# Routes to Long Term Predictability in Multi-Scale Systems – Analysis of the Ocean-Atmosphere System

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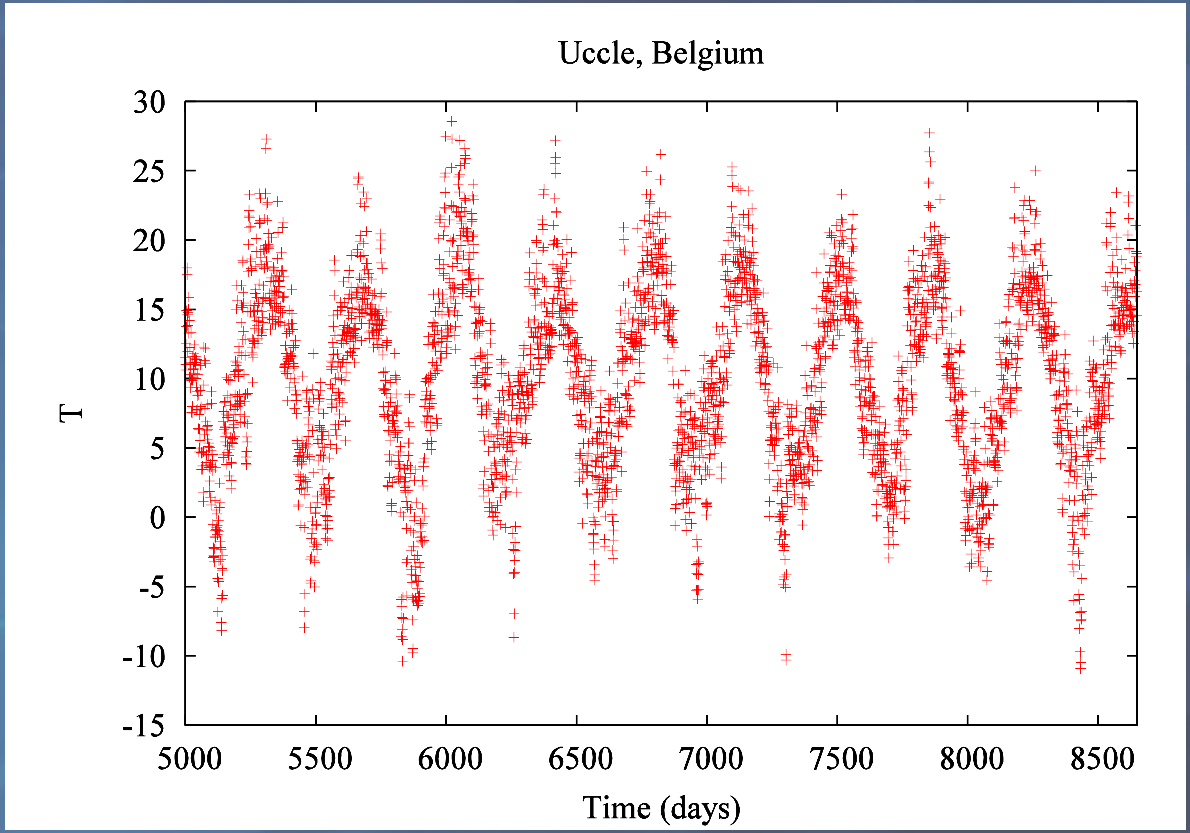
# Introduction

## Weather variability

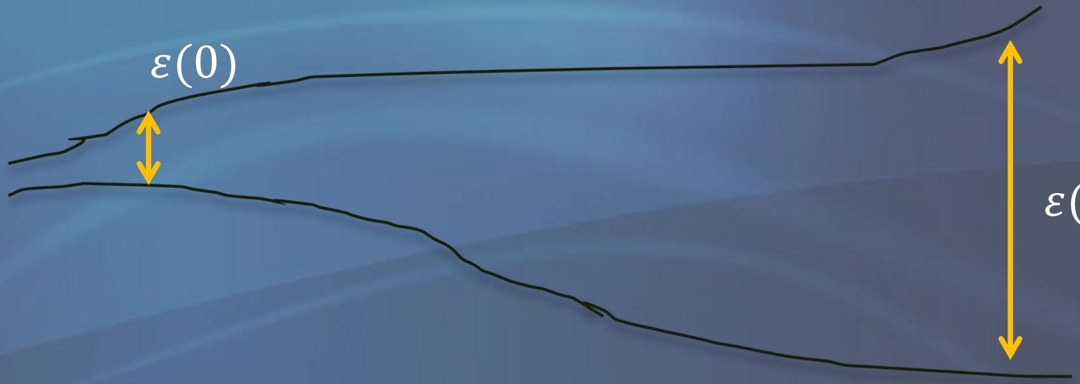


# Predictability

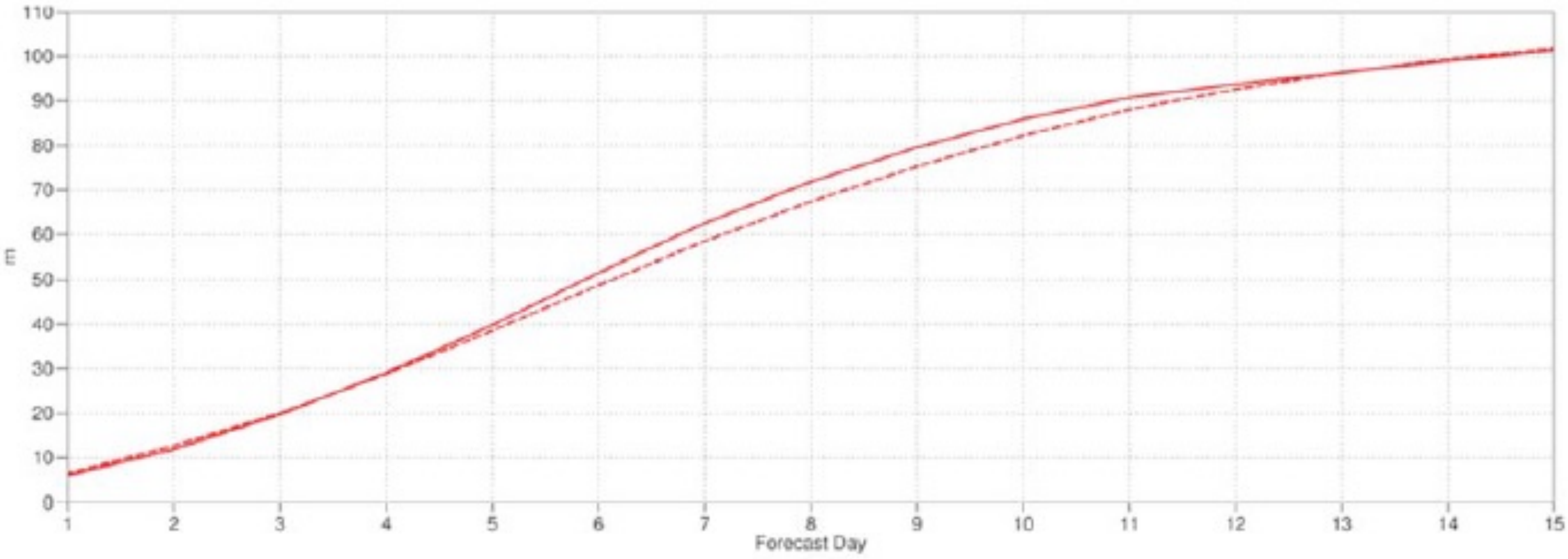
The property of sensitivity to initial (and model) uncertainties at the origin of the degradation of the quality of forecasts of atmospheric flows

Property already recognized by
Thompson (1957, Tellus, 9) and Lorenz (1963)

From a mathematical point of view: Poincaré (1888; 1908, Science et méthode)



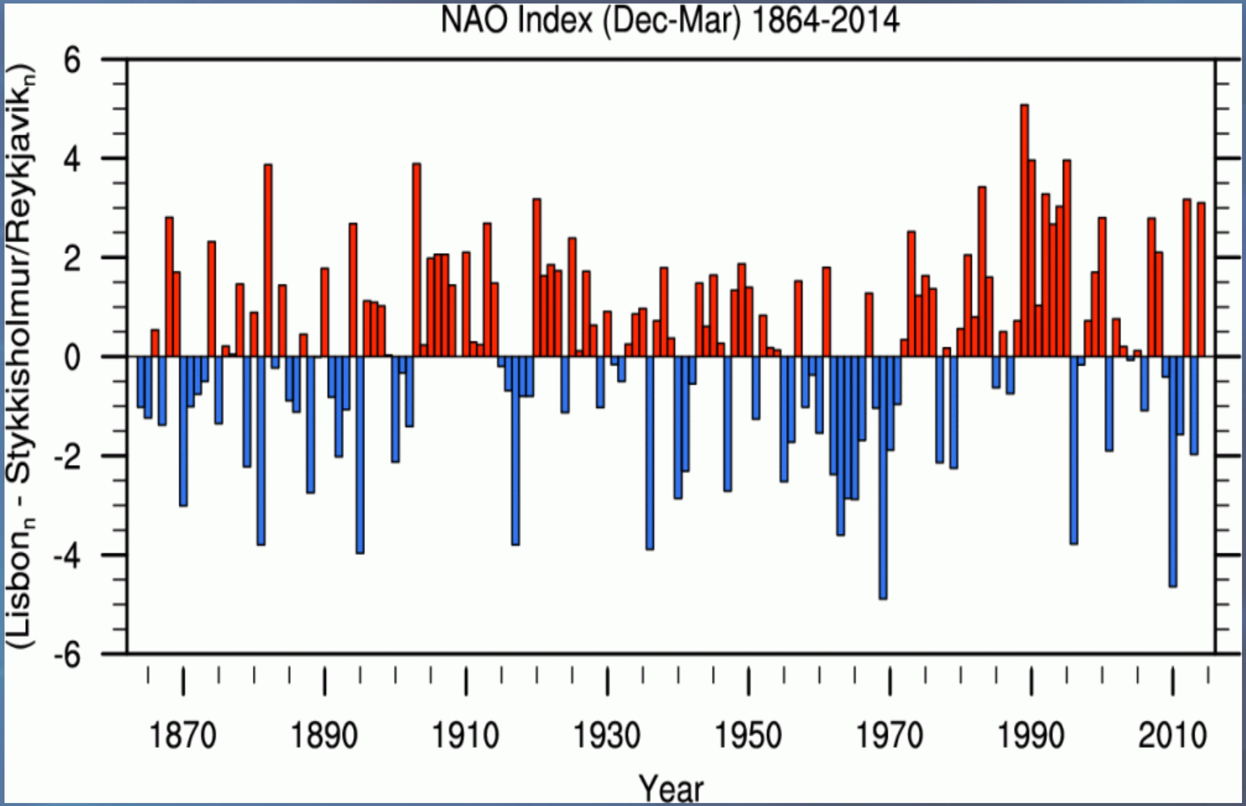
# Predictability: Mean square error evolution for ECMWF forecasts



Haiden T. et al, Evaluation of ECMWF forecasts, including 2014-2015 upgrades, Technical memorandum 765, ECMWF, 2015.

# Climate variability and predictability?

## North Atlantic Oscillation (NAO)



Positive NAO: Larger pressure difference between Lisbon ad Reykjavik
Negative NAO: Smaller pressure difference

Hurrell, 2015, "The Climate Data Guide: Hurrell North Atlantic Oscillation (NAO) Index (station-based)."

# General objective

- To characterize the predictability of the atmosphere on seasonal, inter-annual and decadal time scales

# Strategy

- Development of reduced-order climate models, and in particular coupled ocean-atmosphere models
- Analysis of the predictability of atmospheric and climate models of various resolutions
- Development of schemes for data assimilation in multi-scale systems, for ensemble forecasts …

# Aim of the presentation

Analysis of the properties of the solutions and of
the Lyapunov exponents of a coupled
Reduced order ocean-atmosphere model

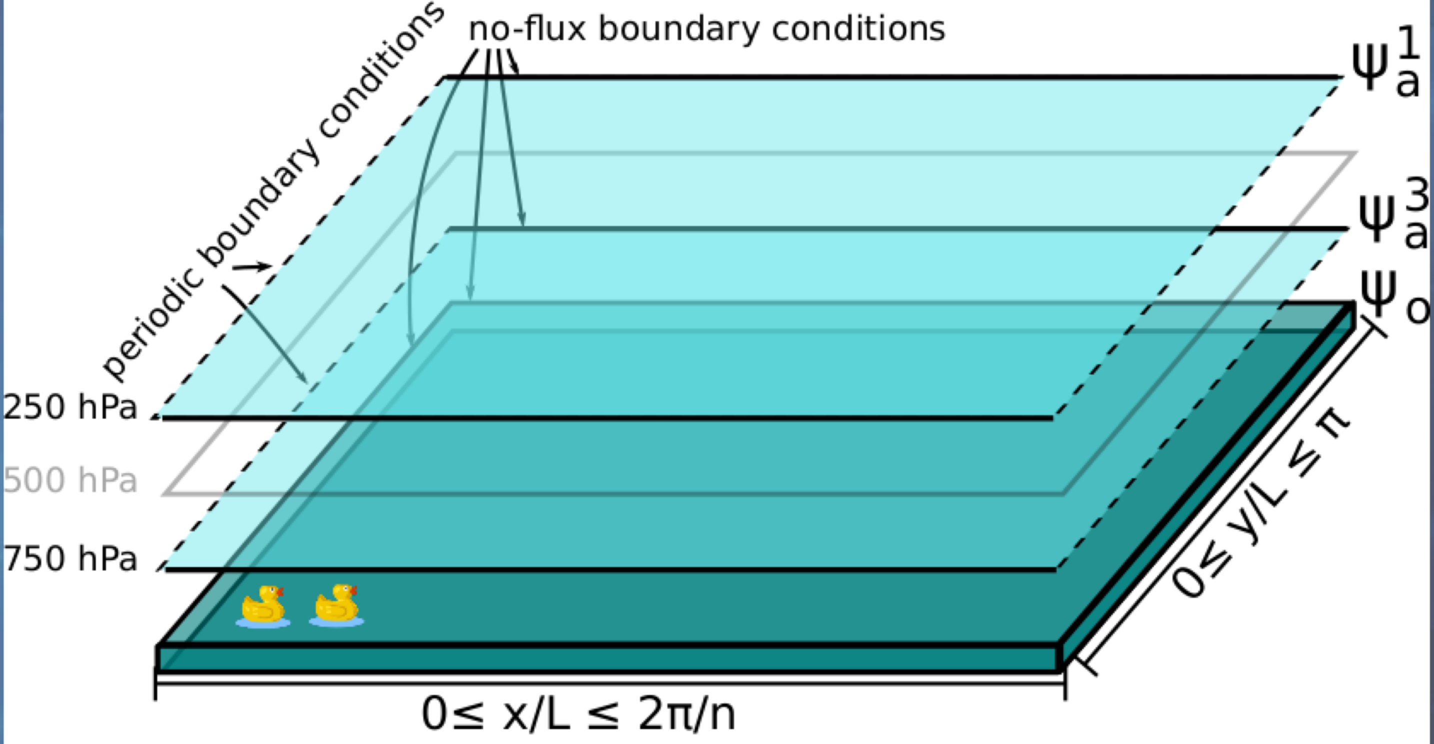
**More specifically we will use:**

- **Different versions of the Modular Arbitrary Order Ocean Atmosphere Model (MAOOAM) over the Atlantic**

- **New version of the coupled ocean-atmosphere model, called MAO(S)OAM with different boundary conditions (Vannitsem, Solé-Pomies and De Cruz, submitted, 2019)**

# An idealized low-order coupled ocean-atmosphere model

- QG model for both the ocean and the atmosphere



Vannitsem et al, 2015, Physica D, 309, 71-85, 2015, (**VDDG**)
De Cruz et al 2016, Geosci. Model Develop, 9, 2793-2808, 2016. (**MAOOAM**)

Latitudinal dependence of the radiative input

$$R_0 + C_0\sqrt{2}\cos y$$

Surface friction strength

$$\delta = \frac{d}{f_0} = \frac{C}{\rho H f_0}$$

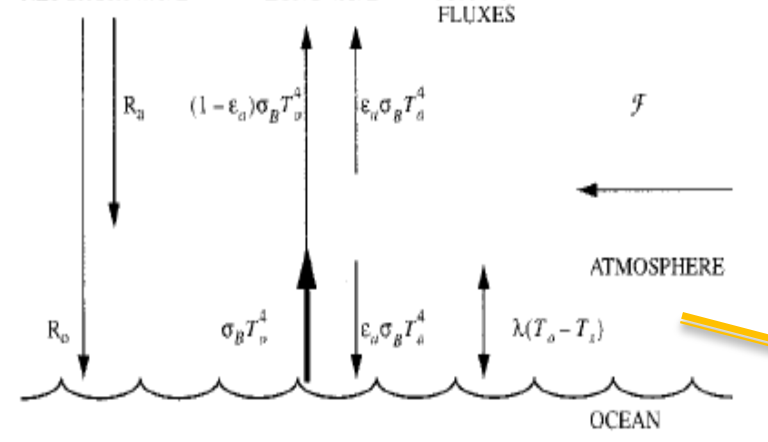
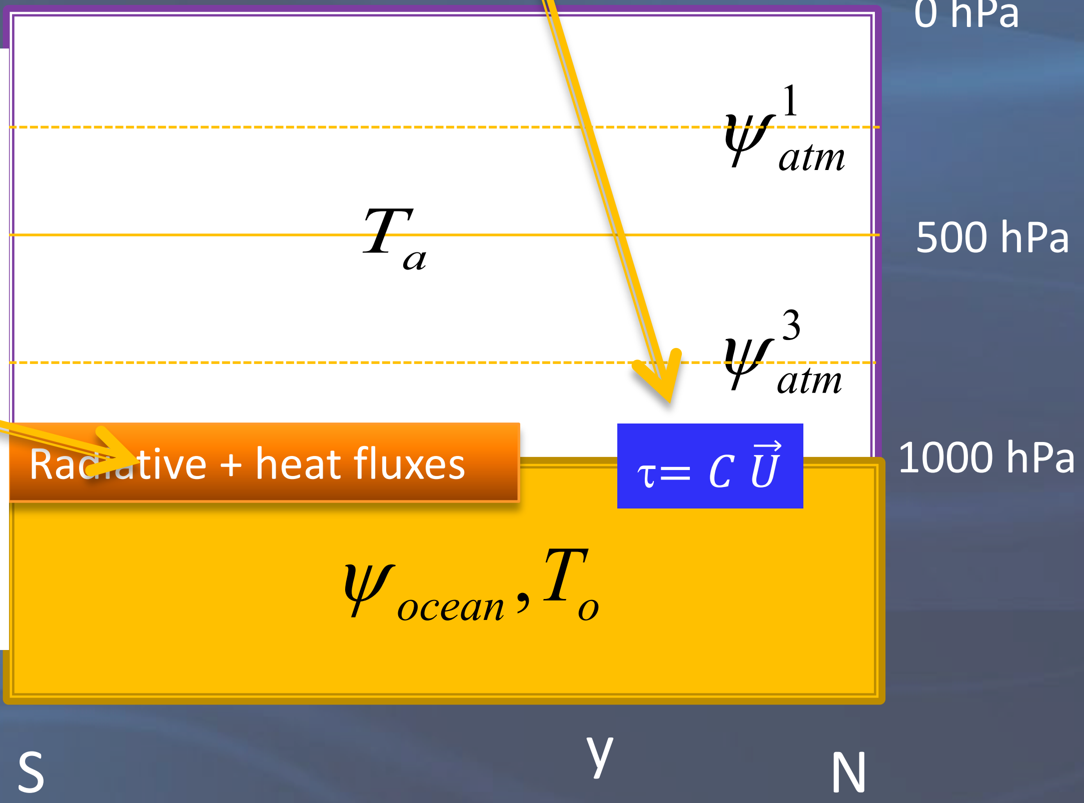


FIG. 2. Diagram of simple energy balance model on which Eqs. (1) and (2) are based. See appendix A for definition of symbols.

Barsugli & Battisti, 1998, JAS



# The dynamical equations for the ocean-atmosphere model

For the atmosphere

$$\begin{aligned}
\frac{\partial}{\partial t}\left(\nabla^2\psi_a^1\right) + J(\psi_a^1, \nabla^2\psi_a^1) + \beta\frac{\partial\psi_a^1}{\partial x} &= -k_d'\nabla^2(\psi^1 - \psi^3) + \frac{f_0}{\Delta p}\omega \\
\frac{\partial}{\partial t}\left(\nabla^2\psi_a^3\right) + J(\psi_a^3, \nabla^2\psi_a^3) + \beta\frac{\partial\psi_a^3}{\partial x} &= +k_d'\nabla^2(\psi_a^1 - \psi_a^3) - \frac{f_0}{\Delta p}\omega \\
&\quad -k_d\nabla^2(\psi_a^3 - \psi_o)
\end{aligned}$$

$$\gamma_a\left(\frac{\partial T_a}{\partial t} + J(\psi_a, T_a) - \sigma\omega\frac{p}{R}\right) = -\lambda(T_a - T_o) + E_{a,R}$$

Friction on a moving surface

$$E_{a,R} = \epsilon_a\sigma_B T_o^4 - 2\epsilon_a\sigma_B T_a^4 + R_a$$

For the ocean

$$\frac{\partial}{\partial t}\left(\nabla^2\psi_o - \frac{\psi_o}{L_R^2}\right) + J(\psi_o, \nabla^2\psi_o) + \beta\frac{\partial\psi_o}{\partial x} = -r\nabla^2\psi_o + \frac{curl_z\tau}{\rho h}$$

$$\gamma_o\left(\frac{\partial T_o}{\partial t} + J(\psi_o, T_o)\right) = -\lambda(T_o - T_a) + E_R$$

Curl of wind stress

$$E_R = -\sigma_B T_o^4 + \epsilon_a\sigma_B T_a^4 + R_o$$

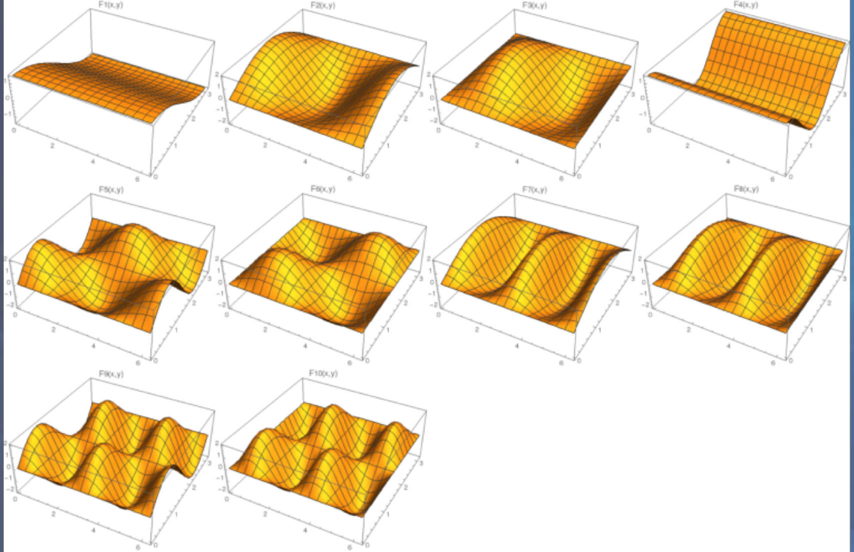
# Truncation of Fourier series

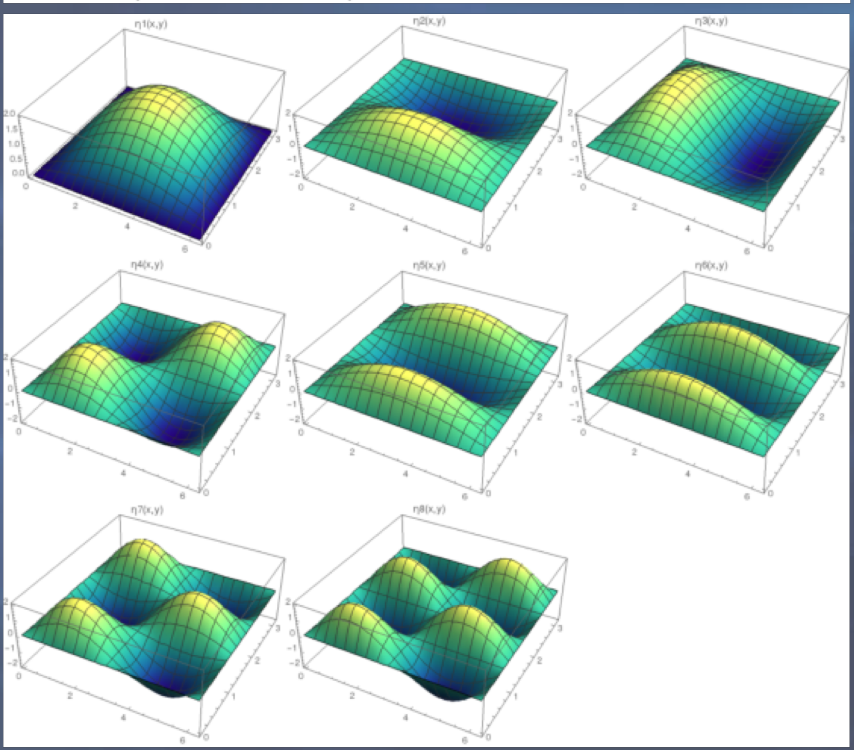
$$\psi = \sum_{k=1}^{K} \psi_k F_k$$

$$\theta = \sum_{k=1}^{K} \theta_k F_k$$

$$\psi_o = \sum_{i=1}^{8} \psi_{o,i}\phi_i, \qquad \delta T_o = \sum_{i=1}^{8} T_{o,i}\phi_i$$

36-variable model



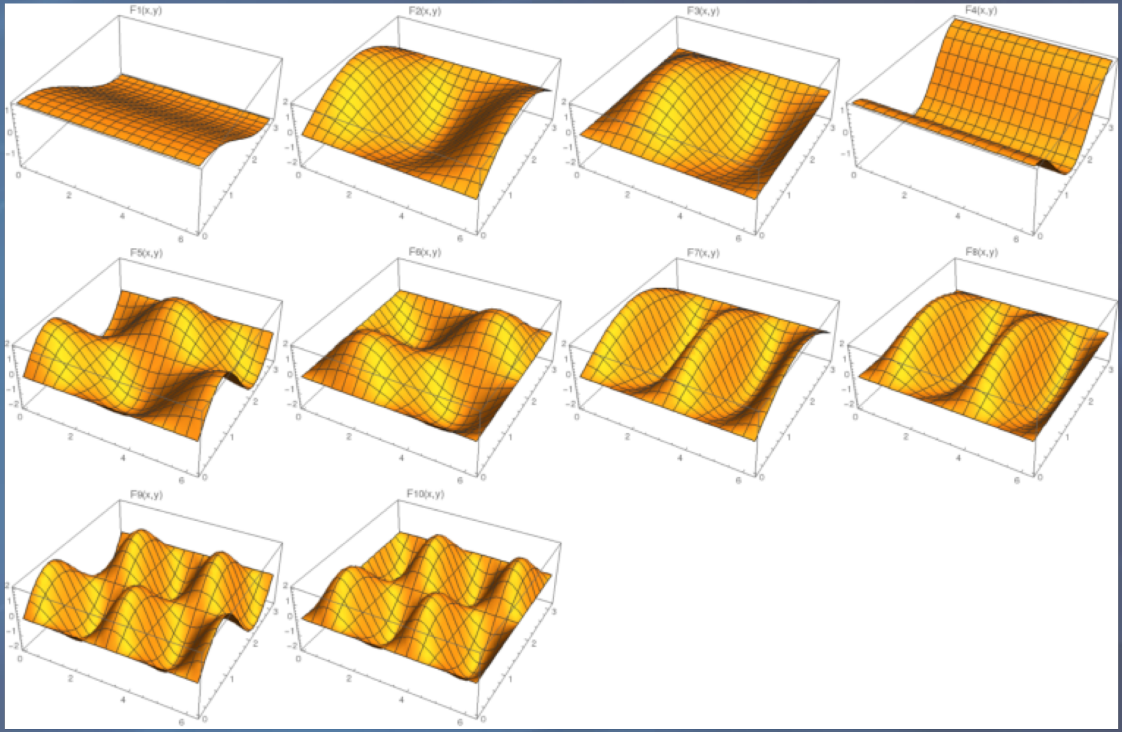


Vannitsem et al, Physica D, 2015

# New version of the model: MAOSOAM

Rectangular geometry

Channel flow for the ocean too



Low-order system of 40 variables, 20 for the ocean and 20 for the atmosphere

# Both model versions are available on Github

The latest version of MAOOAM: https://github.com/Climdyn/MAOOAM/

The latest version of MAOSOAM (with lyapunov code) :

https://github.com/solepomies/MAOOAM/tree/southernocean

Arbitray number of modes can be fixed

# Parameter values

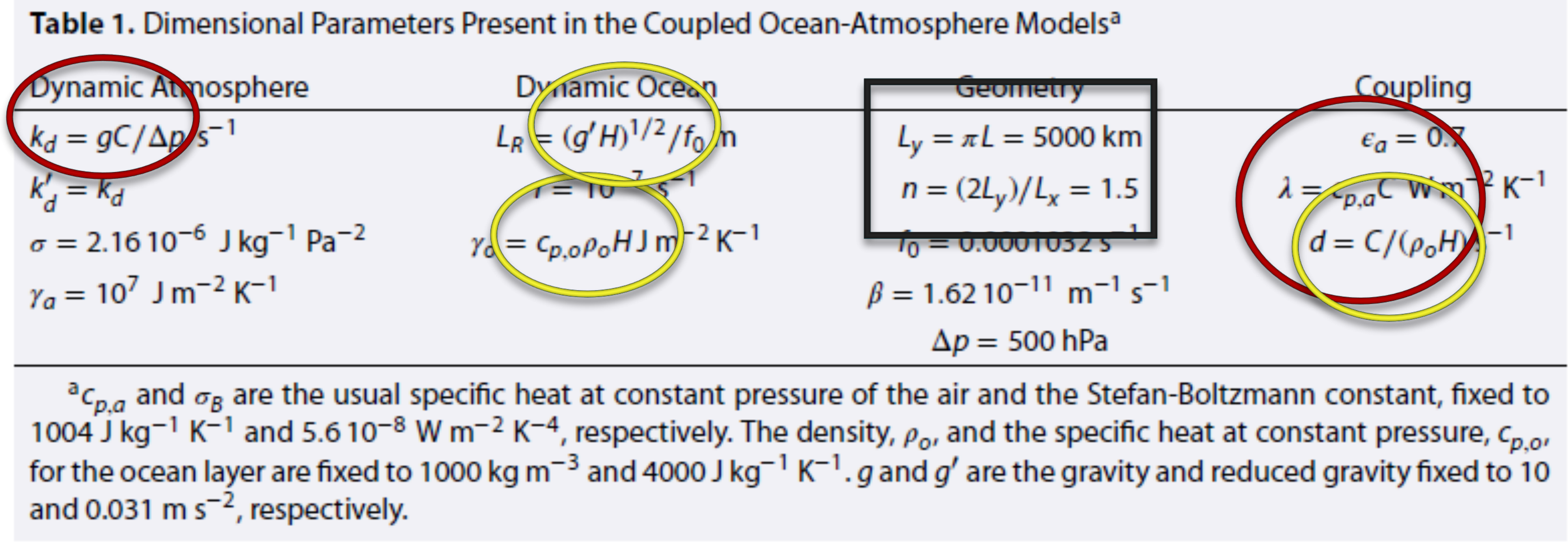


**Table 1.** Dimensional Parameters Present in the Coupled Ocean-Atmosphere Models[a]

| Dynamic Atmosphere | Dynamic Ocean | Geometry | Coupling |
|---|---|---|---|
| $k_d = gC/\Delta p \ \mathrm{s}^{-1}$ | $L_R = (g'H)^{1/2}/f_0 \ \mathrm{m}$ | $L_y = \pi L = 5000 \ \mathrm{km}$ | $\epsilon_a = 0.7$ |
| $k'_d = k_d$ | $r = 10^{-7} \ \mathrm{s}^{-1}$ | $n = (2L_y)/L_x = 1.5$ | $\lambda = c_{p,a}C \ \mathrm{W\,m^{-2}\,K^{-1}}$ |
| $\sigma = 2.16\,10^{-6} \ \mathrm{J\,kg^{-1}\,Pa^{-2}}$ | $\gamma_o = c_{p,o}\rho_o H \ \mathrm{J\,m^{-2}\,K^{-1}}$ | $f_0 = 0.0001032 \ \mathrm{s}^{-1}$ | $d = C/(\rho_o H) \ \mathrm{s}^{-1}$ |
| $\gamma_a = 10^7 \ \mathrm{J\,m^{-2}\,K^{-1}}$ | | $\beta = 1.62\,10^{-11} \ \mathrm{m^{-1}\,s^{-1}}$ | |
| | | $\Delta p = 500 \ \mathrm{hPa}$ | |

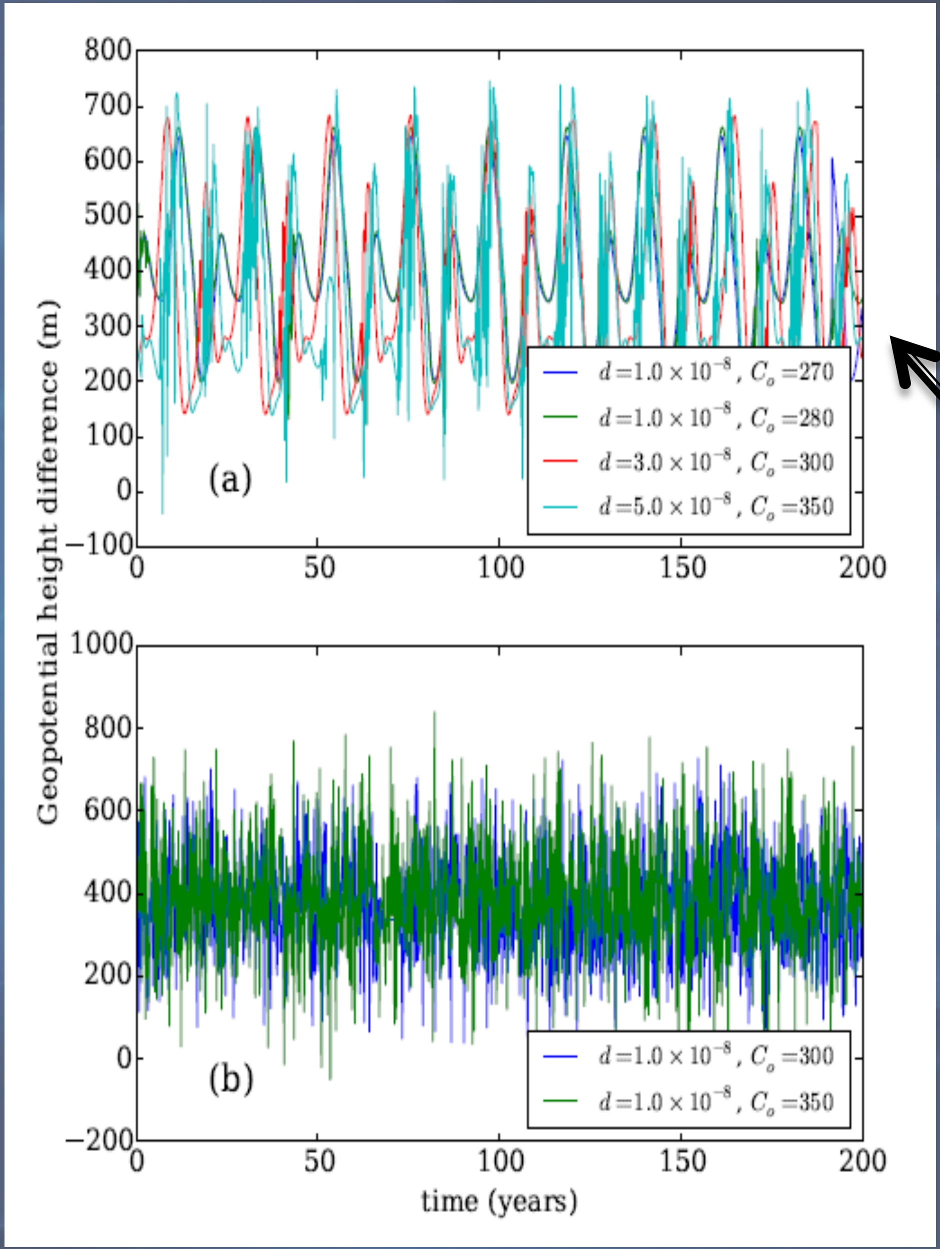
[a] $c_{p,a}$ and $\sigma_B$ are the usual specific heat at constant pressure of the air and the Stefan-Boltzmann constant, fixed to 1004 $\mathrm{J\,kg^{-1}\,K^{-1}}$ and 5.6 $10^{-8}$ $\mathrm{W\,m^{-2}\,K^{-4}}$, respectively. The density, $\rho_o$, and the specific heat at constant pressure, $c_{p,o}$, for the ocean layer are fixed to 1000 $\mathrm{kg\,m^{-3}}$ and 4000 $\mathrm{J\,kg^{-1}\,K^{-1}}$. $g$ and $g'$ are the gravity and reduced gravity fixed to 10 and 0.031 $\mathrm{m\,s^{-2}}$, respectively.

Vannitsem, 2015, Geophys Res Lett

**3 important parameters: n, C and $C_0$**

# Variability and Lyapunov instability properties of the coupled ocean-atmosphere system

# Solution of the VDDG model

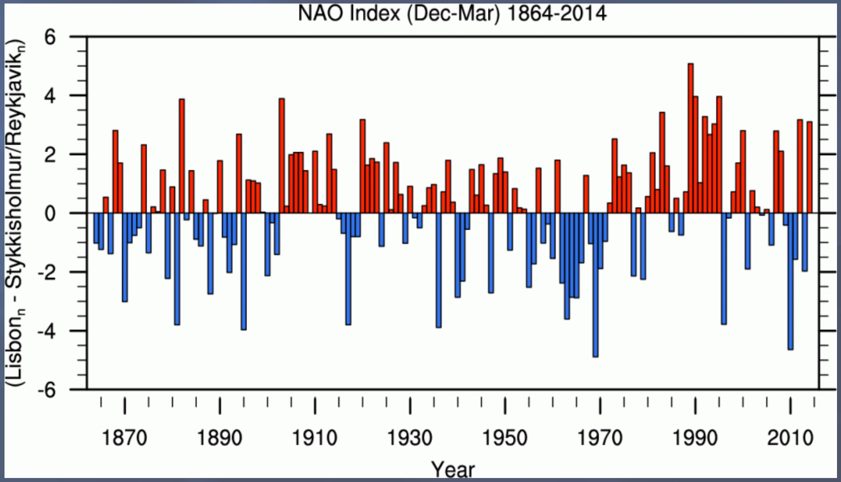


Geop. Height. Diff. between
two points from the North
and South parts of the domain

Vannitsem et al, 2015, Physica D

**n=1.5**
**H=500 m**

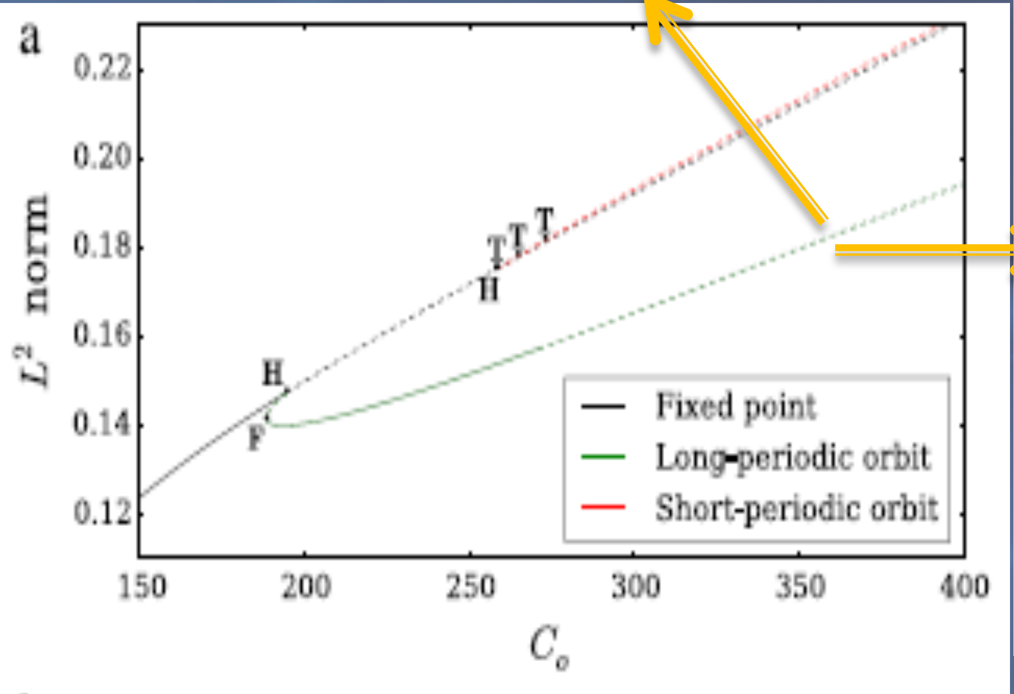
Decadal Variability

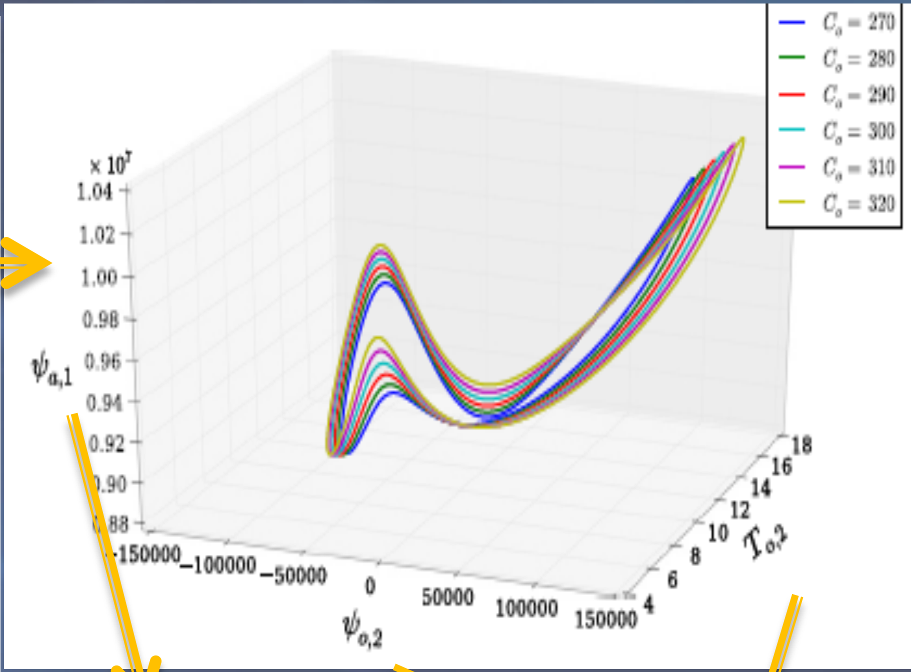


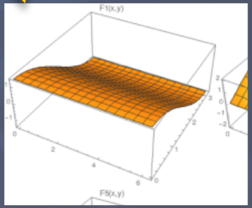
# Bifurcation diagram

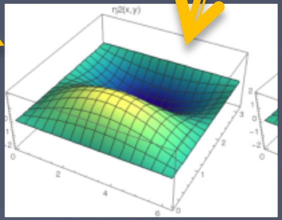
**$n=1.5$**
**$H=500$ m**
**$\lambda=20$ W m$^{-2}$ K$^{-1}$**
**$C=0.005$ kg m$^{-2}$ s$^{-1}$**

Slow branch



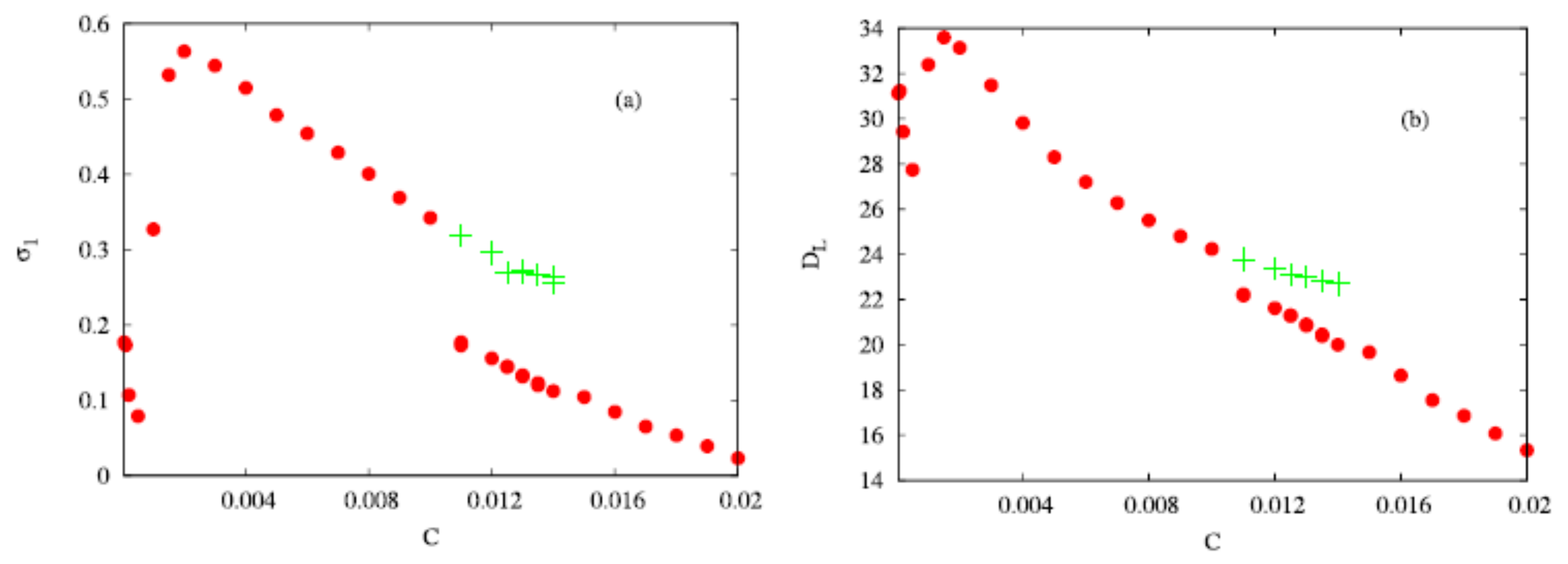






Vannitsem, Demaeyer, De Cruz, Ghil, Physica D, 2015

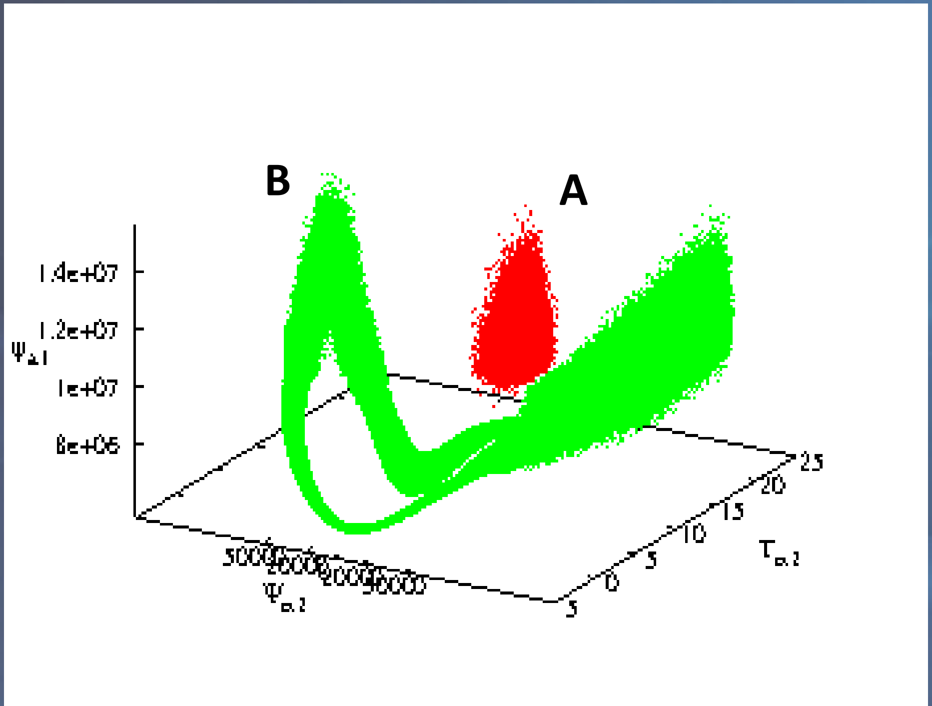
# Changes of the Lyapunov instability as a function of friction



n=1.5
H=164 m
$C_0$=310 W/m$^2$

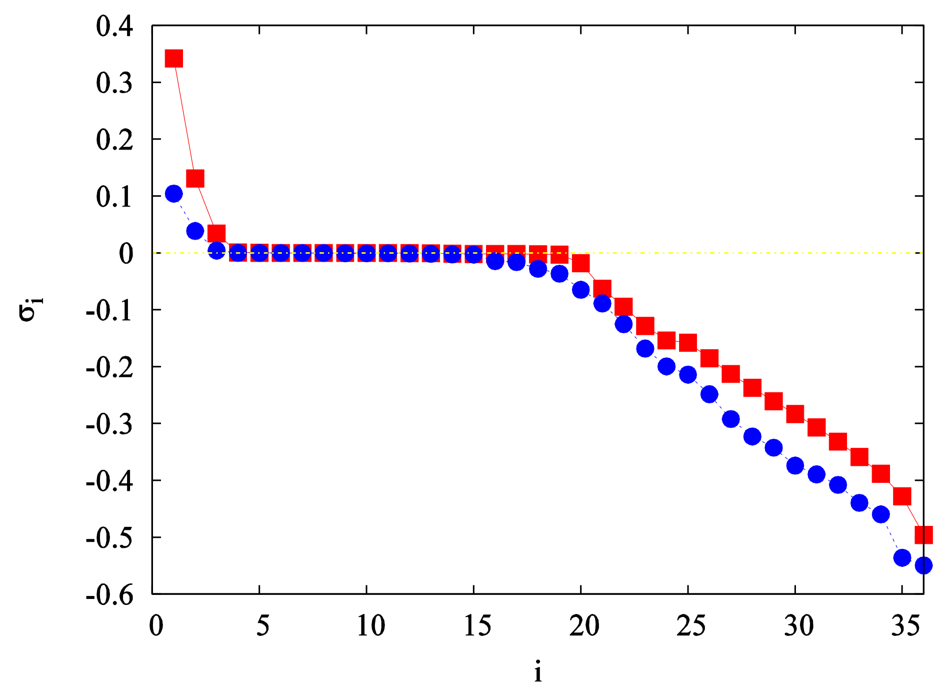
Vannitsem, 2017, Chaos, 27, 032101

Consider 2 different attractors (obtained with different friction parameters)



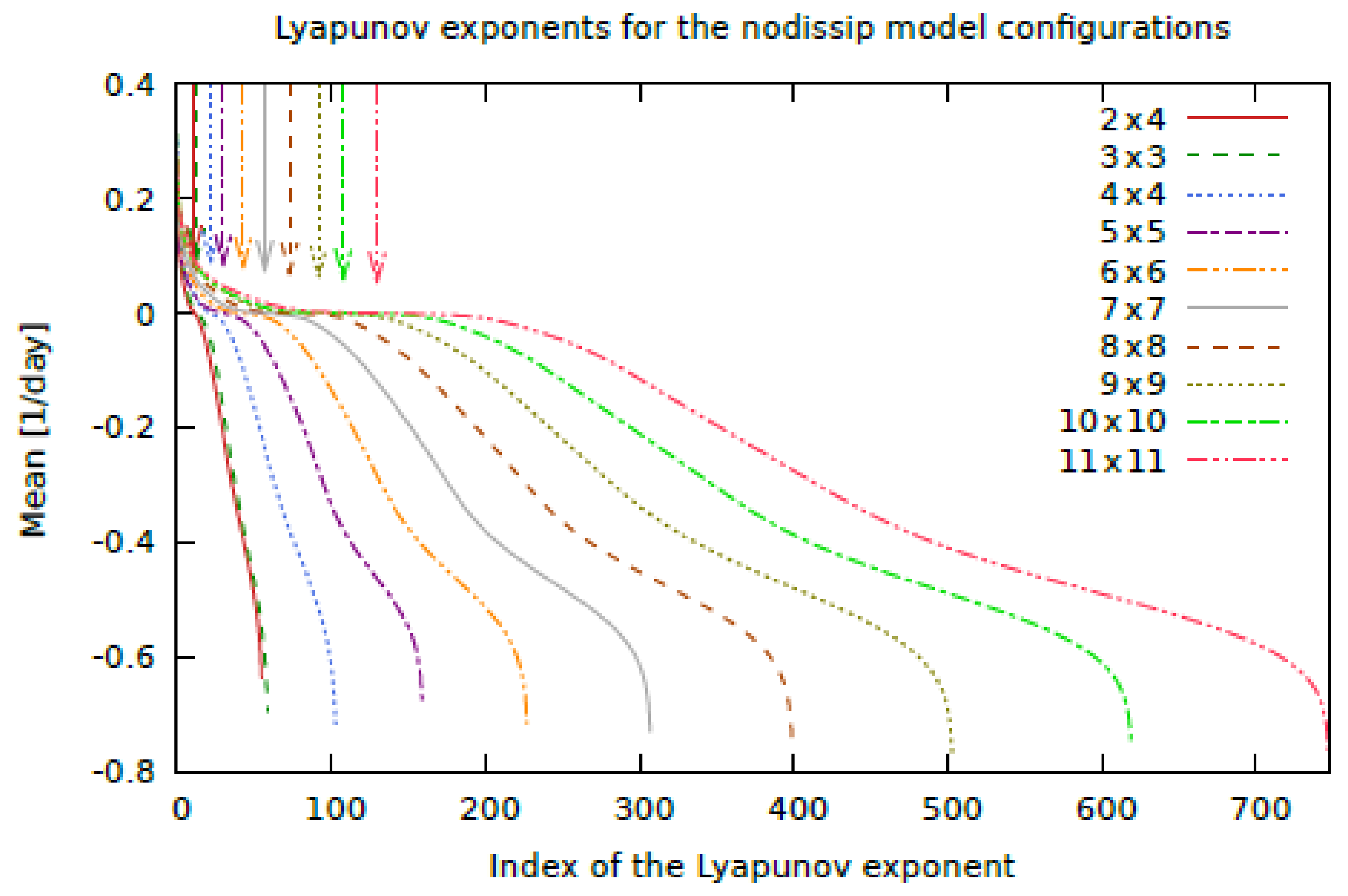
**n=1.5**
**H=164 m**
**$C_0=S_0=310$ W/m$^2$**

## Lyapunov spectra



**Vannitsem, 2017, Chaos, 27, 032101**

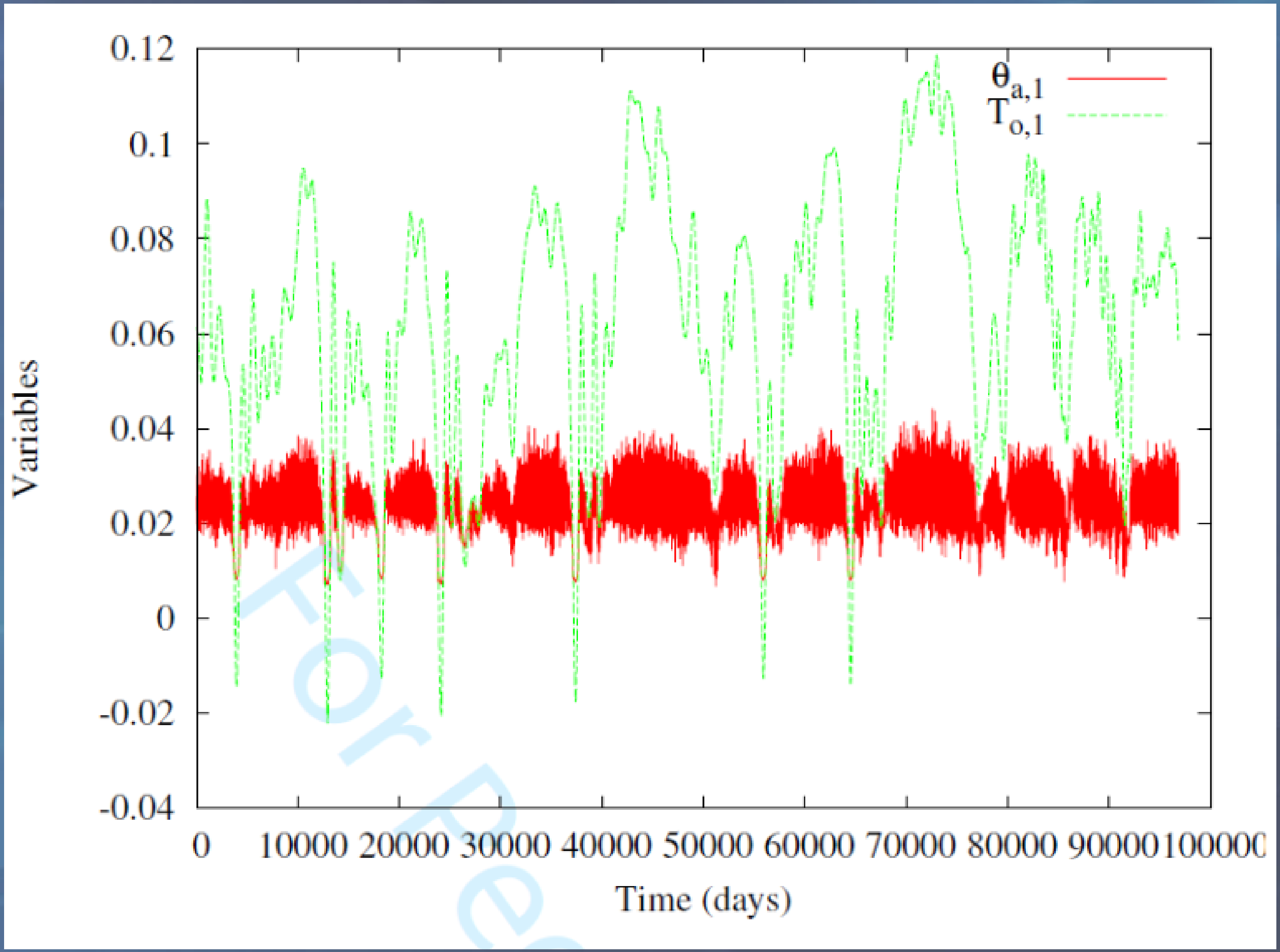
# Increasing the resolution

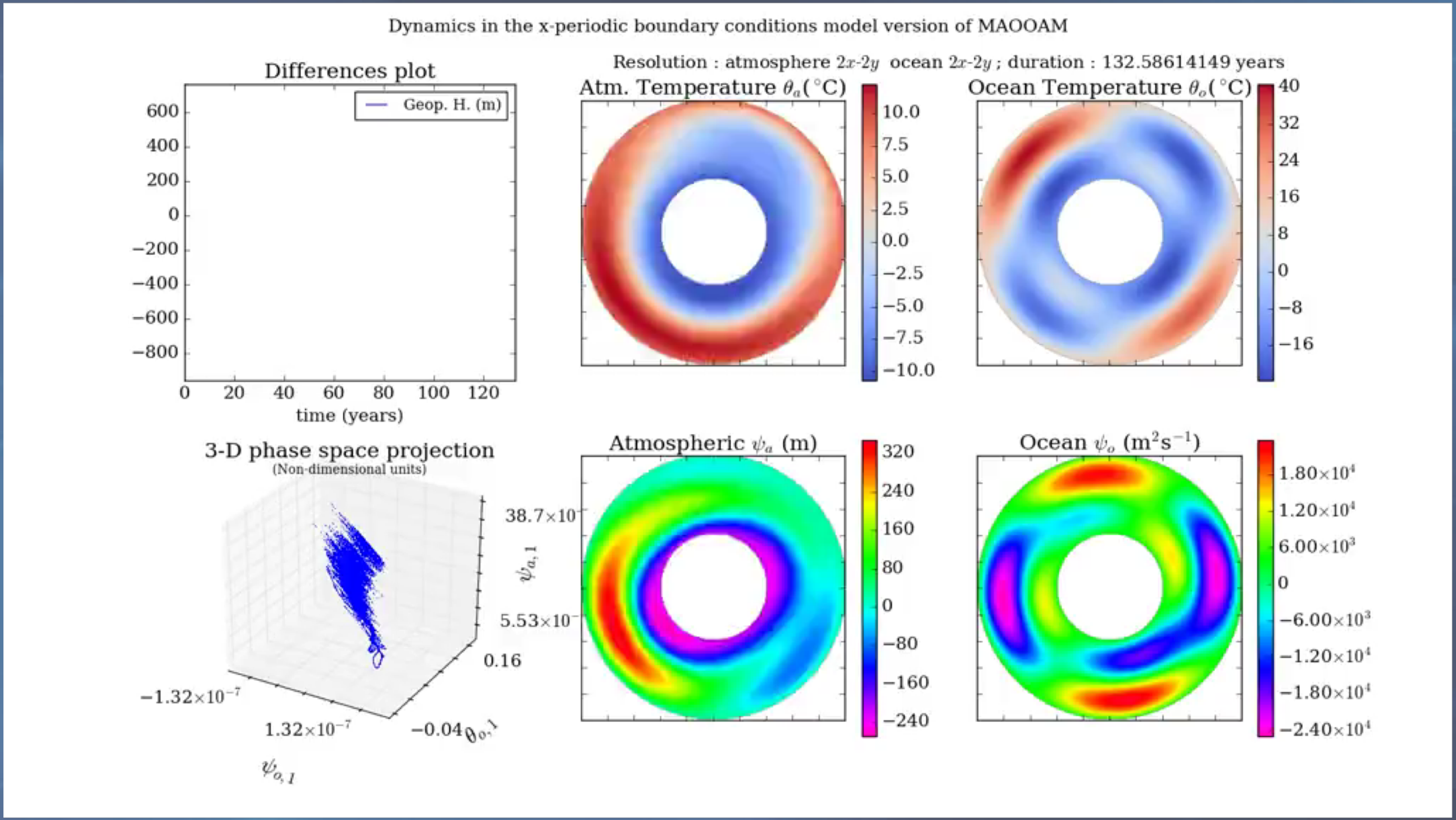


De Cruz, L., S. Schubert, J. Demaeyer, V. Lucarini, et S. Vannitsem, Exploring the Lyapunov instability properties of high-dimensional atmospheric and climate models, *Nonlinear Processes in Geophysics*, 25, 387-412, 2018

# Results with the new version of the model, MAOSOAM

**$n=1.7$**
**$H=1000$ m**
**$Co=350$ W $m^{-2}$**
**$C=0.016$ kg $m^{-2}$ $s^{-1}$**

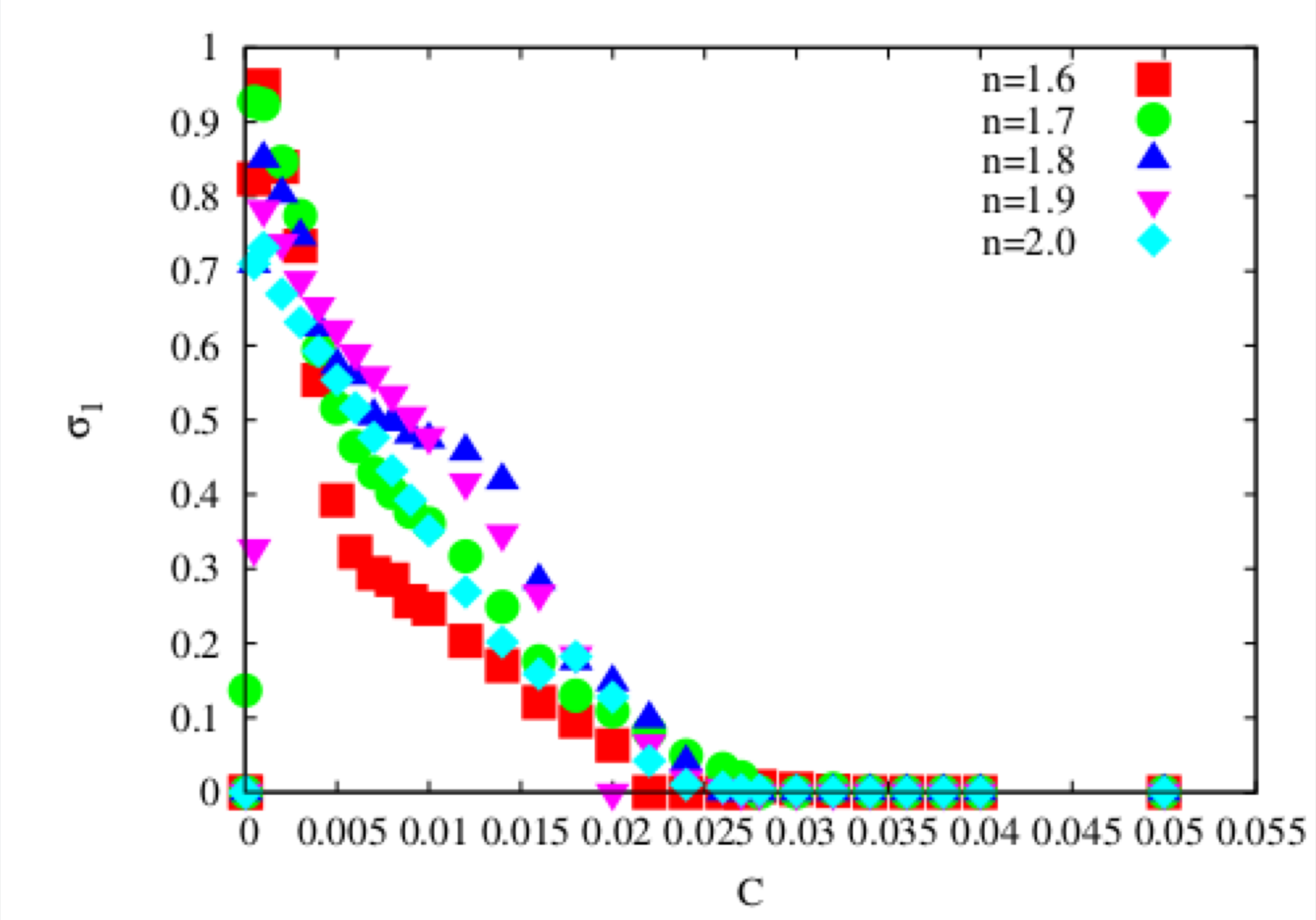



Dynamics in the x-periodic boundary conditions model version of MAOOAM
Resolution : atmosphere $2x$-$2y$ ocean $2x$-$2y$ ; duration : 132.58614149 years
Differences plot
Geop. H. (m)
600
400
200
0
−200
−400
−600
−800
0
20
40
60
80
100
120
time (years)
Atm. Temperature $\theta_a$(°C)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Ocean Temperature $\theta_o$(°C)
40
32
24
16
8
0
−8
−16
3-D phase space projection
(Non-dimensional units)
38.7×10
5.53×10
$\psi_{a,1}$
0.16
−0.04
$\theta_{o,1}$
−1.32×10$^{-7}$
1.32×10$^{-7}$
$\psi_{o,1}$
Atmospheric $\psi_a$ (m)
320
240
160
80
0
−80
−160
−240
Ocean $\psi_o$ (m$^2$s$^{-1}$)
1.80×10$^4$
1.20×10$^4$
6.00×10$^3$
0
−6.00×10$^3$
−1.20×10$^4$
−1.80×10$^4$
−2.40×10$^4$

# Results with the new version of the model, MAOSOAM

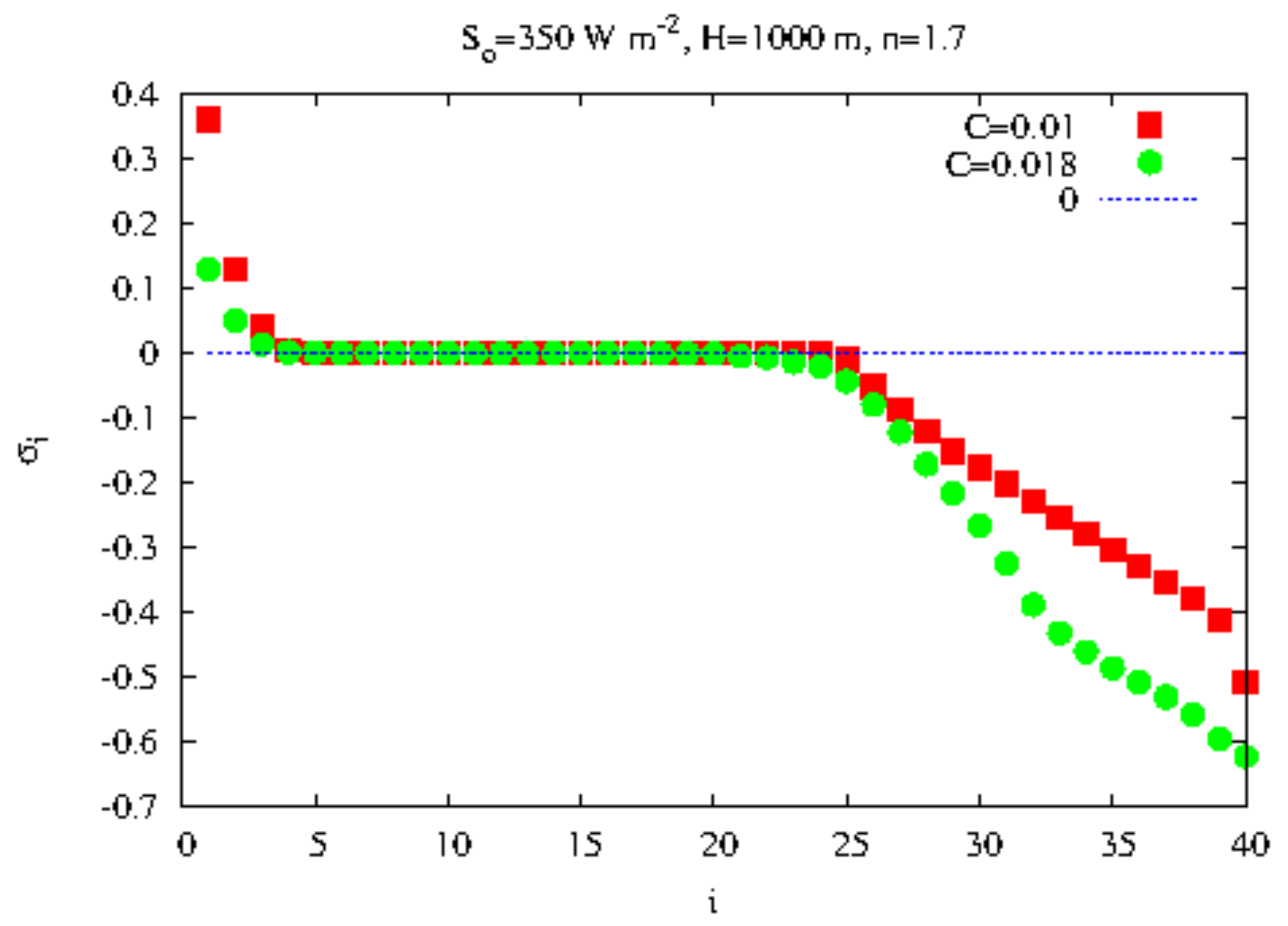
Dominant Lyapunov exponent for different values of C, for different aspect ratios n, $n = 2\,L_y/L_x$

H= 1000 m, $C_o$ = 350 W/m$^2$



# Results with the new version of the model, MAOSOAM

## Lyapunov spectra for 2 different values of the friction



# Some conclusions

In the model version with closed boundaries in the ocean (VDDG or MAOOAM, n=1.5):

- Low frequency variability development associated with the friction and energy balance coupling. The attractor is developing around the unstable periodic orbit
- Large number of Lyapunov exponents close to 0

In the new model version with channel flow:

- Low frequency variability related to excursions close to an unstable periodic orbit
- Large number of Lyapunov exponents close to 0

# Current research in this context

- Data assimilation in such systems (manuscript in preparation)
- Coupling with other components of the climate system

Some references

De Cruz, L., J. Demaeyer and S. Vannitsem, **A modular arbitrary-order ocean-atmosphere model: MAOOAM V1.0**, Geoscientific Model Development, 9, 2793-2808 , 2016. (GITHUB)

De Cruz, L., S. Schubert, J. Demaeyer, V. Lucarini, et S. Vannitsem, Exploring the Lyapunov instability properties of high-dimensional atmospheric and climate models, *Nonlinear Processes in Geophysics*, 25, 387-412, 2018

Vannitsem S. **Predictability of large-scale atmospheric motions: Lyapunov exponents and error dynamics**, Chaos, 27, 032101, 2017, doi: 10.1063/1.4979042

Vannitsem, S., L De Cruz, J Demayer, M Ghil. **Low-frequency variability and heat transport in a low-order nonlinear coupled ocean-atmosphere model**. Physica D, 309, 71-85, 2015. On ArXiv too.

Vannitsem S. and V. Lucarini, **Statistical and Dynamical Properties of Covariant Lyapunov Vectors in a Coupled Atmosphere-Ocean Model - Multiscale Effects, Geometric Degeneracy, and Error Dynamics**, J. Phys. A., 2016. ArXiv: **arXiv:1510.00298v3**

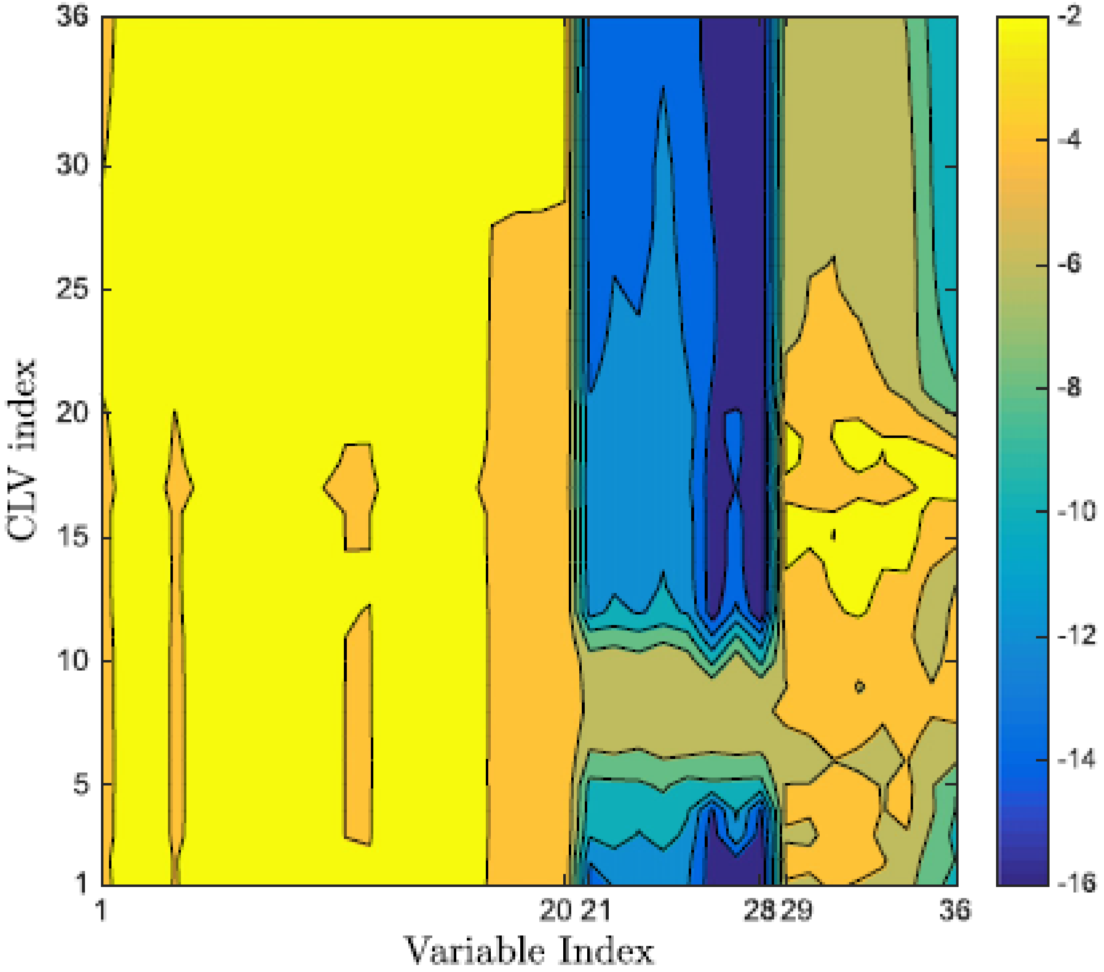
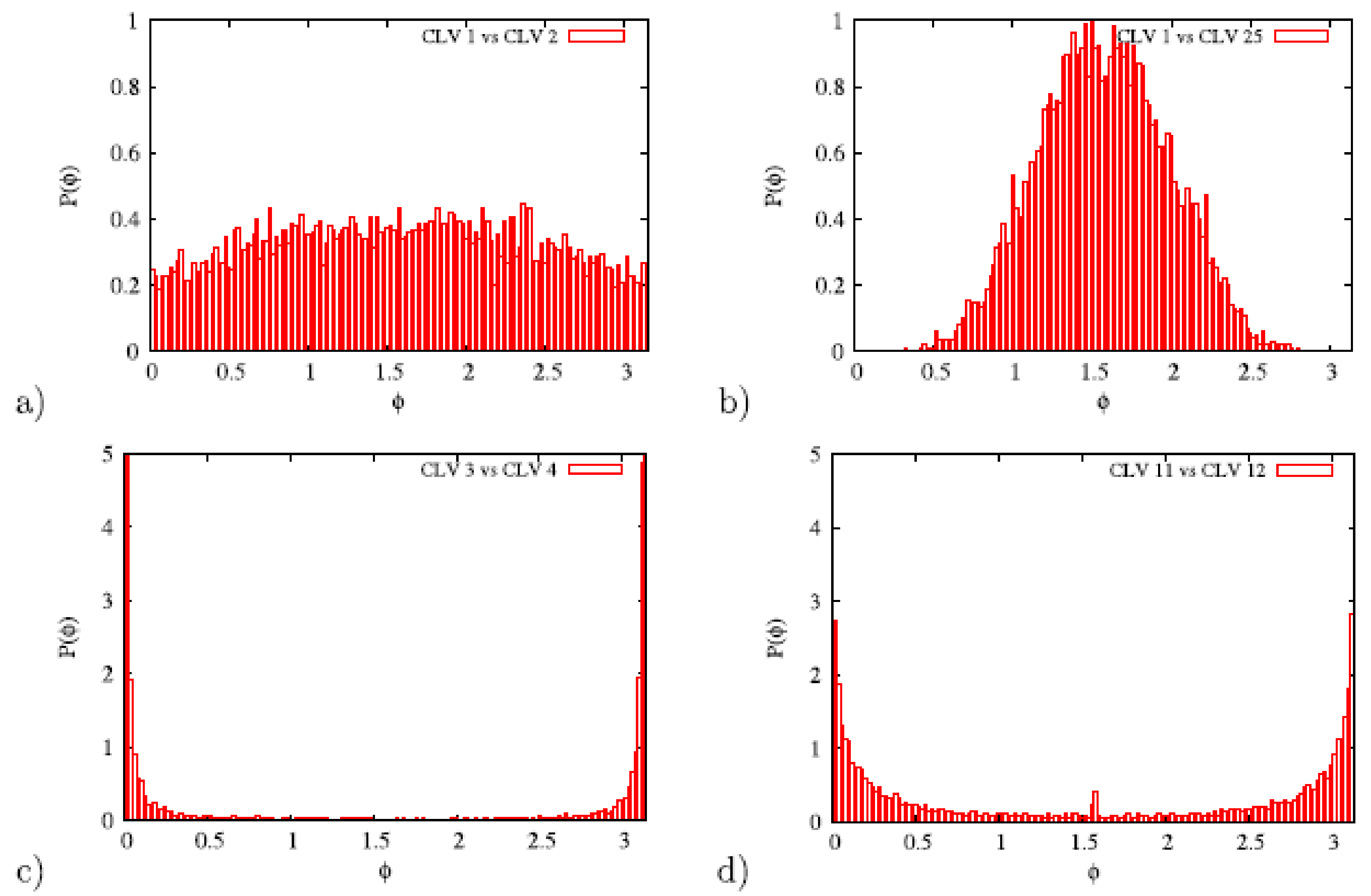
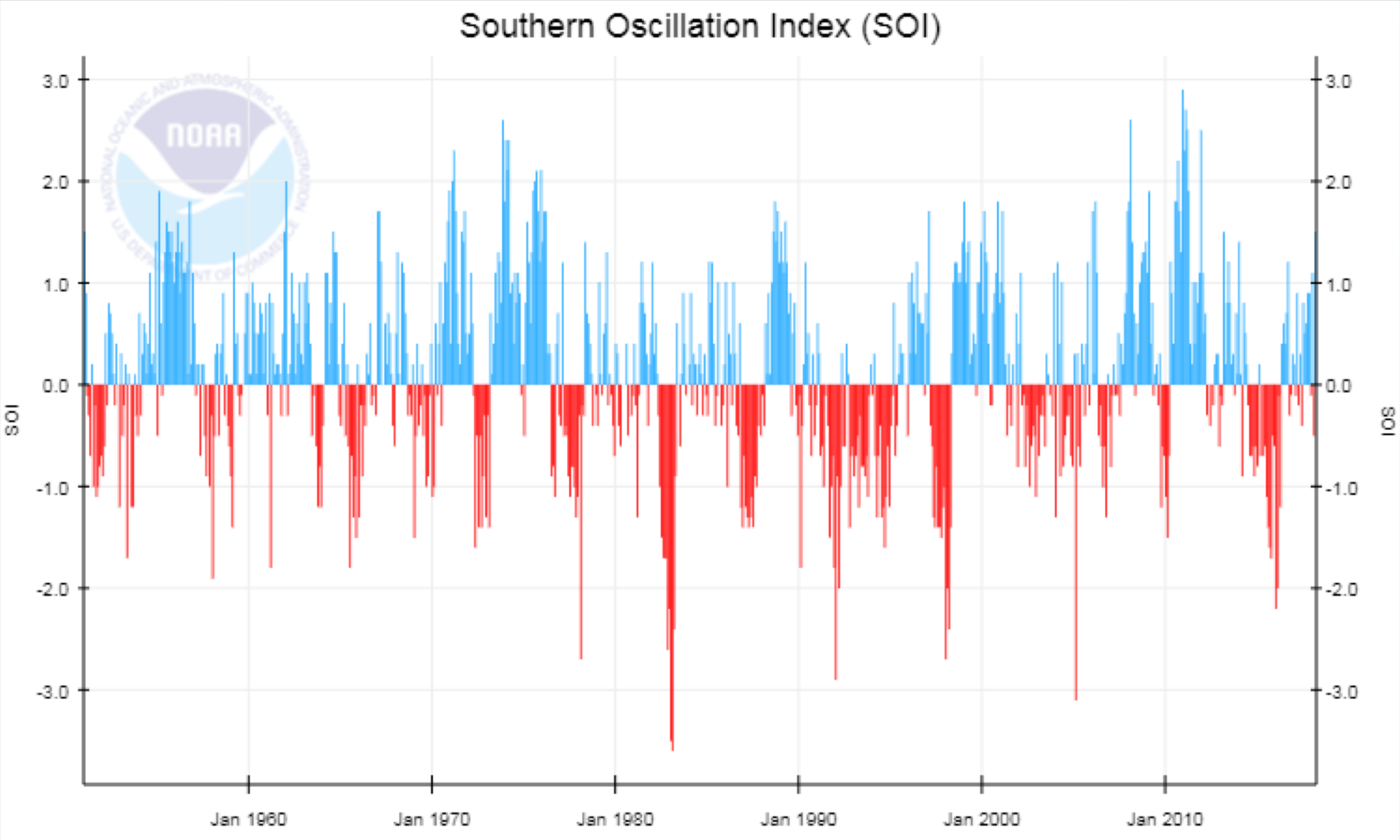


**Figure 2.** Values of the time-averaged and normalized variance of the CLVs as a function of the variables of the model ($\log_{10}$ scale). The 20 first modes corresponds to the variables of the atmosphere, the next 8 ones to the dynamics within the ocean and the last 8 ones to the temperature within the ocean. Parameters' value: $C_o = 350$ W $m^{-2}$ and $d = 1 \times 10^{-8} s^{-1}$. The Euclidean norm is used for all CLVs, and their squared norm is normalized to 1.


CLV 1 vs CLV 2
P(ϕ)
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
ϕ
a)
CLV 1 vs CLV 25
P(ϕ)
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
ϕ
b)
CLV 3 vs CLV 4
P(ϕ)
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
ϕ
c)
CLV 11 vs CLV 12
P(ϕ)
5
4
3
2
1
0
0
0.5
1
1.5
2
2.5
3
ϕ
d)

# Climate variability and predictability?

One important signal: Southern Oscillation Index



Associated with the development of El-Nino and La-Nina

El-Nino-Southern-Oscillation (ENSO)