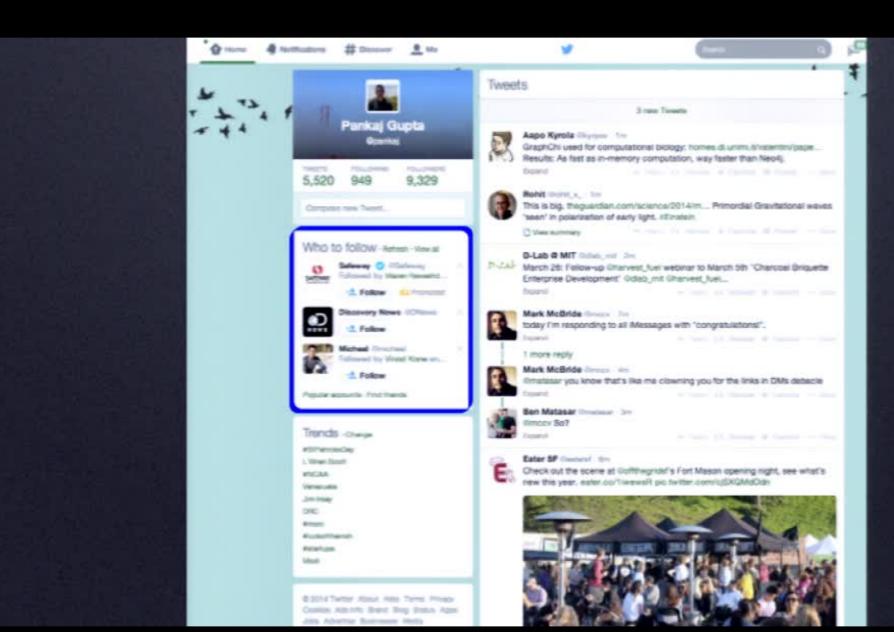
## Efficient Algorithms for Personalization in Social Networks

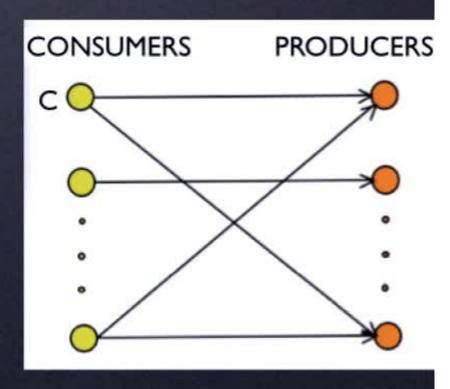
Ashish Goel



# Collaborative Filtering

To get recommendations for C, compute similarity scores for all consumers, and relevance scores for all producers, with respect to C

- 1. Start with sim(C) = 1
- Propagate similarity scores along graph edges to compute relevance scores, and viceversa



Many propagation methods; Often, a linear system of equations

### Collaborative Filter: Love or Money

How should we do this propagation? Two extremes:

LOVE: All the similarity score of a consumer X gets transferred to each producer that X follows, and the same in the reverse direction

Analogous to Singular Value Decompositions in the dense graph limit (HITS)

MONEY: If X follows d producers, then a fraction 1/d of the similarity score of X gets transferred to each producer that X follows (SALSA)

### Personalized PageRank

Given a consumer C, perform a random walk on the Follow graph. If the walk is at node v, then the walk:

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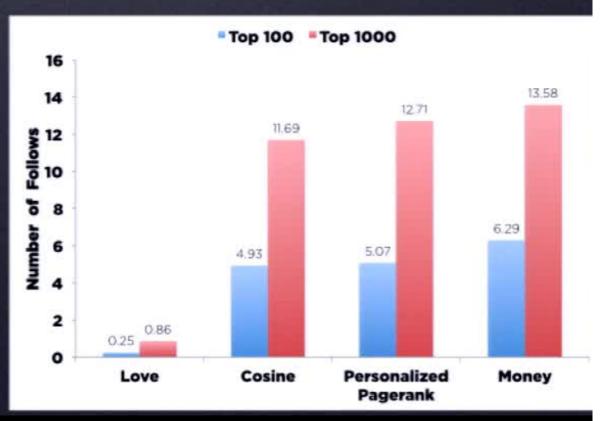
The Personalized PageRank of node Y is the weight of Y in the stationary distribution of this random walk

SALSA/Money is just Personalized PageRank run on the undirected consumer-producer graph

### A Dark Test

Run various algorithms to predict follows, but don't display the results. Instead, just observe how many of the top predictions get followed organically

[Bahmani, Chowdhury, Goel; 2010]



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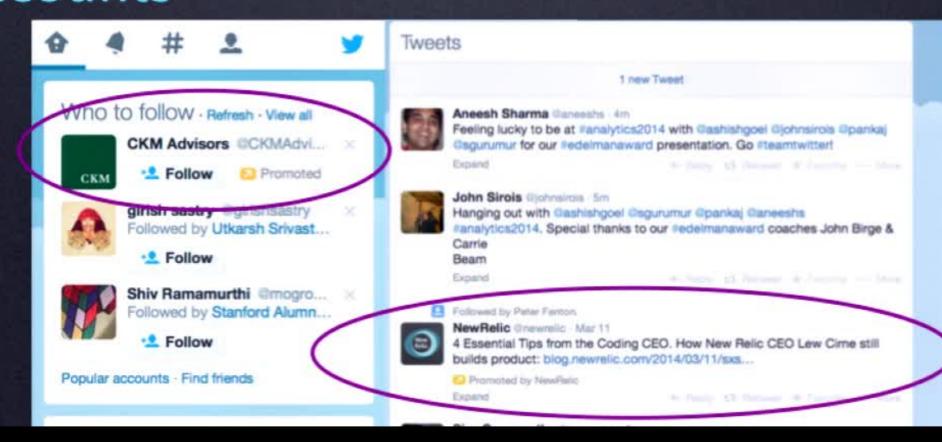


### Strategic Impact

### Creates billions of new follows every year

- More than 1/8 of new follows are directly via the Who-to-Follow module
- More than 15% of active users (> 36 Million users) make at least one follow every month via this module

## Promoted Tweets and Promoted Accounts



### Impact on Revenue

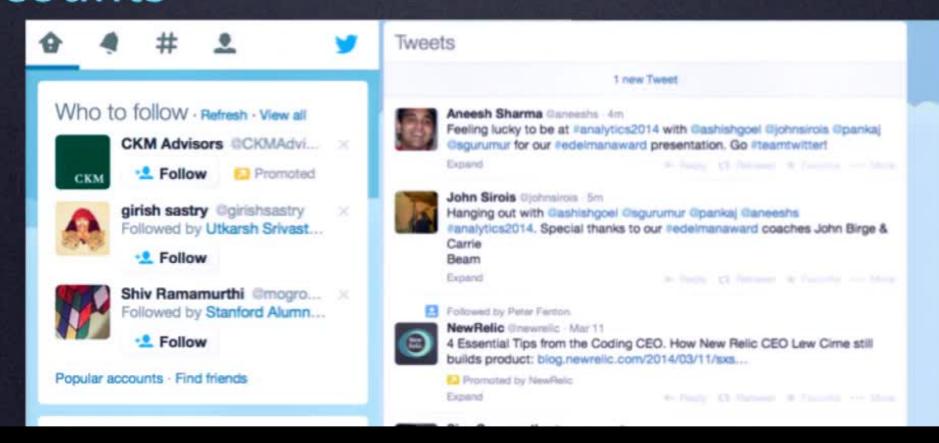
"The Who-To-Follow system was crucial, in a fundamental way, for the Promoted Accounts product, and the Promoted Tweets product also initially used the Who-To-Follow system's targeting"

Alex Roetter (VP of Engineering, Revenue)

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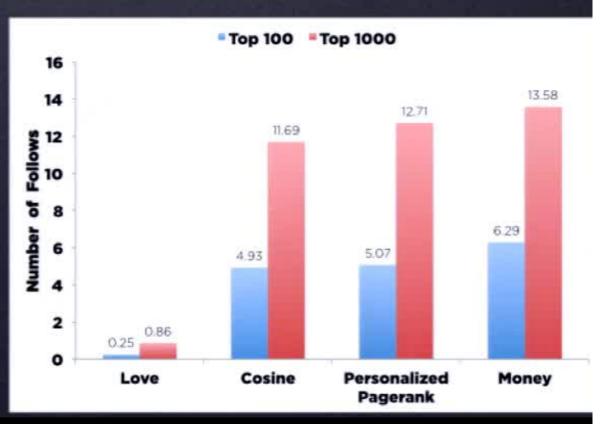
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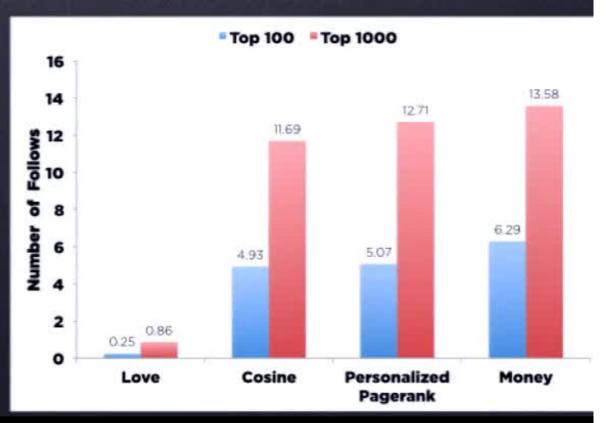
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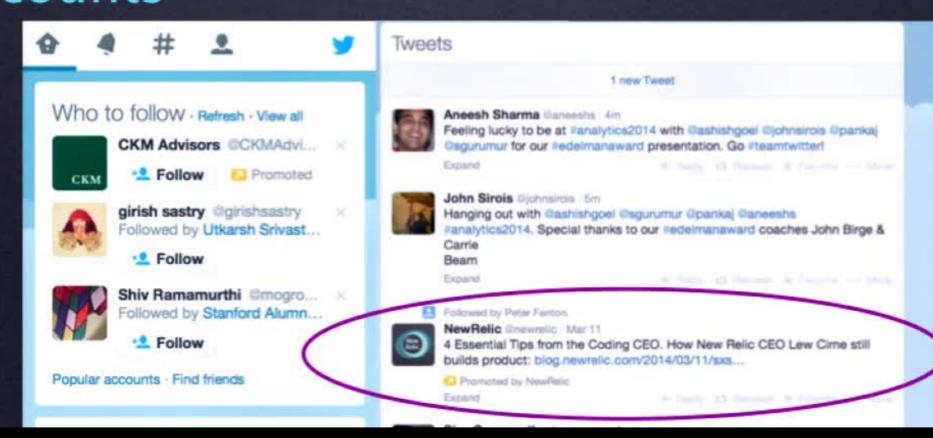


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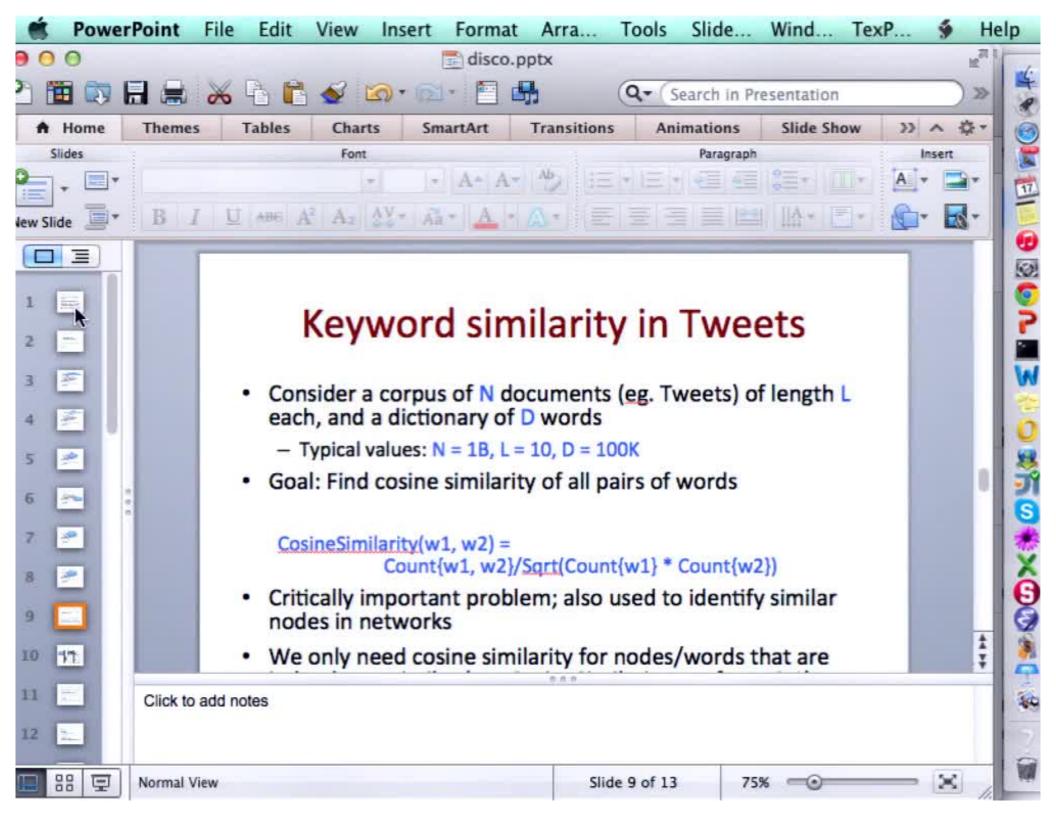
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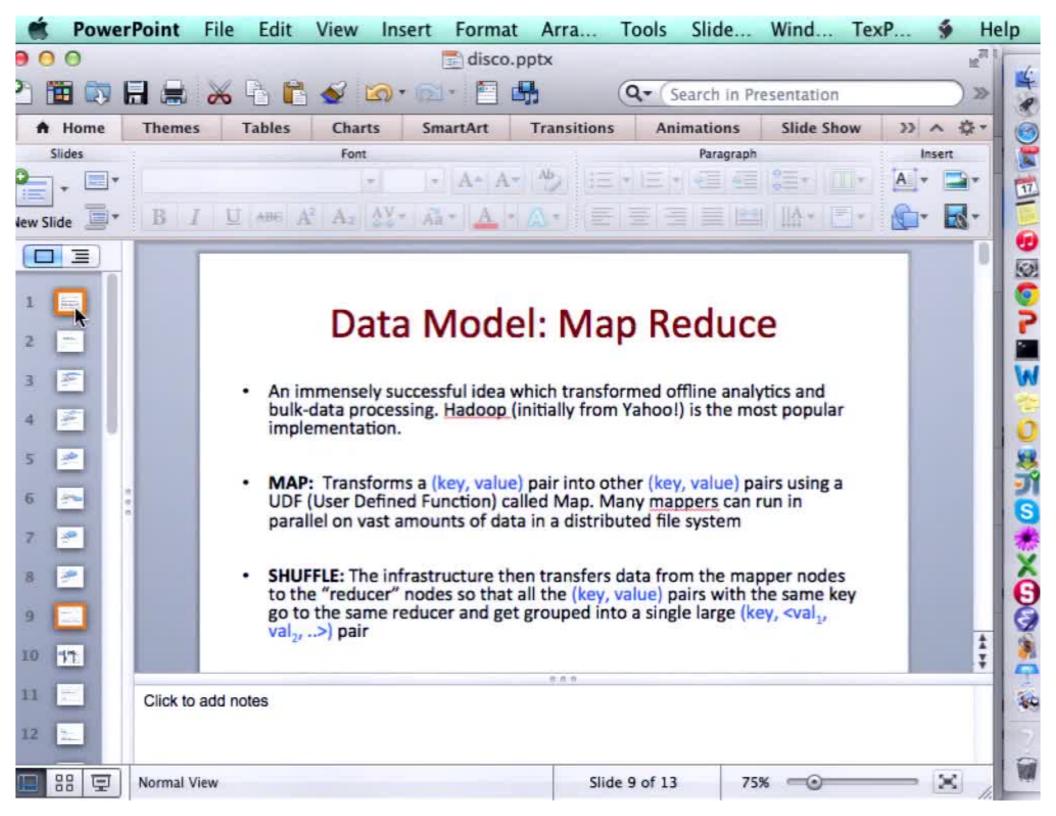
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### Data Model: Map Reduce

- An immensely successful idea which transformed offline analytics and bulk-data processing. Hadoop (initially from Yahoo!) is the most popular implementation.
- MAP: Transforms a (key, value) pair into other (key, value) pairs using a UDF (User Defined Function) called Map. Many mappers can run in parallel on vast amounts of data in a distributed file system
- SHUFFLE: The infrastructure then transfers data from the mapper nodes to the "reducer" nodes so that all the (key, value) pairs with the same key go to the same reducer and get grouped into a single large (key, <val<sub>1</sub>, val<sub>2</sub>, ..>) pair
- REDUCE: A UDF that processes this grouped (key, <val<sub>1</sub>, val<sub>2</sub>, ..>) pair for a single key. Many reducers can run in parallel.

- Key-Complexity:
  - The maximum size of a key-value pair
  - The amount of time taken to process each key
  - The memory required to process each key
- Sequential Complexity:
  - The total time needed by all the mappers and reducers together
  - The total output produced by all the mappers and reducers together
- Number of MapReduce phases

## THE AMOUNT OF WORK DONE PER COMPUTER IF WE HAD INFINITELY MANY COMPUTERS

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### COLLEGE OFTEN, IMPORTANT ONLY FOR REDUCERS

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SHUFFLE SIZE

- The total time needed by all the mapped together
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THE AMOUNT OF WORK

DONE TO AGGREGATE ALL

THE VALUES FOR A SINGLE

KEY (SORTING) IS NOT A

COMPLEXITY MEASURE

"ey

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### Keyword similarity in Tweets

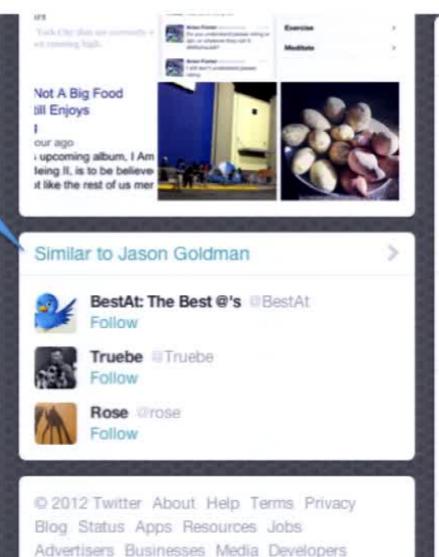
- Consider a corpus of N documents (eg. Tweets) of length L each, and a dictionary of D words
  - Typical values: N = 1B, L = 10, D = 100K
- Goal: Find cosine similarity of all pairs of words

```
CosineSimilarity(w1, w2) = Count{w1, w2}/Sqrt(Count{w1} * Count{w2})
```

- Critically important problem; also used to identify similar nodes in networks
- We only need cosine similarity for nodes/words that are indeed very similar (say CosineSimilarity > ε, for ε=0.1)

### **Example Application**

USER-USER SIMILARITY



#### Tweets



Jason Goldman ■goldman
Tosca, Act I. Many similaritie
Expand



Jason Goldman goldman

©lukester I ate 5lbs of sashi

• View conversation



Jason Goldman ⊜goldman ⊚joshelman ⊜peterpham ca ● View conversation



Jason Goldman goldman
I can feel fantasy sports lurk
obsession that will one day
Expand



### Cosine Similarity [Brute Force]

- Assumption: The tweet corpus is annotated with word counts, so every occurrence of every word is tagged with the frequency of that word as well
  - Need this for other reasons anyway (eg. trends, search)
- Count all co-occurrences between all words
  - MAP(tweet): for every pair of words w1, w2 in the tweet, EMIT({w1, w2}, 1)
  - REDUCE({w1, w2}, <1, 1, ....>): If the size of the value vector(i.e. number of 1's) is larger than ε Sqrt(Count{w1} \* Count{w2}) then EMIT({w1, w2})
- Sequential complexity: Requires shuffling N\*L\*L data across in Hadoop, ≈ 100 Billion records
- Reduce-key complexity: Could be as large as N, or with combining, K (the number of mappers)
- Observation: most of the data shuffled by brute force algorithm was being wasted

### Cosine Similarity [Random Sampling]

```
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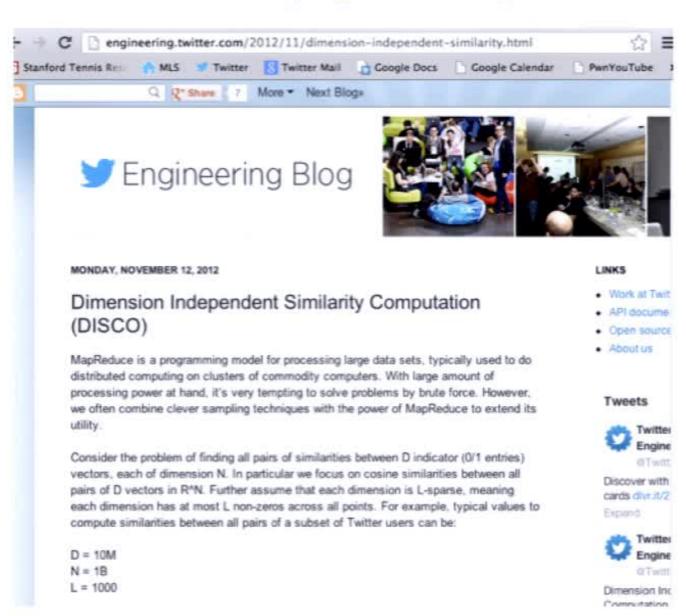
```
where R = (\log D)/\epsilon \approx 100
```

- REDUCE({w1, w2}, <1, 1, ....>): EMIT({w1, w2}, (size of value-vector)/R)
- Unbiased estimate of CosineSimilarity, and accurate whp when CosineSimilarity > ε
- Expected Reduce-Key complexity: At most R
- Sequential complexity: Shuffle size goes down from NL<sup>2</sup> to around DRL . ( $\approx$  100B  $\rightarrow \approx$  100M)

### Cosine Similarity [contd]

- In production at Twitter
- Described in recent twitter engineering blog post

Bogah-Zadeh and Goel: Dimension Independent Similarity Computation

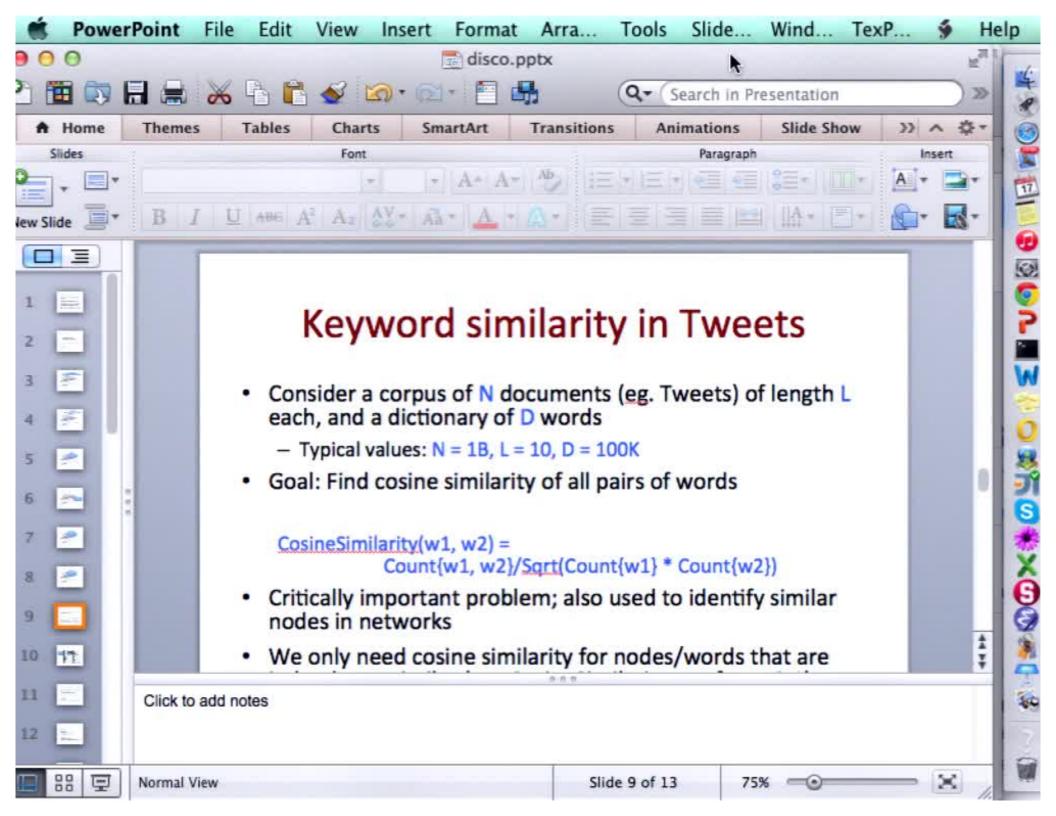


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## Incremental PageRank

#### Updates to social graph are made in real-time

- As opposed to a batched crawl process for web search
- Real-time updates to PageRank are important to capture trending events

## Goal: Design an algorithm to update PageRank incrementally (i.e. upon an edge arrival)

t-th edge arrival: Let  $(u_t, v_t)$  denote the arriving edge,  $d_t(v)$  denote the out-degree of node v, and  $\pi_t(v)$  its PageRank

#### Incremental PageRank via Monte Carlo

Start with R = O(log N) random walks from every node

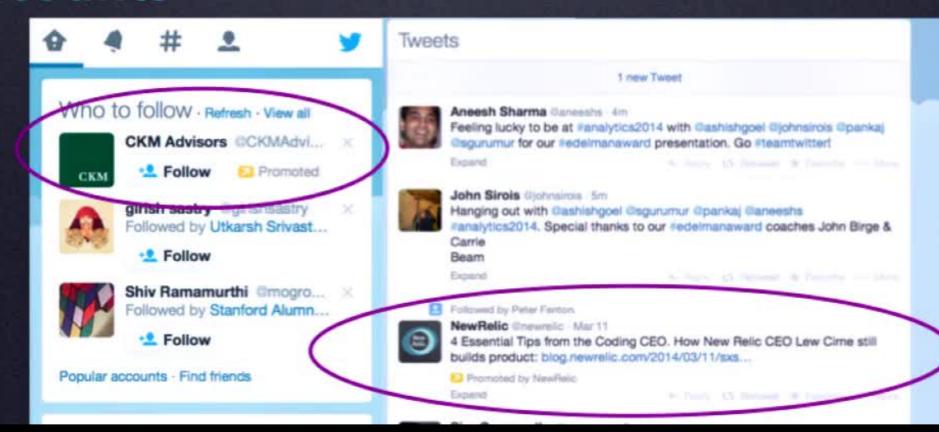
At time t, for every random walk through node  $u_t$ , re-route it to use the new edge  $(u_t, v_t)$  with probability  $1/d_t(u_t)$ 

Time/number of network-calls for each re-routing:  $O(1/\alpha)$ 

Claim: This faithfully maintains R random walks after arbitrary edge arrivals

Need the graph and the stored random walks in fast distributed memory

# Promoted Tweets and Promoted Accounts



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## Incremental PageRank Time

Assume that the edges of the graph are chosen by an adversary, but then presented in random order

Theorem: # of r Power per arrival goes to 0 t-th arrival: # of time M R/ $\alpha$  per arrival goes to 0

 $N \log N/\alpha VS$ 

- Total time over M arrivals =  $O((N R \log N)/\alpha^2)$
- Comparable to doing power iteration/Monte Carlo just once!

[Bahmani, Goel, Chowdhury, VLDB 2010]

Network-based Personalized Search is not yet mature

Missing technical piece: Efficient algorithms for Personalized PageRank Queries

Given source s and target t, estimate the Personalized PageRank of t for s with high accuracy, if it is greater than δ

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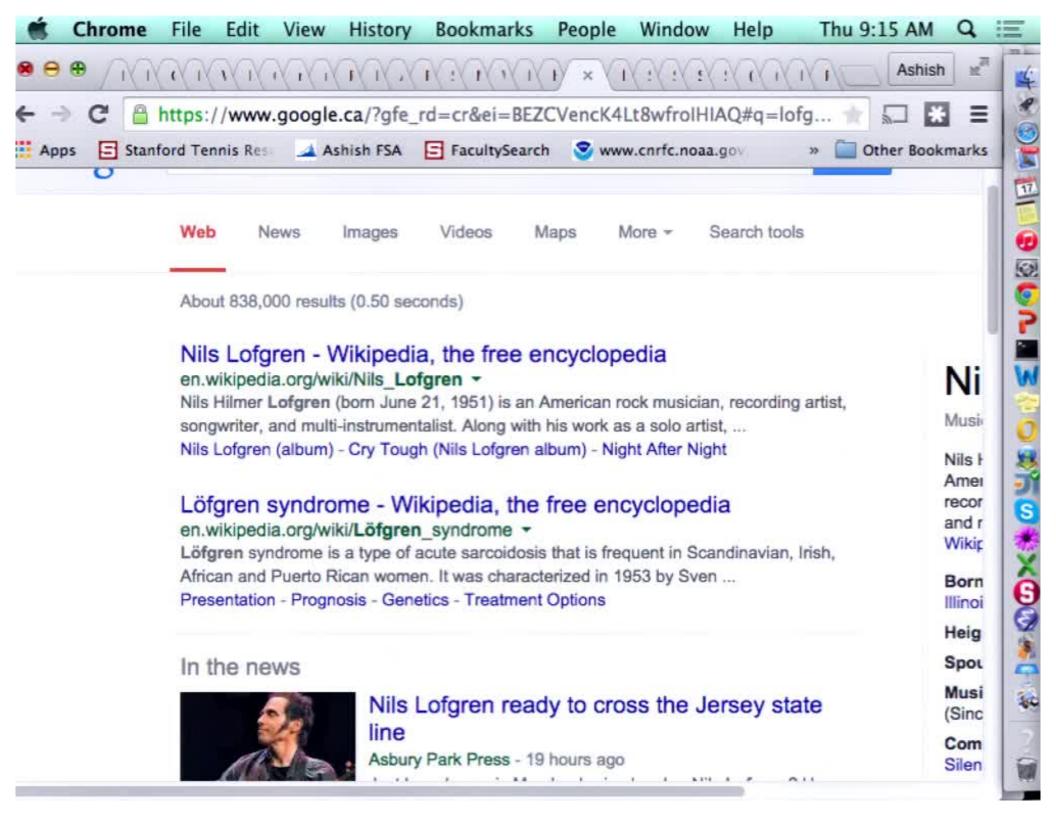
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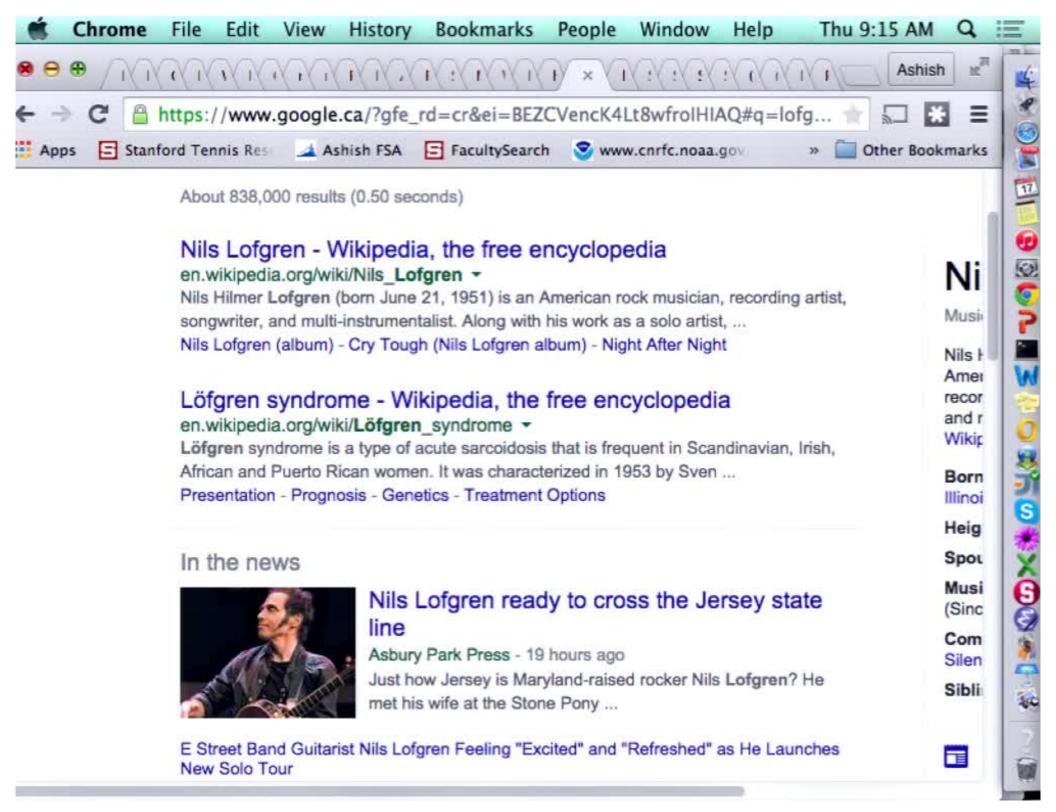
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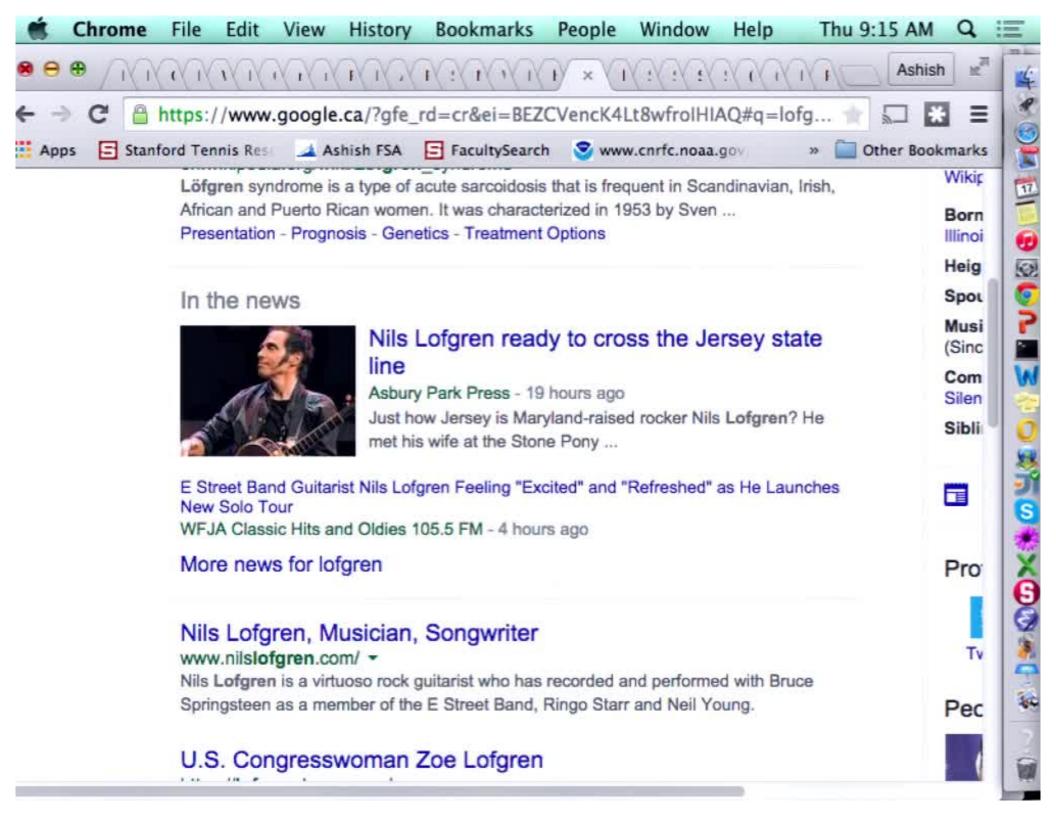
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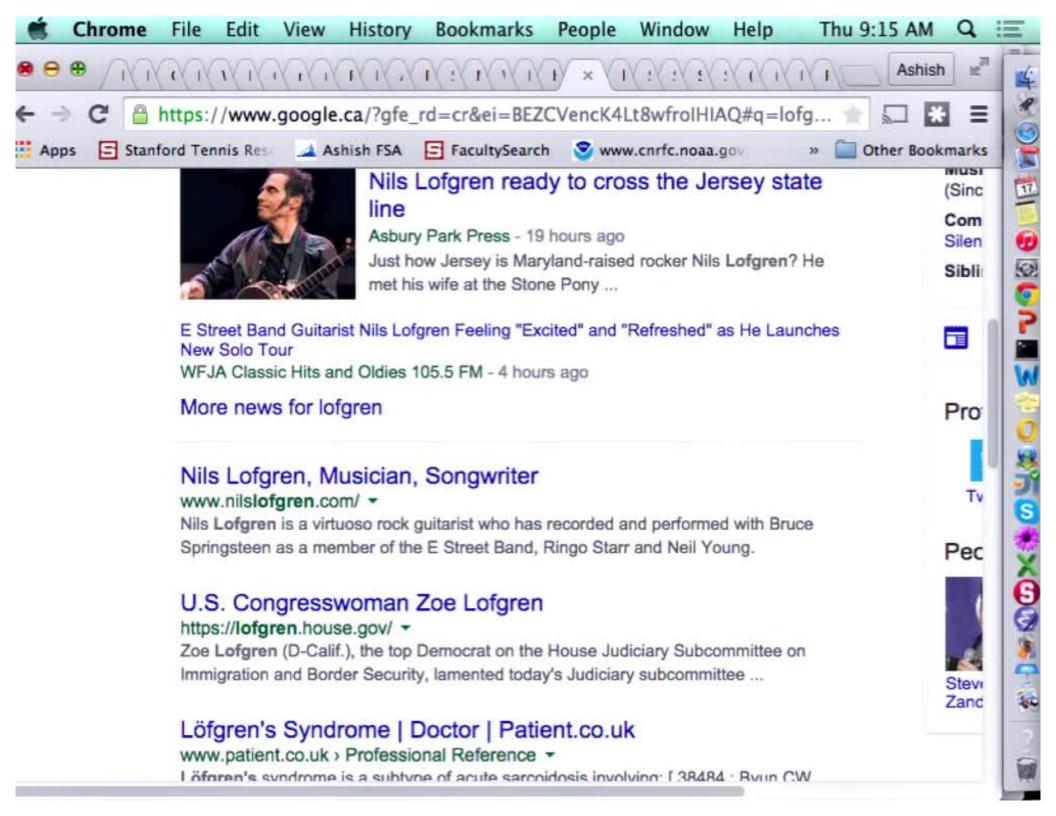
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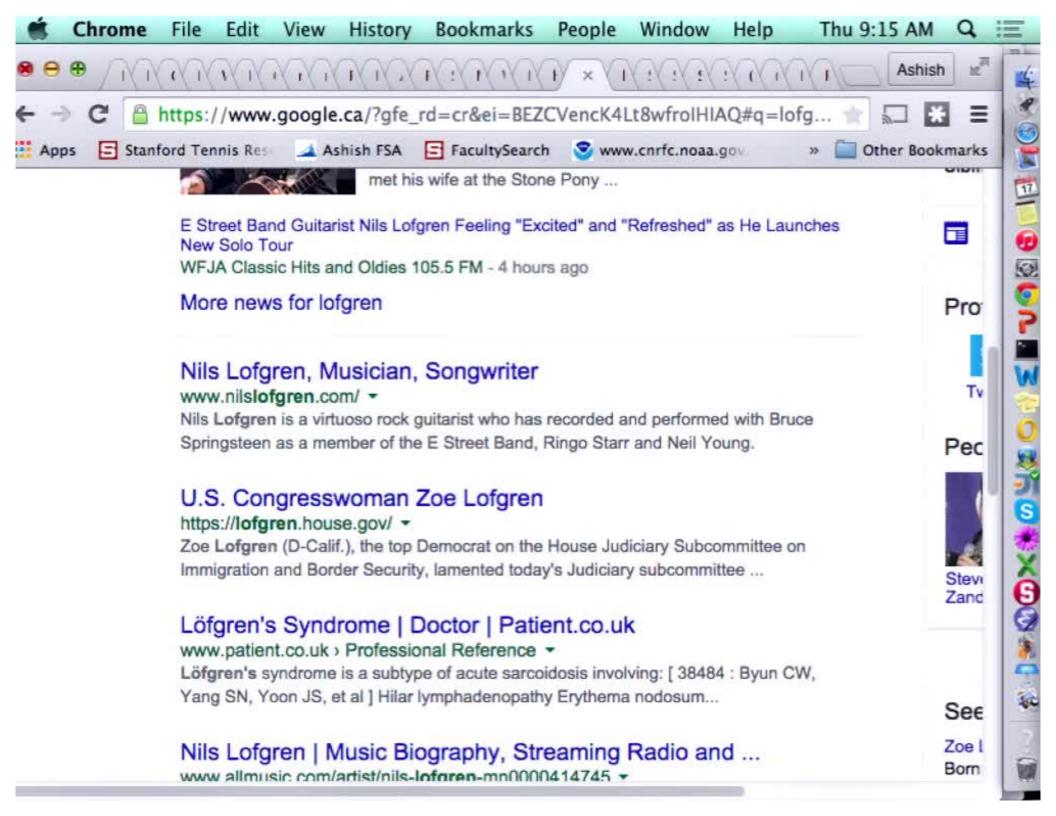
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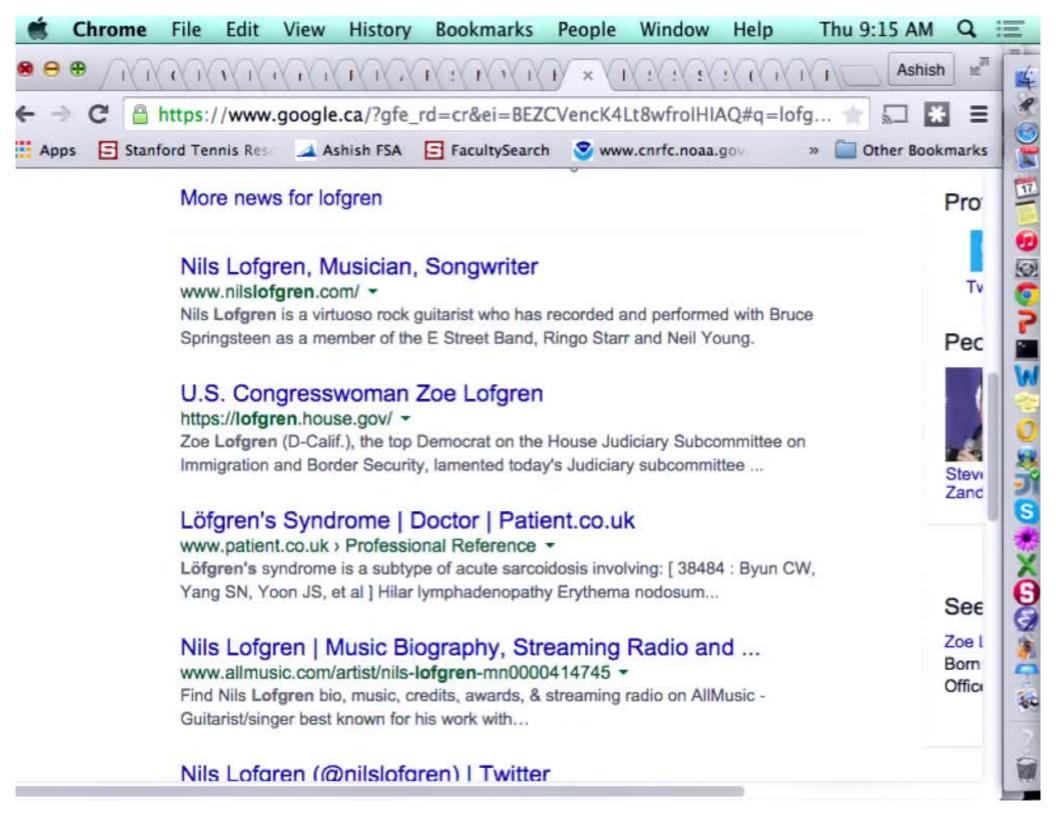


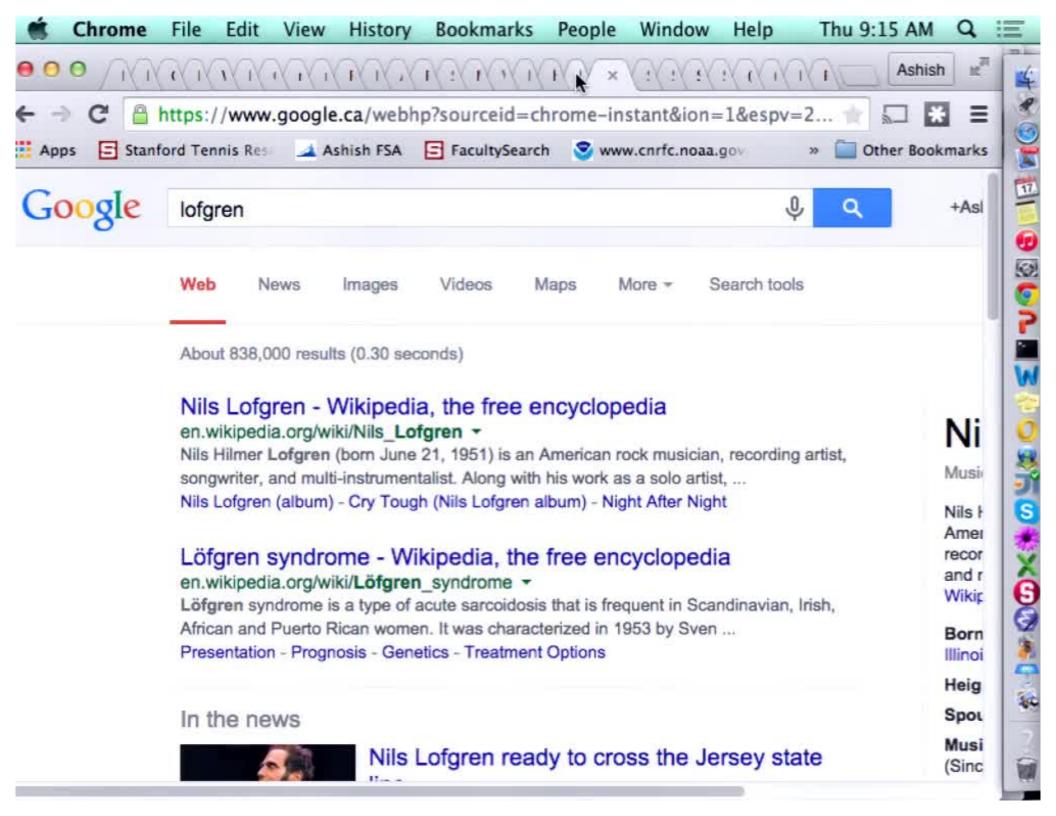


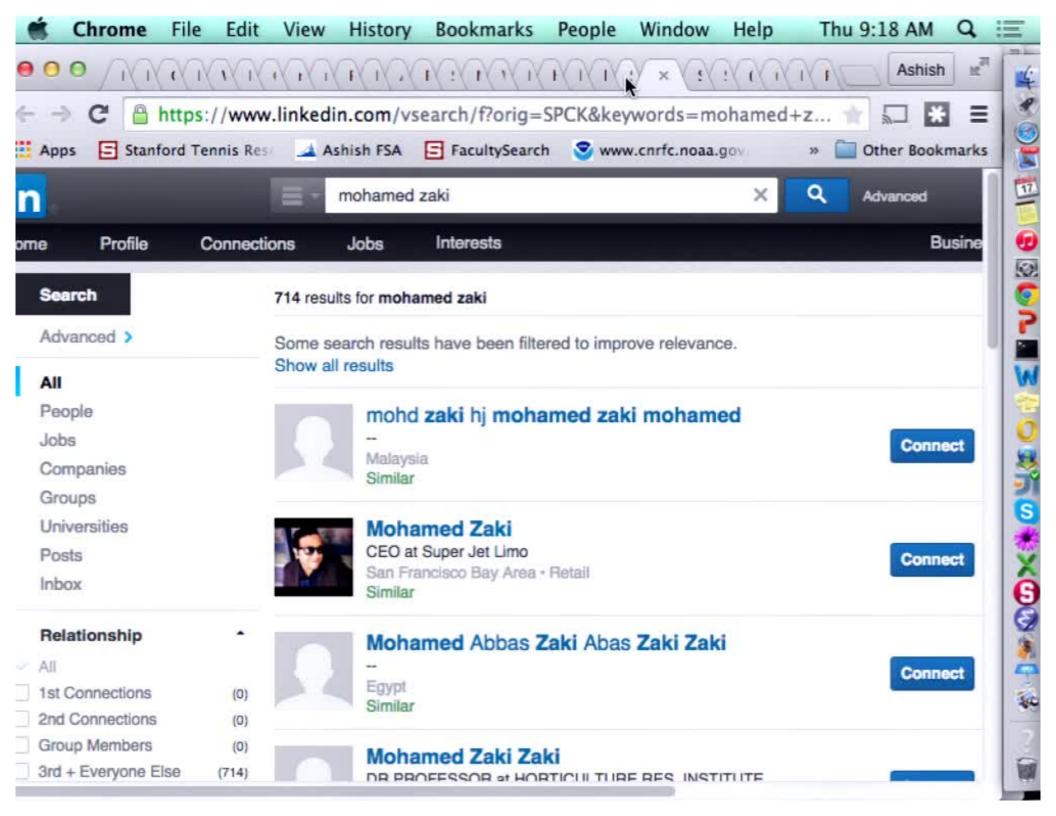


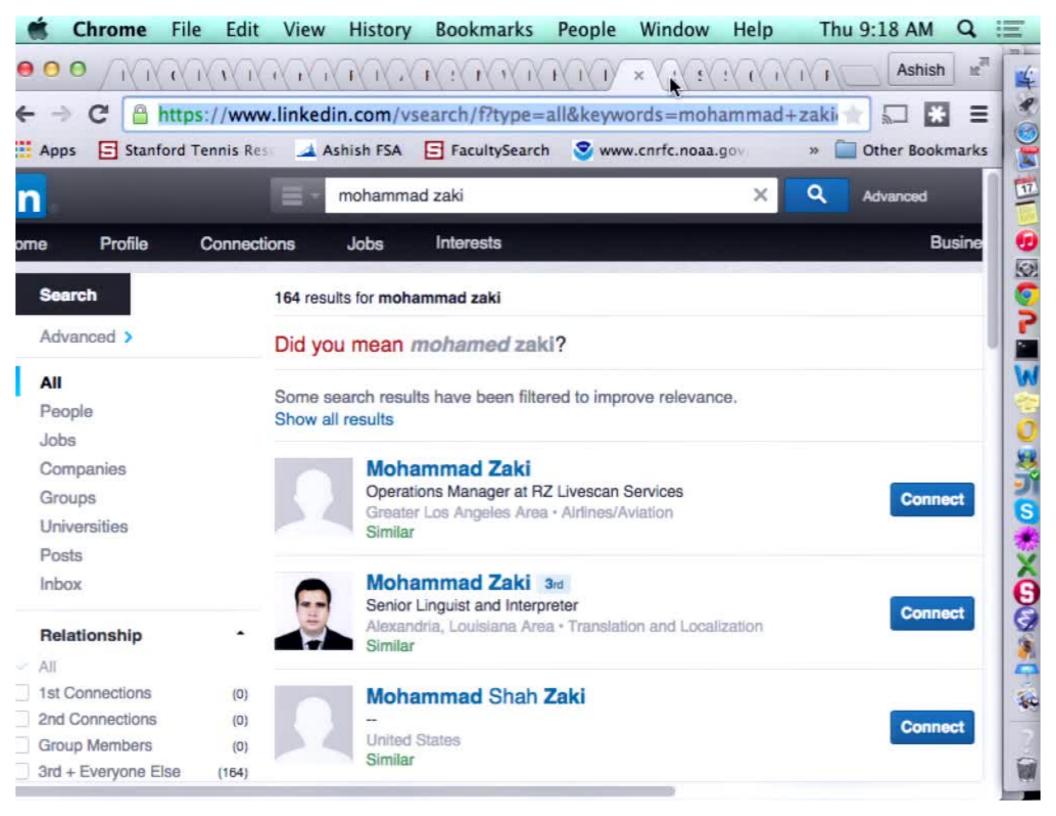


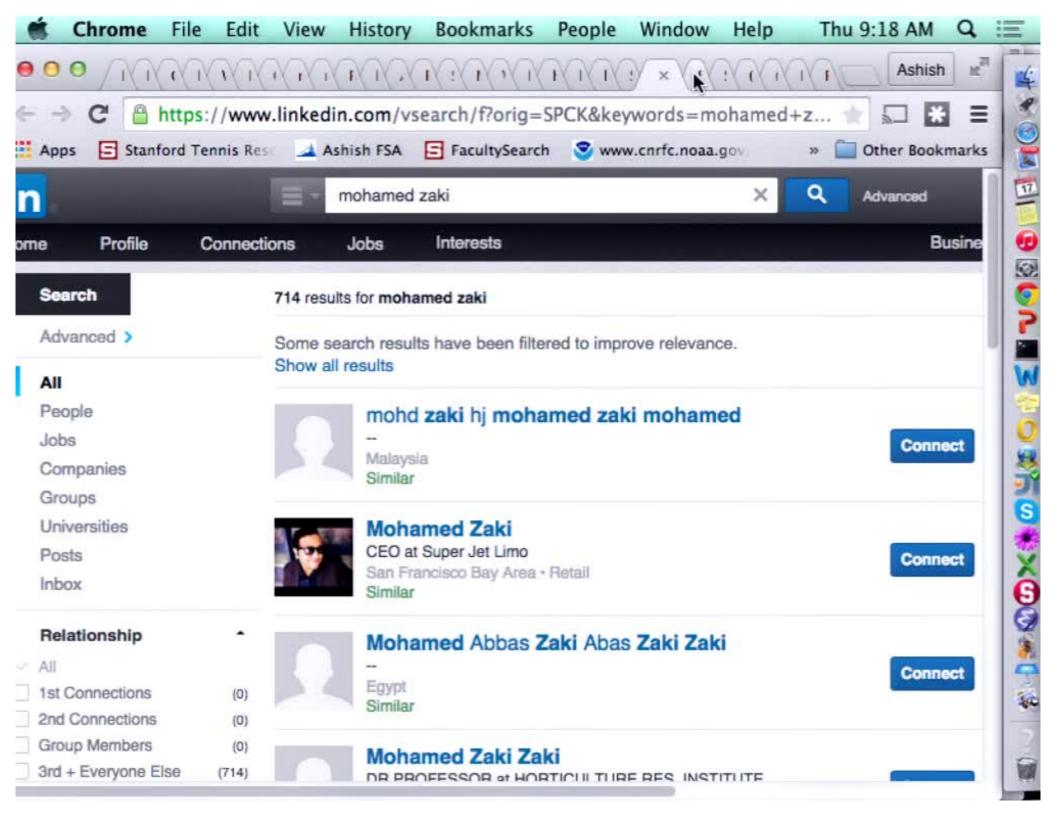


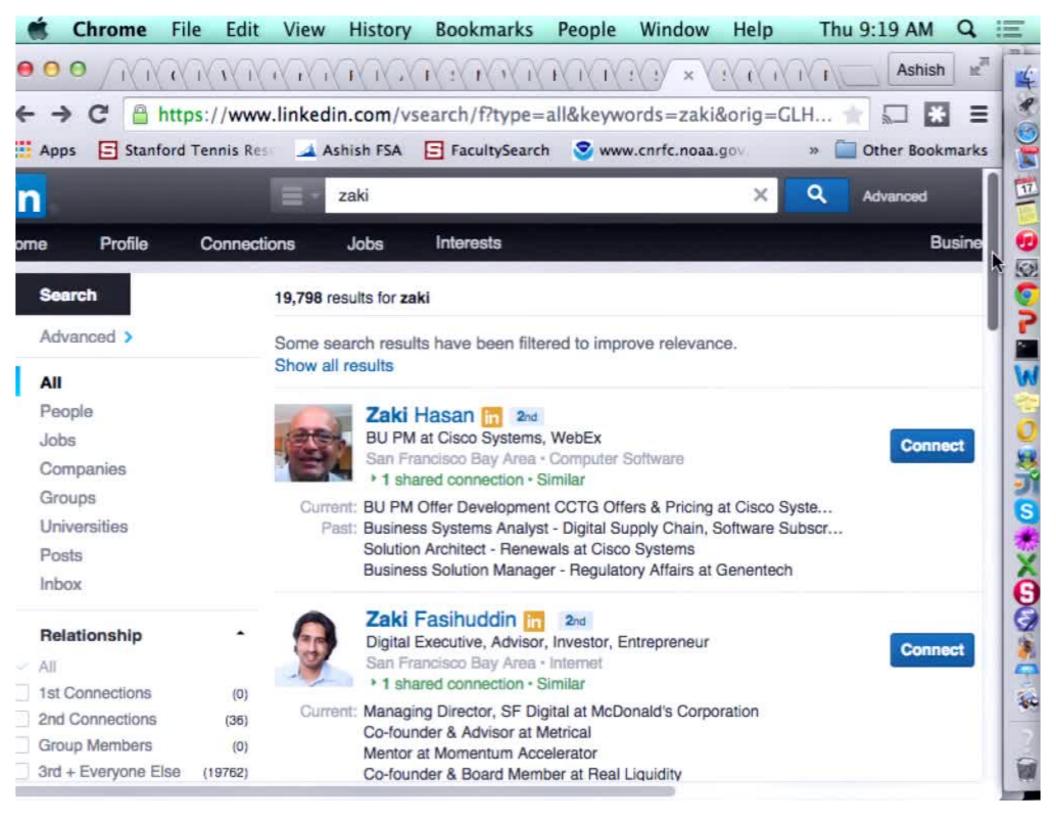


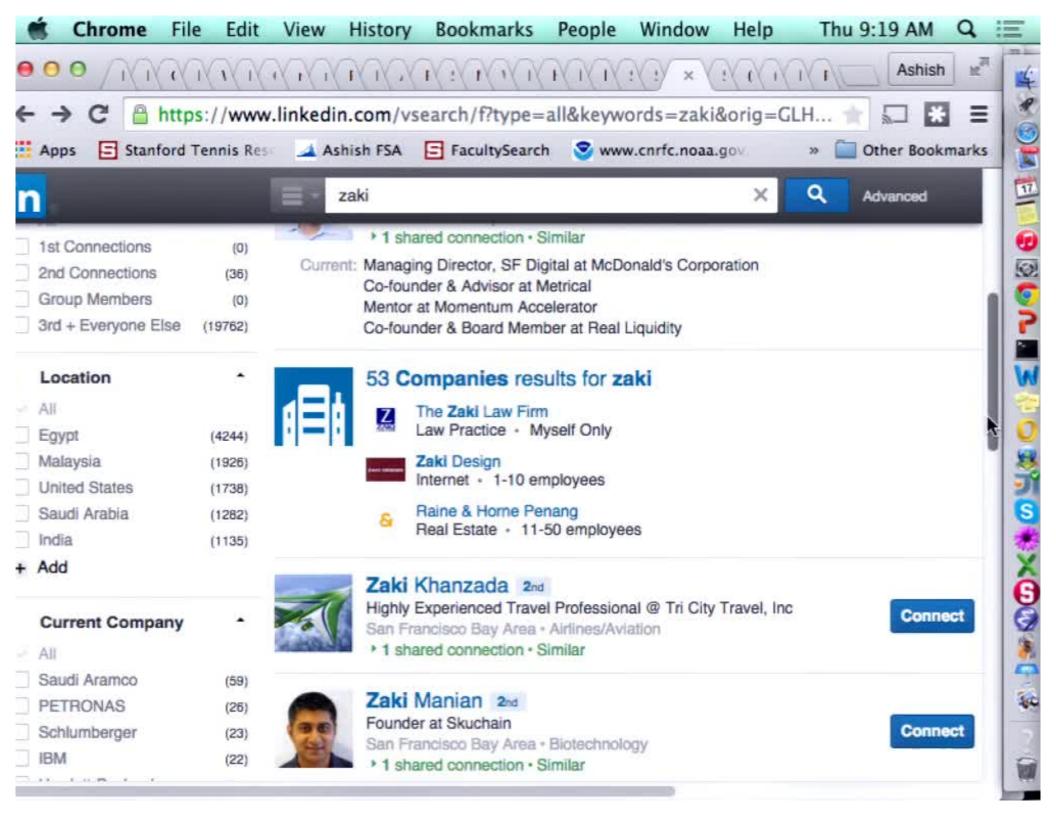


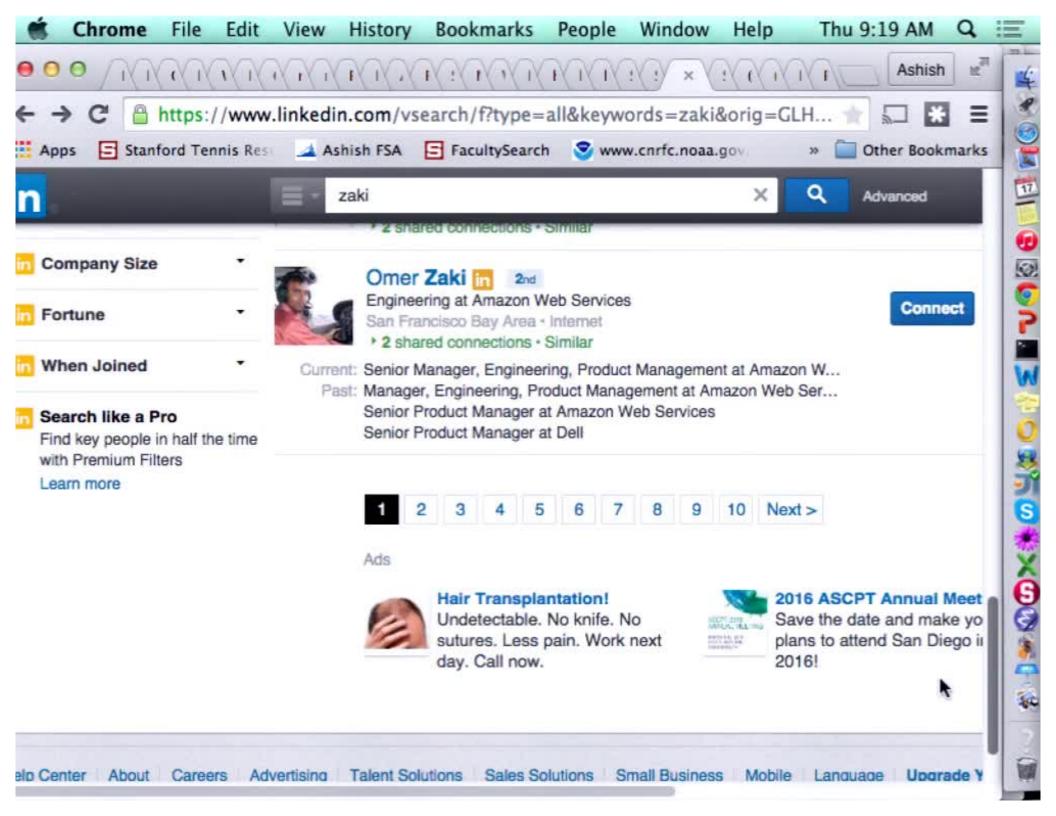


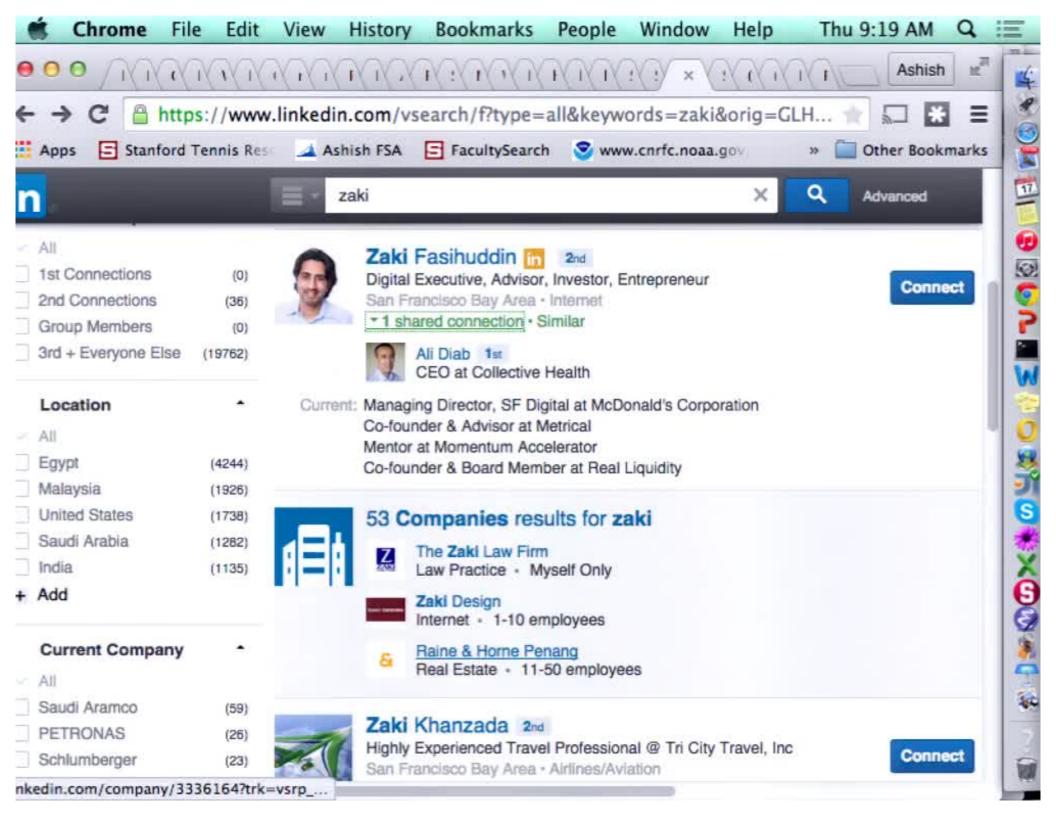


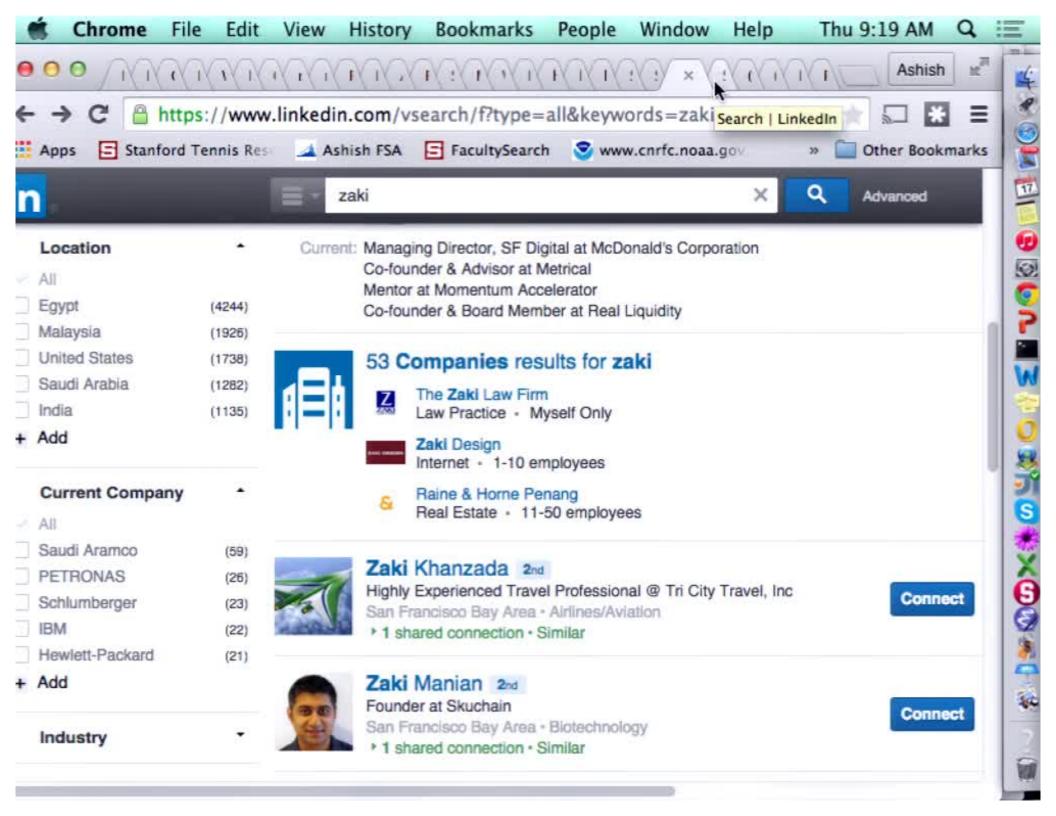


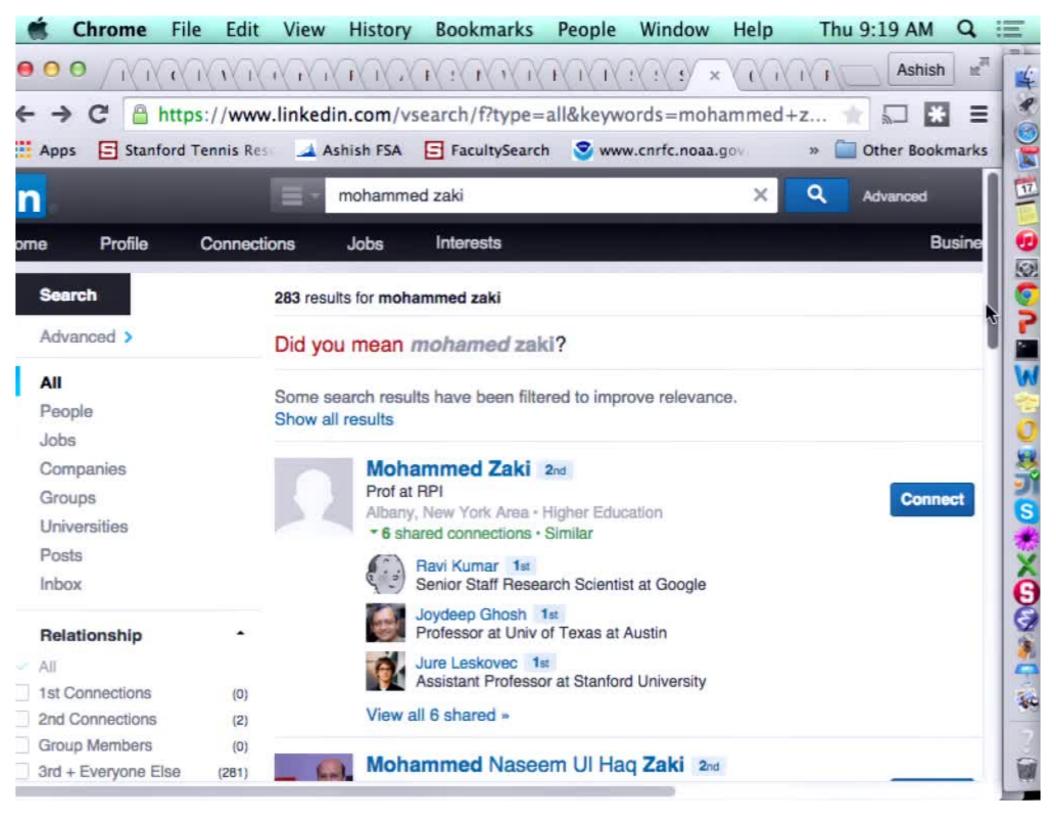


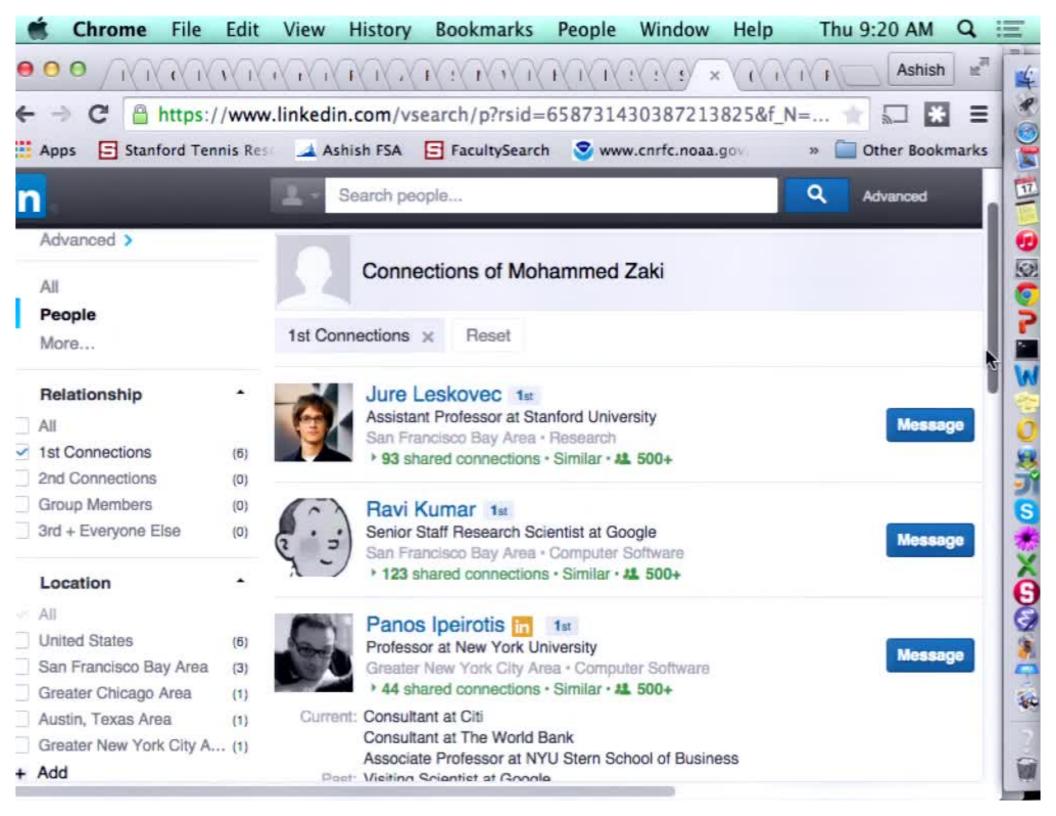


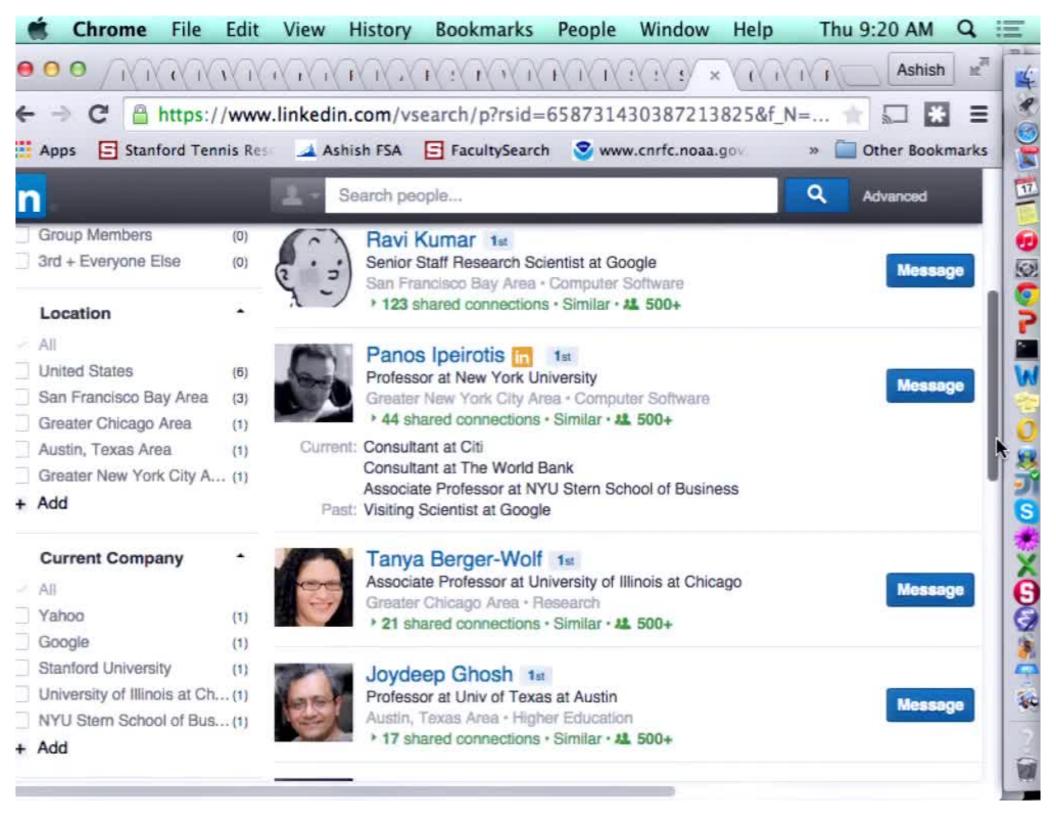












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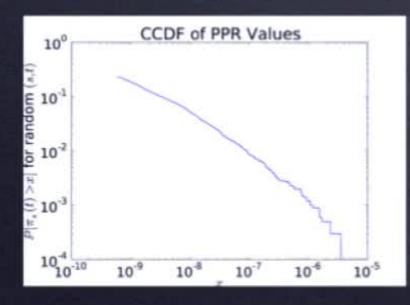
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## Existing Methods for PPR Queries



Monte Carlo uses time >  $1/\delta$ "Local Update" uses time  $d/\delta$ 

[d = M/N is the average degree]

On Twitter-2010, if 
$$\delta = \frac{4}{n} \approx 10^{-7}$$
, then

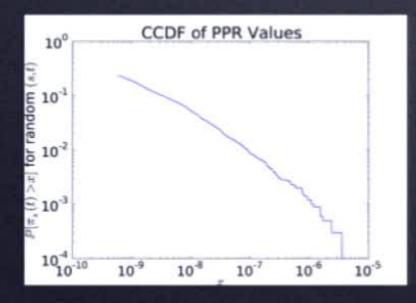
$$\Pr\left[\pi(s,t) > \delta\right] = 1\%$$

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#### FAST PPR

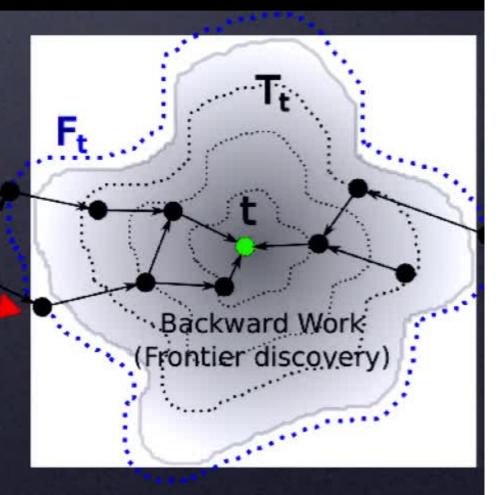
#### We can answer PPR queries in either

- Average time  $\tilde{O}(\sqrt{(d/\delta)})$
- Worst case time  $\tilde{O}(\sqrt{(d/\delta)})$  with  $\tilde{O}(\sqrt{(d/\delta)})$  storage and preprocessing time per node
- Typical values:  $\delta \sim 10^{-8}$ ,  $d \sim 100$ ; results in a > 100-fold decrease

### Basic Idea

#### Intuition: The Birthday Paradox

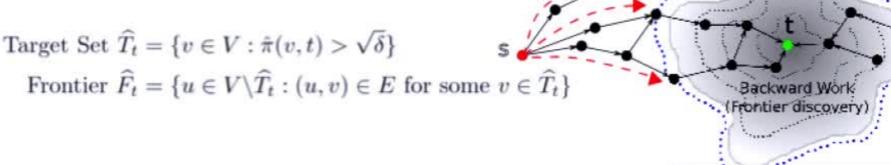
- Do small number of "forward" random walks from s
- Do "reverse" PageRank computation from t using Local Update with low accuracy
- Use number of collisions as an estimator
- Need to "catch" a collision just before it happens



# Simple Version of FAST PPR

- Use Local Update to compute estimates π̂(v, t) to accuracy O(√δ).
- 2. Define

Target Set 
$$\widehat{T}_t = \{v \in V : \widehat{\pi}(v, t) > \sqrt{\delta}\}\$$



3. Take  $O\left(\frac{\log(n)}{\sqrt{\delta}}\right)$  Random Walks  $\{W_i\}$ , terminating each early if it hits  $\hat{F}_t$ . Define

$$X_i = \begin{cases} \hat{\pi}(u, t), & W_i \text{ hits } u \in \widehat{F}_t \\ 0, & W_i \text{ does not hit } \widehat{F}_t \end{cases}$$

Return empirical mean{X<sub>i</sub>}.

# Running Time for Simple Version

For a uniformly random target node t, the average per-query running time is

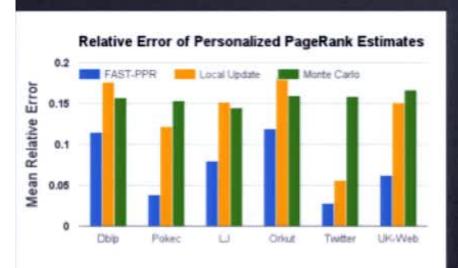
$$O\left(\frac{1}{\sqrt{\delta}}\left(\bar{d} + \log(n)\right)\right).$$

Reverse work (Local Update) Forward work (Monte Carlo)

We get final running time of  $\tilde{O}(\sqrt{(d/\delta)})$  by using different accuracies in forward and reverse computation

We use  $\tilde{O}(\sqrt{(d/\delta)})$  pre-processing/space to go from average to worst case running time

#### Experiments





- Admits Distributed Implementation (much faster)
- Works when source is a set of nodes
- Lower bound of  $1/\sqrt{\delta}$ 
  - → Open problem: do we need the √d?

[Lofgren, Banerjee, Goel, Seshadhri, KDD, 2014]

# Computing via Intersections

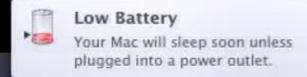
Consider "sharding" a graph among K computers, i.e. the vertices of the graph get randomly partitioned into K sets,  $V_1$ ,  $V_2$ , ...,  $V_K$ . one for each compute node. Vertex V -> Shard s(V)

Typical: For each vertex v, store all its edges in Shard s(v) as a key-value pair (v, ADJ(v))

Additional trick: also store the reverse map, i.e. for each node w, store  $(w, ADJ(w) \cap V_i)$  on the j-th compute node

Advantage: Intersection of the neighborhood of u and v can be computed efficiently with one scatter-gather query

[Gupta, Satuluri et al., 2014]



### Computing via Intersections

Both PPR and All Pairs Cosine Similarity can be efficiently reduced to graph intersections

Also, approximate shortest path reduces to graph intersections

[Lofgren, Goel, Gupta; manuscript]



### Summary

Personalization algorithms can lead to both growth and monetization in social networks

Random walks and Cosine similarity are particularly efficient

Careful consideration of algorithms and architecture together can lead to efficiency

Next frontier: personalized search

