On the Convergence of Recursive Schemes for Wave Shape Functions

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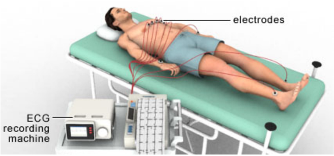
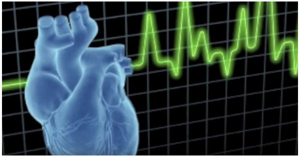
- **Wave shape function analysis** in mode decomposition problems
  - ◇ Motivation
  - ◇ The curse of wave shape functions
  - ◇ Recursive diffeomorphism-based analysis
  - ◇ Numerical examples

Health data

Electrocardiography (ECG) for cardiology test;





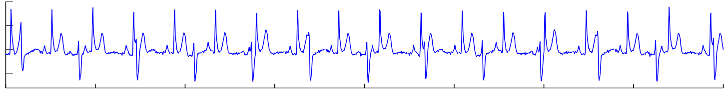
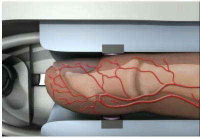


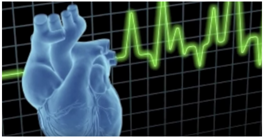
Figure: An ECG signal

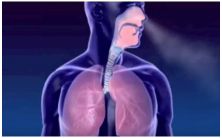
- A periodic signal $s(2\pi Nx)$?
- A quasi-periodic signal $s(2\pi\phi(x))$?
- A quasi-periodic signal with changing amplitude $\alpha(x)s(2\pi\phi(x))$

Health data

Photoplethysmogram (PPG), an optically obtained plethysmogram, a volumetric measurement of an organ;







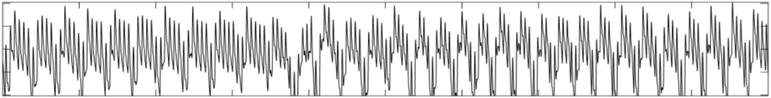


Figure: The PPG signal contains two components: $f(x) = \sum_{k=1}^{2} \alpha_k(x) s_k(2\pi\phi_k(x))$.

Health data

Photoplethysmogram (PPG), an optically obtained plethysmogram, a volumetric measurement of an organ;

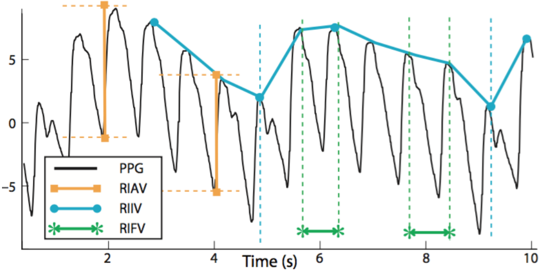


Figure: Existing PPG signal analysis lacks accuracy. Courtesy of W. Karlen, S. Raman, J. Ansermino, G. Dumont, IEEE Trans. Biomed. Eng., 2013.

Problem Statement

A long history, $s(x) = e^{ix}$:

$$f(x) = \sum_{k=1}^{K} \alpha_k(x) e^{2\pi i \phi_k(x)}$$

instantaneous amplitude $\alpha_k(x)$

instantaneous phase $\phi_k(x)$

Litterature review

- Windowed Fourier, wavelet transform;
- Wigner-Ville distribution;
- Empirical mode decomposition;
- Data-driven optimization;
- Synchrosqueezed transform.

Problem Statement

Litterature review

- Windowed Fourier, wavelet transform;
- Wigner-Ville distribution;
- Empirical mode decomposition;
- Synchrosqueezed transform;
- Data-driven optimization.
- All methods work only for $s_k(x) = e^{ix}$, i.e.

$$f(x) = \sum_{k=1}^{K} \alpha_k(x) e^{2\pi i \phi_k(x)}.$$

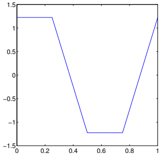
The curse of wave shape functions for more than 20 years

$$f(x) = \sum_{k=1}^{K} \alpha_k(x) s_k(2\pi \phi_k(x))$$

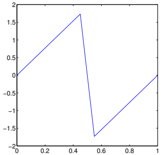
The curse of wave shape functions for more than 20 years

$$f(x)=\sum_{k=1}^{2}\alpha_k(x)s_k(2\pi\phi_k(x))=\sum_{k=1}^{2}\sum_{n}\widehat{s}_k(n)\alpha_k(x)e^{2\pi i n\phi_k(x)}$$

with instantaneous frequencies $n\phi_k'(x)$ for all n such that $\widehat{s}_k(n)\neq 0$.



s_1



s_2

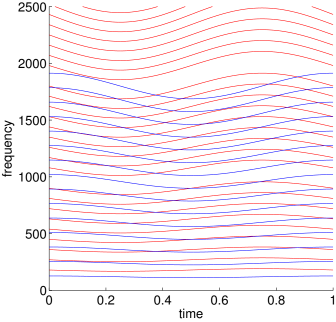


Figure: Instantaneous frequencies: $\{n\phi_1'(x)\}_n$ in blue and $\{n\phi_2'(x)\}_n$ in red.

Problem Statement

- Given data:

$$f(x) = \sum_{k=1}^{K} \alpha_k(x) s_k(2\pi\phi_k(x))$$

- Assume $\{\alpha_k(x)\}$ and $\{\phi_k(x)\}$ are known smooth functions;
- Targets: $s_k(x)$ with $\widehat{s}(0) = 0$.
- Solutions:
  - Diffeomorphism-based spectral analysis, Y., ACHA, 2015;
  - **Recursive diffeomorphism-based regression for shape functions**, Xu, Y., Daubechies, Preprint, 2016.

The idea of diffeomorphism-based regression

A simple case

- $f_1(x) = \alpha_1(x)s_1(2\pi\phi_1(x))$;
- and we know $\alpha_1(x)$ and $\phi_1(x)$.

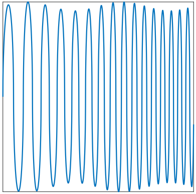


Figure: Original signal: $\alpha_1(x)s_1(2\pi\phi_1(x))$.

The idea of diffeomorphism-based regression

A simple case

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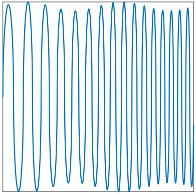

Figure: Original signal: $\alpha_1(x)s_1(2\pi\phi_1(x))$.

Diffeomorphism to create periodicity

$$h_1(v) = \frac{f_1 \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} = s_1(2\pi v).$$

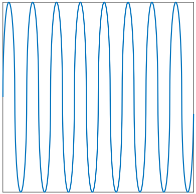

Figure: After deffeomorphism: $s_1(2\pi v)$.

The idea of diffeomorphism-based regression

A simple case

- $f_1(x) = \alpha_1(x) s_1(2\pi\phi_1(x))$;
- and we know $\alpha_1(x)$ and $\phi_1(x)$.

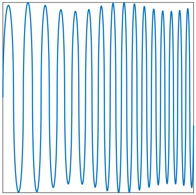

Figure: Original signal: $\alpha_1(x)s_1(2\pi\phi_1(x))$.

Diffeomorphism to create periodicity

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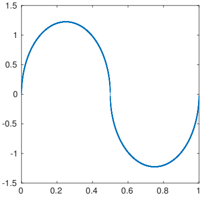


Figure: Regression gives an accurate shape function.

The idea of diffeomorphism-based regression

Difficult case:

- $f(x) = \alpha_1(x)s_1(2\pi\phi_1(x)) + \alpha_2(x)s_2(2\pi\phi_2(x))$,
- we know $\alpha_k(x)$ and $\phi_k(x)$, for $k = 1, 2$.

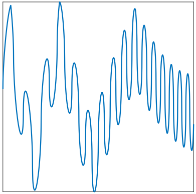


Figure: $f_1(x) + f_2(x)$.

The idea of diffeomorphism-based regression

Difficult case:

- $f(x) = \alpha_1(x)s_1(2\pi\phi_1(x)) + \alpha_2(x)s_2(2\pi\phi_2(x))$,
- we know $\alpha_k(x)$ and $\phi_k(x)$, for $k = 1, 2$.

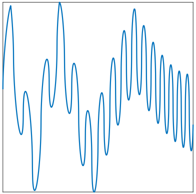


Figure: $f_1(x) + f_2(x)$.

Diffeomorphism to create periodicity? NO!

$$
\begin{aligned}
h_2(v) &= \frac{f \circ \phi_2^{-1}(v)}{\alpha_2 \circ \phi_2^{-1}(v)} \\
&= s_2(2\pi v) + \frac{\alpha_1 \circ \phi_2^{-1}(v)}{\alpha_2 \circ \phi_2^{-1}(v)} s_1(2\pi\phi_1 \circ \phi_2^{-1}(v)) \\
&= s_2(2\pi v) + \kappa_2(v),
\end{aligned}
$$

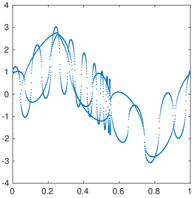
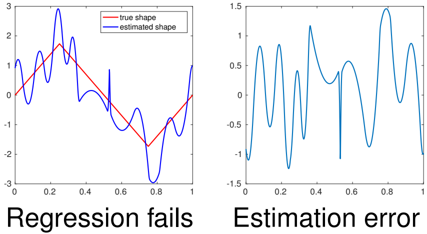


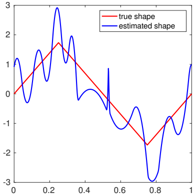
Figure: Regression is not accurate.

The idea of diffeomorphism-based regression

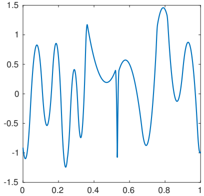


- **Bad news:** fail

The idea of diffeomorphism-based regression



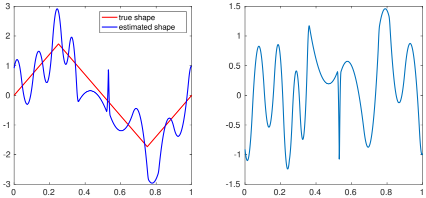
Regression fails



Estimation error

- **Bad news:** fail
- **However:** failure not extrme

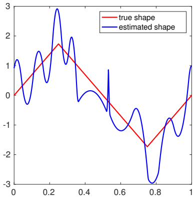
The idea of diffeomorphism-based regression



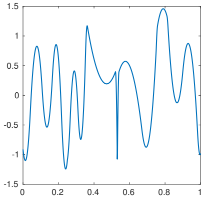
Regression fails   Estimation error

- **Bad news:** fail
- **However:** failure not extrme
  ↪ iterative method?

The idea of diffeomorphism-based regression



Regression fails

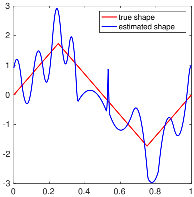


Estimation error

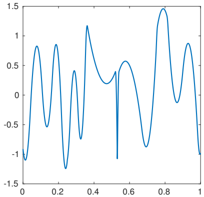
- **Bad news:** fail
- **However:** failure not extrme
  $\hookrightarrow$ iterative method?
- **Key observation:** the residual error is again a new mode decomposition problem

$$r(x) \quad = \quad f(x) - \alpha_1(x)\bar{s}_1(2\pi\phi_1(x)) + \alpha_2(x)\bar{s}_2(2\pi\phi_2(x))$$

The idea of diffeomorphism-based regression



Regression fails

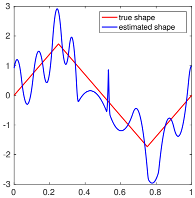


Estimation error

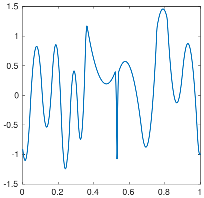
- **Bad news:** fail
- **However:** failure not extrme
  $\hookrightarrow$ iterative method?
- **Key observation:** the residual error is again a new mode decomposition problem

$$
\begin{aligned}
r(x) &= f(x) - \alpha_1(x)\bar{s}_1(2\pi\phi_1(x)) + \alpha_2(x)\bar{s}_2(2\pi\phi_2(x)) \\
&= \alpha_1(x)(s_1 - \bar{s}_1)(2\pi\phi_1(x)) + \alpha_2(x)(s_2 - \bar{s}_2)(2\pi\phi_2(x))
\end{aligned}
$$

The idea of diffeomorphism-based regression



Regression fails



Estimation error

- **Bad news:** fail
- **However:** failure not extrme
  $\hookrightarrow$ iterative method?
- **Key observation:** the residual error is again a new mode decomposition problem

$$\begin{aligned} r(x) &= f(x) - \alpha_1(x)\bar{s}_1(2\pi\phi_1(x)) + \alpha_2(x)\bar{s}_2(2\pi\phi_2(x)) \\ &= \alpha_1(x)(s_1 - \bar{s}_1)(2\pi\phi_1(x)) + \alpha_2(x)(s_2 - \bar{s}_2)(2\pi\phi_2(x)) \\ &:= \alpha_1(x)s_1^{(2)}(2\pi\phi_1(x)) + \alpha_2(x)s_2^{(2)}(2\pi\phi_2(x)) \end{aligned}$$

The idea of diffeomorphism-based regression

Iterative method? Let $\tilde{s}_k$ be the final estimation.

1. Step 1:

$$\begin{aligned} r^{(j)}(x) &= r^{(j-1)}(x) - \alpha_1(x)\bar{s}_1^{(j)}(2\pi\phi_1(x)) - \alpha_2(x)\bar{s}_2^{(j)}(2\pi\phi_2(x)) \\ &= \alpha_1(x)(s_1^{(j)} - \bar{s}_1^{(j)})(2\pi\phi_1(x)) + \alpha_2(x)(s_2^{(j)} - \bar{s}_2^{(j)})(2\pi\phi_2(x)) \\ &:= \alpha_1(x)s_1^{(j+1)}(2\pi\phi_1(x)) + \alpha_2(x)s_2^{(j+1)}(2\pi\phi_2(x)) \end{aligned}$$

2. Step 2: regression for $\bar{s}_k^{(j+1)}$ from $r^{(j)}(x)$.
3. Step 3: update $\tilde{s}_k \leftarrow \tilde{s}_k + \bar{s}_k^{(j+1)}$ for better estimation.

The idea of diffeomorphism-based regression

Iterative method? NO convergence!

1. Step 1:

$$\begin{aligned} r^{(j)}(x) &= r^{(j-1)}(x) - \alpha_1(x)\bar{s}_1^{(j)}(2\pi\phi_1(x)) - \alpha_2(x)\bar{s}_2^{(j)}(2\pi\phi_2(x)) \\ &= \alpha_1(x)(s_1^{(j)} - \bar{s}_1^{(j)})(2\pi\phi_1(x)) + \alpha_2(x)(s_2^{(j)} - \bar{s}_2^{(j)})(2\pi\phi_2(x)) \\ &:= \alpha_1(x)s_1^{(j+1)}(2\pi\phi_1(x)) + \alpha_2(x)s_2^{(j+1)}(2\pi\phi_2(x)) \end{aligned}$$

2. Step 2: regression for $\bar{s}_k^{(j+1)}$ from $r^{(j)}(x)$.
3. Step 3: update $\tilde{s}_k \leftarrow \tilde{s}_k + \bar{s}_k^{(j+1)}$ for better estimation.

The idea of diffeomorphism-based regression

New iterative method

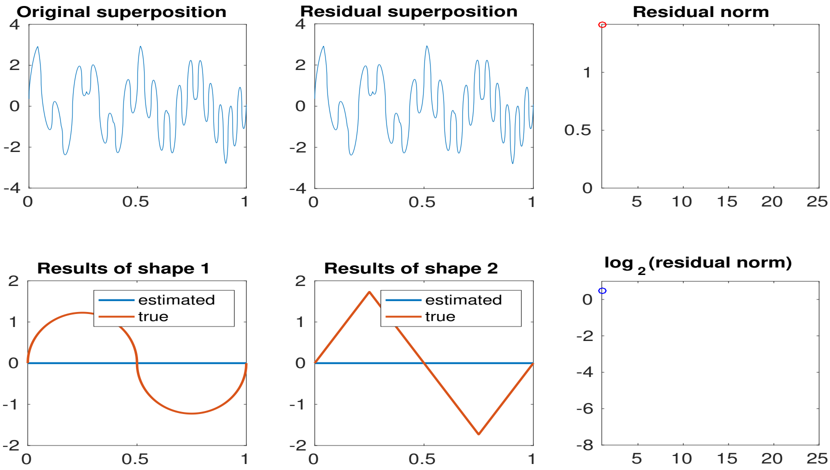
1. Step 1:

$$
\begin{aligned}
r^{(j)}(x) &= r^{(j-1)}(x) - \alpha_1(x)\bar{s}_1^{(j)}(2\pi\phi_1(x)) - \alpha_2(x)\bar{s}_2^{(j)}(2\pi\phi_2(x)) \\
&= \alpha_1(x)(s_1^{(j)} - \bar{s}_1^{(j)})(2\pi\phi_1(x)) + \alpha_2(x)(s_2^{(j)} - \bar{s}_2^{(j)})(2\pi\phi_2(x)) \\
&:= \alpha_1(x)s_1^{(j+1)}(2\pi\phi_1(x)) + \alpha_2(x)s_2^{(j+1)}(2\pi\phi_2(x))
\end{aligned}
$$

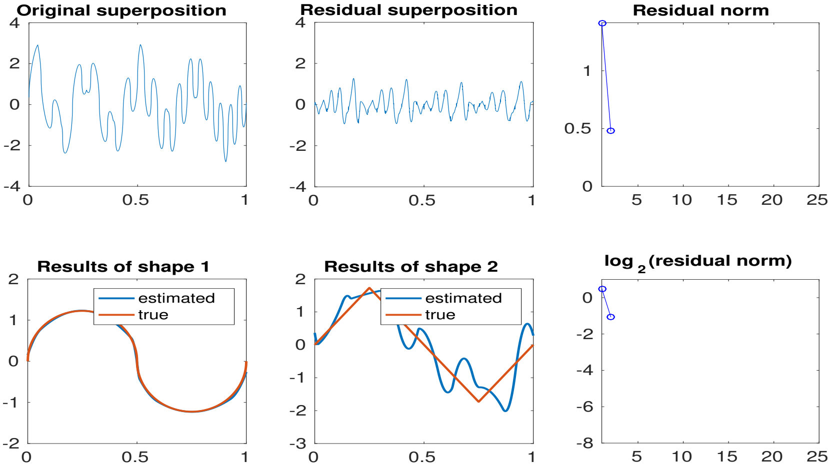
2. Step 2: regression for $\bar{s}_k^{(j+1)}$ from $r^{(j)}(x)$.
3. Step 3: update $\bar{s}_k^{(j+1)} \leftarrow \bar{s}_k^{(j+1)} - \frac{1}{2\pi}\int_0^{2\pi} \bar{s}_k^{(j+1)}(x)dx$ (key step for convergence).
4. Step 4: update $\tilde{s}_k \leftarrow \tilde{s}_k + \bar{s}_k^{(j+1)}$ for better estimation.

Remark: only after careful mathematical analysis, we realize the key step.

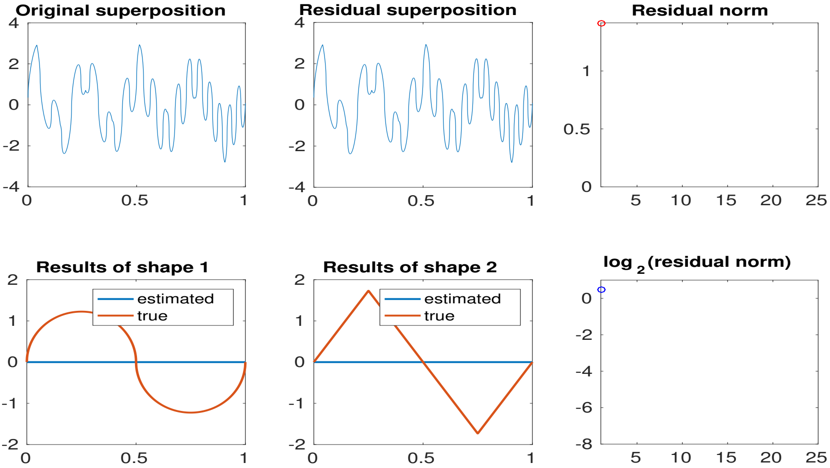
The idea of recursive diffeomorphism-based regression



The idea of recursive diffeomorphism-based regression



The idea of recursive diffeomorphism-based regression



Go back

Convergence analysis

Theorem (Xu, **Y.**, Daubechies, Preprint, 2016)

- Given data:

$$f(x) = \sum_{k=1}^{2} \alpha_k(x) s_k(2\pi\phi_k(x)) + n(t)$$

sampled on $[0, T]$ with N samples, where $n(t)$ is a random nose with a bounded variance.

- Assume $\{\alpha_k(x)\}$ and $\{\phi_k(x)\}$ are known smooth functions, $\{\phi_k(x)\}$ are well-differentiated, and s_k are Lipschitz continuous with $\widehat{s}(0) = 0$.
- The residual data in the jth iteration satisfies

$$\|r^{(j)}\|_{L^2} \leq O(\epsilon + \beta^j).$$

- Convergence is
  - $\beta < 1$, linear, if N and T are large enough;
  - robust against noise as long as N is large enough.

Remark:

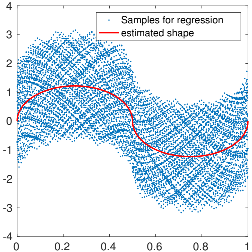
- In fact, the theory works for K components;
- We numerically observed the convergence even though T is small.

Convergence analysis

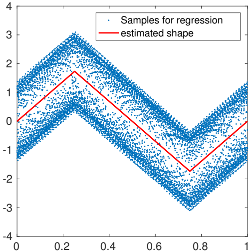
Diffeomorphism for periodicity:

$$
\begin{aligned}
h_k(v) &= \frac{f \circ \phi_k^{-1}(v)}{\alpha_k \circ \phi_k^{-1}(v)} \\
&= s_k(2\pi v) + \sum_{j \neq k} \frac{\alpha_j \circ \phi_k^{-1}(v)}{\alpha_k \circ \phi_k^{-1}(v)} s_j(2\pi \phi_j \circ \phi_k^{-1}(v)) \\
&= s_k(2\pi v) + \kappa_k(v),
\end{aligned}
$$

Folding to create "stochastic processes" $\kappa_k(v)$ for $v \in [0, 1]$:



$s_1(2\pi v) + \kappa_1(v)$



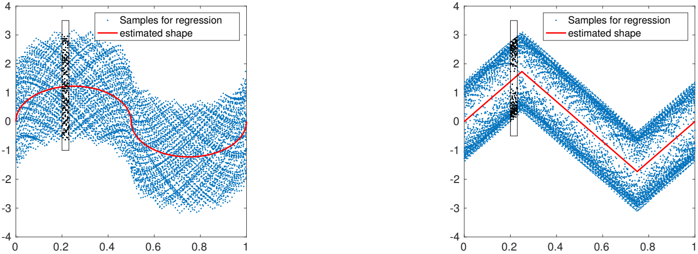
$s_2(2\pi v) + \kappa_2(v)$

Convergence analysis

Diffeomorphism for periodicity:

$$\begin{aligned} h_k(v) &= \frac{f \circ \phi_k^{-1}(v)}{\alpha_k \circ \phi_k^{-1}(v)} \\ &= s_k(2\pi v) + \sum_{j \neq k} \frac{\alpha_j \circ \phi_k^{-1}(v)}{\alpha_k \circ \phi_k^{-1}(v)} s_j(2\pi \phi_j \circ \phi_k^{-1}(v)) \\ &= s_k(2\pi v) + \kappa_k(v), \end{aligned}$$

Folding to create "stochastic processes" $\kappa_k(v)$ for $v \in [0, 1]$:



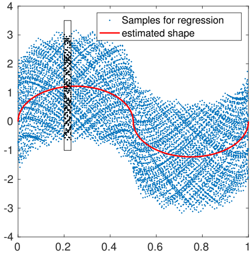
$s_1(2\pi v) + \kappa_1(v)$ $\qquad$ $s_2(2\pi v) + \kappa_2(v)$

Convergence analysis

Diffeomorphism for periodicity:

$$\begin{aligned} h_1(v) &= \frac{f \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} \\ &= s_1(2\pi v) + \frac{\alpha_2 \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} s_2(2\pi \phi_2 \circ \phi_1^{-1}(v)) \\ &= s_1(2\pi v) + \kappa_1(v), \end{aligned}$$

Folding to create "stochastic processes" $\kappa_k(v)$ for $v \in [0, 1]$:



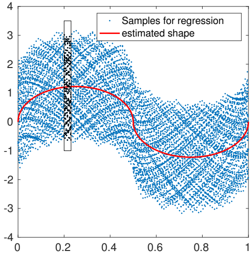
$s_1(2\pi v) + \kappa_1(v)$

Convergence analysis

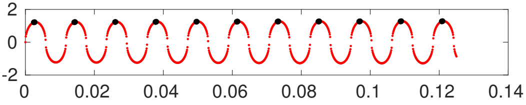
Diffeomorphism for periodicity:

$$\begin{aligned} h_1(v) &= \frac{f \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} \\ &= s_1(2\pi v) + \frac{\alpha_2 \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} s_2(2\pi \phi_2 \circ \phi_1^{-1}(v)) \\ &= s_1(2\pi v) + \kappa_1(v), \end{aligned}$$

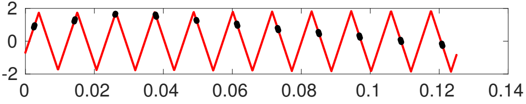
Folding to create “stochastic processes” $\kappa_k(v)$ for $v \in [0, 1]$:



$s_1(2\pi v) + \kappa_1(v)$

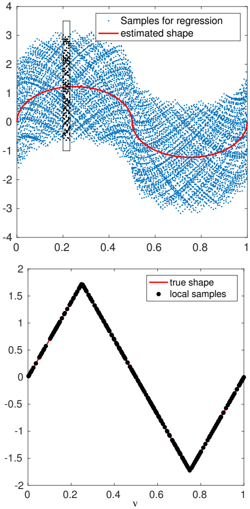


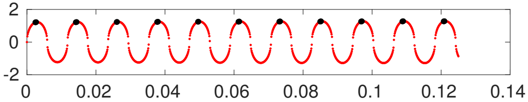
$s_1(2\pi v)$



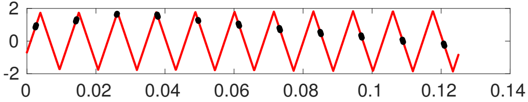
$\kappa_1(v) = \frac{\alpha_2 \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} s_2(2\pi \phi_2 \circ \phi_1^{-1}(v))$

Convergence analysis



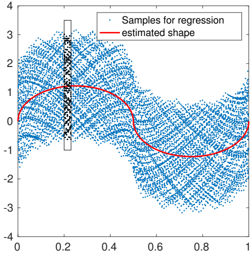


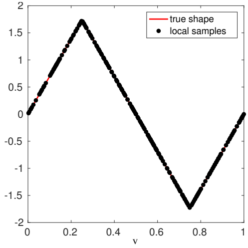
$s_1(2\pi v)$

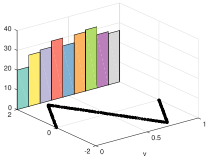


$$\kappa_1(v) = \frac{\alpha_2 \circ \phi_1^{-1}(v)}{\alpha_1 \circ \phi_1^{-1}(v)} s_2(2\pi\phi_2 \circ \phi_1^{-1}(v))$$

Convergence analysis

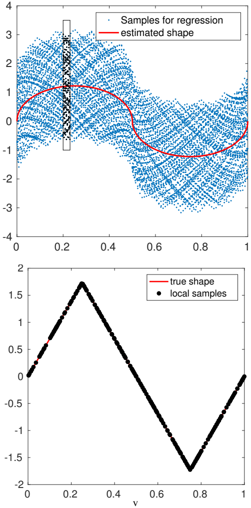


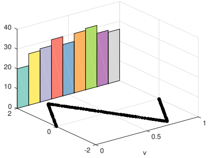




Histogram of black points

Convergence analysis

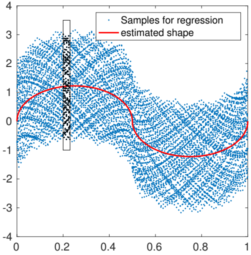


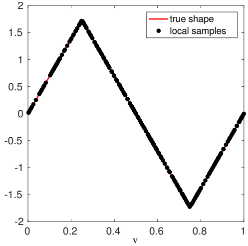


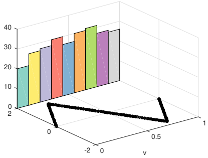
Histogram of black points

$$\bar{s}_1(2\pi v) \quad := \quad \mathrm{E}(s_1(2\pi v) + \kappa_1(v))$$

Convergence analysis



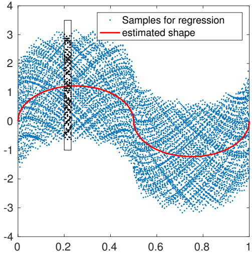


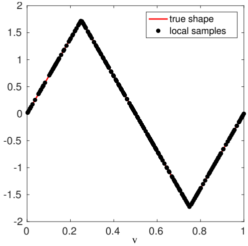


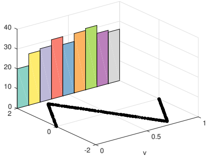
Histogram of black points

$$\begin{aligned}\bar{s}_1(2\pi v) &:= \mathrm{E}(s_1(2\pi v) + \kappa_1(v)) \\ &= s_1(2\pi v) + \mathrm{E}(\kappa_1(v))\end{aligned}$$

Convergence analysis



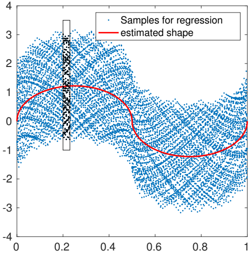


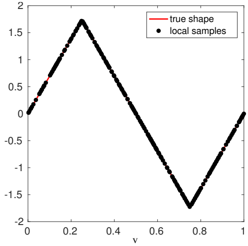


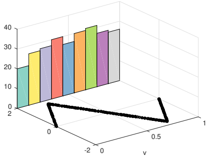
Histogram of black points

$$\begin{aligned} \bar{s}_1(2\pi v) &:= \mathrm{E}(s_1(2\pi v) + {\color{blue}\kappa_1(v)}) \\ &= s_1(2\pi v) + \mathrm{E}({\color{blue}\kappa_1(v)}) \\ &\approx s_1(2\pi v) + \int_0^1 s_2(2\pi v)dv \end{aligned}$$

Convergence analysis



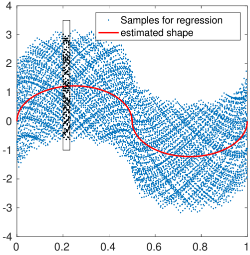


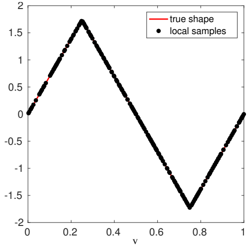


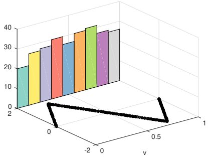
Histogram of black points

$$
\begin{aligned}
\bar{s}_1(2\pi v) &:= \mathrm{E}(s_1(2\pi v) + \kappa_1(v)) \\
&= s_1(2\pi v) + \mathrm{E}(\kappa_1(v)) \\
&\approx s_1(2\pi v) + \int_0^1 s_2(2\pi v)dv \\
&= s_1(2\pi v)
\end{aligned}
$$

Convergence analysis







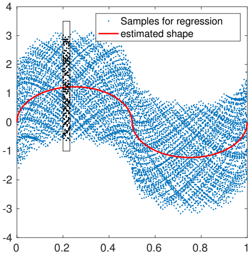
Histogram of black points

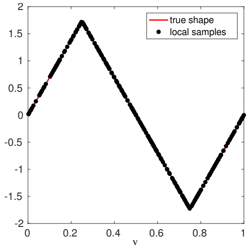
$$
\begin{aligned}
\bar{s}_1(2\pi v) &:= \mathrm{E}(s_1(2\pi v) + \kappa_1(v)) \\
&= s_1(2\pi v) + \mathrm{E}(\kappa_1(v)) \\
&\approx s_1(2\pi v) + \int_0^1 s_2(2\pi v)dv \\
&= s_1(2\pi v)
\end{aligned}
$$

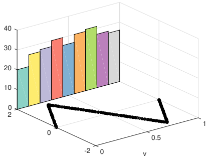
If distribution close to uniform:

$$
\|\bar{s}_1 - s_1\|_{L^2} \leq \left(\int_0^1 |\mathrm{E}(\kappa_1(v))|^2 dv\right)^{1/2} \approx 0
$$

Convergence analysis







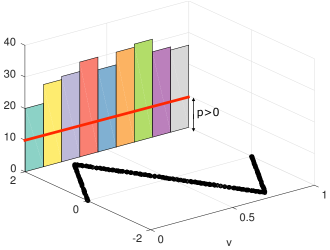
Histogram of black points

$$
\begin{aligned}
\bar{s}_1(2\pi v) &:= \mathrm{E}(s_1(2\pi v) + \kappa_1(v)) \\
&= s_1(2\pi v) + \mathrm{E}(\kappa_1(v)) \\
&\approx s_1(2\pi v) + \int_0^1 s_2(2\pi v)dv \\
&= s_1(2\pi v)
\end{aligned}
$$

Distribution close to uniform? NO!

$$
\|\bar{s}_1 - s_1\|_{L^2} = \left(\int_0^1 |\mathrm{E}(\kappa_1(v))|^2 dv \right)^{1/2} \approx 0?
$$

Convergence analysis



$$\bar{s}_1(2\pi v) := \mathrm{E}(s_1(2\pi v) + \kappa_1(v))$$
$$\approx s_1(2\pi v) + \int_0^1 s_2(2\pi v)dv$$
$$= s_1(2\pi v)$$

As long as $p > 0$, $\exists 0 < \beta < 1$ such that

$$\|\bar{s}_1 - s_1\|_{L^2} = \left(\int_0^1 |\mathrm{E}(\kappa_1(v))|^2 dv\right)^{1/2} \leq \beta \|s_2\|_{L^2}$$

and hence

$$\max\{\|\bar{s}_k - s_k\|_{L^2}\} \leq \beta \max\{\|s_k\|_{L^2}\}.$$

Finally

$$\|r^{(j)}\|_{L^2} \leq O(\epsilon + \beta^j).$$

The idea of diffeomorphism-based regression

New iterative method

1. Step 1:

$$\begin{aligned} r^{(j)}(x) &= r^{(j-1)}(x) - \alpha_1(x)\bar{s}_1^{(j)}(2\pi\phi_1(x)) - \alpha_2(x)\bar{s}_2^{(j)}(2\pi\phi_2(x)) \\ &= \alpha_1(x)(s_1^{(j)} - \bar{s}_1^{(j)})(2\pi\phi_1(x)) + \alpha_2(x)(s_2^{(j)} - \bar{s}_2^{(j)})(2\pi\phi_2(x)) \\ &:= \alpha_1(x)s_1^{(j+1)}(2\pi\phi_1(x)) + \alpha_2(x)s_2^{(j+1)}(2\pi\phi_2(x)) \end{aligned}$$

2. Step 2: regression for $\bar{s}_k^{(j+1)}$ from $r^{(j)}(x)$.
3. Step 3: update $\bar{s}_k^{(j+1)} \leftarrow \bar{s}_k^{(j+1)} - \frac{1}{2\pi}\int_0^{2\pi} \bar{s}_k^{(j+1)}(x)dx$ (key step for convergence).
4. Step 4: update $\tilde{s}_k \leftarrow \tilde{s}_k + \bar{s}_k^{(j+1)}$ for better estimation.

Remark: only after careful mathematical analysis, we realize the key step.

Future works

Theorem for the recursive regression (Xu, **Y.**, Daubechies, Preprint, 2016)

- Given data:

$$f(x) = \sum_{k=1}^{K} \alpha_k(x) s_k(2\pi\phi_k(x)) + n(t)$$

  sampled on $[0, T]$ with N samples, where $n(t)$ is a random nose with a bounded variance.
- Assume $\{\alpha_k(x)\}$ and $\{\phi_k(x)\}$ are known smooth functions, $\{\phi_k(x)\}$ are well-differentiated, and s_k are Lipschitz continuous with $\widehat{s}(0) = 0$.
- The residual data in the jth iteration satisfies

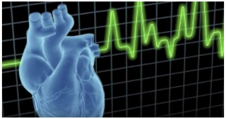
$$\|r^{(j)}\|_{L^2} \leq O(\epsilon + \beta^j) \quad \text{and} \quad s_k^{(j)} \to s_k + \epsilon$$

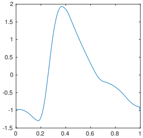
- Convergence is
  - $\beta < 1$, linear, if N and T are large enough;
  - robust against noise as long as N is large enough.

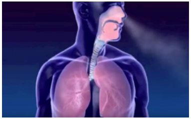
- Fast algorithms to get the shape function in each iteration?
- Fast convergence?
- Lower bound of the number of periods T and samples N?

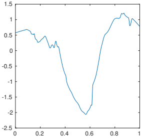
Numerical examples

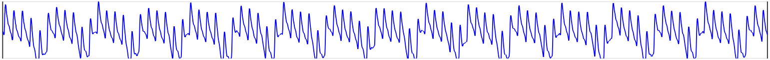
Photoplethysmogram (PPG) signal



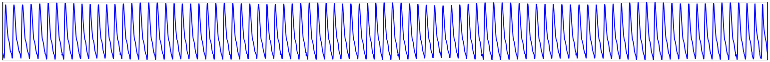




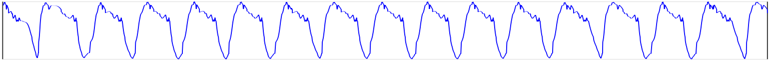




Raw PPG signal



Recovered cardiac component



Recovered respiratory component

Thank you!