Forcing-induced transitions in a Paleoclimate delay model

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MS162 Planetary Motion and its Effects on Climate
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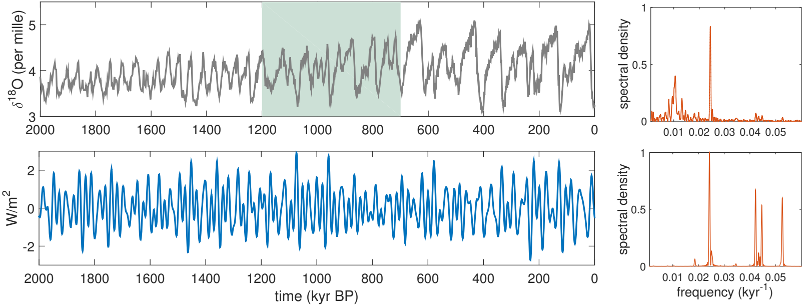
Mid-Pleistocene Transition (MPT)

Change in glacial cycle periods:
41 kyr before 1200 kyr BP, ∼100 kyr after 700 kyr BP



top: Lisiecki, L. E., and M. E. Raymo (2005) **bottom:** Huybers, P. and Eisenman, I. (2006)

Proposed mechanisms for MPT

The MPT due to parameter shift: Saltzman models (1987-1991), Paillard (1998), Paillard & Parrenin (2004), Tzipermen & Gildor (2003), Widiasih, Stuecker, & Baek (2018) **[MS125, next talk]**, Morupisi & Budd **[CP9]**

- slow decrease of background atmospheric CO_2 concentration
- change in bottom water (NADW) formation
- gradual cooling of deep ocean allowing for sea-ice switch mechanism
- change in critical temperature for ice formation
- varying amplitude and frequency of periodic forcing

The MPT as a spontaneous transition: Huybers (2009)

- glacial variability as a chaotic response to obliquity forcing

A delay model for the Pleistocene climate

$$\dot{X}(t) = -pX(t-\tau) + rX(t) - sX(t-\tau)^2 - X(t-\tau)^2 X(t)$$

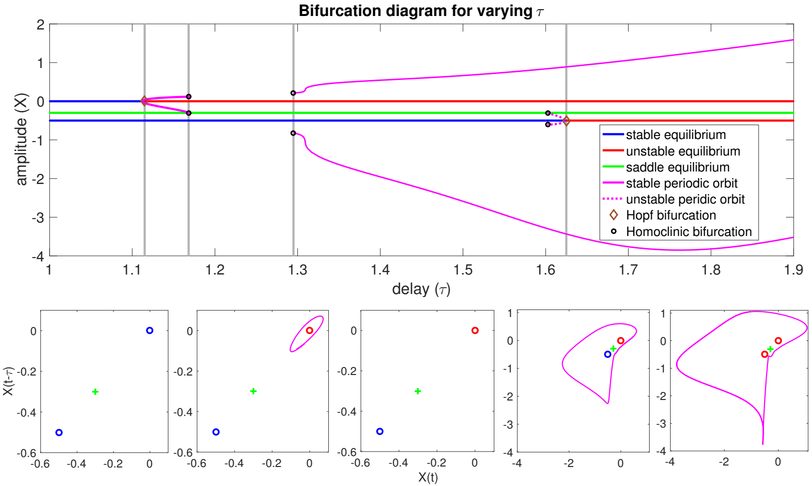
X - Global Ice Mass (anomalies)

Parameters:
p - CO_2 dependence on North Atlantic Circulation
r - balance of CO_2 exchange
s - asymmetry
τ - delay from feedback processes associated with ice accumulation and decay, and carbon storage and transport in the deep ocean

Derived from Saltzman and Maasch (1988)

- Three-dimensional ODE model
- Ice mass, atmospheric CO_2, ocean circulation

Unforced solutions of DDE model



Bistable region: $1.295 < \tau < 1.625$ $\quad (p = 0.95,\ r = 0.8,\ s = 0.8)$

Periodic forcing

$$\dot{X} = -0.95X(t-\tau) + 0.8X(t) - 0.8X(t-\tau)^2 - X(t-\tau)^2X(t) - uF(t)$$

$$F(t) = \sin(\omega t - \phi), \quad \omega = \frac{2\pi}{4.1}, \quad \phi \in [0, 2\pi]$$

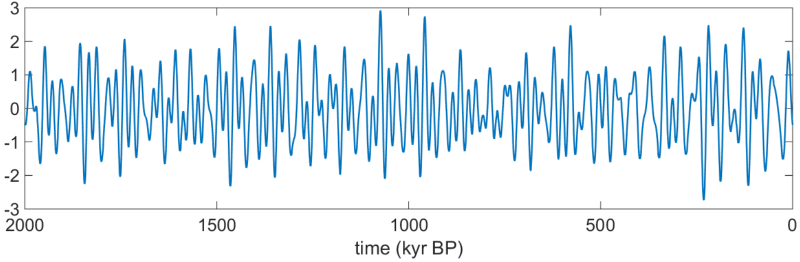
Results:

- Two responses - periodic and quasiperiodic/chaotic
- Transition due to moving basins of attraction
- Calculation of intersection of a stable manifold of a saddle with a slow manifold in a DDE
  - Embedding the algorithm for planar maps (England *et al*, 2004) into the equation-free framework (Kevrekidis *et al*, 2009)

CQ, J. Sieber, & A. S. von der Heydt, (2019). "Effects of forcing on a Paleoclimate delay model" **arXiv: 1808.02310** (to be published in SIADS)

Milankovitch forcing

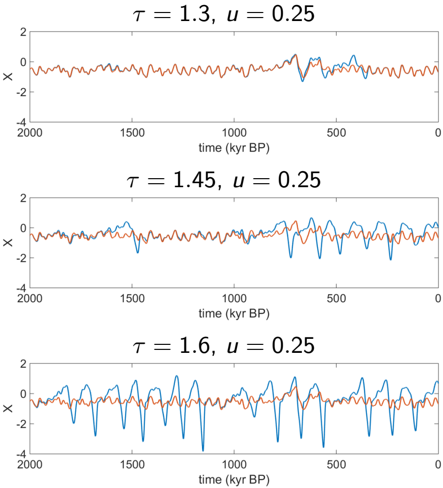
Precession ≈ 19/23 kyr, Obliquity ≈ 41 kyr, Eccentricity ≈ 100/400 kyr



Huybers, P. and Eisenman, I. 2006. Integrated Summer Insolation Calculations.

$$\dot{X} = -0.95X(t-\tau) + 0.8X(t) - 0.8X(t-\tau)^2 - X(t-\tau)^2X(t) - uM(t)$$

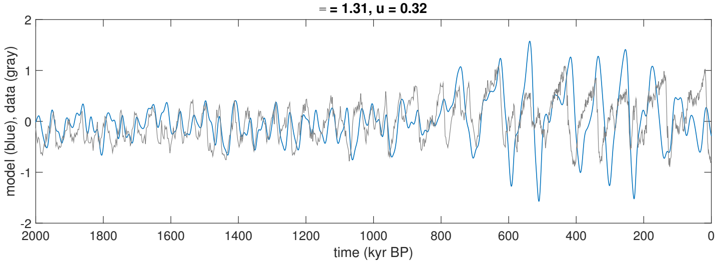
Small- and large-amplitude response



Red trajectory shows quasisteady state taken for $\tau = 1.25,\ u = 0.25$

MPT-like transition

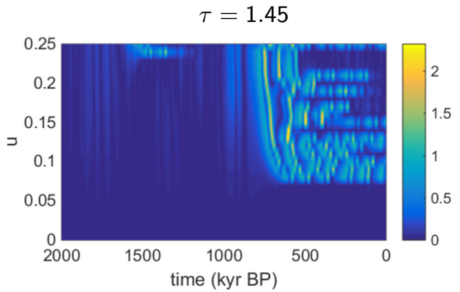
Model output (blue) compared to climate record (grey)



Example of realisation that has similar features to observations:

- small amplitude oscillations with period ≈ 41 kyr
- transition just after 1 Myr BP
- large amplitude oscillations with asymmetric shape and period ≈ 100 kyr

Varying forcing strength



Transition in forcing strength - threshold behaviour

Transition in time - preferred time for transition to large-amplitude response 700-800 kyr BP

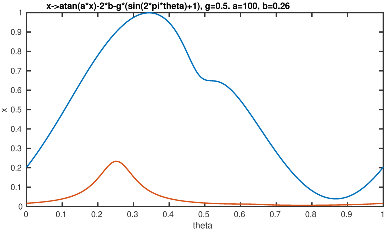
Non-smooth saddle-node bifurcation in quasiperiodically forced maps

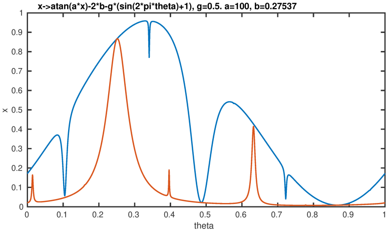
Jäger, 2009 - Invariant circles approach each other with an exponential evolution of peaks, “strange non-chaotic attractor”

Example of map that undergoes non-smooth saddle-node bifurcation:

$$(\theta, x) \mapsto (\theta + \omega, f_\beta(\theta, x))$$

$$\omega = \frac{\sqrt{5}-1}{2}(2\pi), \quad f_\beta(\theta, x) = \arctan(\alpha x) - 2\beta - \gamma[\sin(2\pi\theta) + 1]$$



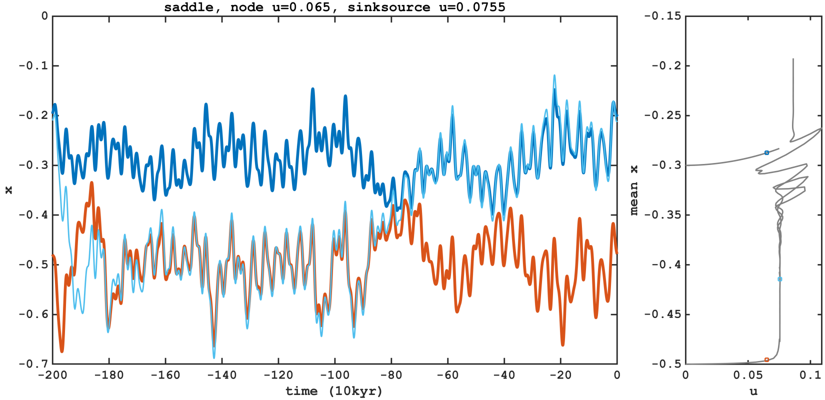


Figures created by J. Sieber based on Furhmann, Gröger, Jäger, 2017

Finite-time truncation of non-smooth saddle-node bifurcation

Video of pullback attractor and nonautonomous saddle for increasing u

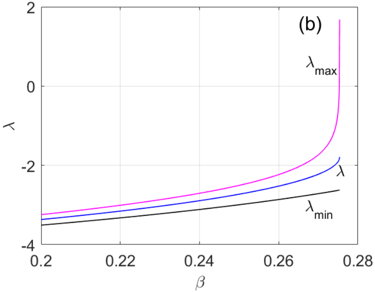
Finite-time truncation of non-smooth saddle-node bifurcation



Figures created by J. Sieber

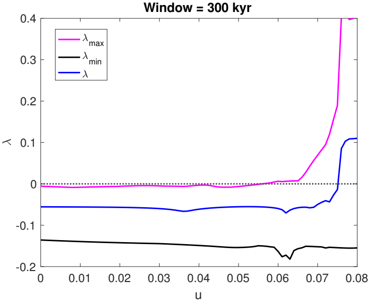
Finite-time Lypapunov Exponents (FTLEs)

F. Remo, G. Furhmann, & T. Jäger (2019, arXiv:1904.06507) studied the behaviour of FTLEs when approaching the non-smooth saddle-node bifurcation in quasiperiodically forced map **[PP2]**



bifurcation parameter $\beta \approx 0.2752$

F. Remo, G. Furhmann, & T. Jäger (2019)



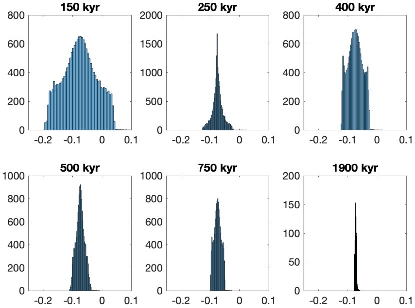
bifurcation parameter $u \approx 0.0755$

Quinn *et al* DDE

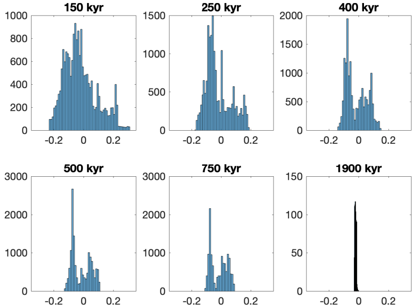
Distribution of FTLEs for increasing window length

Bifurcation parameter $u \approx 0.0755$

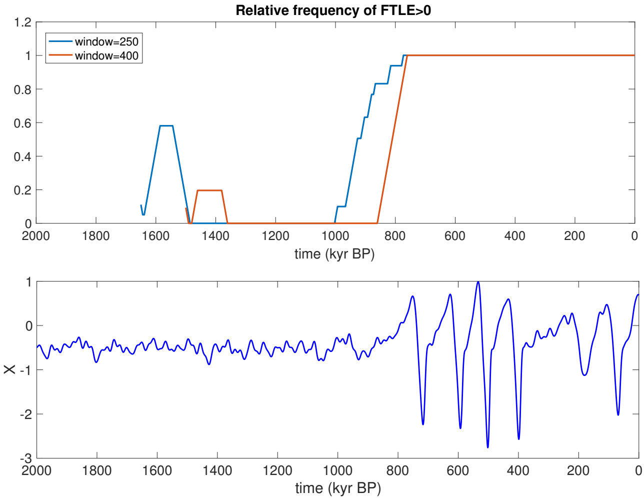
$u = 0.01$



$u = 0.075$



Relative frequency of positive FTLEs



Conclusions

- Dynamics of glacial cycles during the Pleistocene can be modelled through scalar DDE for ice mass.
- Existence of bistable region with equilibrium and large amplitude periodic orbit.
- The quasiperiodically forced model consistently transitions within the time window for the MPT (large range of parameters and noise) - no parameter shift necessary.
- This transition resembles a finite-time truncation of a non-smooth saddle-node bifurcation observed in some quasiperiodically forced maps
- Relative frequency of positive FTLEs can potentially be used as an identification of bifurcation ocurrence and early warning signal for transition (work in progress)

Thank you for your attention.

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