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Modeling and Controlling the Power Grid - Part II of II

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Exploring state estimation techniques to accommodate non-Gaussian noises

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Gaussian noise assumption in power grid dynamic state estimation

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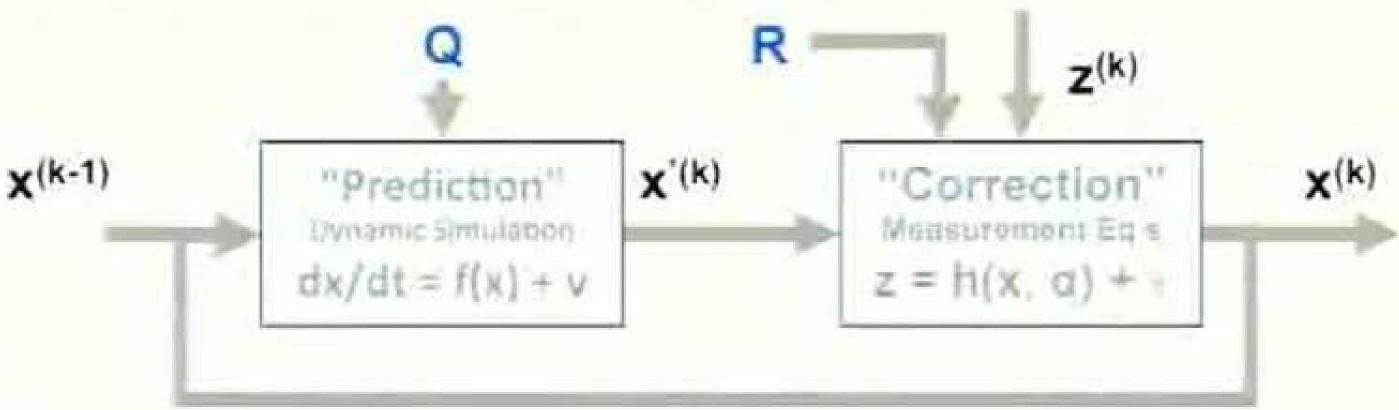
- Power grids trend to be more dynamic… Estimating dynamic states is necessary vs. static states in the traditional function.
- The problem is formulated using phasor measurement and Kalman Filter.
  - Noise are assumed Gaussian for both the process and the measurement.
  - Our prior work has examined the non-Gaussian nature in measurement noise.
  - Today's focus: exploring methods to accommodate non-Gaussian noises.



Excellent state tracking with realistic evaluation conditions

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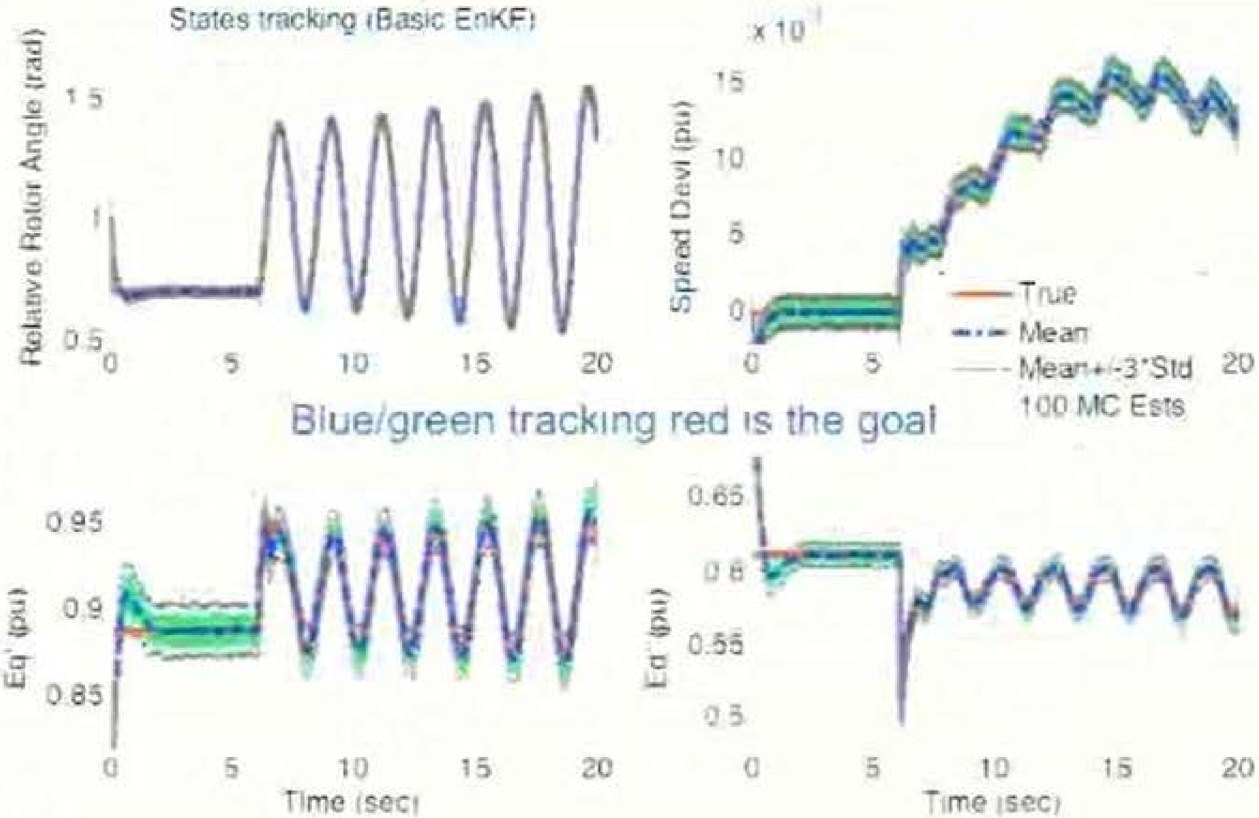
- 3% Gaussian measurement noise; 40 ms measurement cycle (phasor measurement)
- 5 ms interpolation cycle; modeling errors considered; unknown inputs; unknown initial states



Non-Gaussian noise observed from real phasor measurements

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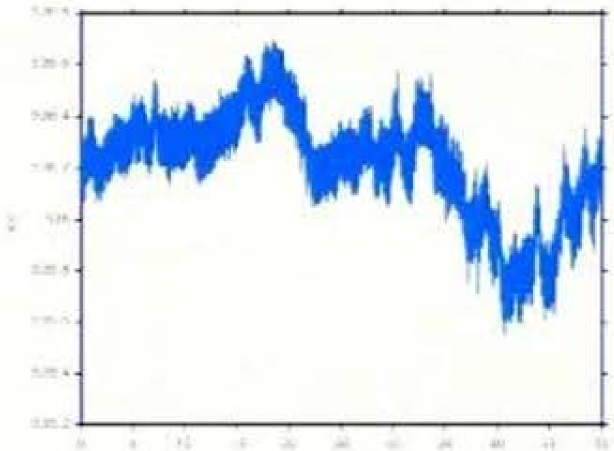
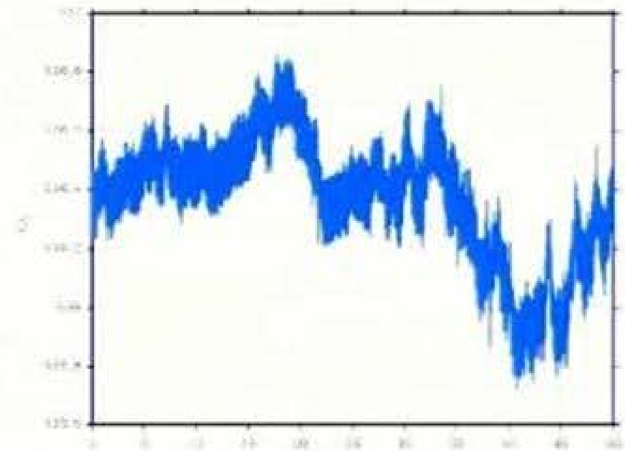
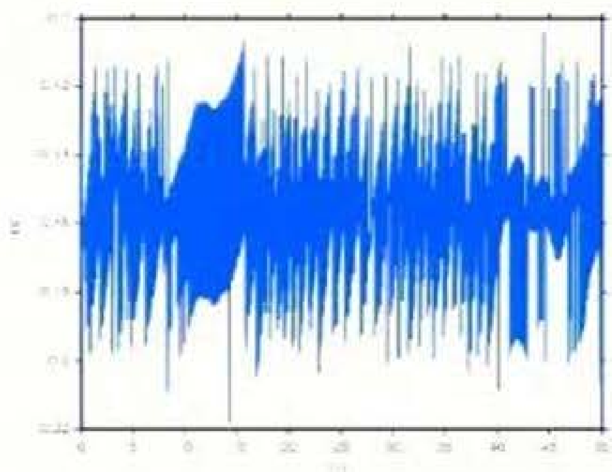


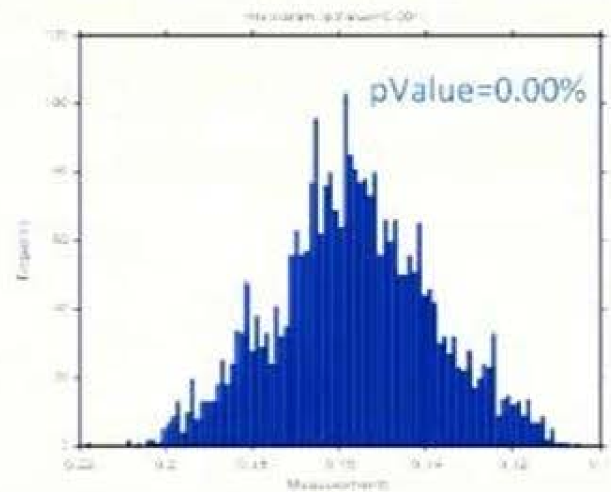
Measurement 1



Measurement 2



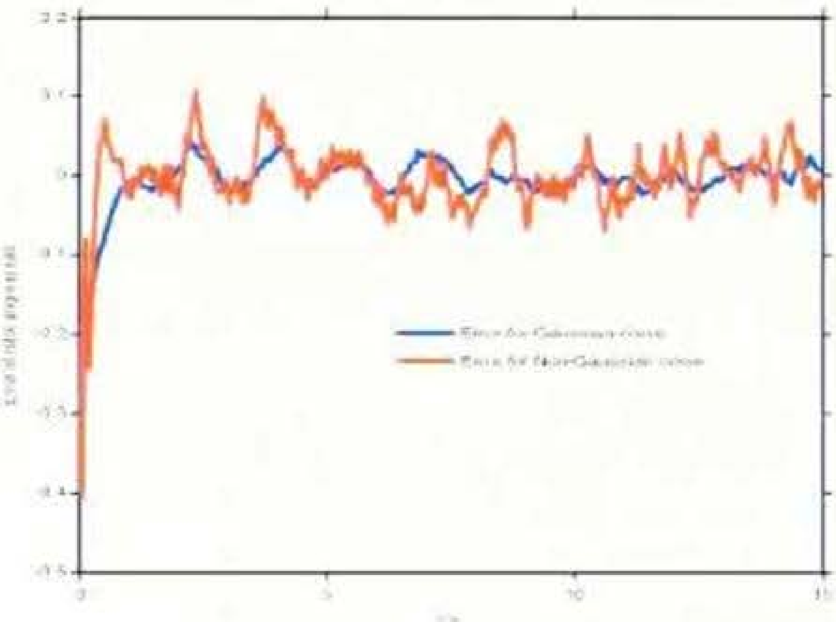
Difference between two measurements

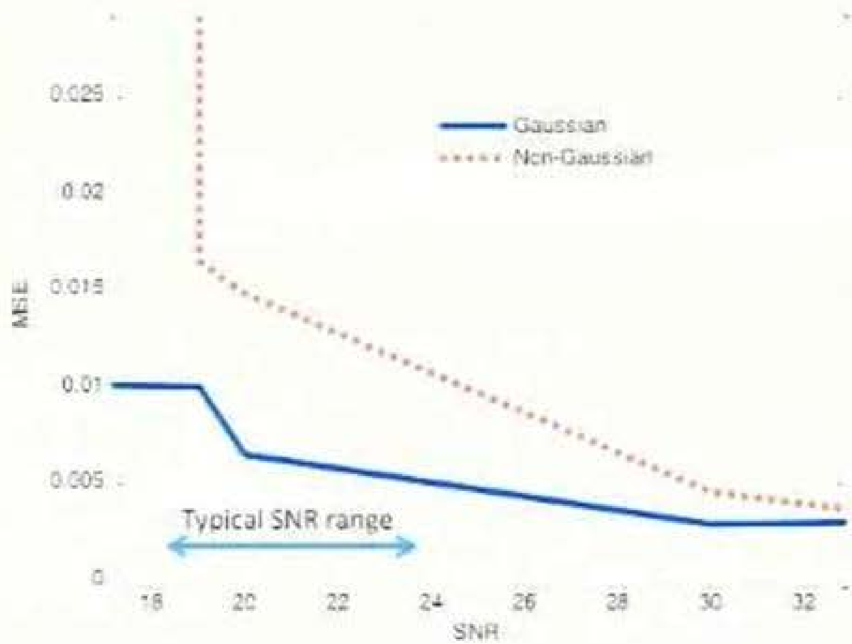


Results of normal distribution test

Impact of non-Gaussian noises on dynamic state estimation

- Non-Gaussian noise results in larger estimation errors.
- The errors could mean 10% power flow difference (100s MW).
- Typical signal noise ratio (SNR) falls in the sensitive range.





Gaussian noise propagation in non-linear systems

- Central Limit Theorem

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} X_i \Rightarrow Gaussian$$

- Mathematical convenience for a *linear* system

$$\vec{y} \sim Gaussian \quad \Rightarrow \quad A\vec{y} + b \sim Gaussian$$

- Non-Gaussian if propagation through non-linear system

Non-linearity of the power grid

- Nonlinear State Transition Functions

$$
\begin{cases}
\dot{\delta} = \omega_0 \Delta\omega \\
\Delta\dot{\omega} = \frac{1}{2H}\left(T_m - T_e - K_D \Delta\omega\right) \\
\dot{e}'_q = \frac{1}{T'_{d0}}\left(E_{fd} - e'_q - (x_d - x'_d) i_d\right) \\
\dot{e}'_d = \frac{1}{T'_{q0}}\left(-e'_d + (x_q - x'_q) i_q\right)
\end{cases}
$$

$$T_e \approx P_e = (e'_d + i_q x'_q) i_d + (e'_q - i_d x'_d) i_q$$

$$i_q = i_I \sin\delta + i_R \cos\delta$$

$$i_q = i_I \sin\delta + i_R \cos\delta$$

- Nonlinear Measurement Functions

$$e_R = (e'_d + i_q x'_q)\sin\delta + (e'_q - i_d x'_d)\cos\delta$$

$$e_I = (e'_q - i_d x'_d)\sin\delta - (e'_d + i_q x'_q)\cos\delta$$

Non-linearity index to quantify the linearization errors

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- 1st order Taylor approximation

$$\begin{cases} x_k = \Phi(x_{k-1}, u_{k-1}) + w_{k-1} \\ z_k = h(x_k, u_k) + v_k \end{cases}$$

$$\Phi(x_k + \delta x) \approx \Phi(x_k) + \left.\frac{\partial \Phi(x)}{\partial x}\right|_{x_k} \delta x$$

$$h(x_k + \delta x) \approx h(x_k) + \left.\frac{\partial h(x)}{\partial x}\right|_{x_k} \delta x$$

- Non-linearity indices

$$\varepsilon_\Phi = \Phi(x_k + \delta x) - \left[\Phi(x_k) + \left.\frac{\partial \Phi(x)}{\partial x}\right|_{x_k} \delta x\right]$$

$$\varepsilon_h = h(x_k + \delta x) - \left[h(x_k) + \left.\frac{\partial h(x)}{\partial x}\right|_{x_k} \delta x\right]$$

$$n(\Phi) = \varepsilon_\Phi^T Q_k^{-1} \varepsilon_\Phi$$

$$n(h) = \varepsilon_h^T R_k^{-1} \varepsilon_h$$

$$Q_k = E(w_k w_k^T) \qquad R_k = E(v_k v_k^T)$$

Smaller prediction steps improves quality of linearization, requiring interpolation

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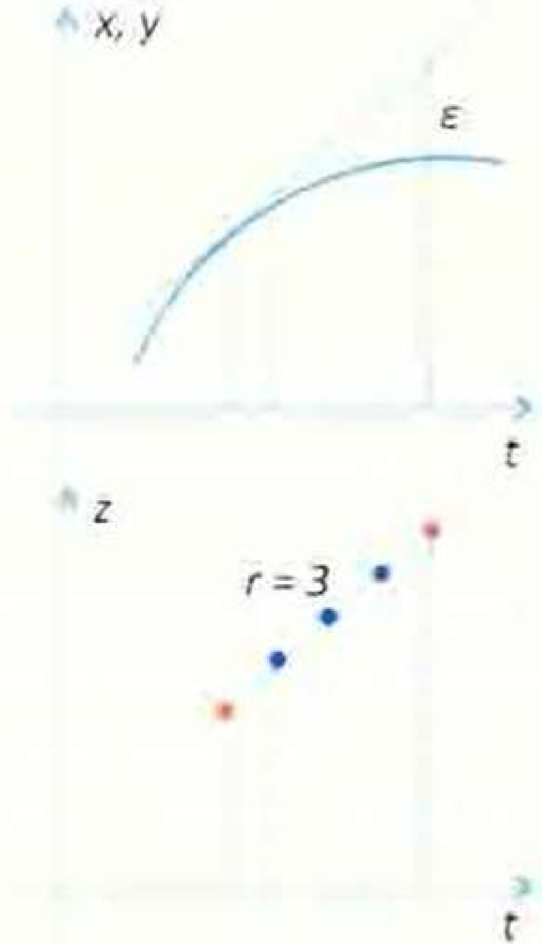
- Smaller prediction steps reduces the linearization error of the nonlinear process.

$$n(\Phi) = \varepsilon_{\Phi}^{T} Q_{k}^{-1} \varepsilon_{\Phi}$$

$$n(h) = \varepsilon_{h}^{T} R_{k}^{-1} \varepsilon_{h}$$



- Interpolation is required to match the measurement rate with the prediction steps
  - Interpolation factor r = number of added pseudo measurements between two consecutive measurements
  - New sampling rate $F_s = F_{s0}\,(r + 1)$

Interpolation reduces the non-linearity and thus the impact of non-Gaussian noises

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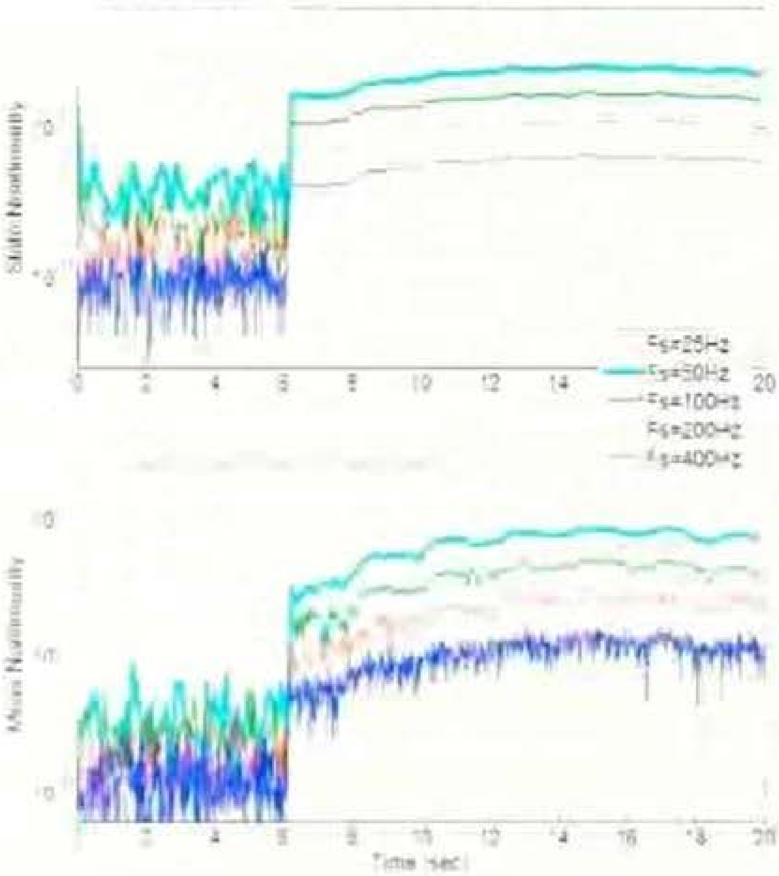
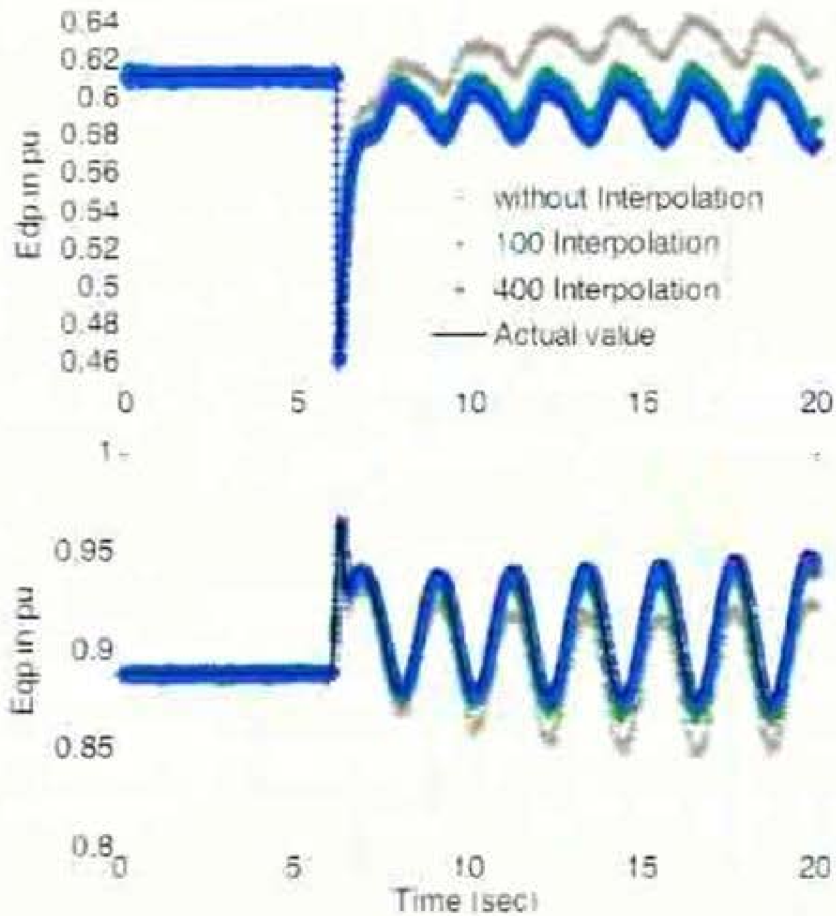
Non-linearity decreases with the interpolation factors

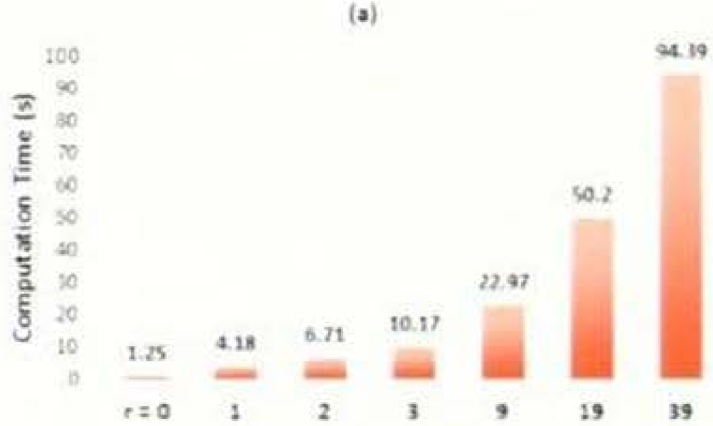


Estimation errors decreases with the interpolation factors

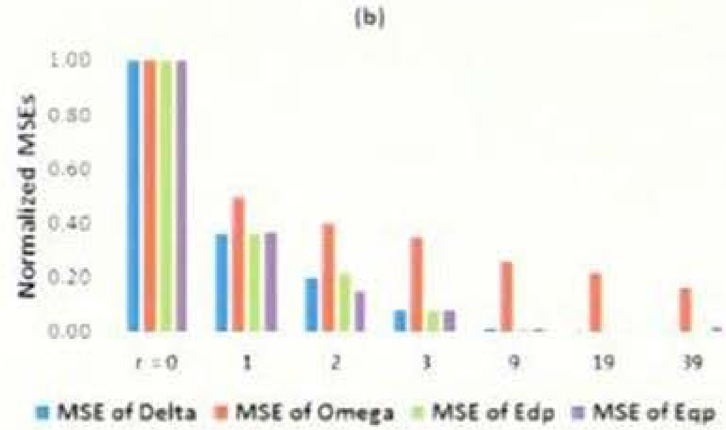


Trade-off between estimation accuracy and computation time

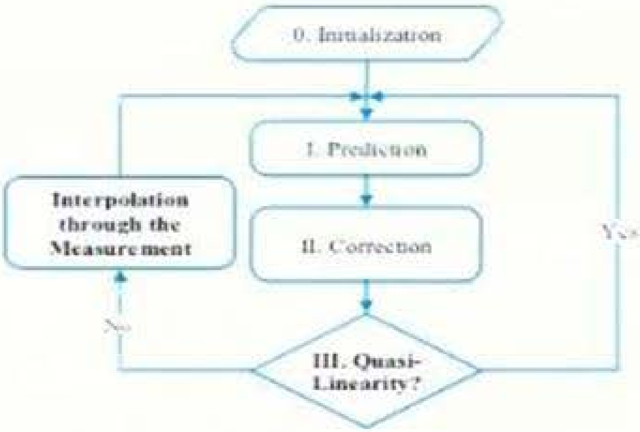
Computation time increases with the interpolation factors (*r*)

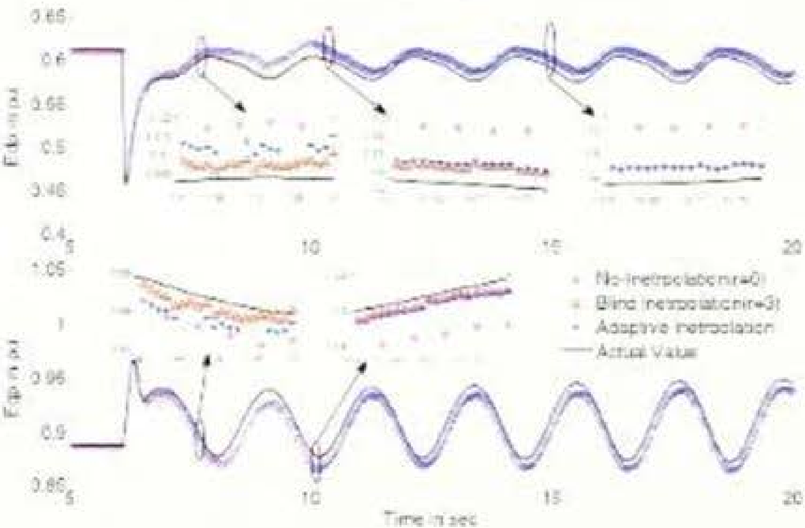


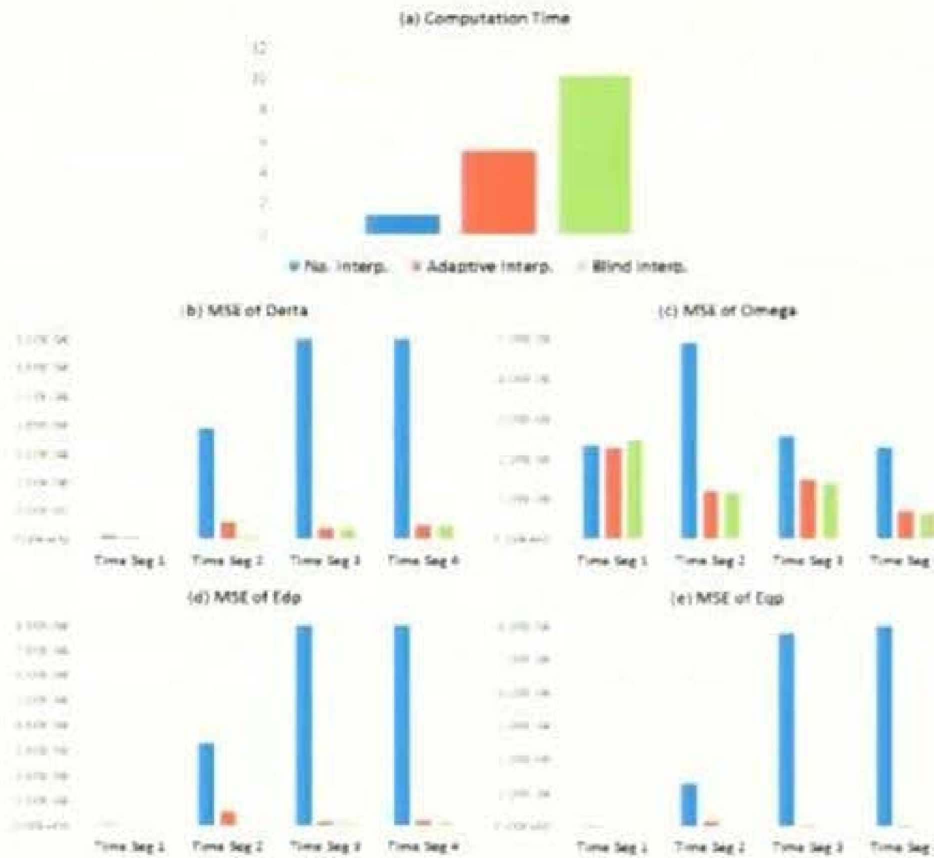
MSEs of the EKF estimates decreases with interpolation factors (*r*)



Adaptive interpolation based on the nonlinearity index







Additional ideas to accommodate non-Gaussian noises

- Particle filtering (PF): Sequential Monte Carlo technique which does not require Gauss assumption
- Gaussian mixture approach
- Adjustment of the parameters of Kalman filter to minimize the impact of non-Gaussian noise

Summary

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- Noises in power grid dynamic state estimation exhibit non-Gaussian nature.
- The impact of non-Gaussian noises can be significant enough that new mathematical methods need to be developed.
- Interpolation methods are showing improvement for non-Gaussian noises.
- Other methods (Gaussian mixture, adaptive parameters) are being explored.

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Questions?