Uncertainty Quantification for Volcanic Hazards

E. Bruce Pitman

The University at Buffalo

SIAM Geoscience

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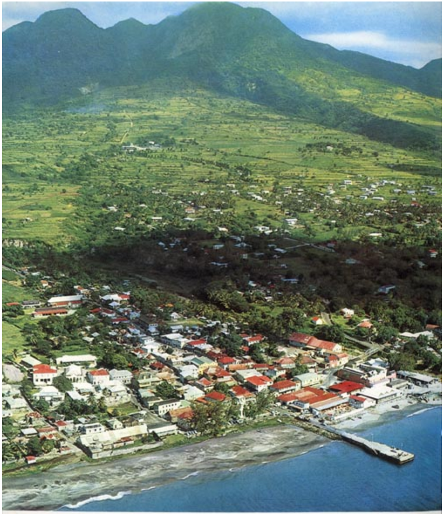
Montserrat

Plymouth 1977



MONTSERRAT: CAPITAL PLYMOUTH WITH SOUFRIERE (3002 FT.)

Pyroclastic Flows

Plymouth 1998

How bad is it?

The Cast of Characters

This project began as a SAMSI Working Group in the 'Computer Models' program 2006-2007. Over the years several wonderful graduate students at Buffalo, Duke, and Marquette have participated in the effort. The research has been continuously supported by the NSF through its FRG, BigData, and CDSE programs.

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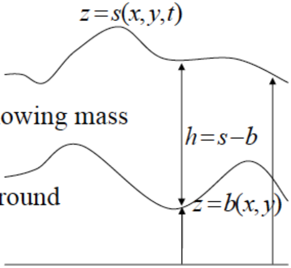

Upper free surface

$$F^s(\mathbf{x}, t) = s(x, y, t) - z = 0,$$

Basal material surface

$$F^b(\mathbf{x}, t) = b(x, y) - z = 0$$

Kinematic BC:

at $F^s(\mathbf{x}, t) = 0: \ \partial_t F^s + \mathbf{v} \cdot \nabla F^s = 0$

at $F^b(\mathbf{x}, t) = 0: \ \partial_t F^b + \mathbf{v} \cdot \nabla F^b = e_s$

$$\nabla \cdot \mathbf{u} = 0$$

$$\partial(\rho_0 \mathbf{u}) + \nabla \cdot (\rho_0 \mathbf{u} \otimes \mathbf{u}) = \nabla \cdot \mathbf{T} + \rho_0 \mathbf{g}$$

Assume $H/L \ll 1$ and scale governing equations, and depth average.

Final System

$$\mathbf{U} = \begin{pmatrix} h \\ hv_x \\ hv_y \end{pmatrix} \qquad \mathbf{f}(\mathbf{U}) = \begin{pmatrix} hv_x \\ hv_x^2 + \frac{1}{2} k_{ap} g_z h^2 \\ hv_x v_y \end{pmatrix}$$

$$S_x = g_x h - hk_{ap}\mathrm{sgn}(\frac{\partial v_x}{\partial y})\partial_y(g_z h)\sin\varphi_{int}$$
$$- \frac{v_x}{\sqrt{v_x^2 + v_y^2}} \left[g_z h \left(1 + \frac{v_x}{r_x g_z} \right) \right] \tan\varphi_{bed}$$

Final System

$$\frac{\partial U}{\partial t} + \frac{\partial f(U)}{\partial x} + \frac{\partial g(U)}{\partial y} = S(U)$$

A system of hyperbolic conservation laws.

Input parameters ϕ_b, ϕ_{int} could, in principle, be measured in lab

TITAN2D is a computational environment for solving this system.

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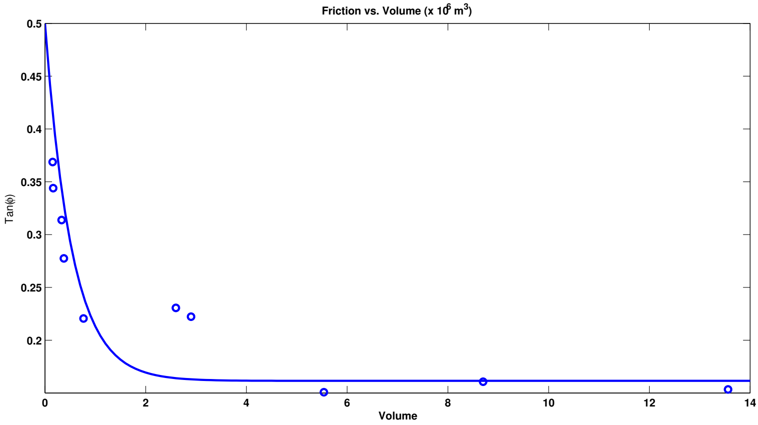
Inputs

TITAN-2D requires a model of the topography at every lat-lon location.

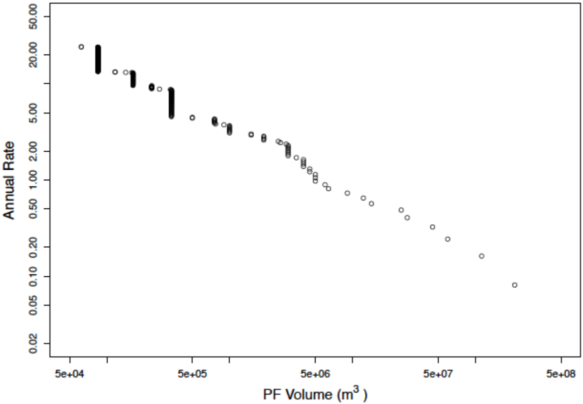
Other inputs include ϕ_b, ϕ_{int}, initial volume and location, direction and velocity of the flow at the start.

Sensitivity analysis: ϕ_{int} less important than ϕ_b.
Discover $\phi_b = \phi_b(V)$!

Effective Friction

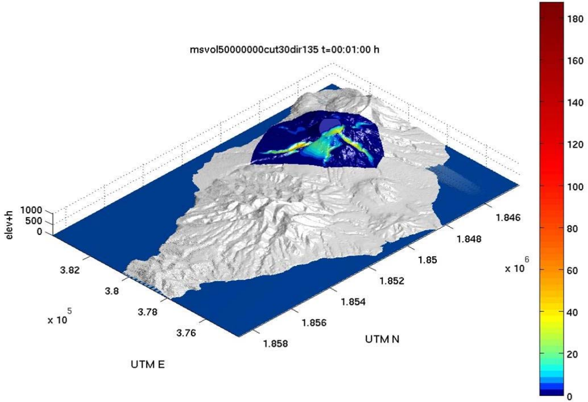


Volume-Frequency



Montserrat Computation

Fix ϕ_{int}, choose V, ϕ_b, ζ.



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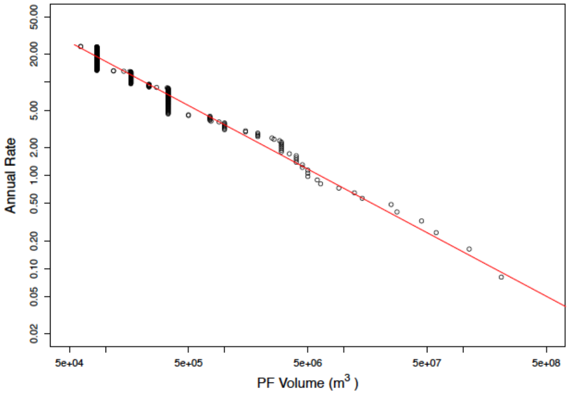
Volume-Frequency



Volume-Frequency

Pareto volume-frequency $f(v|\alpha) = \frac{\alpha\epsilon^{\alpha}}{v^{\alpha+1}}$

Best estimate: $\alpha < 1$ which means expected volume of flows, and expected variance, are both infinity. That is, there is a non-trivial chance of extremely large event (larger than the total mass of the island!).

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Little data out near the shoulder to fit a cut-off.

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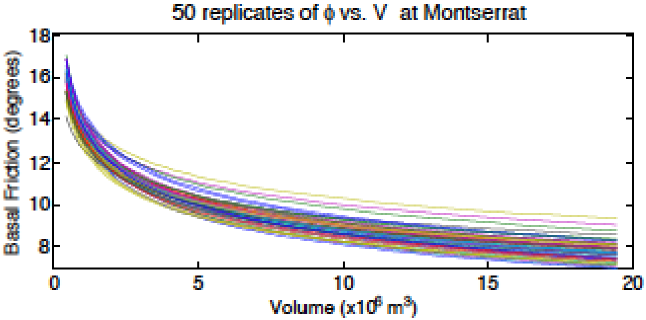
Hierarchical Friction Model



$$\phi_{bed} = arctan(a + b\log(V))$$

A First Step

- Consider what happens at a single location on the island
- Select initial volume and direction from prescribed distributions obtained from historical data, make a random selection of $\phi_b(V)$, and fix other parameters
- Many TITAN2D simulations
- Define a "catastrophic curve" $\psi(\zeta)$ in this plane, calling more simulations as necessary
- Construct a GaSP emulator
- Assume Poisson process in time with rate λ; use a distribution of αs consistent with the data
- Assume uniform distribution of flow directions

Catastrophic Event

If we define $y^M(V, \zeta)$ as the TITAN2D output, a catastrophic event is a $y^M \in \psi^C$ where

$$\psi = \psi(\zeta) = \inf(V : y^M(V, \zeta) \geq h_{crit})$$

Finding ψ is an inverse problem.

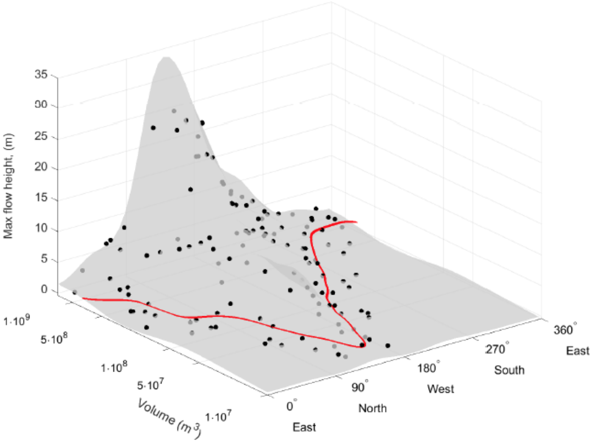
Emulator

GaSP

$$g(y^M(V,\zeta)) = \delta + mV + Z(V,\zeta)$$

with squared exponential correlation in V and ζ

GaSP



35
30
25
20
15
10
5
0
Max flow height, (m)
1·10^9
5·10^8
1·10^8
5·10^7
1·10^7
Volume (m^3)
0°
East
90°
North
180°
West
270°
South
360°
East

Hazard Probability

$$E(\#\text{catastrophic flows in } t \text{ yrs}) = \frac{t\lambda}{2\pi} \int_0^{2\pi} \psi(\zeta)^{-\alpha} d\zeta$$

Hazard Probability

Then the posterior distribution is given as

$P(\text{at least one flow } > \psi(\zeta) \text{ in } t \text{ yrs}) =$

$$1 - \int \int \exp\left(\frac{-\lambda t}{2\pi} \int_0^{2\pi} \psi(\zeta)^{-\alpha} d\zeta\right) \Pi(\alpha, \lambda | \text{data}) d\lambda d\alpha$$

Hazard Probability

Note: the simulations and GaSP construction are divorced from the determination of ψ which is divorced from the hazard calculation.

Map Construction

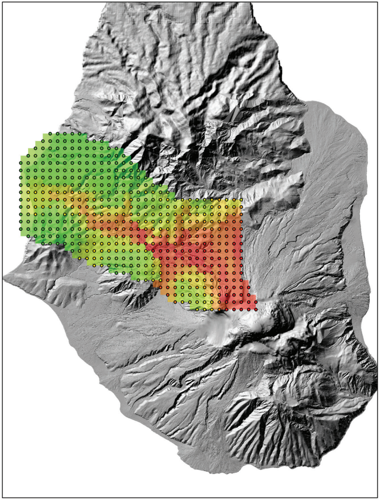
To compute the hazard probability for many points in a region, compute the hazard at a point, for many locations.

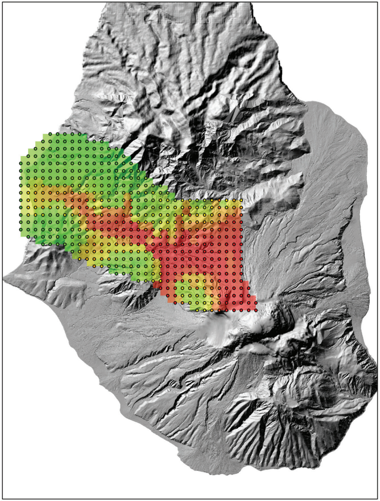
One large set of simulations ($\sim$2000), done beforehand.

Draws of about 50 runs to construct GaSP for each unique location. Add some additional simulations if necessary to find ψ curve accurately enough. This can be done for all locations in parallel.

We also have data on which valleys were hit with which frequency, and when, showing an intereting switching phenomena after large events.

Flow Distribution





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Summary

- We have outlined one approach for the rapid construction of a hazard map, one which separates the flow simulations from the construction of the hazard map
- We turned an inverse problem for parameters into a series of forward problems
- We have discovered uncertain inputs in our models and accounted for them
- We have explored Bayes model averaging – because we don't know the correct constitutive relation
- We used a GaSP emulator in an interesting construction. Since the first work we did, Gu and Berger - Parallel Partial Emulation.
- Can emulate time series, which we are using to locate rockfall locations.

References

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5. Gu and Berger, *Annals Applied Stats* **10** (2016)
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