Mimetic Finite Difference Methods

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Why we need mimetic methods?

- Capture most important features of physical, geometrical, and mathematical model
  - The preservation of the conservation laws in a discrete model is necessary for modeling flows with strong shocks
  - The preservation of the spatial symmetries in numerical simulation of implosion is critically important in the inertial confinement fusion program
- Provide reliability, accuracy, and efficiency
- Lead to significant advances in computer models based on PDE's
- Enable to solve new and challenging physical problems, that require all facets of the modeling process to be addressed

Discrete Vector and Tensor Analysis

- Discrete scalar, vector and tensor functions on different type of grids
- Discrete analogs of differential operators like divergence, gradient and curl
- Discrete analogs of the theorems of the vector analysis: Gauss', Stokes', Orthogonal decomposition (Hodge)
- Constrained data transfer between meshes

Mimetic Discretizations for Maxwell's Equations

The system of first-order Maxwell's curl equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -\mathbf{curl}\,\mathbf{E}, \quad \frac{\partial \mathbf{E}}{\partial t} = \epsilon^{-1}\,\mathbf{curl}\,\mu^{-1}\mathbf{E}.$$

The "divergence-free" conditions

$$\mathbf{div}_{\epsilon}\,\mathbf{E} = 0, \quad \mathbf{div}\,\mathbf{B} = 0$$

Curl identity

$$\int_V (\mathbf{curl}\,\mathbf{E} \cdot \mathbf{H})\, dV = \int_V (\mathbf{E} \cdot \mathbf{curl}\,\mathbf{H})\, dV$$

Modified curl identity

$$\int_V \mu^{-1}\,(\mathbf{curl}\,\mathbf{E} \cdot \mathbf{B})\, dV = \int_V \epsilon\,(\mathbf{E} \cdot \epsilon^{-1}\,\mathbf{curl}\,\mu^{-1}\,\mathbf{B})\, dV$$

Mimetic Discretizations for Maxwell's Equations

- Divergence-free conditions
  - $\mathbf{DIV} : \mathcal{H}F \to HC; \quad \mathbf{DIV}\,\mathbf{B} = 0$
  - $\mathbf{DIV}' = \mathbf{GRAD}^* : \mathcal{H}E \to HN; \quad \mathbf{DIV}'\,\mathbf{E} = 0;$

$$(\mathbf{GRAD} : HN \to \mathcal{H}E; \mathbf{GRAD}\,u)_e = (u_{ne} - u_{nb})/L_e$$

- Scalar and vector potentials:

$$\mathbf{B} = \mathbf{CURL}\,\mathbf{A}\,,\ \mathbf{A} \in \mathcal{H}E$$

$$\mathbf{E} = \mathbf{GRAD}\,\varphi\,,\ \varphi \in HN$$

Discrete Calculus

- Discrete analogs of integrals, discrete paths
- Discrete Gauss' theorem
- $\mathbf{DIV}\,\mathbf{A} = 0$ if and only if $\mathbf{A} = \mathbf{CURL}\,\mathbf{B}$, $\mathbf{A} \in \mathcal{H}F$, $\mathbf{B} \in \mathcal{H}E$
- $\overline{\mathbf{DIV}}\,\mathbf{A} = 0$ if and only if $\mathbf{A} = \overline{\mathbf{CURL}}\,\mathbf{B}$, $\mathbf{A} \in \mathcal{H}E$, $\mathbf{B} \in \mathcal{H}F$
- $\mathbf{CURL}\,\mathbf{A} = 0$ if and only if $\mathbf{A} = \mathbf{GRAD}\,\varphi$, $\mathbf{A} \in \mathcal{H}E$, $\varphi \in HN$
- $\overline{\mathbf{CURL}}\,\mathbf{A} = 0$ if and only if $\mathbf{A} = \overline{\mathbf{GRAD}}\,\varphi$, $\mathbf{A} \in \mathcal{H}F$, $\varphi \in HC$
- Discrete Orthogonal Decomposition Theorems
  - $\mathbf{DIV}\,\mathbf{A} = \psi$, $\overline{\mathbf{CURL}}\,\mathbf{A} = \omega \rightarrow \mathbf{A} = \overline{\mathbf{GRAD}}\,\varphi + \mathbf{CURL}\,\mathbf{B}$
    $\mathbf{A} \in \mathcal{H}F$, $\varphi \in HC$, $\mathbf{B} \in \mathcal{H}E$
  - $\overline{\mathbf{DIV}}\,\mathbf{A} = \psi$, $\mathbf{CURL}\,\mathbf{A} = \omega \rightarrow \mathbf{A} = \mathbf{GRAD}\,\varphi + \overline{\mathbf{CURL}}\,\mathbf{B}$
    $\mathbf{A} \in \mathcal{H}E$, $\varphi \in HN$, $\mathbf{B} \in \mathcal{H}F$

Mimetic Discretizations for Lagrangian Hydrodynamics

Equations in Lagrangian Form

$$\frac{d\rho}{dt} = -\rho\, \mathbf{div}\, \mathbf{u}, \quad \rho \frac{d\mathbf{u}}{dt} = -\mathbf{grad}\, p, \quad \rho \frac{d\varepsilon}{dt} = -p\, \mathbf{div}\, \mathbf{u}, \quad \frac{\partial \mathbf{r}}{dt} = \mathbf{u}$$

Conservation laws follows from following properties of the operators

$$\mathbf{div}\, \mathbf{W} = \lim_{\delta V \to 0} \frac{\frac{d}{dt}(\delta V)}{\delta V}, \quad \int_V \mathbf{div}\, \mathbf{W}\, dV = \oint_{\partial V} (\mathbf{W}, \mathbf{n})\, dS$$

$$\int_V \mathbf{grad}\, p\, dV = \oint_{\partial V} p\, \mathbf{n}\, dS$$

$$\int_V p\, \mathbf{div}\, \mathbf{W}\, dV + \int_V (\mathbf{W}, \mathbf{grad}\, p)\, dV = \oint_{\partial V} p\, (\mathbf{W}, \mathbf{n})\, dV$$

Applications

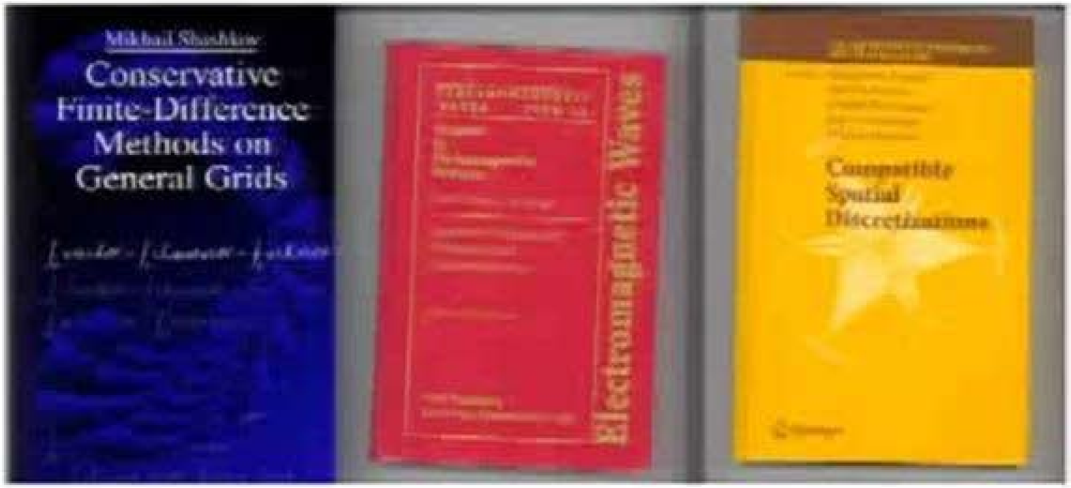
- Fluid and solid dynamics
- Shock physics
- Electromagnetism
- Radiation transport
- General relativity
- Flow in porous media
- Laser plasma simulations
- Computational geometry
- Image analysis
- Astrophysics

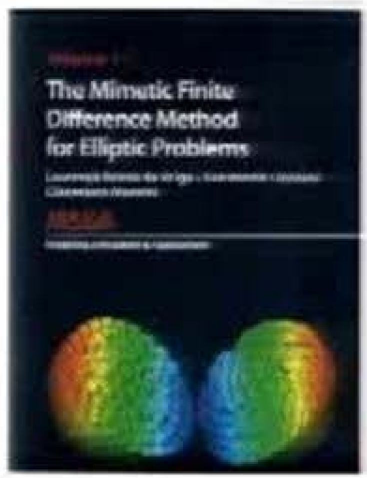
Information

https://www.researchgate.net/profile/Mikhail_Shashkov

http://scholar.google.com/citations?user=AP9k_o8AAAAJ&hl=en

Publications





Conclusion

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