

Near-Krylov

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Multigrid Arnoldi

Rank-1 Update
Eigenvalues

Multiple
Eigenvalues

Subspace
Recycling

Convergence for Arnoldi(30,15)

Residual Norm curves for the smallest 10 eigenvalues

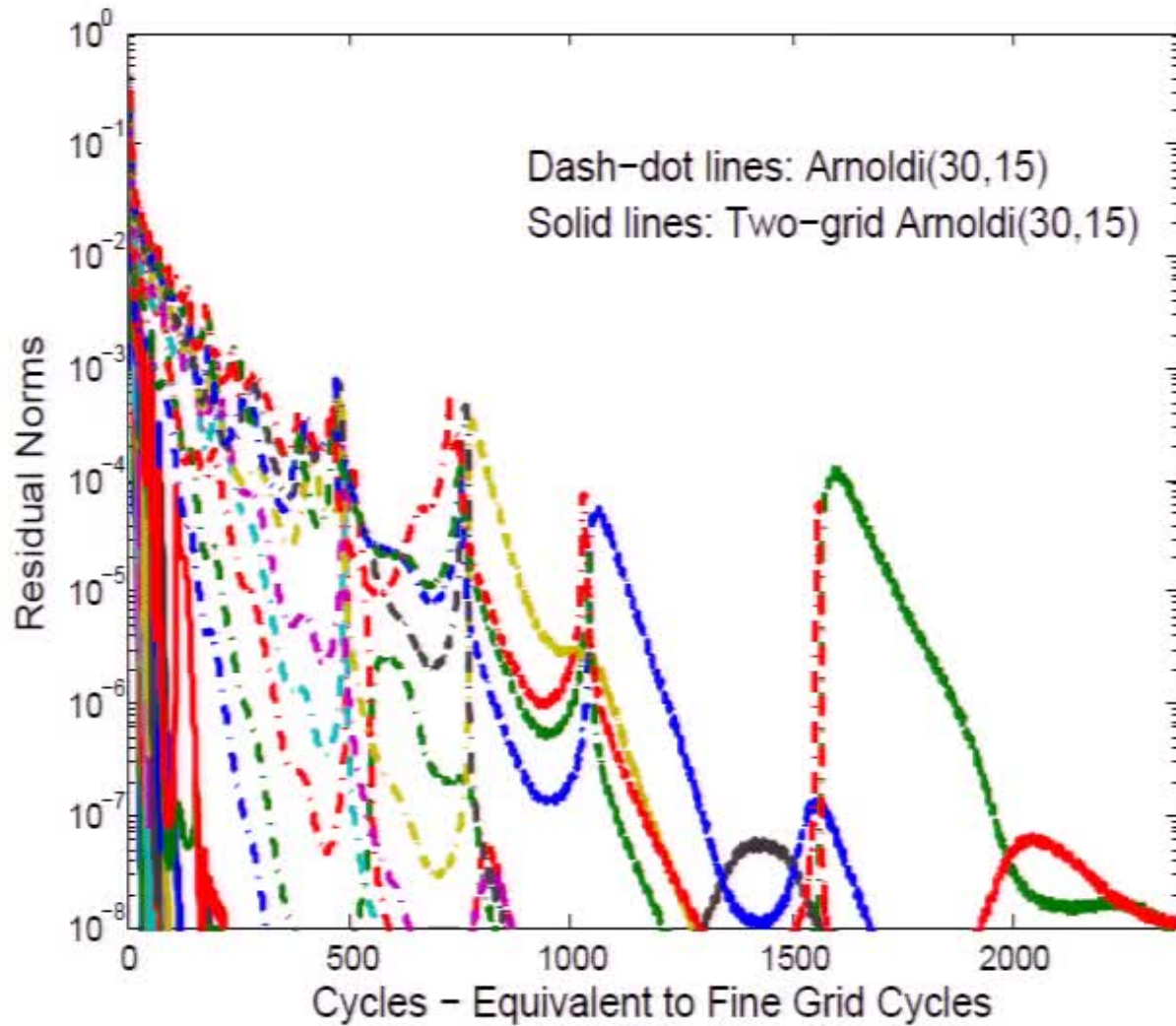


Figure: 2-D Convection-Diffusion, $n = 488,601$

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Large Matrices are like the Fat Giraffe



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Try Multigrid to knock the giraffe down to size



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Helmholtz Example: Fine Grid $n = 1023$

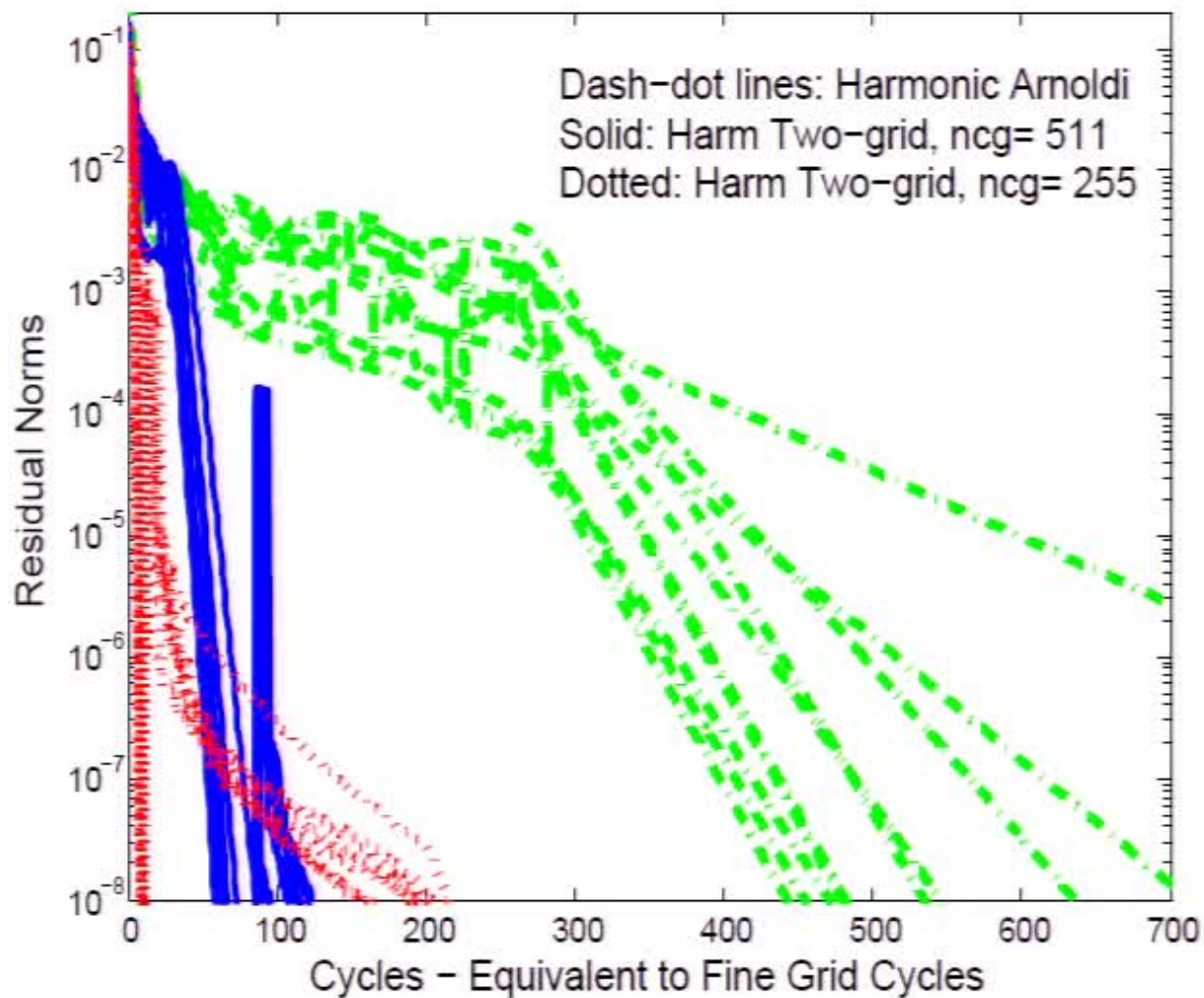


Figure: Simple 1-D Helmholtz, $\gamma = 40,000$, $n = 1023$

Harmonic Arnoldi(30,15) does not find all 10 eigenvalues.

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Rank-1 Update Eigenvalues

Joint with Mark Embree



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Is Rank-1 perturbation for eigenvalue problem a big change
or a little change?

Is it a zorse or an okapi?

Arnoldi(35,15) for Rank-1 Update of norm 1.

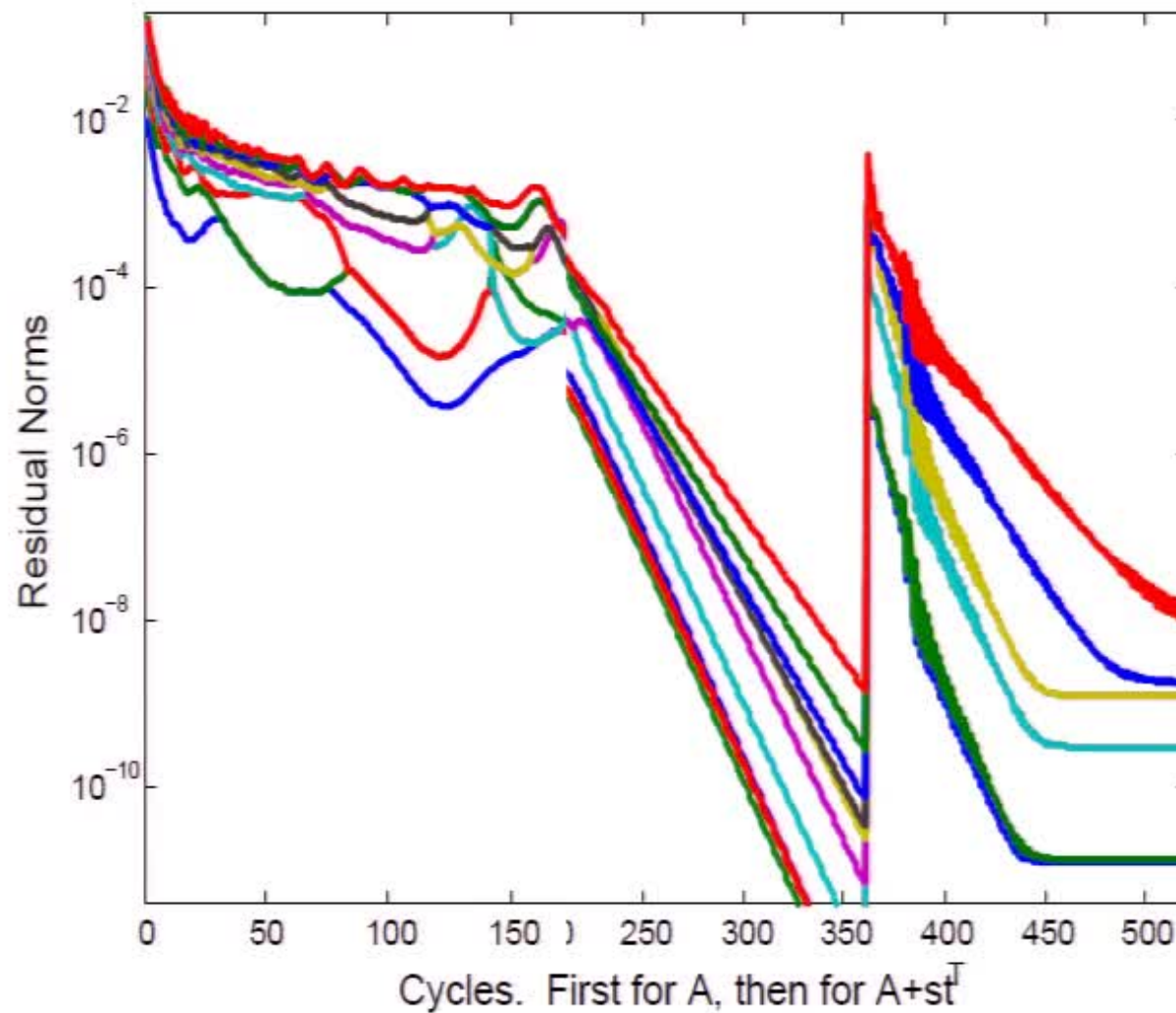


Figure: Rank-1 perturbation of norm 1. 1-D conv-diff, $-u'' + 10u' = \lambda u$, $n = 2000$.

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Multiple Eigenvalues - Use Arnoldi with perturbed matrix

Joint with Mark Embree, Thomas Gibson, Kevin Mendoza



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RDB12

Residual Norms with A

Method	Cycles
Rank-1	193
Roundoff	288
Block	451

Compare 3 methods

Rank-1: 193 cycles

Roundoff: 288 cycles

Block: 451 cycles

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B1250L. Rank-1 Perturbed Arnoldi vs. roundoff & block.

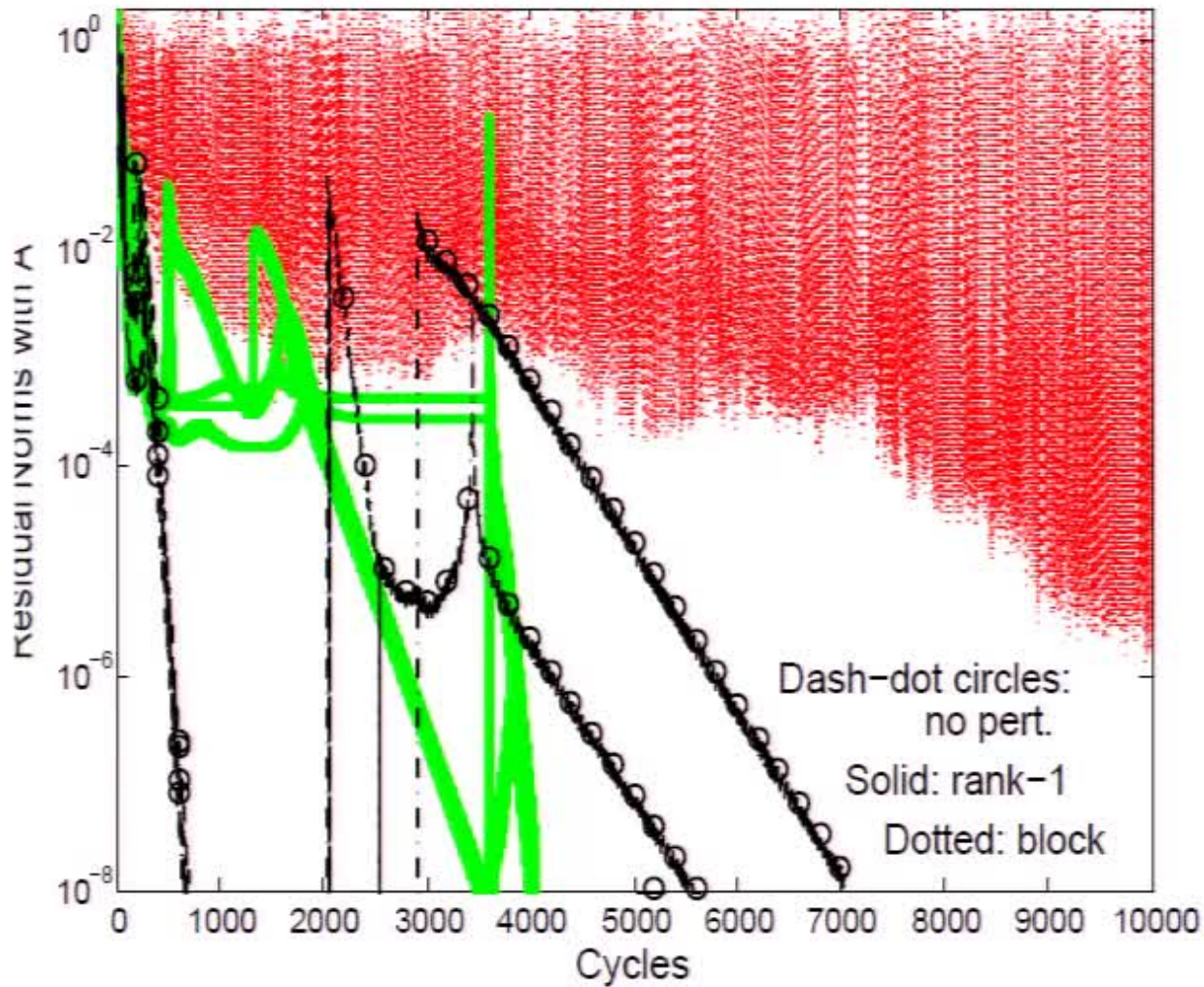


Figure: Arnoldi(16,6), matrix RDBL1250, $n = 1250$.

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**Subspace
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Subspace Recycling

Joint with Huy Nguyen



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Subspace Recycling

Build Krylov subspace with $(I - CC^*)A$, where $C = AU$, with U spanning the subspace of approximate eigenvectors.

Another way: $\text{Span}\{r, Ar, A^2r, \dots, A^{m-k}r, y_1, y_2, \dots, y_k, \}$

Near Krylov does not work well for eigenvectors here, but often good enough.



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