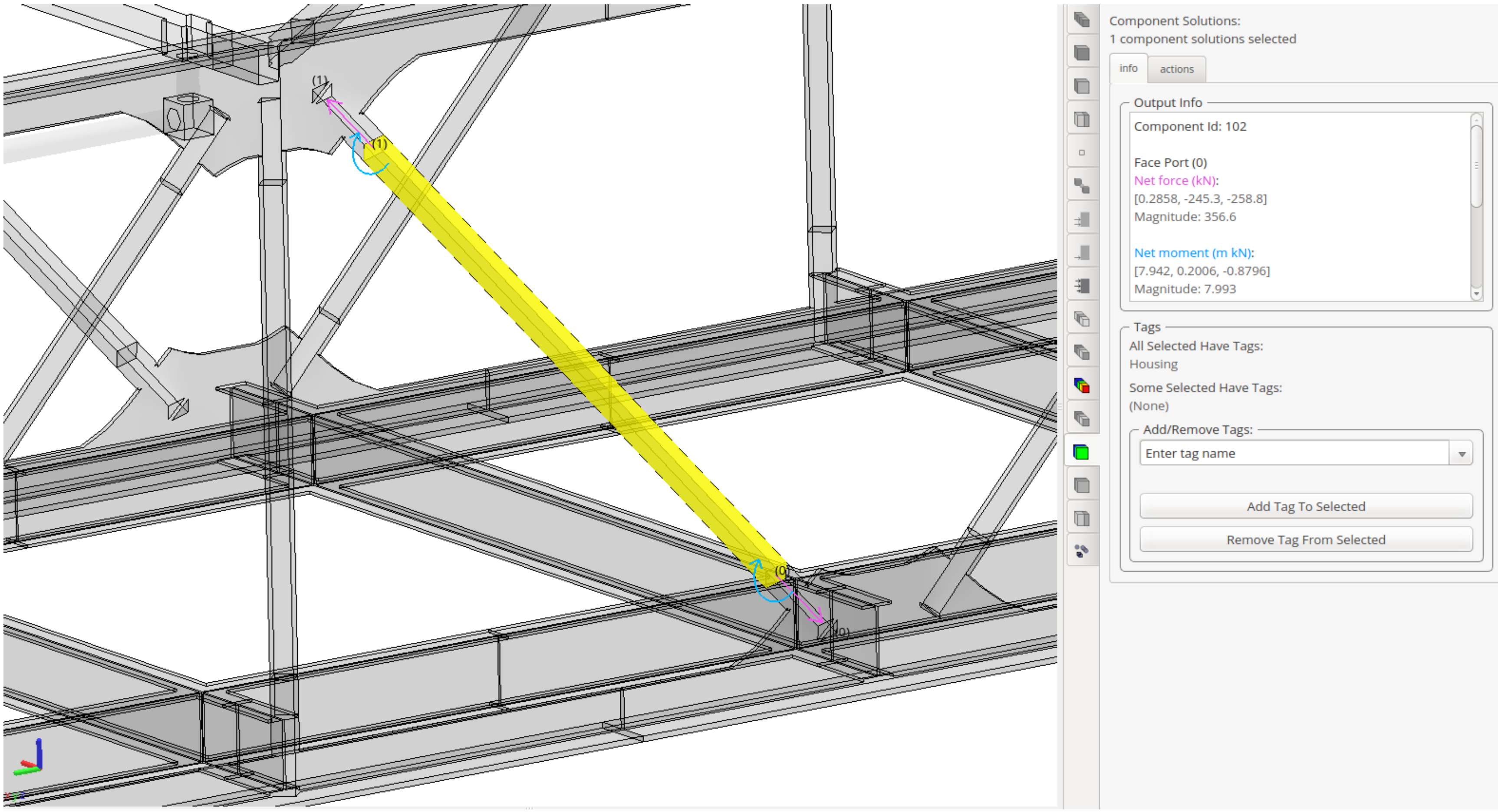
Stresses on beam elements and 3D SCRBE components





Shiploader: Hybrid 1D/3D

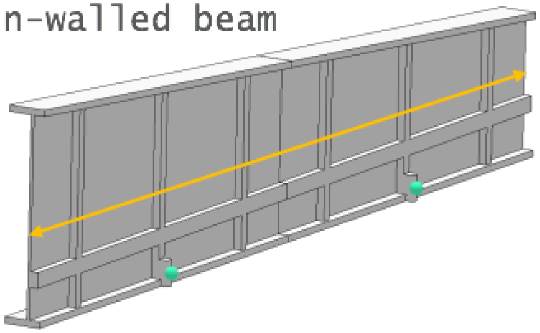
"Flux integral output" (actually stress integral) approach gives forces/moments on ports of 3D components



Shiploader: Engineering Standards

Forces/moments enable standards-based analysis (cf. Kotsalos EPFL Masters thesis)

Thin-walled beam



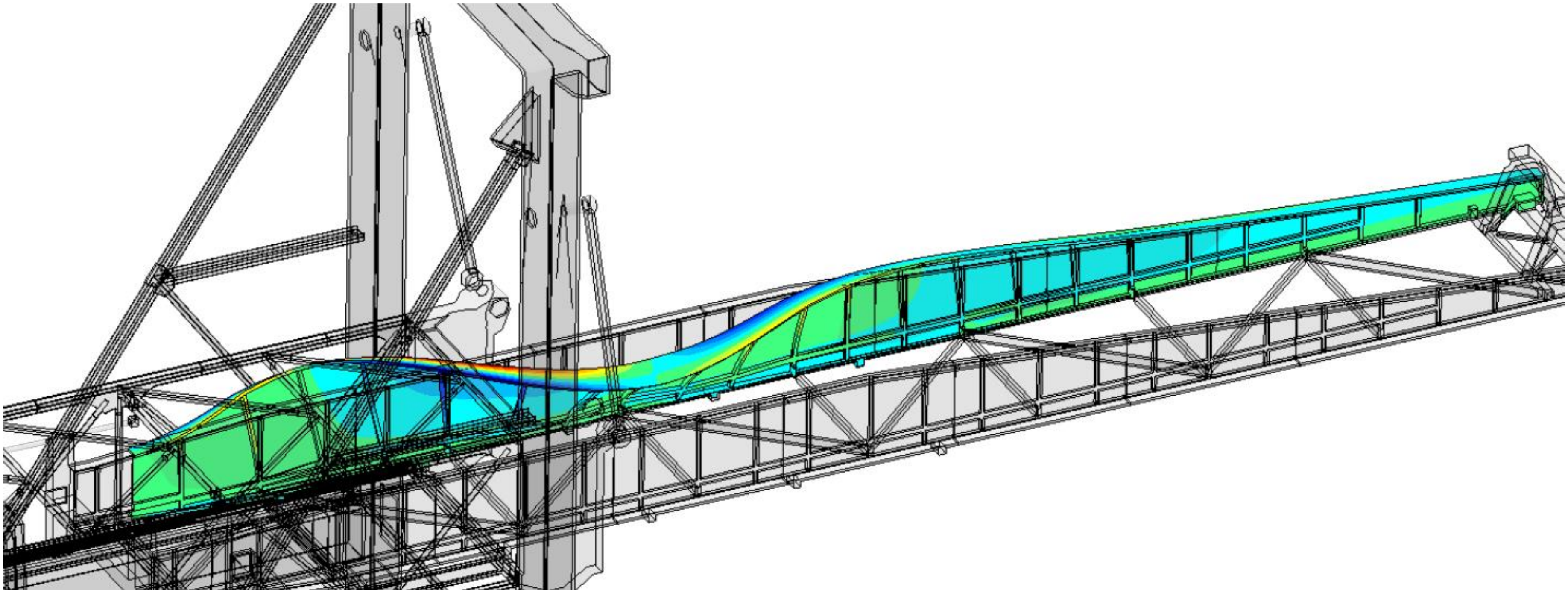
Truss

```
ID: superbeam[1] | Type: Thin-walled Beam Element
------------------------------------------------------------------------------
Cross-section type : Non-standard shape
Cross-section class: class 1
Analysis of the Web based on: Buckling Strength of Plated Structures
                                 DNV-RP-C201 (2010)
Analysis of the Flanges: We check only webs
--
Cross-section properties:
A: 1000.81 cm^2 | It1: 2680513.35 cm^4 | It2: 75699.53 cm^4
W.el.t1: 37313.32 cm^3 | W.el.t2: 3236.36 cm^3
Number of aligned webs: 1 | D: 220.00 cm | t: 3.16 cm
max W/t (outstand):1.00 | max W/t (internal):1.00
--
Actual Length: 28.37 m
Reduced Length about t1: 28.37 m | about t2: 14.67 m
--
Axial loading & Web Stresses: Undefined
Compression: 6 MPa | Max Bending: 42 MPa | Shear: 10 MPa
--
Web Buckling Capacity:
Compression: 235 MPa | Shear: 135 MPa
--
Coefficients:
psi: -0.76
--
                      Critical Factor ( > 1 | safe)
Web Buckling Factor: 21
```

Shiploader: FE Buckling

SCRBE + local FE* for detailed buckling analysis (cf. Kotsalos EPFL Masters Thesis):

- Run global SCRBE solve to get displacement field
- Run local FE solve to get buckling modes



*Note that we use FE instead of RB for buckling because it requires displacement field as an input, difficult to parametrize in general

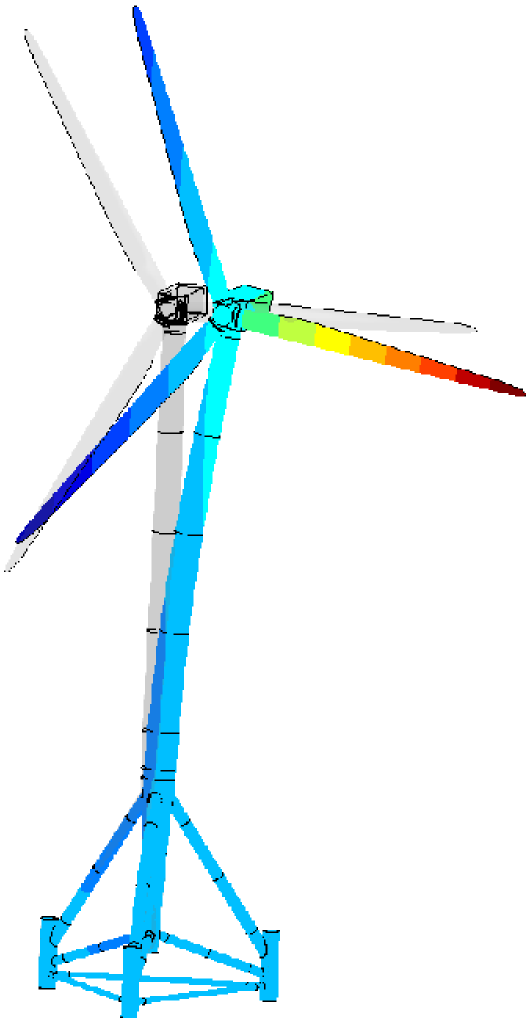
Wind Turbine: SCRBE Eigenmodes

We can use SCRBE for eigenvalue problems too

(Vallaghé, Huynh, Knezevic, Nguyen, Patera, Component-based reduced basis for parametrized symmetric eigenproblems. Advanced Modeling and Simulation in Engineering Sciences, 2015)

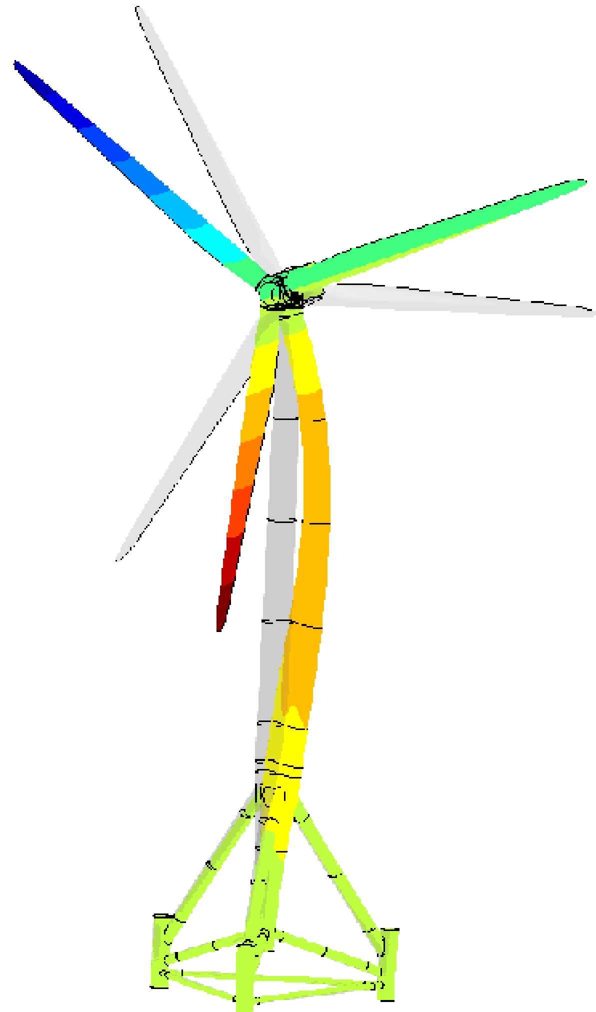
Model of offshore wind turbine
(joint work with LIC Engineering)

Wind Turbine: SCRBE Eigenmodes



0.275Hz

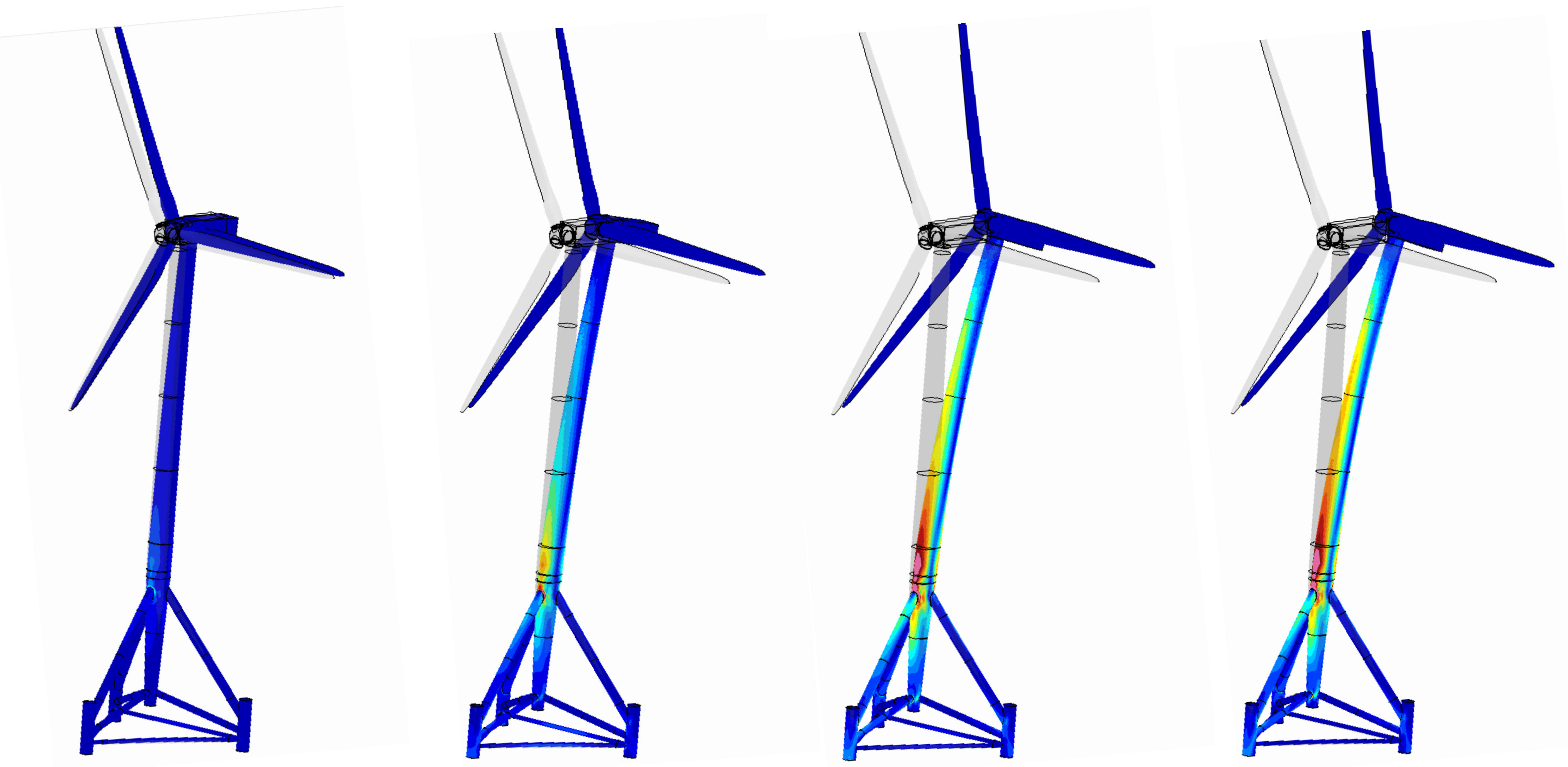
0.792Hz



1.651Hz

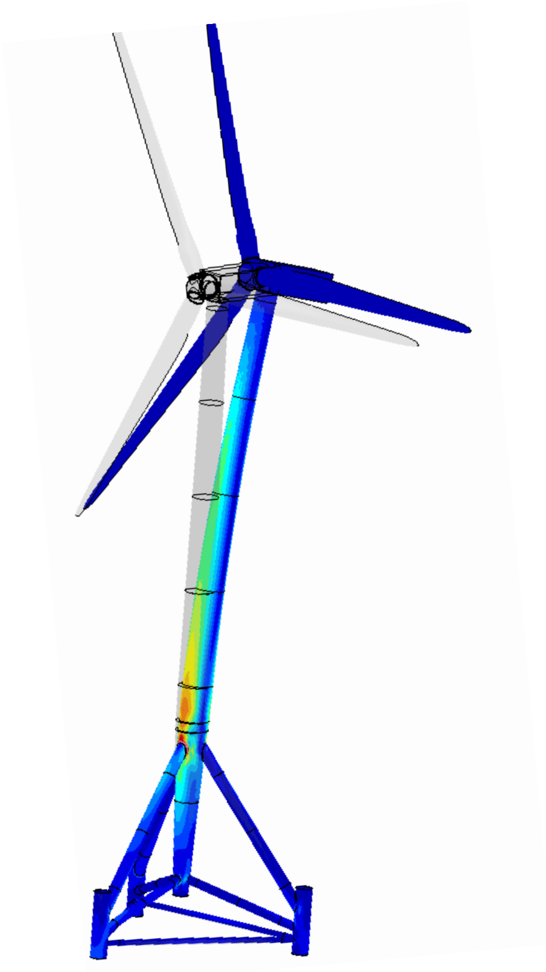
Wind Turbine: SCRBE Modal Dynamic

Can use SCRBE modal expansion for dynamic analysis (a common approach in engineering practice)



0.5s 1.0s 1.5s 2.0s

Wind Turbine: SCRBE Modal Dynamic



2.5s

3.0s

3.5s

3

1. Context

2. SCRBE

3. Coupled SCRBE/FE for Nonlinear

Coupled SCRBE/FE for Nonlinear

All results presented so far have been for linear PDEs

SCRBE is inherently limited to linear analysis, due to static condensation approach

In order to incorporate nonlinear analysis we provide a coupled SCRBE/FE solver:

- SCRBE in “linear regions”
- FE in “nonlinear regions”

For localized nonlinearities we still obtain significant speedup compared to global FE

Coupled SCRBE/FE for Nonlinear

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Core idea: Formulate as a coupled nonlinear system

$$G(U) = 0, \quad U \in \mathbb{R}^{N_{\mathrm{SCRBE}} + \mathcal{N}_{\mathrm{FE}}}$$

Use constraint matrix to rewrite residual and Jacobian in terms of (linear) SCRBE and (nonlinear) FE terms:

$$G(U) = C^T \begin{bmatrix} \mathbb{F}(\mu) - \mathbb{A}(\mu)\mathbb{U}(\mu) \\ G_{\mathrm{FE}}(U) \end{bmatrix}$$

$$J_G(U) = C^T \begin{bmatrix} -\mathbb{A}(\mu) & 0 \\ 0 & J_{G_{\mathrm{FE}}}(U) \end{bmatrix} C$$

Coupled SCRBE/FE for Nonlinear

Apply Newton's method (with line search) to the coupled system:

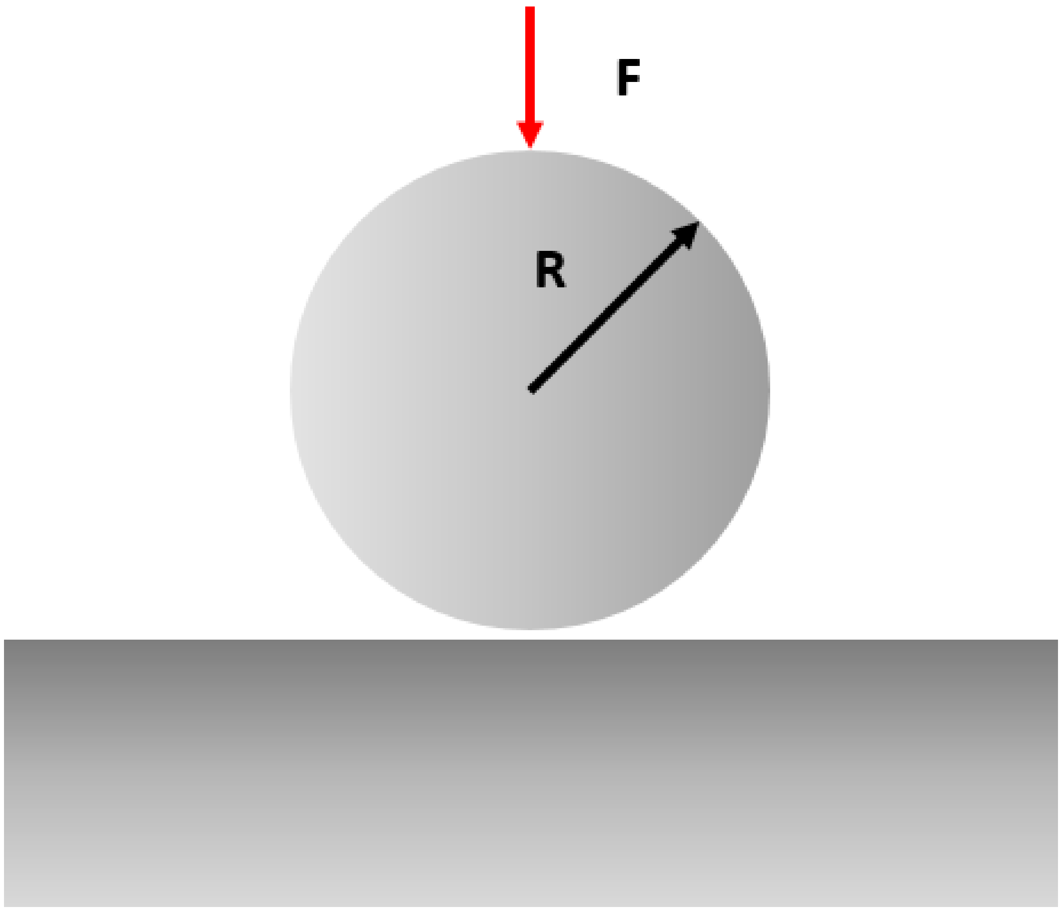
$$\begin{aligned} J_G(U^k)\Delta U^k &= -G(U^k) \\ U^{k+1} &= U^k + \Delta U^k \end{aligned}$$

Key points:

- Arbitrary nonlinearities in FE region (e.g. contact, plasticity, finite strain)
- SCRBE accelerates the linear region, fast for "localized nonlinearities"
- Formulation is fully conforming, numerically robust

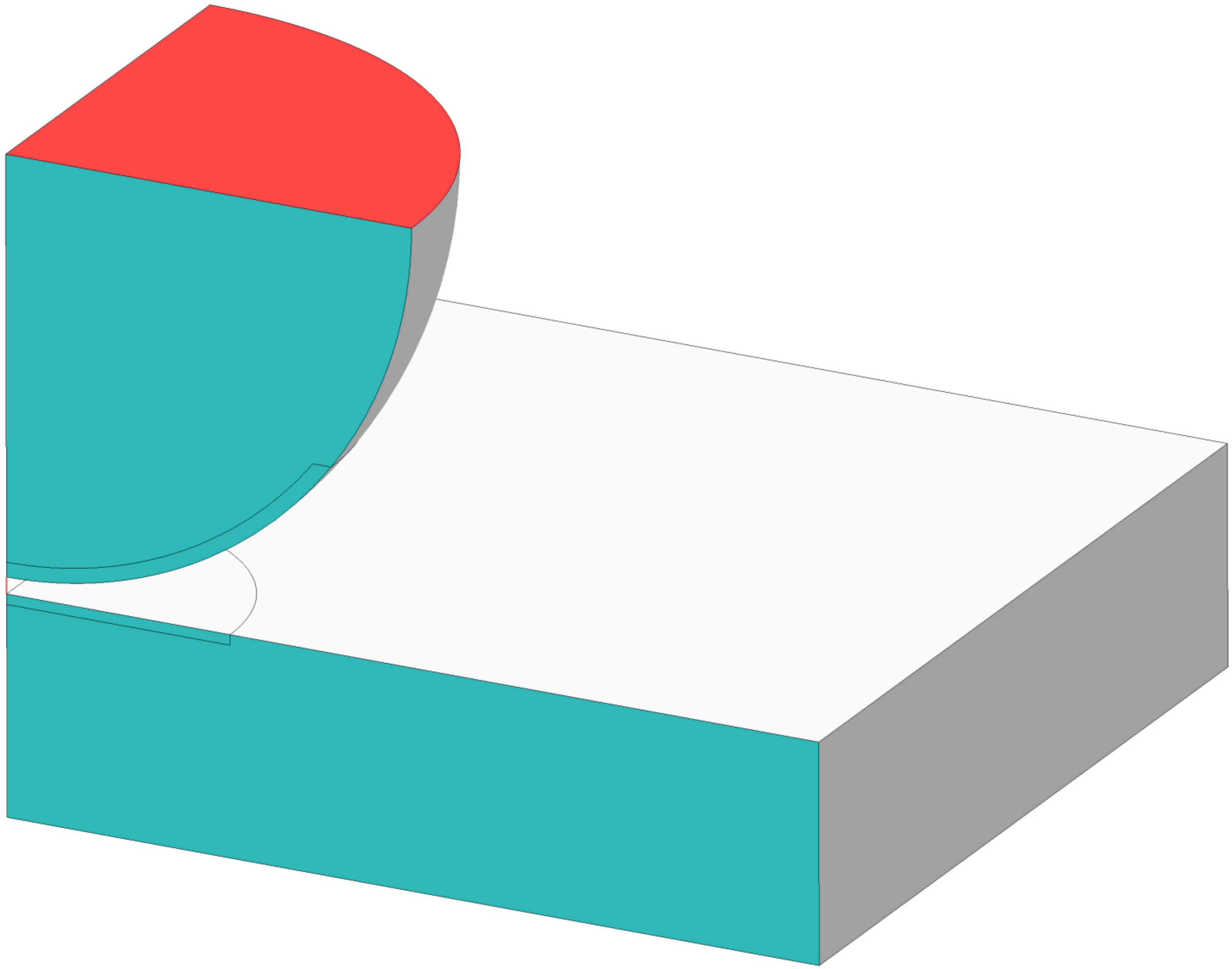
Paper in preparation (Knezevic, Huynh, Nguyen, Patera)

Benchmark problem: sphere and half-space contact (Hertzian analysis gives analytical solution)



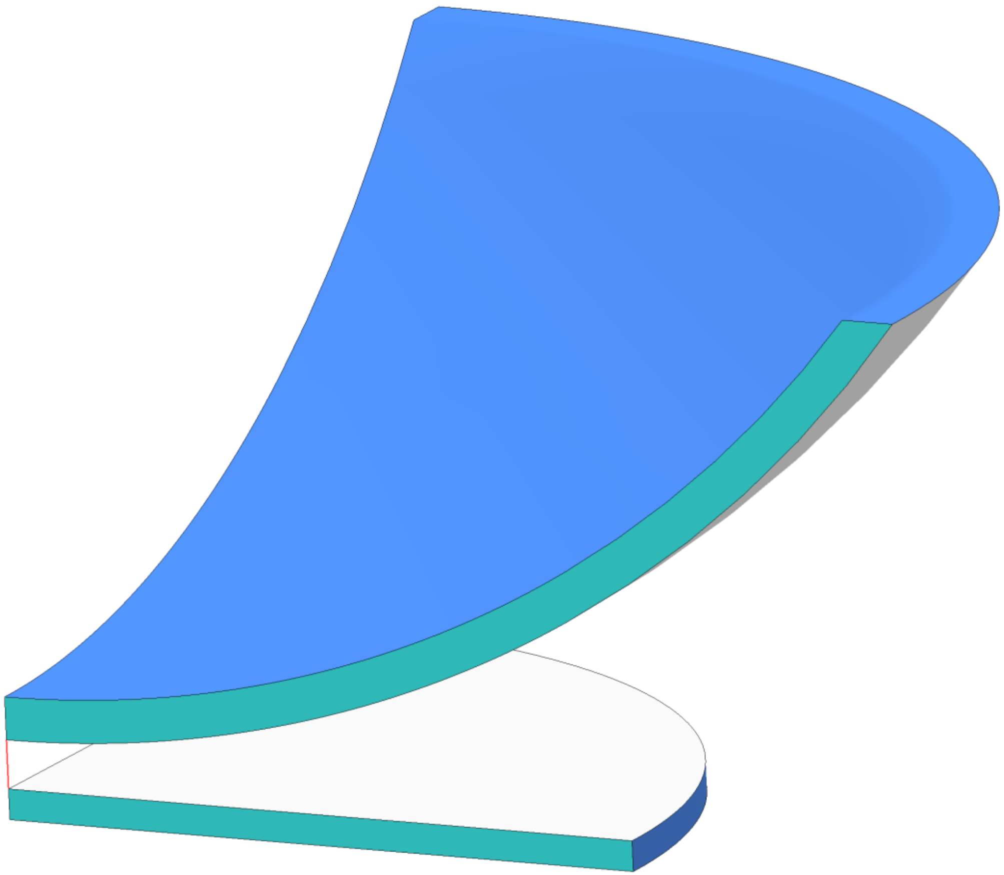
Contact Analysis

Four components, symmetry boundary conditions

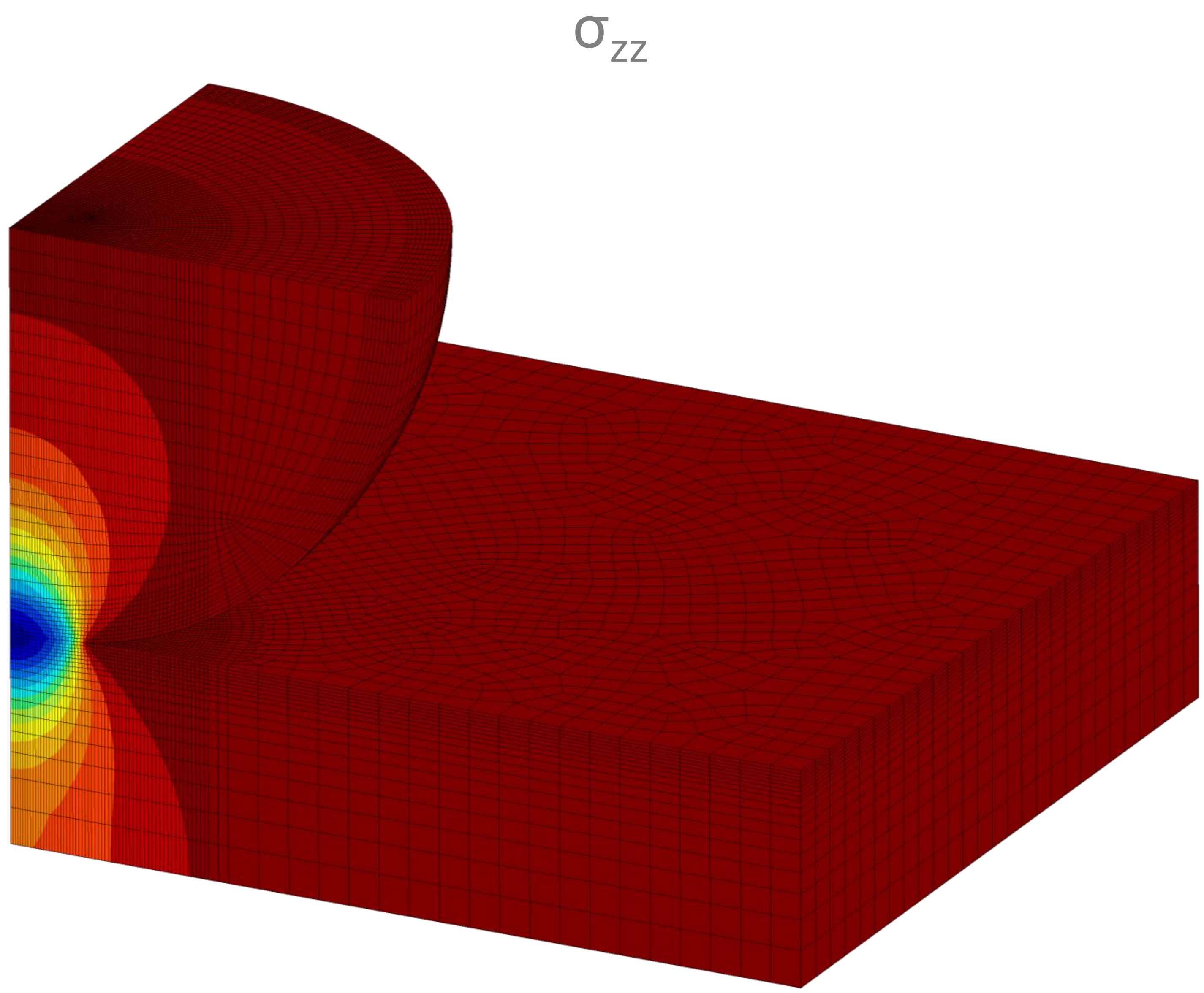


Contact Analysis

Two small “FE components” in the contact zone

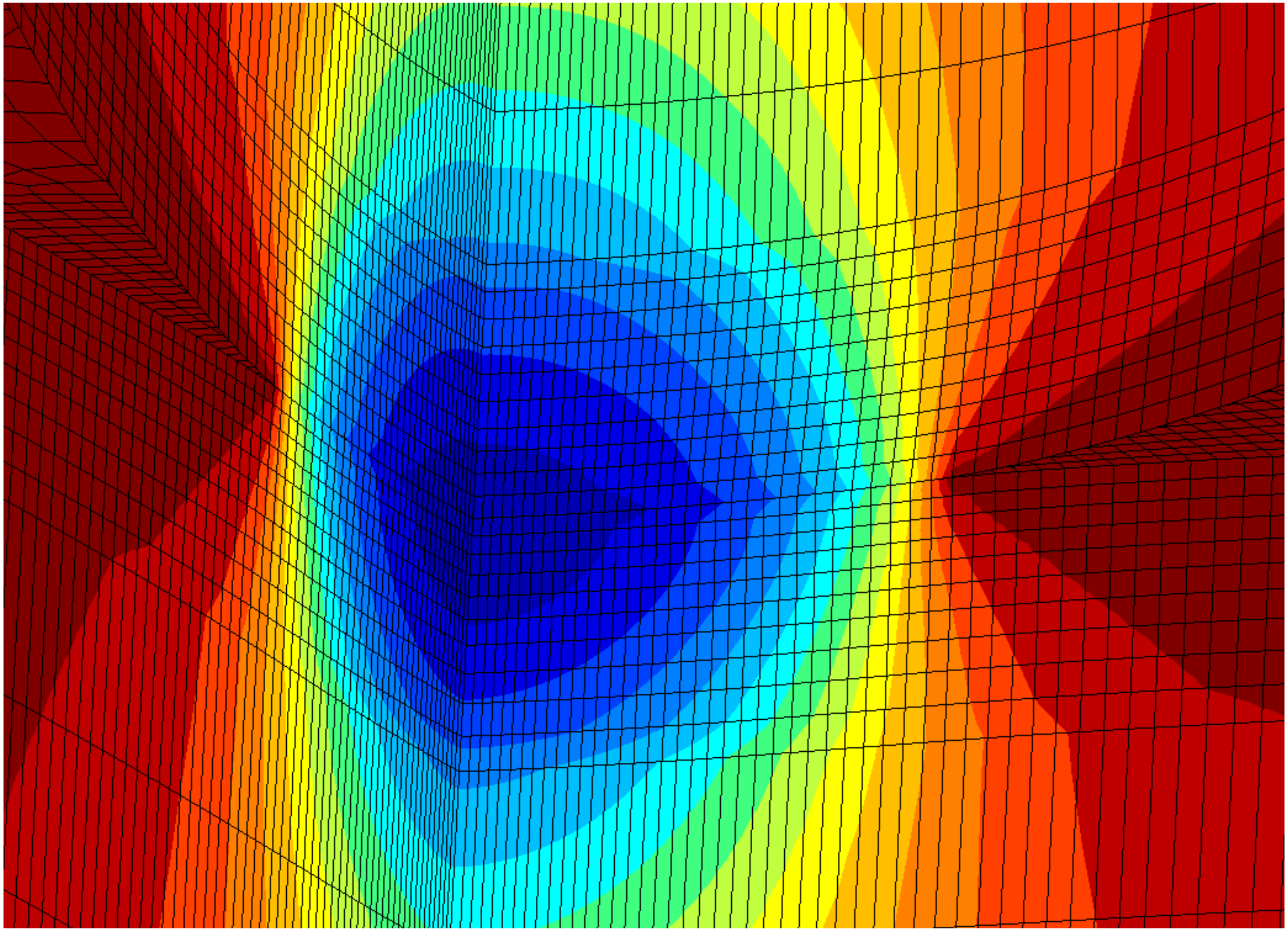


Contact Analysis



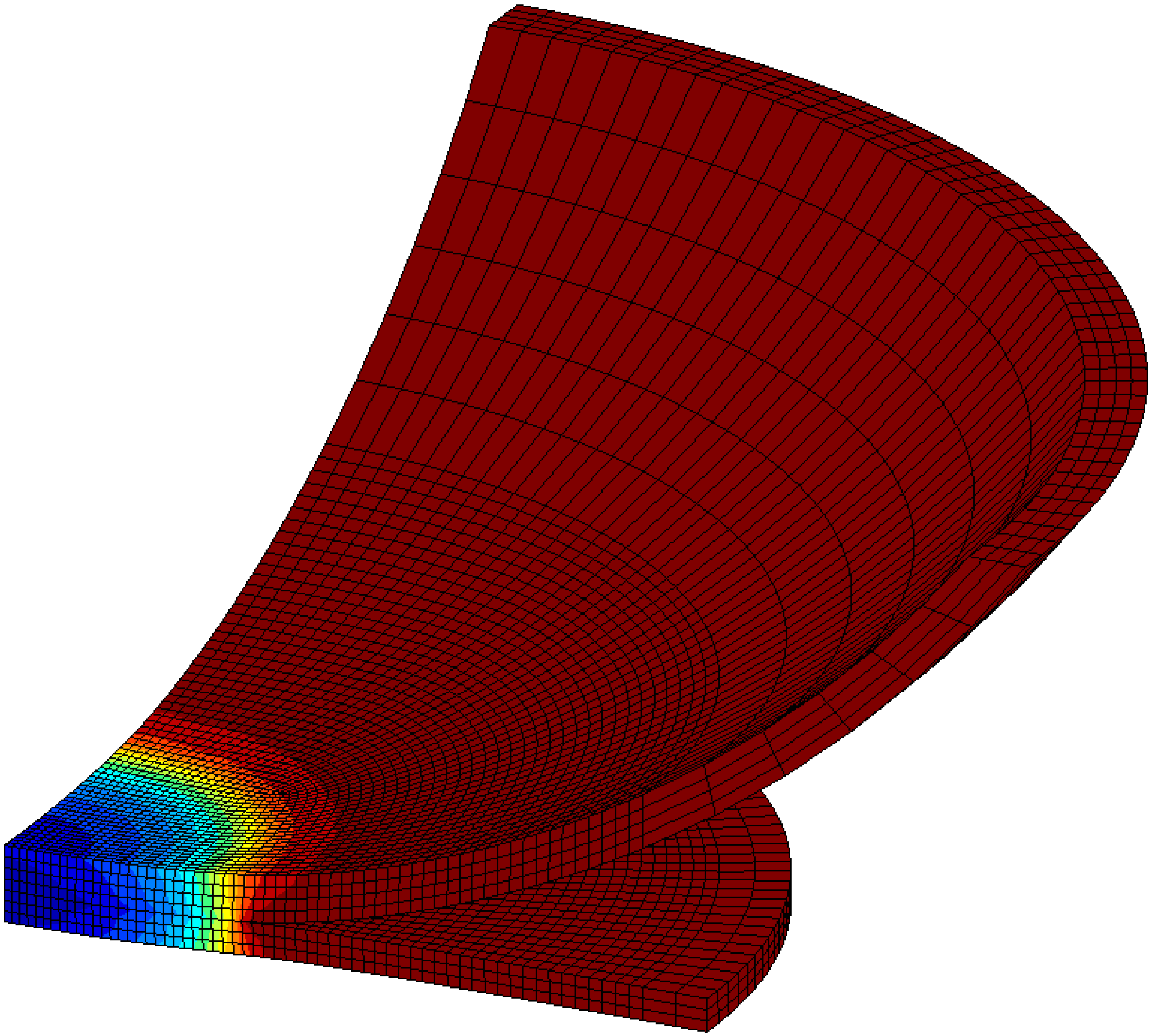
Contact Analysis

σ_{zz} (zoomed in)



Contact Analysis

σ_{zz} in FE components only



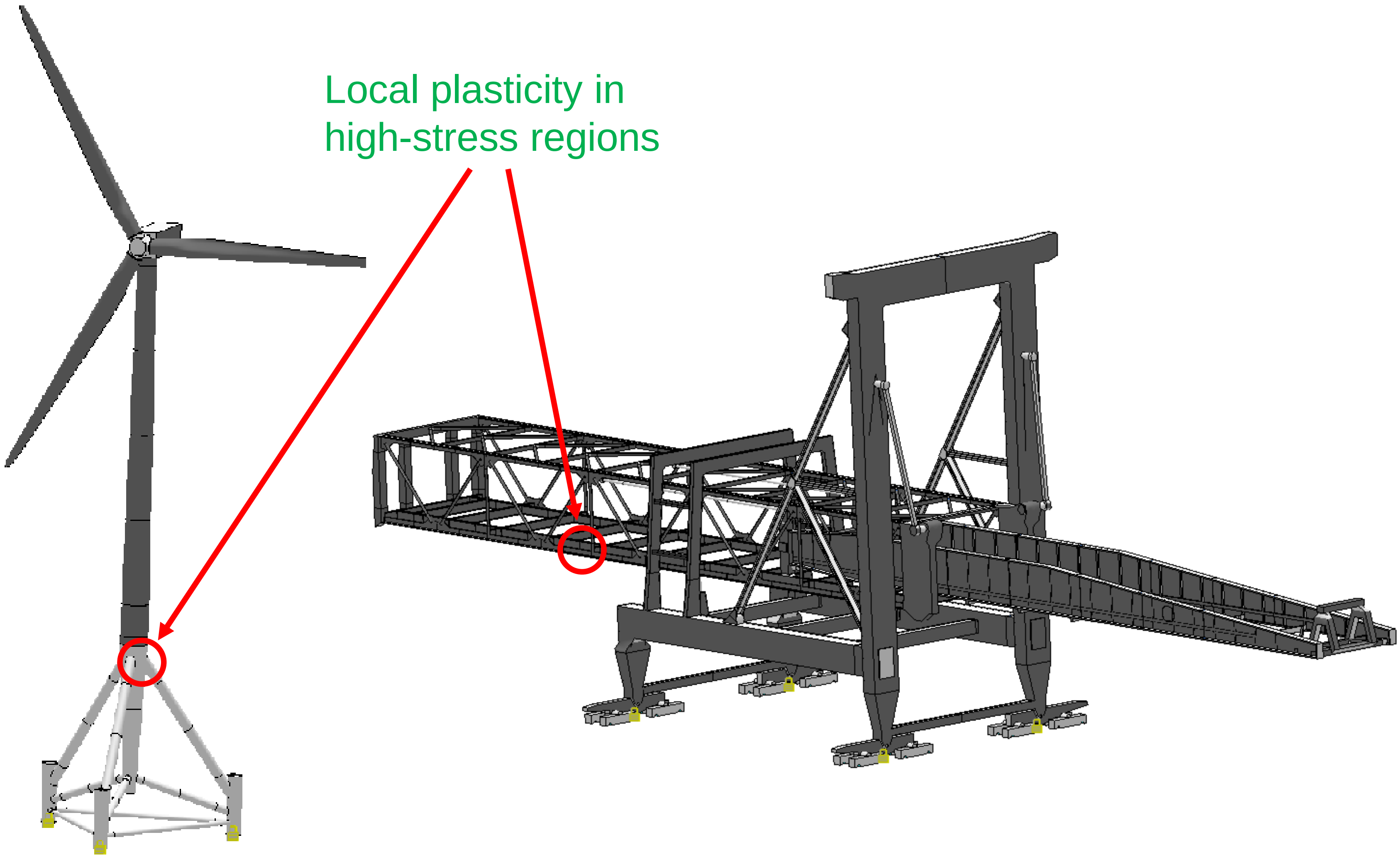
Contact Analysis

	Analytical Value	FE Value	SCRBE/FE Value
Contact Radius (m)	0.1897m	0.1885	0.1885
Max. Pressure (Pa)	1.327×10^{10}	1.366×10^{10}	1.366×10^{10}

FE DOFs	440,514
SCRBE/FE DOFs	70,089
SCRBE/FE Speedup	~19x

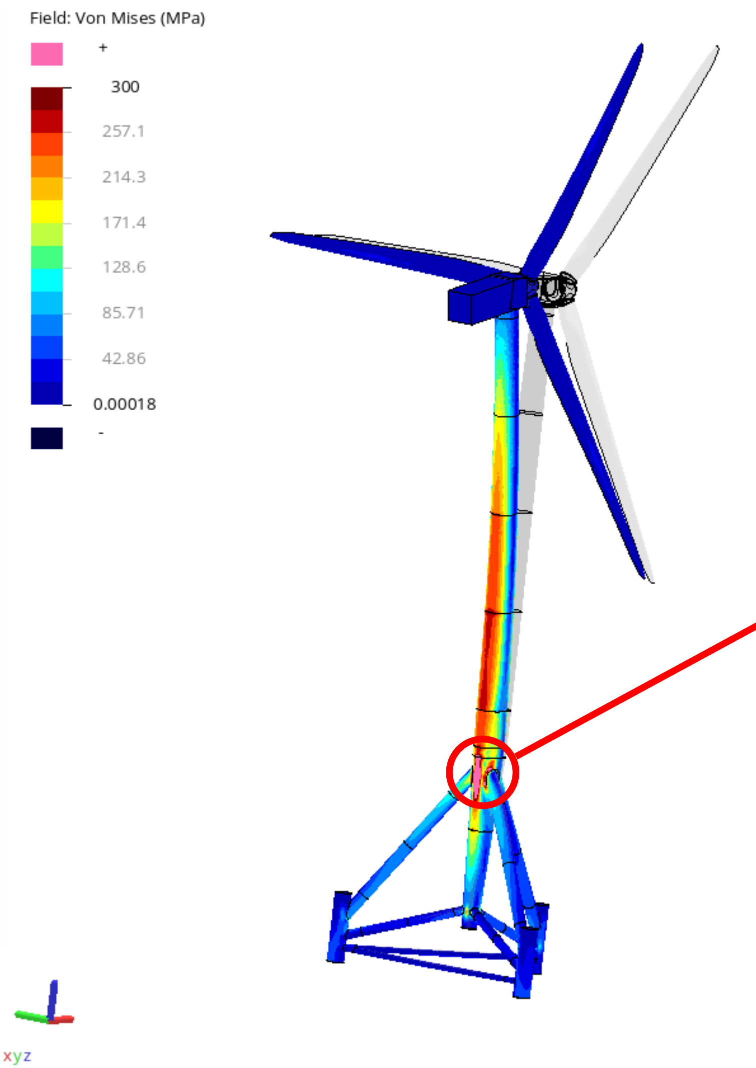
Local Nonlinearities in Large Systems

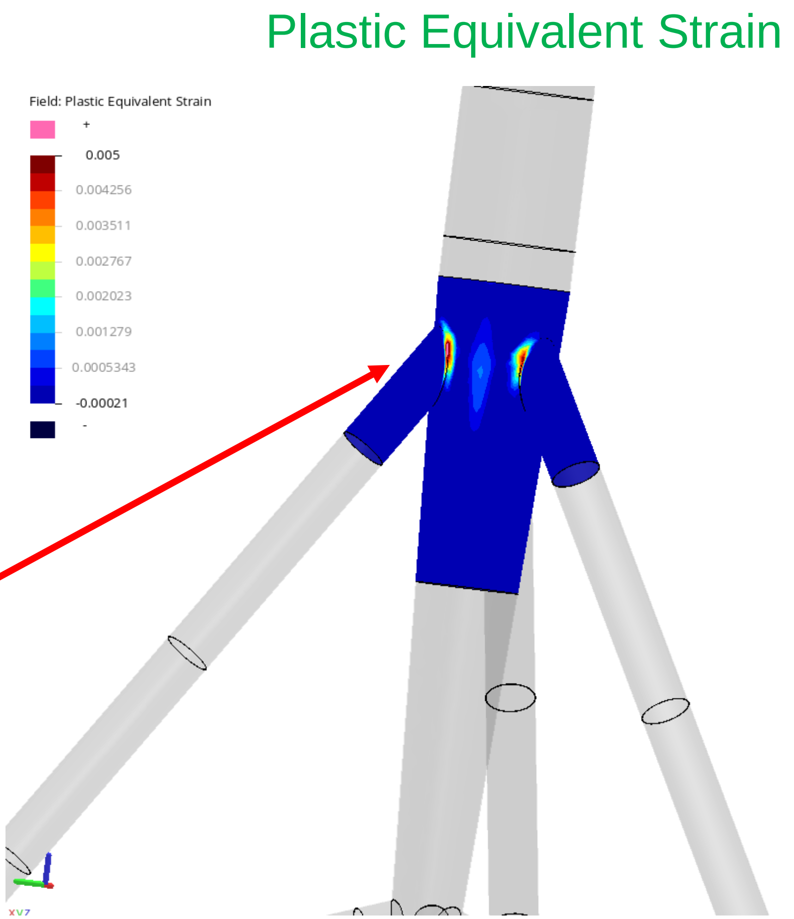
We achieve much greater speedup (e.g. >100x) when the nonlinear region is small compared to overall model



Local Nonlinearities in Large Systems

For example, local plasticity in wind turbine





Nonlinear Analysis: Looking Ahead

Coupled SCRBE/FE works very well for **localized nonlinearities**

In some systems nonlinearities are not localized, and SCRBE/FE "degenerates" to global FE

Looking ahead: Strong interest in using model reduction in the nonlinear regions as well…