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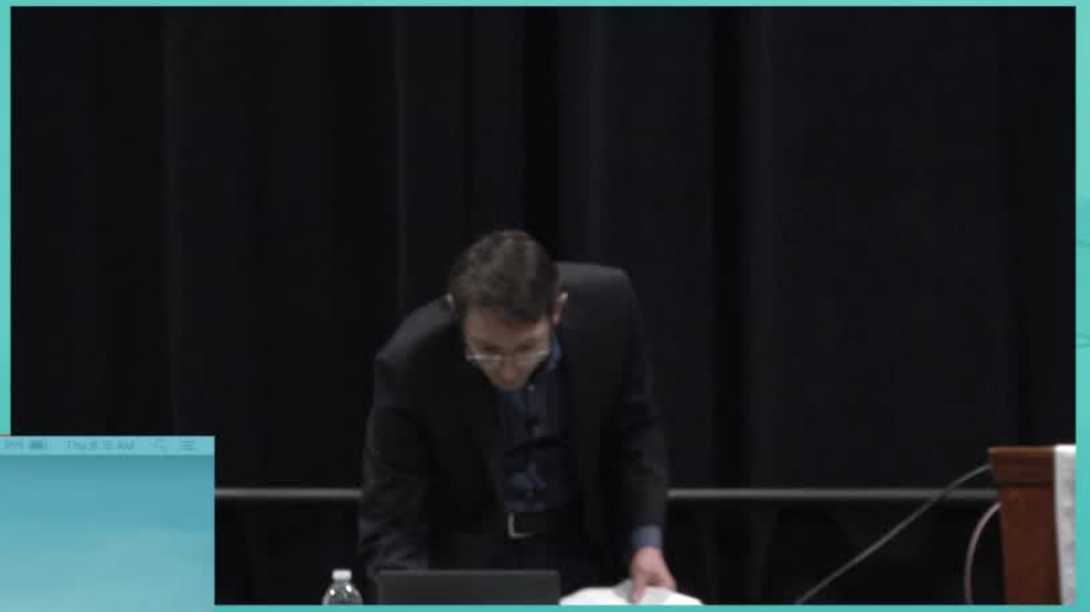

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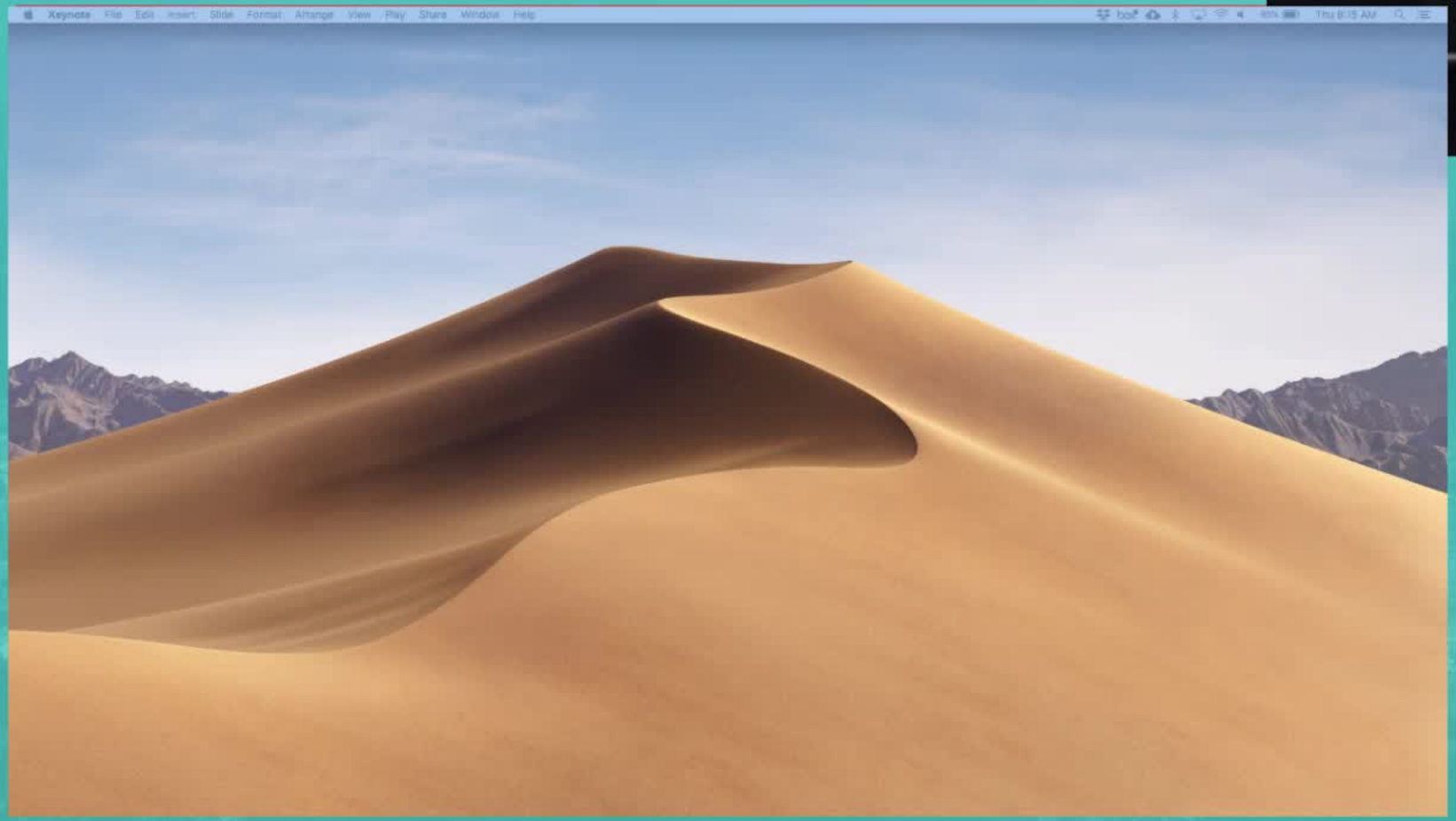

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Data-driven discovery and control of complex systems: uncovering interpretable and generalizable models

Steve Brunton
University of Washington

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GORDON AND BETTY MOORE FOUNDATION

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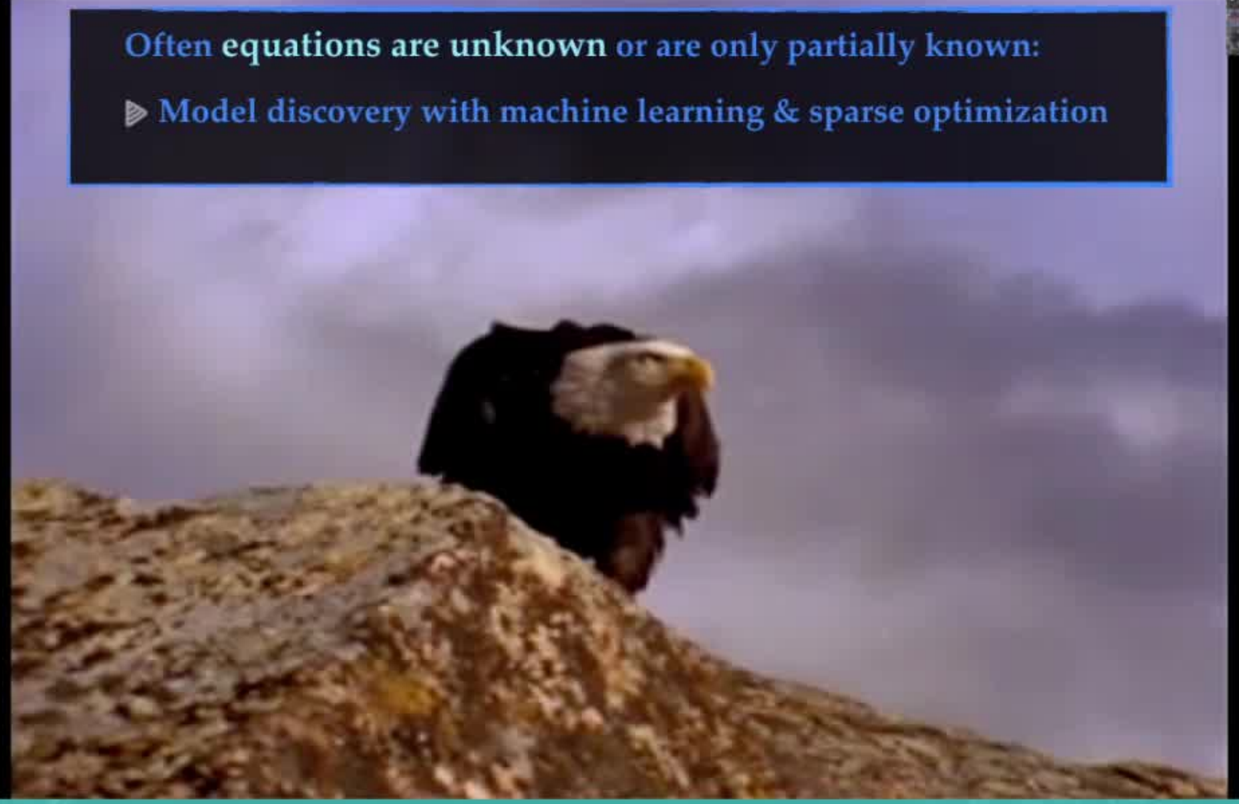

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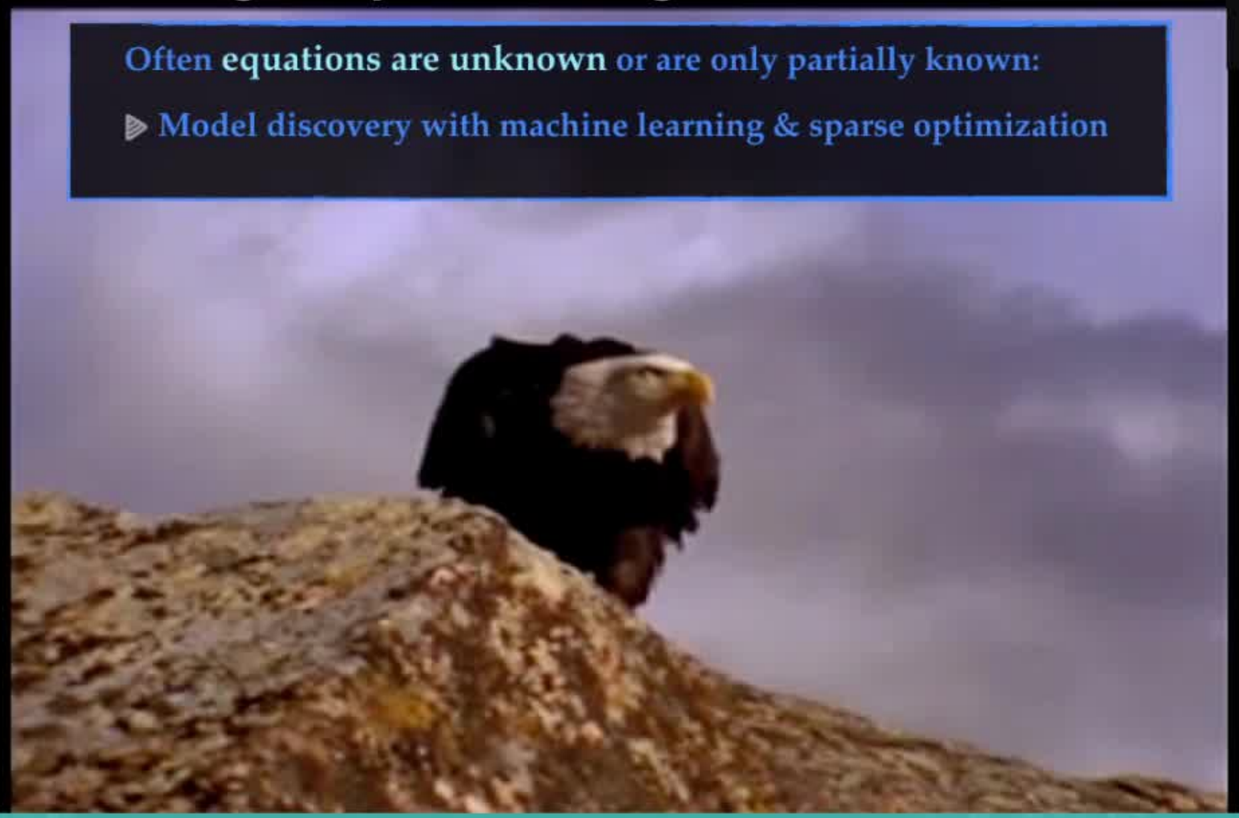

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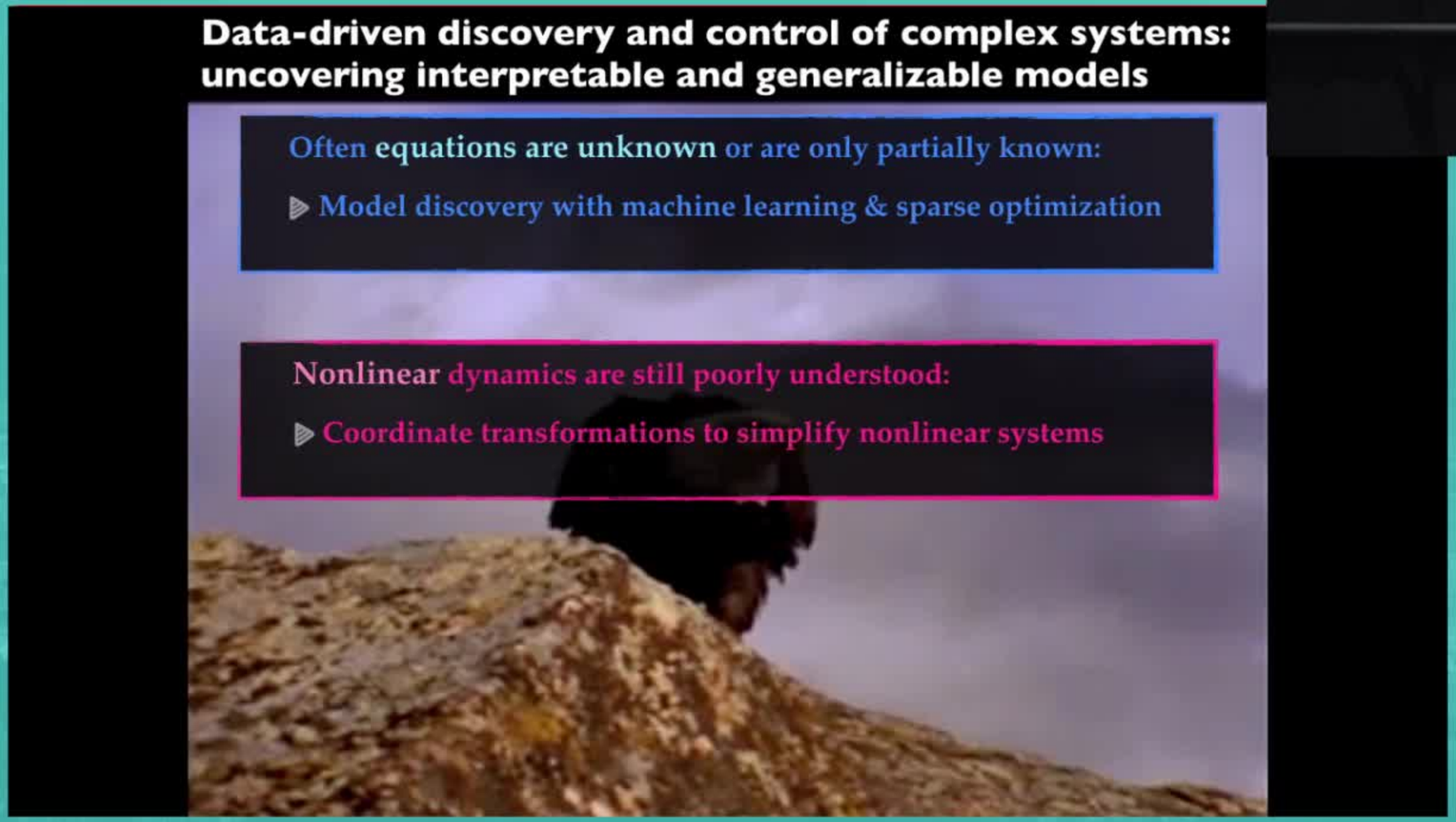

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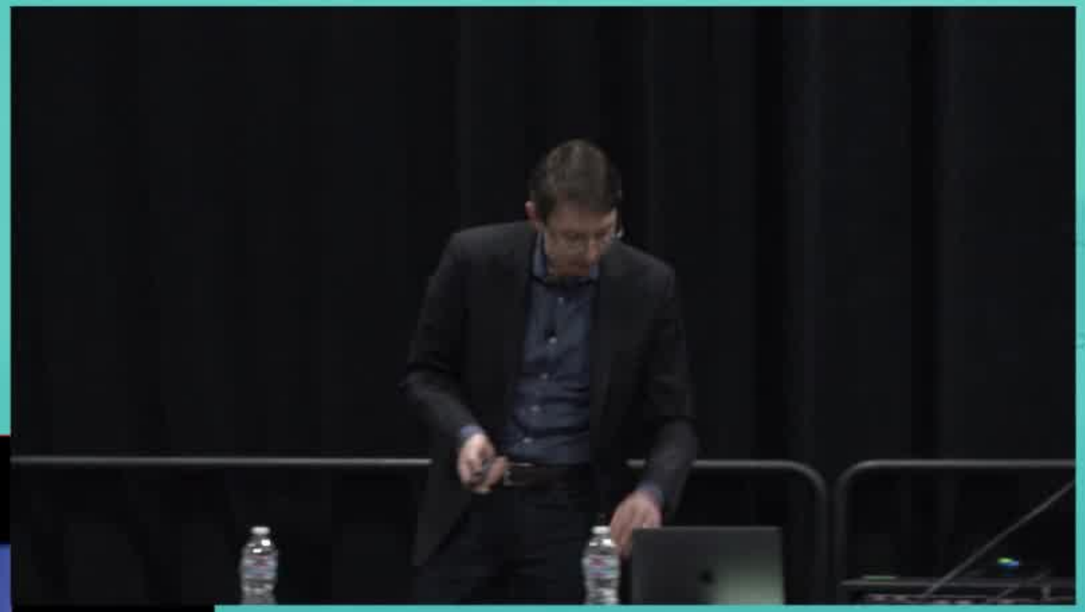

Data-driven discovery and control of complex systems: uncovering interpretable and generalizable models

Often equations are unknown or are only partially known:

- Model discovery with machine learning & sparse optimization

Nonlinear dynamics are still poorly understood:

- Coordinate transformations to simplify nonlinear systems

Our approach:

- Learn physics from data: interpretable and generalizable
- Respect known, or partially known, physics
- The existence of patterns facilitate sparse (few) measurements
- Machine learning is high-dimensional optimization with data

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MODEL DISCOVERY

Lots of great work:

- Gonzalez-Garcia, Rico-Martinez, Kevrekidis, *Comp. Chem. Eng.* 1998
- Yao and Bollt, *Physica D*, 2007
- Bongard and Lipson, *PNAS* 2007
- Schmidt and Lipson, *Science* 2009
- Wang, Yang, Lai, Kovanis, Grebogi, *PRL* 2011
- Bright, Lin, Kutz, *Phys. Fluids*, 2013
- Schaeffer, Caflisch, Hauck, Osher, *PNAS* 2013
- Noe, et al., Molecular dynamics, 2013-2016
- Schaeffer, *PRSA*, 2017
- Schaeffer, Tran, Ward, *SIAP*, 2018
- Raissi, Perdikaris, Karniadakis, *JCP* 2019
- … and many more!!!

Sparsity/parsimony in dynamics

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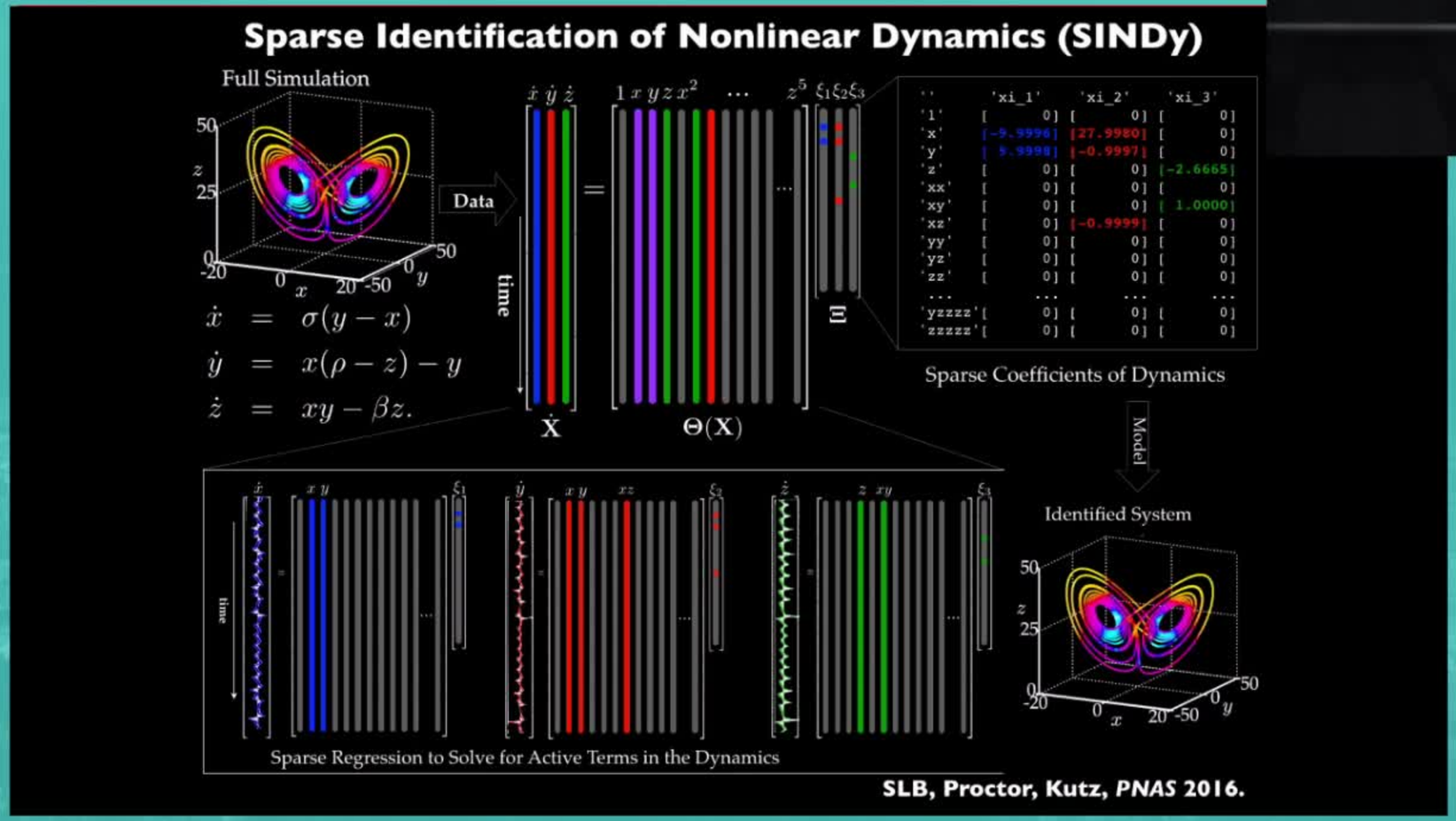

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FLUIDS

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Constrained Sparse Galerkin Regression

Innovation 1: Enforcing known constraints

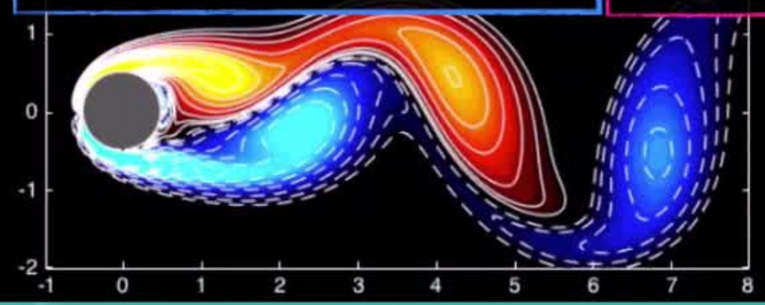
- Skew-symmetric quadratic nonlinearities to enforce energy conservation
- Improved stability

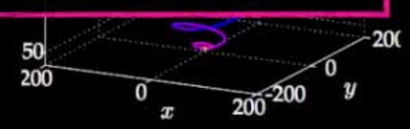
$$\min_{\xi,z} \|\Theta(X)\Xi - \dot{X}\|_2^2 + z^T(C\xi - d)$$

Innovation 2: Higher-order Nonlinearities

- Cubic, Quintic, Septic terms approximate truncated terms in Galerkin expansion

$$\begin{aligned} \dot{x} &= \mu x - \omega y + Axz \\ \dot{y} &= \omega x + \mu y + Ayz \\ \dot{z} &= -\lambda(z - x^2 - y^2). \end{aligned}$$





Loiseau & SLB, *JFM* 838, 2018

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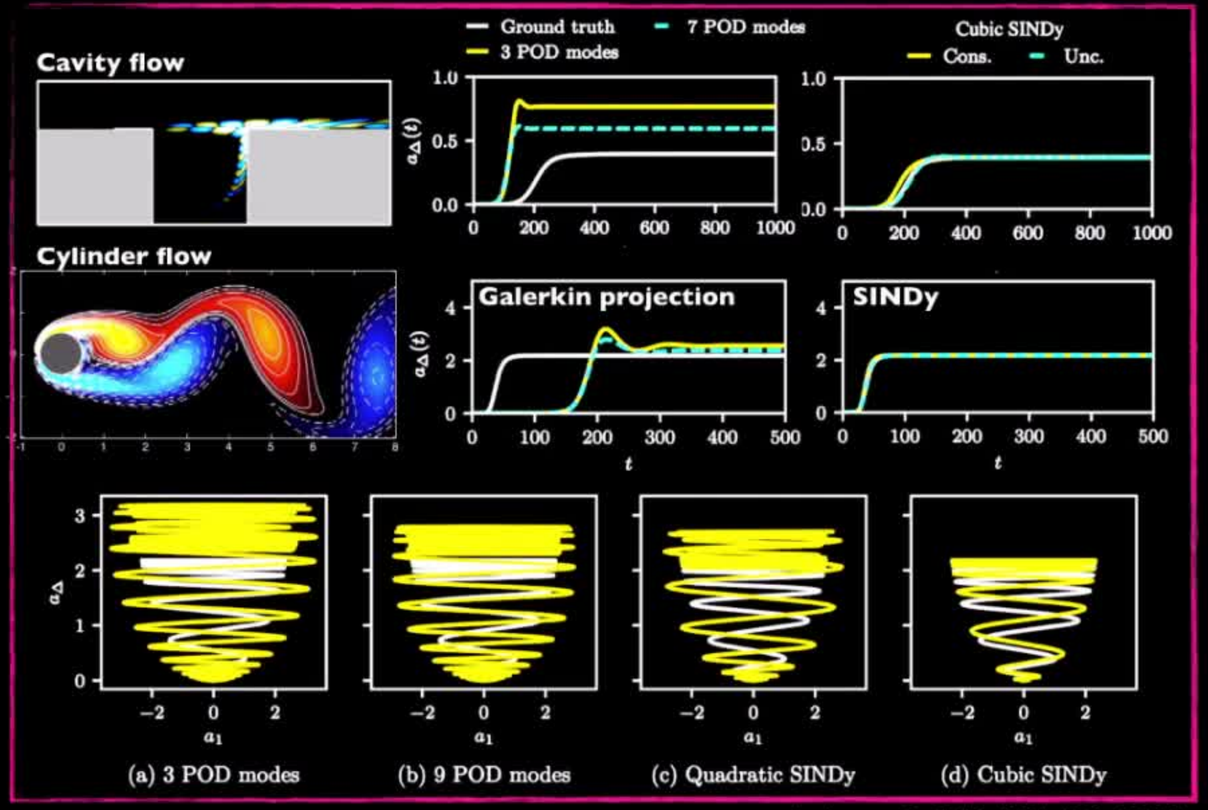
Constrained Sparse Galerkin Regression



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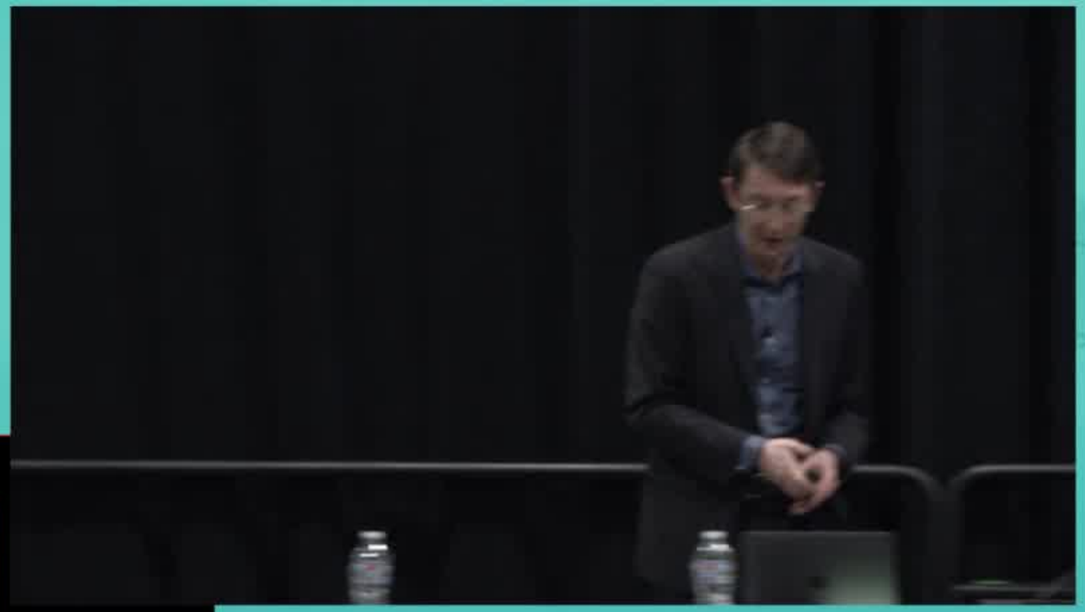

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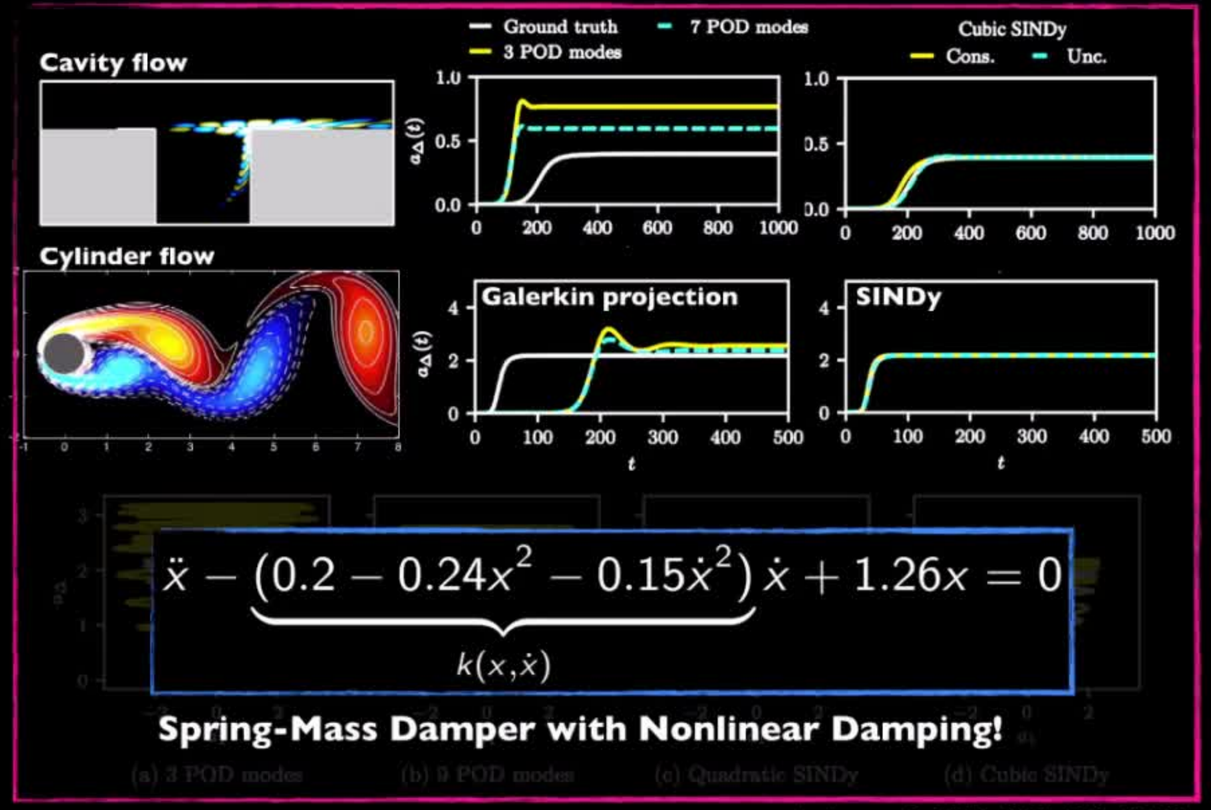

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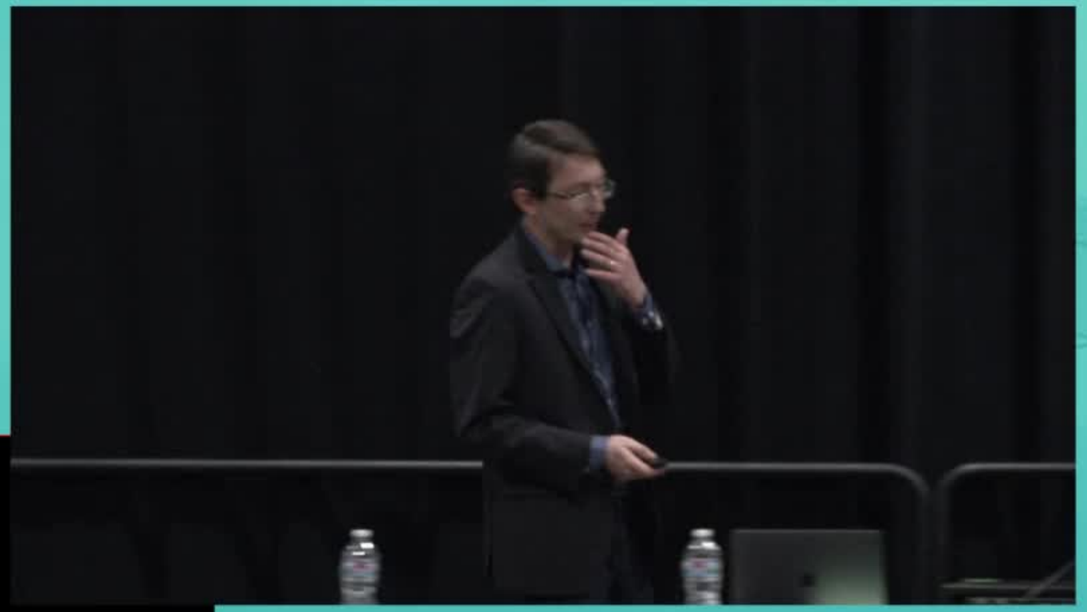

PDES

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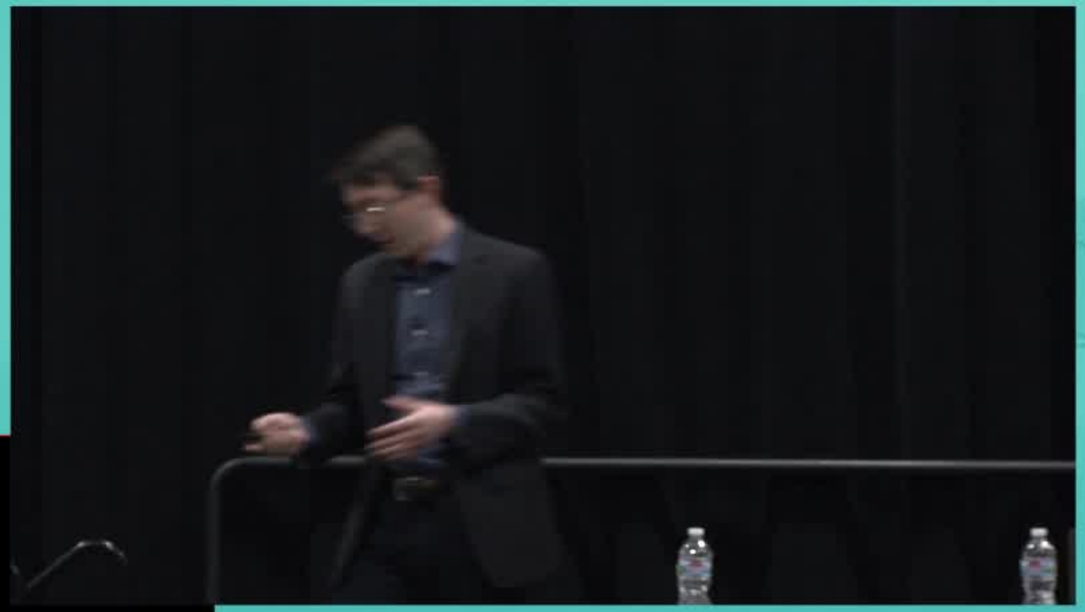
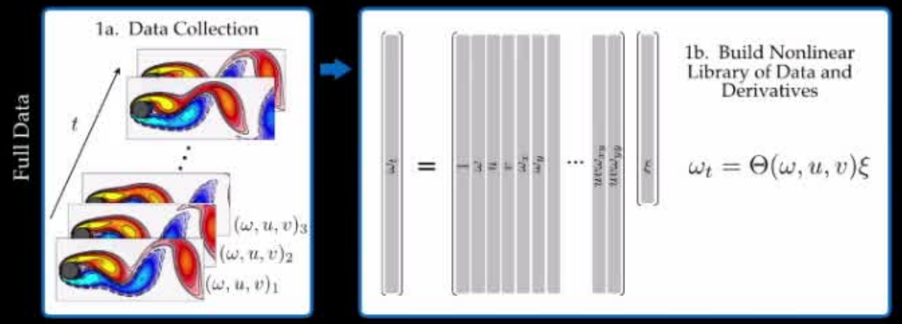


SINDy: Partial Differential Equations



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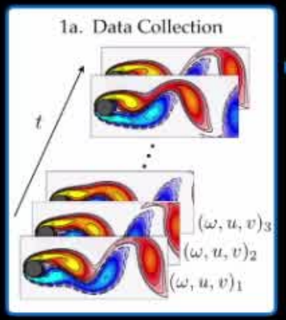
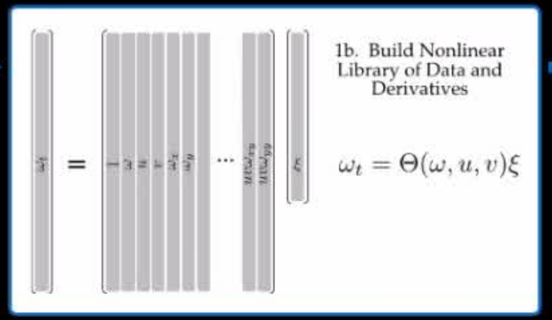
SINDy: Partial Differential Equations

Full Data

1a. Data Collection





1b. Build Nonlinear Library of Data and Derivatives

$$\omega_t = \Theta(\omega, u, v)\xi$$

1c. Solve Sparse Regression

$$\arg\min_{\xi} \|\Theta\xi - \omega_t\|_2^2 + \lambda\|\xi\|_0$$

d. Identified Dynamics

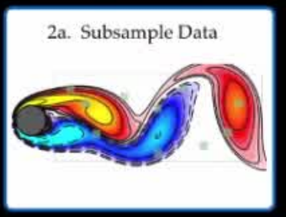
$$\omega_t + 0.9931 u\omega_x + 0.9910 v\omega_y = 0.0099\omega_{xx} + 0.0099\omega_{yy}$$

Compare to True Navier Stokes ($Re = 100$)

$$\omega_t + (\mathbf{u}\cdot\nabla)\omega = \frac{1}{Re}\nabla^2\omega$$

Compressed Data

2a. Subsample Data



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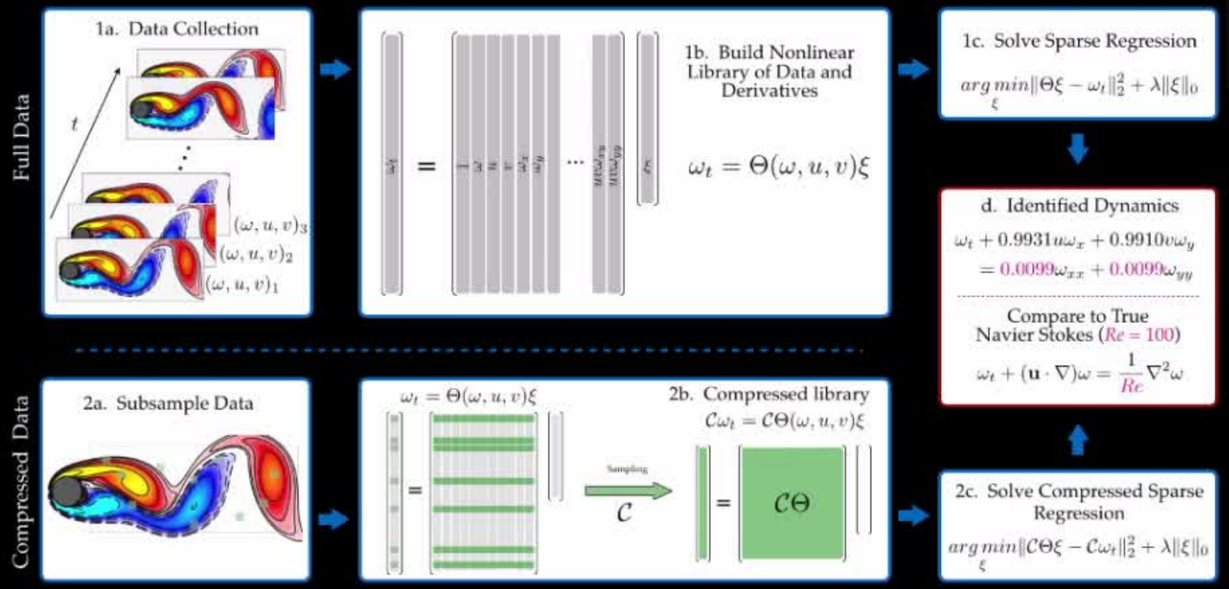

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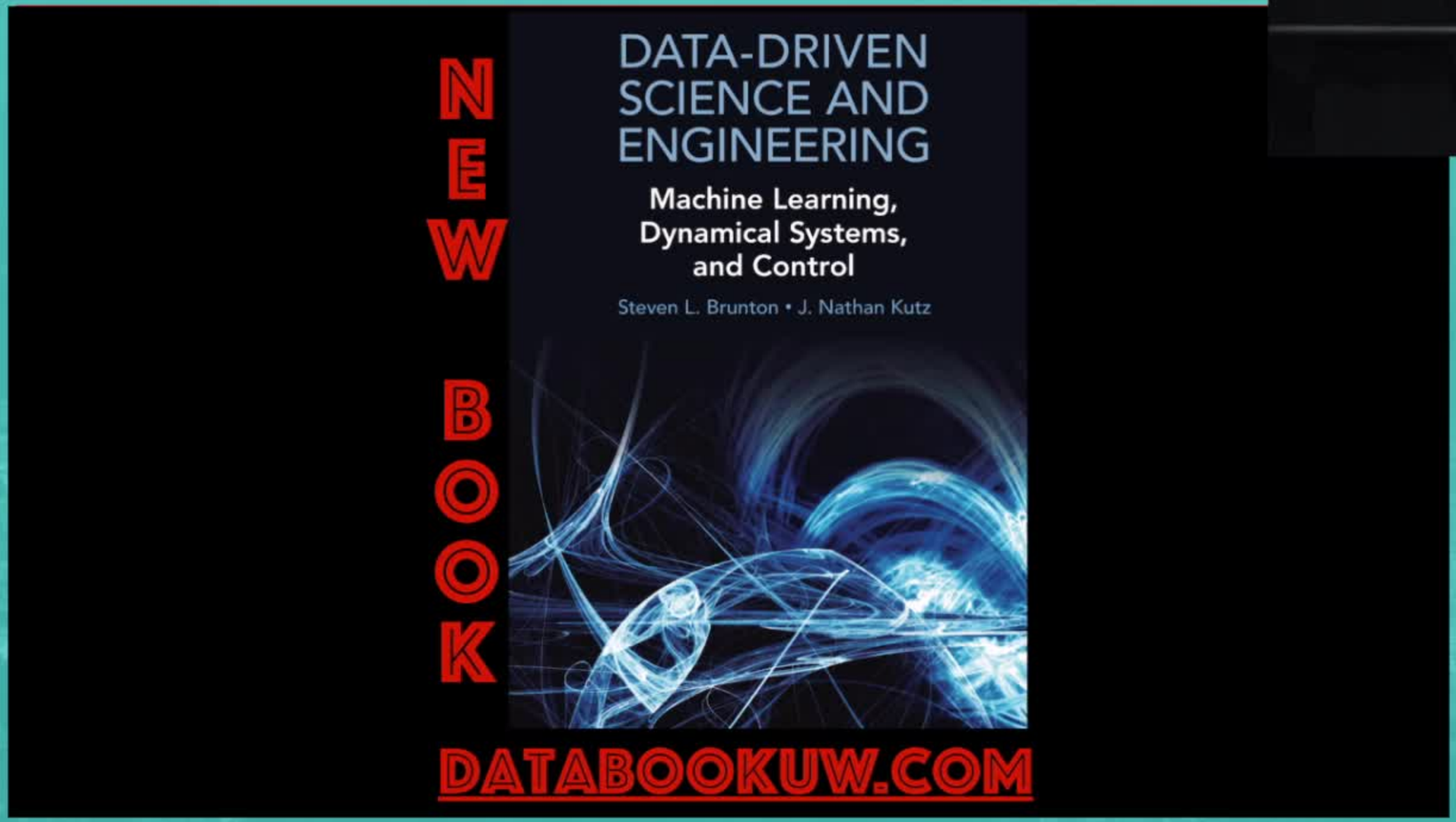

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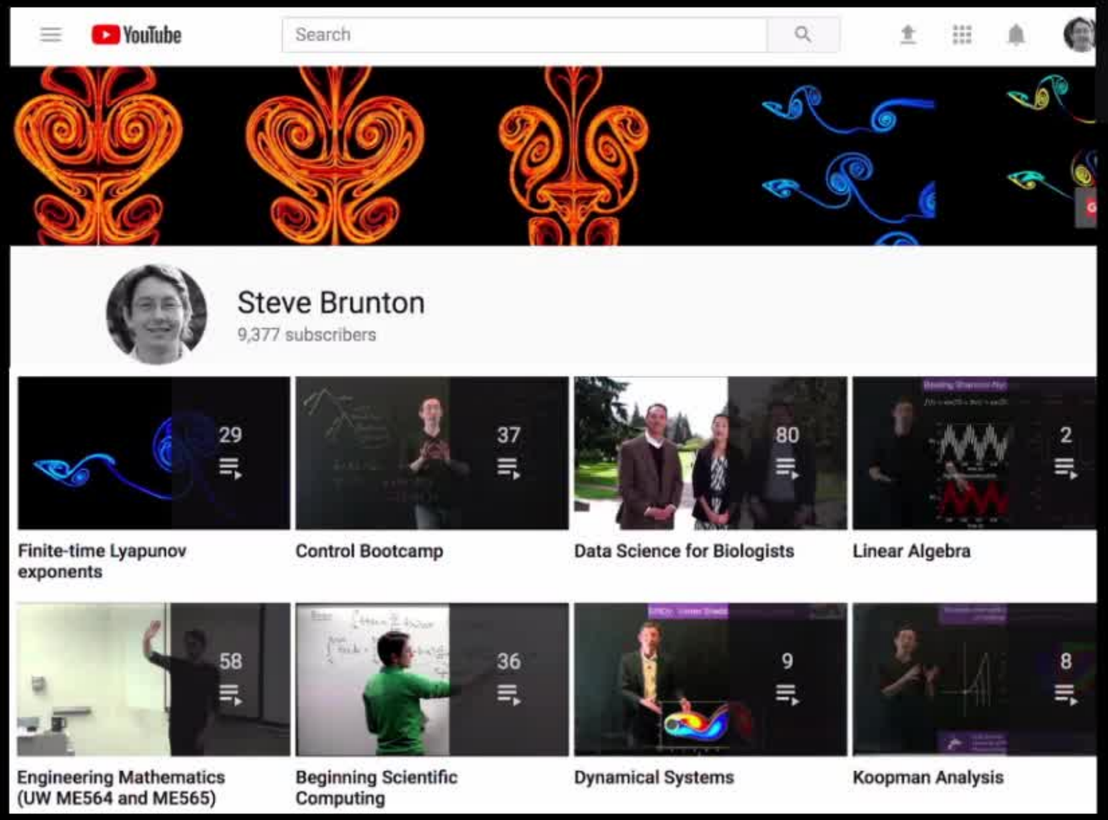
VIDEOS



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