Mesohyperbolicity, Mix-Norm and the Hunt for MH370

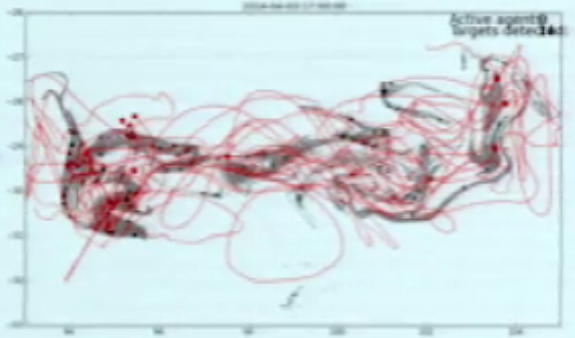
Sophie Loire

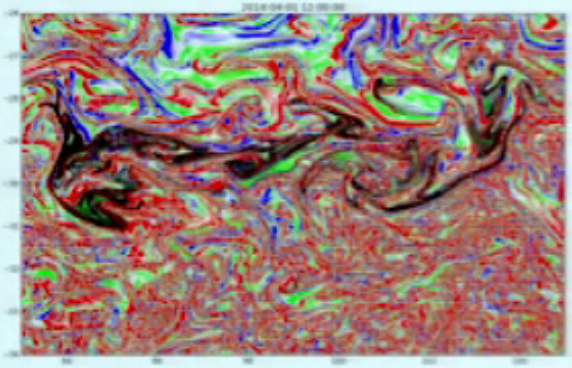
Hassan Arbabi, Patrick Clary, Igor Mezić
Department of Mechanical Engineering,
University of California, Santa Barbara

Bojan Crnković, Stefan Ivić
Rijeka University, Croatia

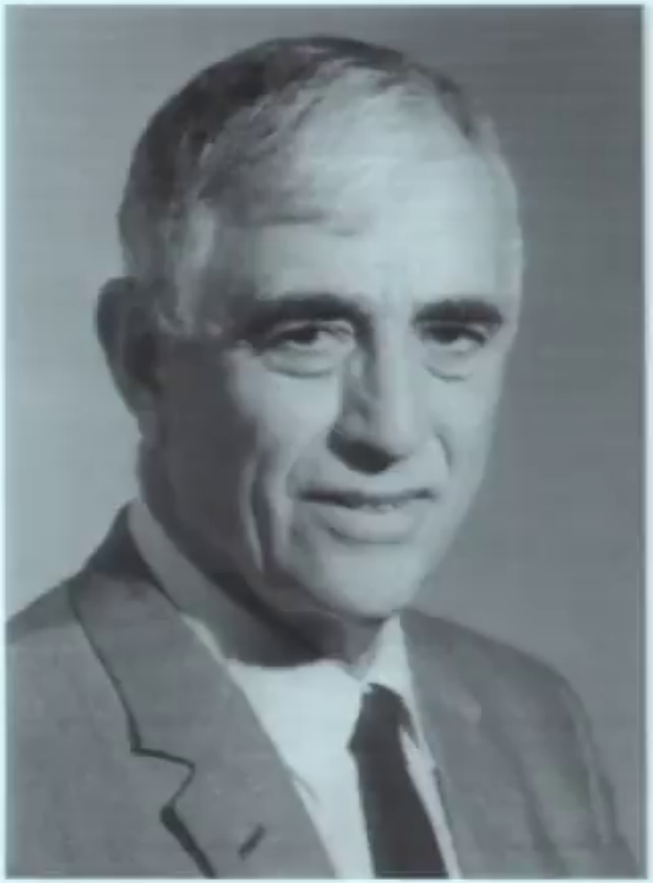
The Indian Ocean Case and the hunt for MH370 floating objects

Talk Outline:





The other Koopman Legacy



Bernard Osgood Koopman

Bernard Osgood Koopman (1900-1981):

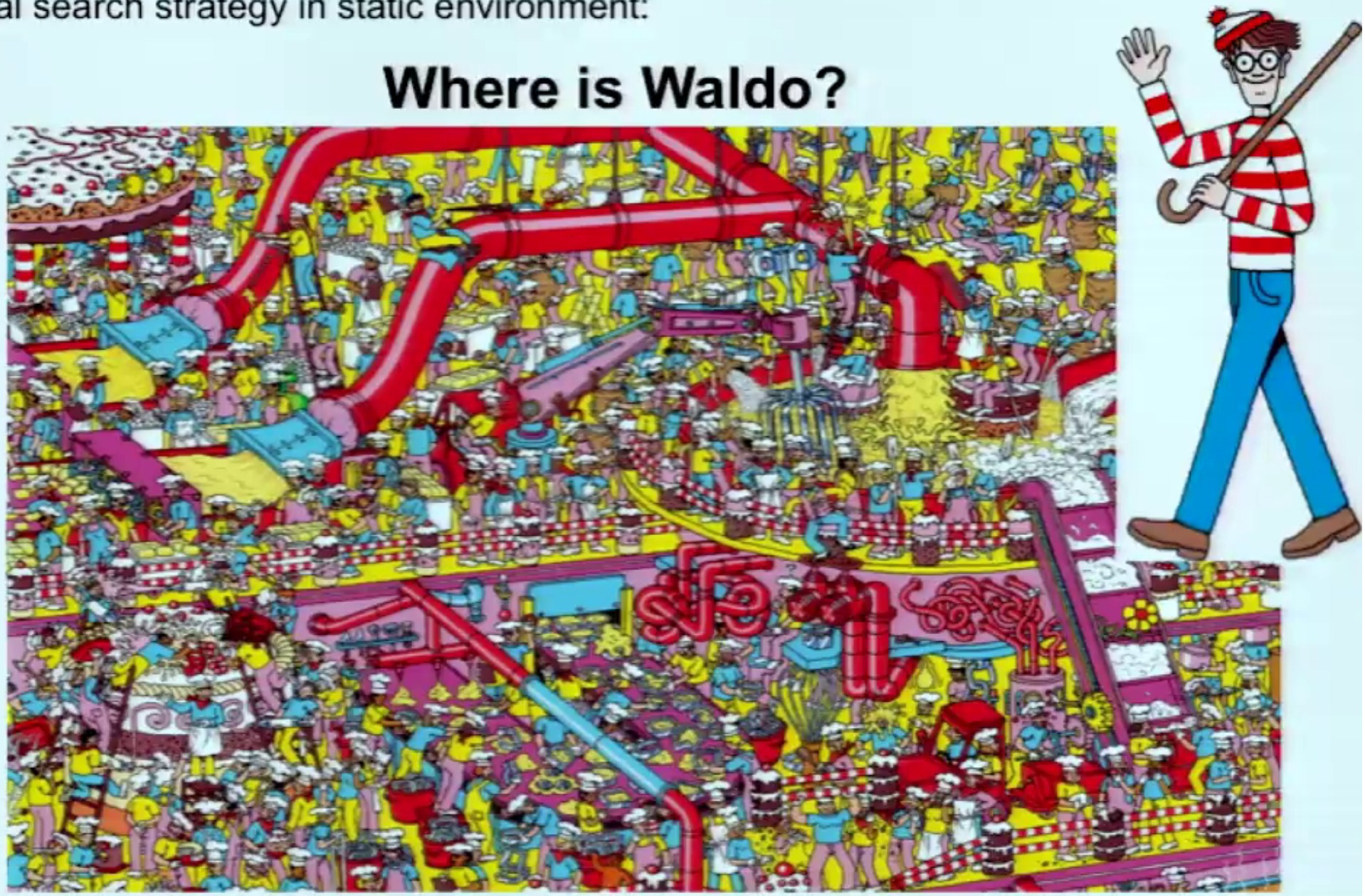
- Better known for the Koopman Operator Theory
- But also the father of the "Theory of Search":

[1] The Theory of Search; I. Kinematic Bases. Opns. Res. 4, 324-346 (1956).

[2] The Theory of Search; II. Target Detection. Opns. Res. 4, 503-531 (1956).

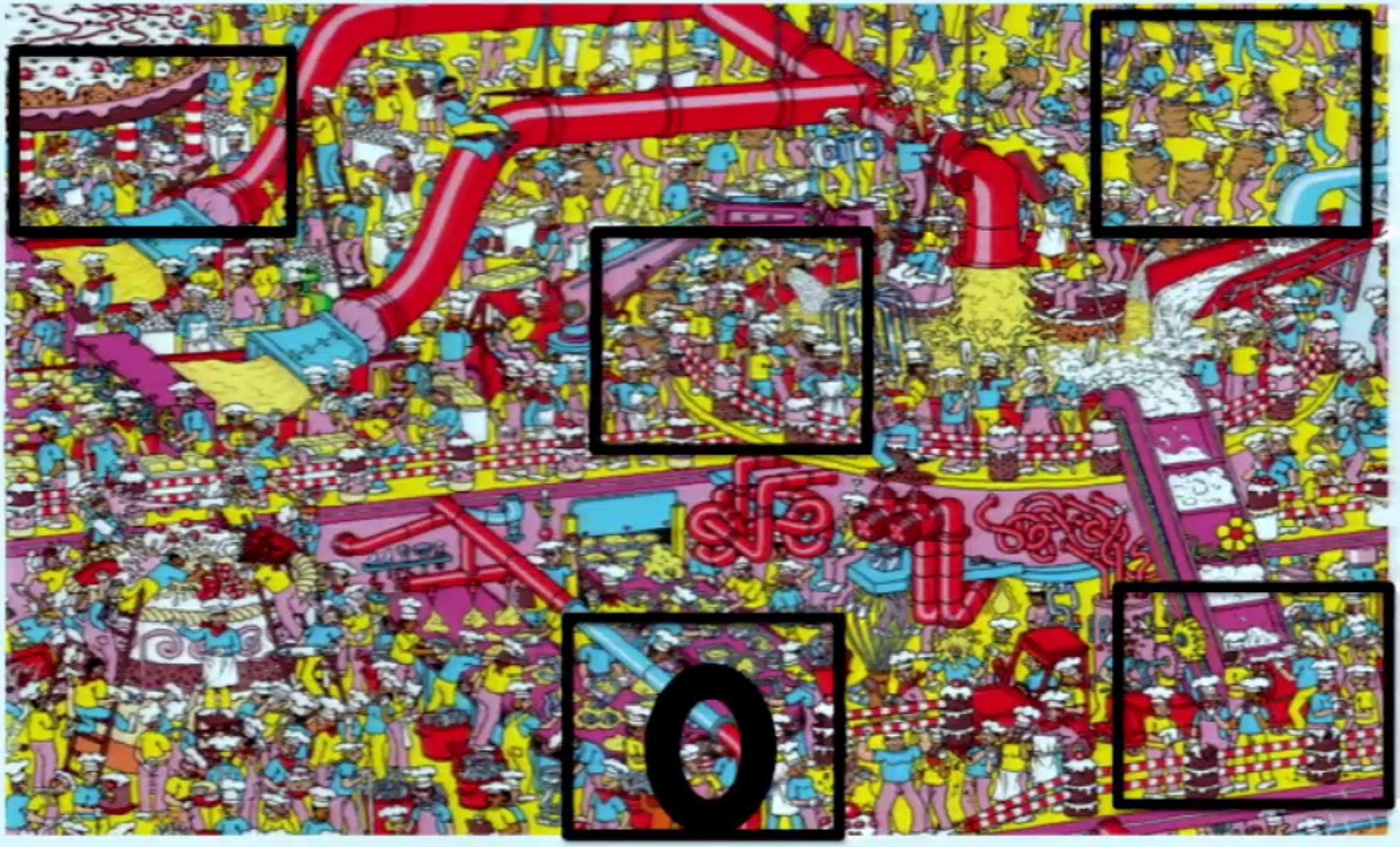
[3] The Theory of Search; III. The Optimum Distribution of Searching Effort. Opns. Res. 5, 613-656 (1957).

Typical search strategy in static environment:



Typical search strategy in static environment

Random Search

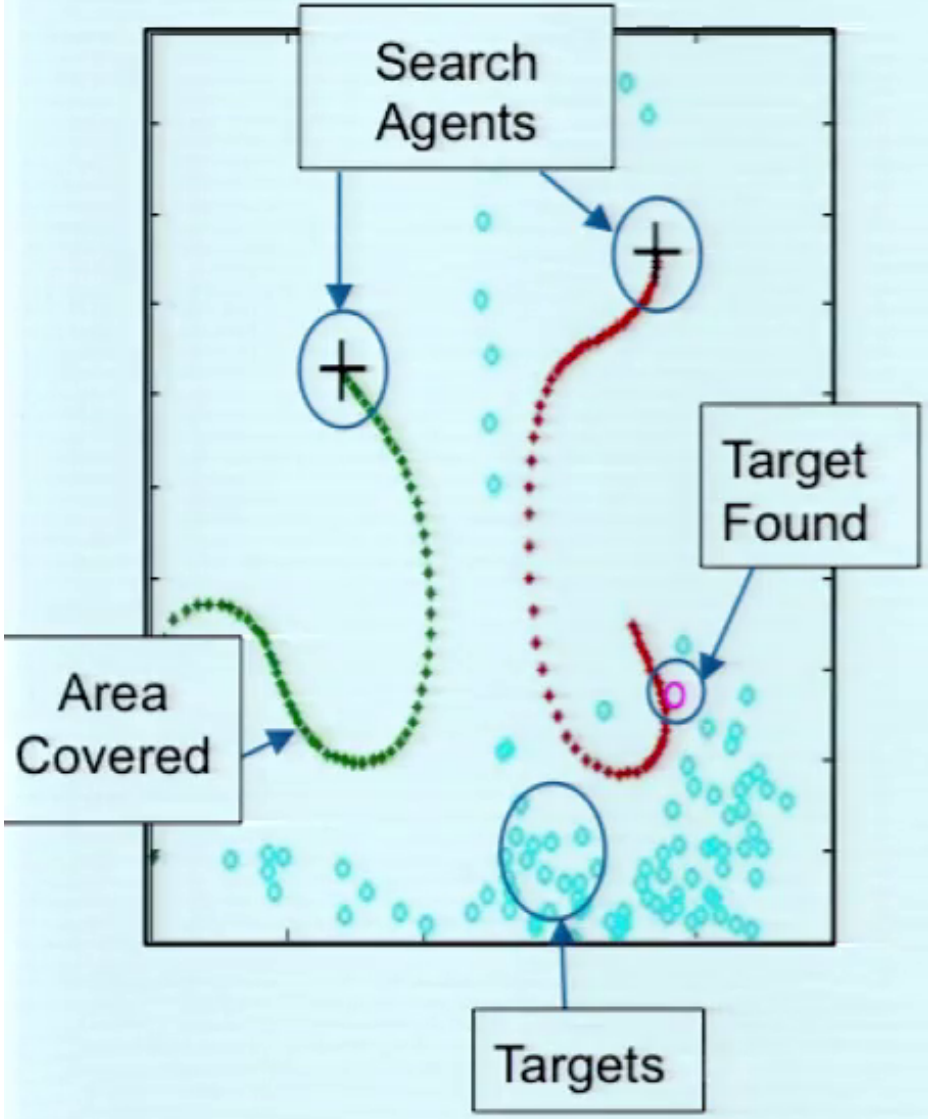


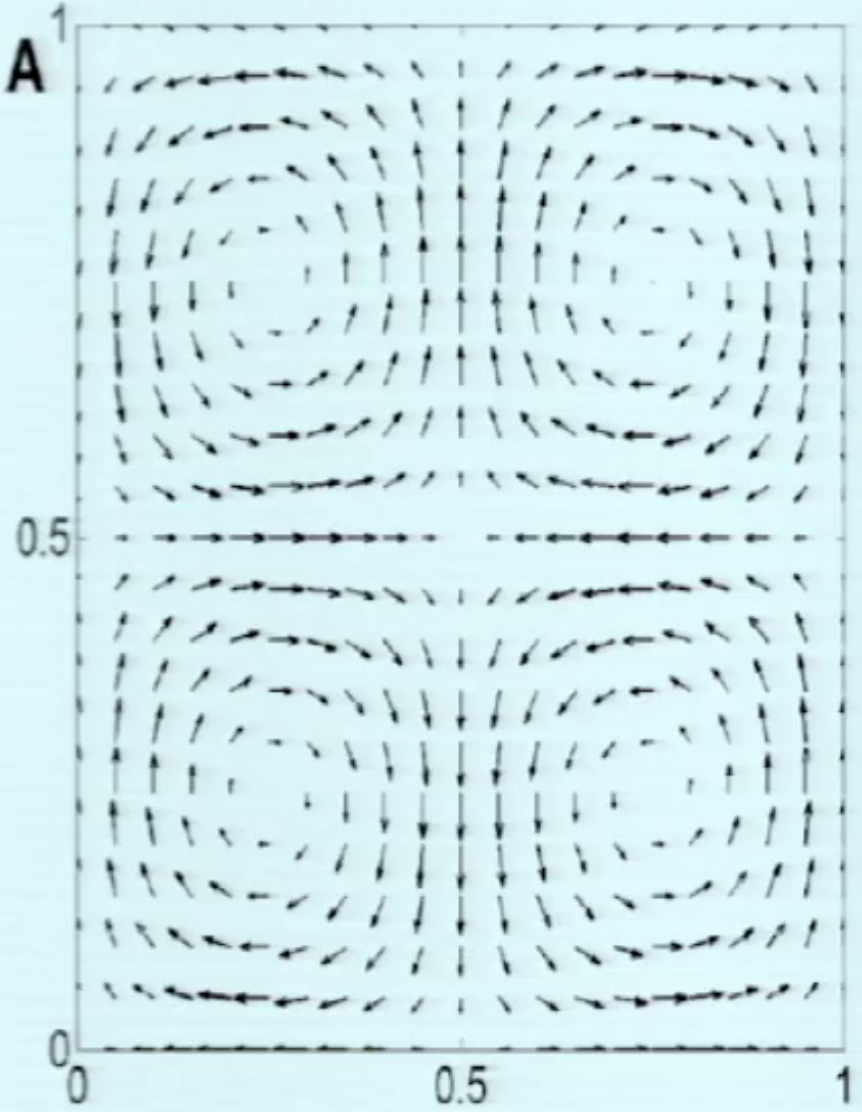
Typical search strategy in static environment

Lawnmower Search

Search of targets moving in a double gyre

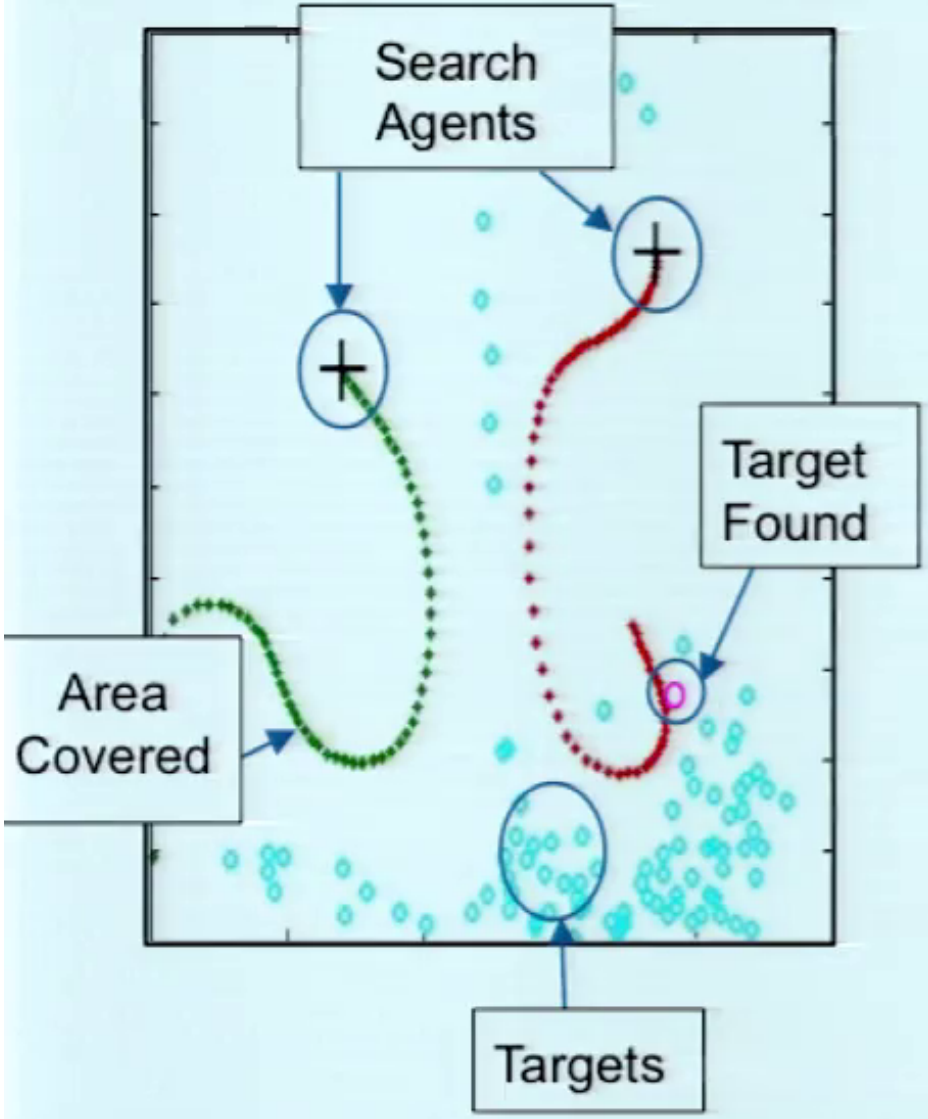
Lawnmower strategy

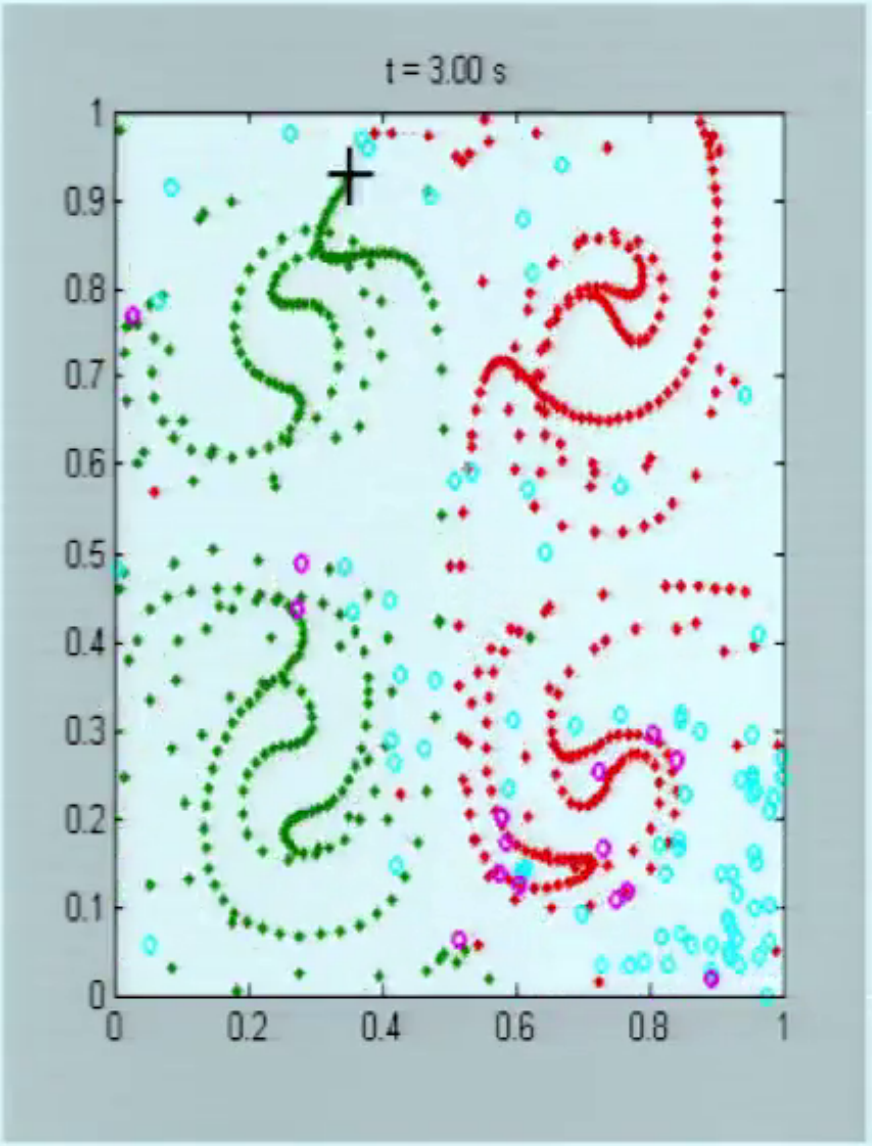




Search of targets moving in a double gyre

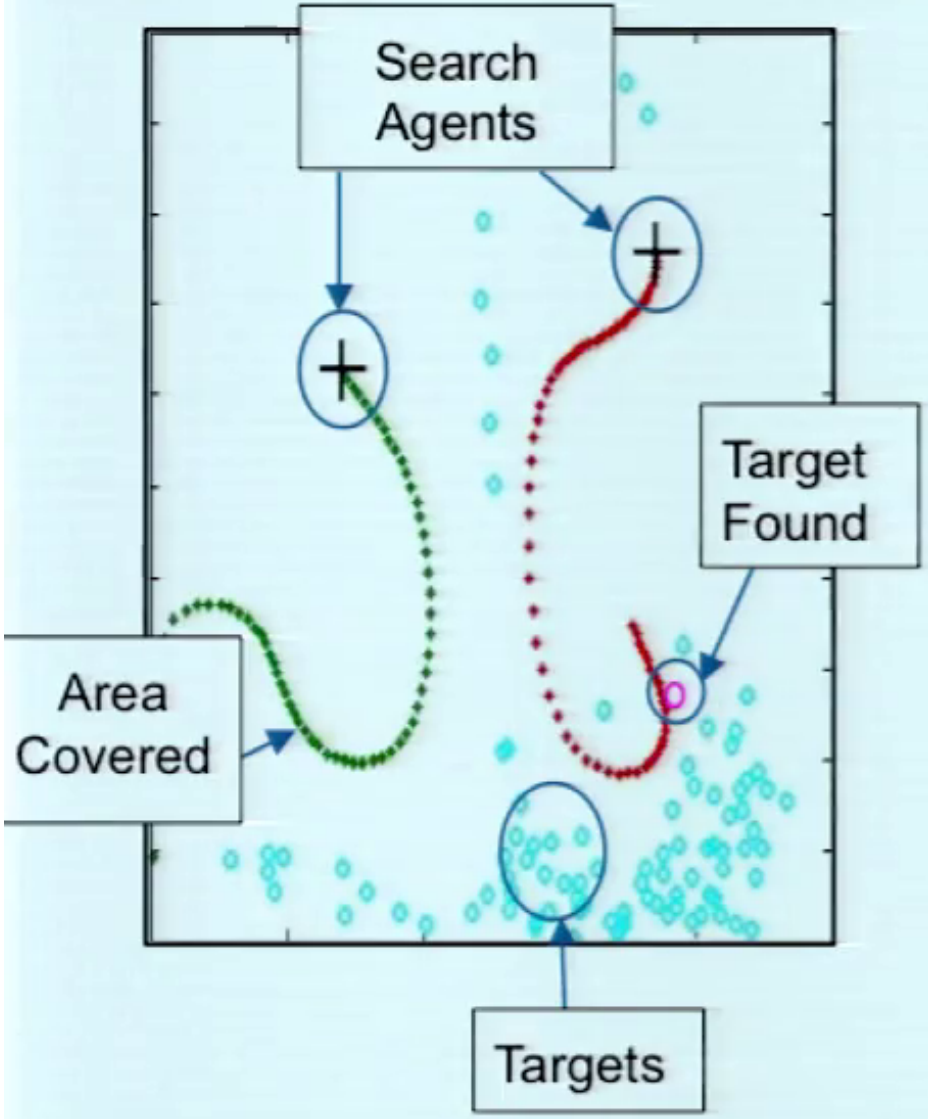
Lawnmower strategy

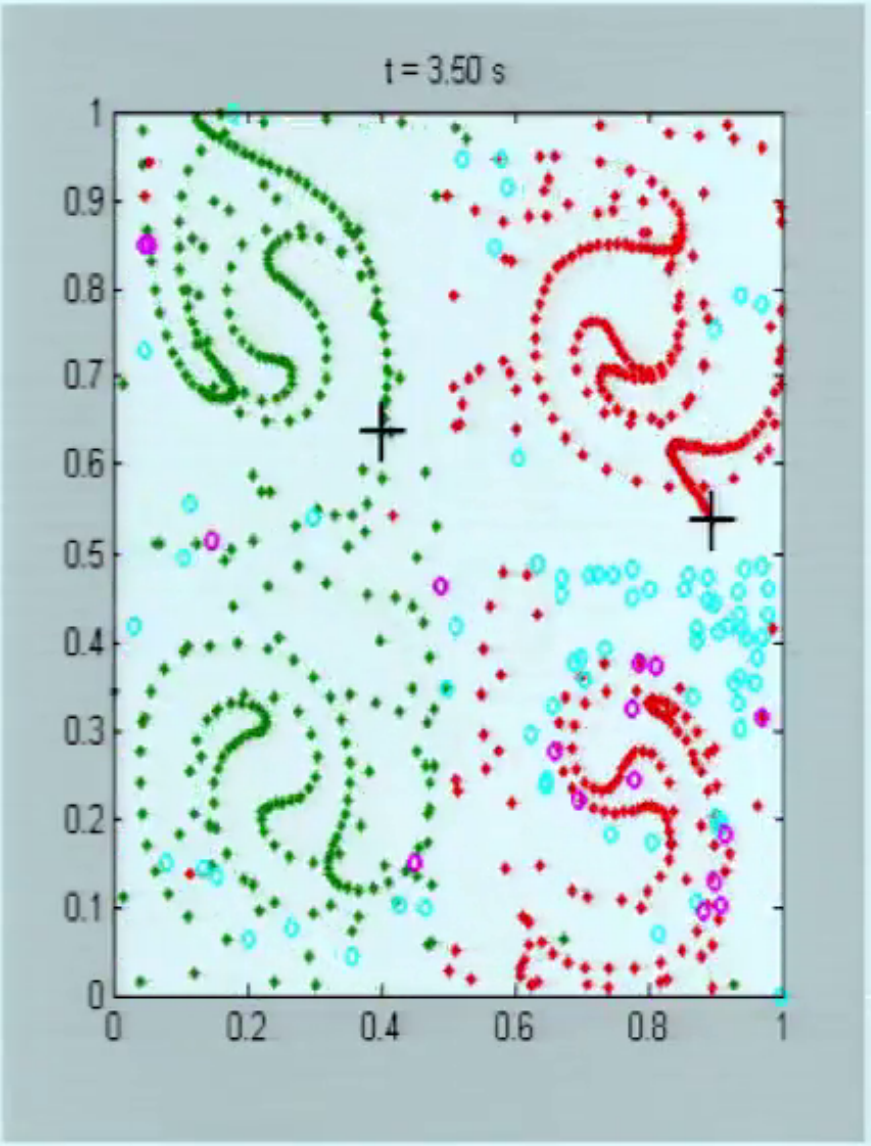




Search of targets moving in a double gyre

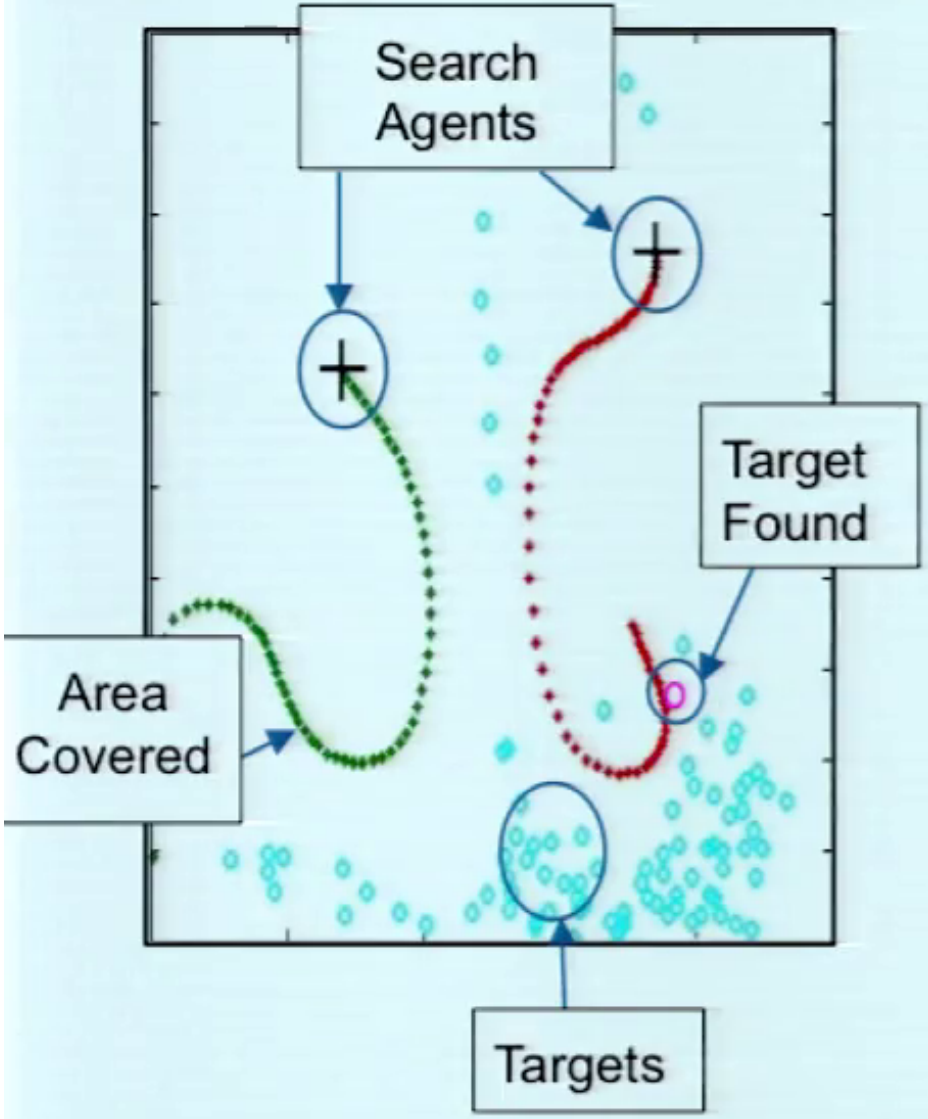
Lawnmower strategy

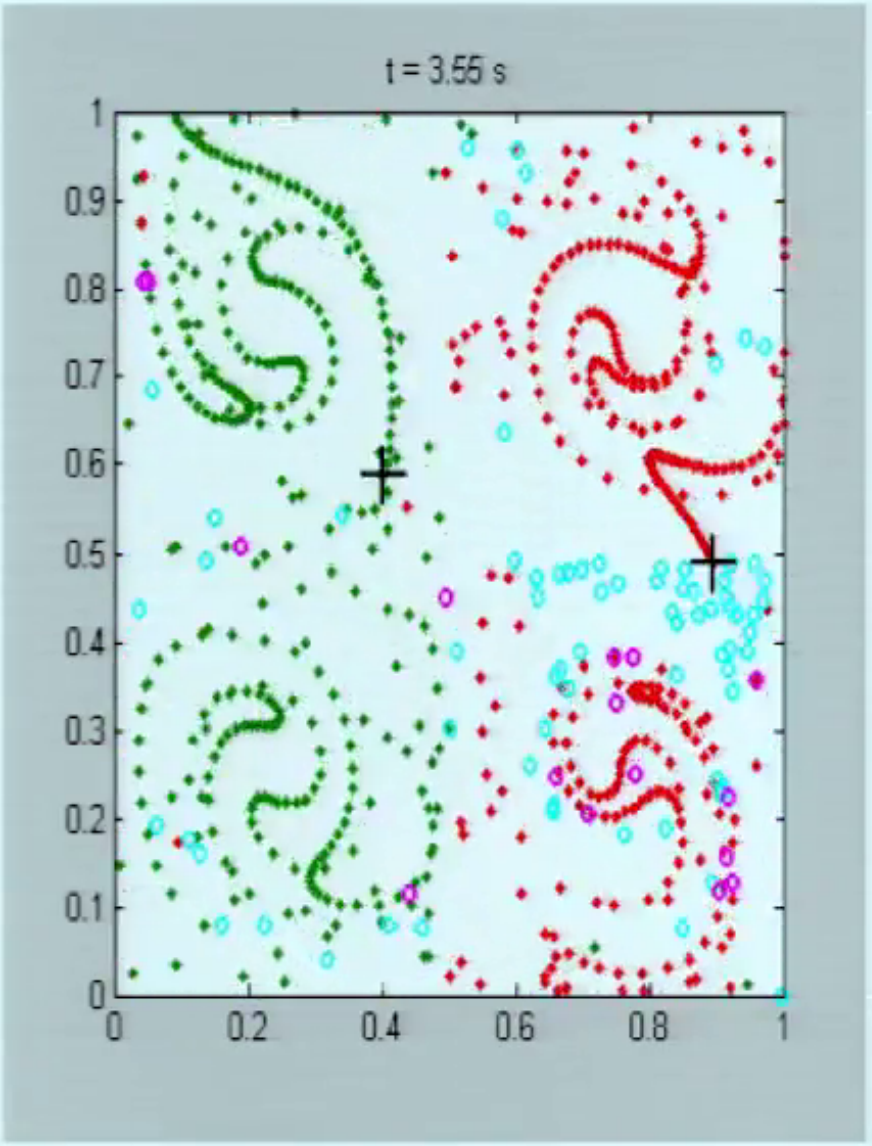




Search of targets moving in a double gyre

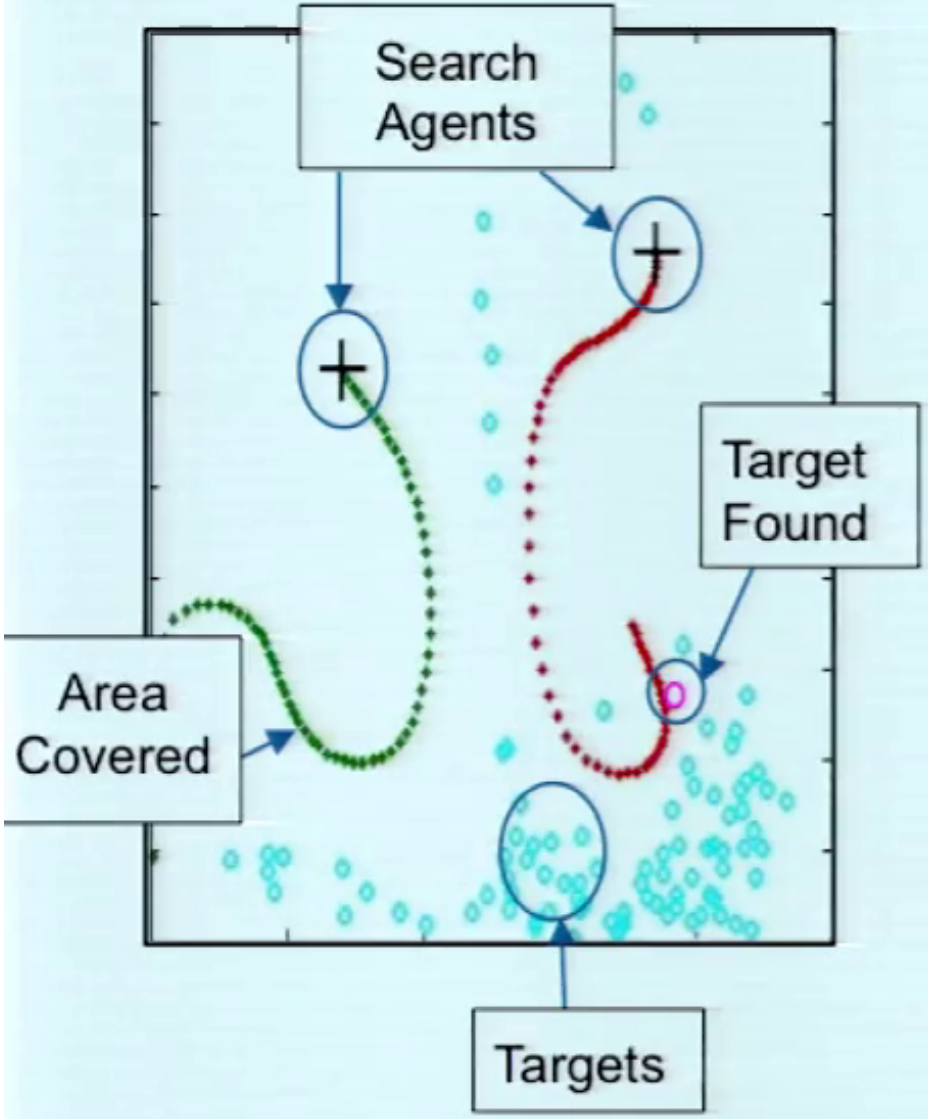
Lawnmower strategy

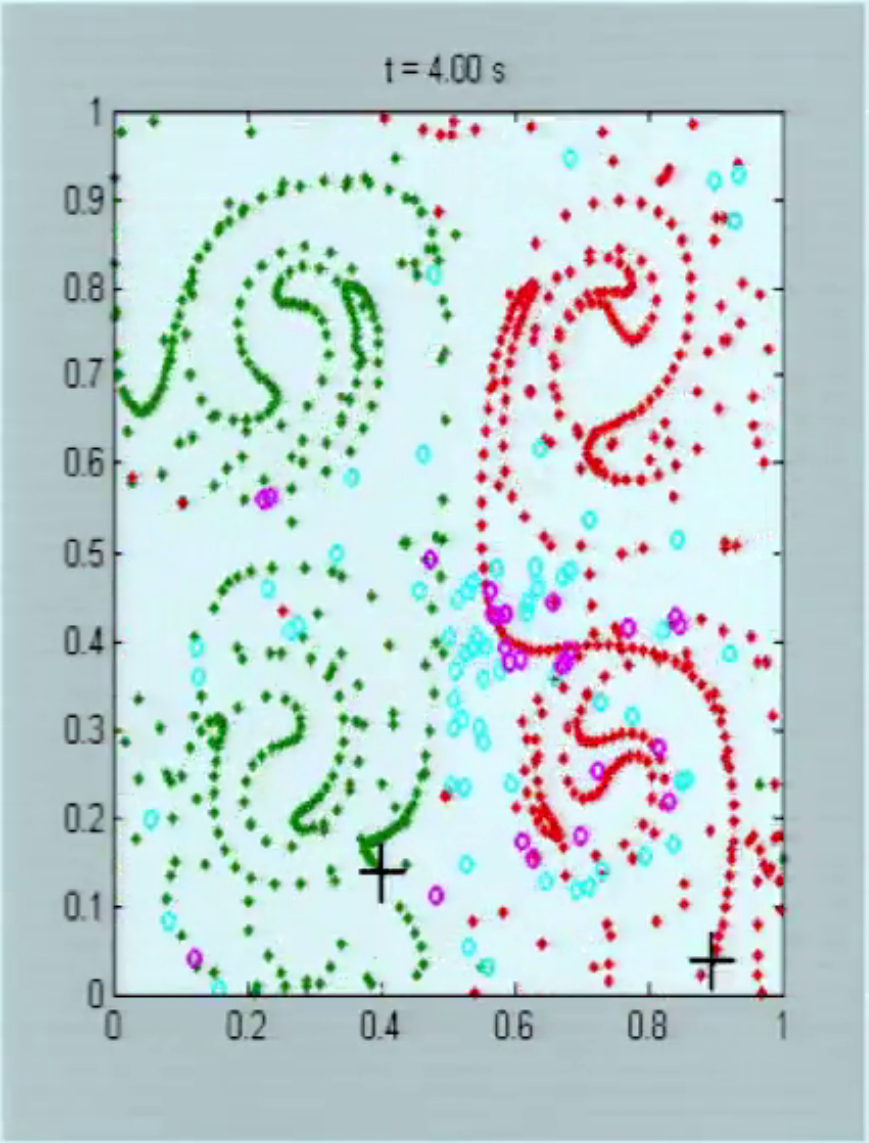




Search of targets moving in a double gyre

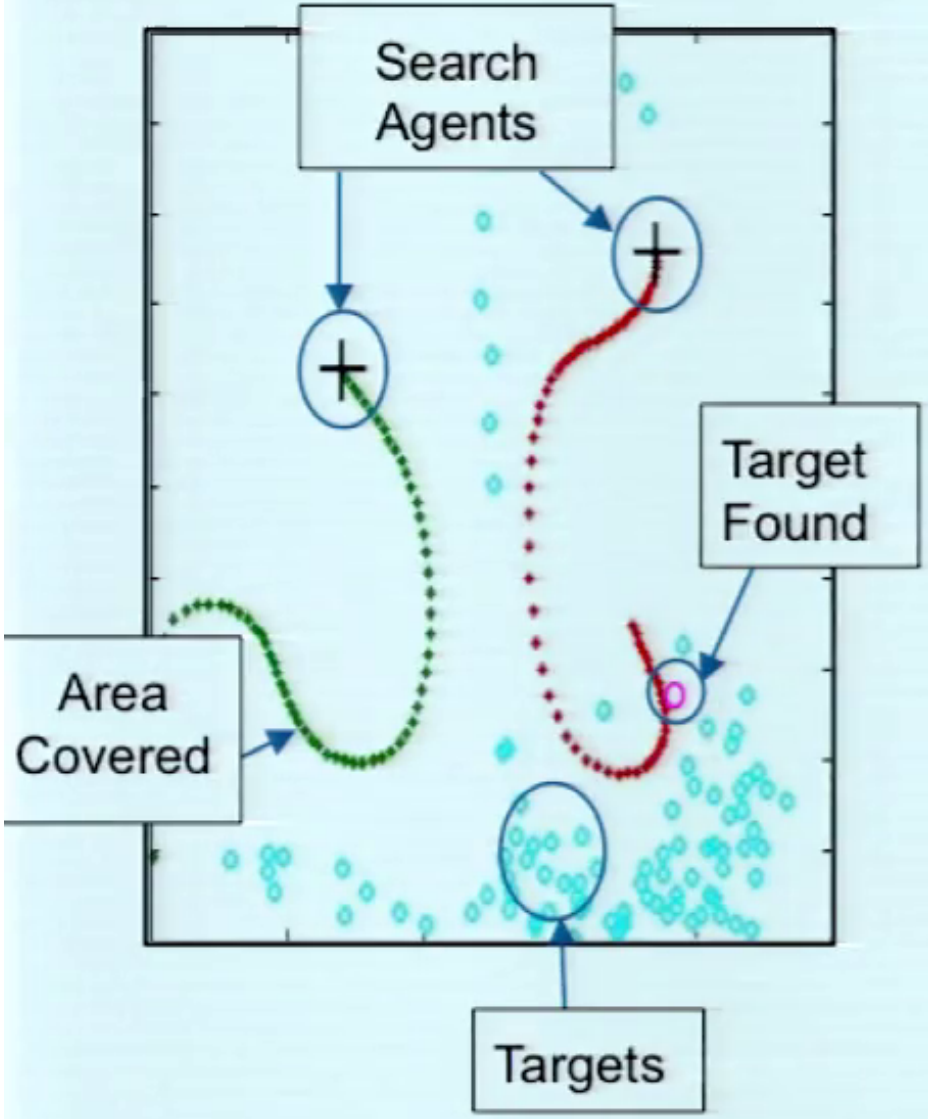
Lawnmower strategy

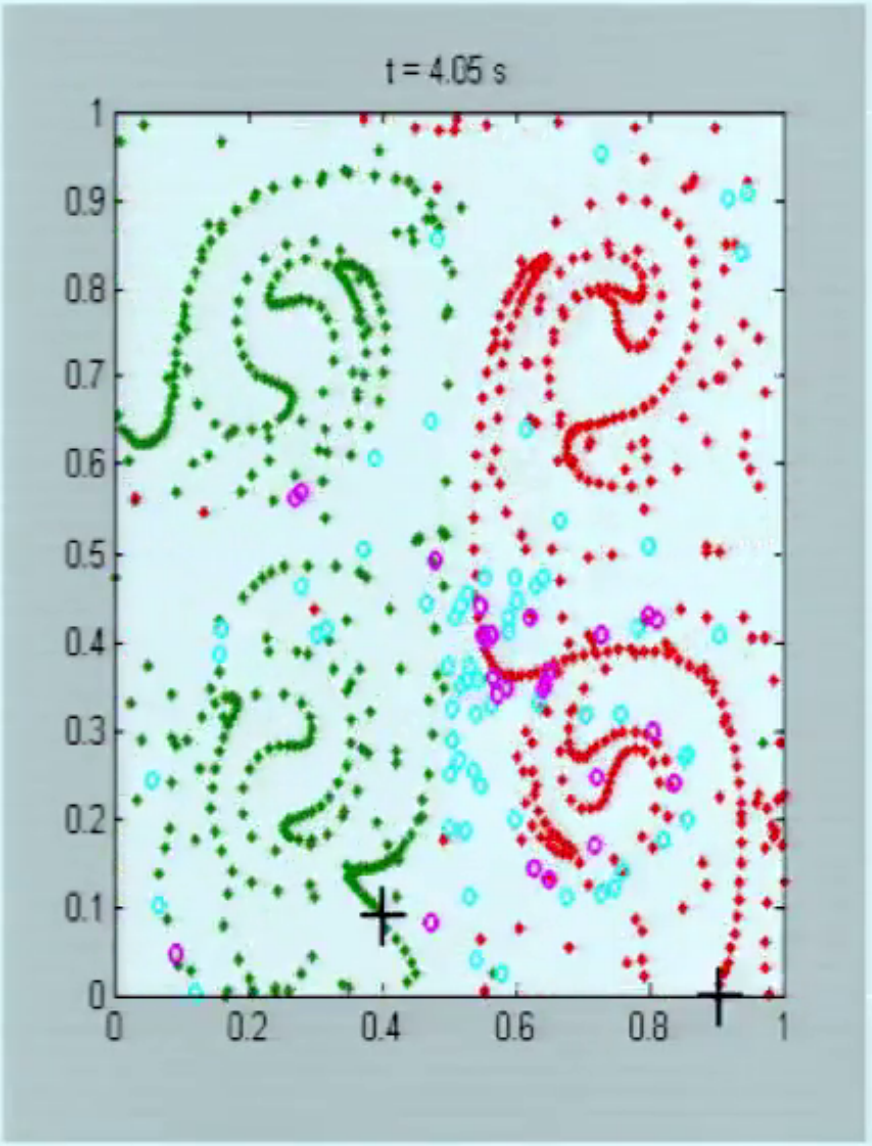




Search of targets moving in a double gyre

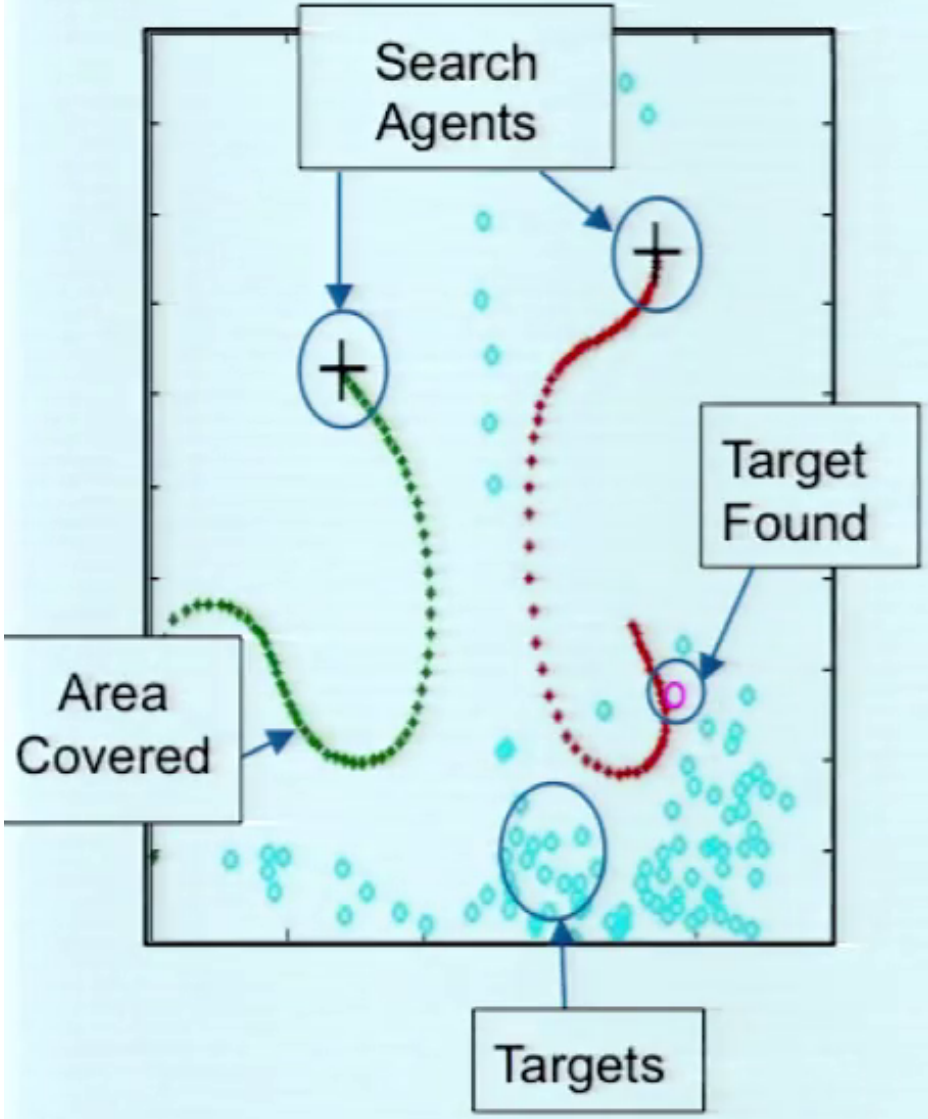
Lawnmower strategy

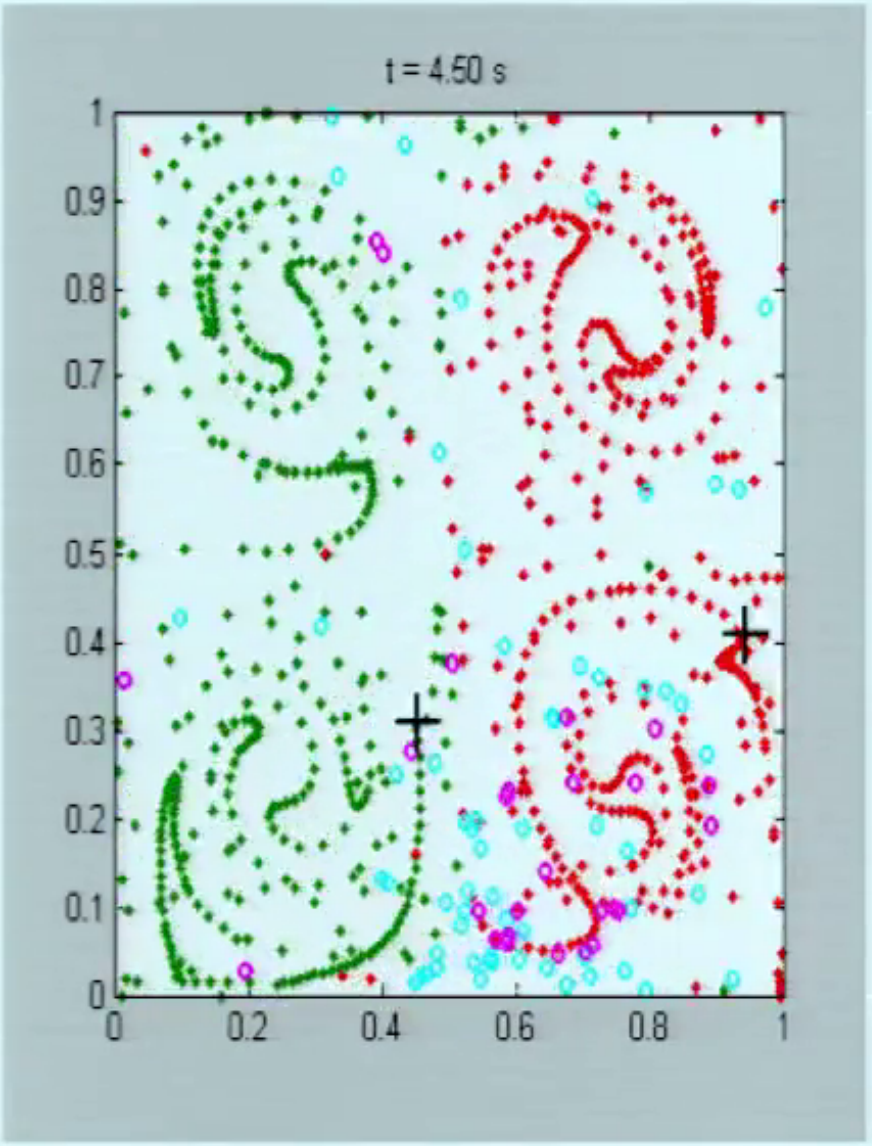




Search of targets moving in a double gyre

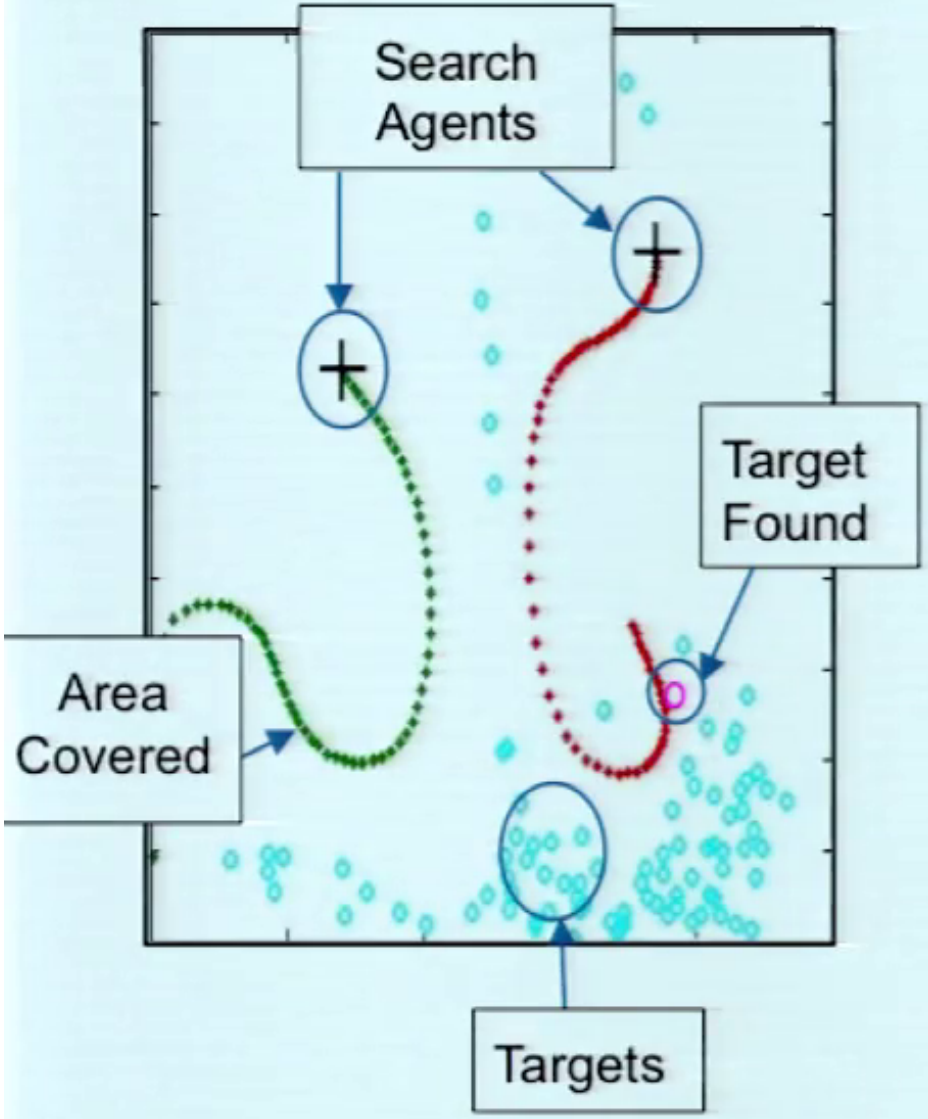
Lawnmower strategy

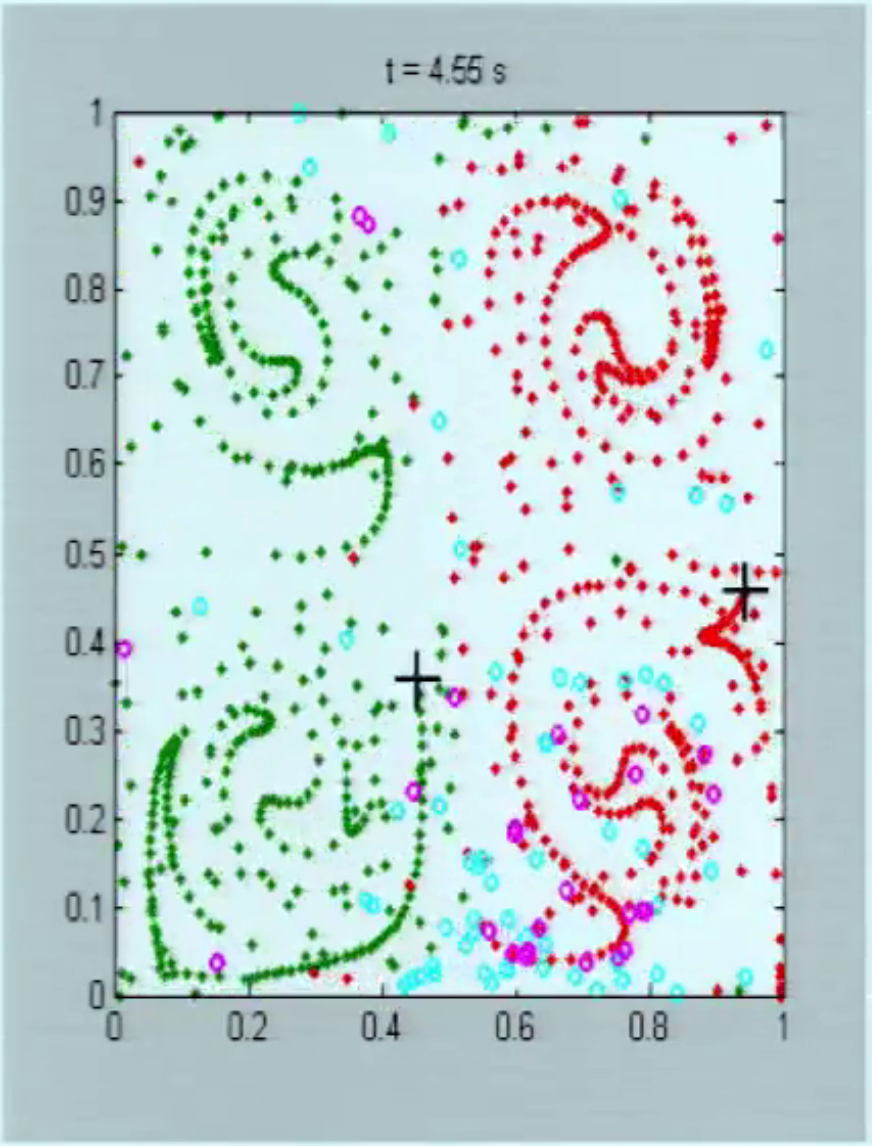




Search of targets moving in a double gyre

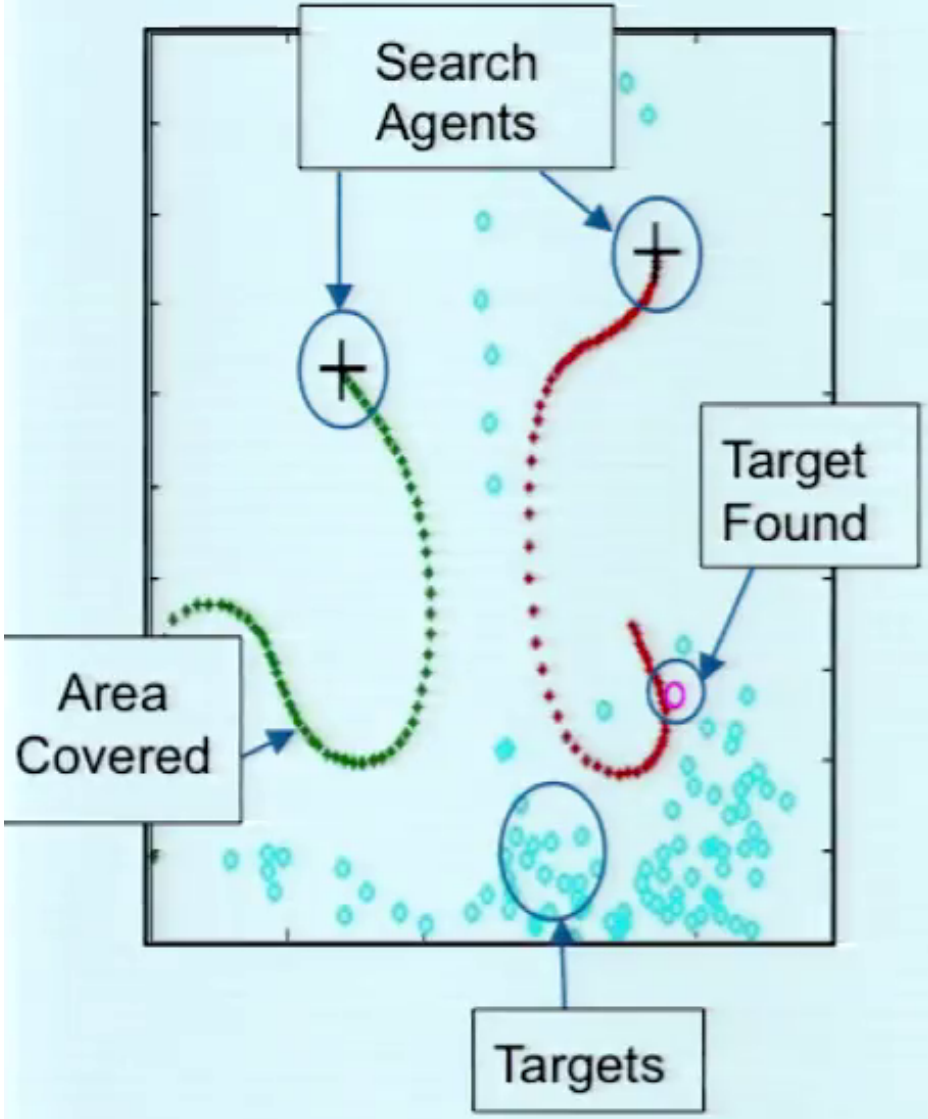
Lawnmower strategy

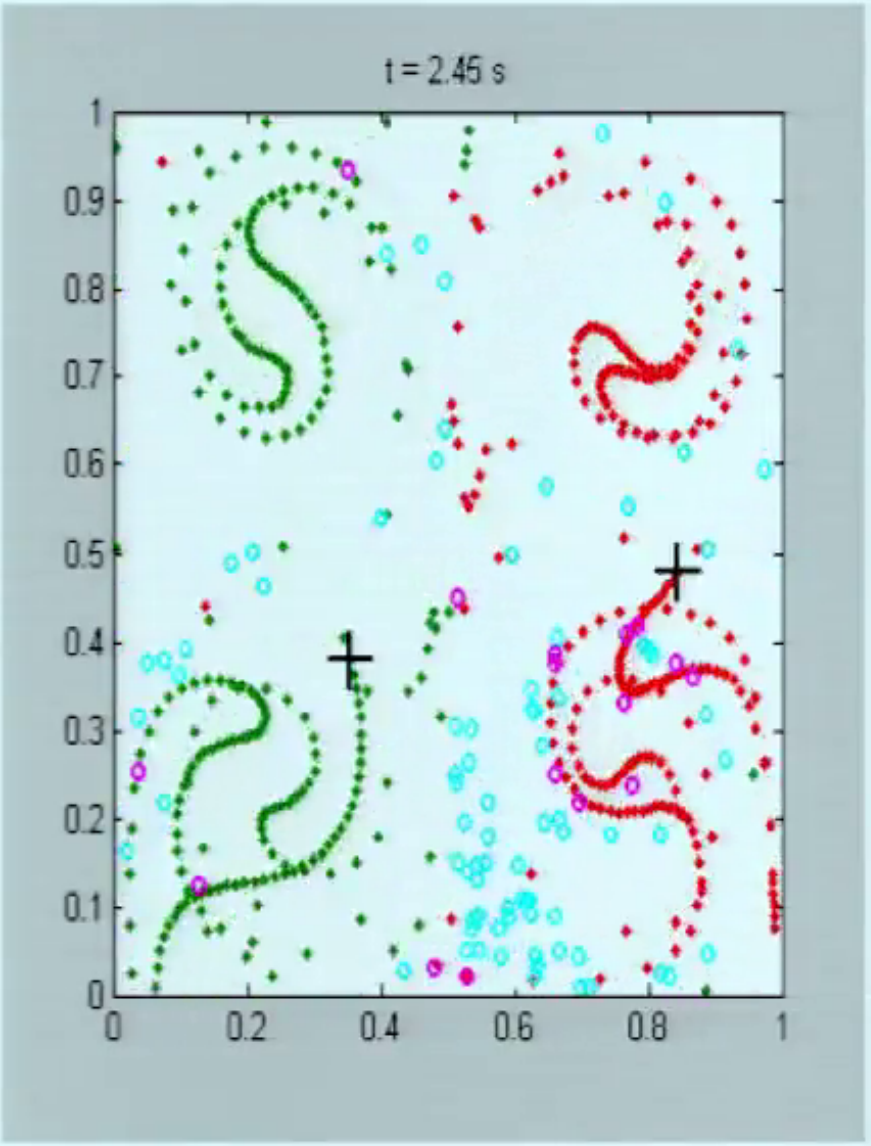




Search of targets moving in a double gyre

Lawnmower strategy





Problem Statement

Model: X(t)=T(X(0),0,t)

Dynamic of initial target density:

$$\mu^t(B(x,r)) = \int_{B(x,r)} d\mu^t = \int_{T^{-1}(.,0,t)(B(x,r))} d\mu^0.$$

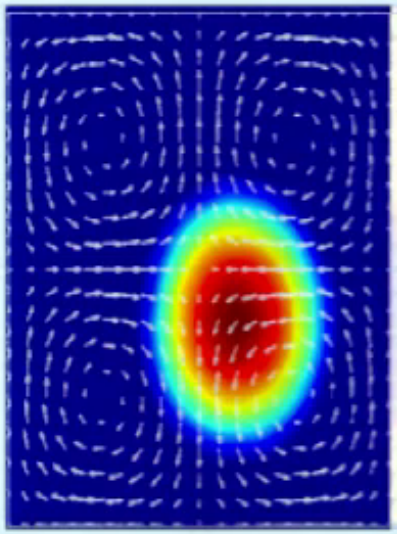
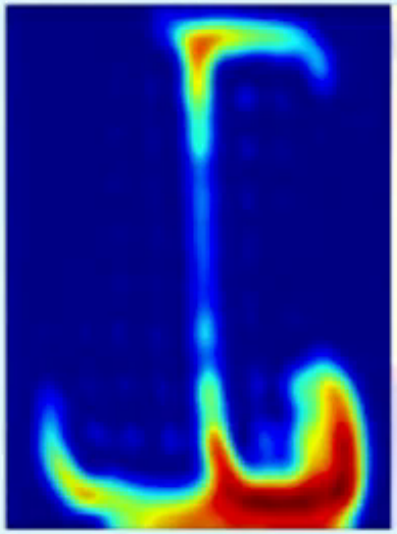
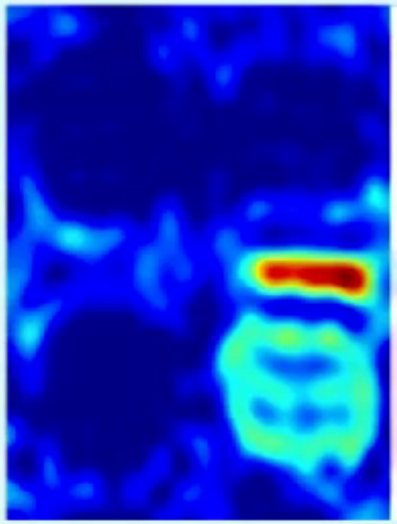
$$v^t(x) = \frac{\mathcal{F}(t,\mu^t(x))}{\int_U \mathcal{F}(t,\mu^t(x))dx}$$

Dynamic of searchers coverage:

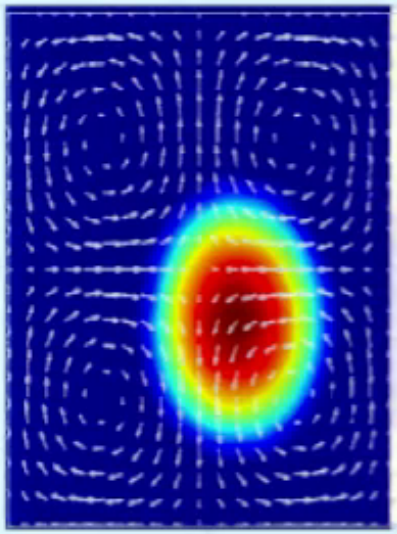
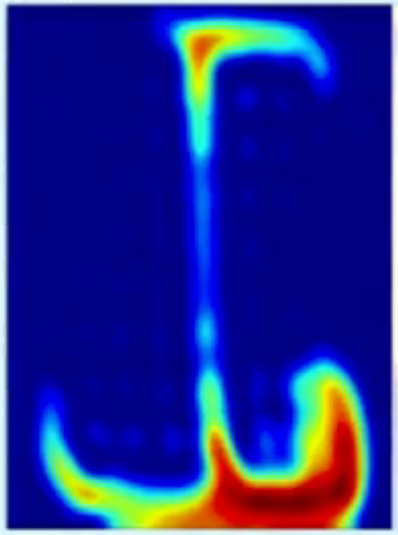
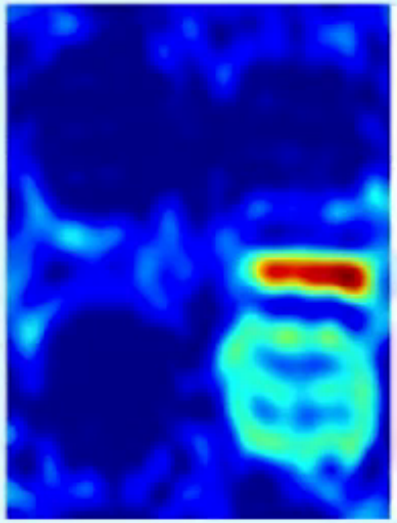
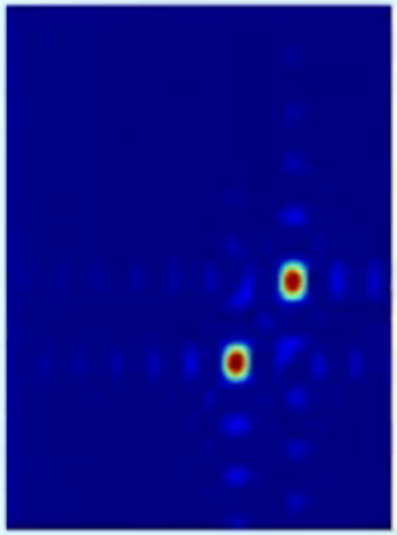
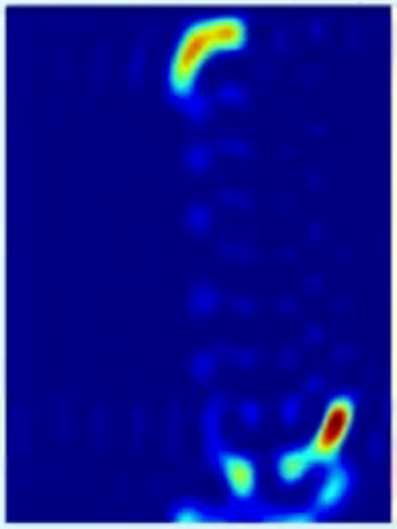
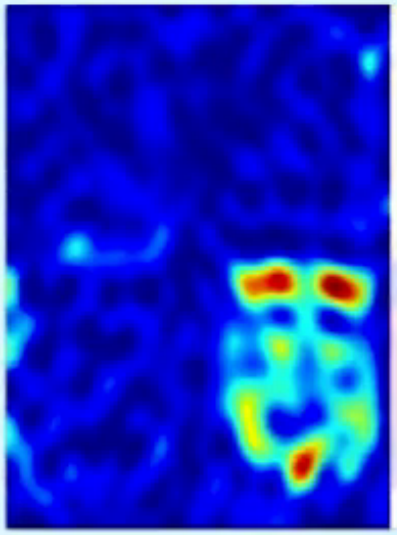
$$C^t(x) = \frac{1}{Nt}\sum_{j=1}^{N}\int_0^t \left[P^{\tau,t}\delta_{x_j(\tau)}\right](x)d\tau,$$

Goal: Minimize the difference between Target Density and Coverage

Dynamical Spectral Multiscale Coverage (DSMC)

	t=0.01s	t=0.61s	t=19.68s
μ^t Density			

Dynamical Spectral Multiscale Coverage (DSMC)

	t=0.01s	t=0.61s	t=19.68s
μ^t Density			
C^t Coverage			

Searching using the Mix-Norm aka Dynamical Spectral Multiscale Coverage (DSMC)

Searcher trajectories:

- Minimize instantaneously the measure of difference between coverage and density
  - Sobolev space norm of negative index (s=(n+1)/2)
  - Weight on the energy contained in various spectral modes.
  - Favors the large spatial scales as long as their energy is large

$$\phi^2(t) = \|C^t - v^t\|_{H^{-s}}^2 = \sum_{K \in \mathbb{Z}^{*n}} \Lambda_k |s_k(t)|^2,$$

$$\text{where } s_k(t) = c_k(t) - v_k(t), \quad \Lambda_k = \frac{1}{(1+\|k\|^2)^s}$$

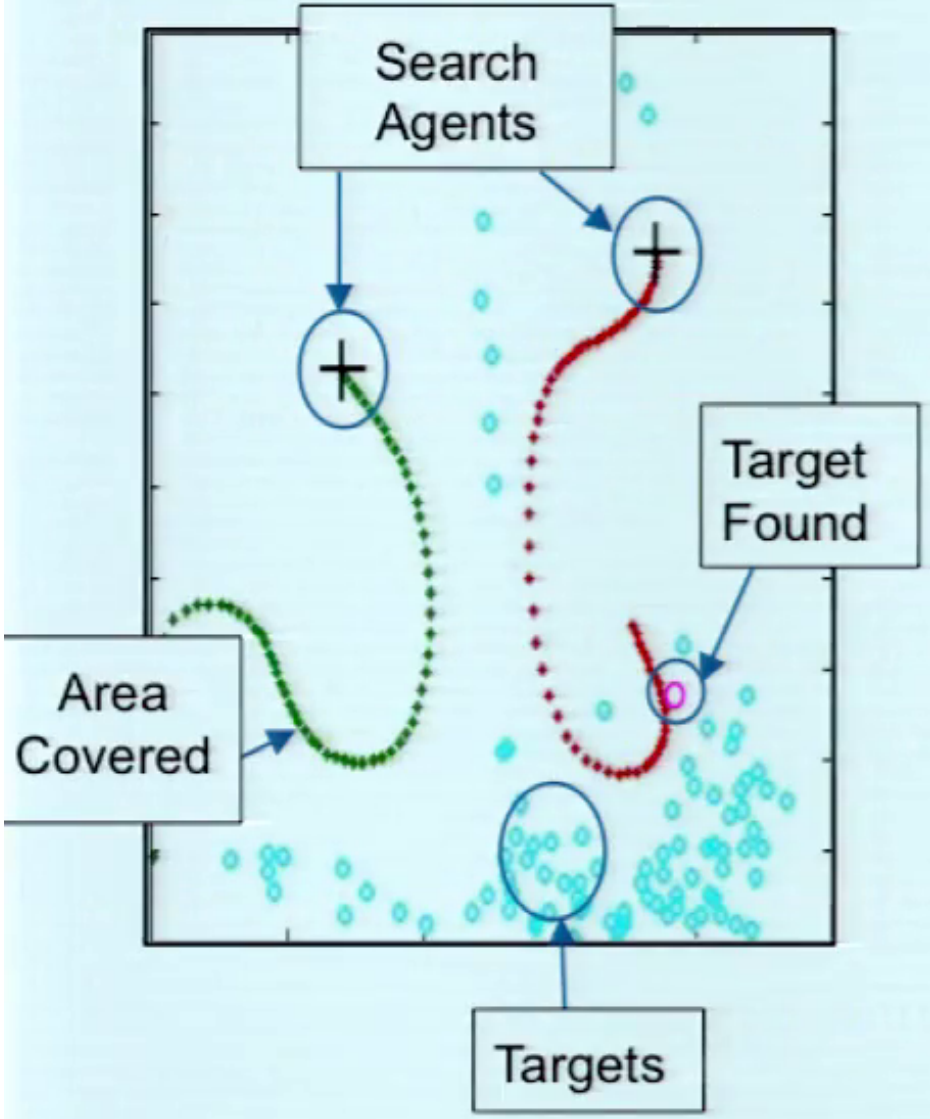
$$c_k(t) = \langle C^t, f_k \rangle \text{ and } v_k(t) = \langle v^t, f_k \rangle.$$

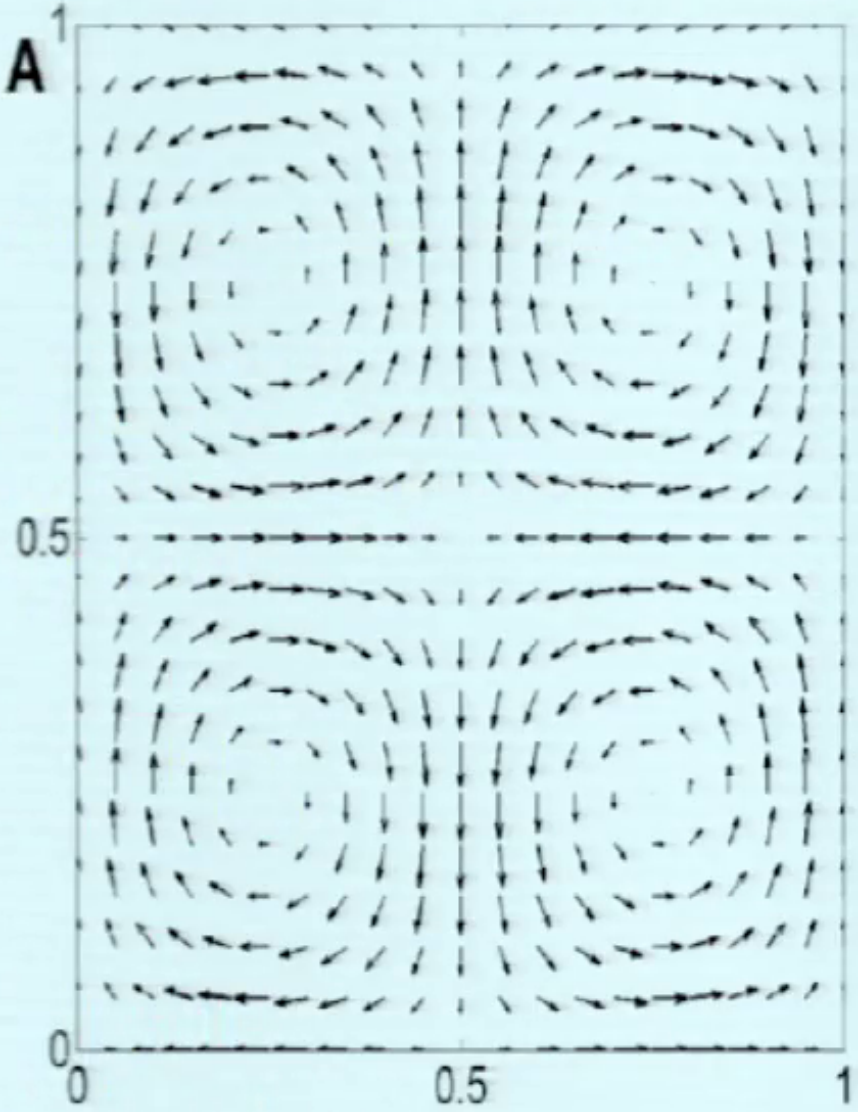
- Simple feedback control law: $u_j^*(t) = -u_{max} \frac{B_j}{\|B_j(t)\|_2}$,

$$\text{where } B_j(t) = \sum_K \Lambda_k S_k(t) \nabla f_k(x_j(t)).$$

Search of targets moving in a double gyre

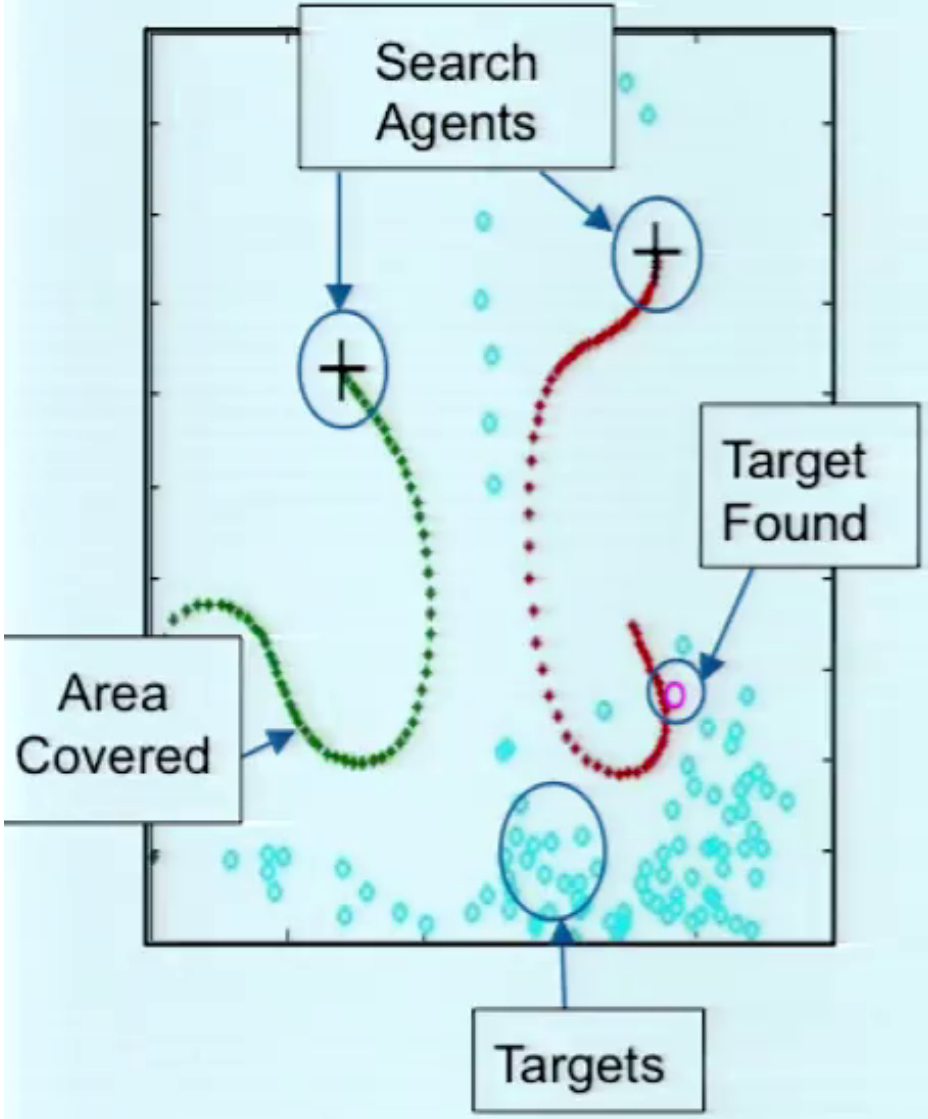
DSMC strategy

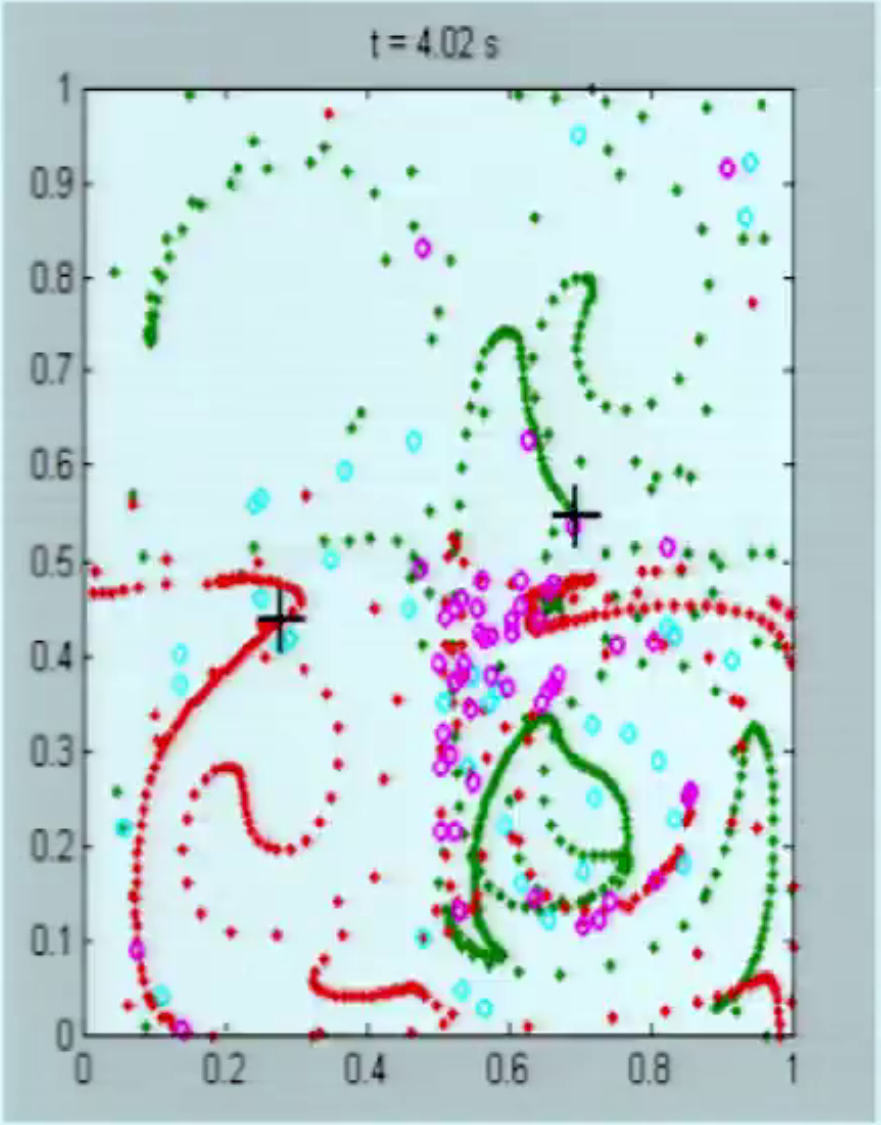




Search of targets moving in a double gyre

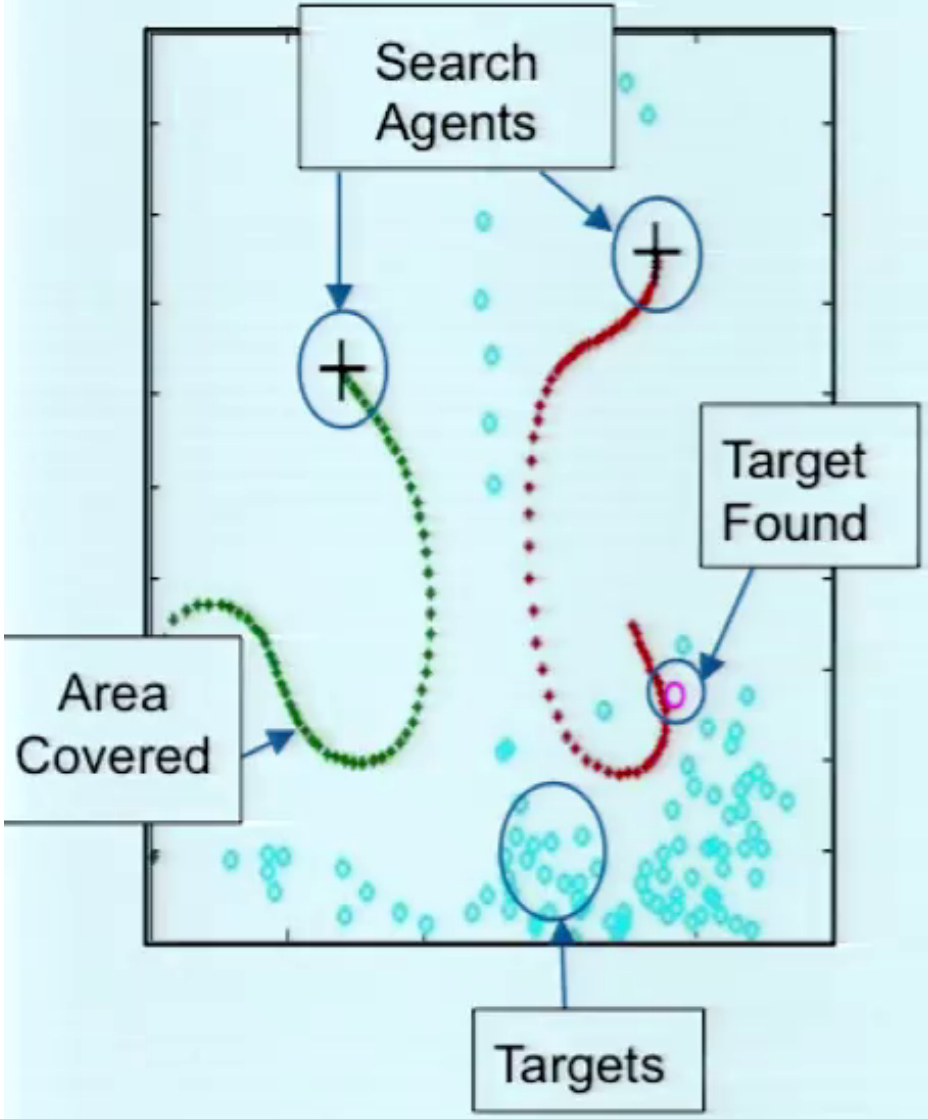
DSMC strategy

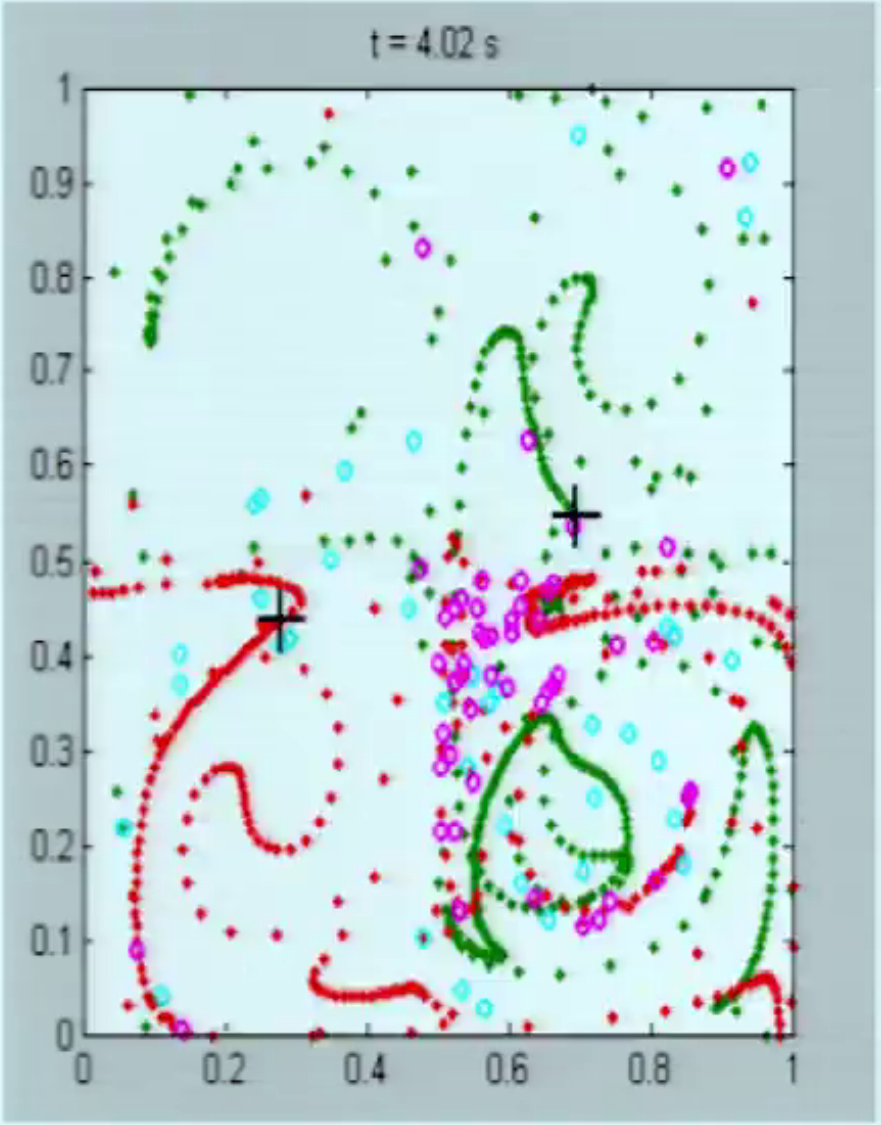




Search of targets moving in a double gyre

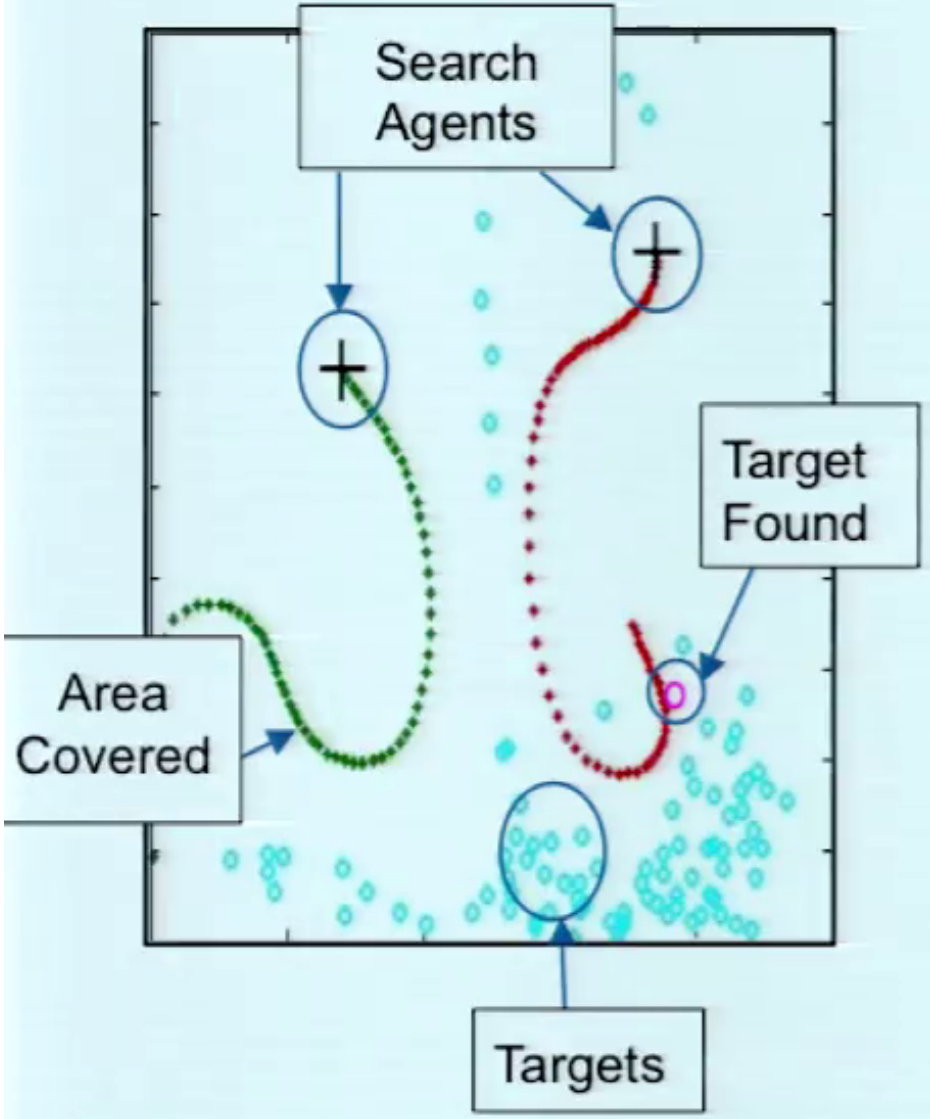
DSMC strategy

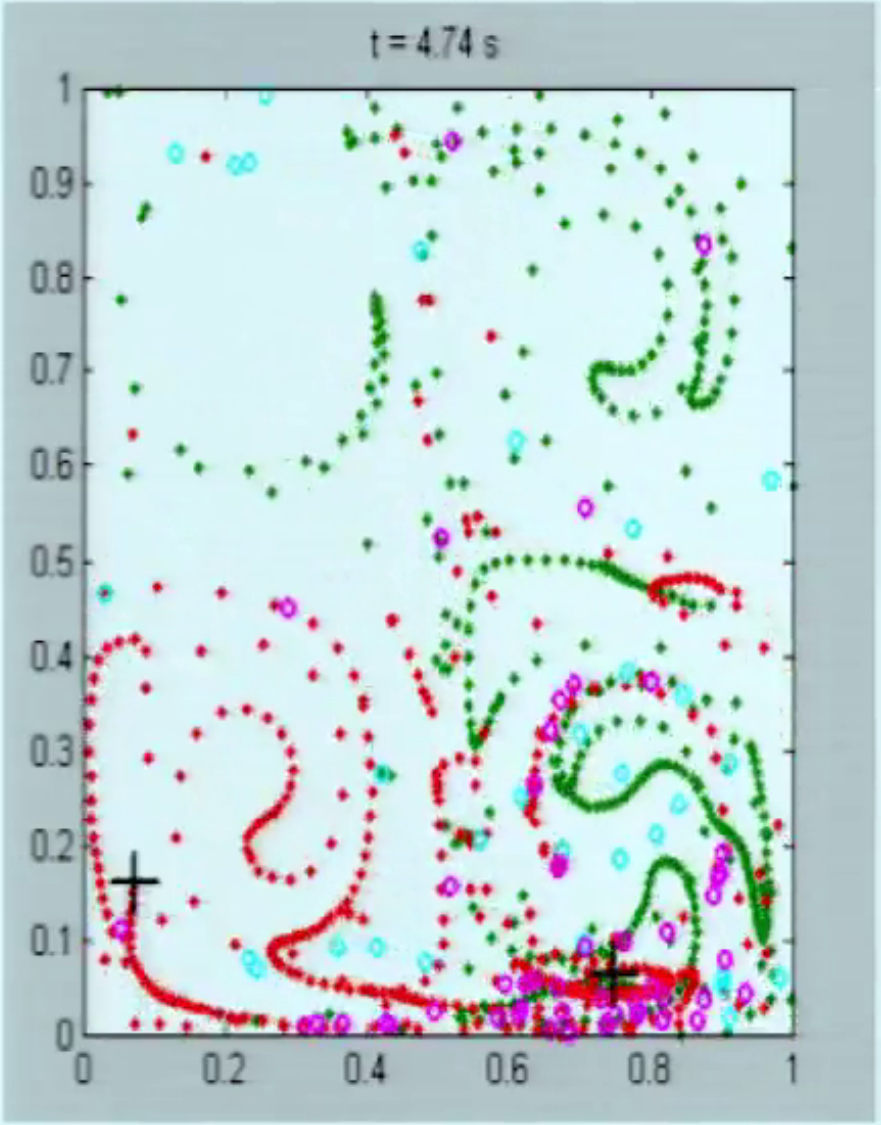




Search of targets moving in a double gyre

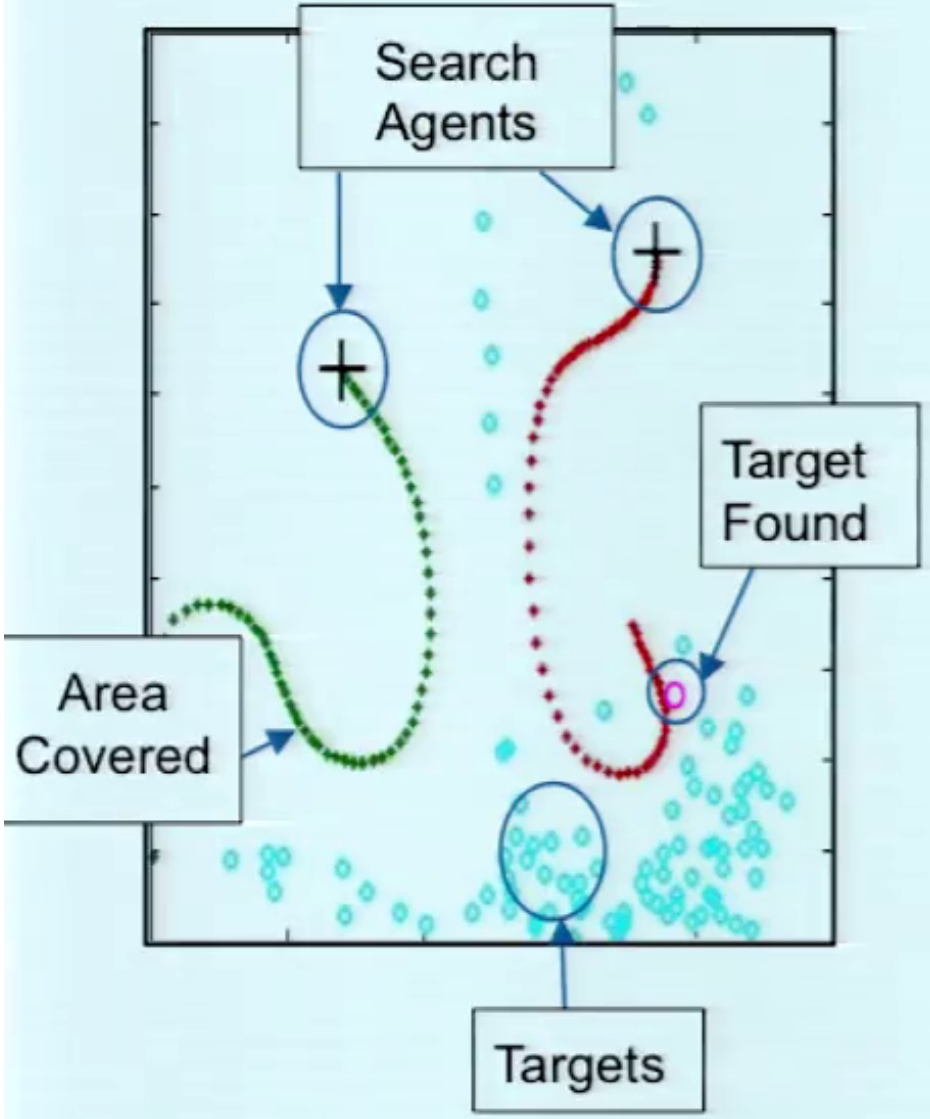
DSMC strategy

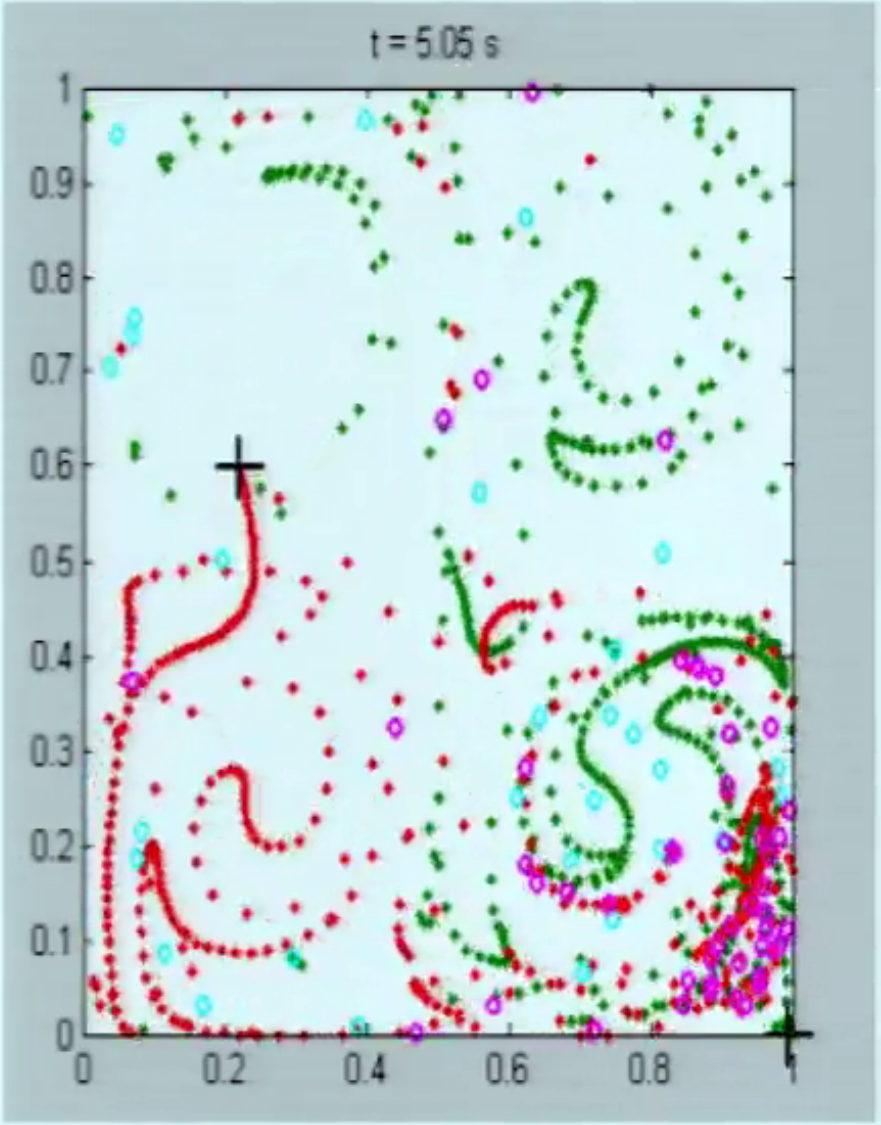




Search of targets moving in a double gyre

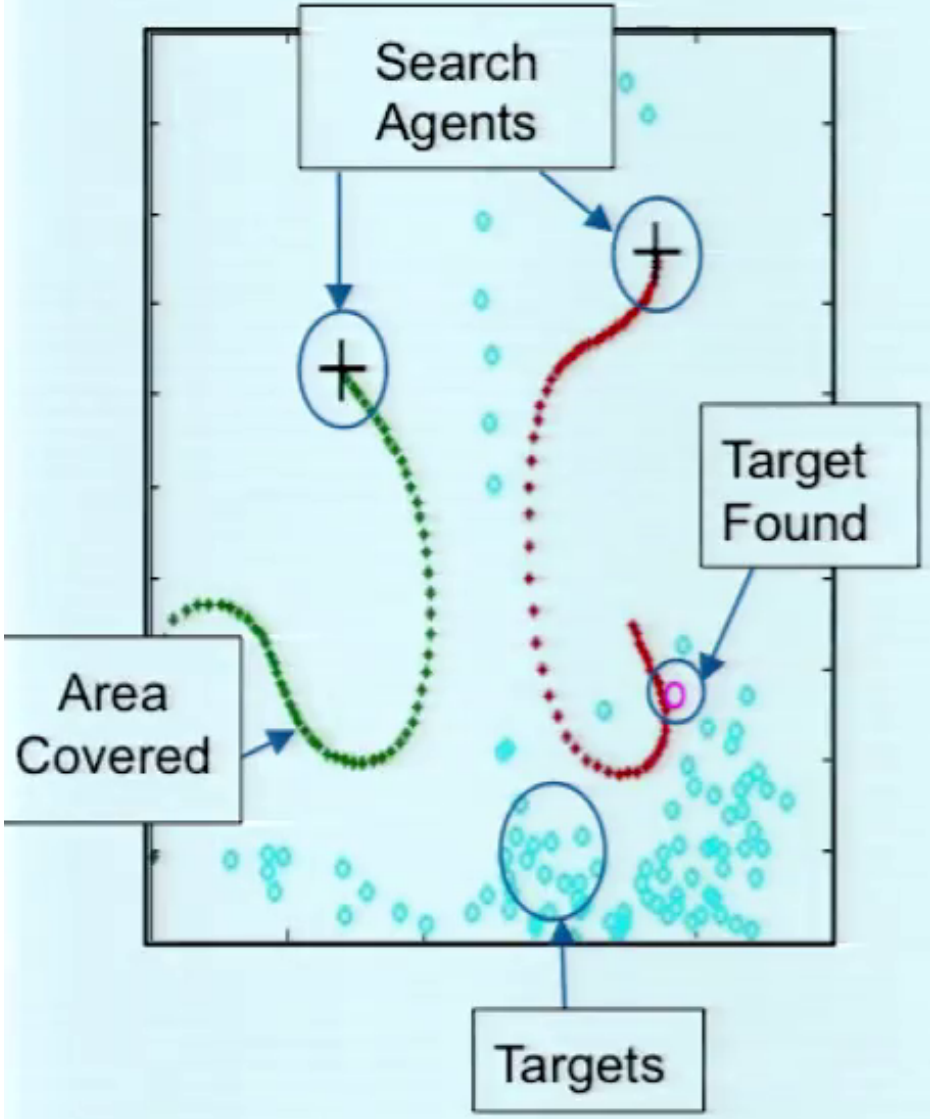
DSMC strategy

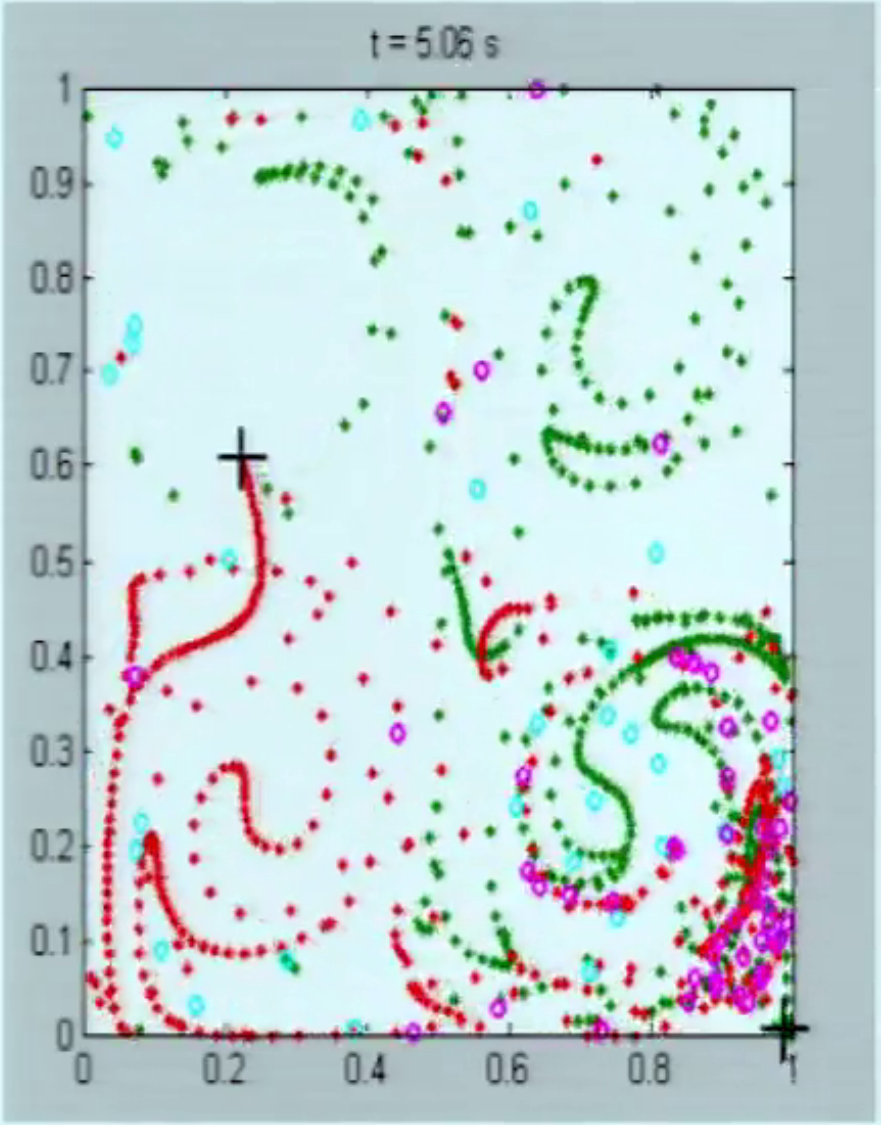




Search of targets moving in a double gyre

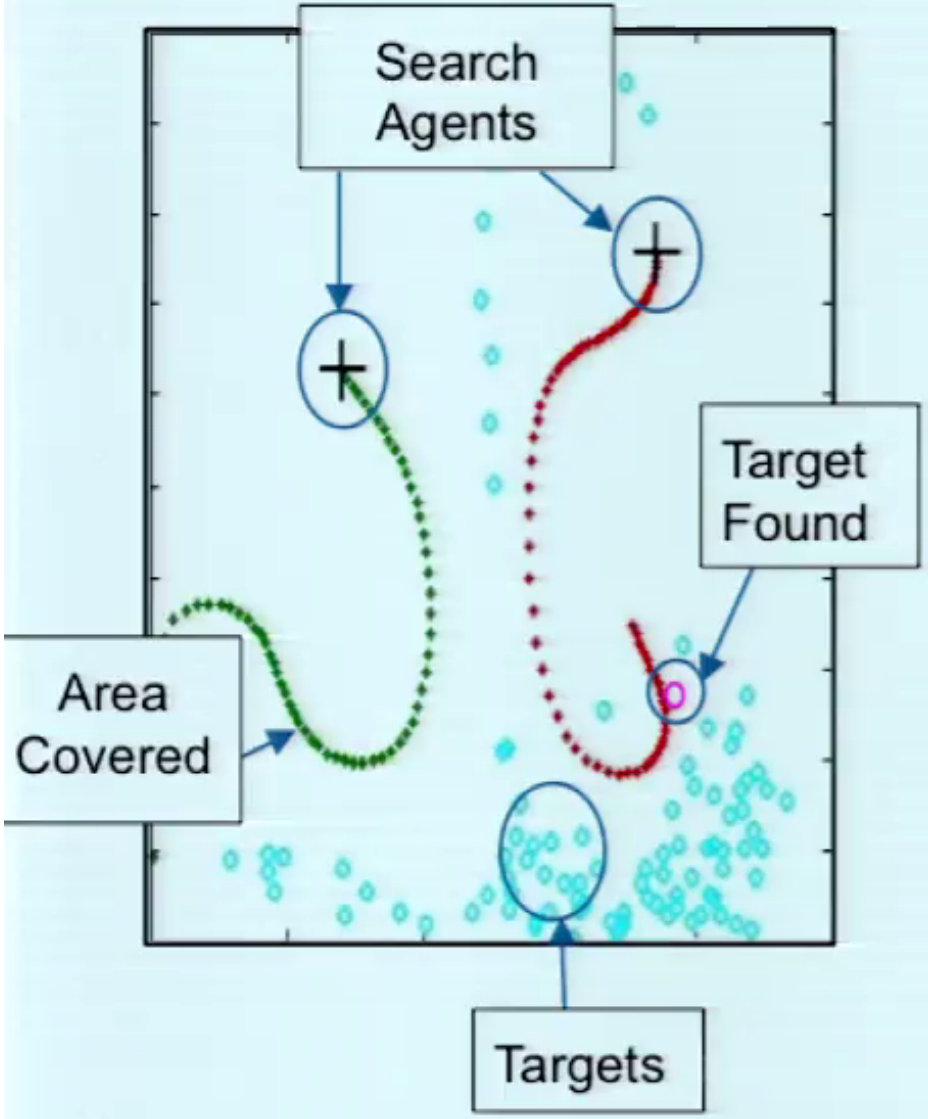
DSMC strategy

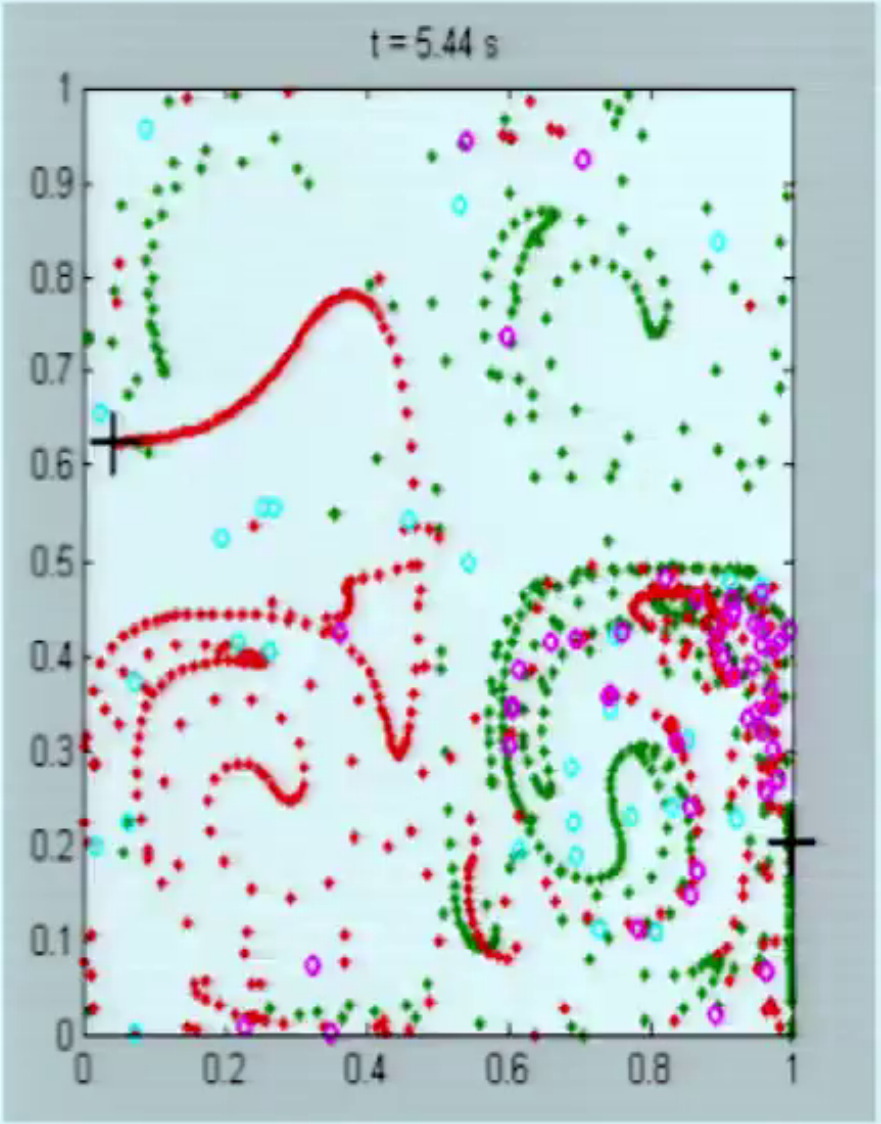




Search of targets moving in a double gyre

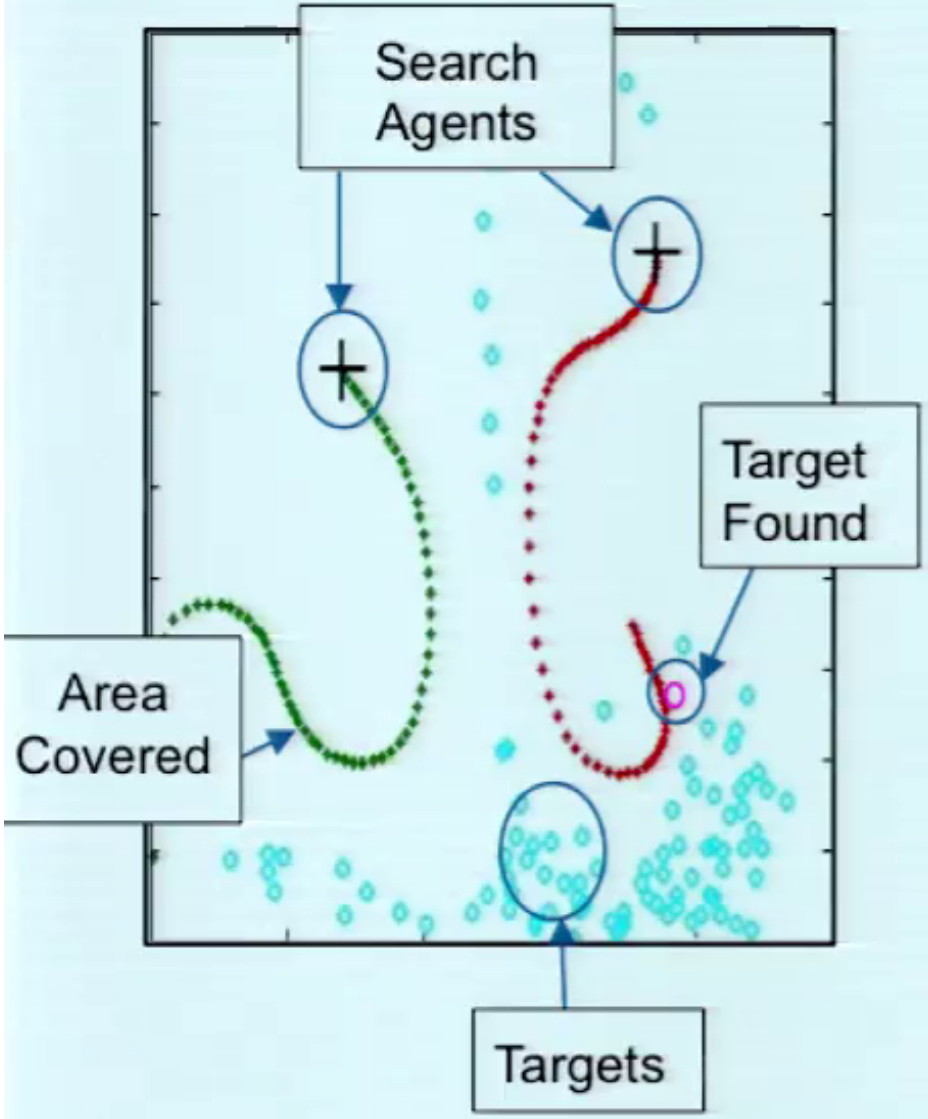
DSMC strategy

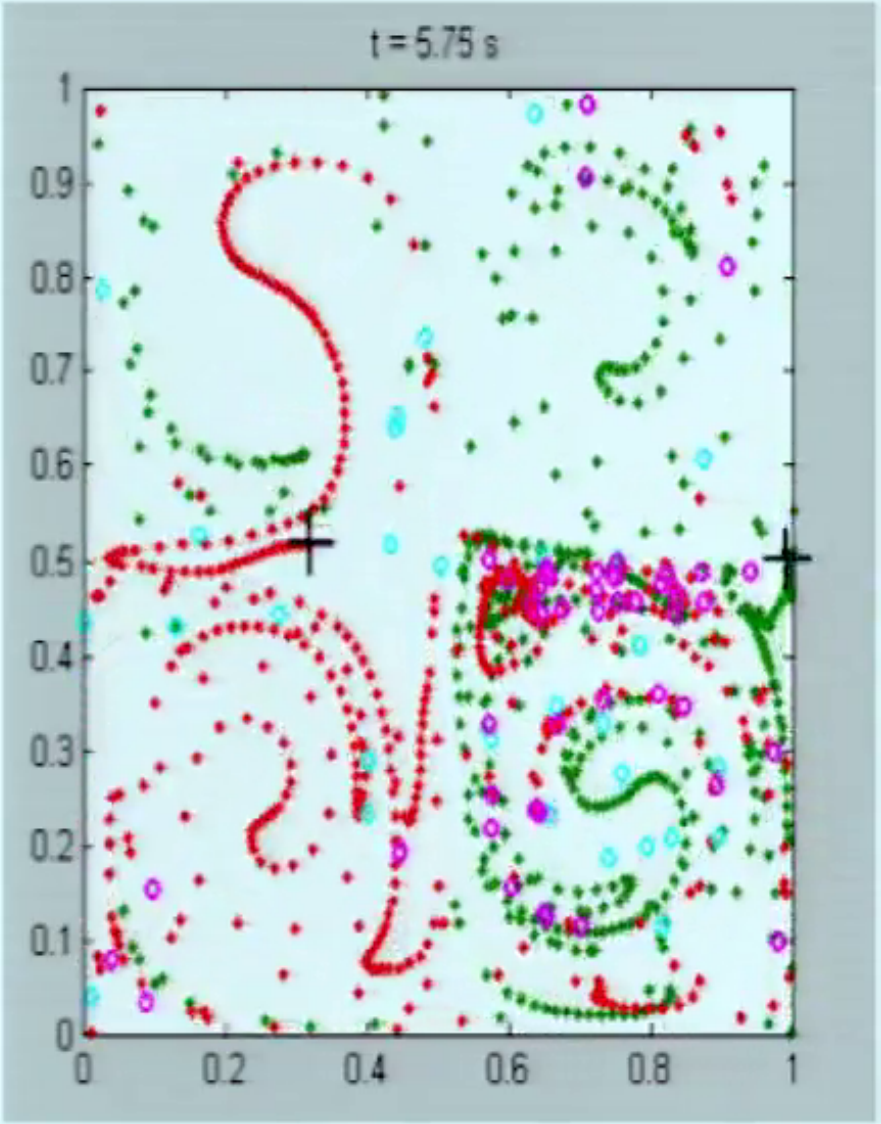




Search of targets moving in a double gyre

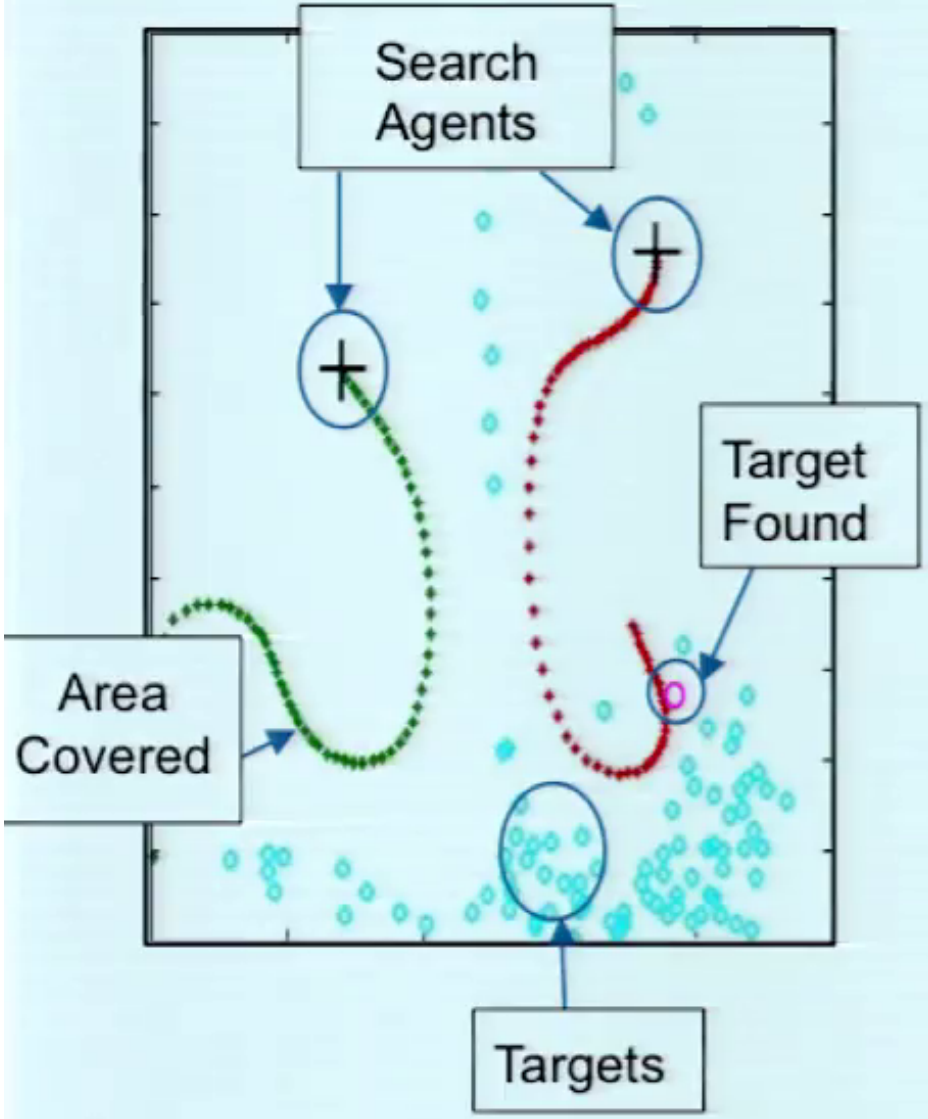
DSMC strategy

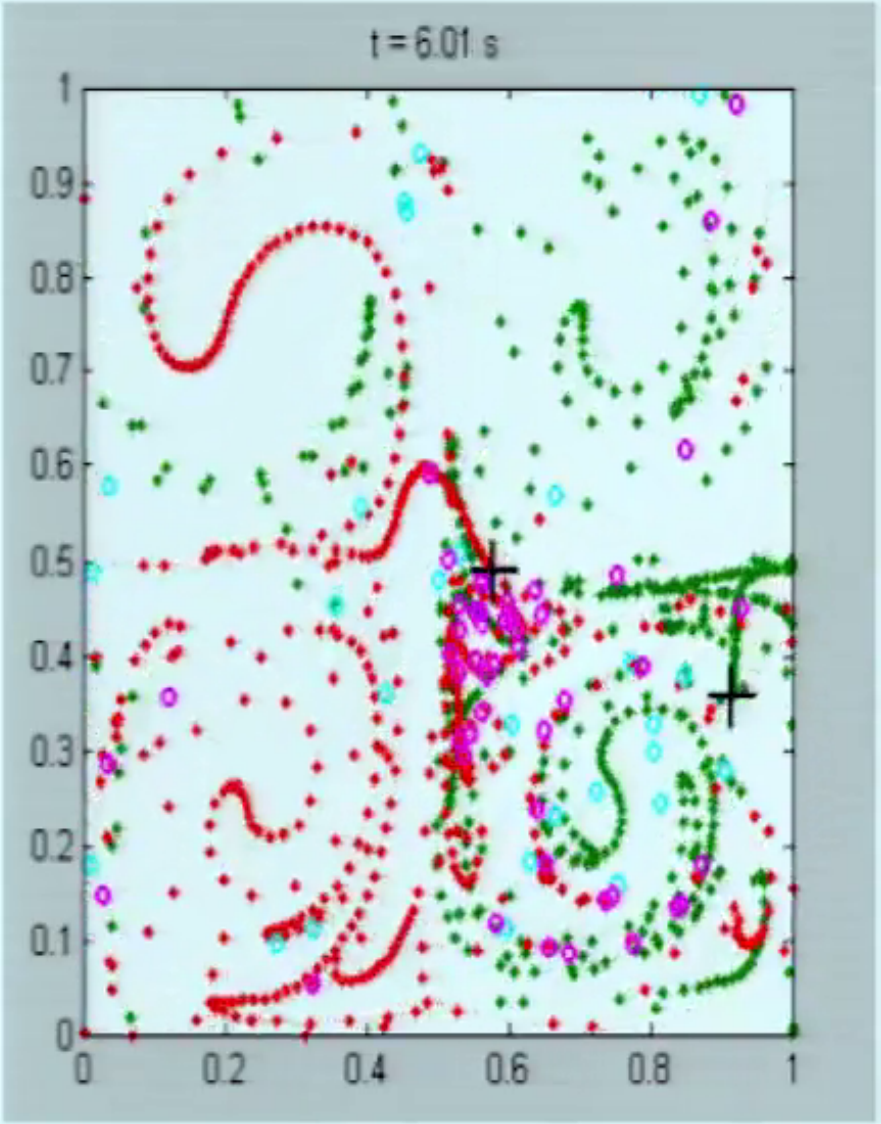




Search of targets moving in a double gyre

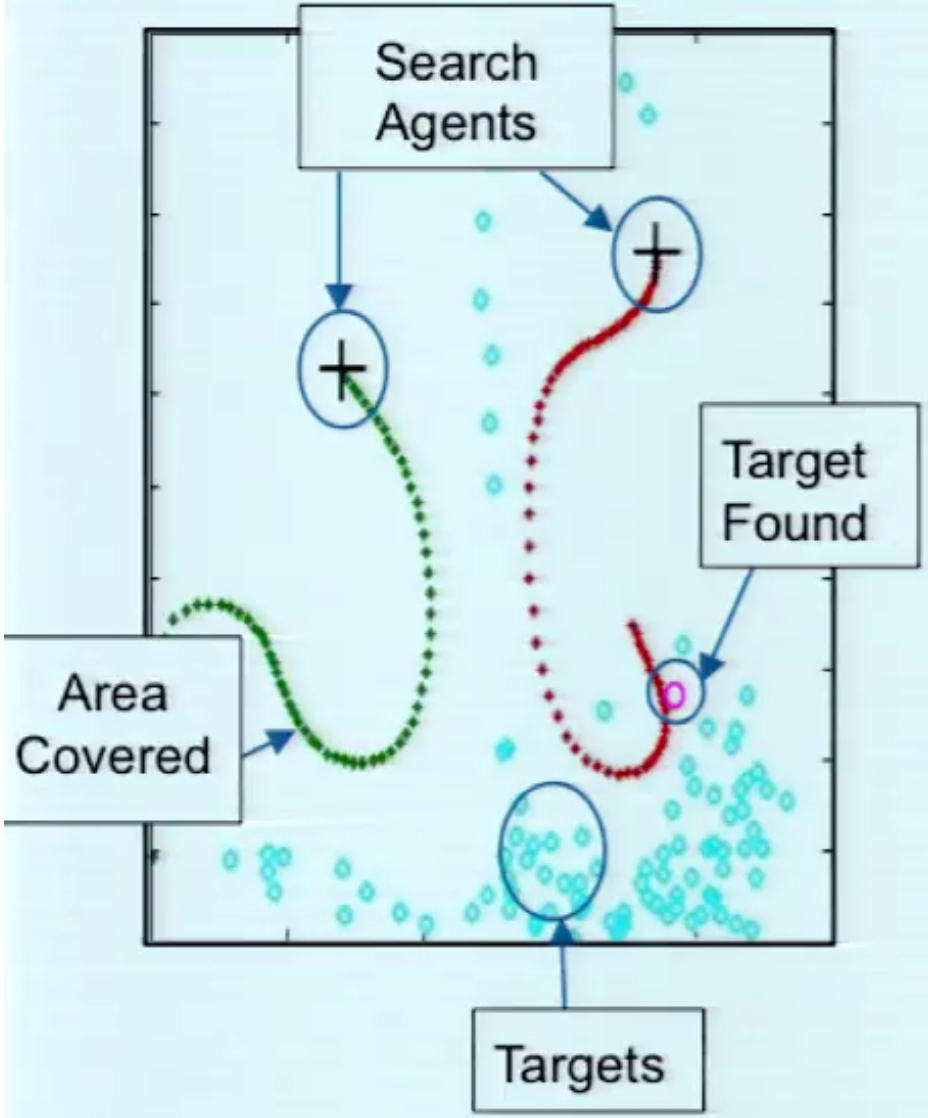
DSMC strategy

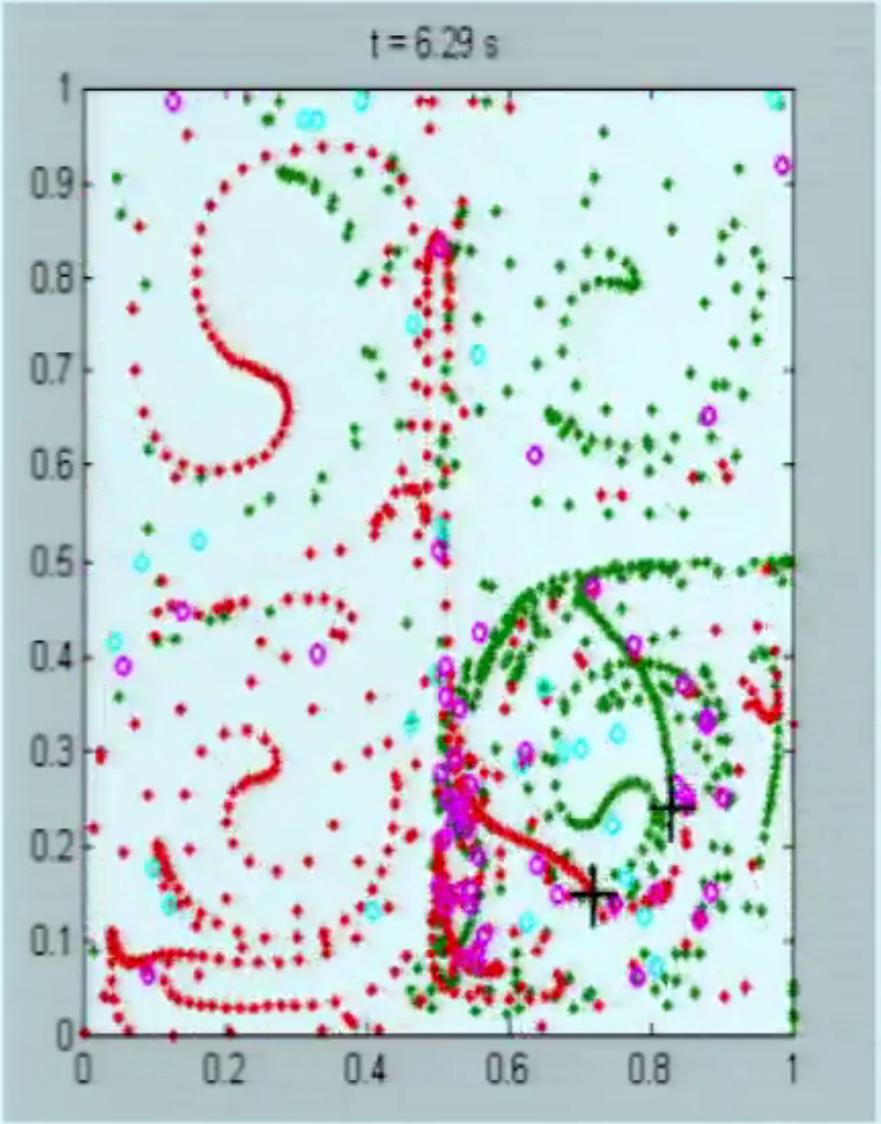




Search of targets moving in a double gyre

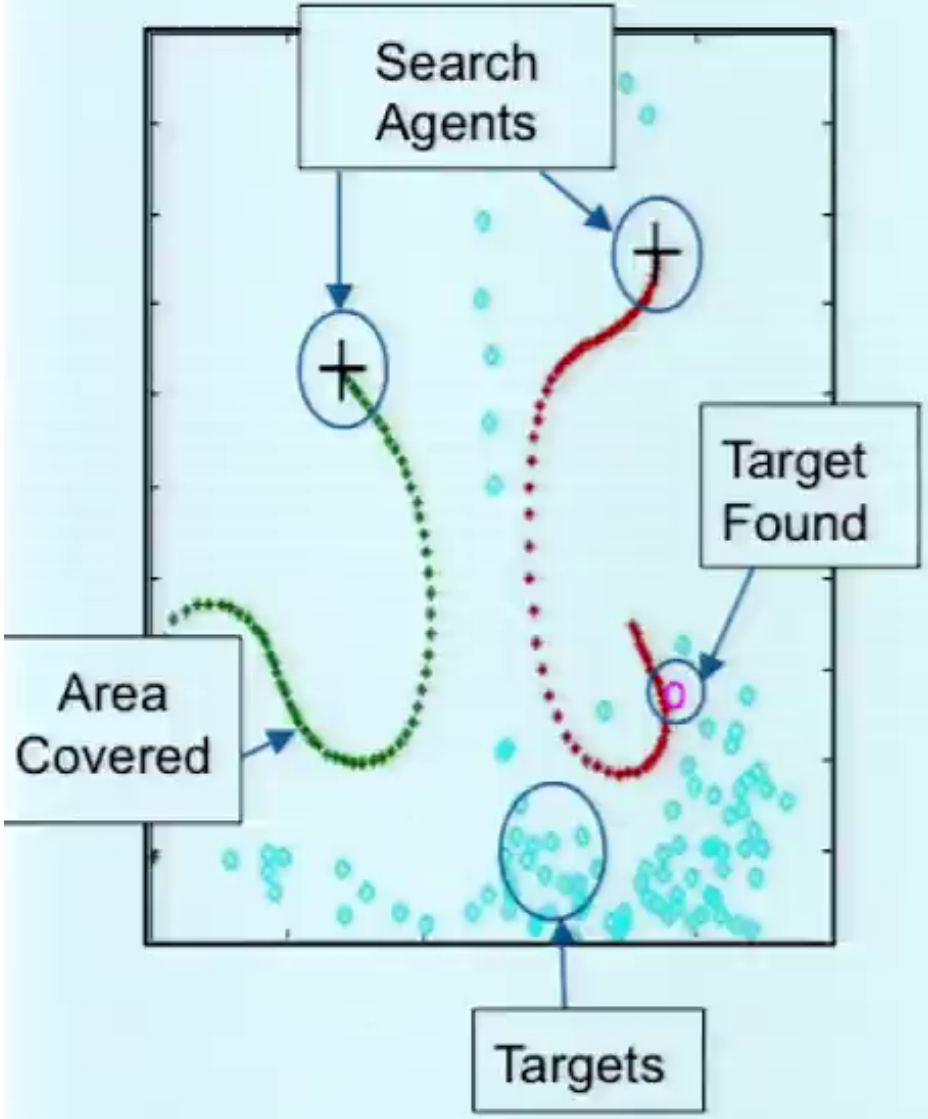
DSMC strategy

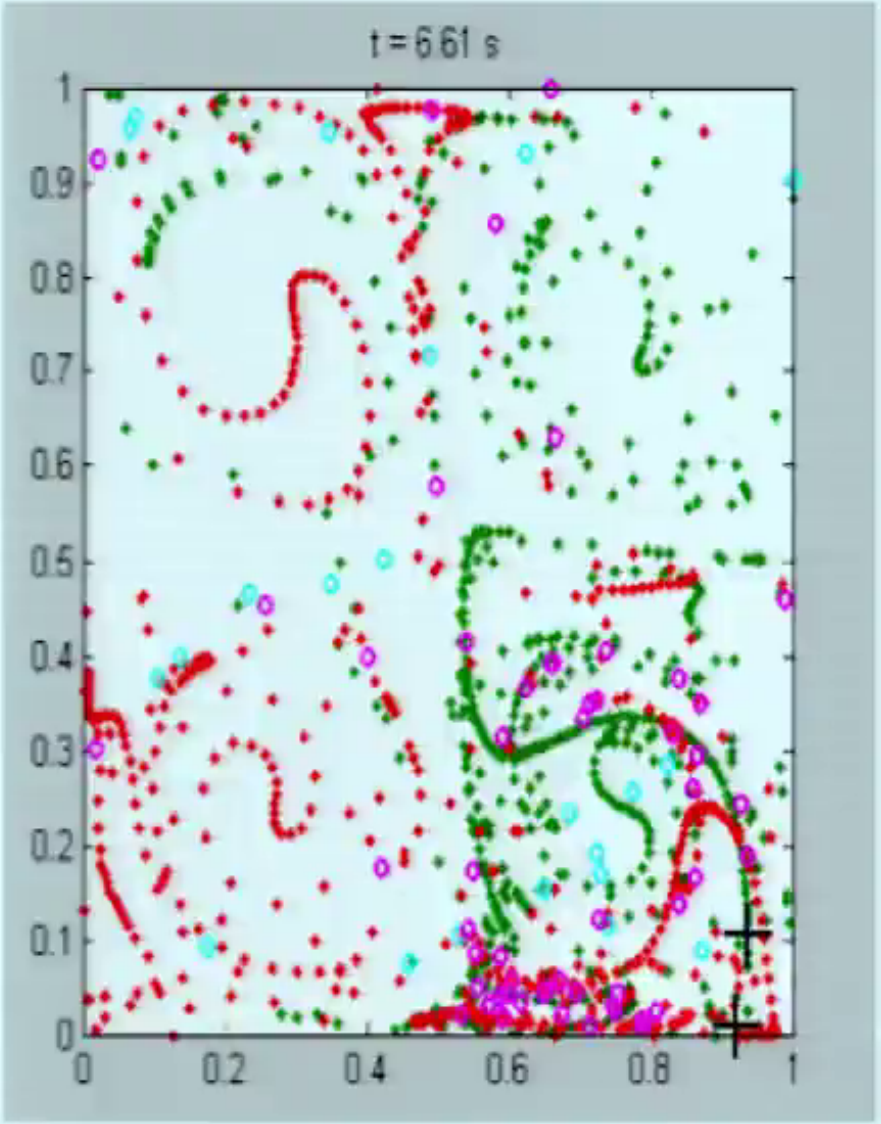




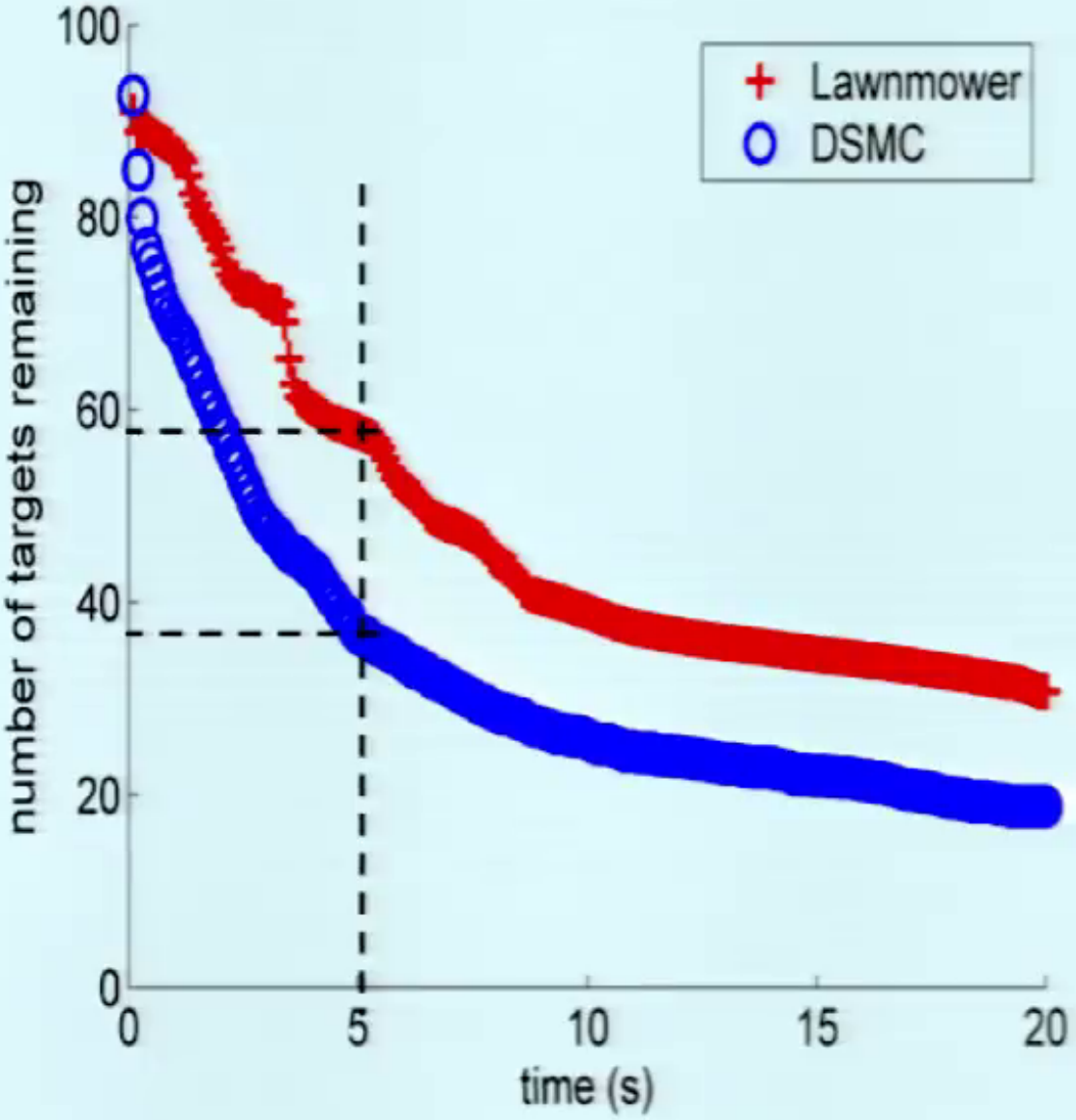
Search of targets moving in a double gyre

DSMC strategy



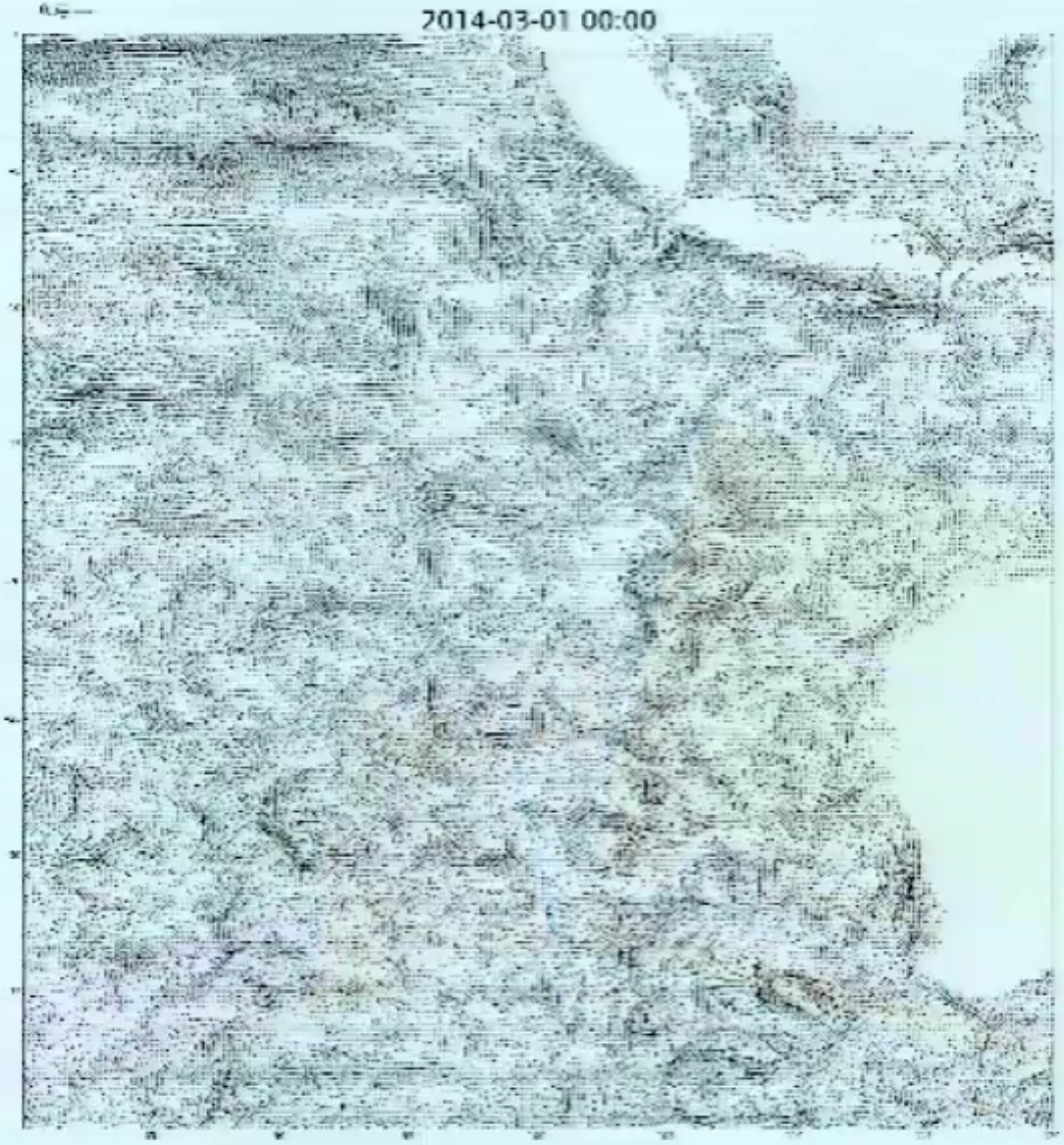


Efficiency comparison



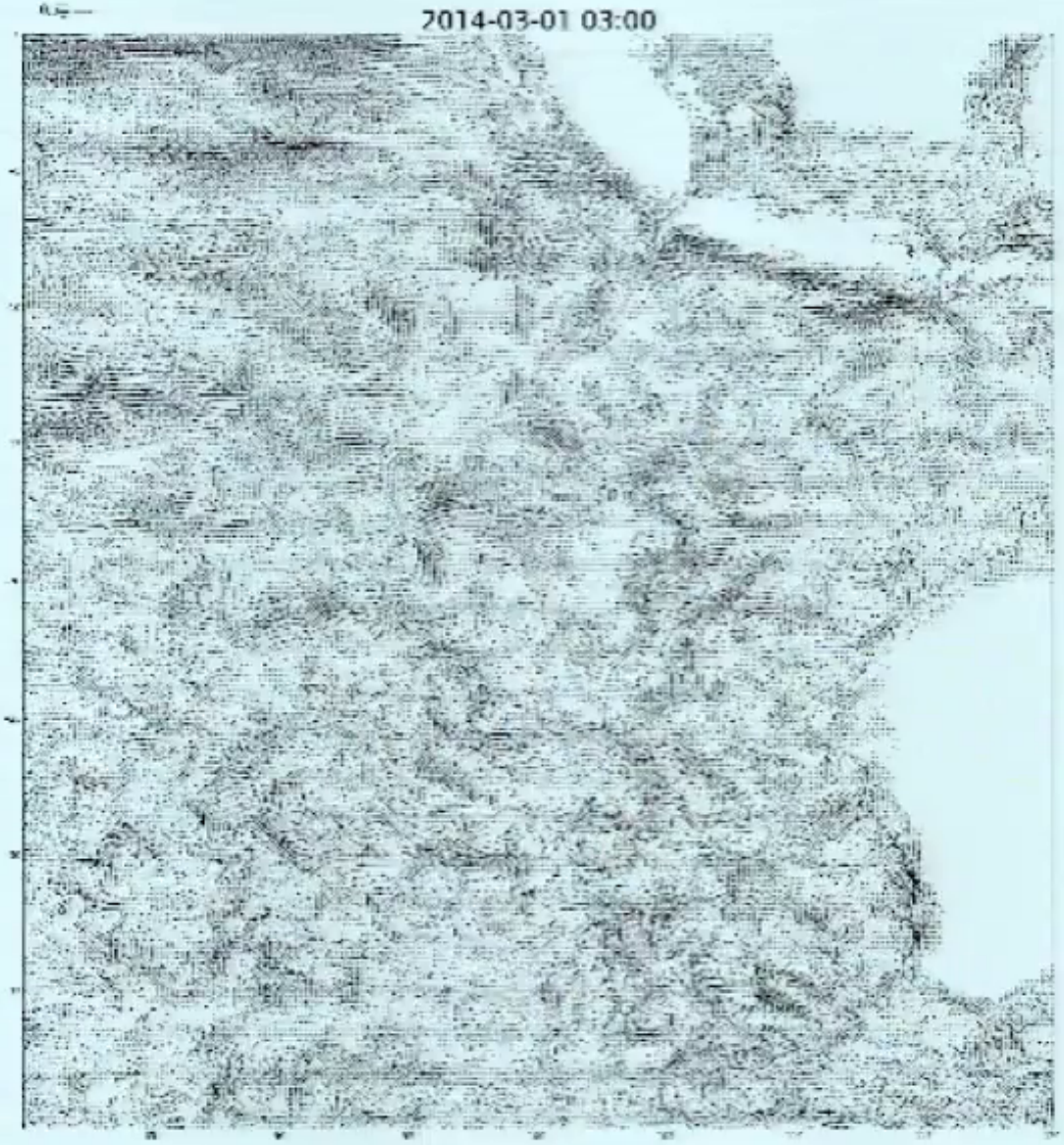
The Indian Ocean Case and the hunt for MH370 floating objects

First Step: Where to search?



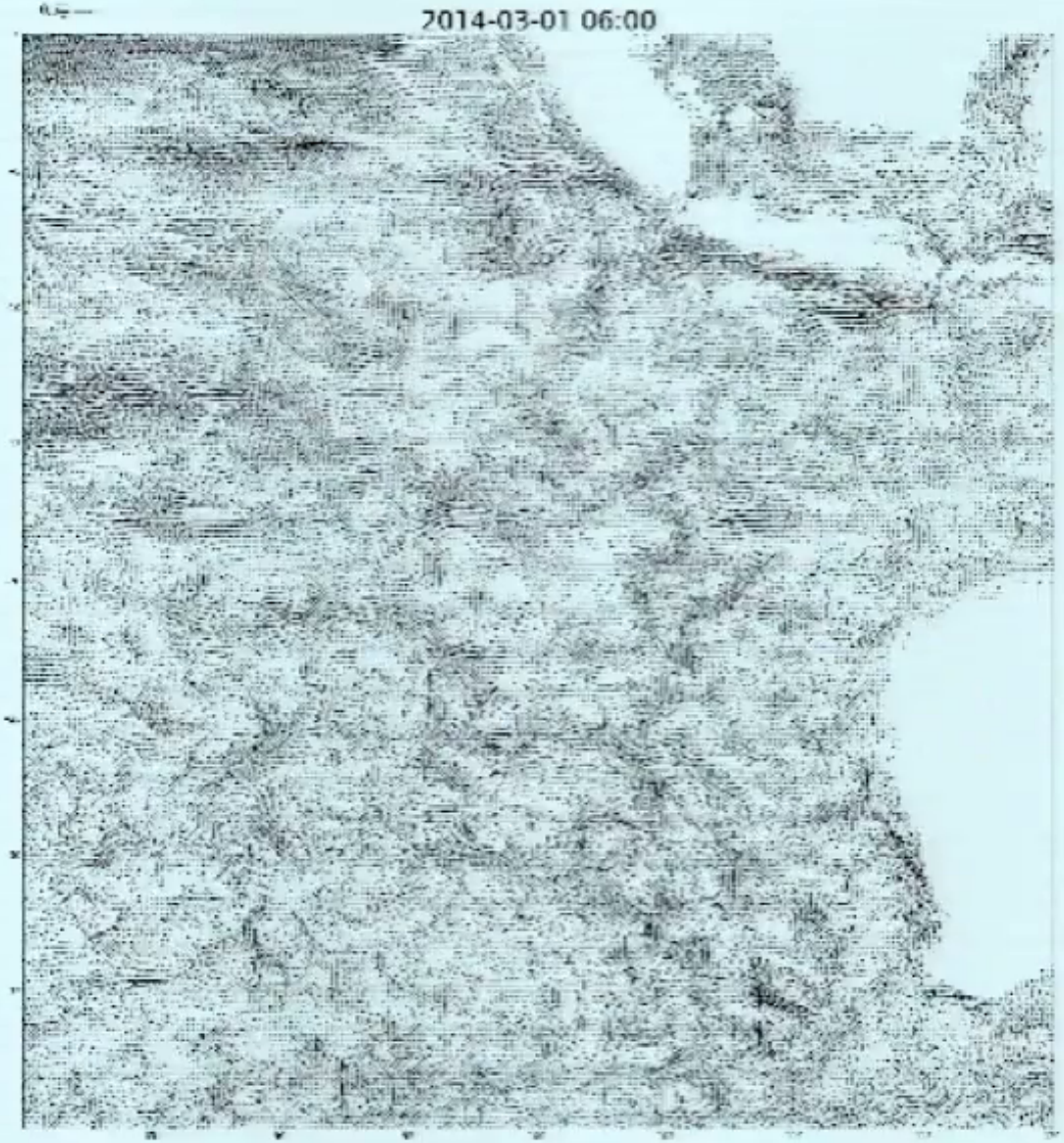
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?



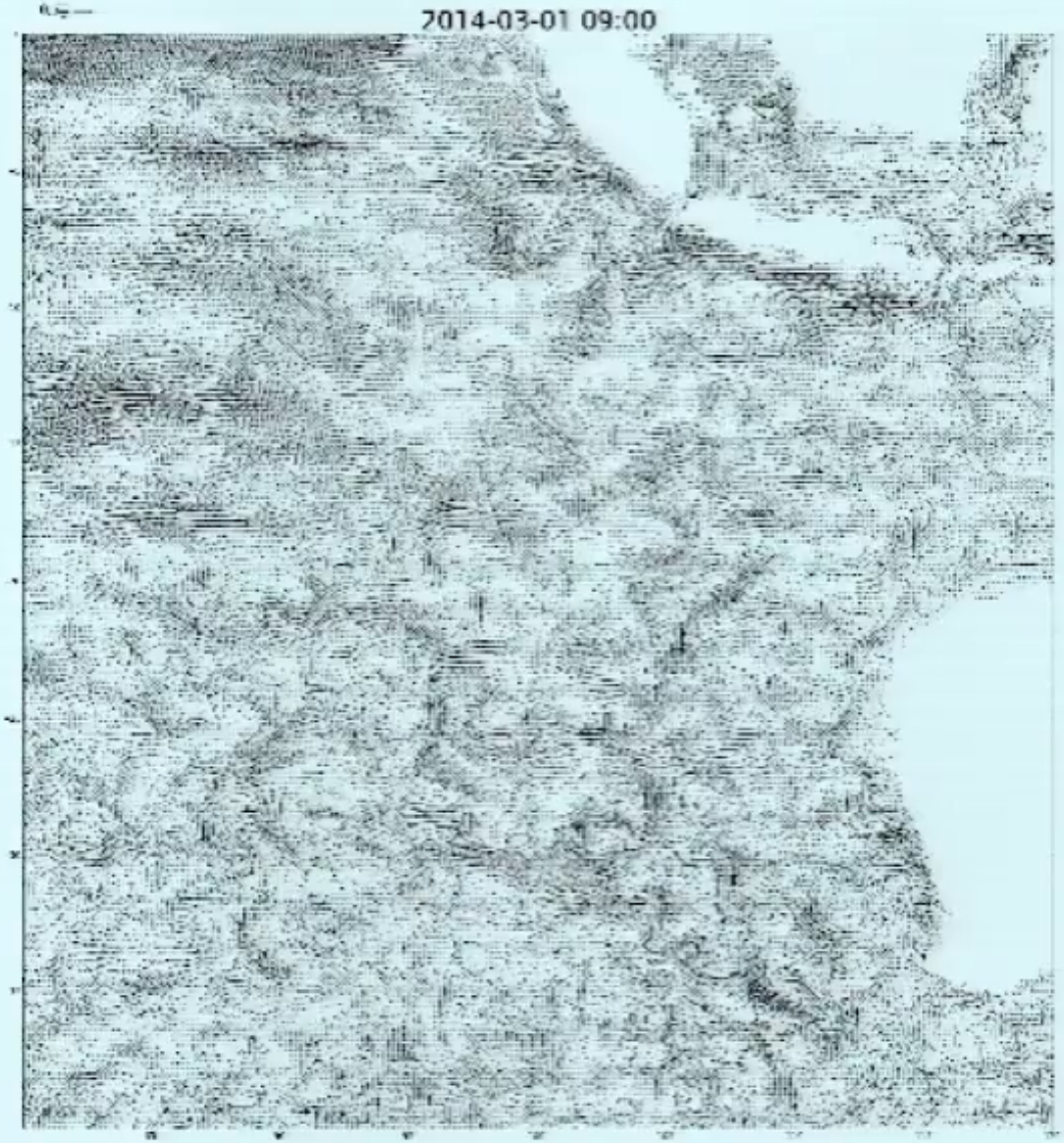
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?



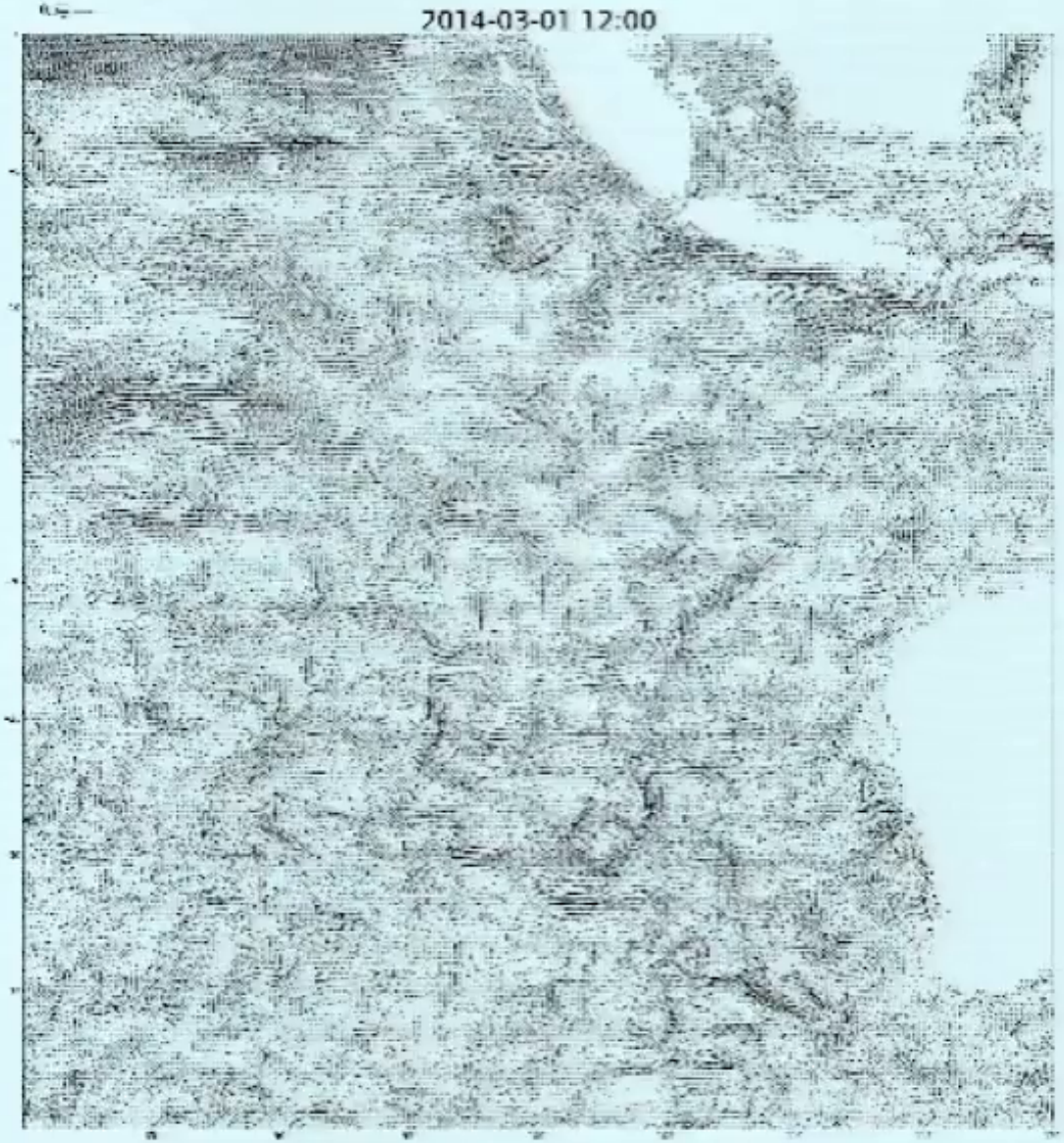
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?



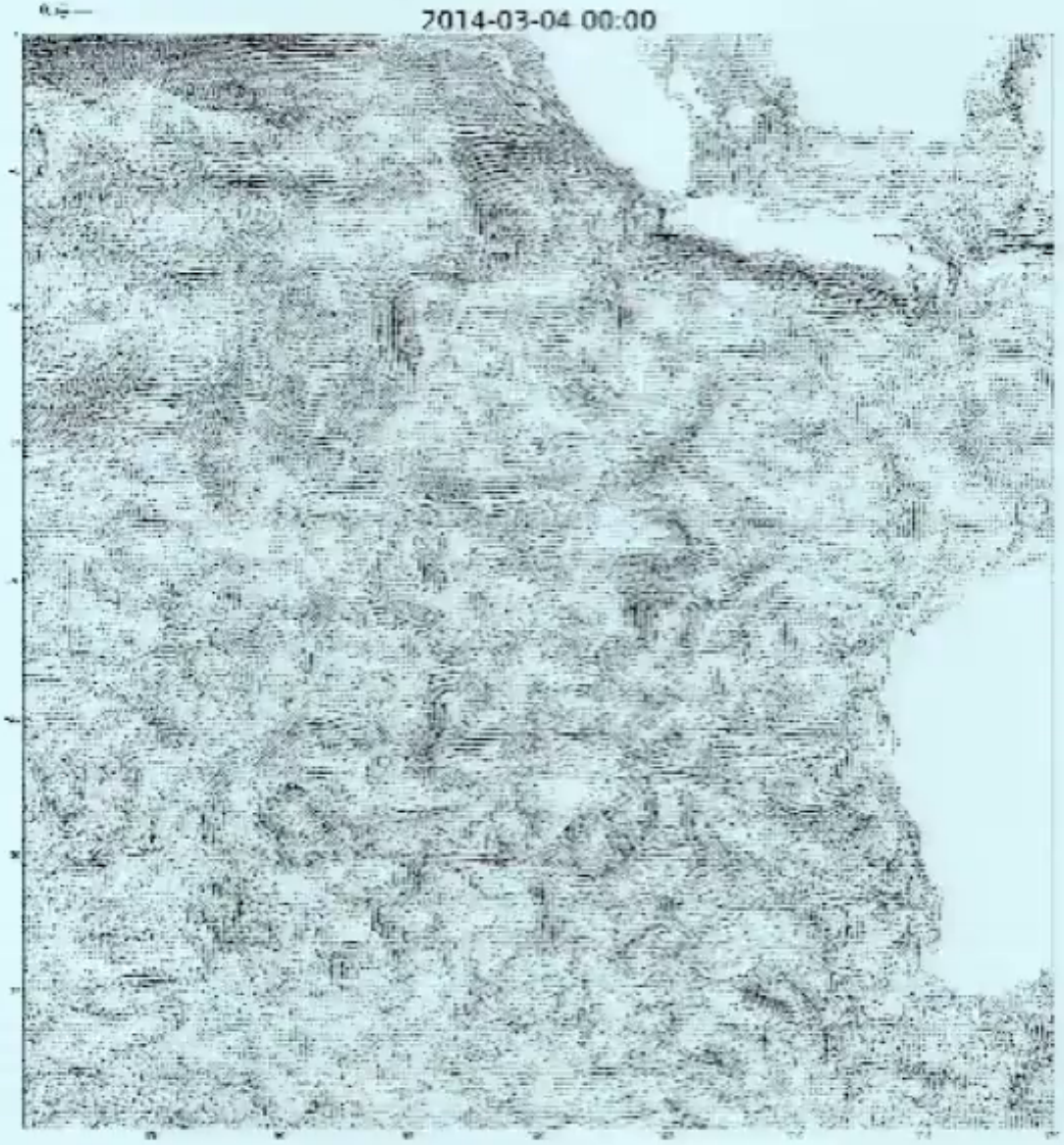
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?



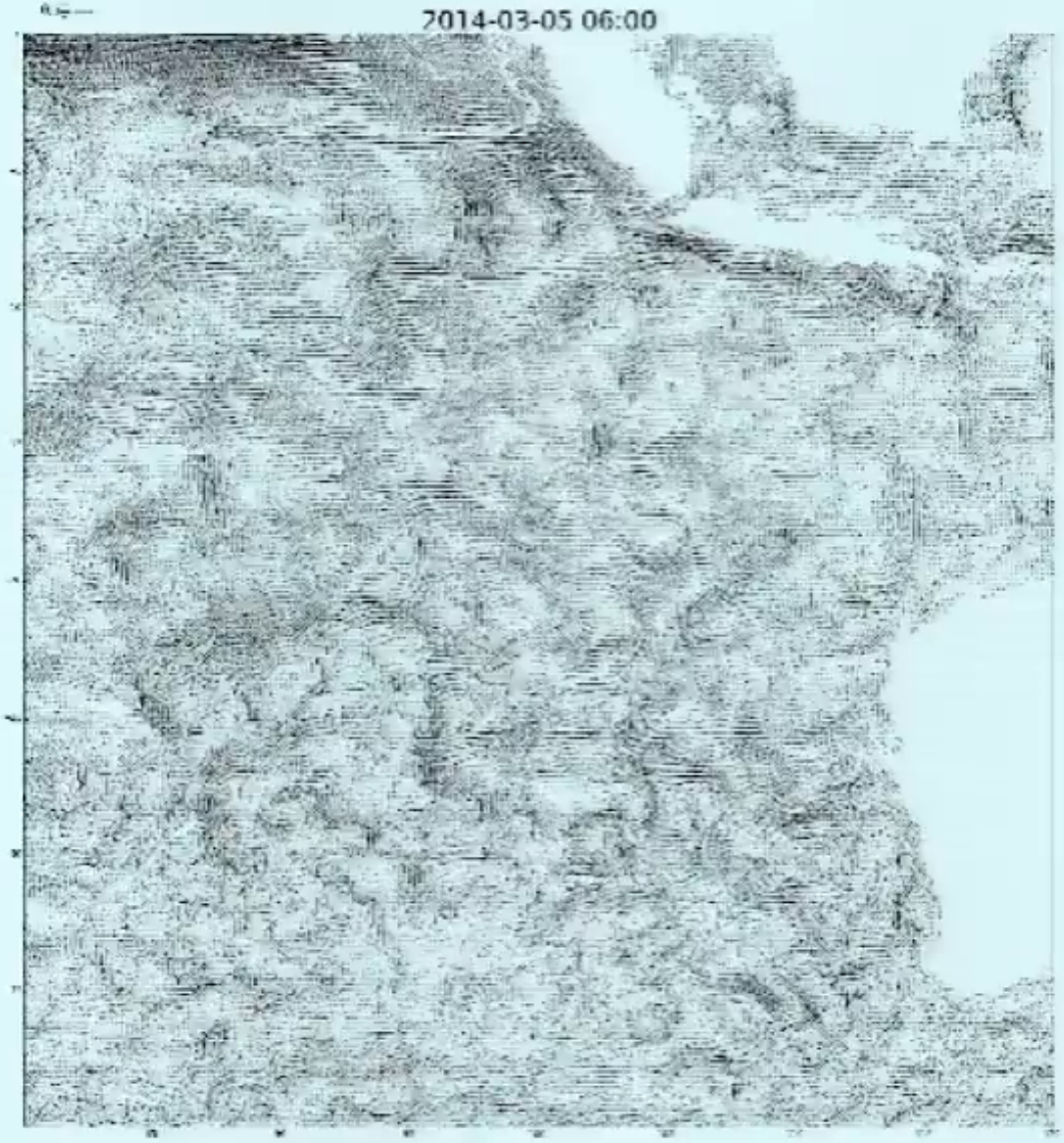
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?



Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?

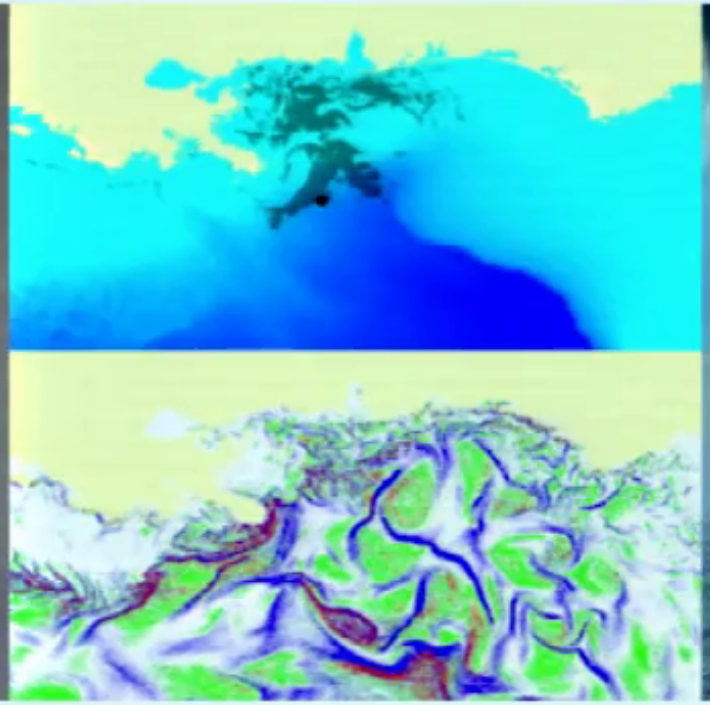


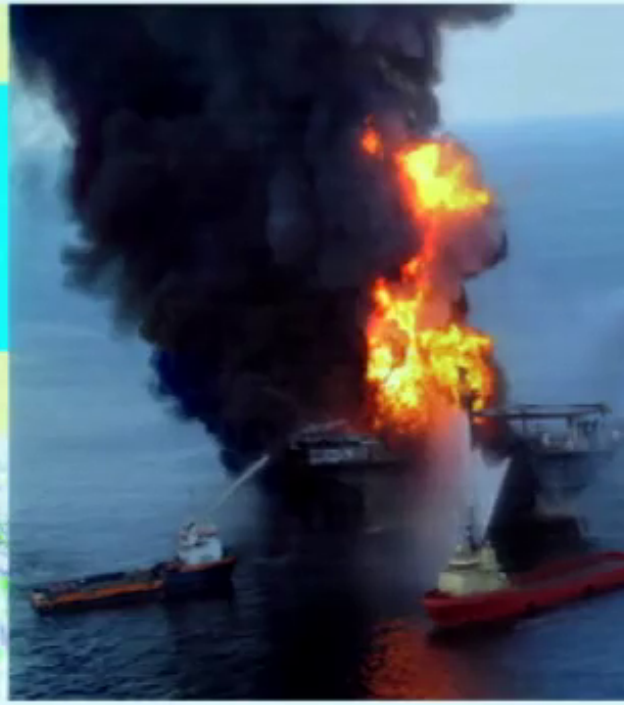
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?

What about using Hypergraph Maps
to design a strategy for the hunt of MH370?

Spread of Deepwater Horizon oil slick predicted using Hypergraph Calculation

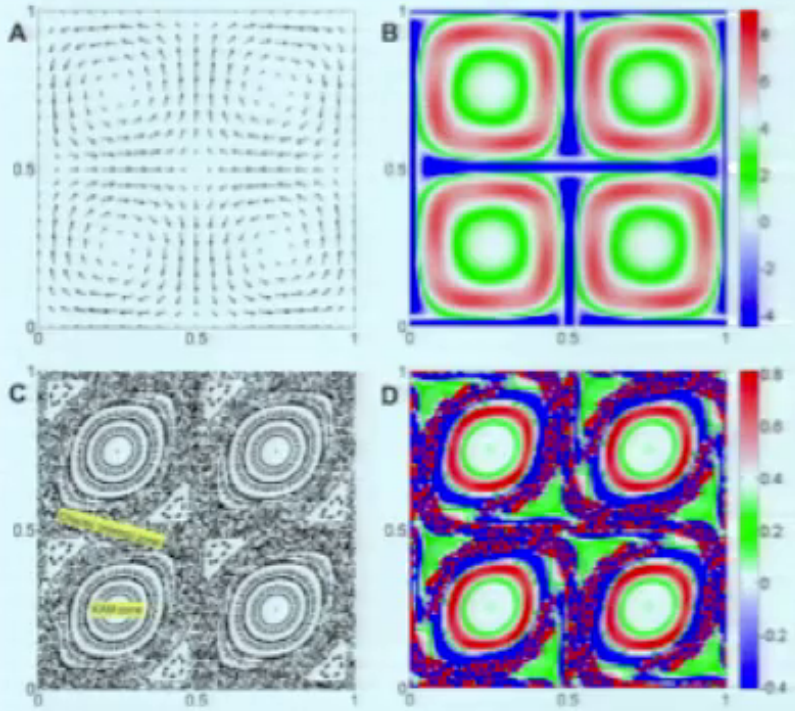




A New Mixing Diagnostic and Gulf Oil Spill Movement, Igor Mezić, S. Loire, Vladimir A. Fonoberov, and P. Hogan, Science 22 October 2010

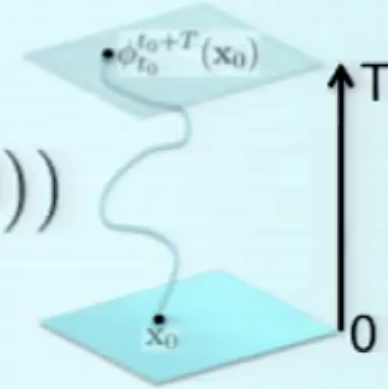
Dynamic autoinoculation and the microbial ecology of a deep water hydrocarbon irruption, David L. Valentine, Igor Mezić, Senka Maćešić, Nelida Črnjarić-Žic, Stefan Ivić, Patrick J. Hogan,Vladimir A. Fonoberov, and Sophie Loire, PNAS 2012

Hypergraph: Meso-hyperbolicity, Meso-ellipticity,Meso-Helical



Average Lagrangian velocity

$$\mathbf{v}^* = \frac{1}{T}(\mathbf{X}(T) - \mathbf{X}(0))$$



Eigenvalues of its gradient

$$\nabla \mathbf{v}^*$$

$\det |\nabla \mathbf{v}^*|$

Meso-Hyperbolic	Meso-Elliptic	Meso-Helical
strain	rotation	rotation + strain
$\det \lvert\nabla \mathbf{v}^*\rvert < 0$	$0 < \det \lvert\nabla \mathbf{v}^*\rvert < \frac{4}{T^2}$	$\frac{4}{T^2} < \det \lvert\nabla \mathbf{v}^*\rvert$

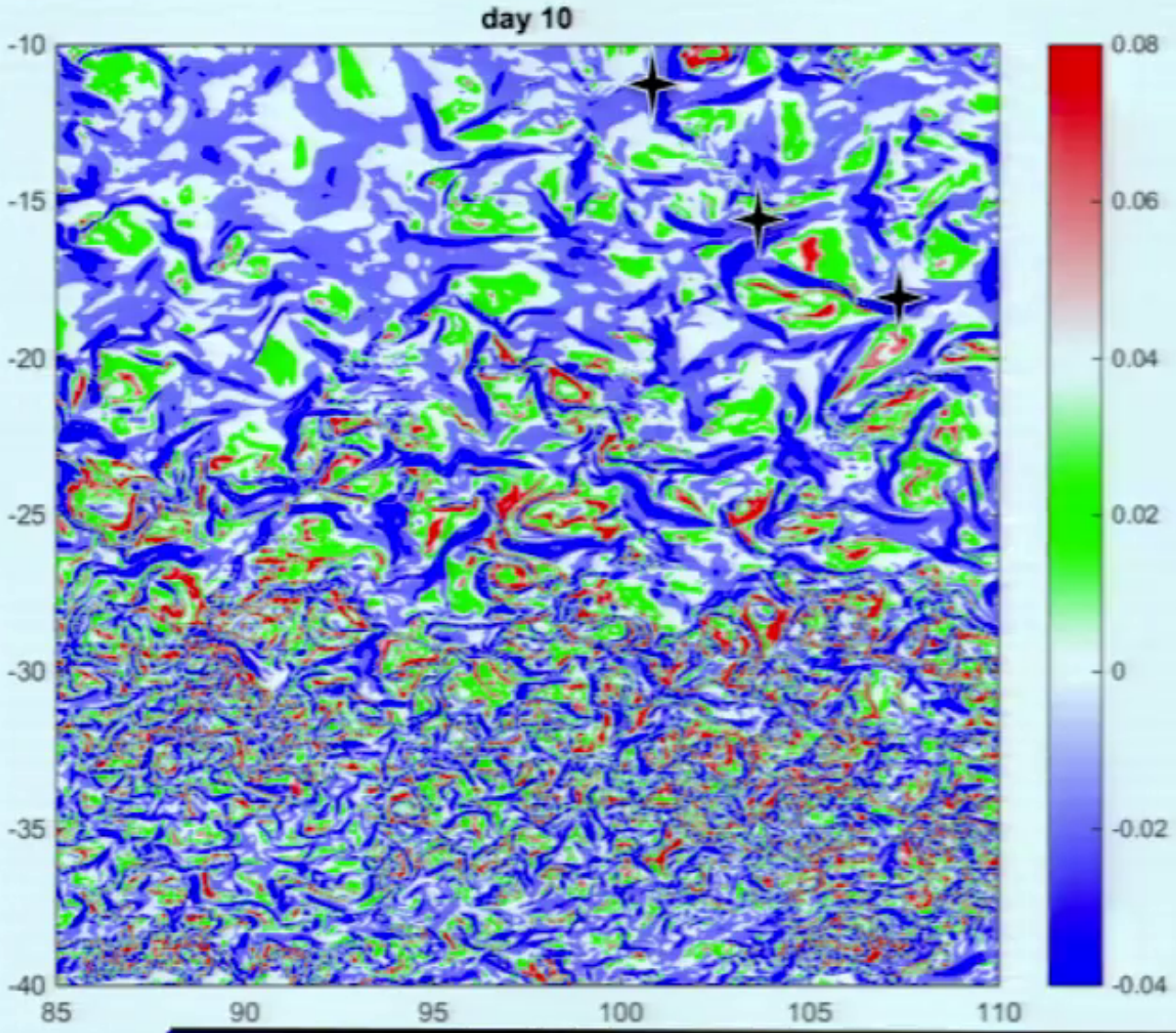
Hypergraph: Meso-hyperbolicity, Meso-ellipticity,Meso-Helical
For Complex and dynamic environment:

Classification of regions of
- Mixing
- High shear
- Ellipticity (including location of possible coherent vortices)

For search strategies:
- Predict future location of targets and define search area
- Help define initial target position probability distribution
- Prioritize search in large shear regions

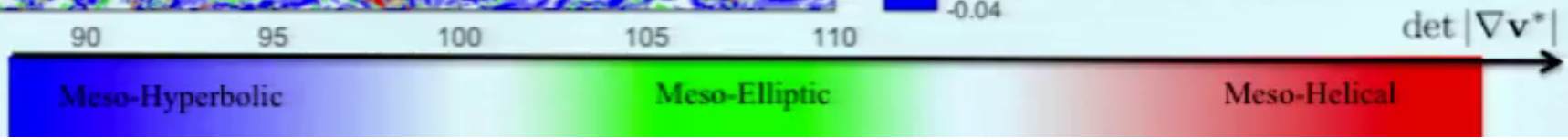
Based on the gradient of the average Lagrangian velocity

First Step: Where to search?



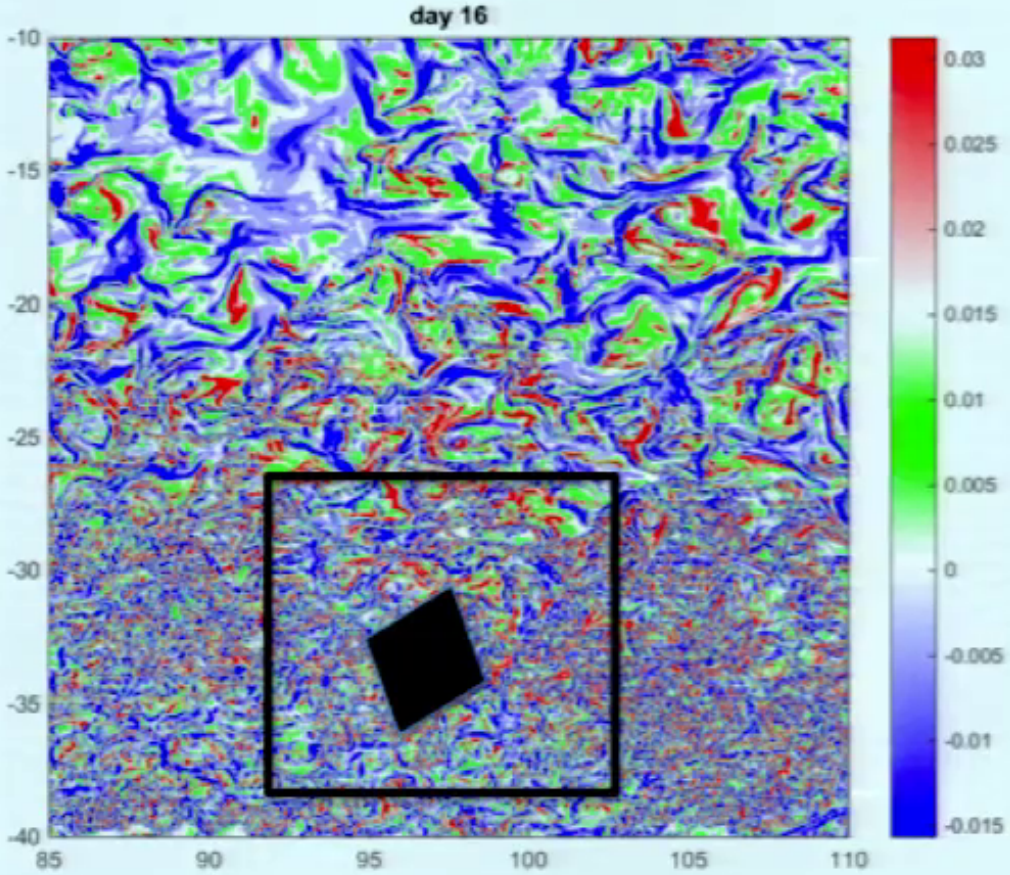
With No additional Information:

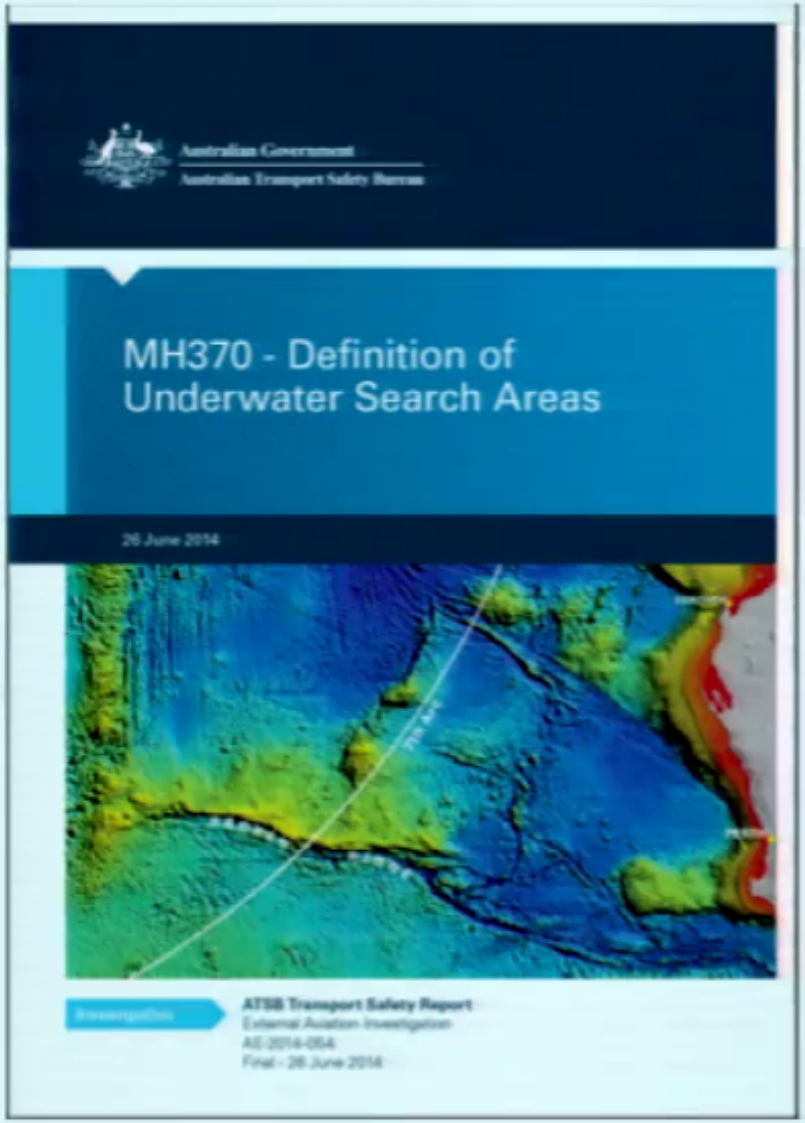
the best strategy is to allocate search efforts in the Mesohyperbolic region



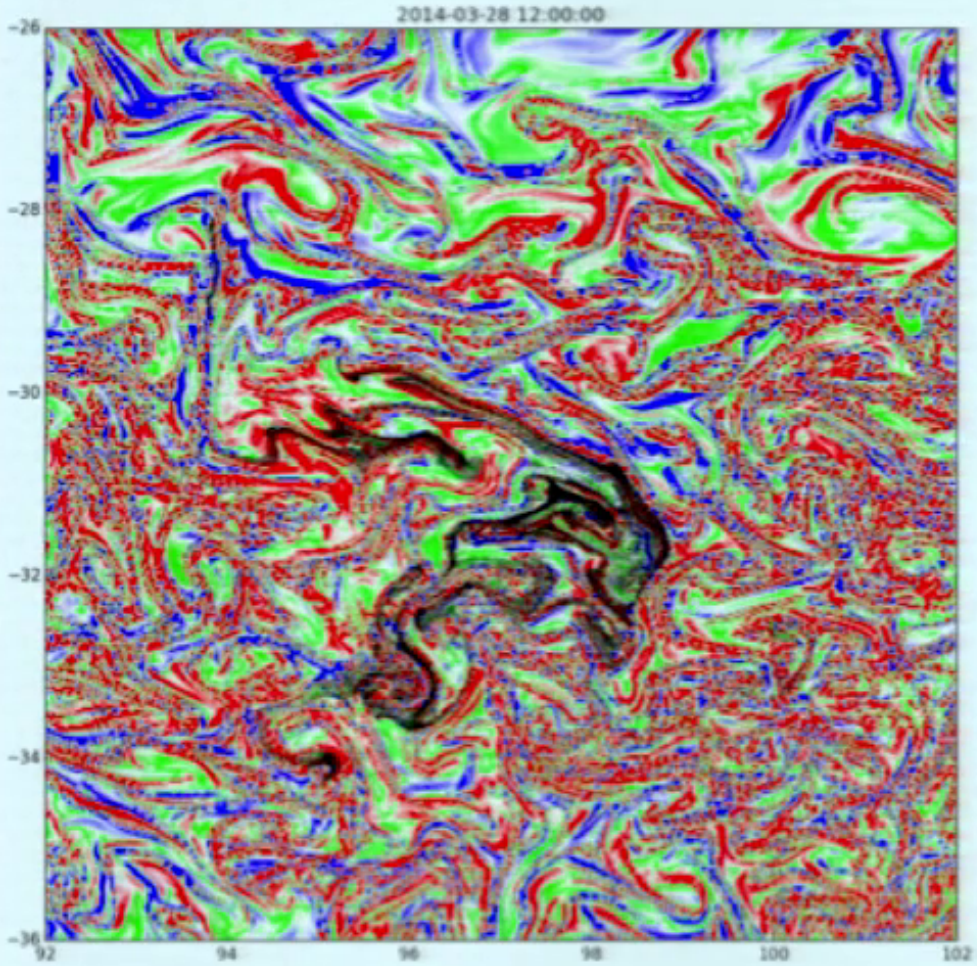
The Indian Ocean Case and the search for MH370 floating objects

Inmarsat satellite: March 24 2014
First Announced Splash Location





The Indian Ocean Case and the search for MH370 floating objects



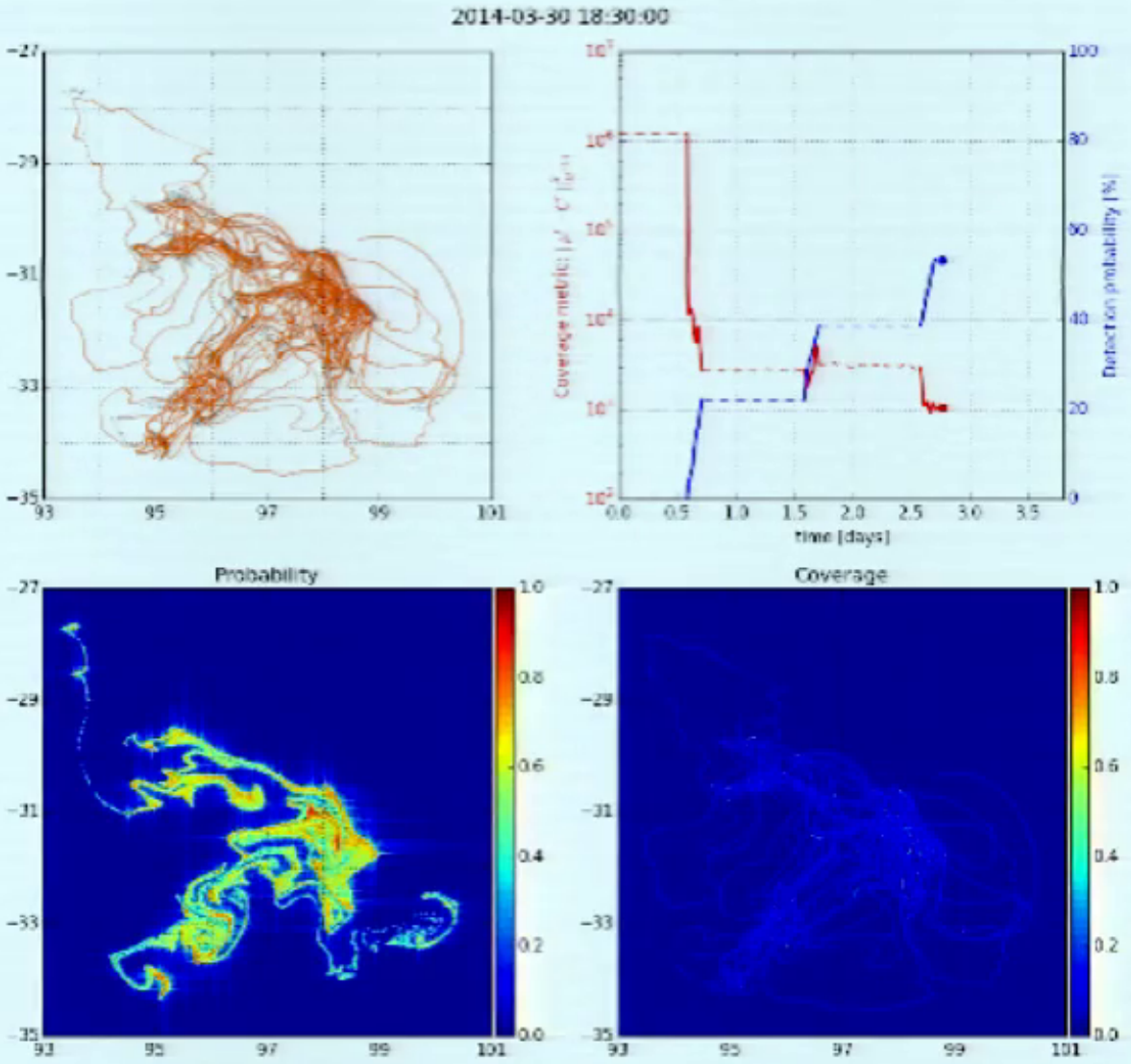
Search area A

2014-03-31 (Search day 4)

Search period: 14:00 to 17:00

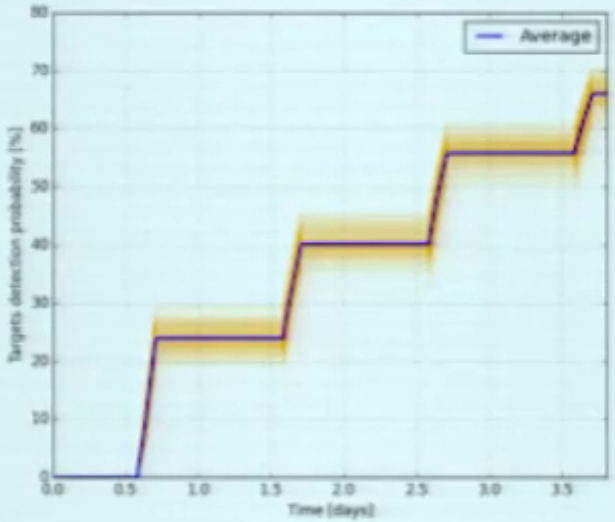
Number of agents: 9

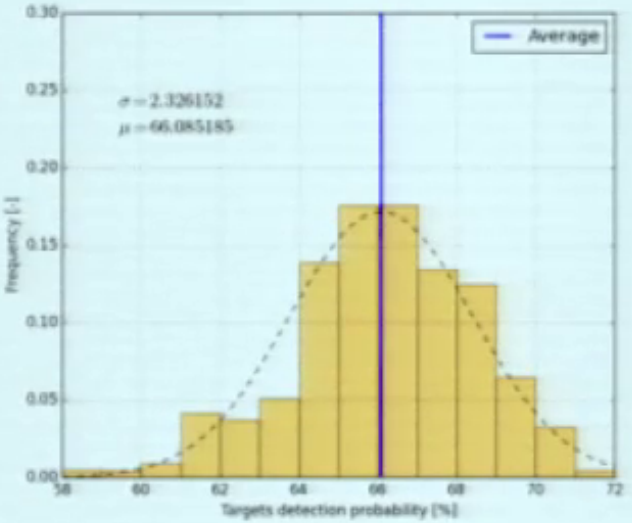
DSMC search area A



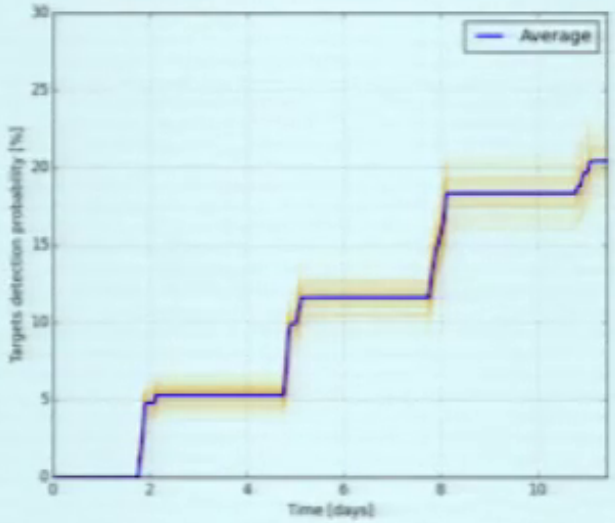
Comparison:

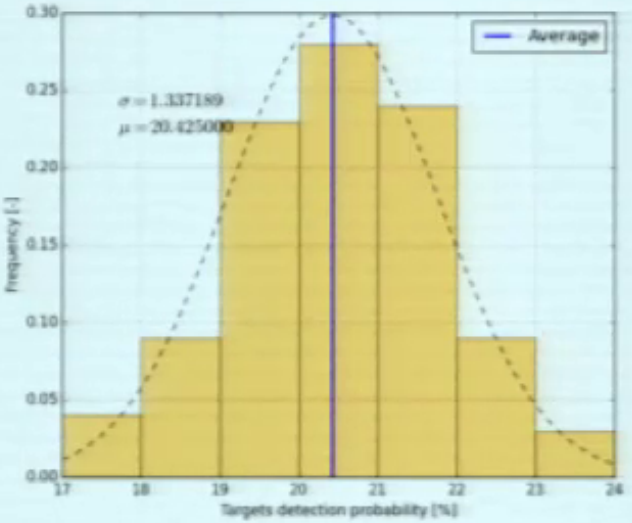
DSMC



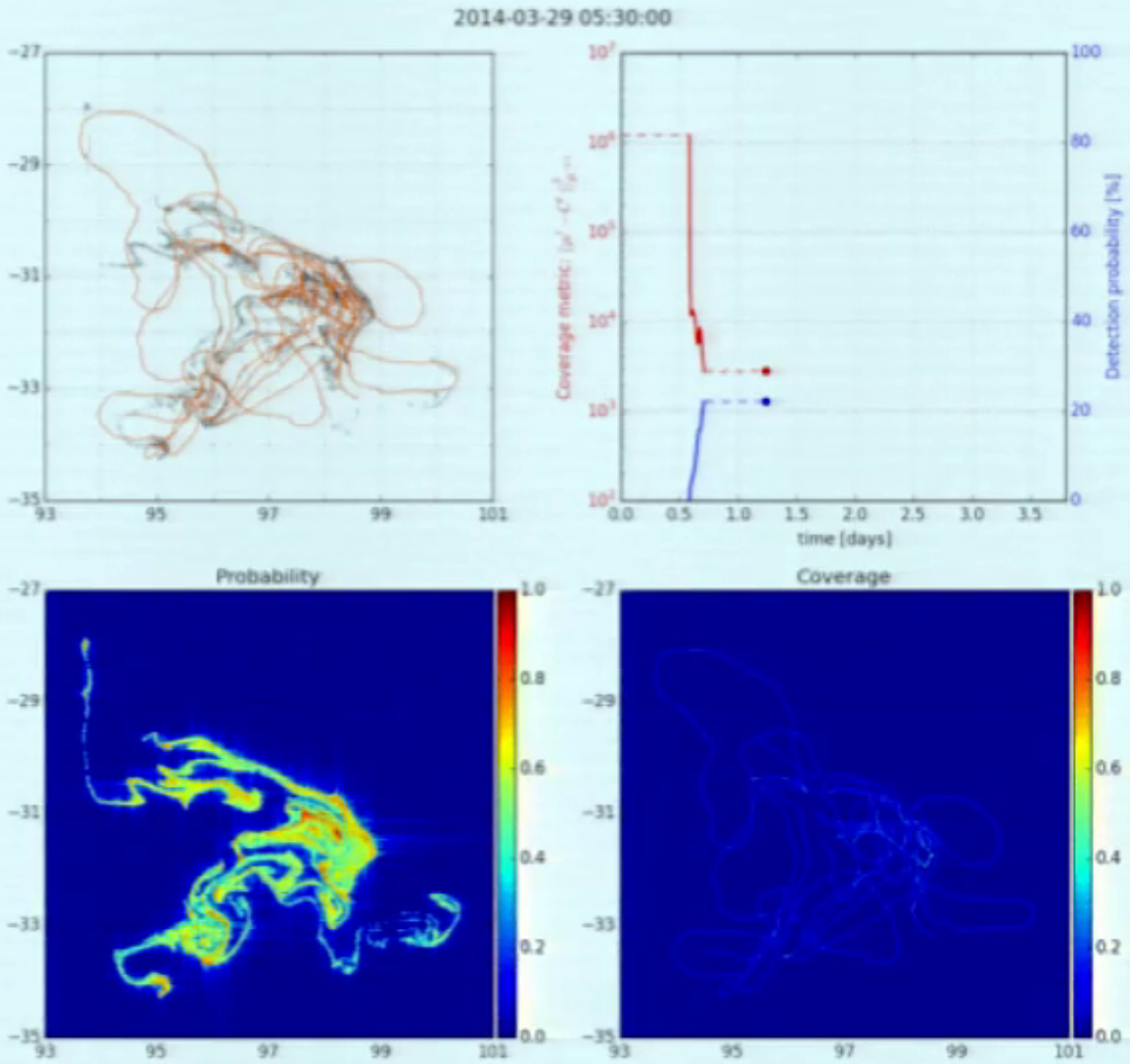


Lawnmower

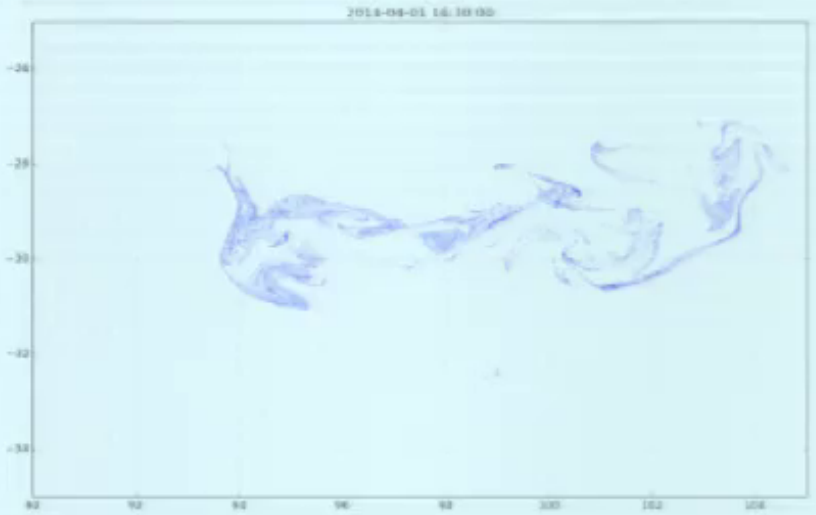




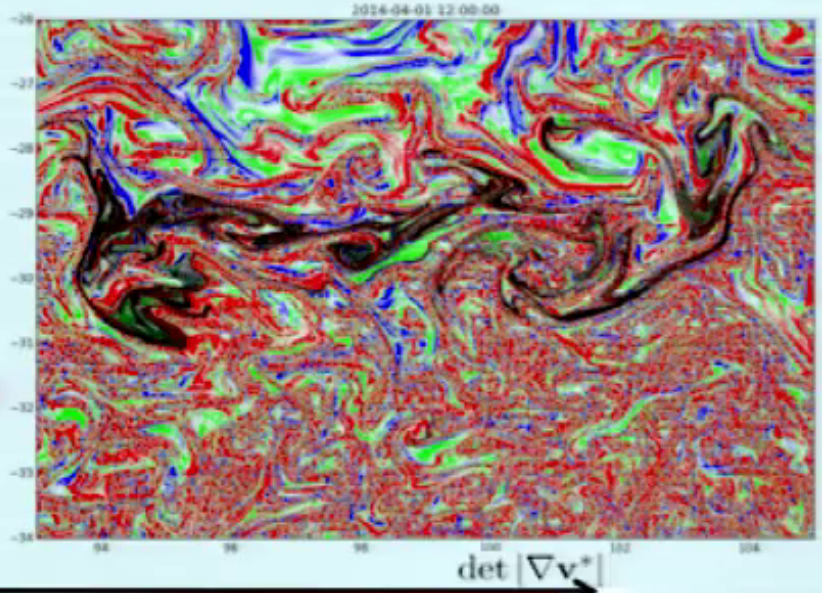
DSMC search area A



Hypergraph: Backward-time hypergraphs from April 01 to March 08 2014



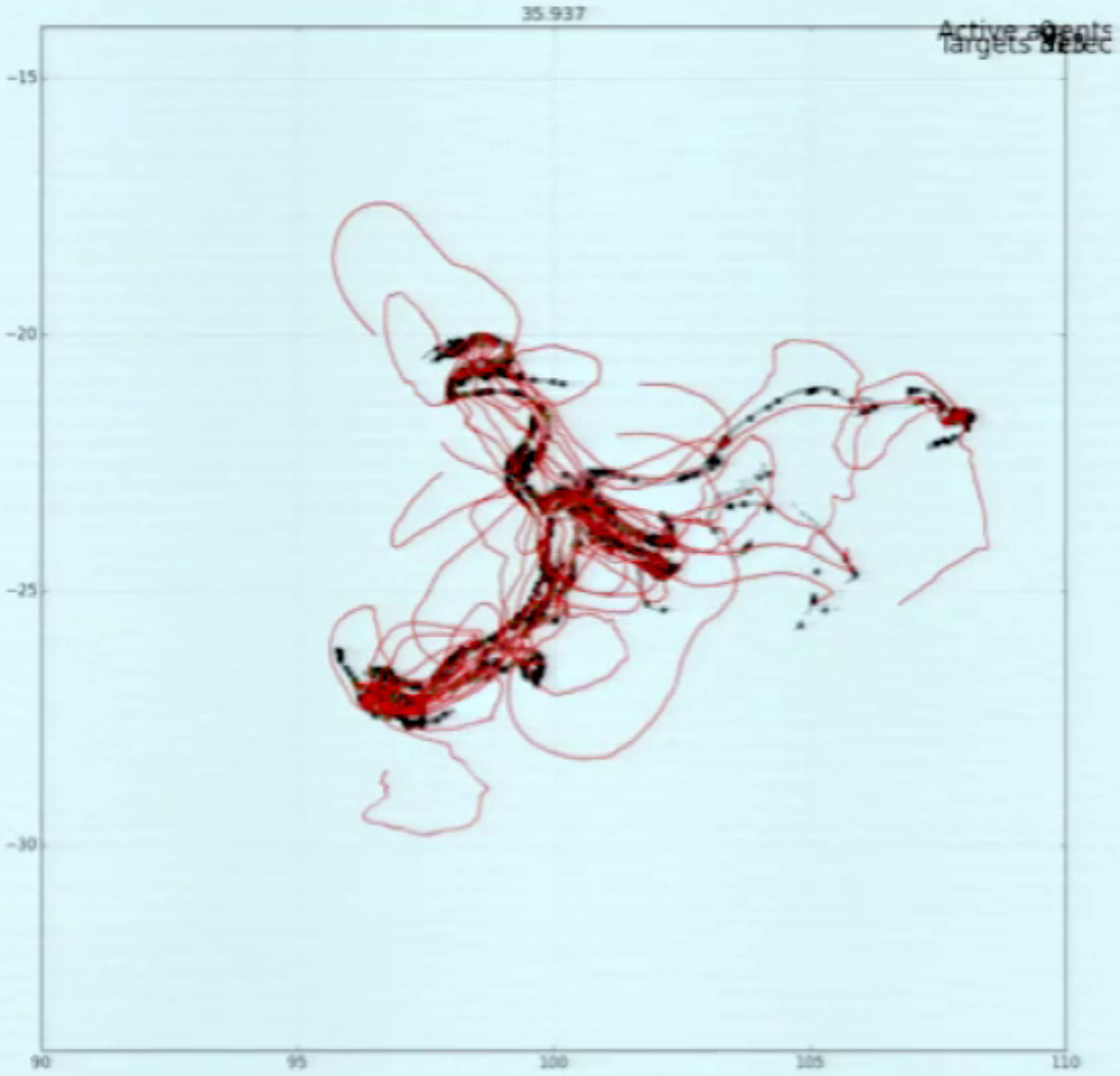
Advected Area B



Meso-Hyperbolic | Meso-Elliptic | Meso-Helical

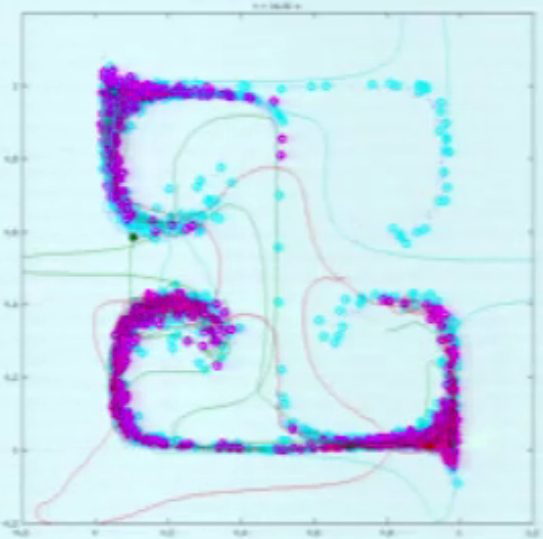
strain | rotation | rotation + strain

What if the target density is very filamented?

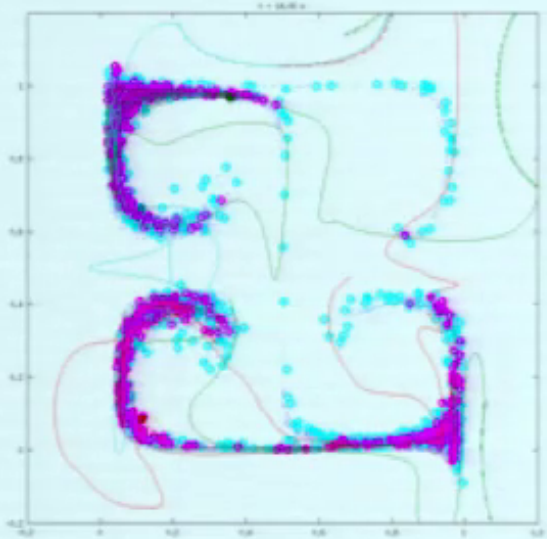


Wavelet vs changing the weight on spatial modes

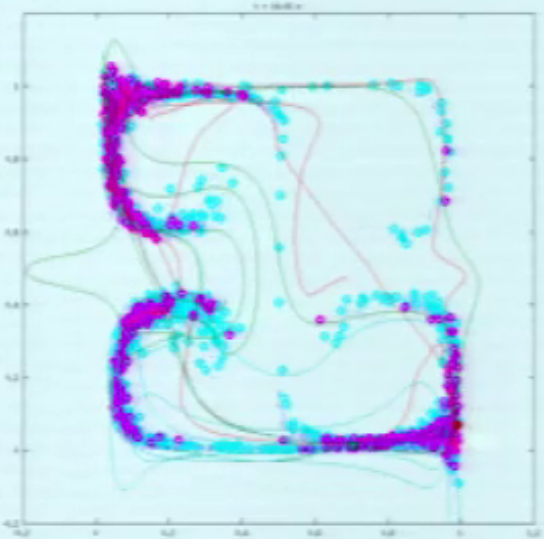
DCT s=1

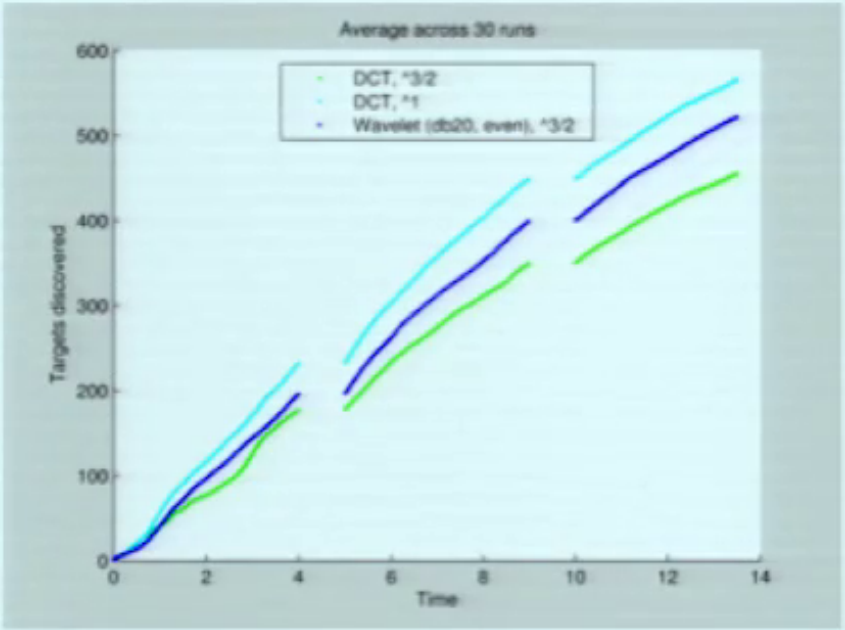


Wavelet



DCT s=3/2





Conclusion

- Mesohyperbolicity: Hypergraph maps
  - prediction of target dynamic
  - help define a search strategy
  - give useful information on the dynamic of the targets
- Mix-Norm: DSMC search strategies
  - searches the region with high probability
  - searches from large to low spatial scale
  - large spatial scale can be weighted in the filamented case
- The Hunt for MH370 is still going on!