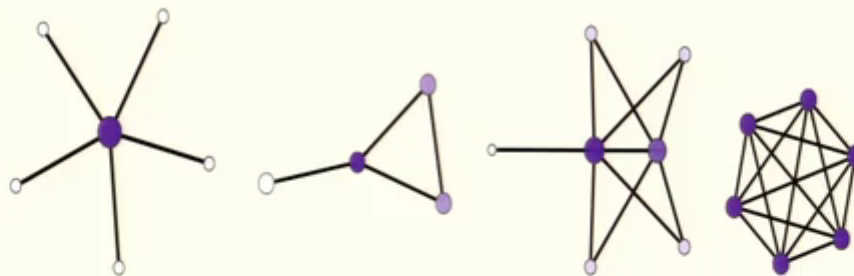


Stars, Neighborhood Inclusion, and Network Centrality

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“Traditional”

Betweenness-like

Friedkin Measures

Miscellaneous

Path-based

Specific Network Type

Spectral-based

Closeness-like

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- Traditional
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characterization of centrality indices

when is a mapping a measure of centrality?

axiomatization (based on intuitively plausible ideas):

- ▶ behavior under graph transformation (adding/switching edges)
- ▶ focused on change of values

⇒ too restrictive

conceptual (based on practical properties):

- ▶ reasoning about features embodied in centrality indices
- ▶ relates formal definitions with substantive motivations

⇒ does not allow for sharp distinctions and provable statements

*"There is certainly **no unanimity on exactly what centrality is** or its conceptual foundations, and there is very little agreement on the proper procedure for its measurement."* [Freeman, 1979]



justification of new indices

star property:

*"A person located in the **center of a star** is universally assumed to be structurally **more central** than any other person in any other position in any other network of similar size."*

[Freeman, 1979]

correlation:

*"If centralities are **not highly correlated**, they indicate **distinctive measures**, associated with different outcomes."*

[Valente, 2006]

empirical evidence:

*"The degree of a network is a very simple measure, and **more sophisticated measures** may result in better results."*

[Hong, 2015]



centrality in empirical research

as explanatory variable — centrality effects

“We have [...] adopted an ‘agnostic’ perspective by looking at some of the most common centrality/peripherality measures.”

[Pozzi et al., 2013]

“The centrality measures based on graph spectral properties [...], in particular the subgraph centrality, show the best performance in identifying essential proteins [...].”

[Estrada, 2005]

issues:

- ▶ justification for appropriateness?
- ▶ underlying process?
- ▶ data fitting!

trial and error vs. substantive theory



shared meaning of centrality concepts

and what we can learn from it

better relations \iff better position

neighborhood-inclusion preorder: $N[u] \supseteq N(v) \implies u \succcurlyeq v$

theorem [Schoch & Brandes, in preparation]

standard centrality indices preserve neighborhood-inclusion preorder

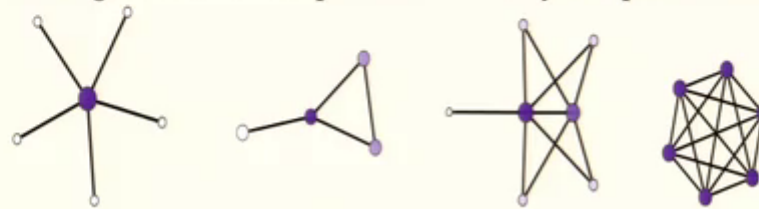
\implies many derivable statements about centrality



uniquely ranked graphs

aka threshold graphs

all neighborhoods comparable \Rightarrow only one possible ranking

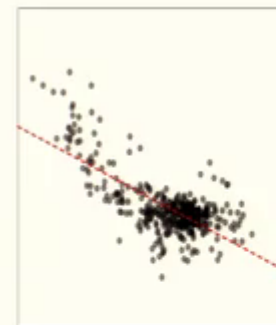


agreement of indices on threshold graphs

\Rightarrow strengthening star property

relations among indices **contingent on data**

\Rightarrow beware of testing on generated data!



distance vs. correlation



preorders in networks

from quantitative to ordinal scale of measurement

equivalence relations " \sim " (structural, automorphic):

$u \sim v \implies u$ and v equally central (in any sense!)

dominance relations " \succsim " (structural, automorphic):

$u \succsim v \implies u$ more central than v (in any sense!)

benefits:

- ▶ progressive tightening of feasible rankings
- ▶ no analytically inconvenient differentials after graph transformation

