A direct method for computing failure boundaries of non-autonomous systems

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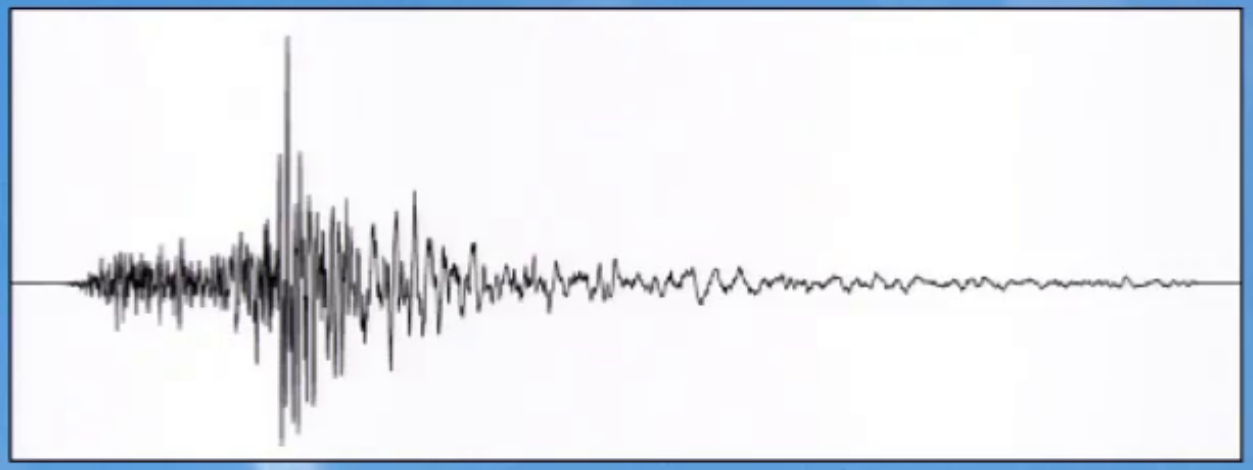

Joint work with
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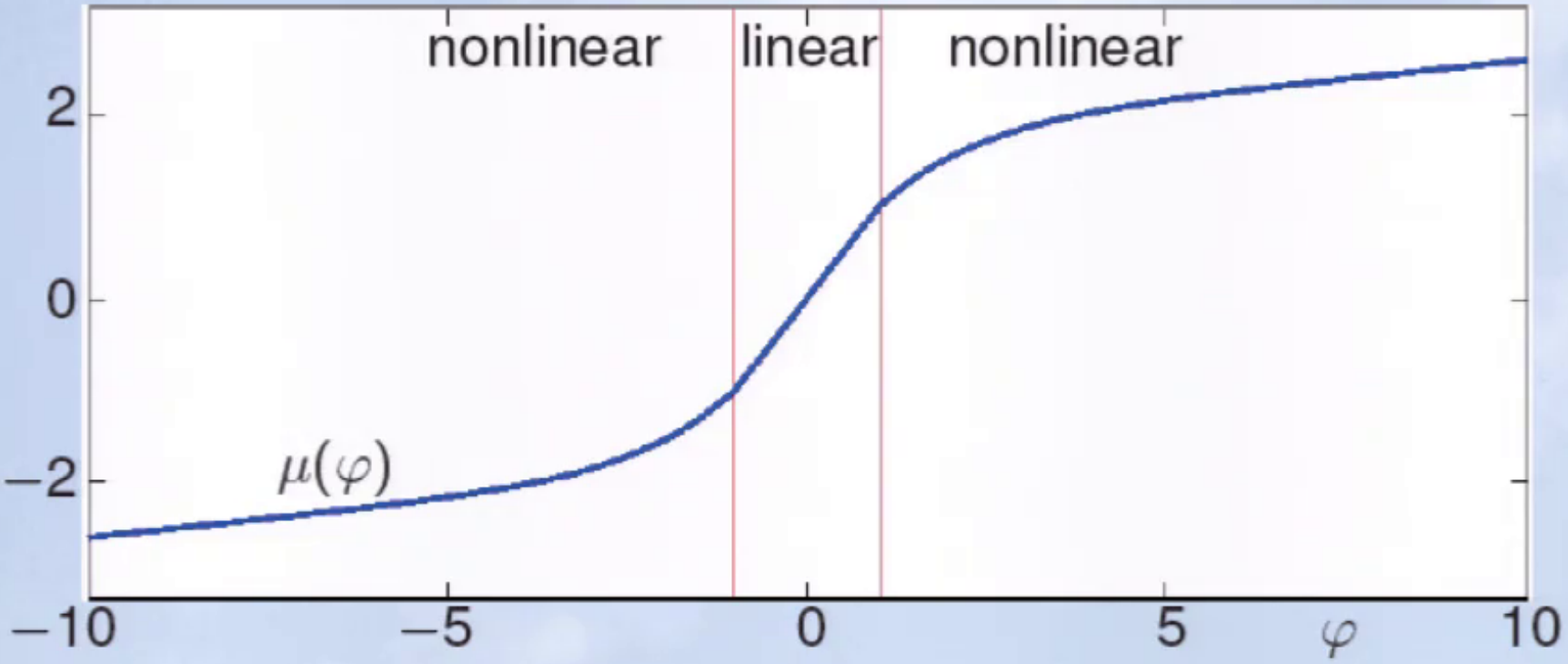
Motivation

[Alexander, Taylor, HMO & Kelly
J. Sound and Vibration **330** (2011)]

- precast concrete frame with post-tensioned tendons that connect elements
- model equivalent to tied rocking block on elastic foundation
- earthquake modelled by sine wave

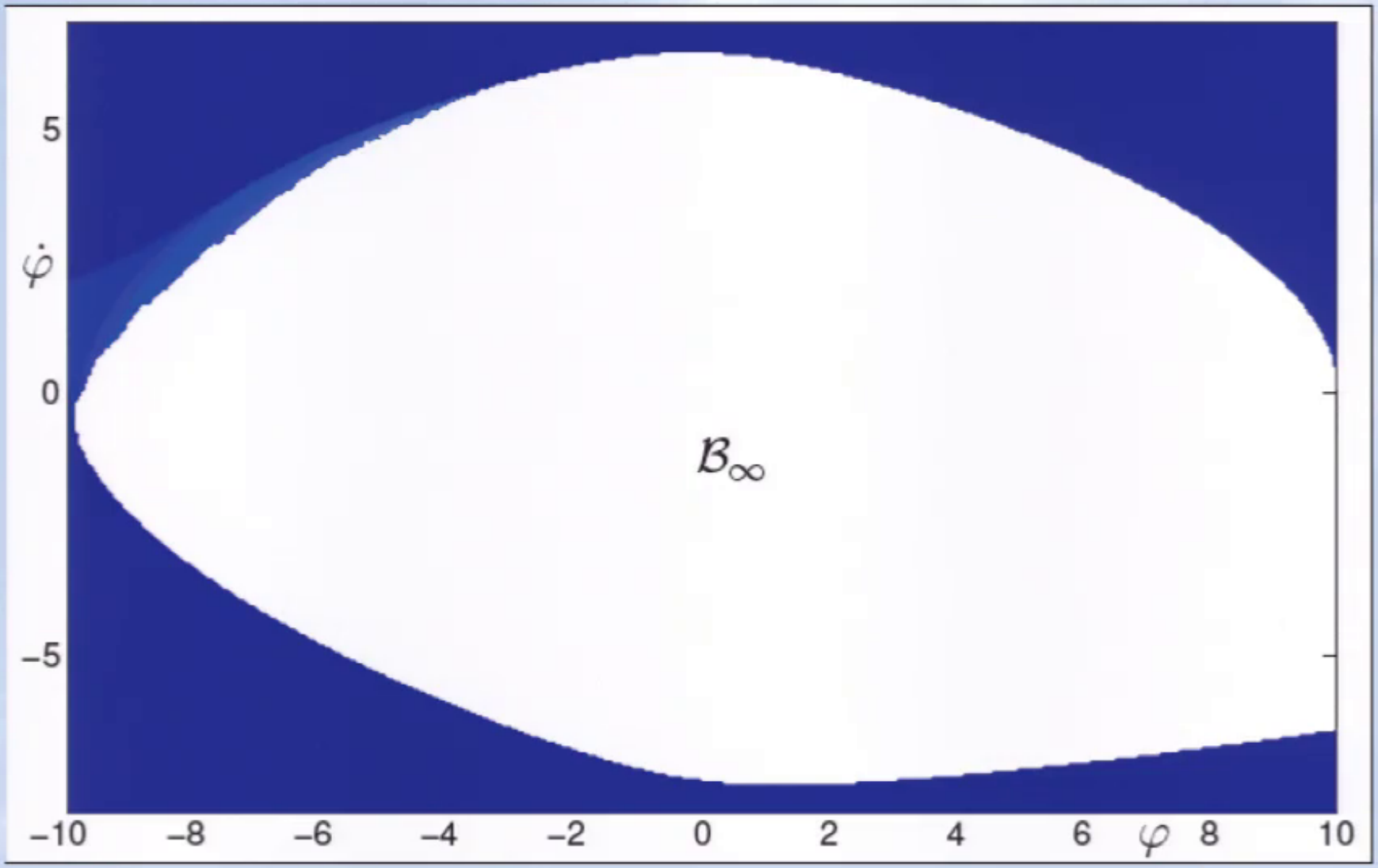
Model equations

$$\ddot{\varphi} + 2\gamma\dot{\varphi} + \mu(\varphi) = A\sin\omega t$$



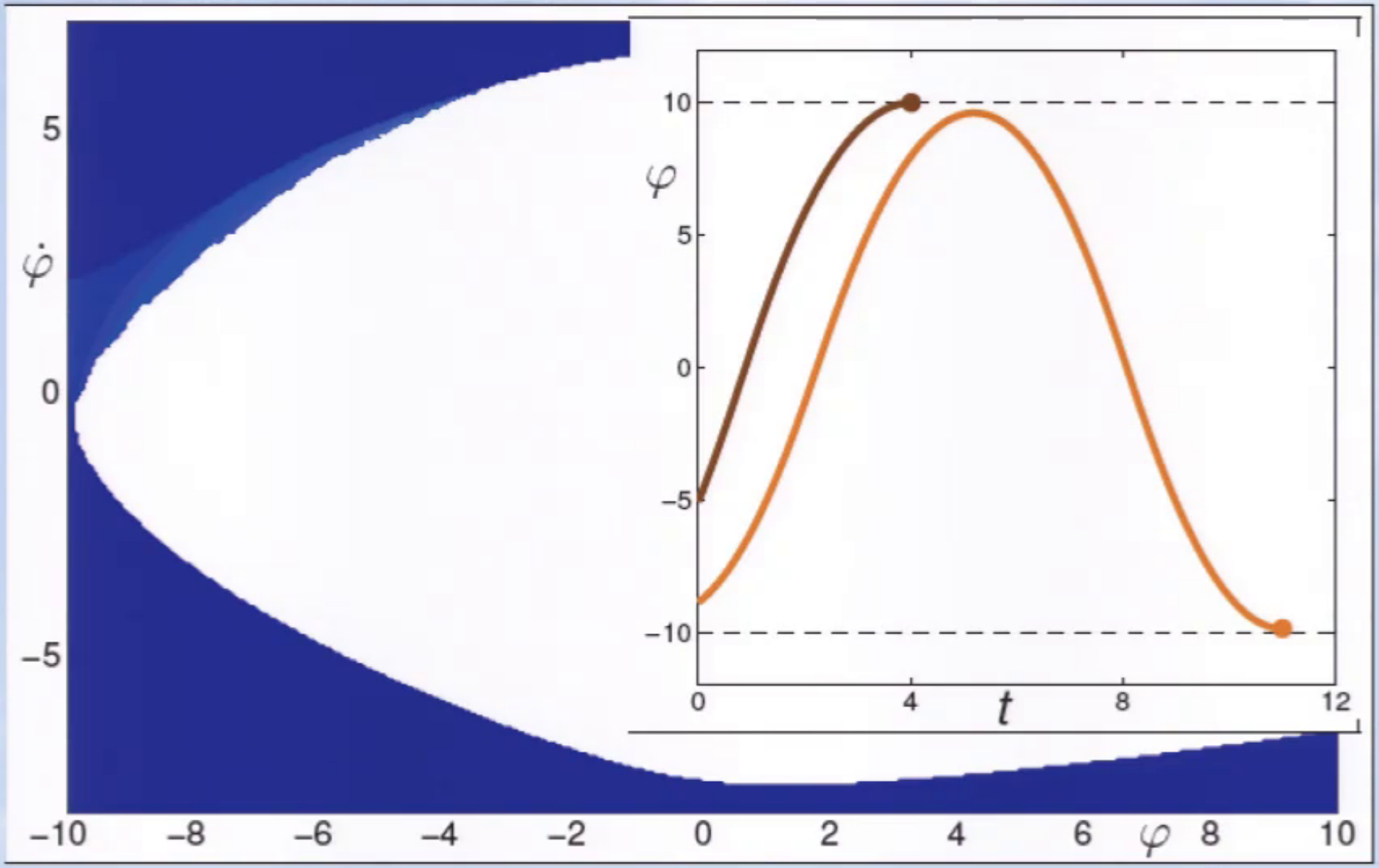
- function $\mu(\varphi) = \varphi$ on $[-1, 1]$, but is nonlinear outside $[-1, 1]$
- we fix $\gamma = 0.05$
- structural failure occurs as soon as $|\varphi| > 10$

Fixed $\omega = 0.575$ and $A = 0.600$



leave $\varphi \in [-10, 10]$ within first 4 periods

Fixed $\omega = 0.575$ and $A = 0.600$



leave $\varphi \in [-10, 10]$ within first 4 periods

Grazing maximal angle

Consider two-point boundary value problem with a solution family of right-grazing orbits

$$\begin{cases} \dot{\mathbf{u}}(s) &= T\,\mathbf{f}(\mathbf{u}(s), t(s)) \\ \dot{t}(s) &= T \end{cases}$$

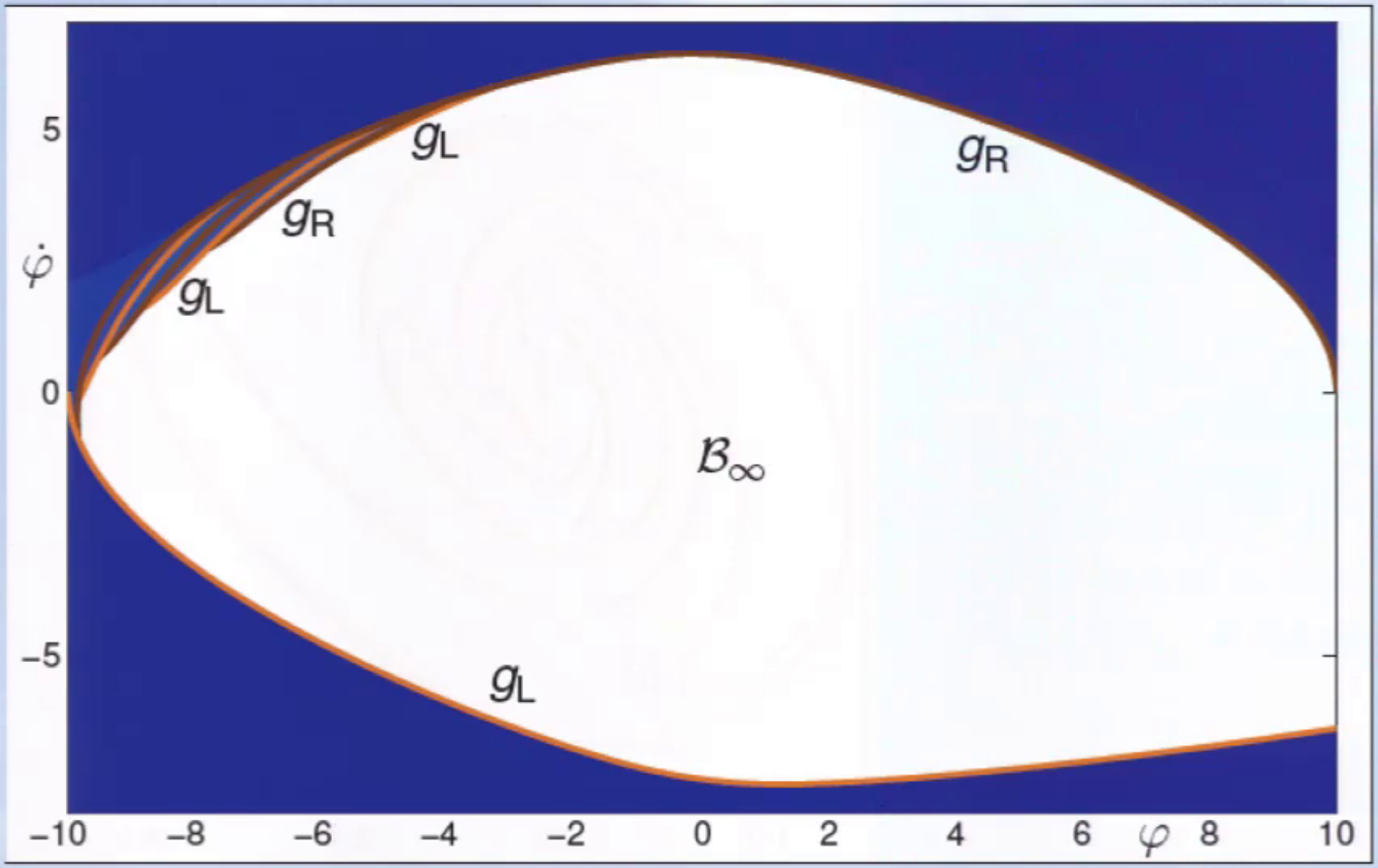
with $\mathbf{u} = \{\mathbf{u}(s) = (\varphi(sT), \dot{\varphi}(sT)) \mid 0 \leq s \leq 1\}$

The boundary conditions are:

$$\begin{aligned} \mathbf{u}(1) &= (+10, 0) \\ t(0) &= 0 \end{aligned}$$

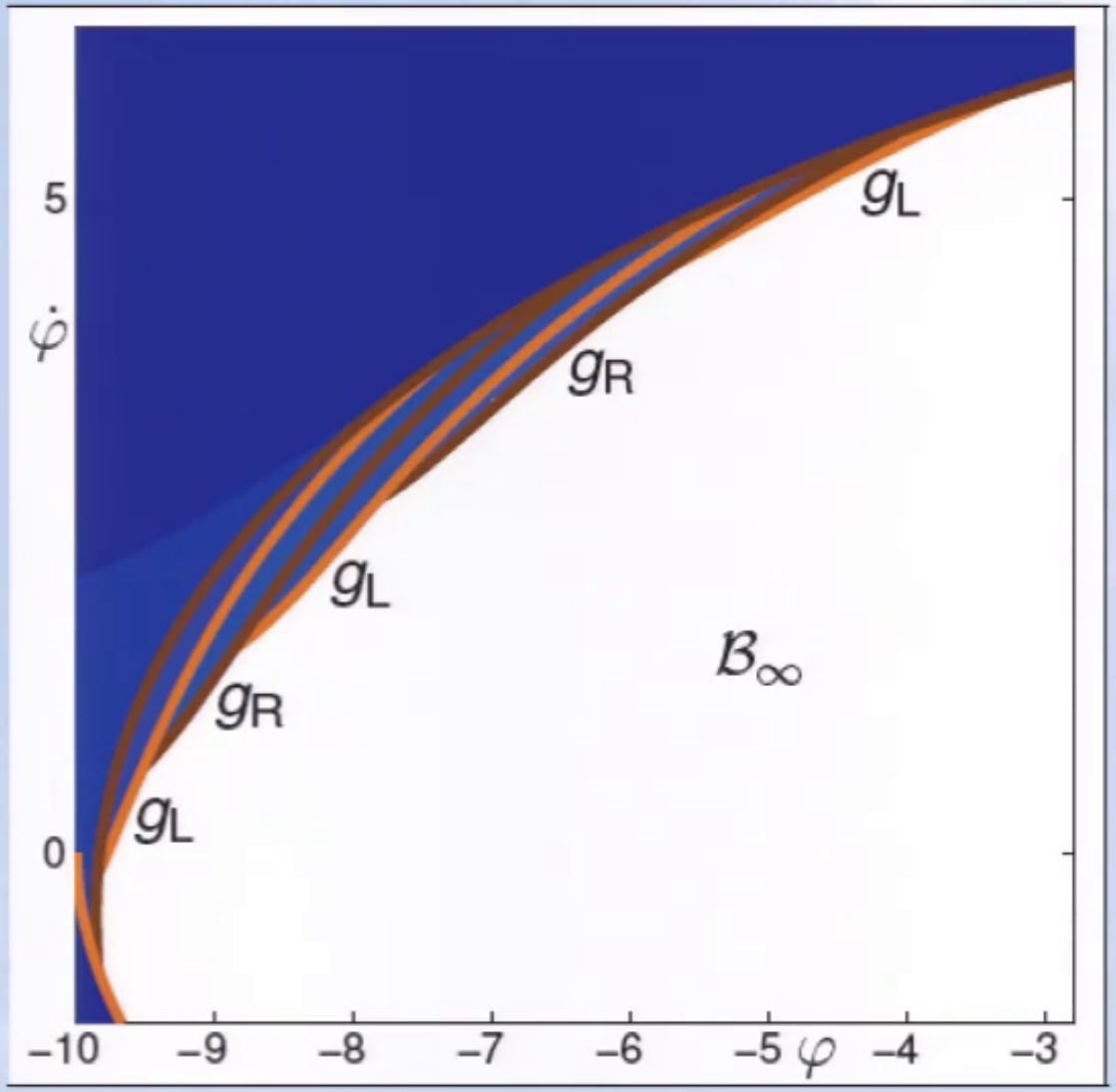
the total integration time T is treated as a parameter.

Admissible grazing orbits



leave $\varphi \in [-10, +10]$ within first 4 periods

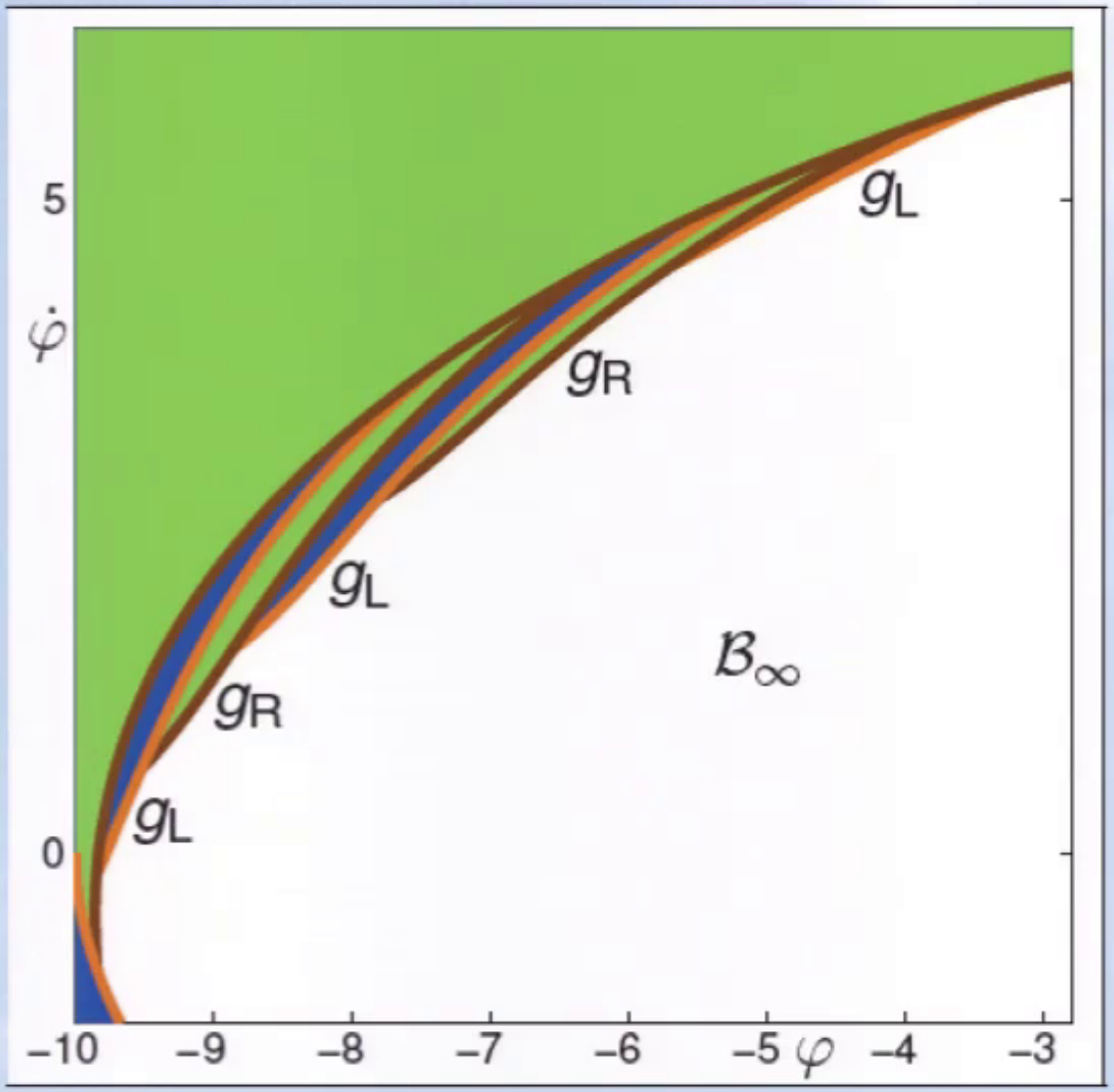
Results overlayed



- failure boundary consist of alternating segments from right-grazing (▬) and left-grazing (▬) families
- due to this switching failure boundary is piecewise smooth

▬ leave $\varphi \in [-10, +10]$ within first 4 periods

Failures left or right

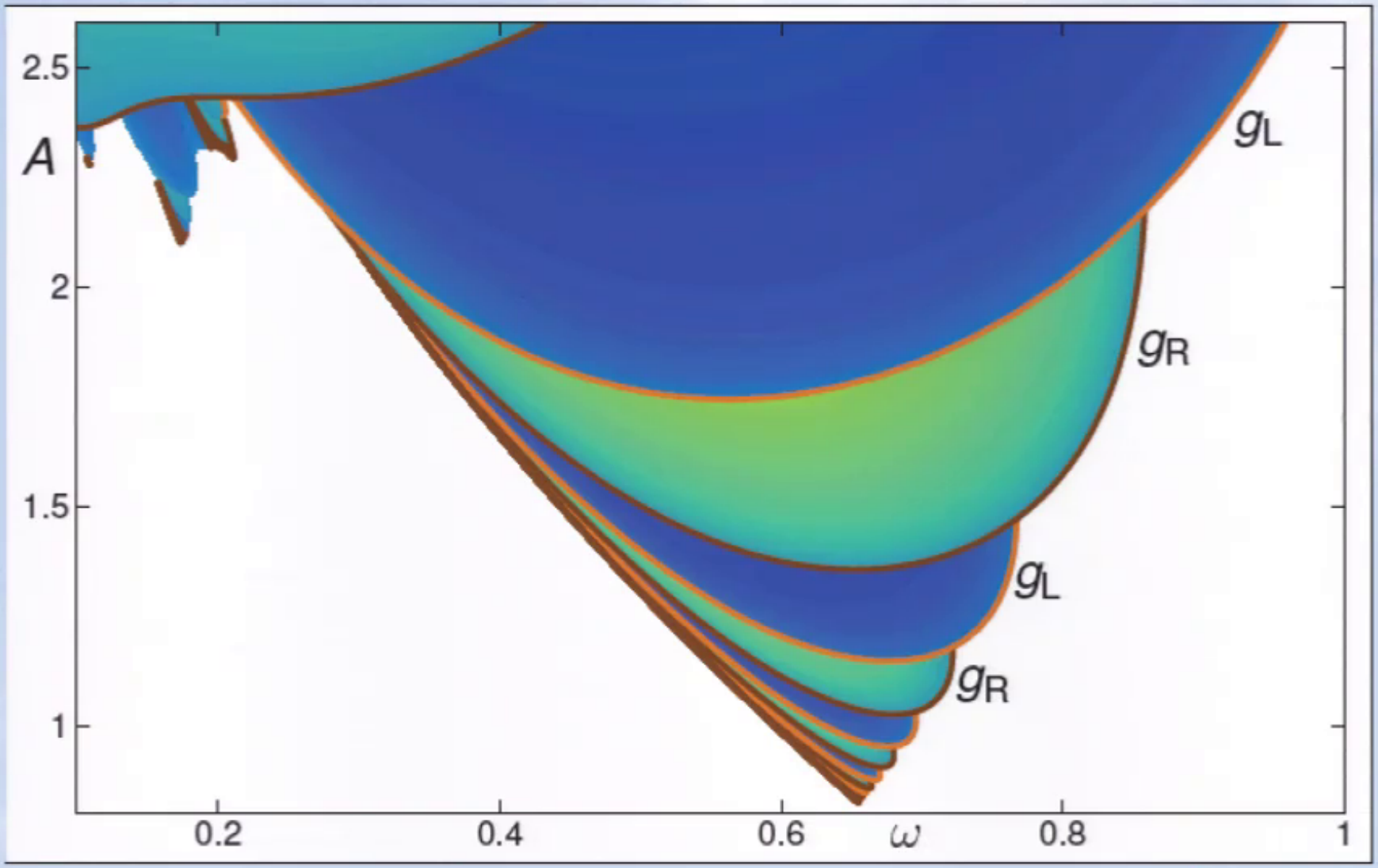


leave $\varphi = +10$ ($\dot{\varphi} > 0$)

leave $\varphi = -10$ ($\dot{\varphi} < 0$)

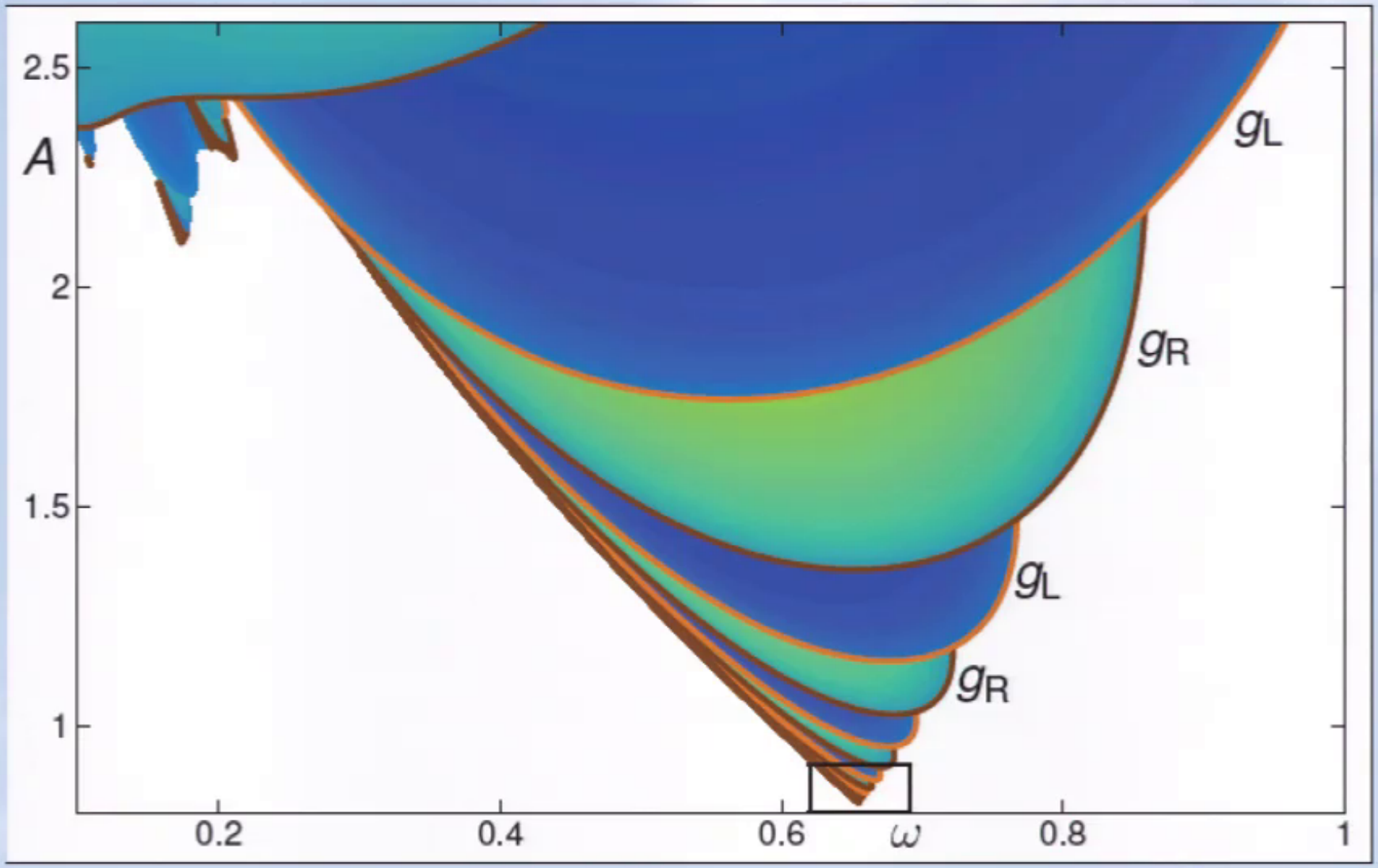
- right-grazing ( ) and left-grazing ( ) solution families define discontinuity curves in the time it takes to reach $|\varphi| = 10$

Frequency-Amplitude plane



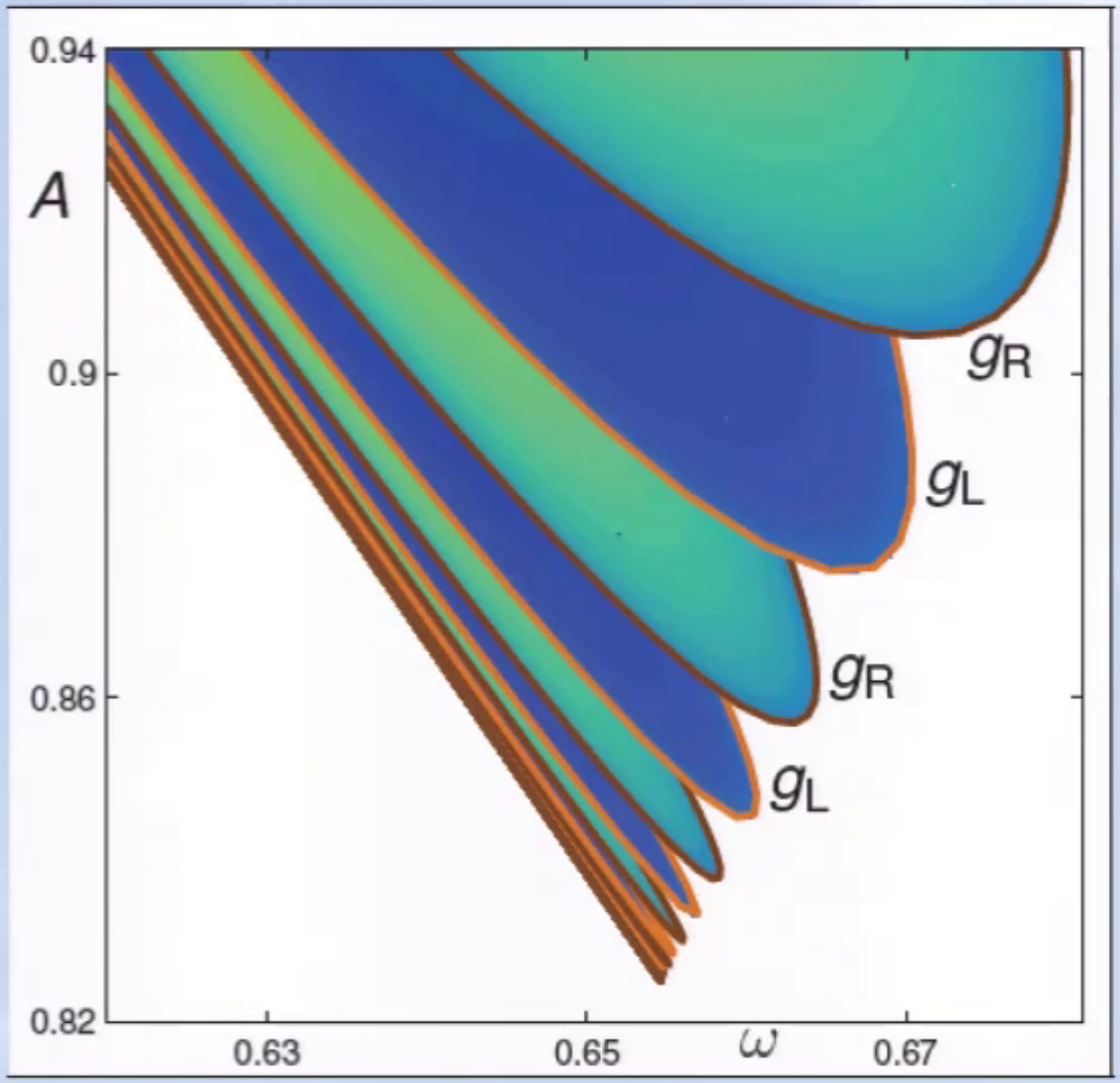
- Grazing trajectories always start at $(\varphi, \dot{\varphi}) = (0, 0)$

Frequency-Amplitude plane



- Grazing trajectories always start at $(\varphi, \dot{\varphi}) = (0, 0)$

Tip of main resonance

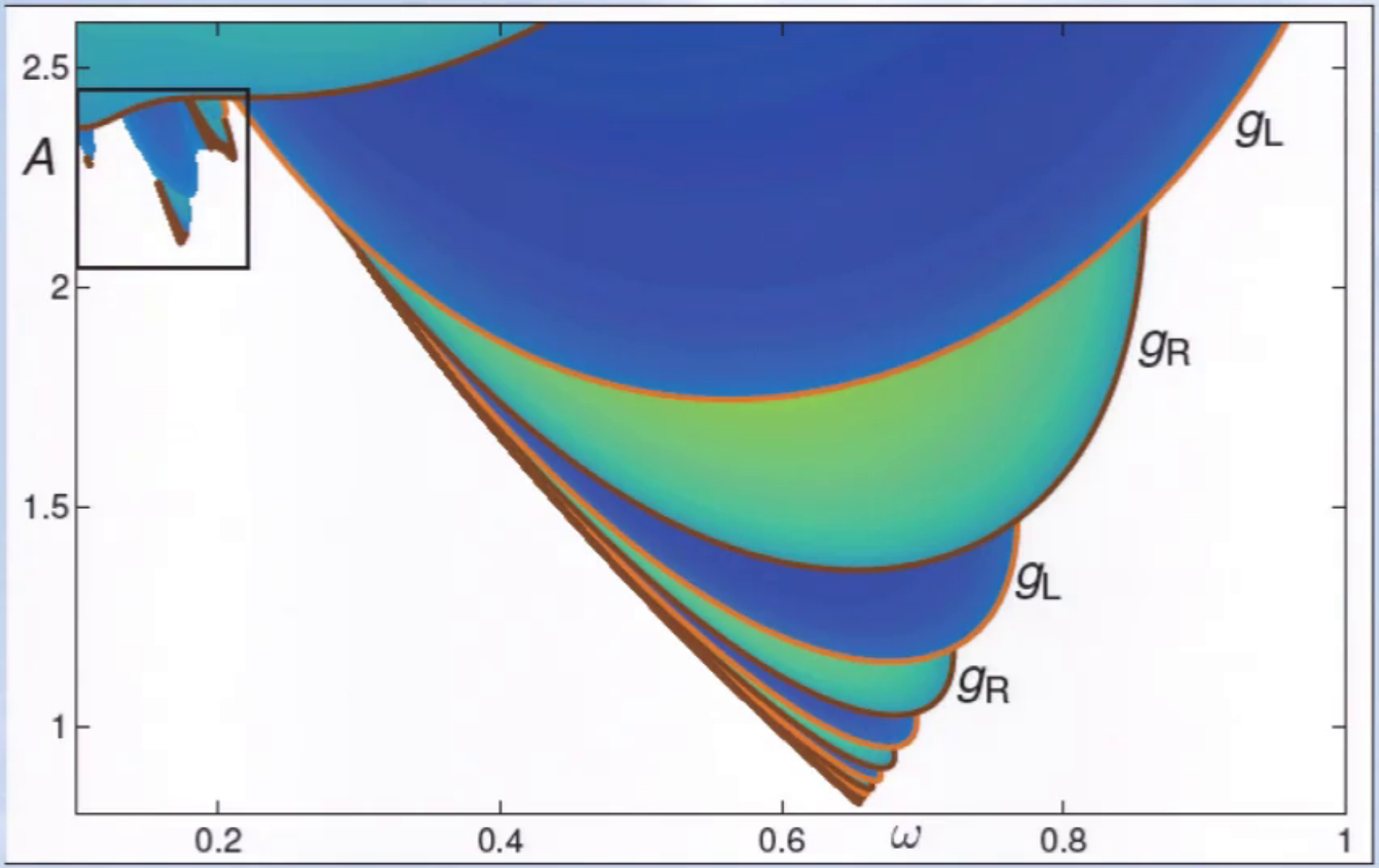


$\dot{\varphi} > 0$

$\dot{\varphi} < 0$

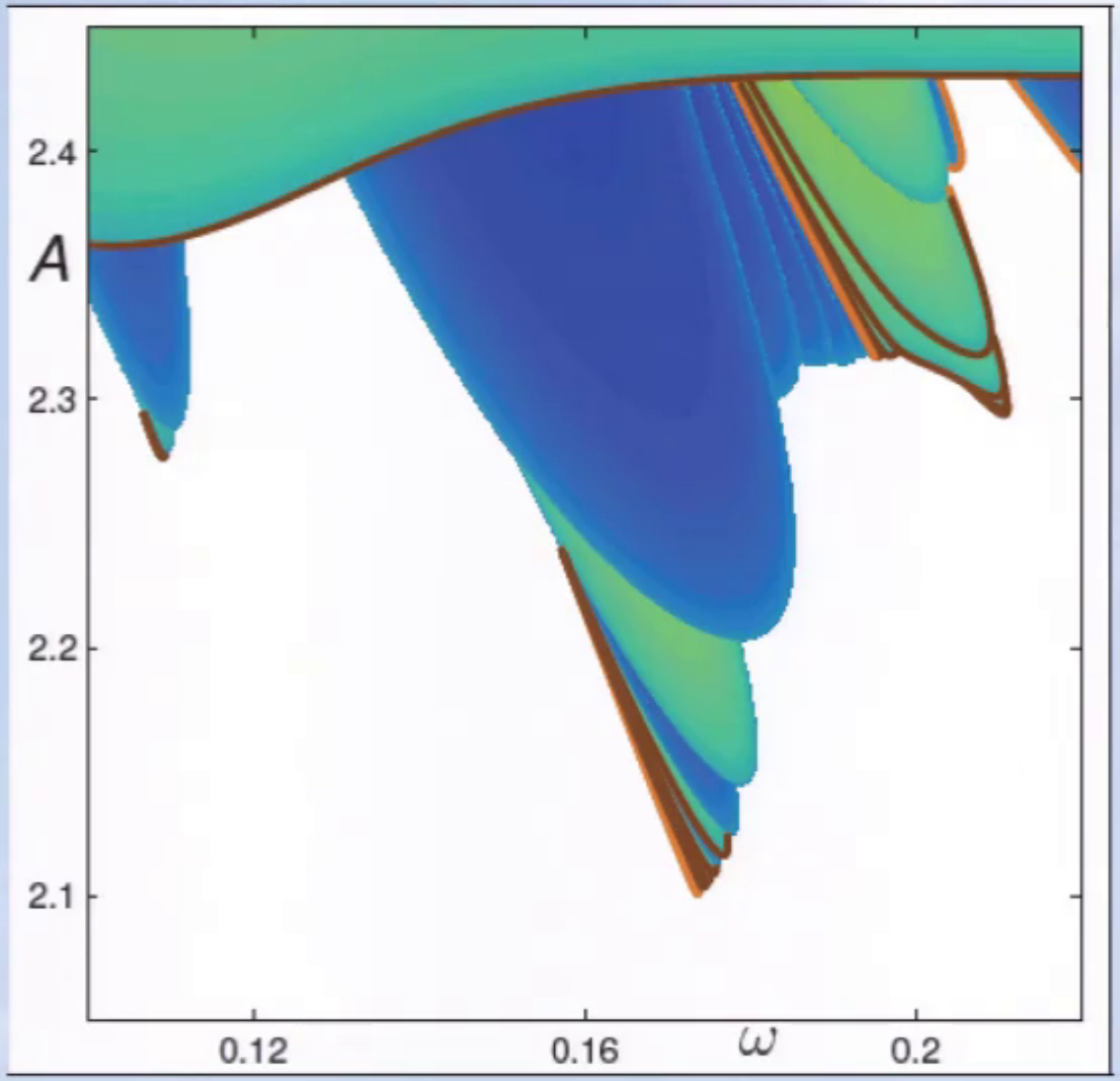
- Fingers in (ω, A)-plane define similar alternatingly right-grazing ( ) and left-grazing ( ) solution families

Subharmonic regime



- Grazing trajectories always start at $(\varphi, \dot{\varphi}) = (0, 0)$

Different grazing structure



$\dot{\varphi} > 0$

$\dot{\varphi} < 0$

- Right-grazing (▬) and left-grazing (▬) solution families do not (always) alternate in subharmonic regime