Developing Geometric Imagination With the Aid of 3D Printed Models

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SIAM ED16
Enhancing Mathematical Learning Experiences with 3D Printing

Our UTD Calculus III Team

- **Faculty:** Sue Minkoff, Farid Khafizov, Changsong Li
- **GTA's:** Sonny Skaaning, Yanping Chen, Fatih Gelir, Jing Guo, Abdullah Helal, Elvira Kadaub, Arafat Khan
- **UTeach TA's:** Henry Curtis, Carl Finley, Dalia Franco Cortes, Andrew Marder, Mikaela McMurtry, Nikunj Patel, Matthew Portman, Erik Ringqvist, Jonathan Sok, Josilyn Valencia
- **Model Design:** Stephanie Taylor, Ximone Willis, SME Interns

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P.I.: Mary Urquhart (SME)

Motivation for Project

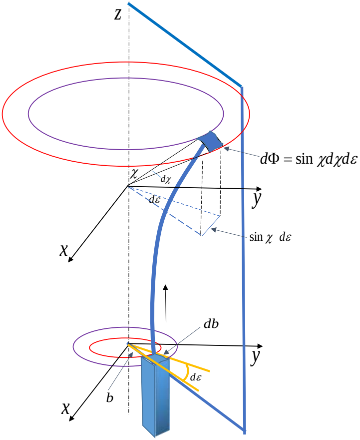
Freshman:

"Math $=$ Formulaic Algebraic Calculation"

Motivation for Project

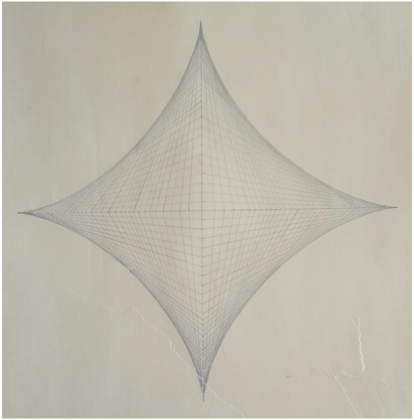
Researchers:

"Math = Geometry + Algebra + Logic"



My Journey to Land of 3D Printing

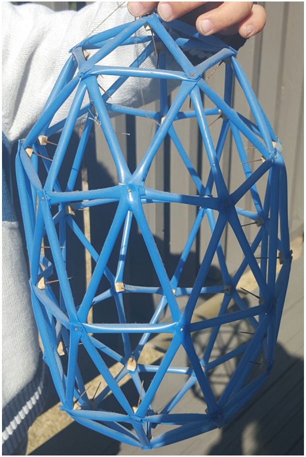
Early Explorations [1980's]



Astroid as Envelope

Polyhedron Models

Early Explorations [1980's]



Geodesic Domes

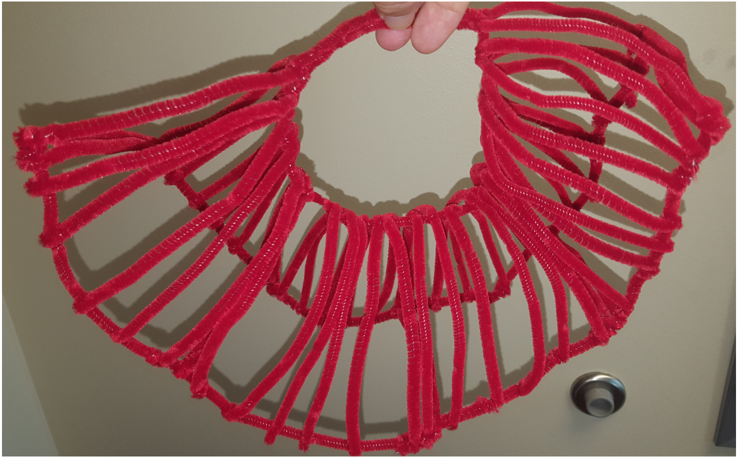
My Journey to Land of 3D Printing

Groping for visualization tools [2011]

Virtual Reality is massive overkill!

My Journey to Land of 3D Printing

Groping for models [2012]



Pipe Cleaners are a bust

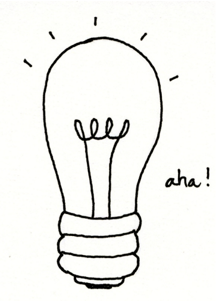
Groping for models [2013]

$z \neq x^2 - y^2$**!**

My Aha! Moment [2014]

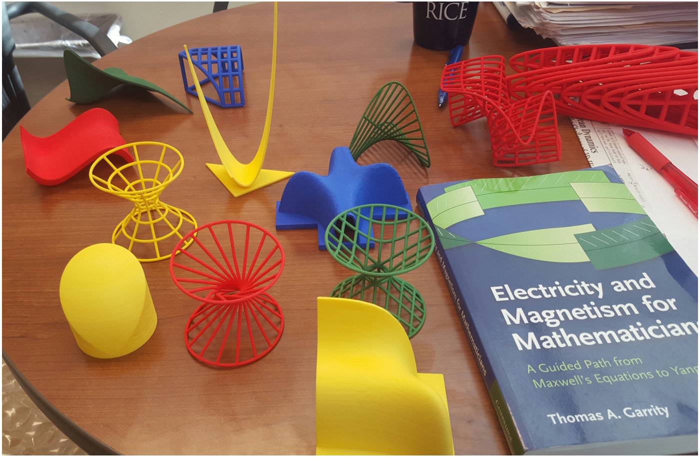
- Marty Ross, Blog Post in *Melbourne Age*:

“Print your own flip-flops.”



“If you can print your own flip-flops you can print anything!"

Realizing the Dream [2015][1]



[1] ... with some help from Mathematica and the NSF.

What are 3D-Printed Models Good For?

3D-printed models enrich student learning of geometric mathematics

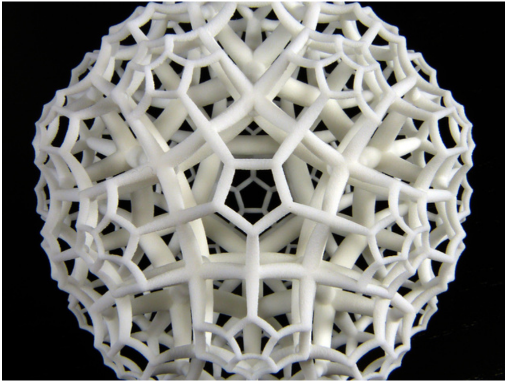
- In the classroom: Low-tech is good!
- Students interact with models at their own pace
- Precise rendering of geometric structure

Former students:

"Wow! I wish we had these models when I took the course."

What are 3D-Printed Models Good For?

Mathematical Art



Henry Segerman, Oklahoma State University

“Visualizing Mathematics with 3D Printing”, 2016.

What are 3D-Printed Models Good For?

Mathematical Art

David Bachman, Pitzer College

"Mathematical Tools for 3-Dimensional Design", Forthcoming, 2017.

What is 3D-Printing Good For?

Having students design and 3D-print models[2]

- Connects algebraic and geometric thinking
- Motivates learning
- Fosters creativity
- Provides experience in computational mathematics

Segerman Course:

Geometry and Algorithms in 3D Modeling

[2] Denne, MS 3, Knill, MS 3, Aboufadel, MS 6

What are 3D-Printed Models Good For?

3D models as learning aids in Calculus

- **What to teach?**
  1. **Geometric Imagination:**

     The ability to form and manipulate images of geometric objects "in the mind's eye".
  2. **Geometry⟷Algebra:**
     - Geometric structure guides algebraic calculation
     - Algebraic calculation reveals geometric structure

What are 3D-Printed Models Good For?

3D models as learning aids in Calculus

- **How to learn?**
  1. Fill-in-the-blank worksheets [akin to Physics Labs]
  2. Drawing and measuring on surfaces[3]
  3. Open-ended Inquiry-based learning:

     "Can be intellectually paralyzing"[4]
  4. **Small-group, guided active learning projects**

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[3]Wangberg, Samuels, MS 6
[4]Hitchman, MS 3

What are 3D-Printed Models Good For?

3D models as learning aids in Calculus

- **How to assess?**[5]
  1. Assessment shouldn't hinder interaction with models
  2. **Student comment:**

     "Spending time working with models doesn't help me do my homework."
  3. To fully integrate 3D models must modify homework!

[5] Fukawa-Connelly, MS 6

Calculus III at UT Dallas

- Two Calculus sequences:
  1. Fast pace: 2417, 2419
  2. Regular pace: 2413, 2414, **2415**
- **MATH 2415**, Fall 2016 [230 students]:[6]
  1. 3 Lecture Sections [2x75mins, 75 students]
  2. 7 Problem Sections [1 hr 50 mins, 33 students]
  3. Peer-Led Team Learning [80 mins, 70 students total]
  4. 3 Graduate TA's
  5. 6 Undergraduate TA's [Math majors in UTeach][7]

From vectors to Divergence Theorem

[6] The course coordinator is a manager
[7] NSF funded

Active Learning (AL) Problem Sections[9]

- TA starts with 10 minute summary of lectures
- Then students actively solve assigned problems
- Students
  - Work in small groups of 3-4 at white boards
  - Explain solutions to each other and to TA's[8]
  - Photograph their solutions
- Teaching Assistants
  - Can't hold white-board markers
  - Only ask questions

The room is buzzing with conversation.

[8] Undergraduates are **pre-verbal** mathematicians
[9] Lectures use a traditional format

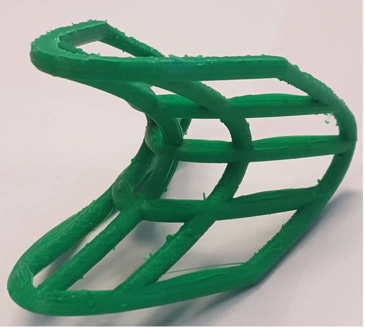
AL Projects with 3D Printed Models

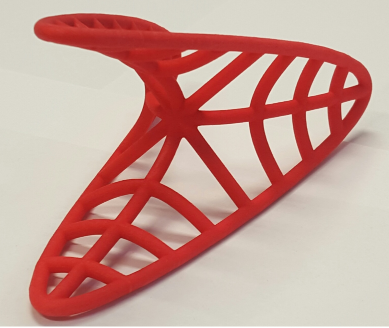
Students do a few active learning projects
in the problem sections

1. Circular Paraboloids
2. Saddle Surfaces
3. Helices
4. Limits
5. Parametrized Surfaces
6. Ruled Surfaces
7. Hills and Valleys

Project web site: http://www.utdallas.edu/~zweck/

AL Project: Saddle Surfaces





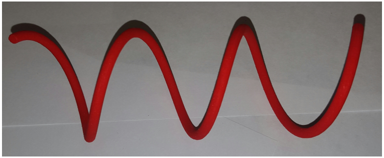
$$z = x^2 - y^2$$

Surfaces represented using families of curves

AL Project: Saddle Surfaces

- Identify axes
- Cast shadow on table using **flashlight app**
- Sketch 2D grids
- Explain how model is constructed from slices
- Sketch surface from model
- Reorient surface to be graph of
  - $x = g(y, z) = y^2 - z^2$
  - $z = h(x, y) = 2xy$
- Visualize $z = 4x^2 - y^2$

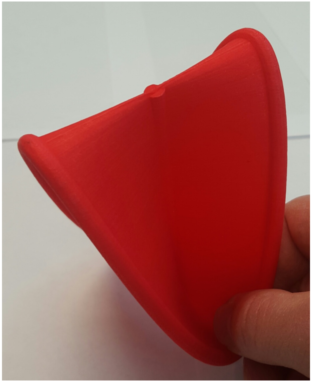
AL Project: Helices



$$\mathbf{r}(t) = (\cos t, \sin t, t)$$

- Explain why helix lies on cylinder
- Distinguish left- and right-handed helices
- Left-handed helix as reflection of right-handed helix
- Parametrize left-handed helix
- Parametrize DNA double helices

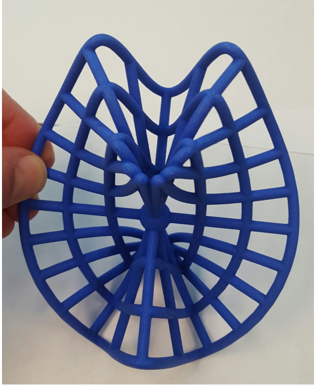
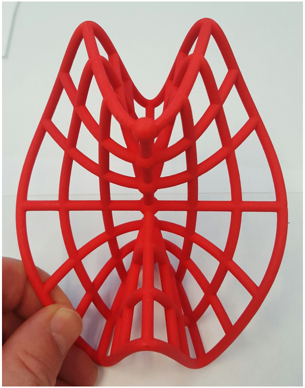
AL Project: Limits



$$f(x,y) = \frac{2xy}{x^2+y^2}$$

f is constant along lines $y = kx$

AL Project: Limits

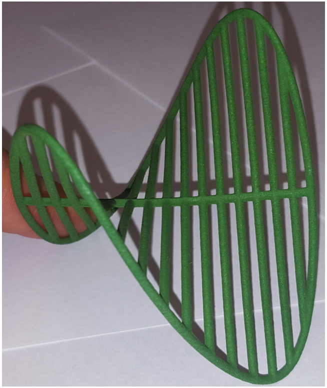
 

$$f(x,y) = \frac{2x^2y}{x^4+y^2}$$

$f \to 0$ **along** $y = kx$

f **is constant along parabolas** $y = kx^2$

AL Project: Ruled Surfaces

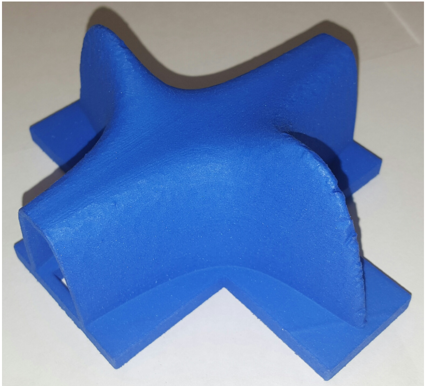


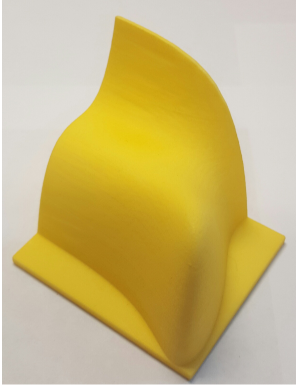
Sweep out a saddle by sliding one broomstick along another, rotating as you go

$$\mathbf{x}(t,s) = (t,0,0) + s\,(0,\cos\theta(t),\sin\theta(t))$$

Use $z = xy$ to solve for $\theta(t)$

AL Project: Hills and Valleys

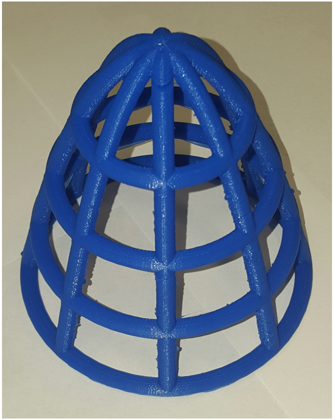


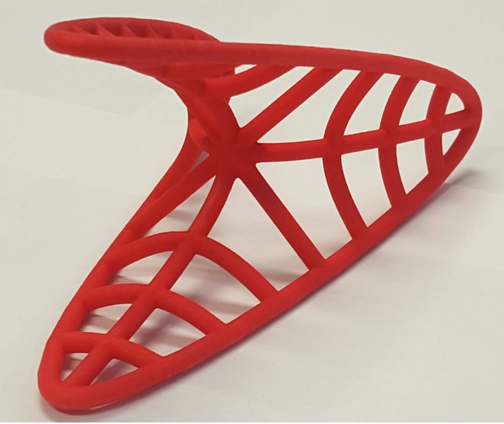


Two critical points: both are local maxima

One critical point: a local maxima but not global

Multi-Use Models





Multi-Use Models

1. Quadric surfaces
2. Cylindrical coordinates and symmetry
3. Level curves
4. Parametrized surfaces
5. Intersections of surfaces
6. Partial derivatives
7. Gradient and directional derivative
8. Local max/min
9. Lagrange multipliers
10. Surface area and integrals

Suggestion:

Base pedagogy on a few fundamental models.

Concluding Thoughts

Surface Parametrizations

- We do calculus on surfaces using curves
- Surface meshes highlight this geometric structure

Geometric Imagination

- Geometric arguments often involve
  1. Manipulation of geometry “in the minds eye”
  2. Corresponding algebraic calculations
- Need exercises to strengthen geometric imagination
- Suggestion: Active learning projects with 3D models