Stars, Neighborhood Inclusion, and Network Centrality

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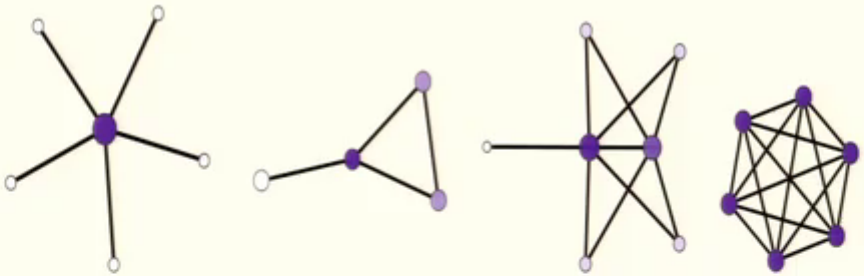

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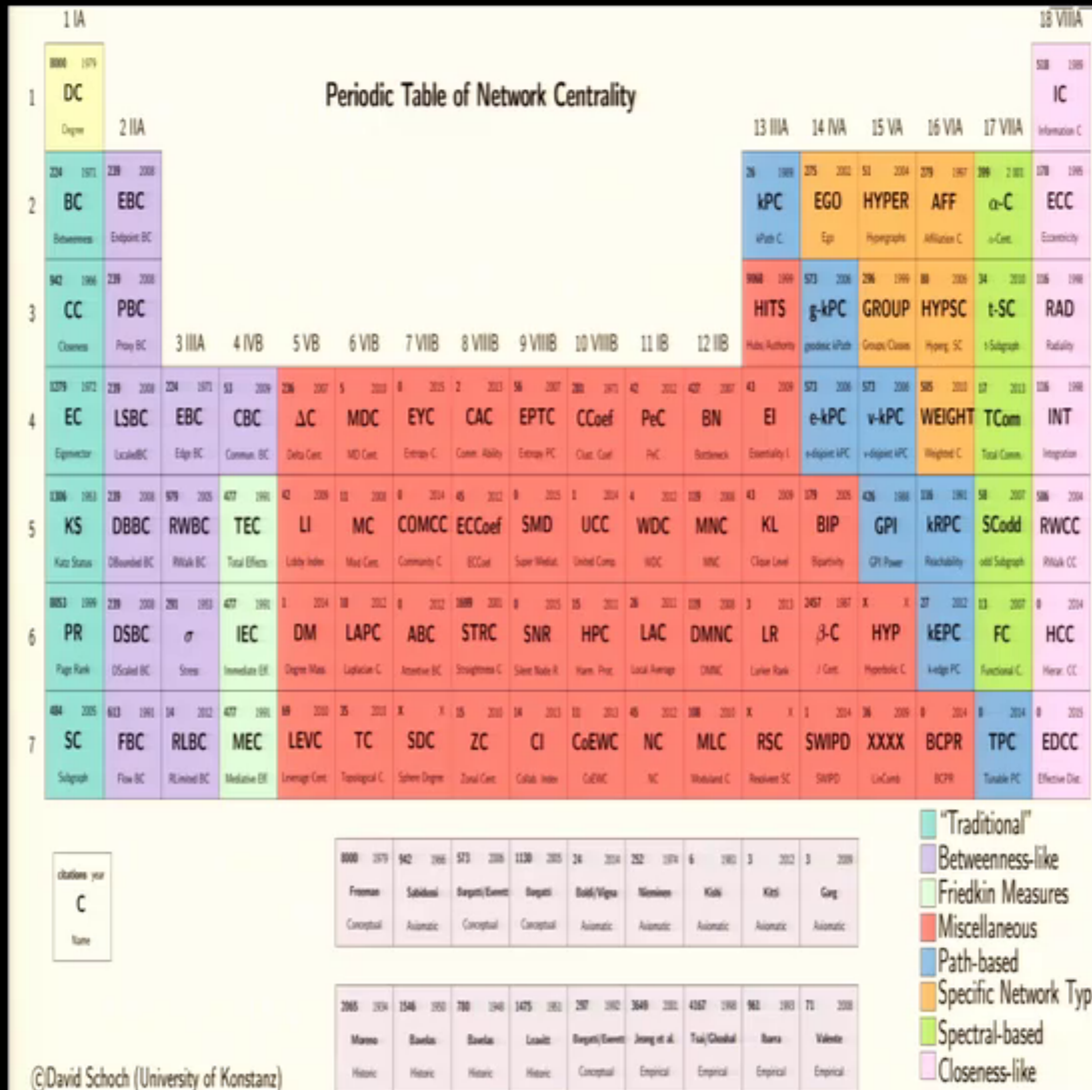
Periodic Table of Network Centrality
1 IA
2 IIA
3 IIIA
4 IVB
5 VB
6 VIB
7 VIIB
8 VIIIB
9 VIIIB
10 VIIIB
11 IB
12 IIB
13 IIIA
14 IVA
15 VA
16 VIA
17 VIIA
18 VIIIA
DC
IC
BC EBC kPC EGO HYPER AFF α-C ECC
CC PBC HITS g-kPC GROUP HYPSC t-SC RAD
EC LSBC EBC CBC ΔC MDC EYC CAC EPTC CCoef PeC BN EI e-kPC v-kPC WEIGHT TCom INT
KS DBBC RWBC TEC LI MC COMCC ECCoef SMD UCC WDC MNC KL BIP GPI kRPC SCodd RWCC
PR DSBC σ IEC DM LAPC ABC STRC SNR HPC LAC DMNC LR β-C HYP kEPC FC HCC
SC FBC RLBC MEC LEVC TC SDC ZC CI CoEWC NC MLC RSC SWIPD XXXX BCPR TPC EDCC
C
"Traditional"
Betweenness-like
Friedkin Measures
Miscellaneous
Path-based
Specific Network Type
Spectral-based
Closeness-like
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characterization of centrality indices

when is a mapping a measure of centrality?

axiomatization (based on intuitively plausible ideas):

- behavior under graph transformation (adding/switching edges)
- focused on change of values

$\Longrightarrow$ too restrictive

conceptual (based on practical properties):

- reasoning about features embodied in centrality indices
- relates formal definitions with substantive motivations

$\Longrightarrow$ does not allow for sharp distinctions and provable statements

> *"There is certainly no unanimity on exactly what centrality is or its conceptual foundations, and there is very little agreement on the proper procedure for its measurement."* [Freeman, 1979]

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justification of new indices

star property:

"A person located in the center of a star is universally assumed to be structurally more central than any other person in any other position in any other network of similar size."

[Freeman, 1979]

correlation:

"If centralities are not highly correlated, they indicate distinctive measures, associated with different outcomes."

[Valente, 2006]

empirical evidence:

"The degree of a network is a very simple measure, and more sophisticated measures may result in better results."

[Hong, 2015]

centrality in empirical research

as explanatory variable — *centrality effects*

> *"We have [...] adopted an 'agnostic' perspective by looking at some of the most common centrality/peripherality measures."*
>
> *[Pozzi et al., 2013]*

> *"The centrality measures based on graph spectral properties [...], in particular the subgraph centrality, show the best performance in identifying essential proteins [...]"* *[Estrada, 2005]*

issues:

- justification for appropriateness?
- underlying process?
- data fitting!

trial and error vs. substantive theory

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shared meaning of centrality concepts

and what we can learn from it

better relations $\iff$ better position

neighborhood-inclusion preorder: $N[u] \supseteq N(v) \implies u \succcurlyeq v$

theorem [Schoch & Brandes, in preparation]
standard centrality indices preserve neighborhood-inclusion preorder

$\implies$ many derivable statements about centrality

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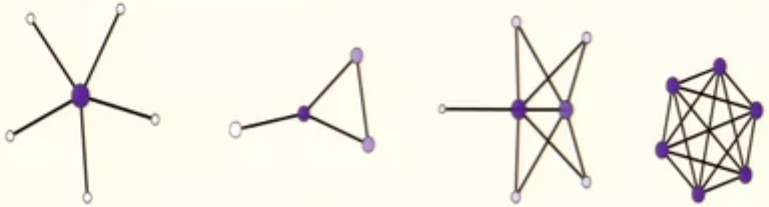
uniquely ranked graphs

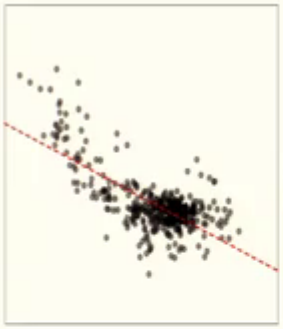
aka threshold graphs

all neighborhoods comparable $\Longrightarrow$ only one possible ranking



agreement of indices on threshold graphs
$\Longrightarrow$ strengthening star property

relations among indices contingent on data
$\Longrightarrow$ beware of testing on generated data!



distance vs. correlation

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preorders in networks

from quantitative to ordinal scale of measurement

equivalence relations "$\sim$" (structural, automorphic):
$u \sim v \implies u$ and v equally central (in any sense!)

dominance relations "$\succcurlyeq$" (structural, automorphic):
$u \succcurlyeq v \implies u$ more central than v (in any sense!)

benefits:

- progressive tightening of feasible rankings
- no analytically inconvenient differentials after graph transformation