On the response of autonomous sweeping processes to periodic perturbations

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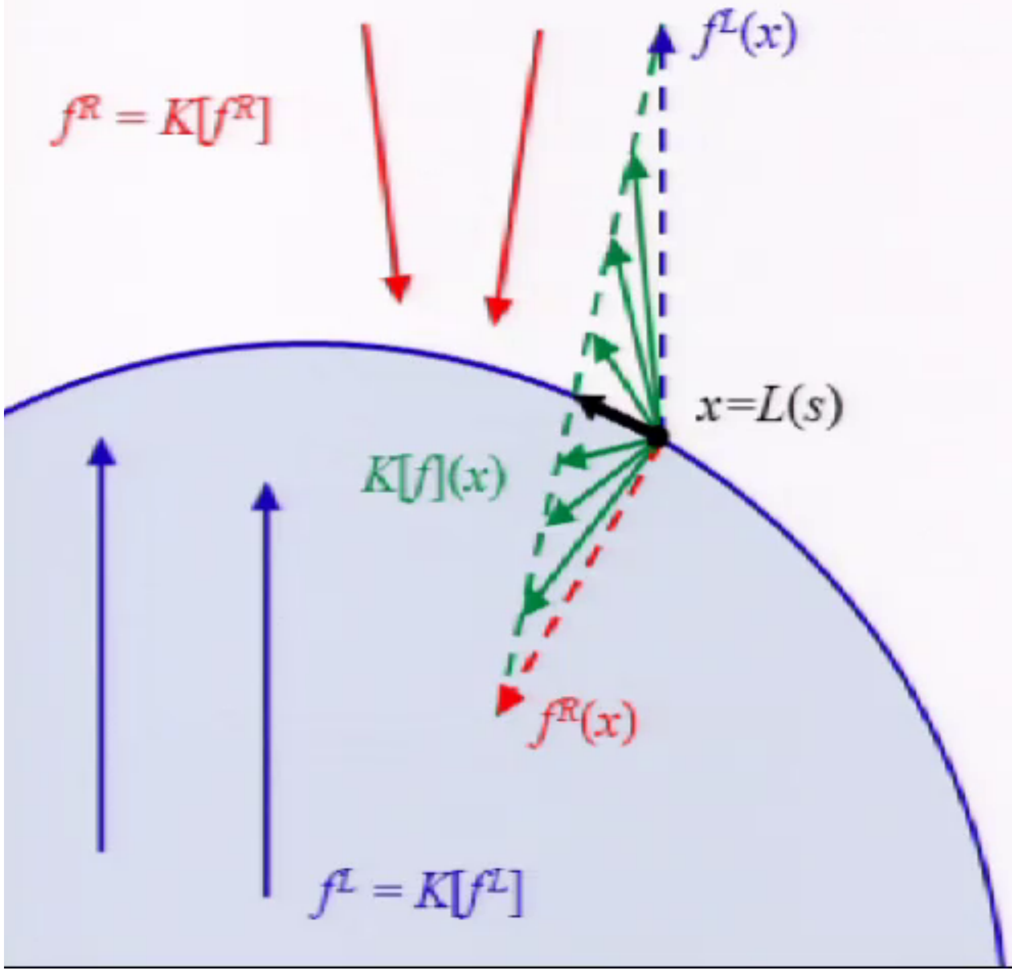
Filippov differential inclusion

$x' = f^L(x)$, if $x \in \text{int}B$

$x' = f^R(x)$, if $x \notin B$

$x' \in K[f](x)$, $x \in \mathbb{R}^2$

Filippov differential inclusion



Filippov differential inclusion

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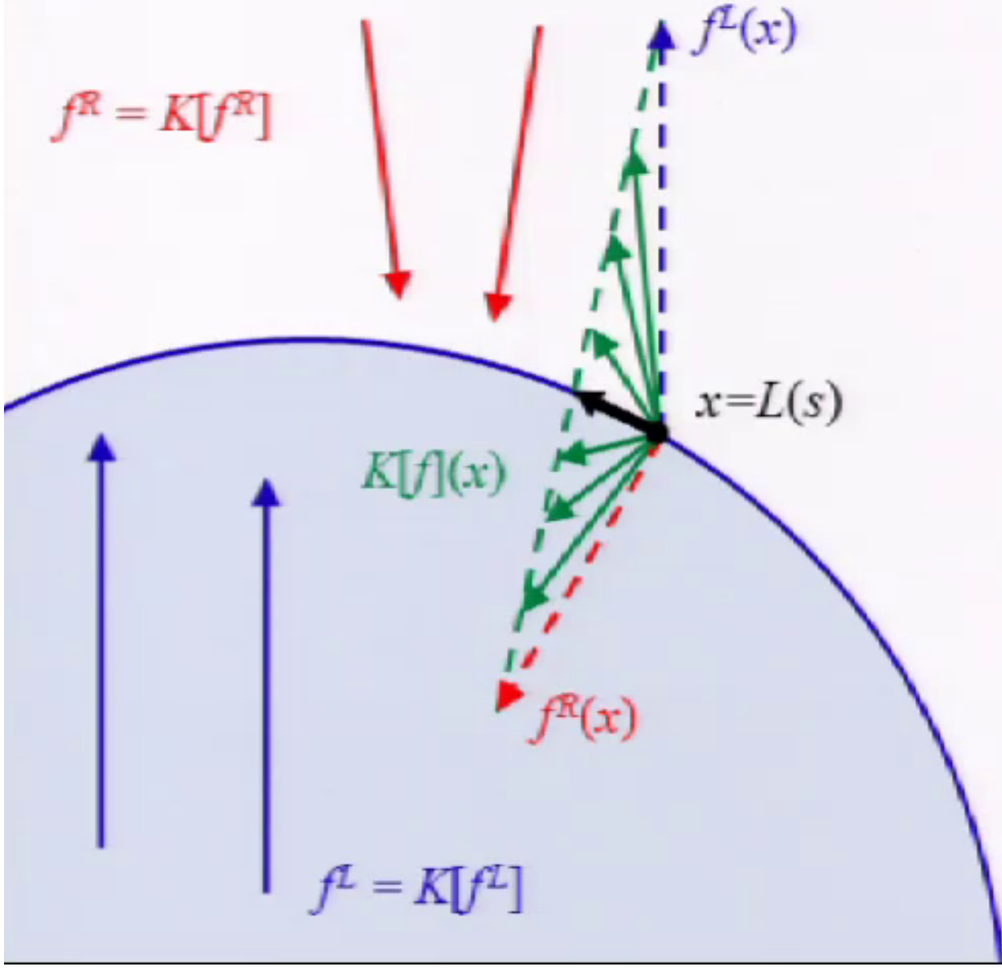
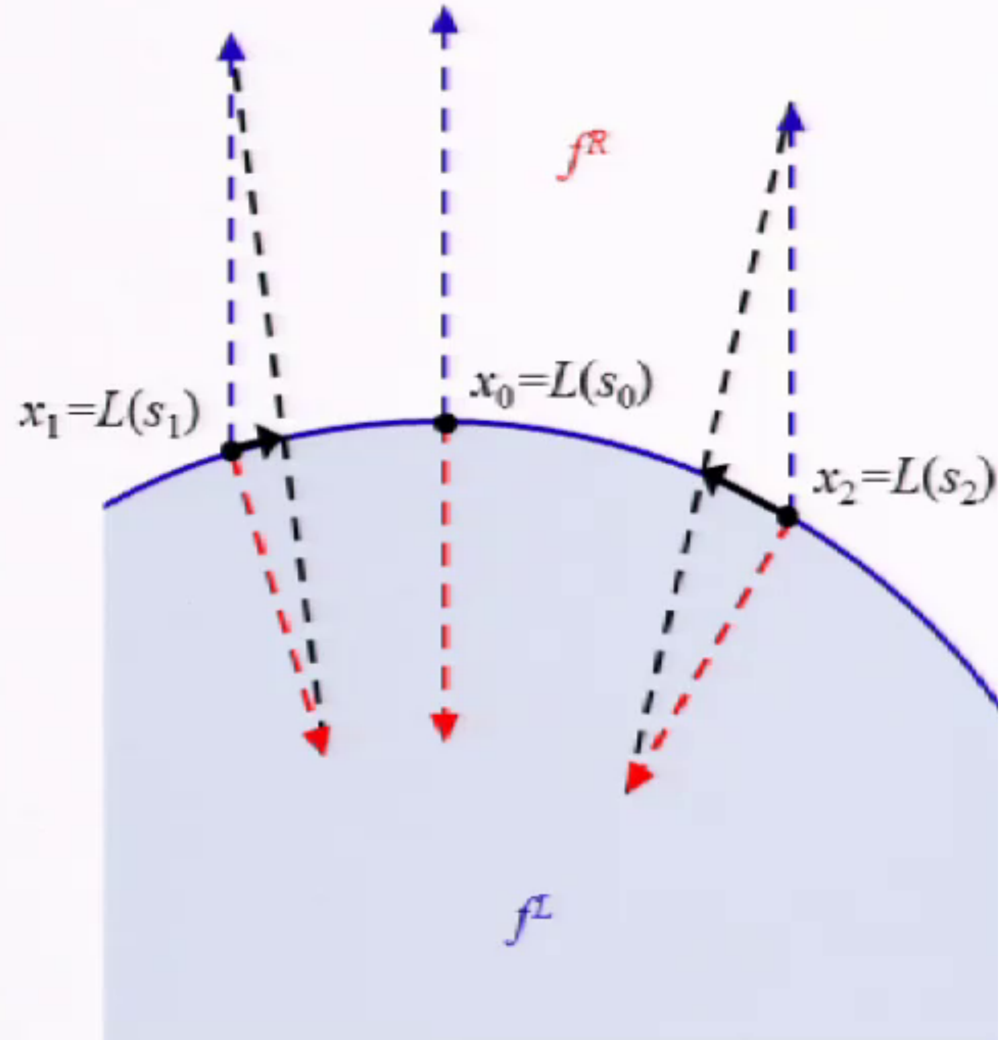
Filippov differential inclusion

Equation of sliding motion: $s' = G(s)$, $s \in \mathrm{R}^1$

$G(s_0)=0$ and $G'(s_0)<0$ $\Rightarrow$ asymptotic stability of x_0

$G(s_0)=0$ and $G'(s_0)=0$ $\Rightarrow$ nonsmooth bifurcation from x_0





Moreau differential inclusion

$x' = f^L(x),\quad$ if $x \in \mathrm{int}B$

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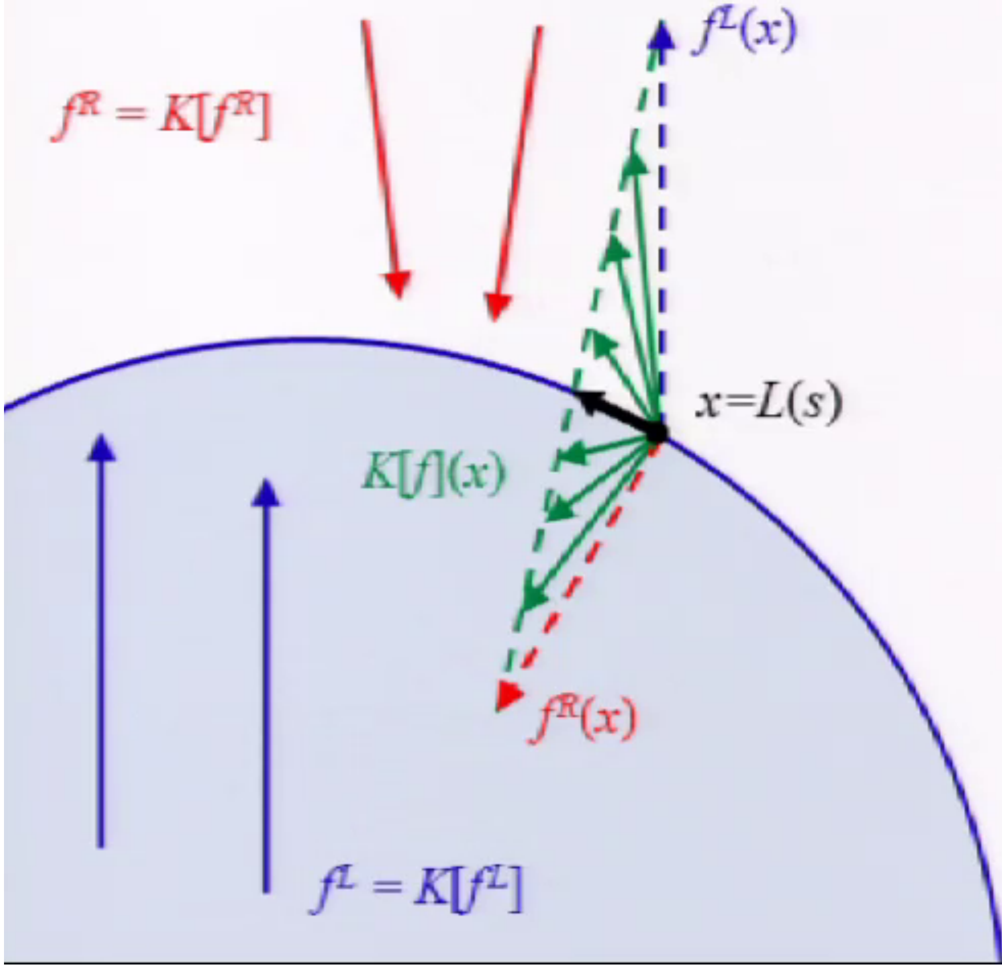
$x' \in K[f](x),\quad x \in \mathrm{R}^2$

Filippov differential inclusion



$x' = f^L(x),\quad$ if $x \in \mathrm{int}B$

$x' \in -N_B(x) + f^L(x),\quad$ if $x \in \partial B$

Moreau differential inclusion

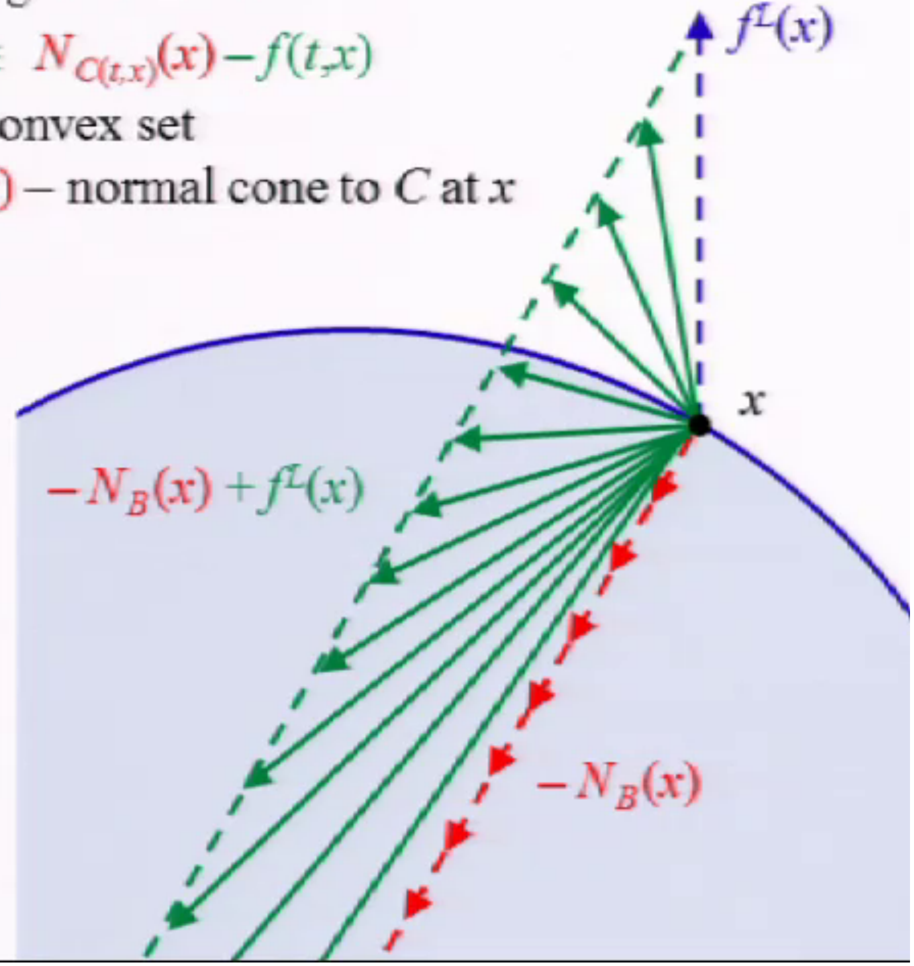
or Moreau sweeping process

Most general form:

$-x' \in N_{C(t,x)}(x) - f(t,x)$

C – convex set

$N_C(x)$ – normal cone to C at x



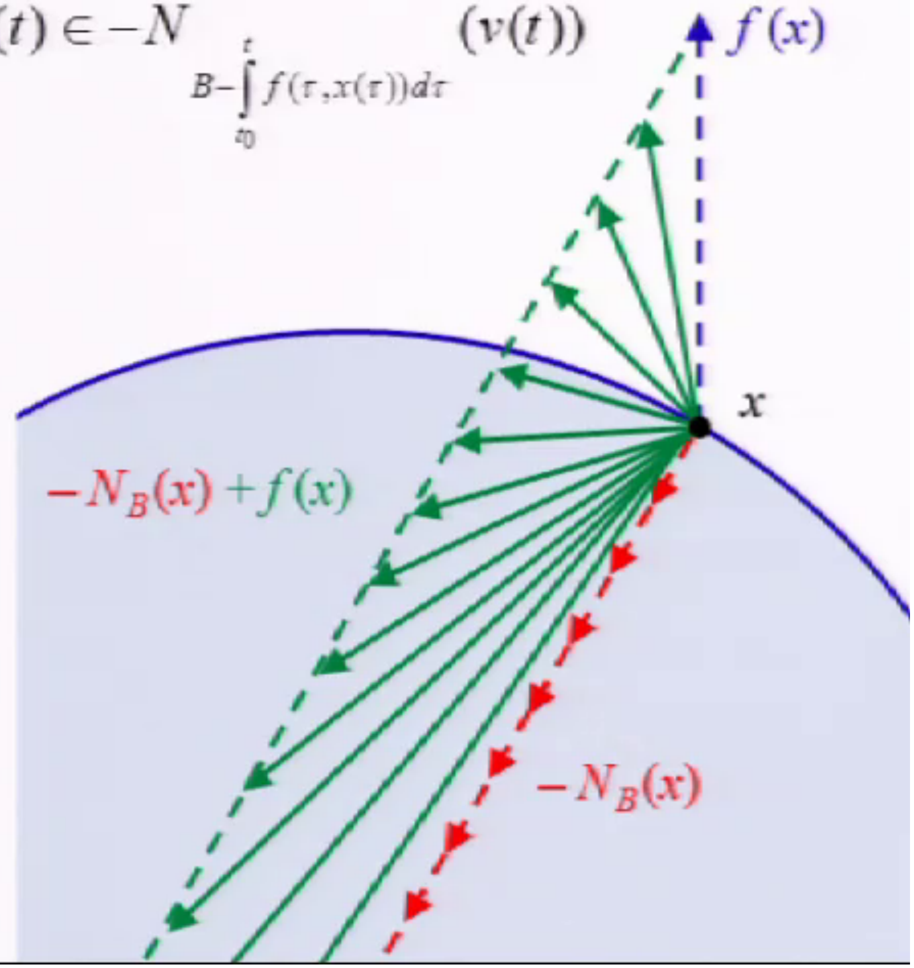
Construction of the solution

$x' \in -N_{B(t,x)}(x) + f(t,x)$

x is a solution iff

$$x(t) = v(t) + \int_{t_0}^{t} f(\tau, x(\tau))\,d\tau$$

$$v'(t) \in -N_{B-\int_{t_0}^{t} f(\tau, x(\tau))\,d\tau}(v(t))$$

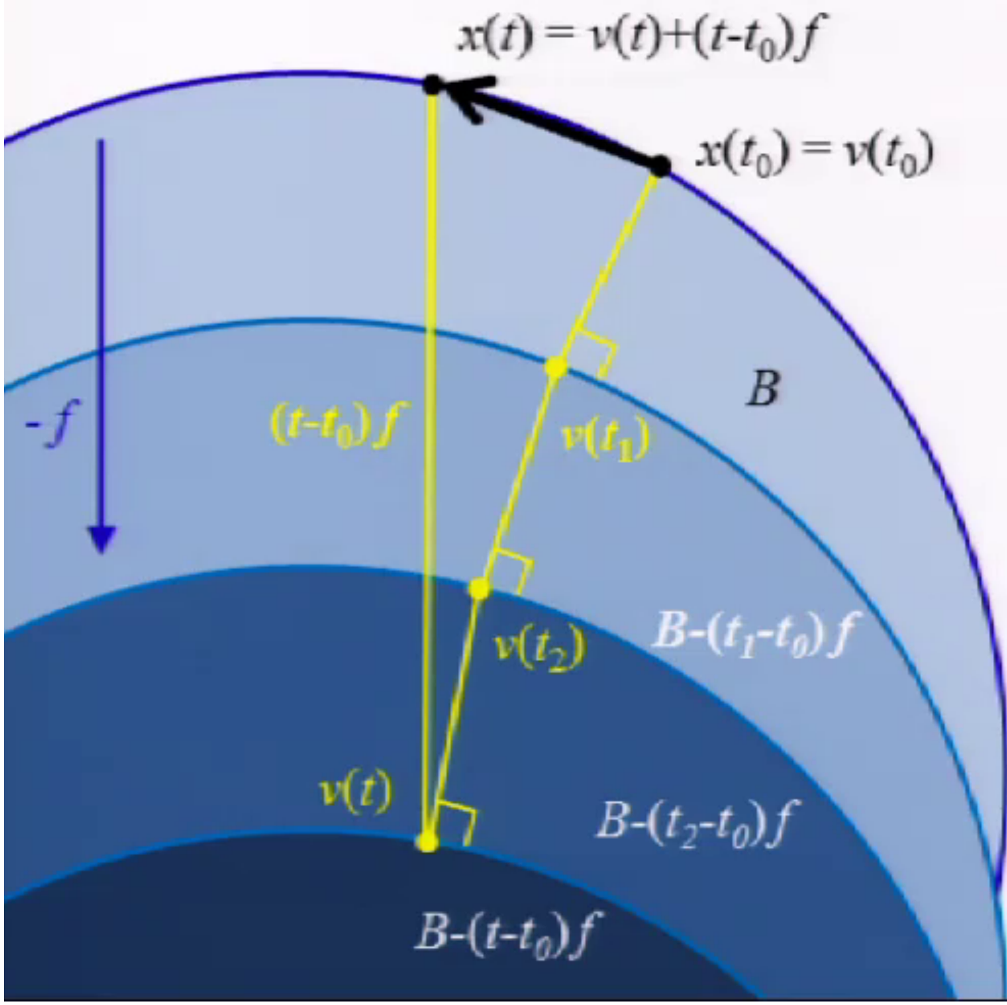


Construction of the solution

$x' \in -N_B(x) + f,$ where $f = \text{const}$

$$x(t) = v(t) + (t - t_0) f$$

$$v'(t) \in -N_{B-(t-t_0)f}(v(t))$$



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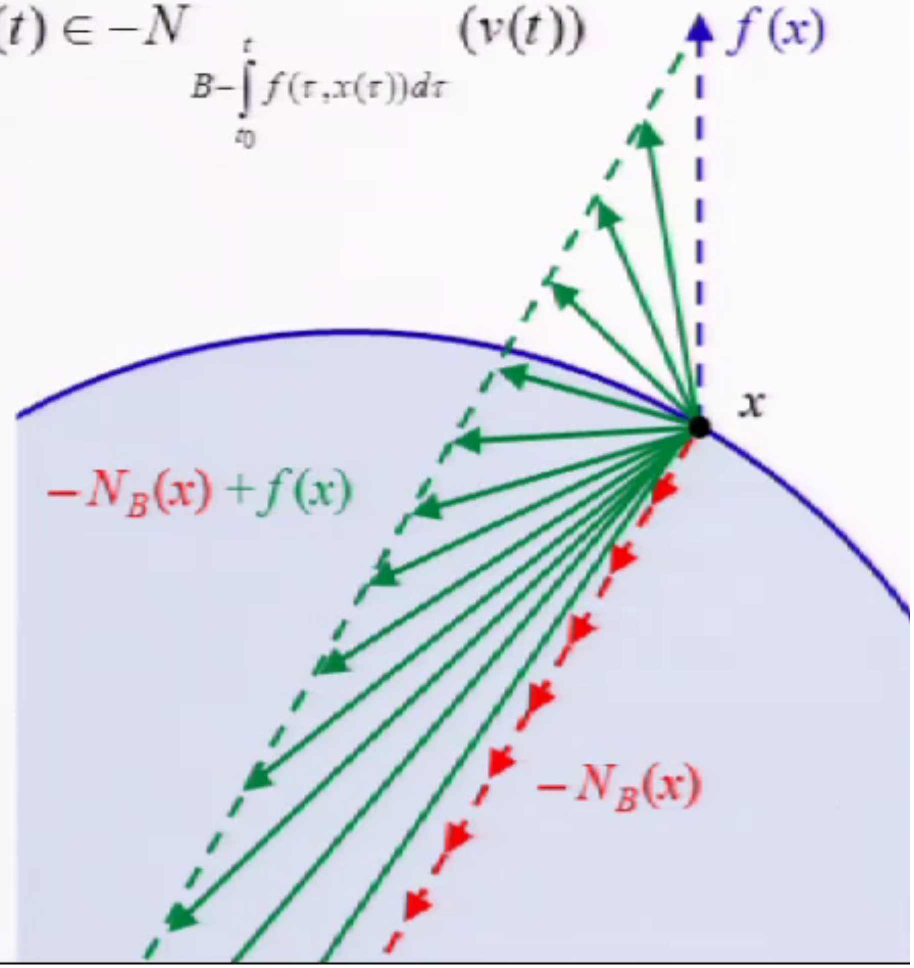

Results

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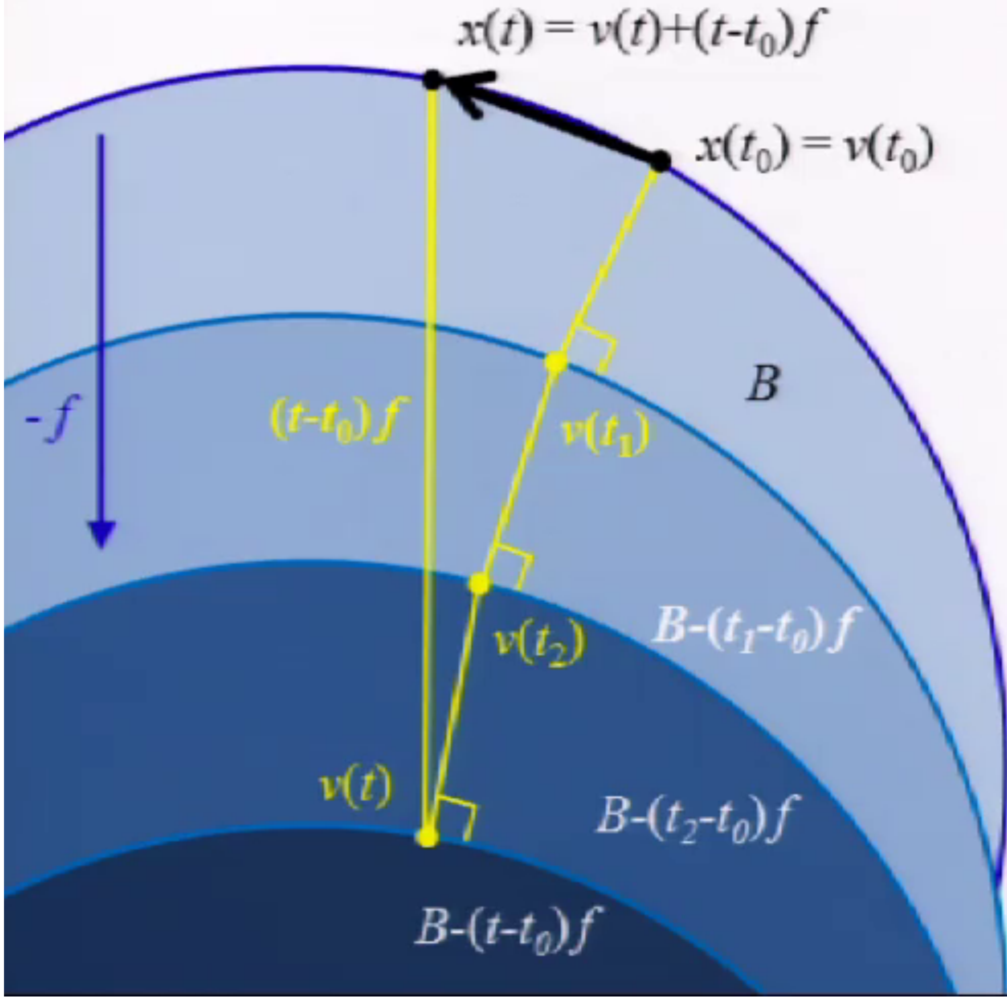
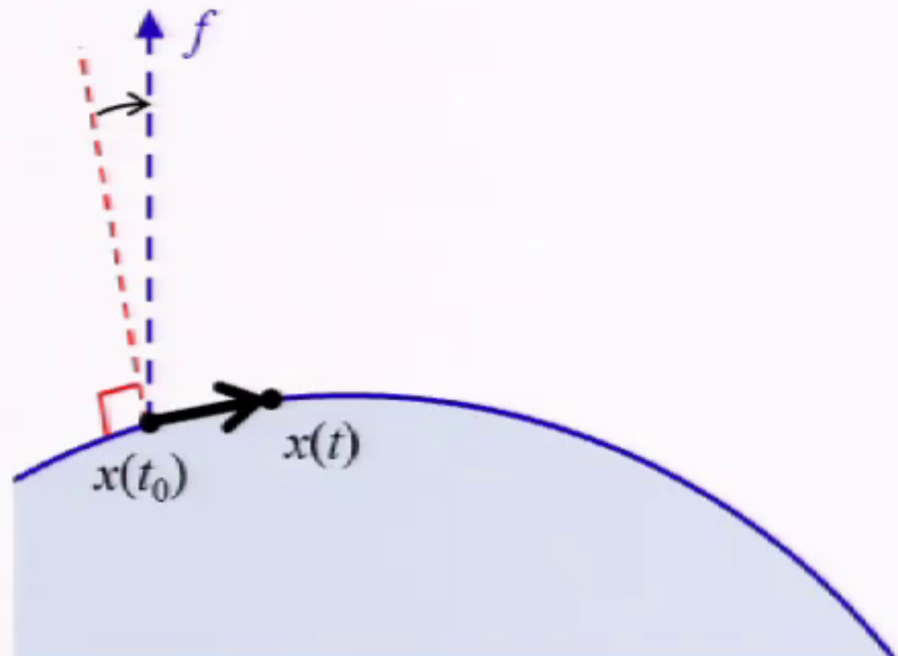
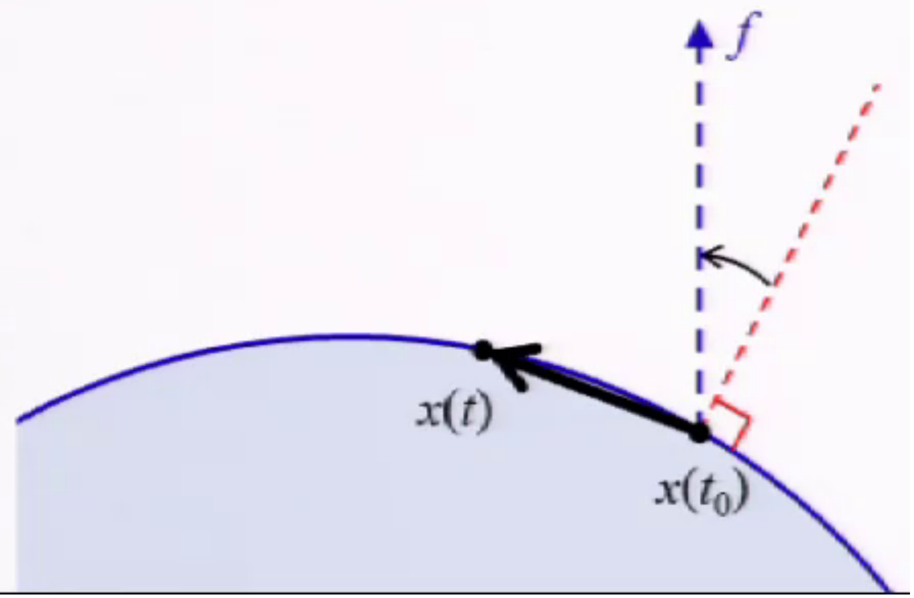






Results

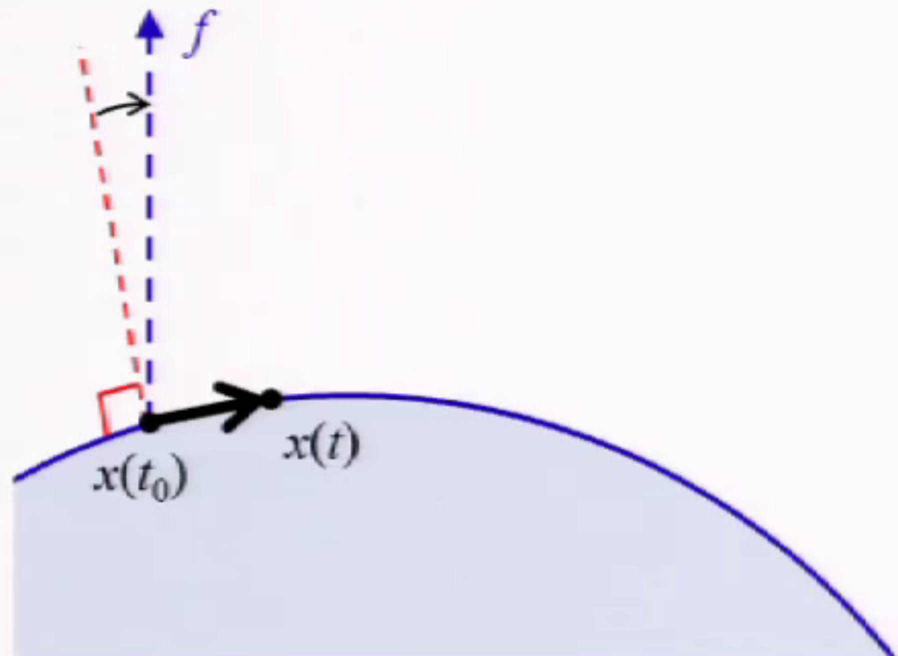
$x' \in -N_B(x) + f(x)$, $L(s)$ parameterizes ∂B

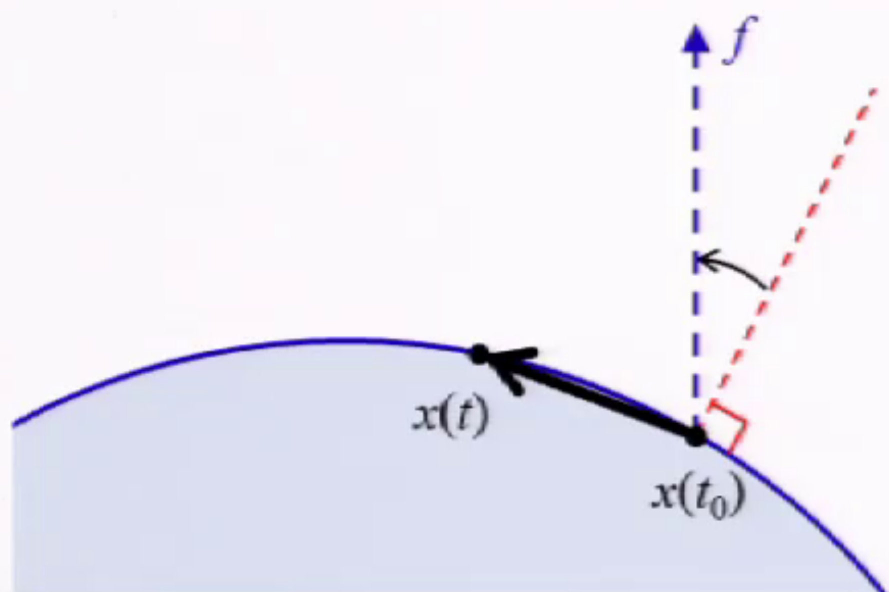
$$\bar{f}(s) = \left\langle f(L(s)), \begin{pmatrix} L_2(s) \\ -L_1(s) \end{pmatrix} \right\rangle$$

$\bar{f}(s_0) = 0 \quad \Rightarrow \quad s_0$ is invisible equilibrium

Statement1: $\bar{f}(s_0) = 0, \; \bar{f}'(s_0) \neq 0 \quad \Rightarrow$

invisible equilibrium persists under perturbations





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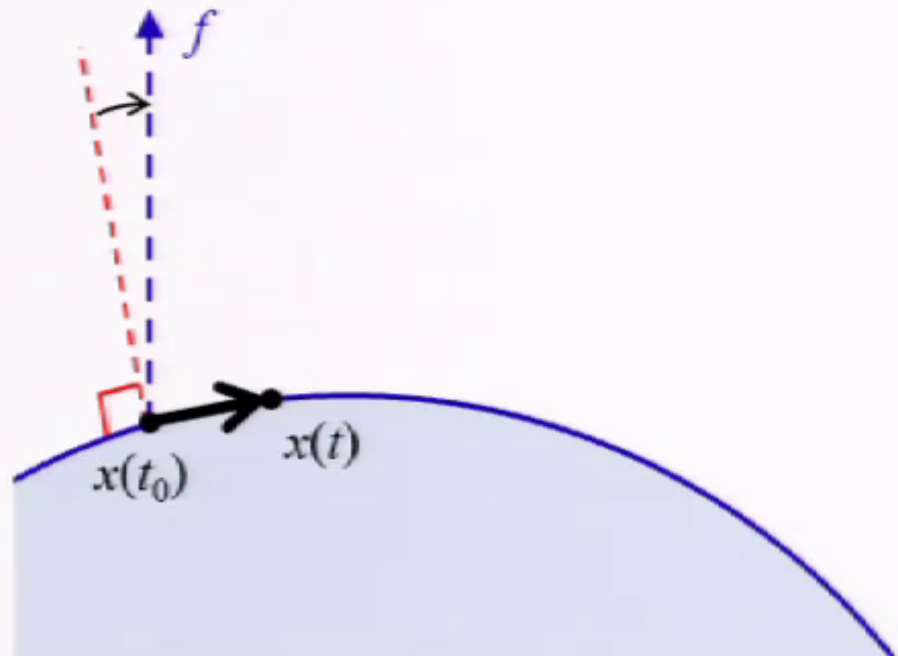

$x' \in -N_B(x) + f(x) + \varepsilon g(t,x),$
where g is T-periodic in time

Statement 2: $\bar{f}(s_0) = 0,\ \ \bar{f}'(s_0) \neq 0 \ \ \Rightarrow$

for all $|\varepsilon|$ sufficiently small the T-periodic sweeping process admits a T-periodic solution that converges to $L(s_0)$ when ε approaches zero.