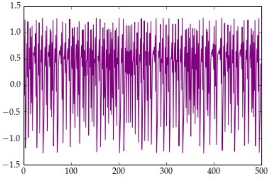
Applications of Persistence to Time Series Analysis

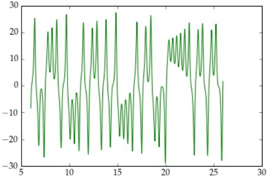
Elizabeth Munch

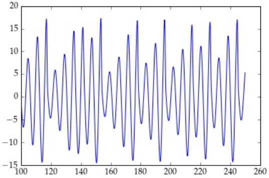
University at Albany - SUNY :: Department of Mathematics & Statistics

May 23, 2017

Time series classification





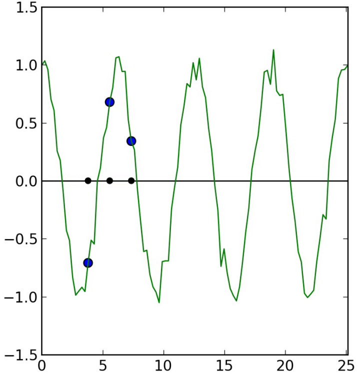


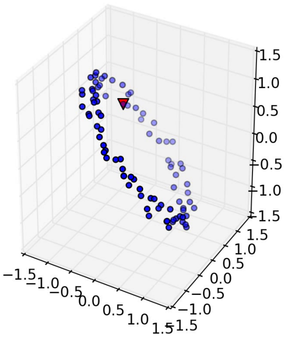
Takens embedding

Definition

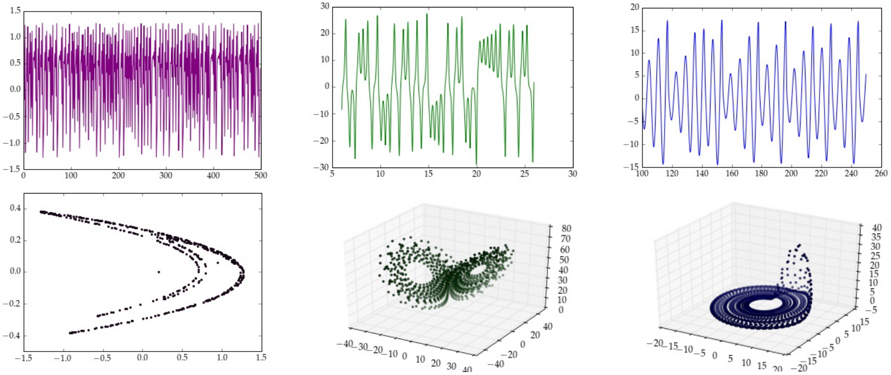
Given a time series $X(t)$, the Takens embedding is

$$\psi_\eta^m : t \longmapsto (X(t), X(t+\eta), \cdots, X(t+(m-1)\eta)).$$





Classification based on attractor

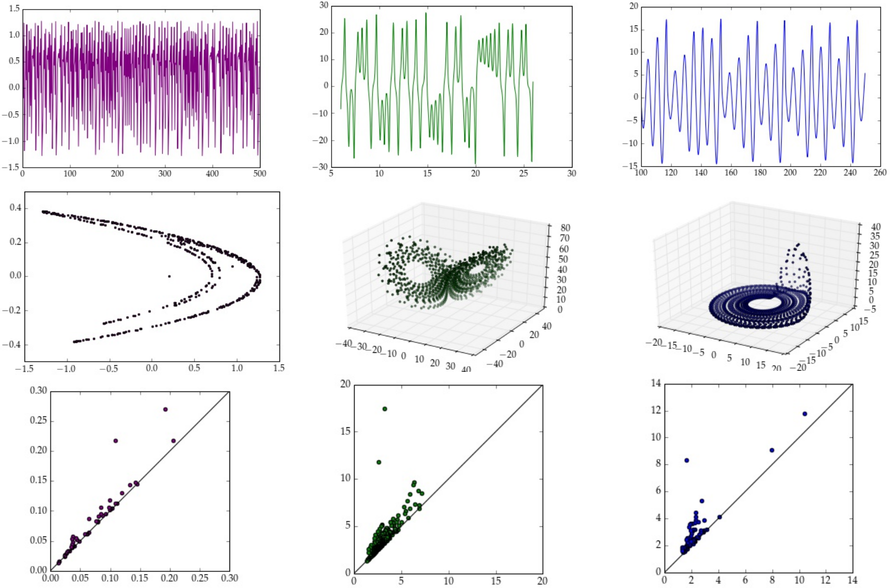


Main question

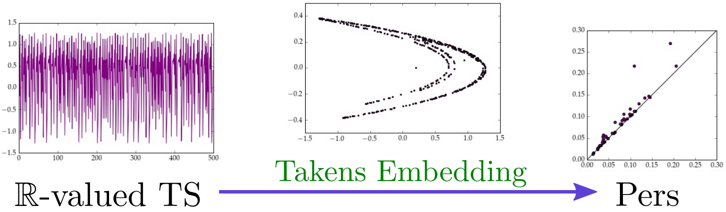
Question:

How do we measure invariants of the resulting embedding?

Classification based on persistent homology



Overview

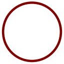
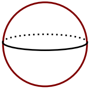
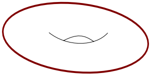
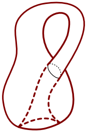


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500
0.4
0.2
0.0
−0.2
−0.4
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
ℝ-valued TS
Takens Embedding
Pers

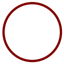
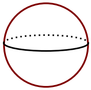
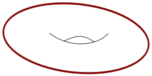
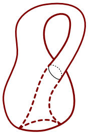
1. Persistent Homology for Time Series Analysis

2. Classification and Machining Dynamics

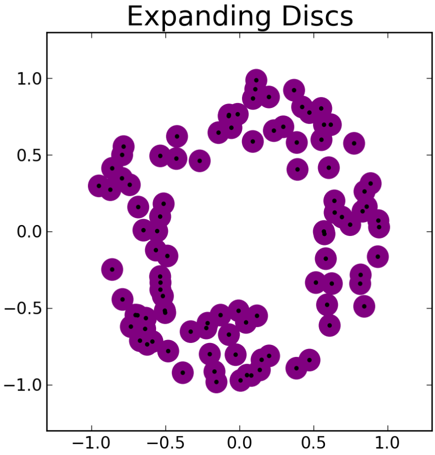
Homology

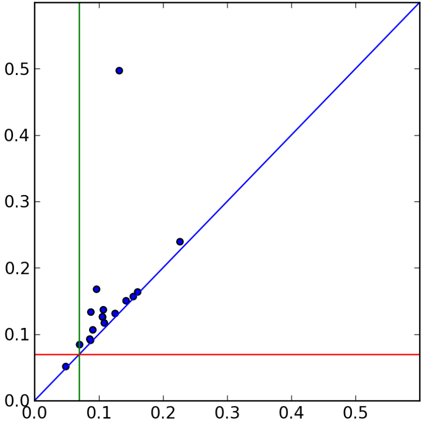
	$H_0(X)$	$H_1(X)$	$H_2(X)$	$H_3(X)$
	$\mathbb{Z}_2$	$\bullet$	$\bullet$	$\bullet$
	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\bullet$	$\bullet$
	$\mathbb{Z}_2$	$\bullet$	$\mathbb{Z}_2$	$\bullet$
	$\mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2$	$\bullet$
	$\mathbb{Z}_2$	$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\mathbb{Z}_2$	$\bullet$

Homology

	β_0	β_1	β_2	β_3
	1	•	•	•
	1	1	•	•
	1	•	1	•
	1	2	1	•
	1	2	1	•

Persistent homology in one slide





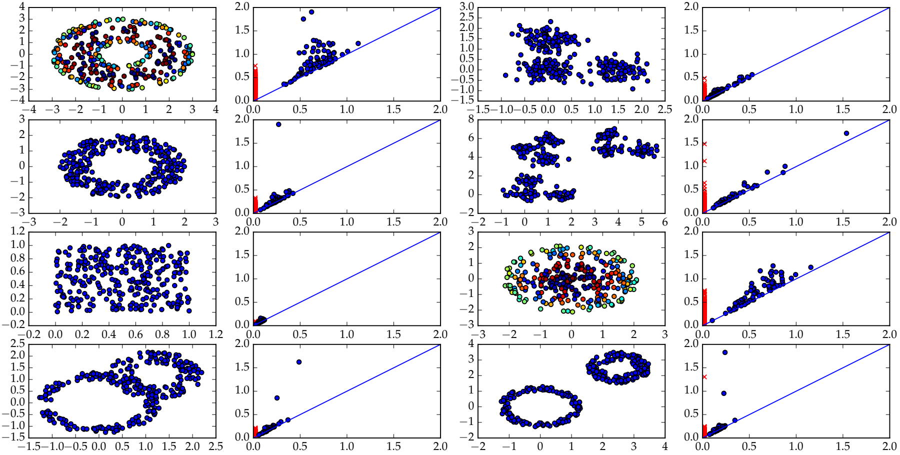
Persistence diagram

x-axis → Birth of feature

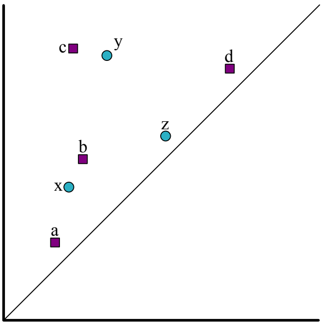
y-axis → Death of feature

point → feature

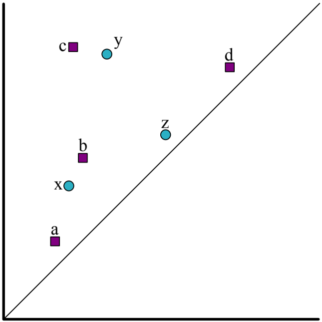
Examples of point clouds and persistence diagrams



Bottleneck Distance on $\mathcal{D}$



Bottleneck Distance on $\mathcal{D}$

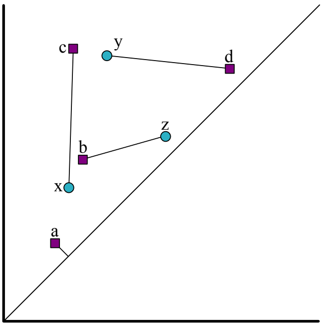


Bottleneck distance for diagrams

Given diagrams X and Y, the distance between them is

$$d_B(X, Y) = \inf_{\varphi: X \to Y} \sup_{x \in X} \|x - \varphi(x)\|$$

Bottleneck Distance on $\mathcal{D}$

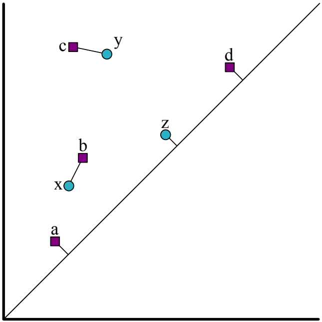


Bottleneck distance for diagrams

Given diagrams X and Y, the distance between them is

$$d_B(X, Y) = \inf_{\varphi: X \to Y} \sup_{x \in X} \|x - \varphi(x)\|$$

Bottleneck Distance on $\mathcal{D}$

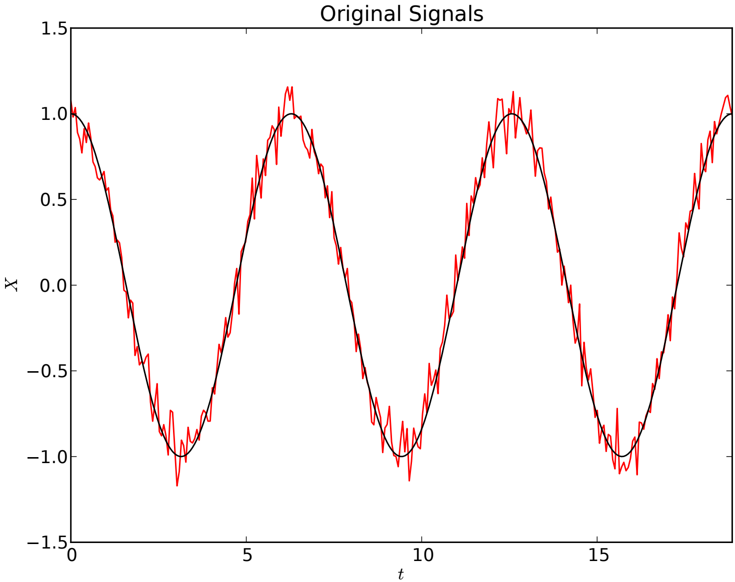


Bottleneck distance for diagrams

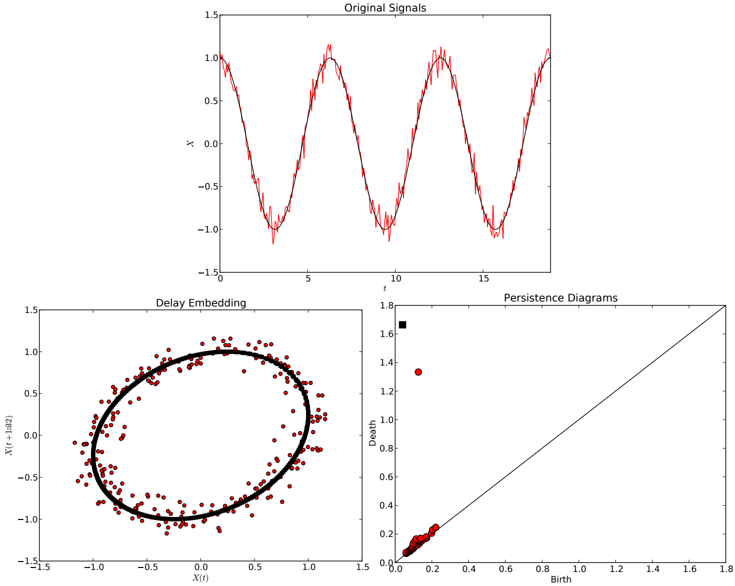
Given diagrams X and Y, the distance between them is

$$d_B(X, Y) = \inf_{\varphi : X \to Y} \sup_{x \in X} \|x - \varphi(x)\|$$

Noise resilience



Noise resilience

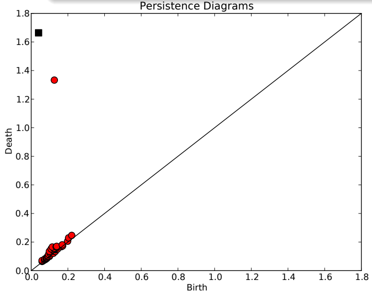


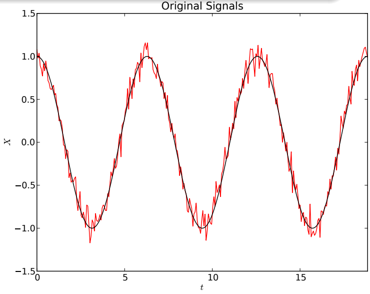
Noise resilience

Theorem (Cor of Cohen-Steiner et al., 2007)

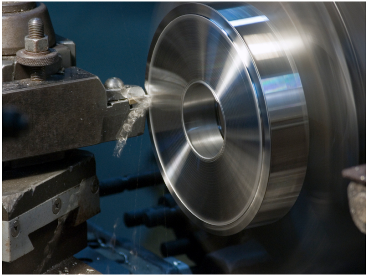
Given two time series $f, g : \mathbb{R} \to \mathbb{R}$.
Let $\mathrm{dgm}(\mathrm{Takens}(f))$ be the persistence diagram of the Takens embedding of f, likewise for g, embedded with same η, m. Then

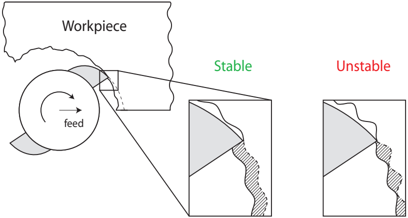
$$d_B(\mathrm{dgm}(\mathrm{Takens}(f)), \mathrm{dgm}(\mathrm{Takens}(g))) \leq \|f - g\|_\infty.$$

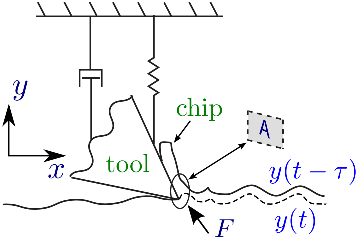


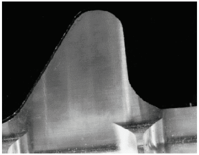


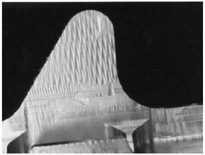
Machining Dynamics







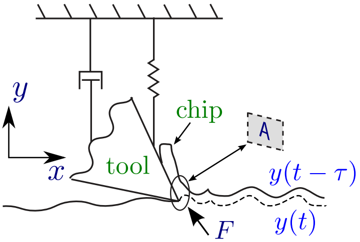




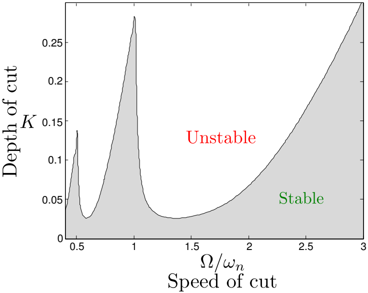
Images courtesy Firas Khasawneh, SUNYIT; and Boeing.

Deterministic model:

$$\ddot{y} + 2\zeta\dot{y} + y = K\rho^{\alpha-1}(1 + y(t-\tau) - y(t))^{\alpha}$$

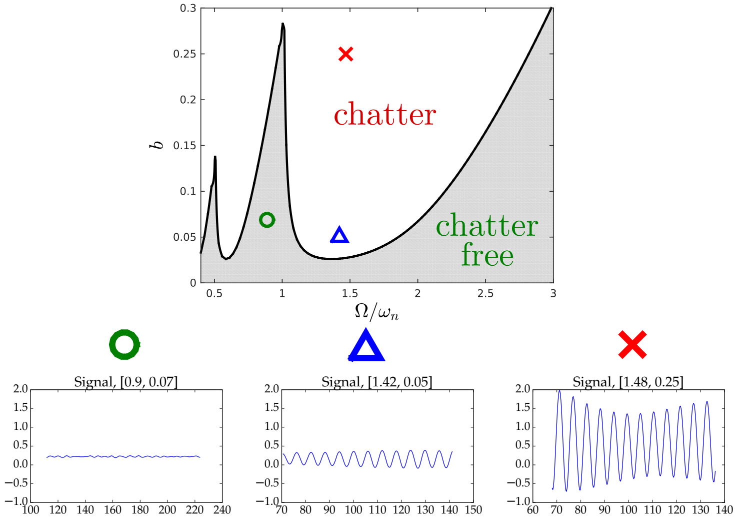


- Left side: standard linear oscillator
- Right side: input based on cutting forces



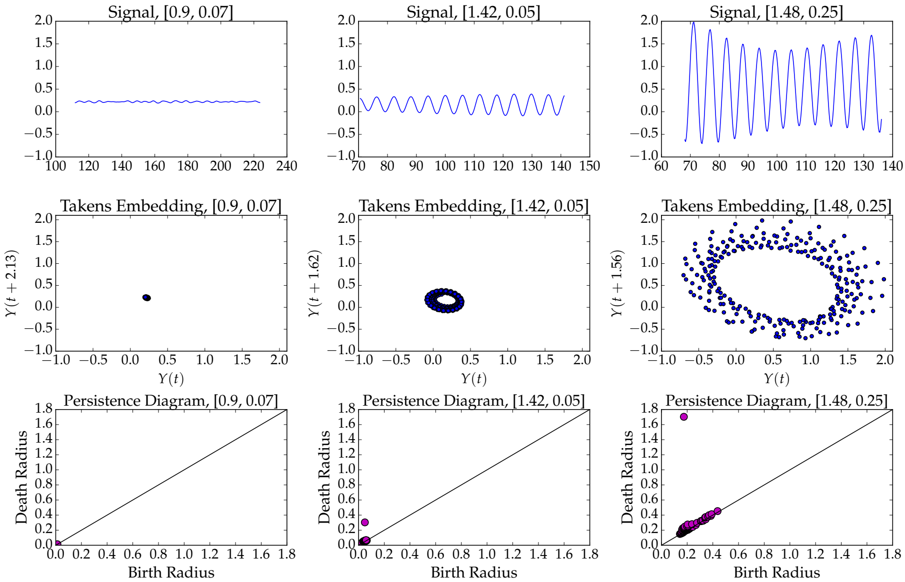
Khasawneh, F.A. & Mann, B. P. A spectral element approach for the stability of delay systems, International Journal for Numerical Methods in Engineering, 2011, 87, 566-592

Chatter

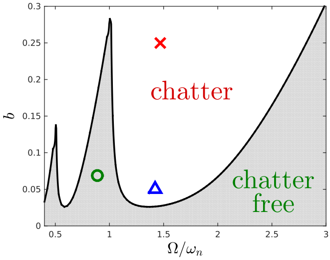


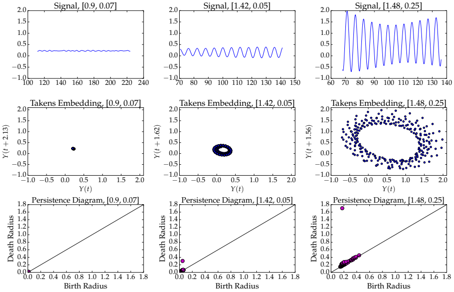
chatter
chatter
free
b
Ω/ω_n
Signal, [0.9, 0.07]
Signal, [1.42, 0.05]
Signal, [1.48, 0.25]

Comparing signals using persistence

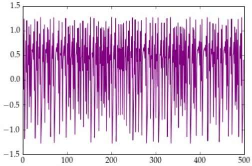


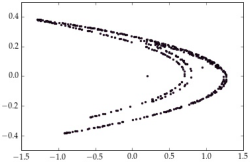
Comparing signals using persistence

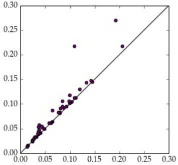




Overview



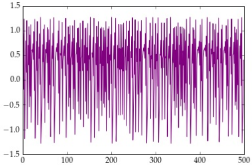


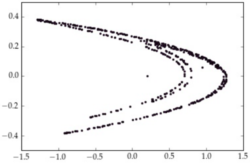


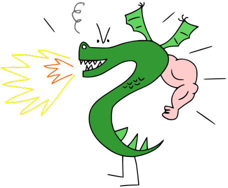
Takens Embedding

$\mathbb{R}$-valued TS → Pers

Overview

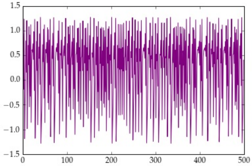


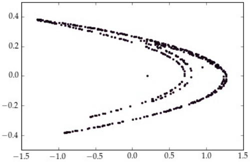


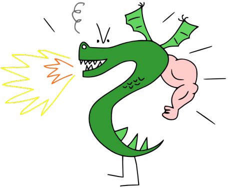


$\mathbb{R}$-valued TS → Takens Embedding → Pers

Overview



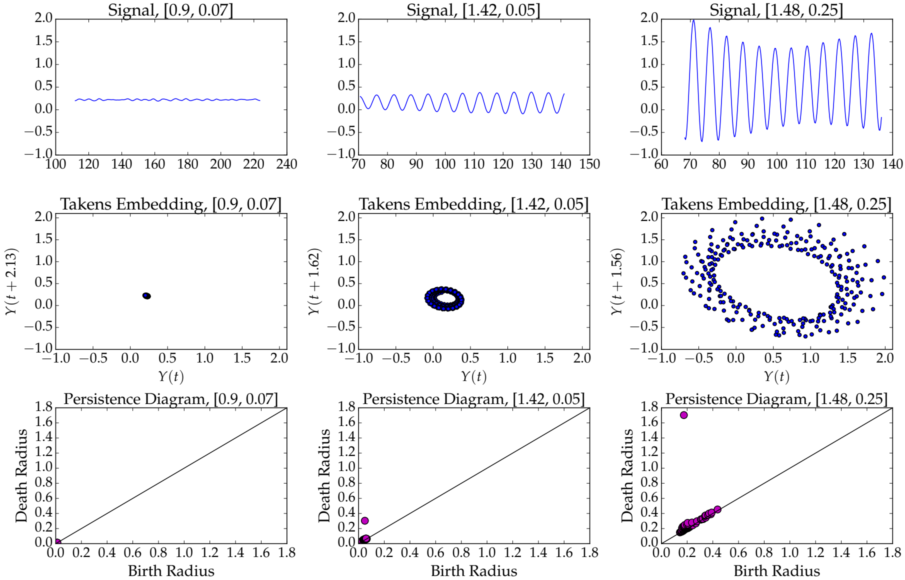




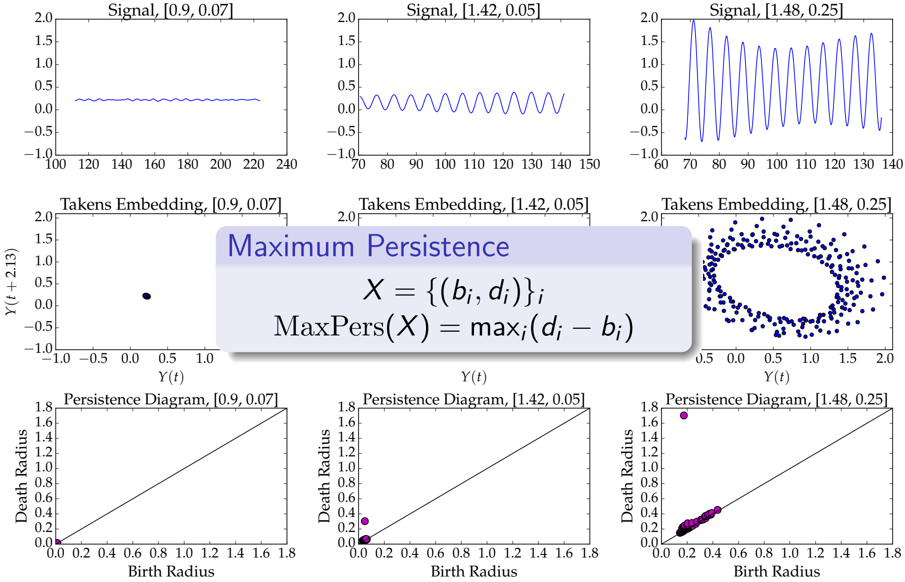
$\mathbb{R}$-valued TS → Takens Embedding → Pers

Pers → Max Pers → $\mathbb{R}$

Differentiation by Max Persistence



Differentiation by Max Persistence

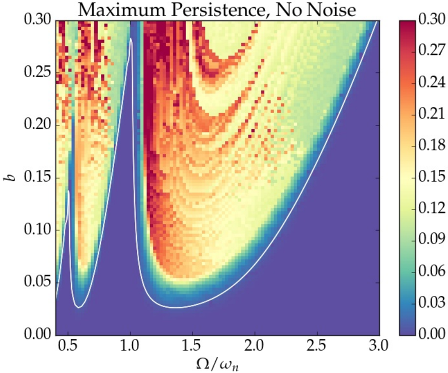


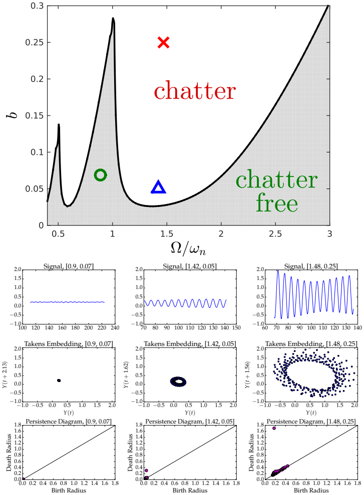
Maximum Persistence

$$X = \{(b_i, d_i)\}_i$$

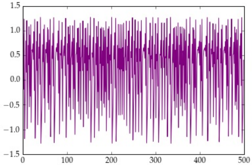
$$\mathrm{MaxPers}(X) = \max_i (d_i - b_i)$$

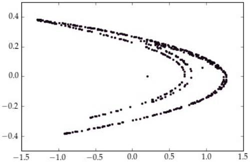
Turning Model

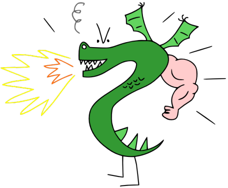




Overview



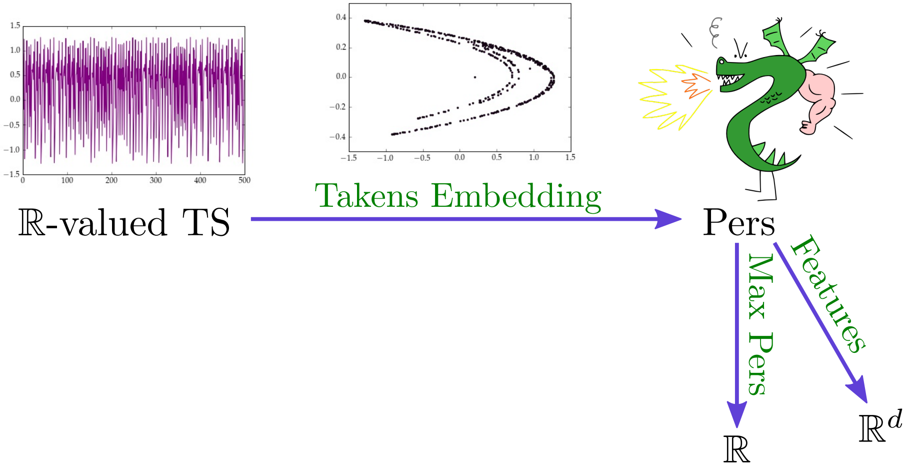




$\mathbb{R}$-valued TS → Takens Embedding → Pers

Pers → Max Pers → $\mathbb{R}$

Overview



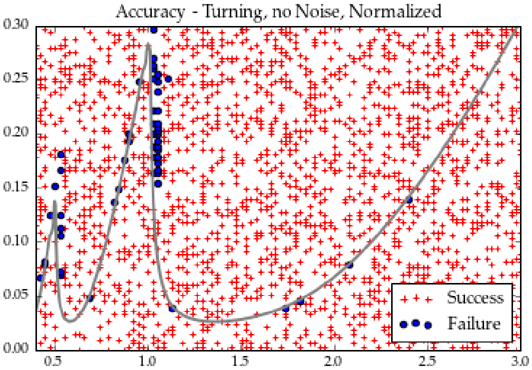
Machine Learning

Adcock et al. 2016 – Coordinates

Diagrams 0 and 1-dimensional of the form $X = \{(x_i, y_i)\}$.
Features:

- $\sum x_i(y_i - x_i)$
- $\sum (y_{max} - y_i)(y_i - x_i)$
- $\sum x_i^2(y_i - x_i)^4$
- $\sum (y_{max} - y_i)^2(y_i - x_i)^4$
- $\max\{(y_i - x_i)\}$

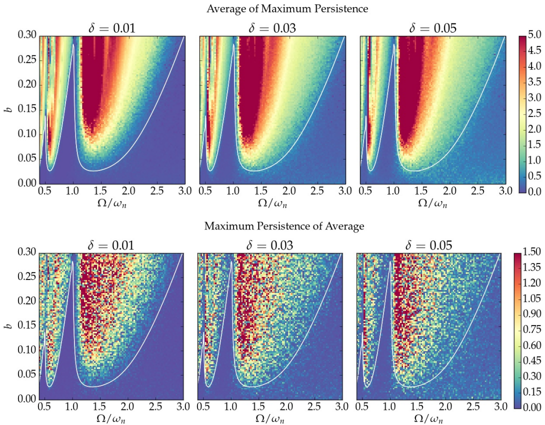
Machine Learning

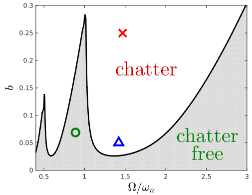


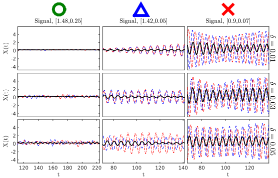
Results (Khasawneh, M, Perea 2017)

- Theoretical stability boundary for training
- Standard logistic classifier
- 97% accuracy

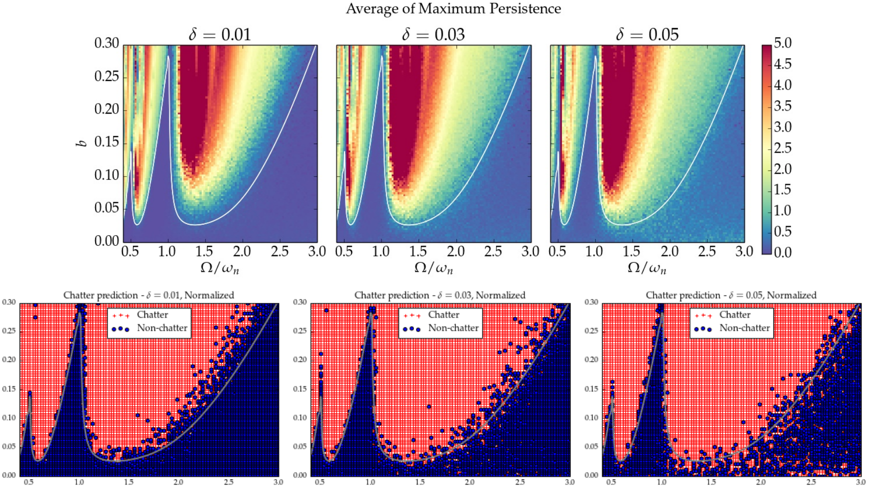
Additive Noise Model







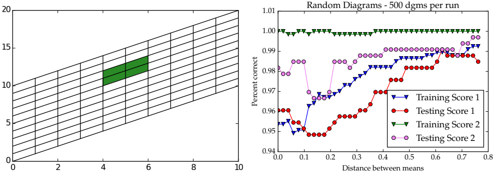
Transfer learning



Future work

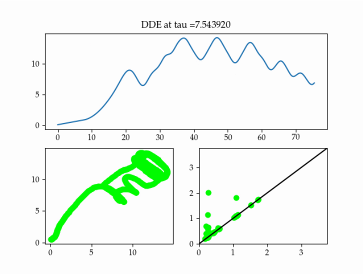
Extending the ML Theory

EM, Jose Perea



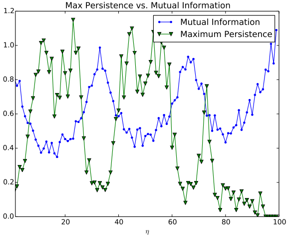
Chaos Characterization

Brian Bollen, Firas Khasawneh, EM



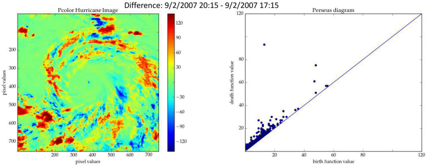
Parameter identification

Firas Khasawneh, EM, Chris Sukhu



Matrix-valued time series

EM, Bill Dong, Kristen Corbosiero, Ryan Torn, Jason Dunion



Thank you!

Firas Khasawneh

elizabethmunch.com

emunch@albany.edu

Jose Perea

NSF

UNIVERSITY AT ALBANY

State University of New York

DARPA