

Mesohyperbolicity, Mix-Norm and the Hunt for MH370

Sophie Loire

Hassan Arbabi, Patrick Clary, Igor Mezić

Department of Mechanical Engineering,
University of California, Santa Barbara

Bojan Crnković, Stefan Ivić

Rijeka University, Croatia

The Indian Ocean Case and the hunt for MH370 floating objects

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News > World news > Malaysia Airlines flight MH370

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TRENDING TOPICS AND NEWS ANALYSIS

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Flight MH370: out of the headlines, but the search goes on

While families demand answers, it seems that the rest of the world has forgotten the lost Malaysian Airlines plane. But the task appears to be more daunting than ever



Posted by Gersh To
Sunday, 13:20
The

SET EDITOR: U.S. INTERNATIONAL MEXICO ASIA

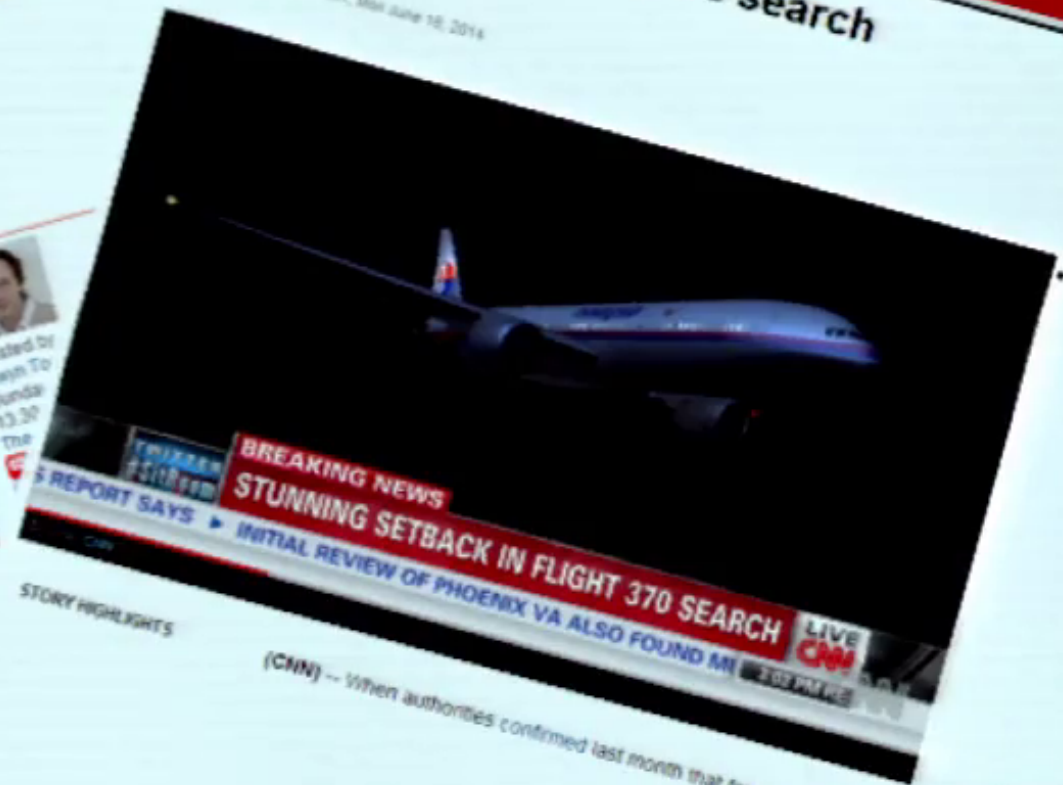
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MH370: How long will the search continue?

By Sophie Brown, CNN
Updated 11:34 AM EDT, Mon June 16, 2014



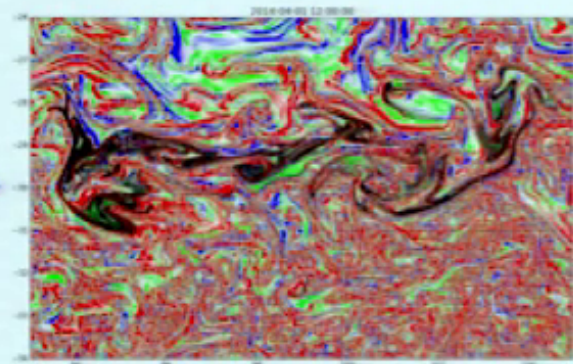
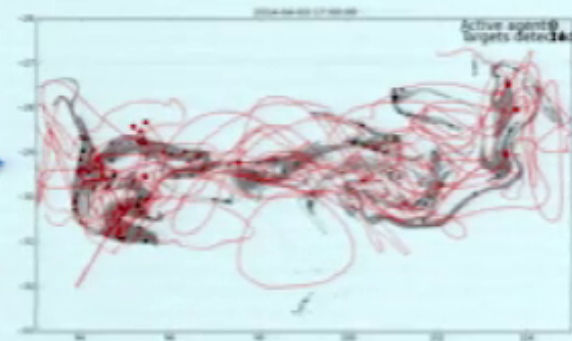
BREAKING NEWS
STUNNING SETBACK IN FLIGHT 370 SEARCH
REPORT SAYS INITIAL REVIEW OF PHOENIX VA ALSO FOUND M

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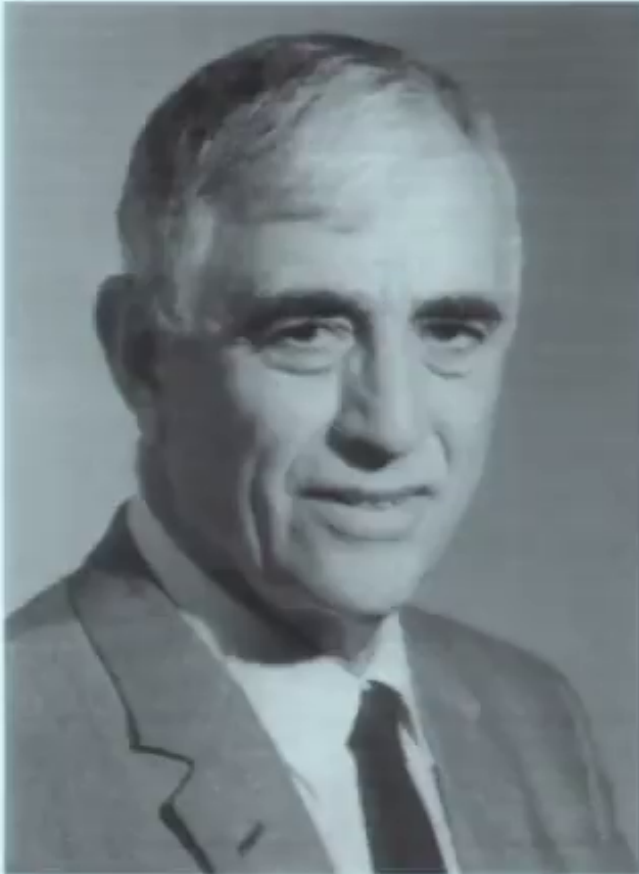
STORY HIGHLIGHTS

(CNN) -- When authorities confirmed last month that four "pings"

Talk Outline:



The other Koopman Legacy



Bernard Osgood Koopman

Bernard Osgood Koopman (1900-1981):

- Better known for the Koopman Operator Theory
- But also the father of the “Theory of Search”:

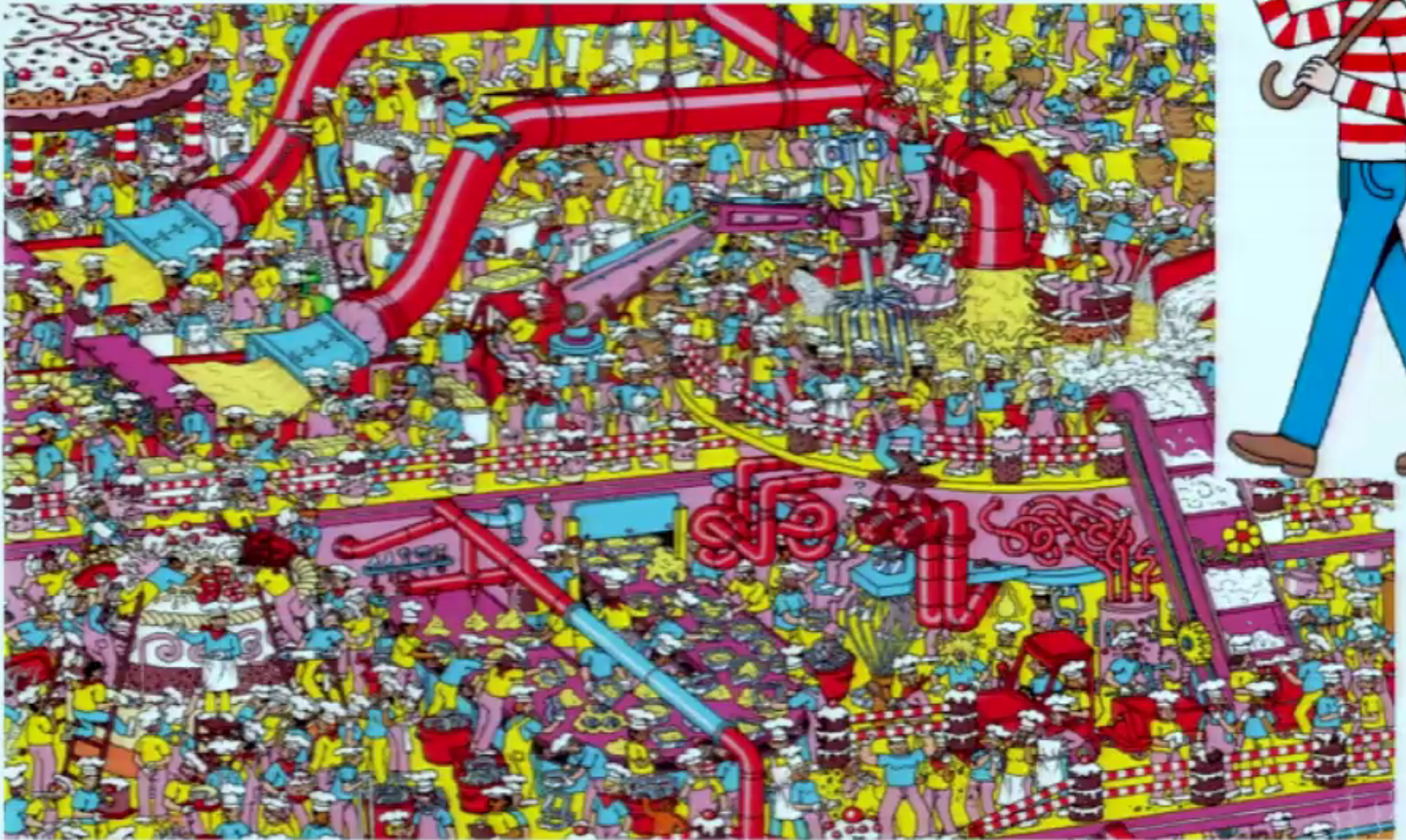
[1] The Theory of Search; I. Kinematic Bases. *Opns. Res.* 4, 324-346 (1956).

[2] The Theory of Search; II. Target Detection. *Opns. Res.* 4, 503-531 (1956).

[3] The Theory of Search; III. The Optimum Distribution of Searching Effort. *Opns. Res.* 5, 613-656 (1957).

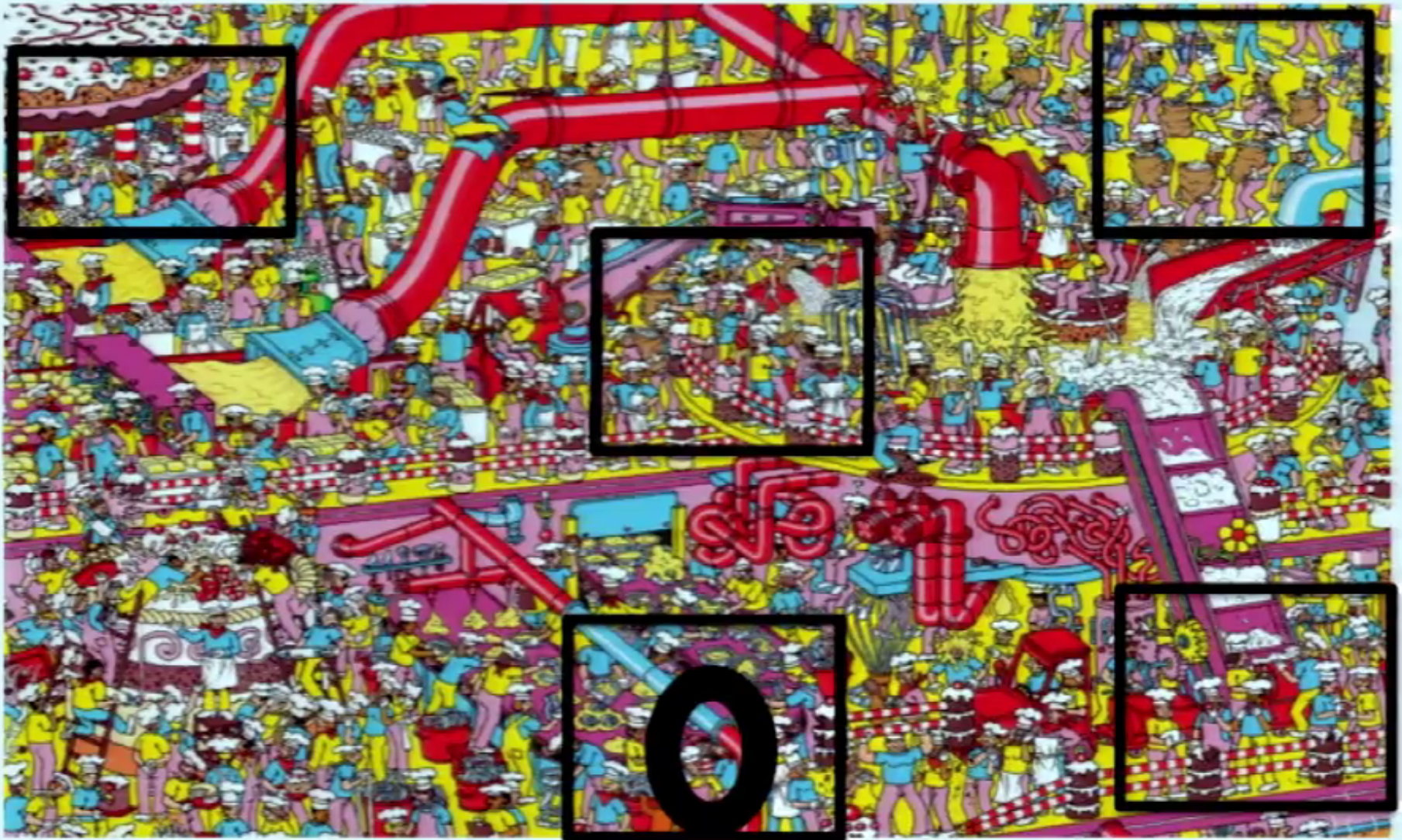
Typical search strategy in static environment:

Where is Waldo?



Typical search strategy in static environment

Random Search



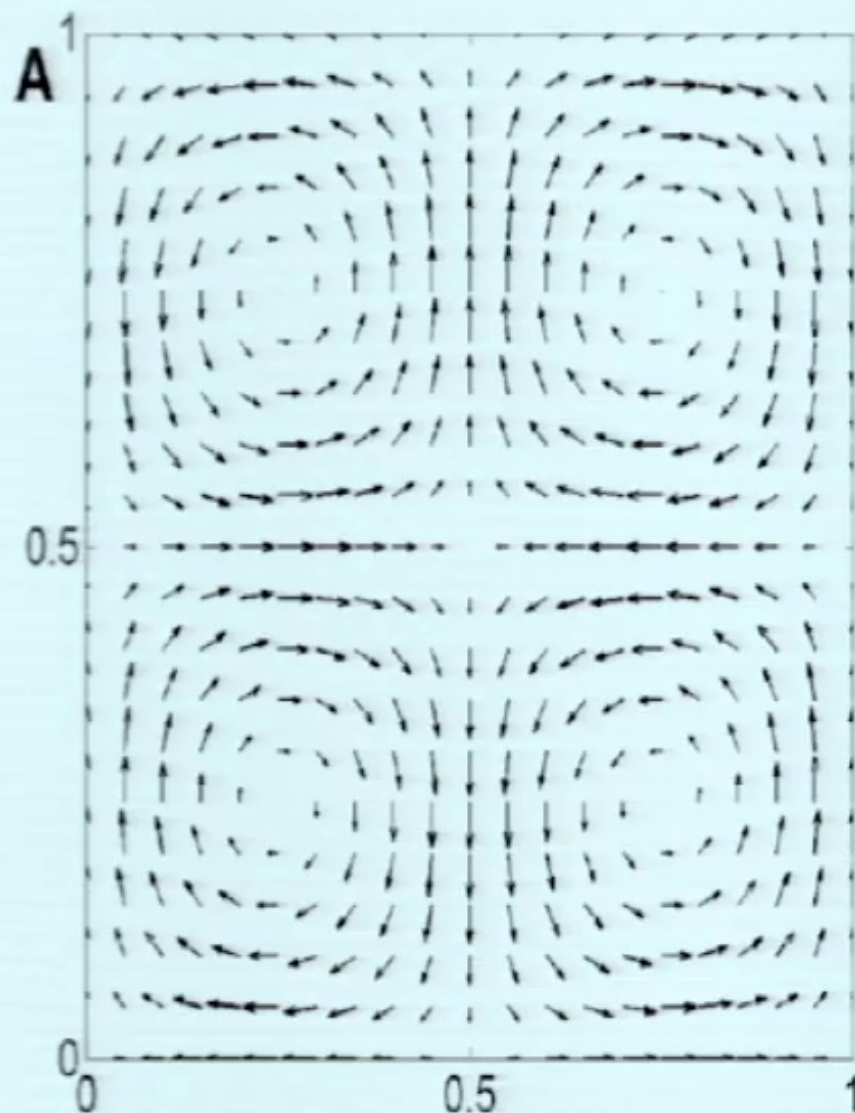
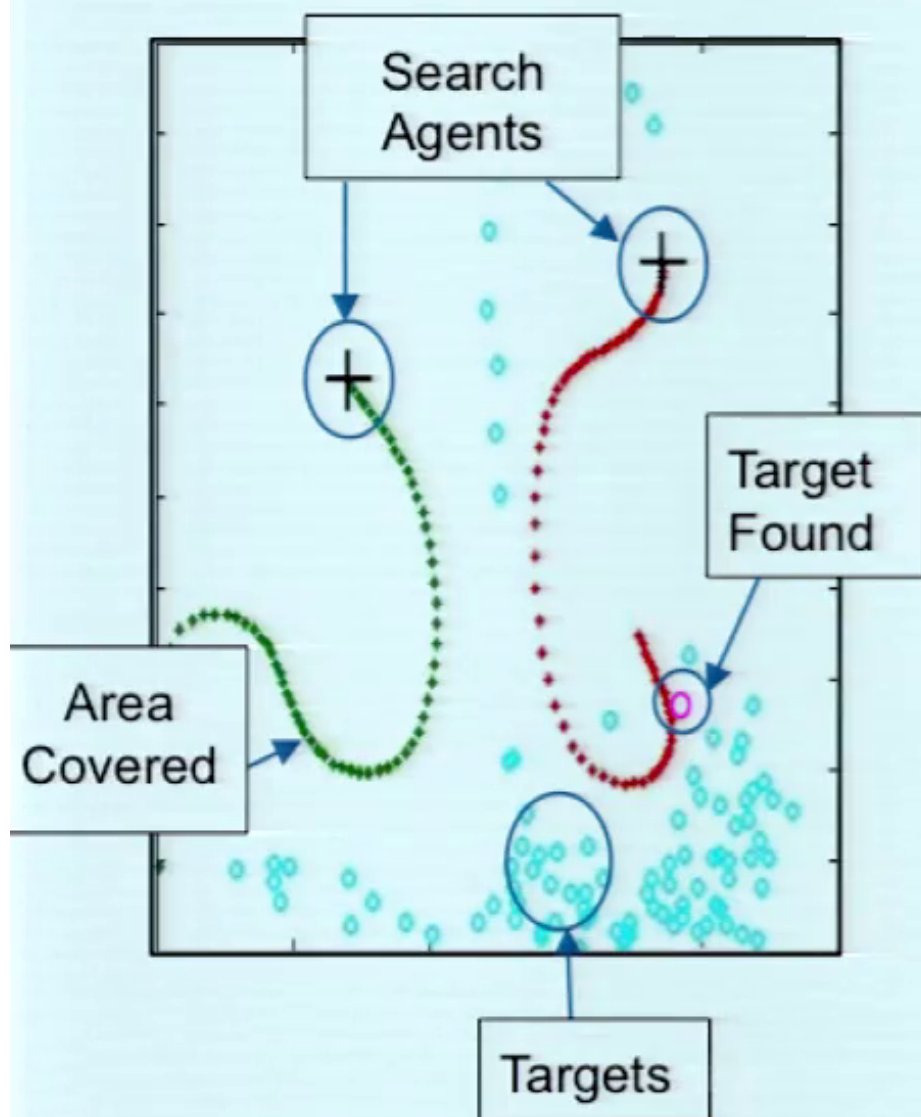
Typical search strategy in static environment

Lawnmower Search



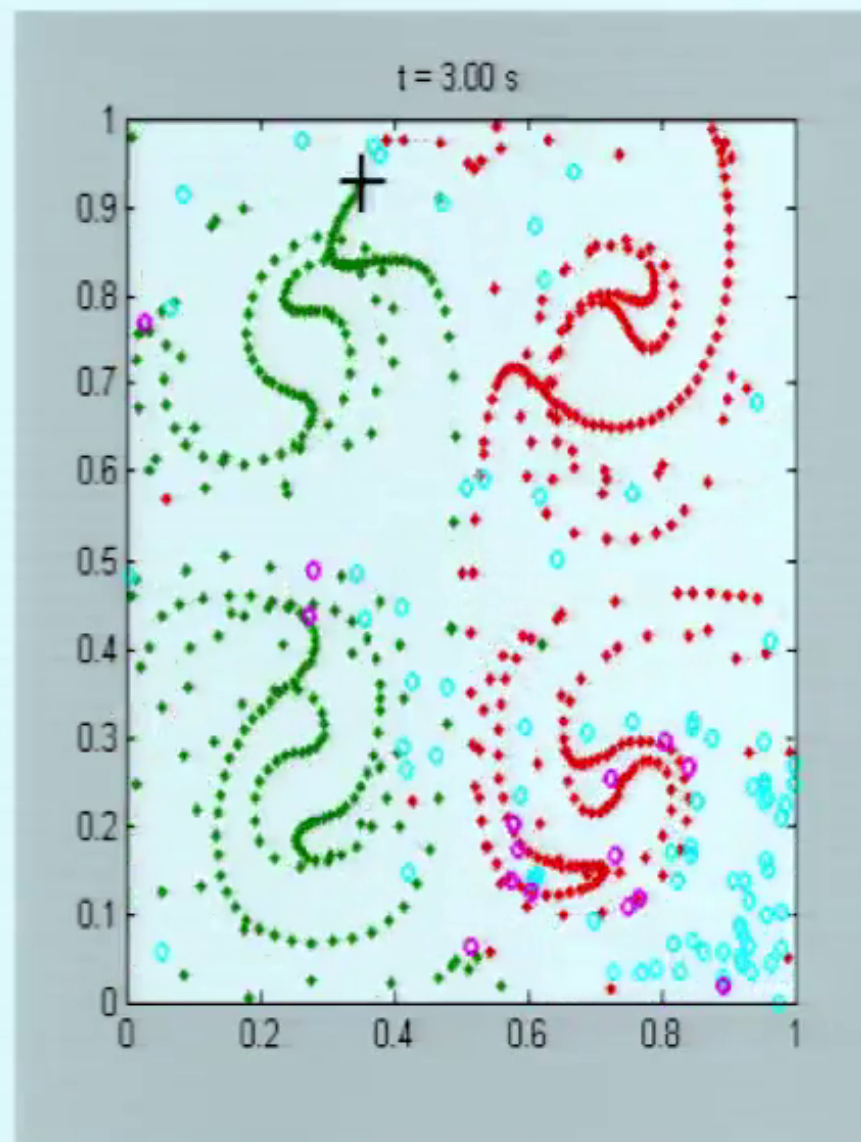
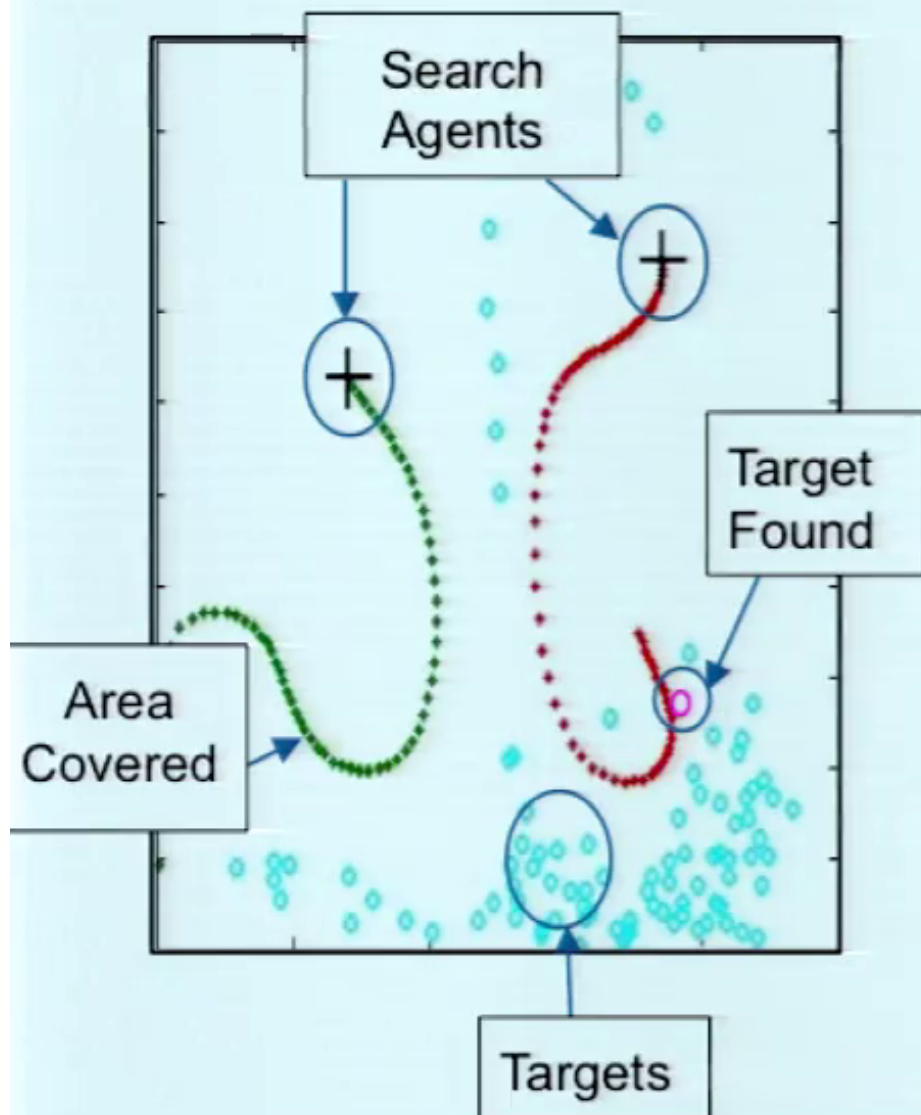
Search of targets moving in a double gyre

Lawnmower strategy



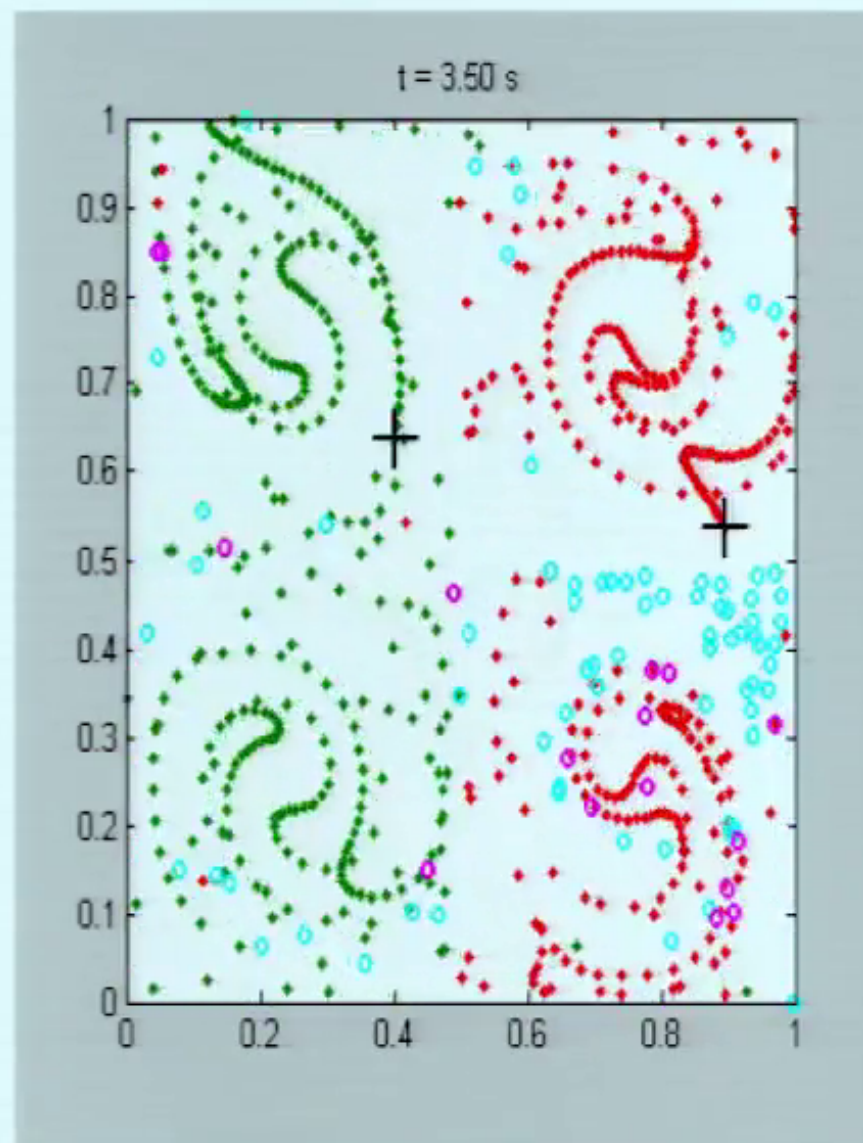
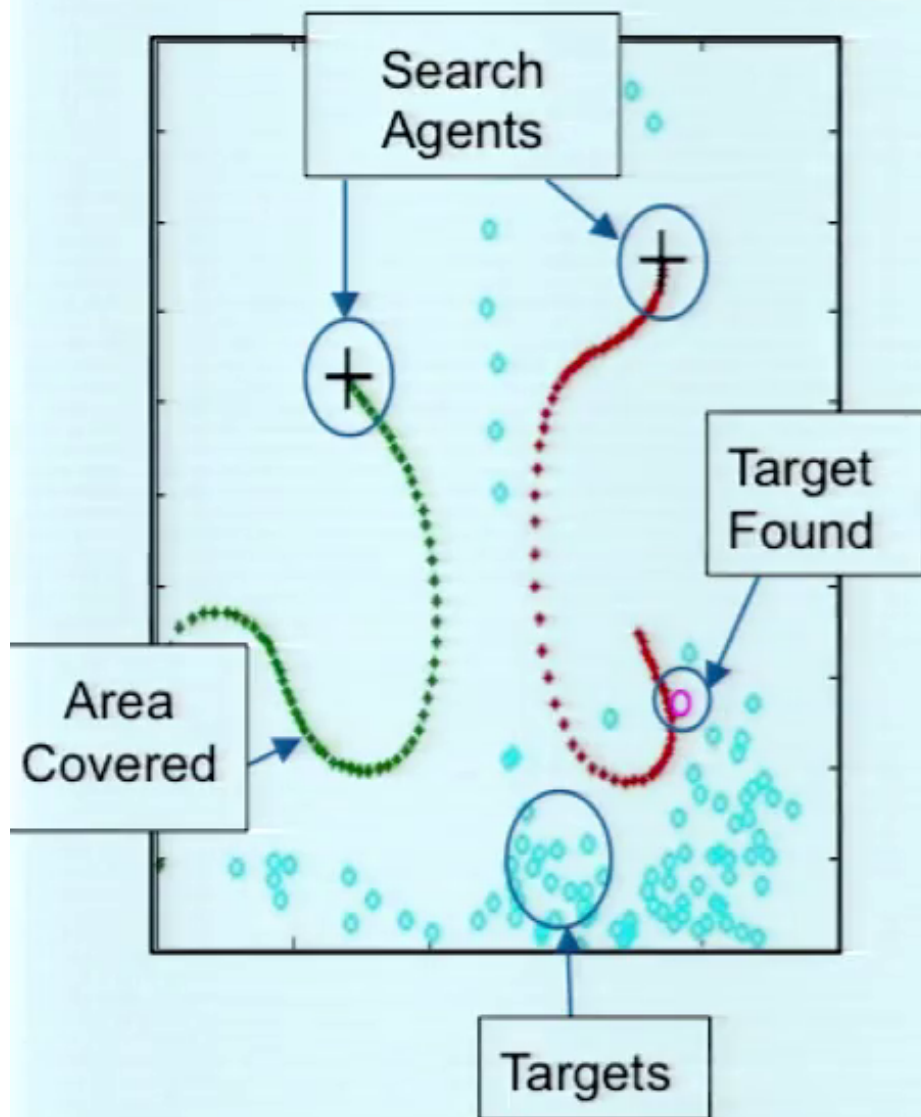
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Lawnmower strategy



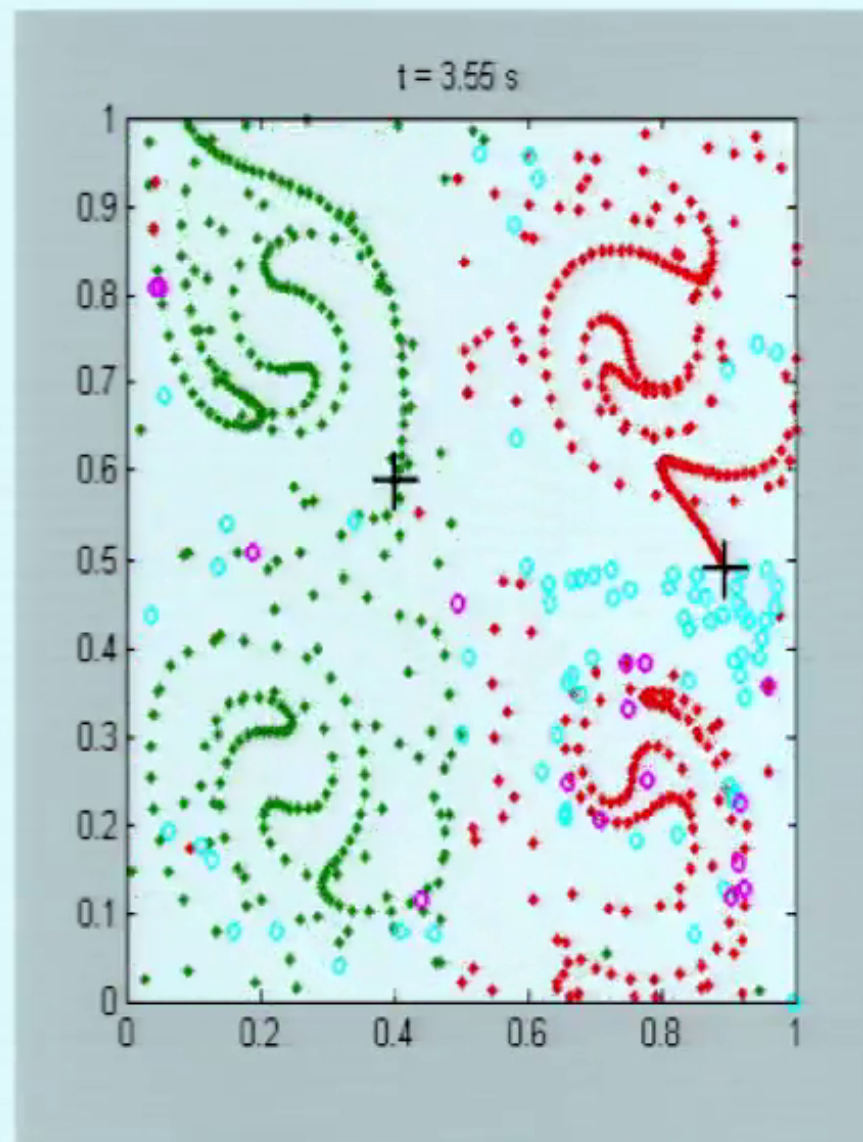
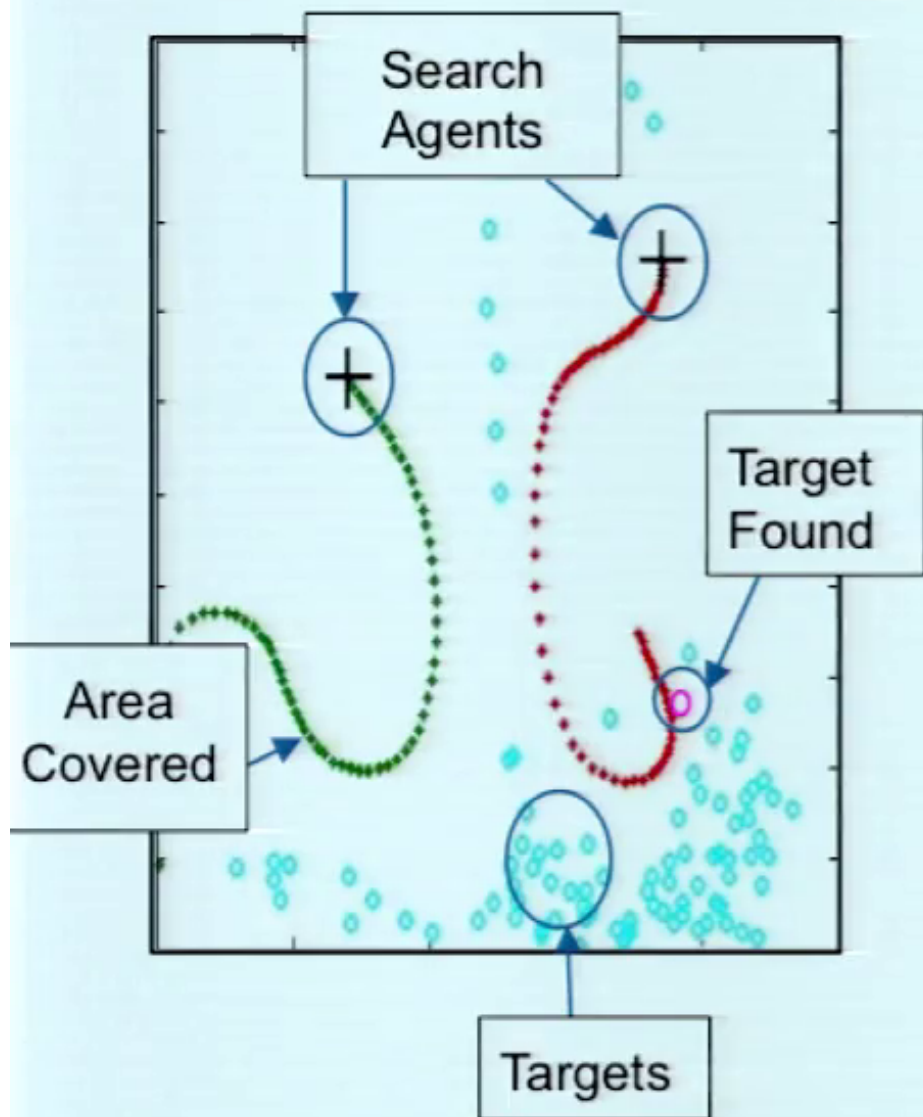
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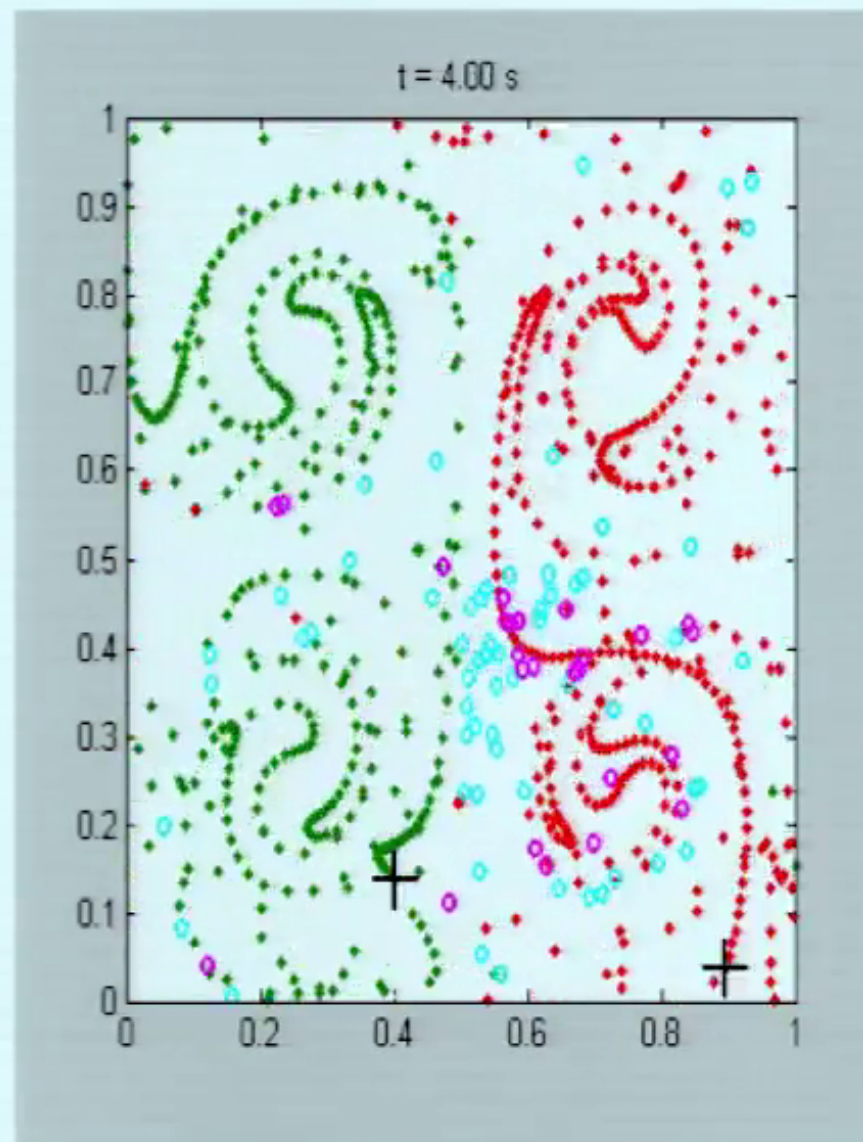
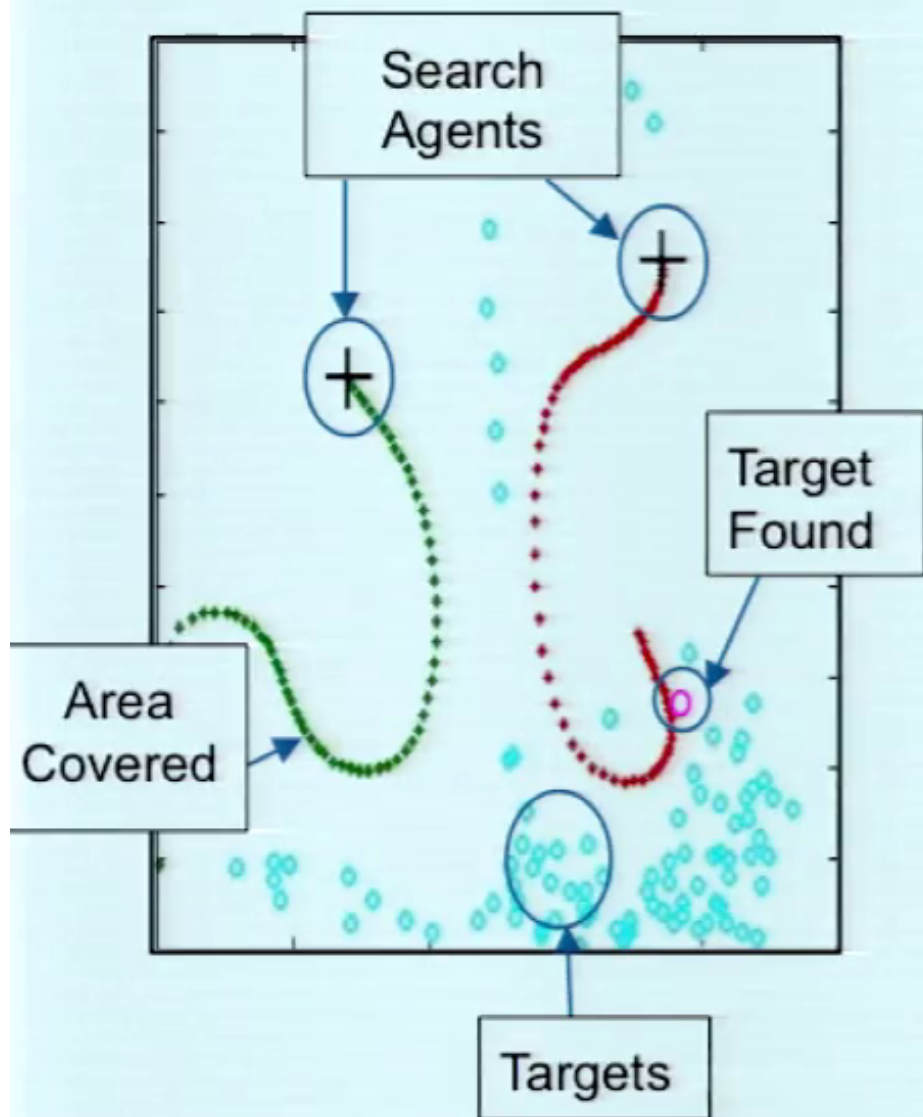
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Lawnmower strategy



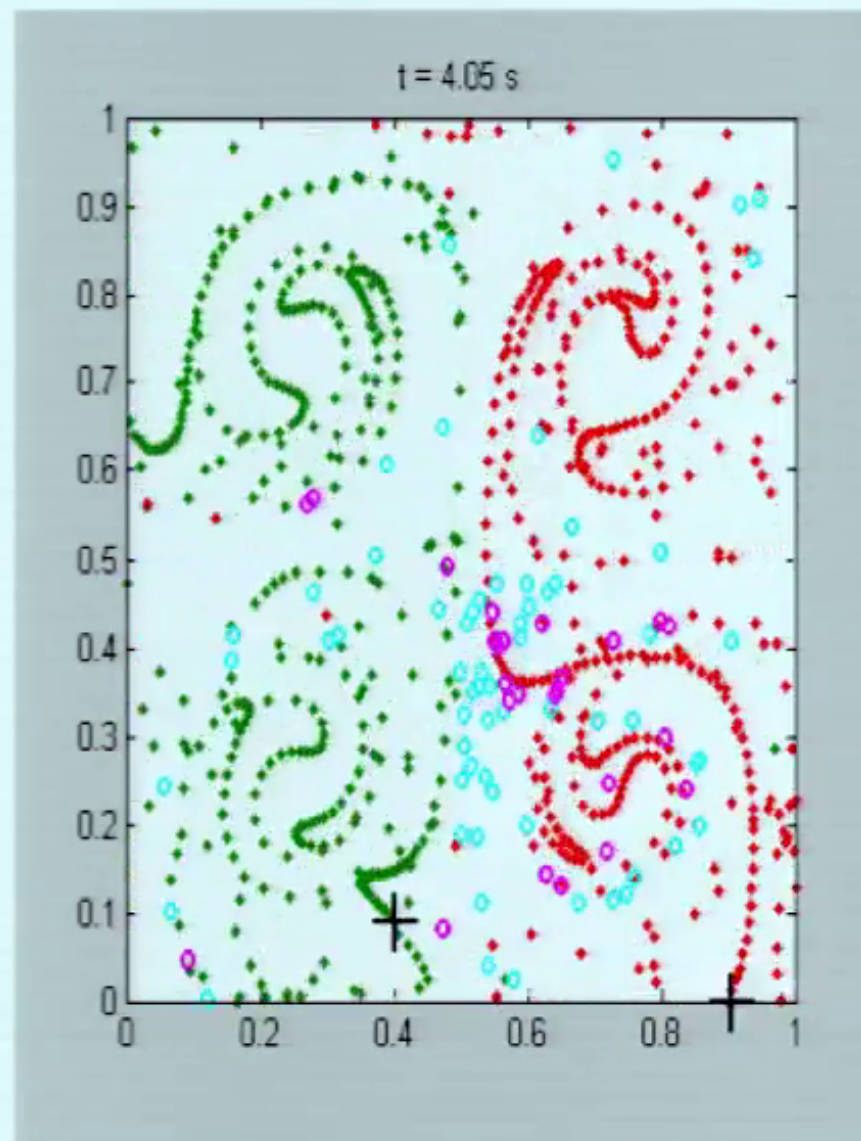
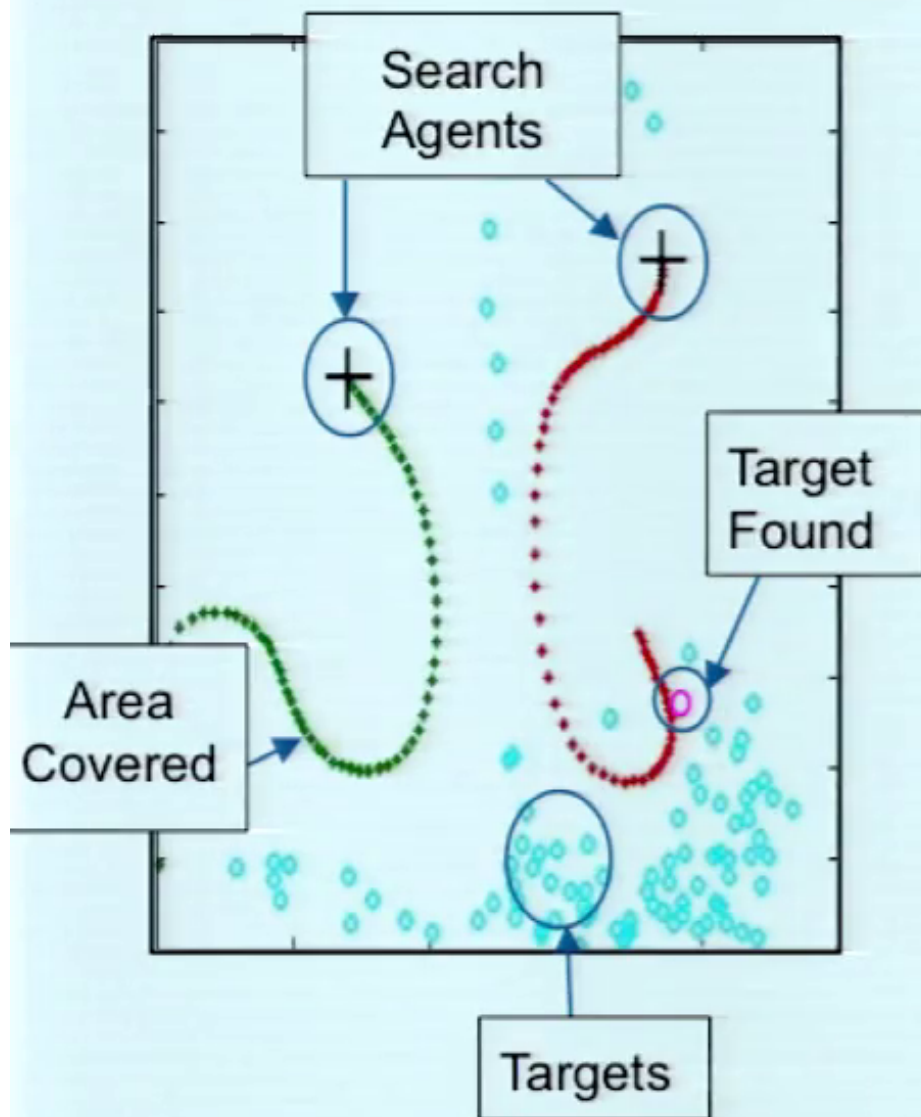
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Lawnmower strategy



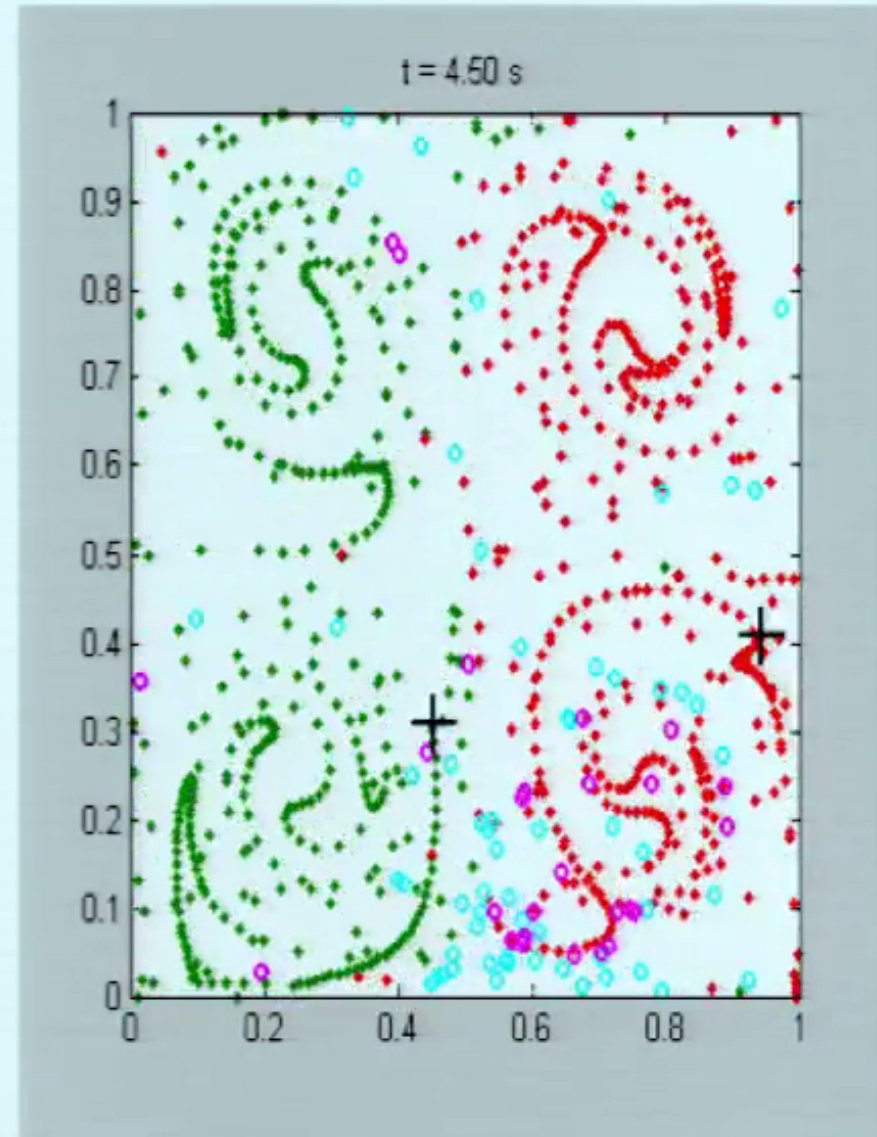
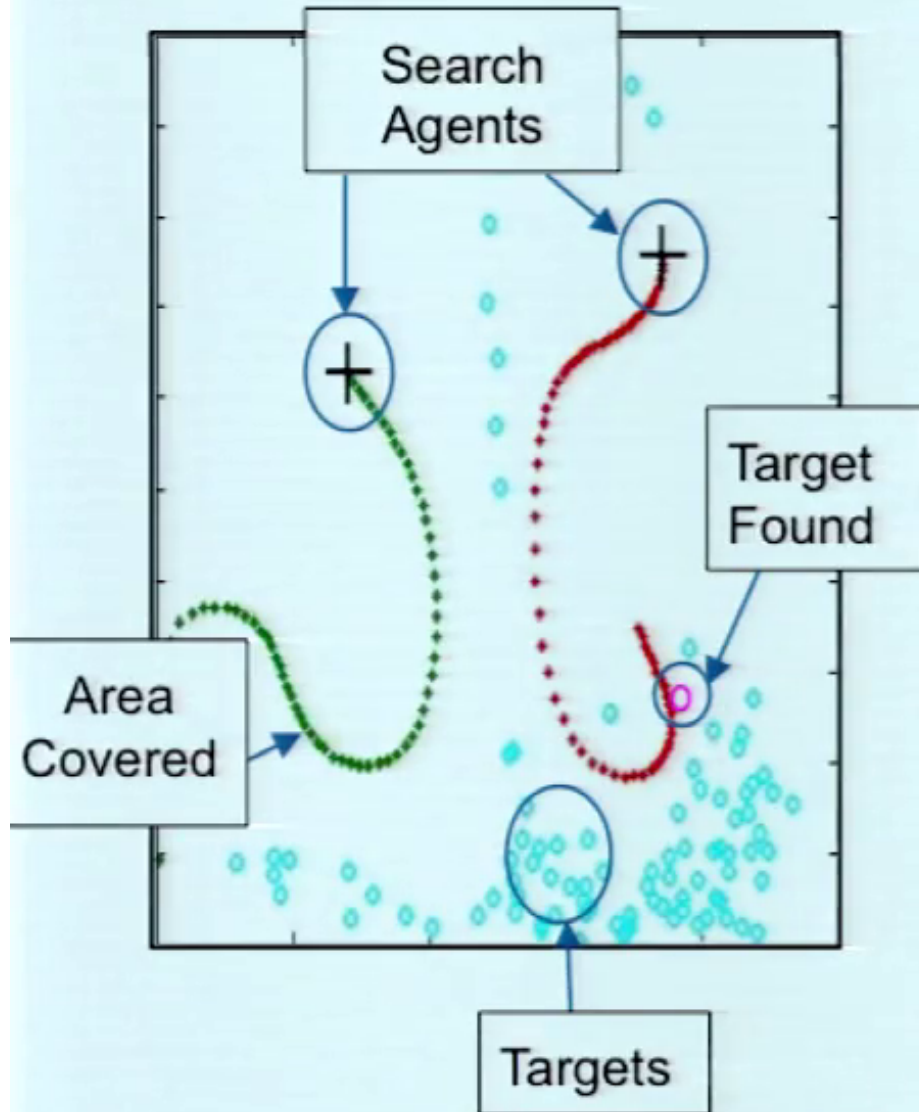
Search of targets moving in a double gyre

Lawnmower strategy



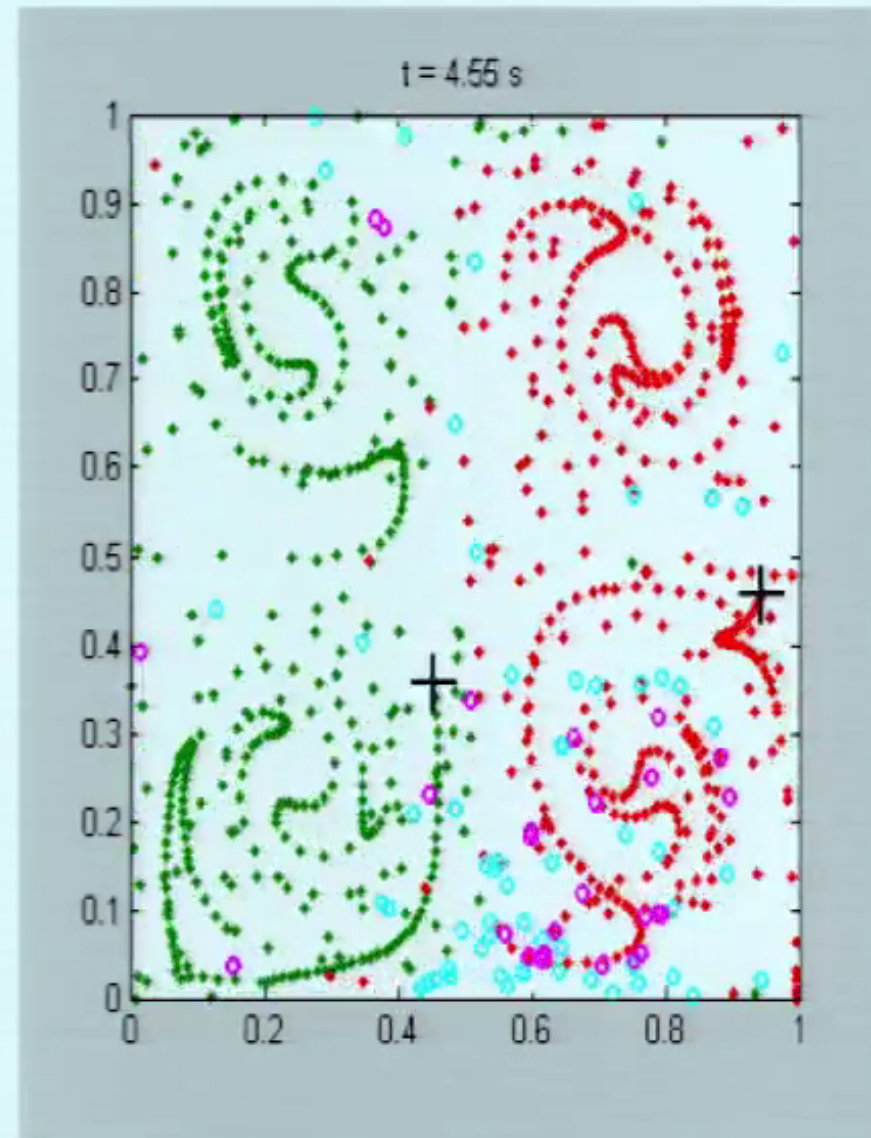
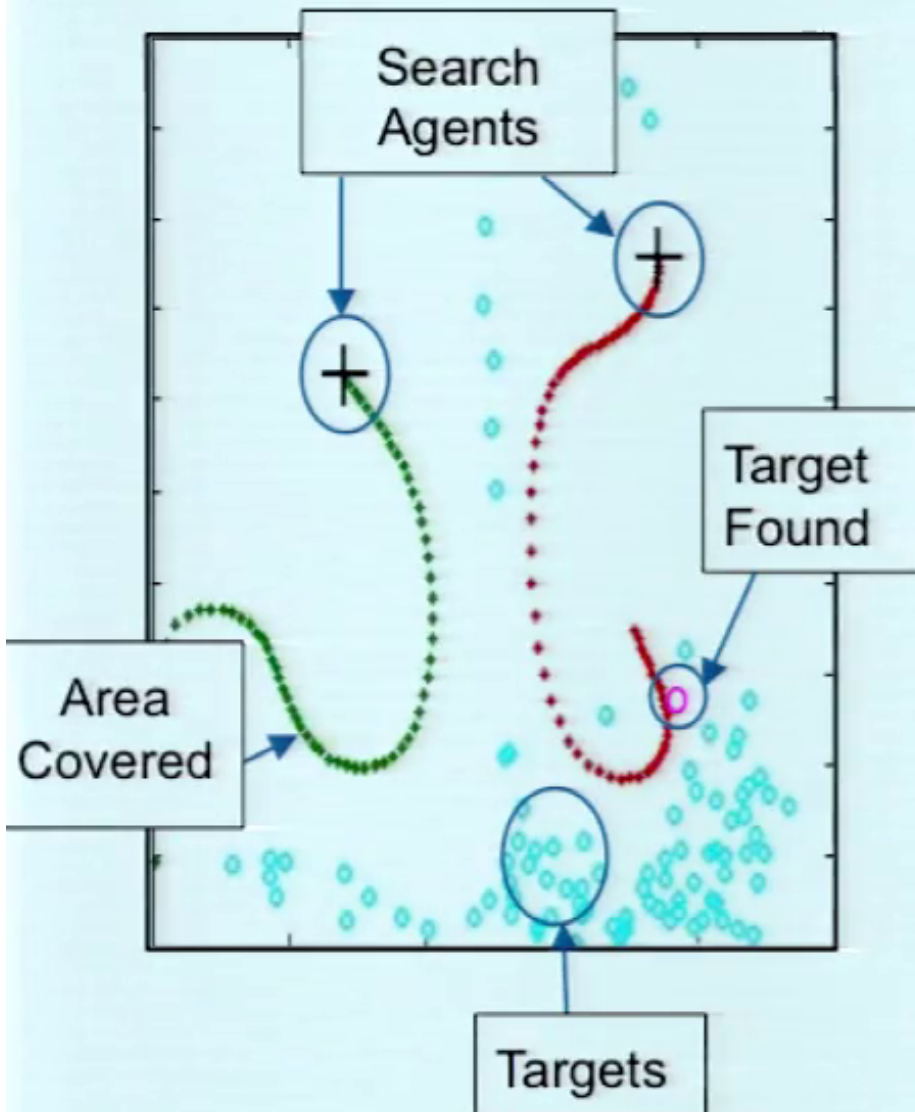
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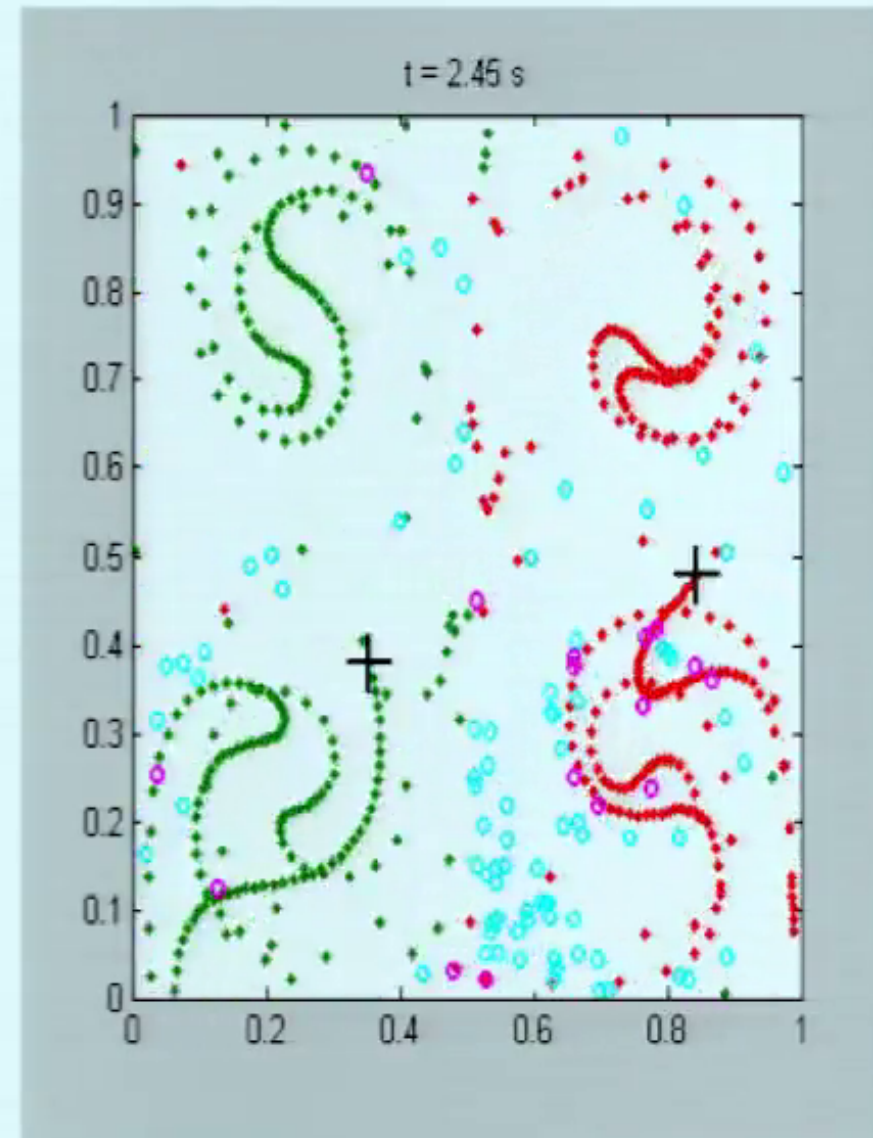
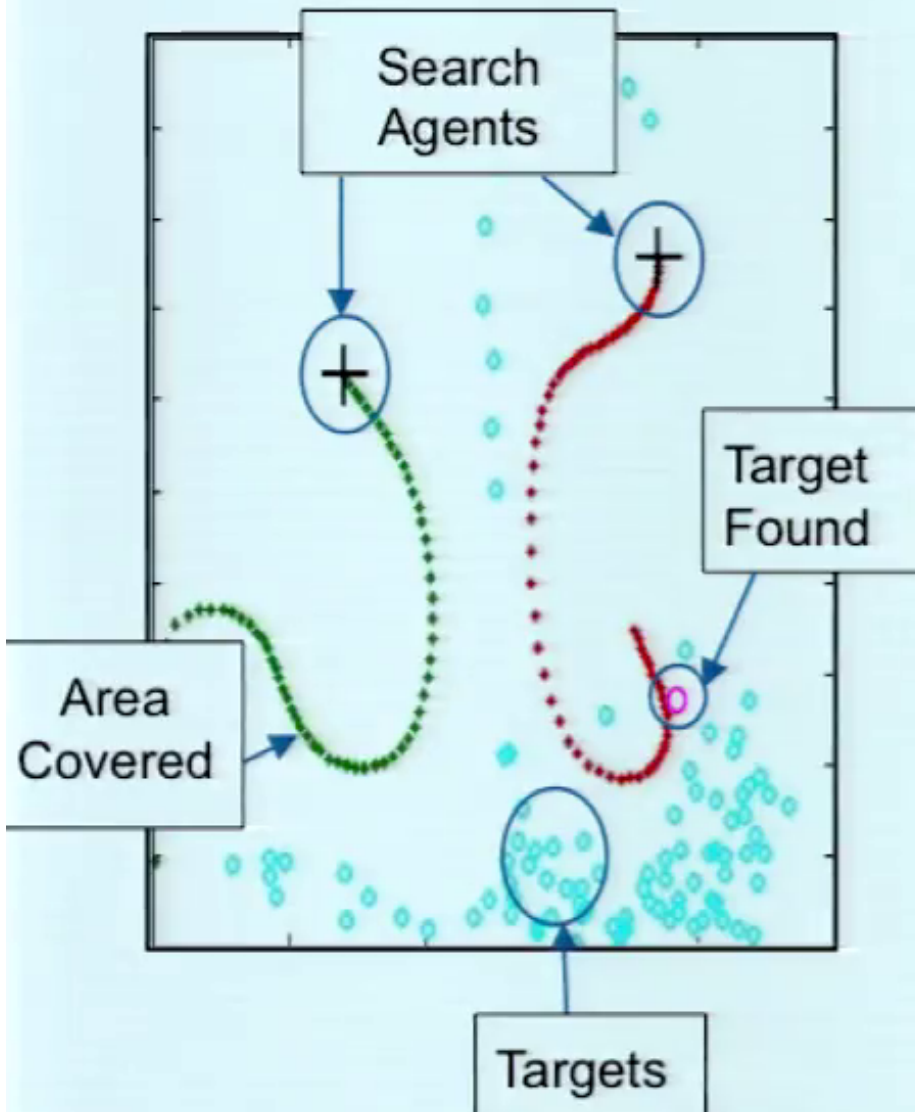
Search of targets moving in a double gyre

Lawnmower strategy



Search of targets moving in a double gyre

Lawnmower strategy



Problem Statement

Model: $X(t)=T(X(0),0,t)$

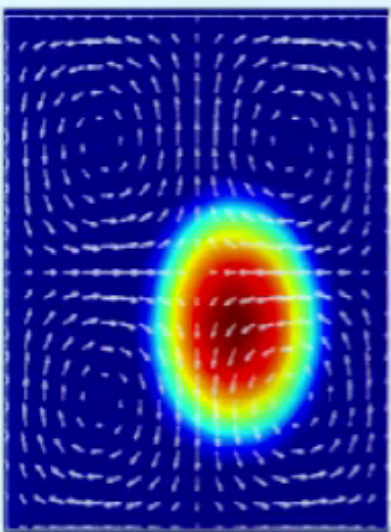
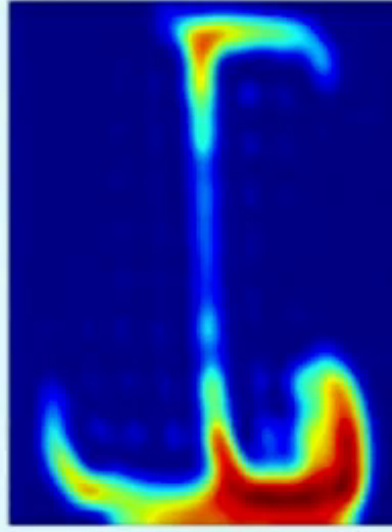
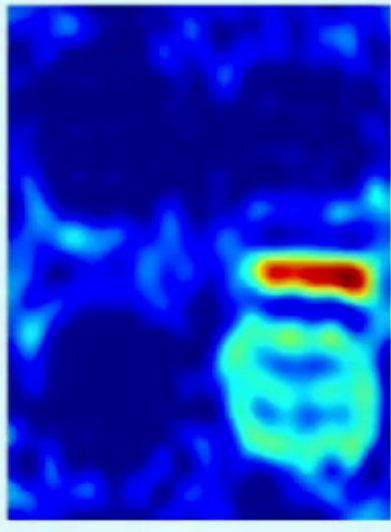
Dynamic of initial target density: $\mu^t(B(x,r)) = \int_{B(x,r)} d\mu^t = \int_{T^{-1}(.,0,t)(B(x,r))} d\mu^0.$

$$v^t(x) = \frac{\mathcal{F}(t, \mu^t(x))}{\int_U \mathcal{F}(t, \mu^t(x)) dx}$$

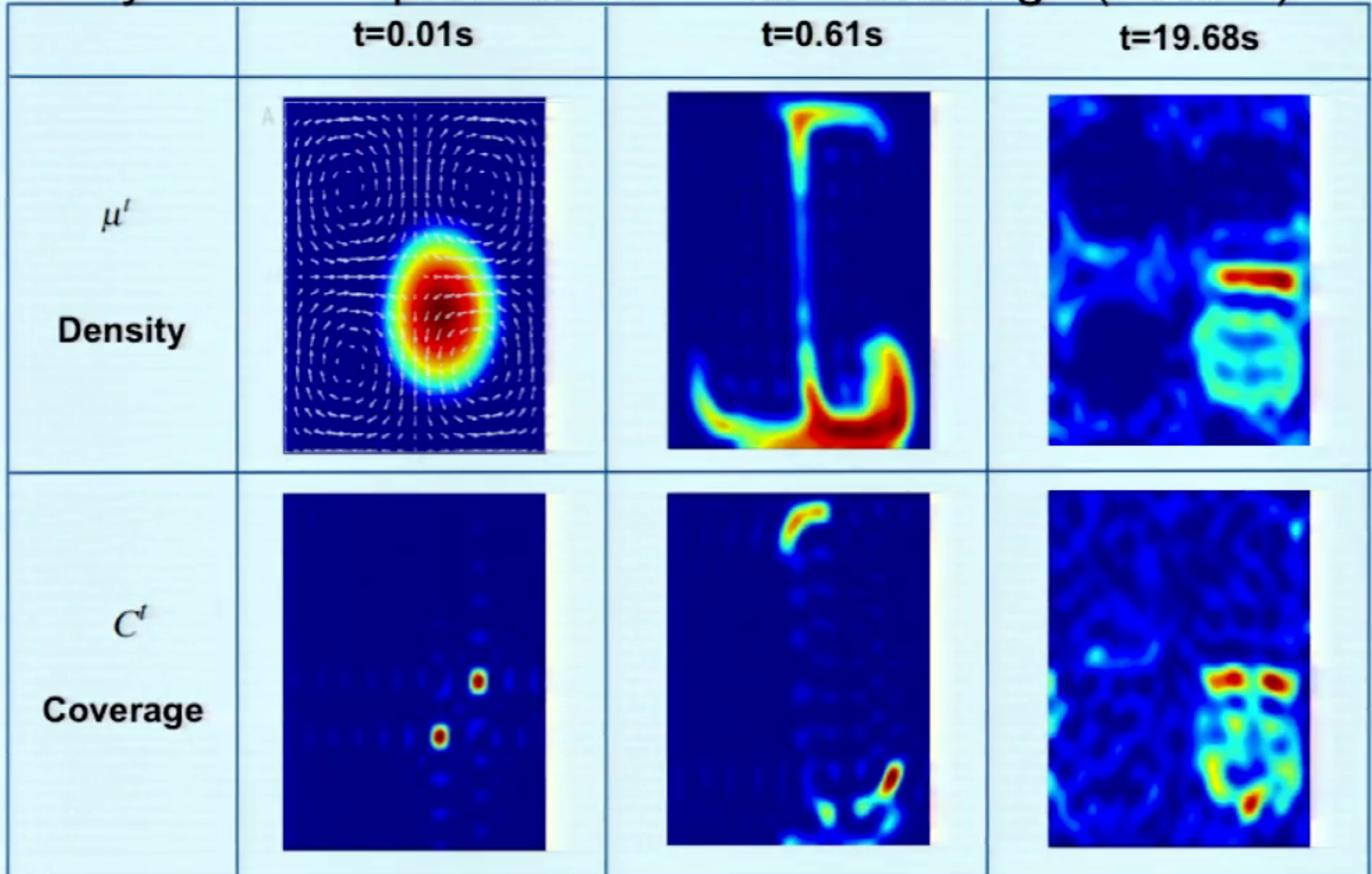
Dynamic of searchers coverage: $C^t(x) = \frac{1}{Nt} \sum_{j=1}^N \int_0^t [P^{\tau,t} \delta_{x_j(\tau)}] (x) d\tau,$

Goal: Minimize the difference between Target Density and Coverage

Dynamical Spectral Multiscale Coverage (DSMC)

	t=0.01s	t=0.61s	t=19.68s
μ^f Density			

Dynamical Spectral Multiscale Coverage (DSMC)



Searching using the Mix-Norm aka Dynamical Spectral Multiscale Coverage (DSMC)

Searcher trajectories:

- Minimize instantaneously the measure of difference between coverage and density
 - Sobolev space norm of negative index ($s=(n+1)/2$)
 - Weight on the energy contained in various spectral modes.
 - Favors the large spatial scales as long as their energy is large

$$\phi^2(t) = \|C^t - v^t\|_{H^{-s}}^2 = \sum_{K \in \mathbb{Z}^{*n}} \Lambda_k |s_k(t)|^2,$$

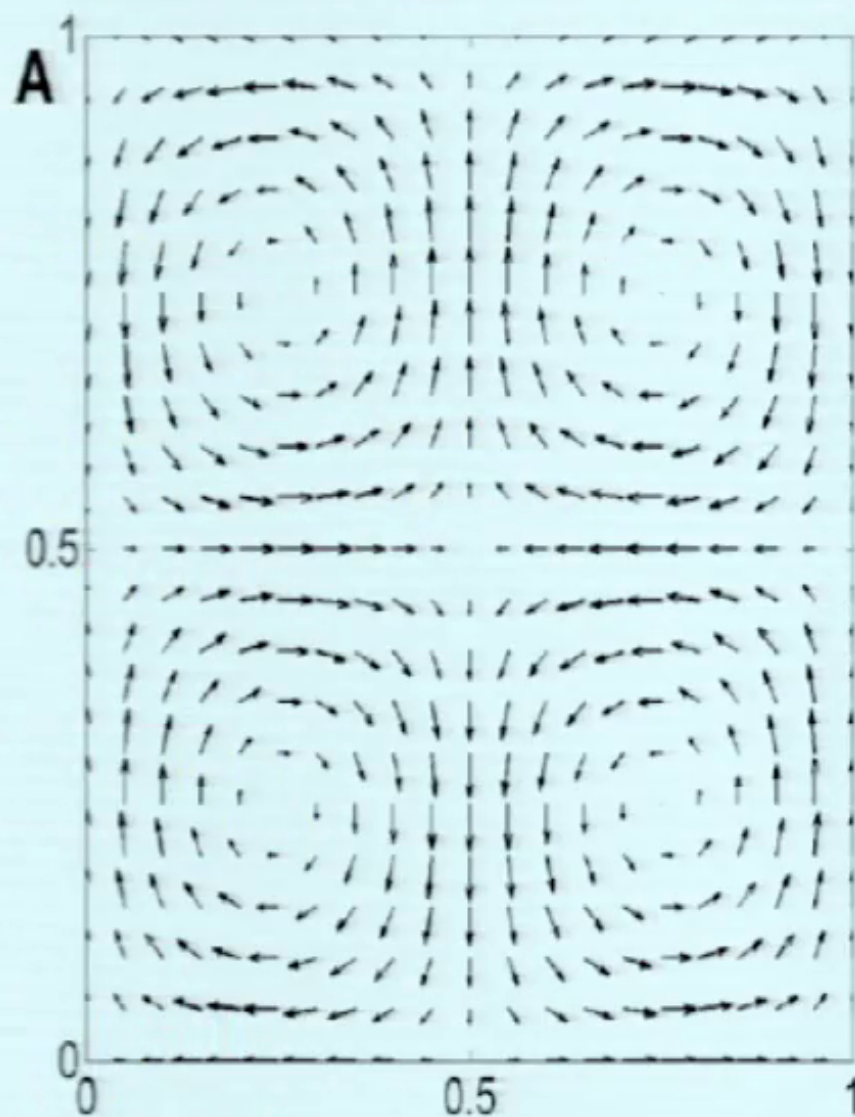
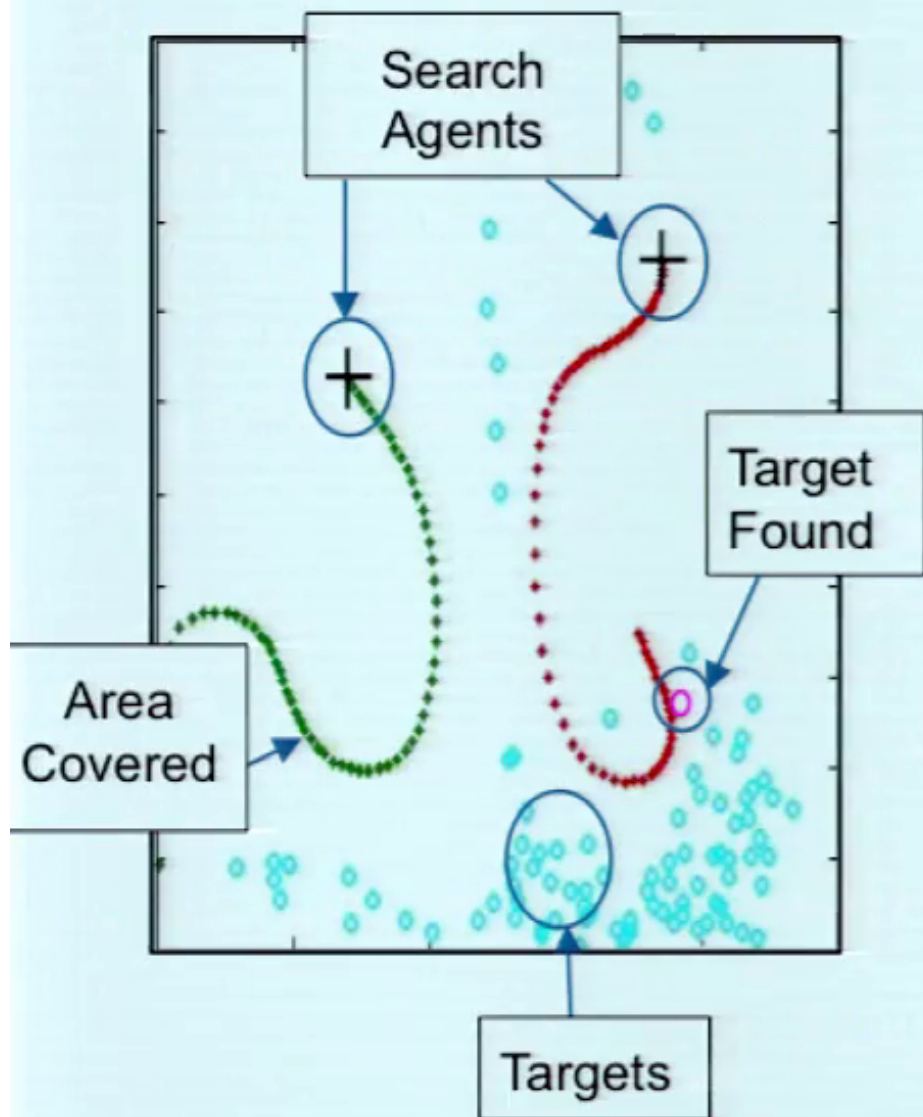
$$\text{where } s_k(t) = c_k(t) - v_k(t), \quad \Lambda_k = \frac{1}{(1 + \|k\|^2)^s}$$

$$c_k(t) = \langle C^t, f_k \rangle \text{ and } v_k(t) = \langle v^t, f_k \rangle.$$

- Simple feedback control law: $u_j^*(t) = -u_{max} \frac{B_j}{\|B_j(t)\|_2}$,
 where $B_j(t) = \sum_K \Lambda_k S_k(t) \nabla f_k(x_j(t))$.

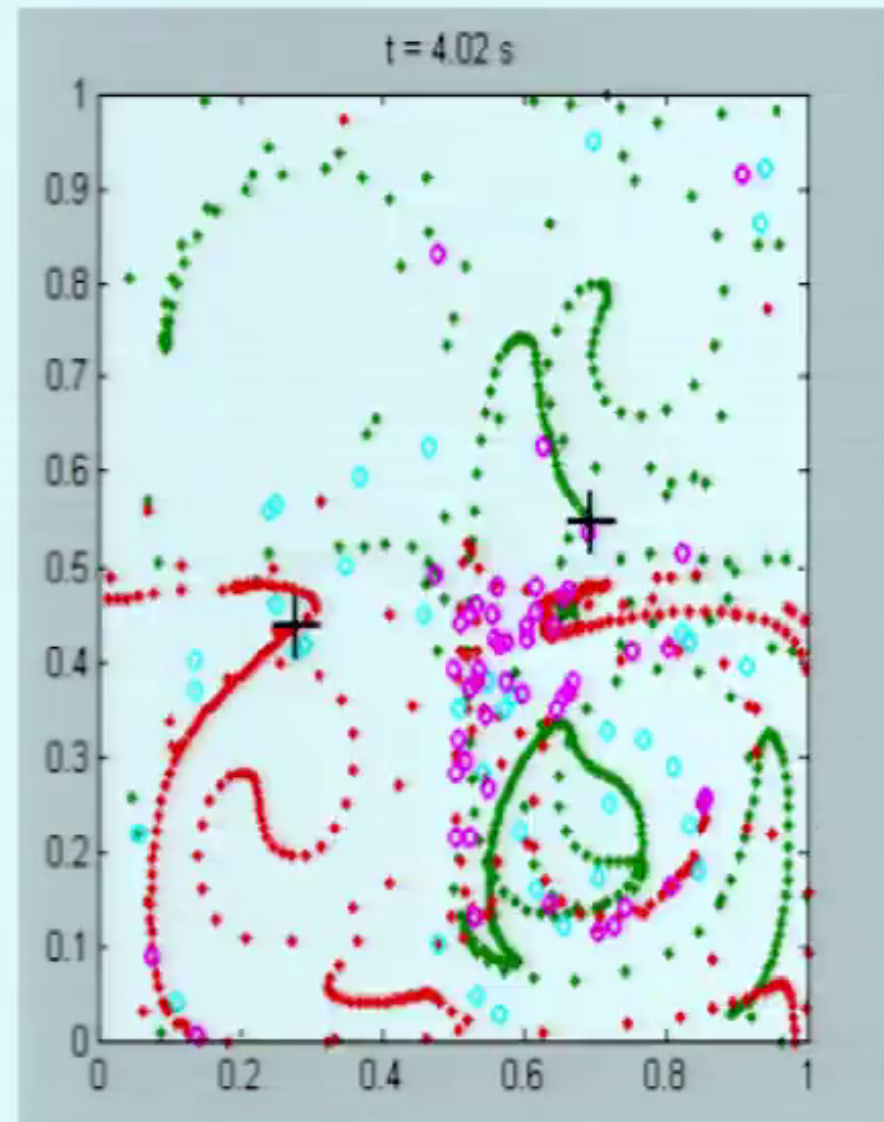
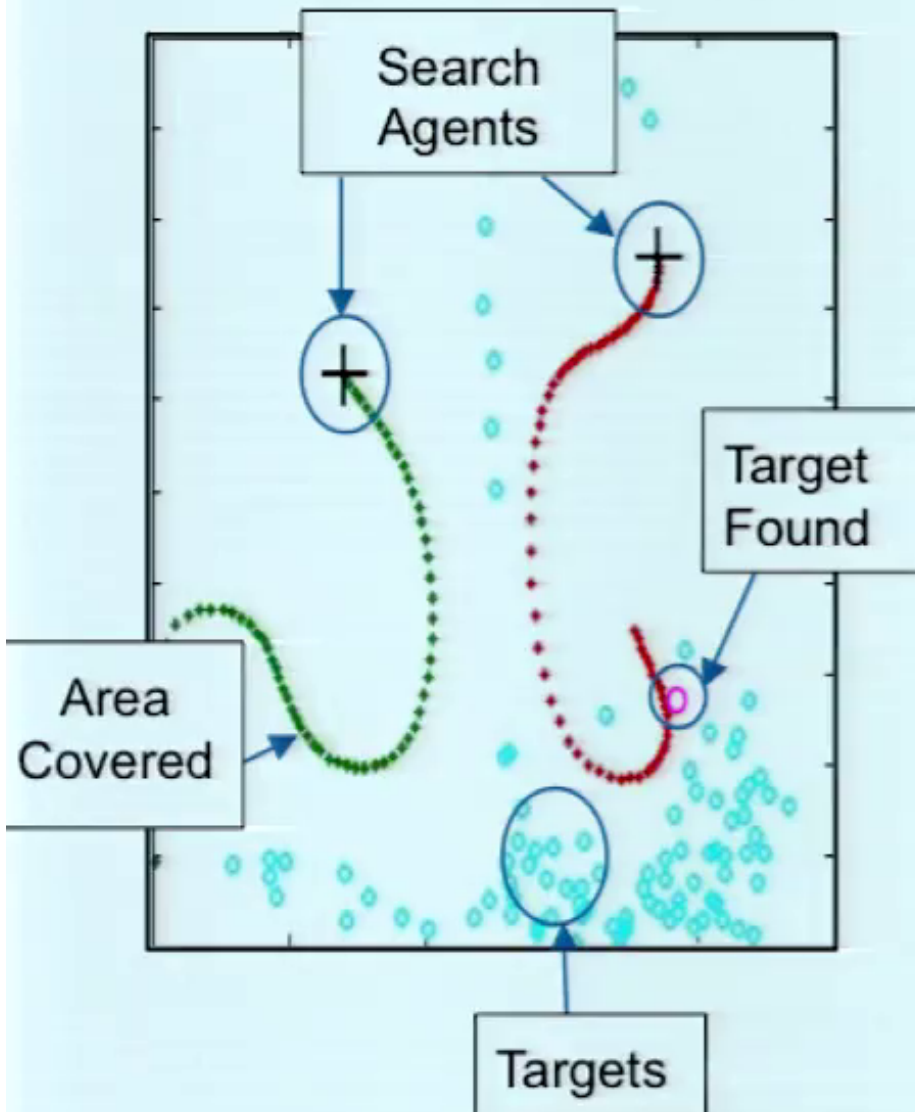
Search of targets moving in a double gyre

DSMC strategy



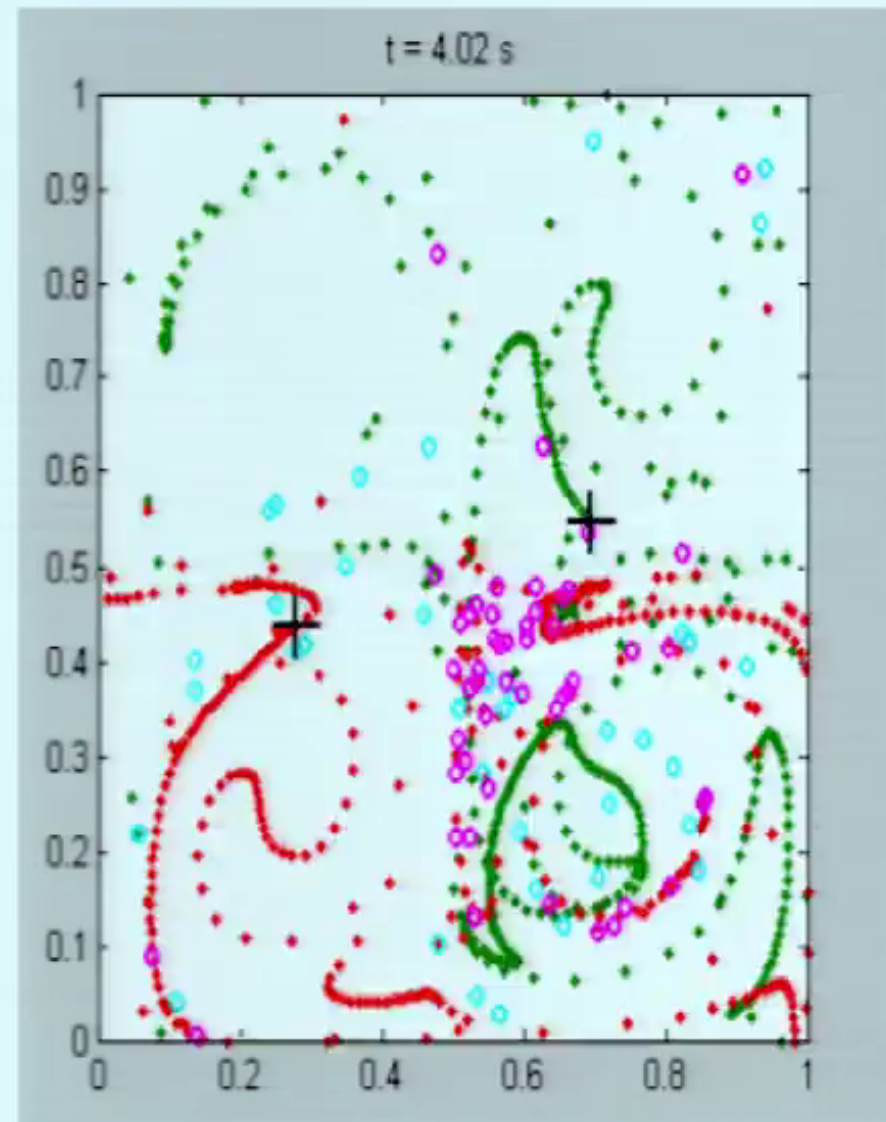
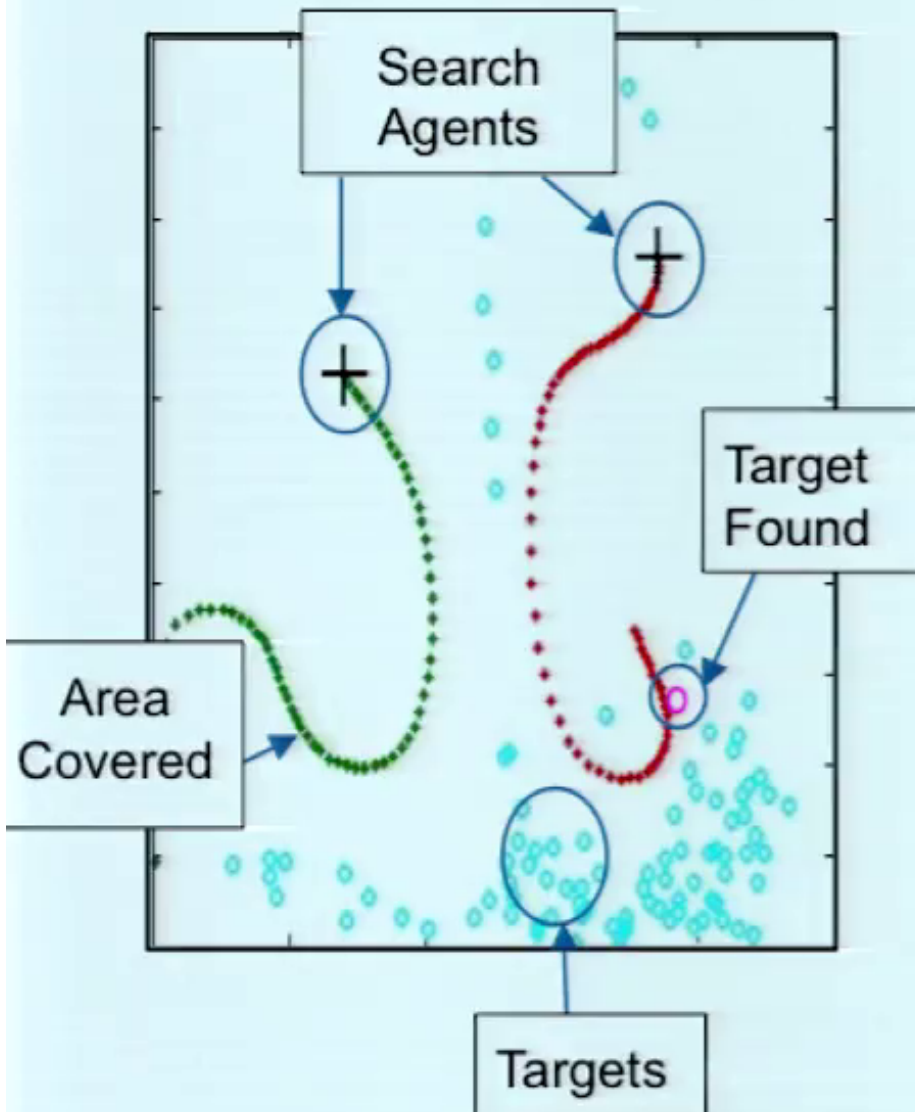
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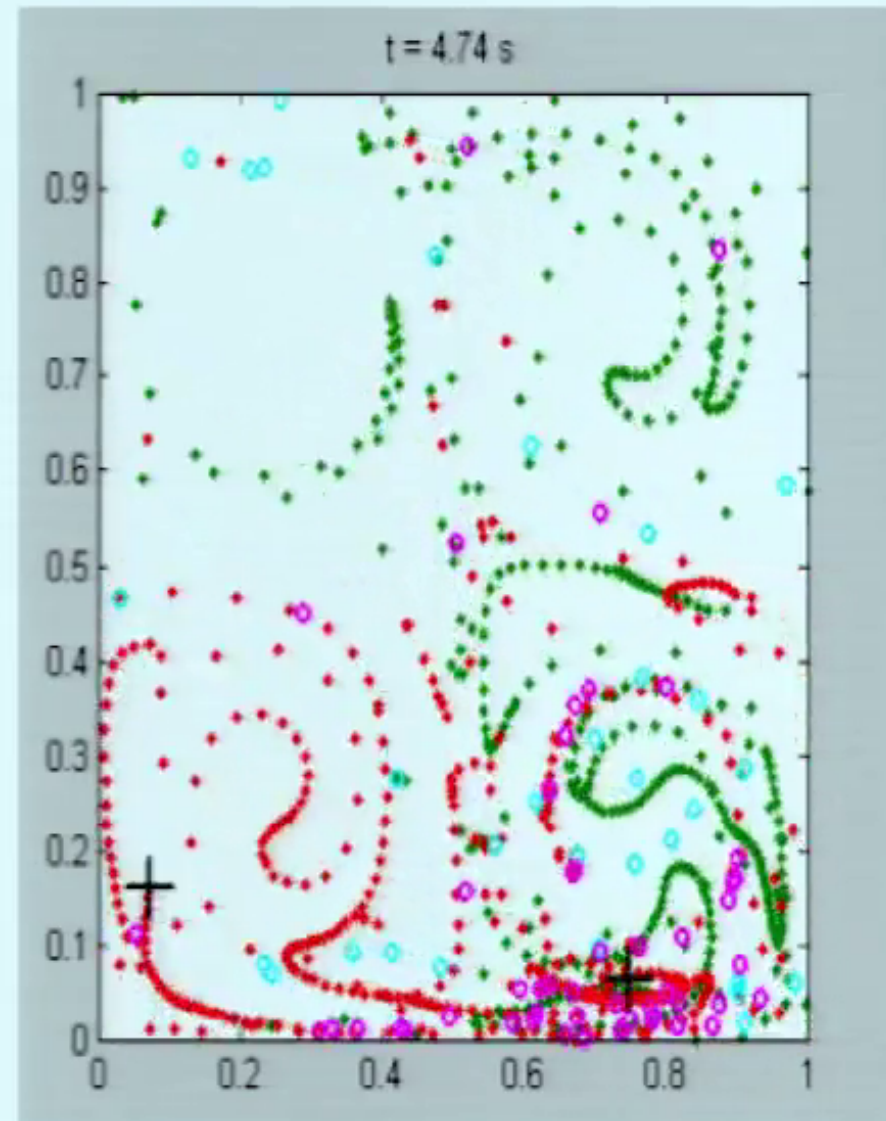
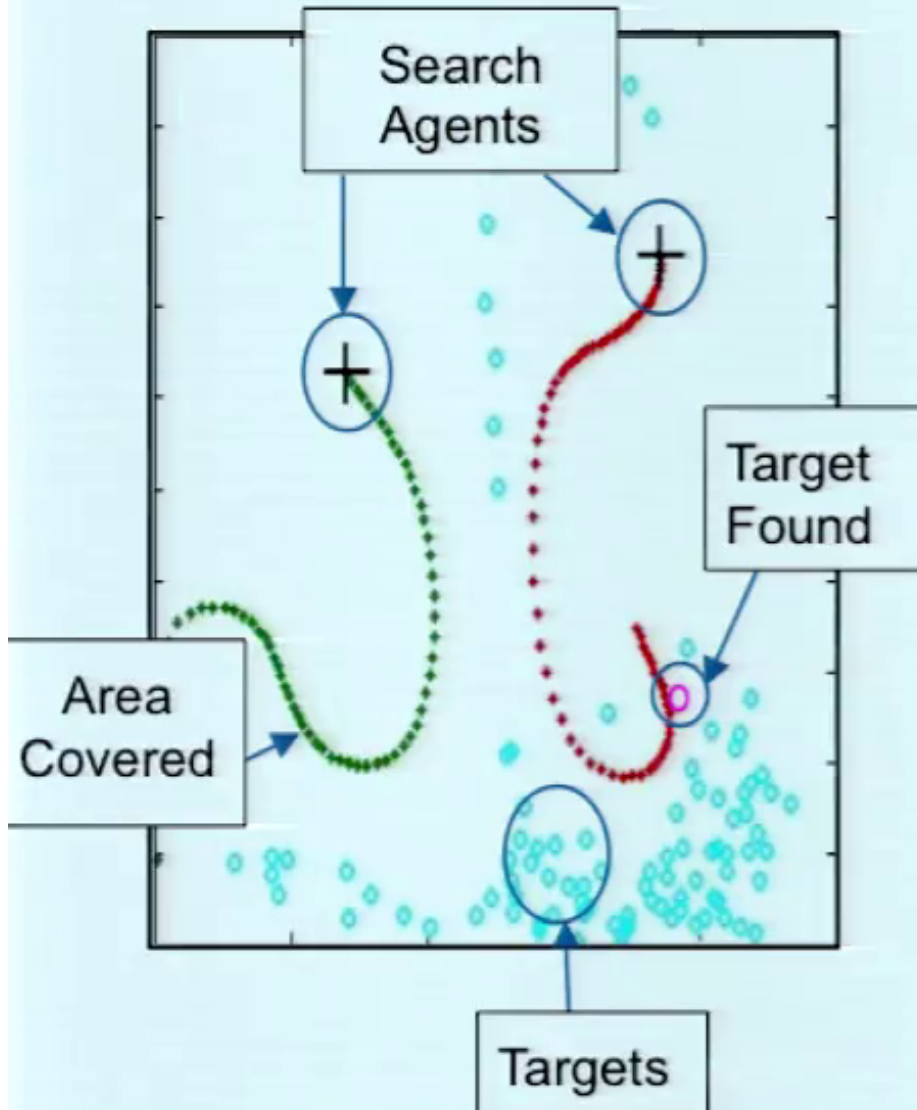
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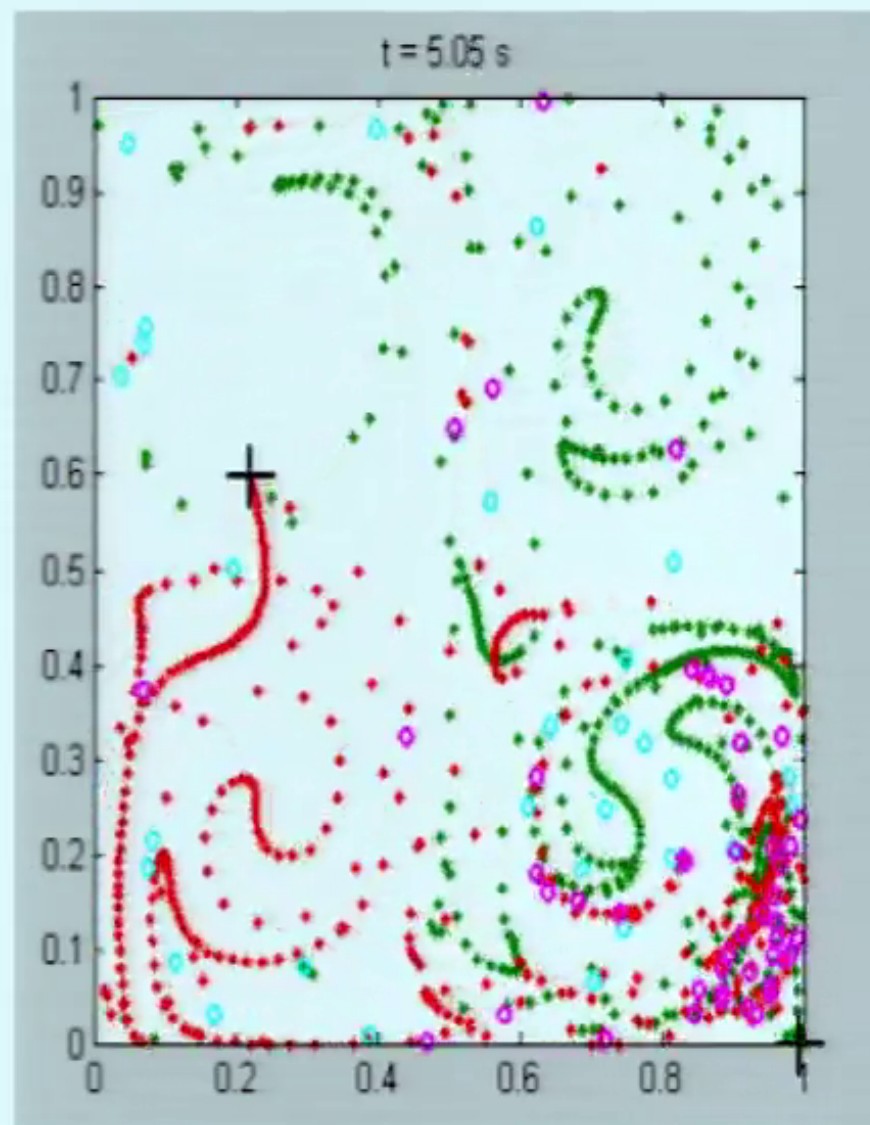
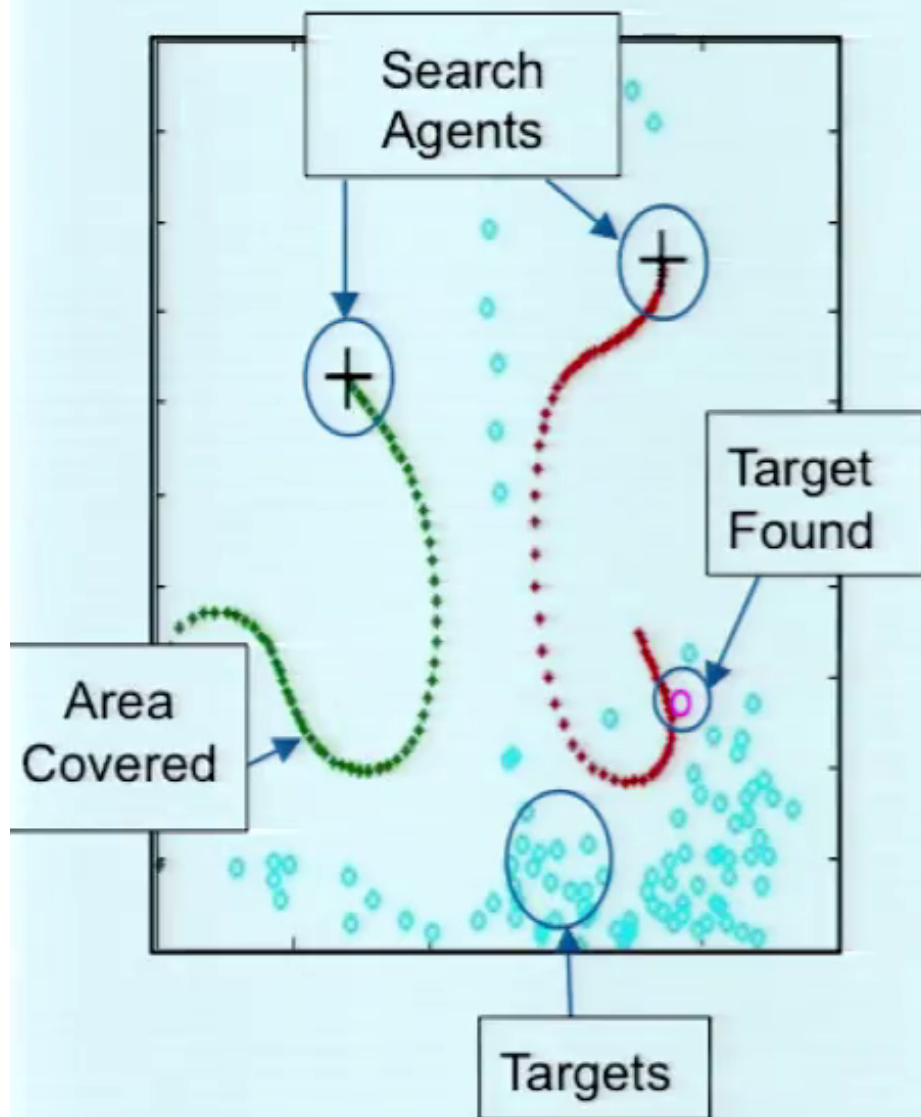
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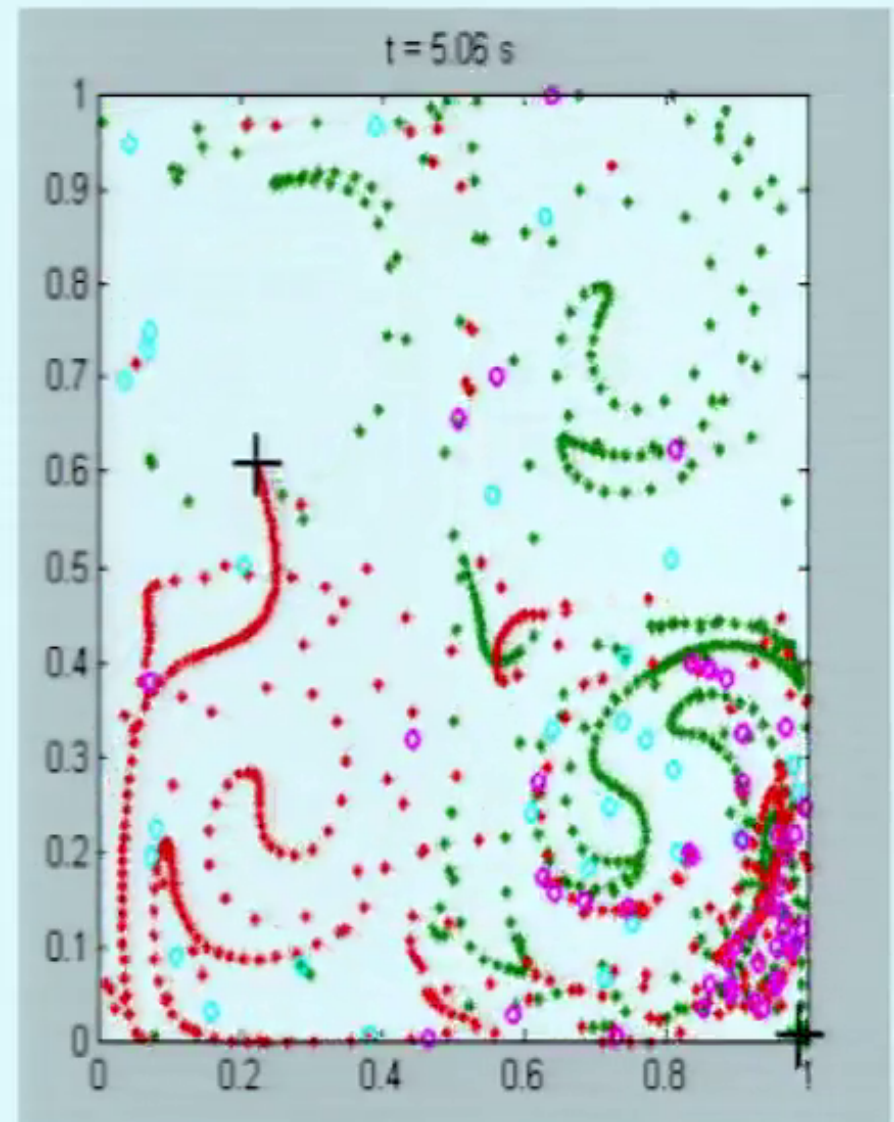
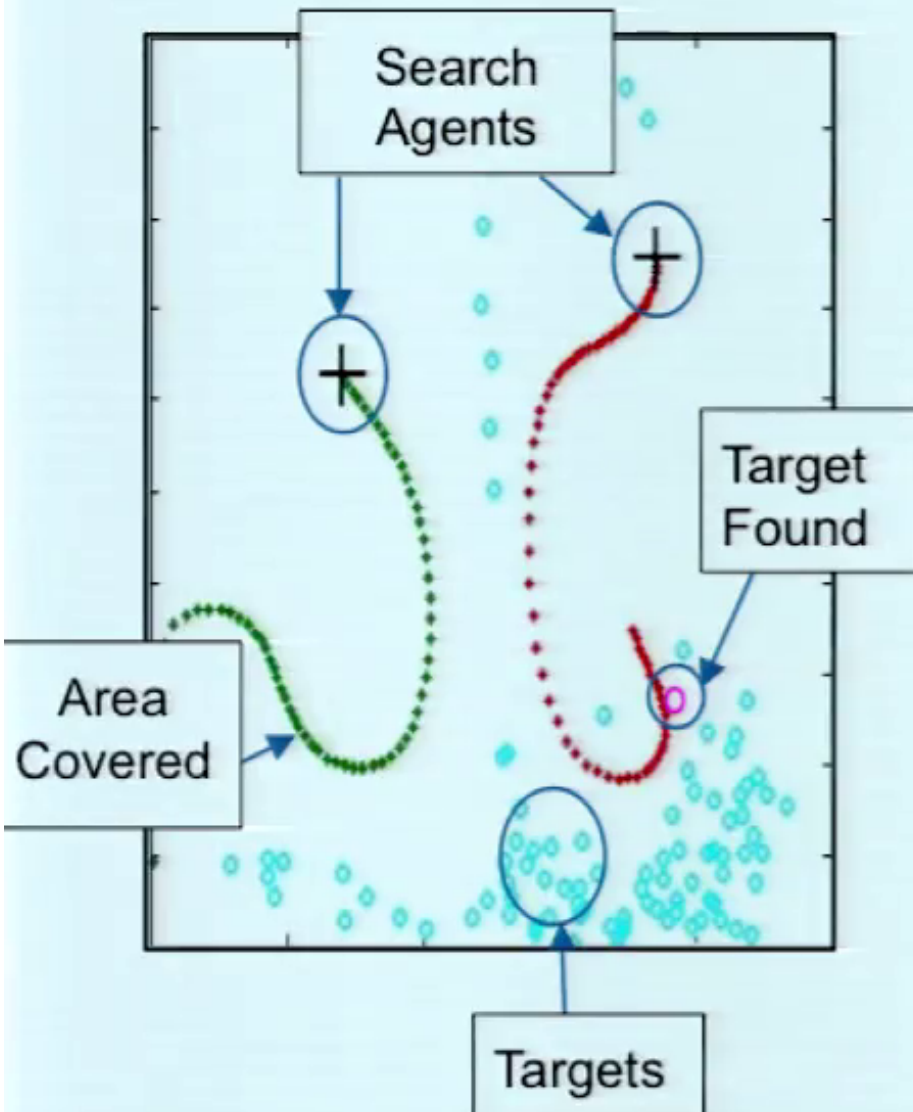
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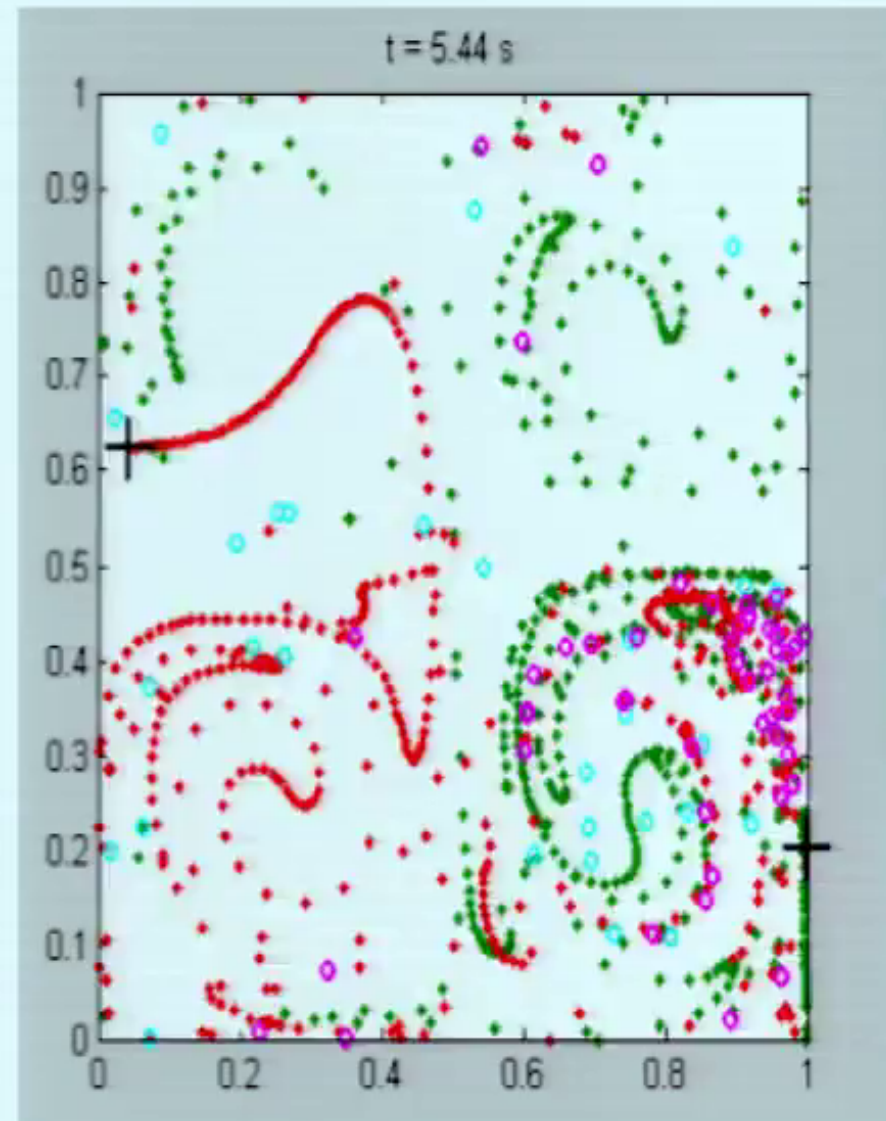
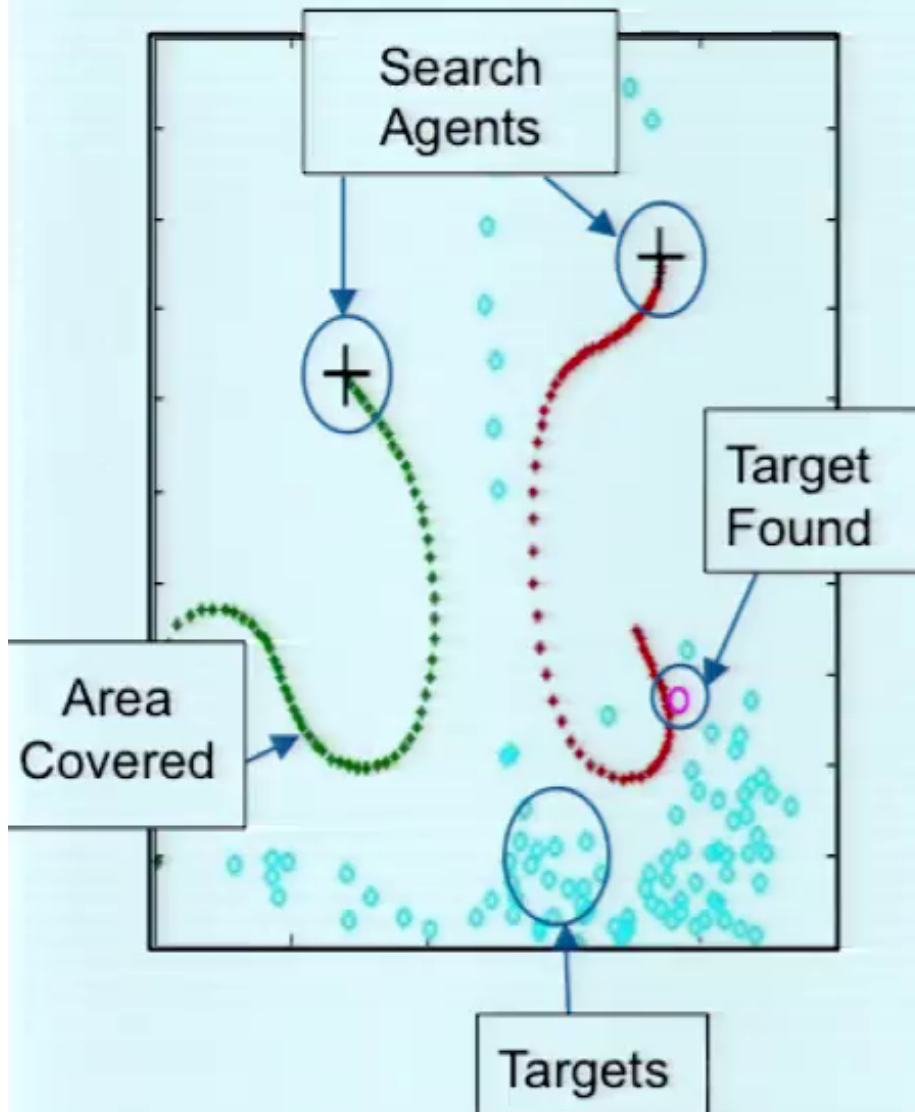
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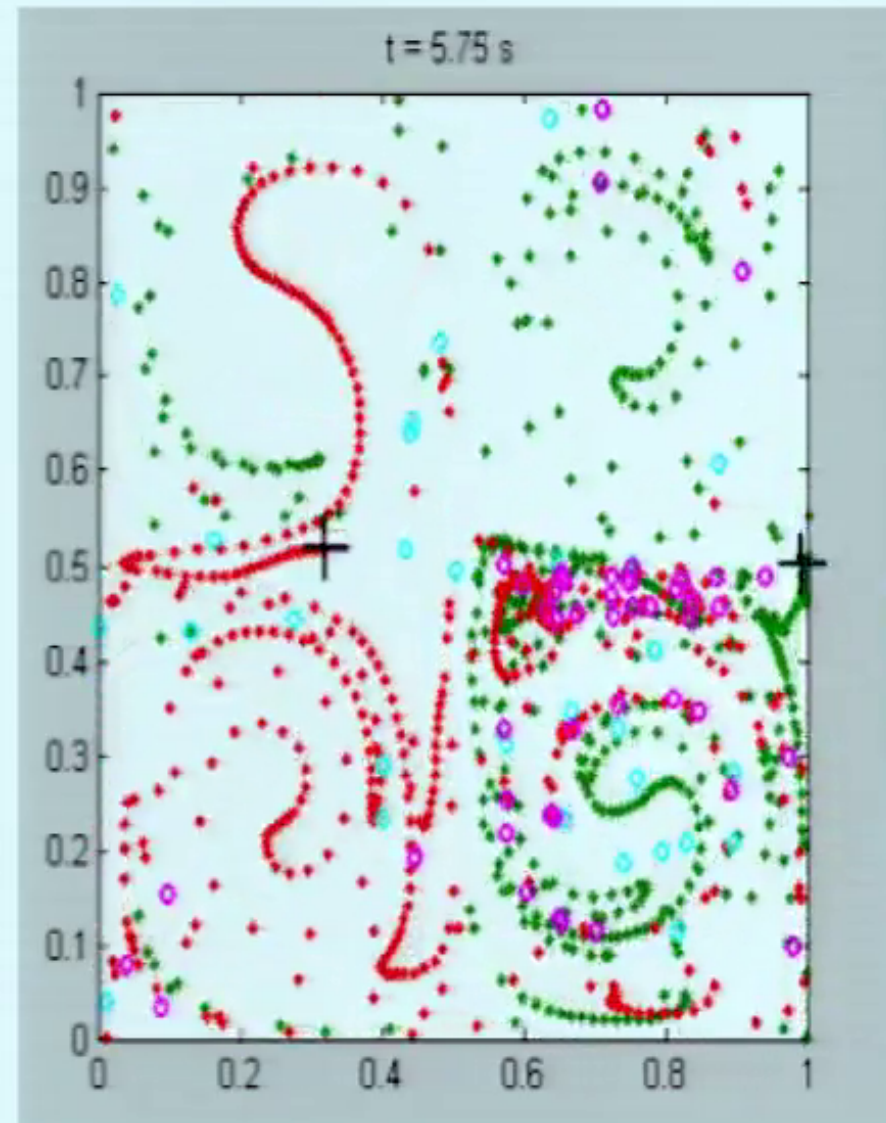
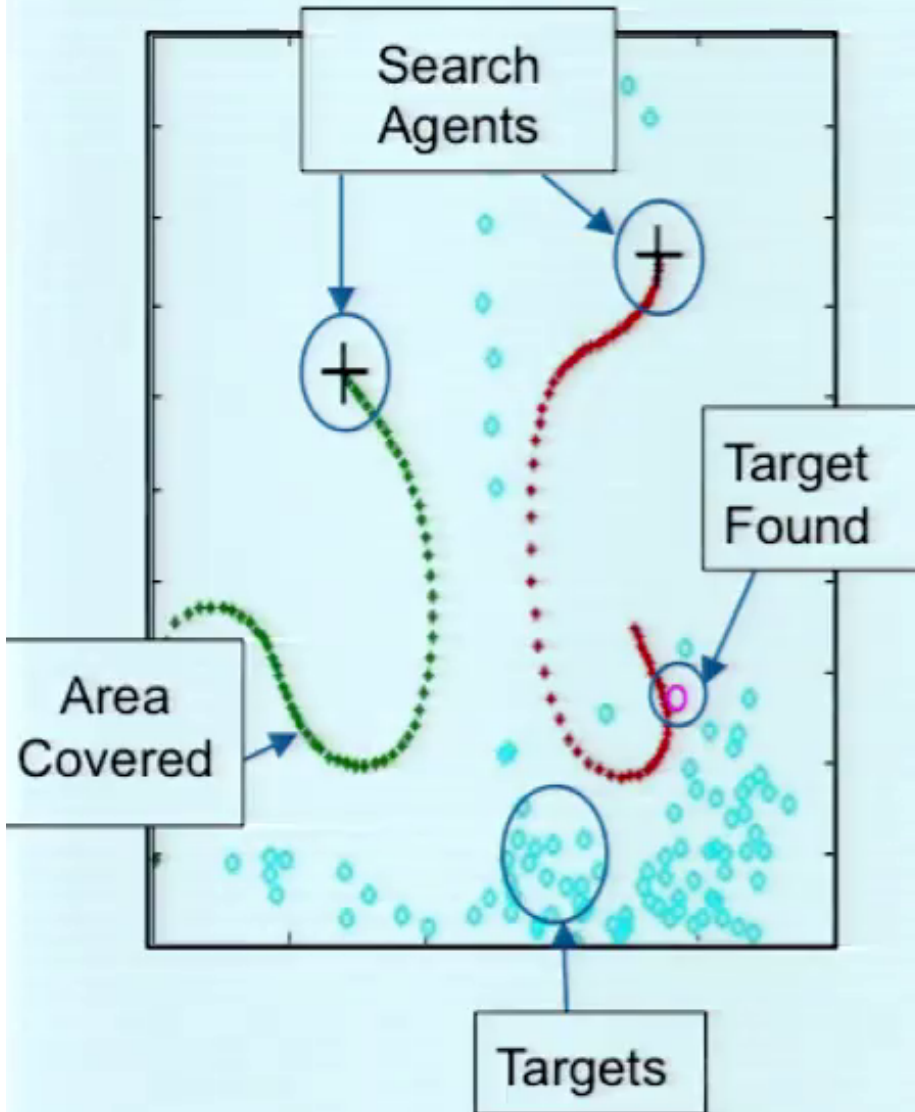
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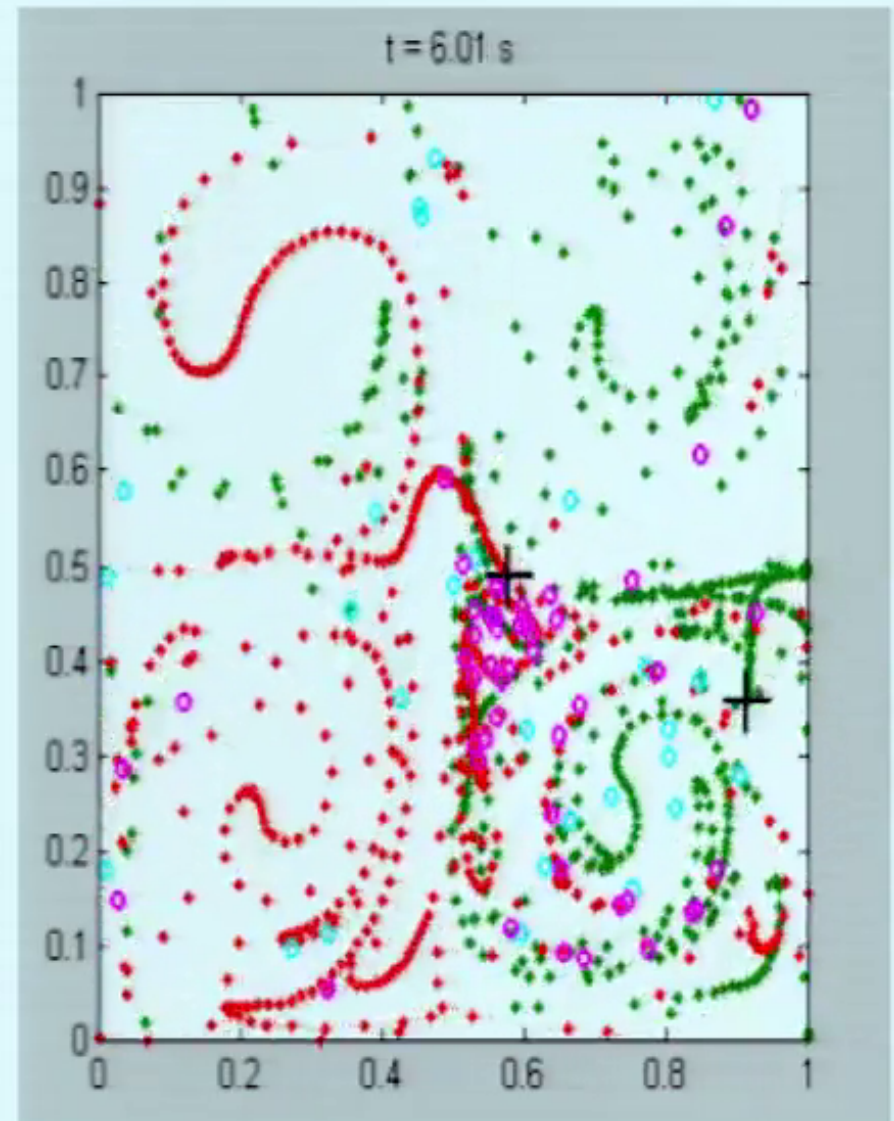
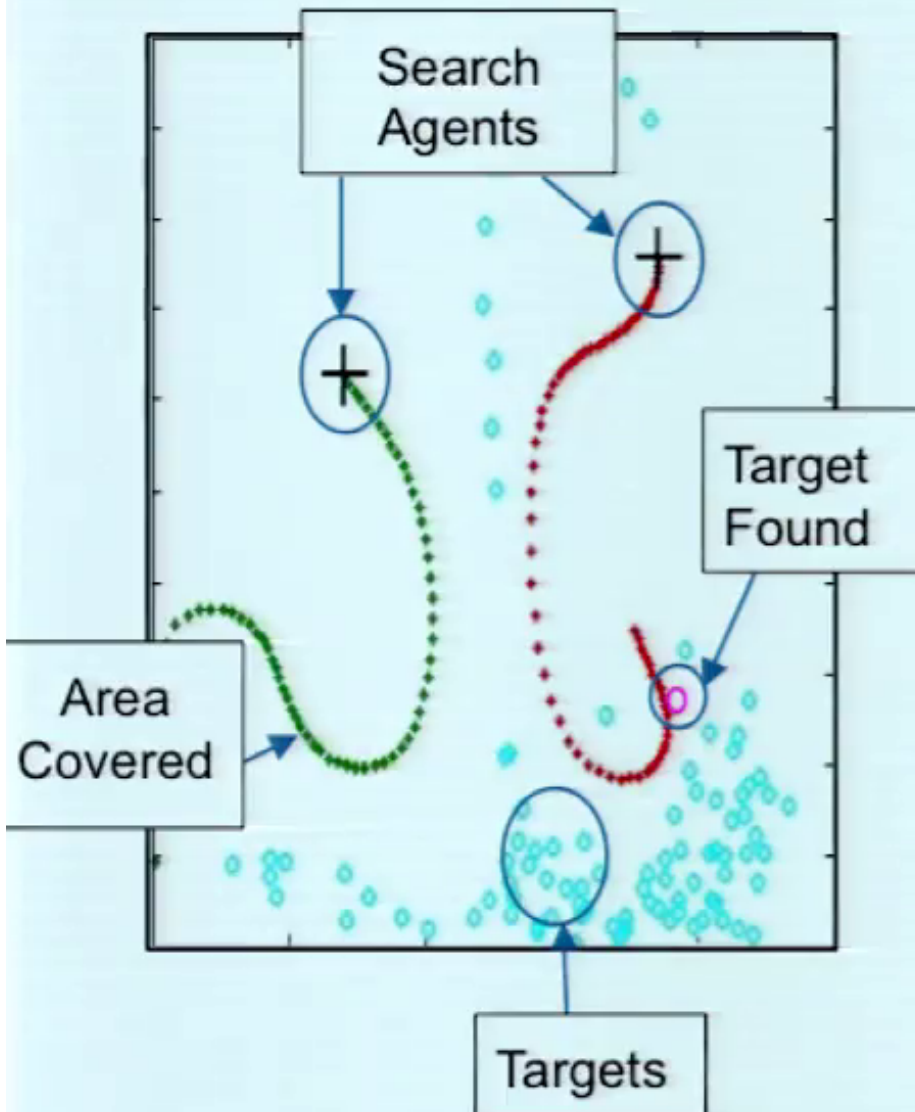
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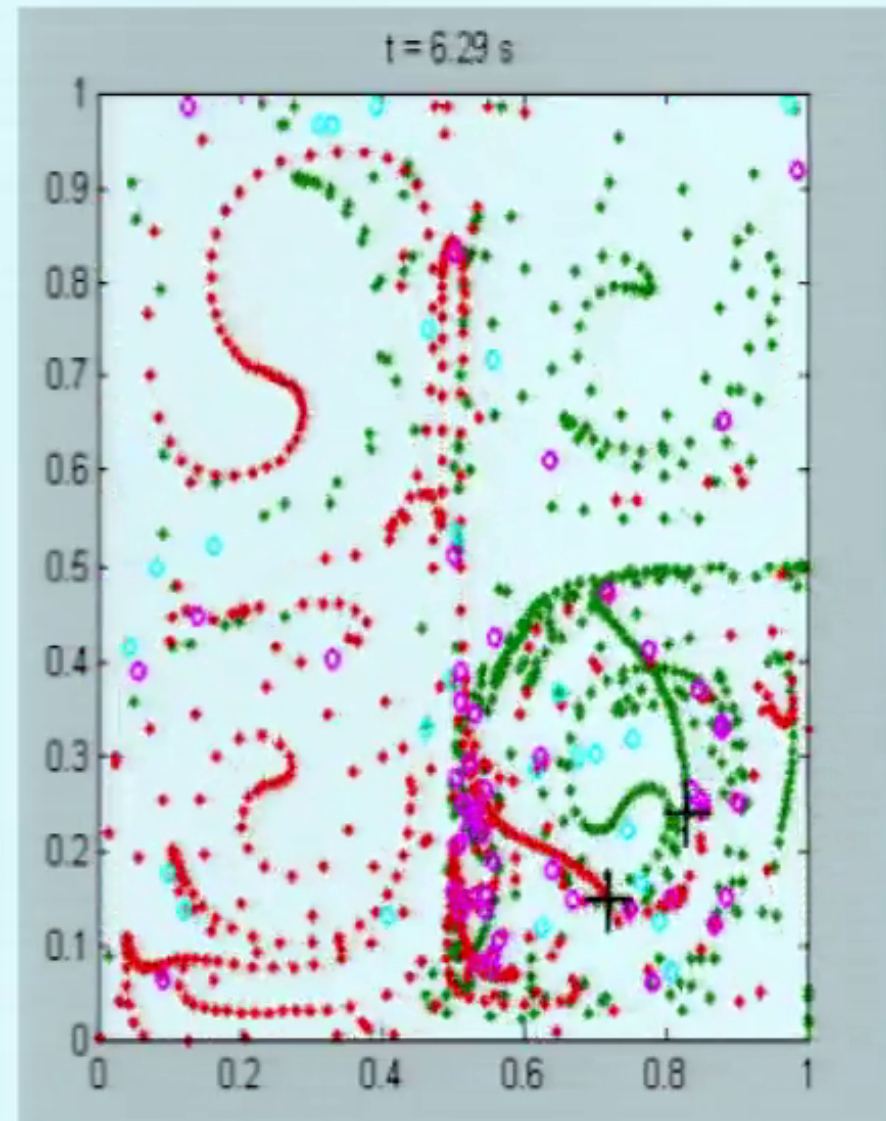
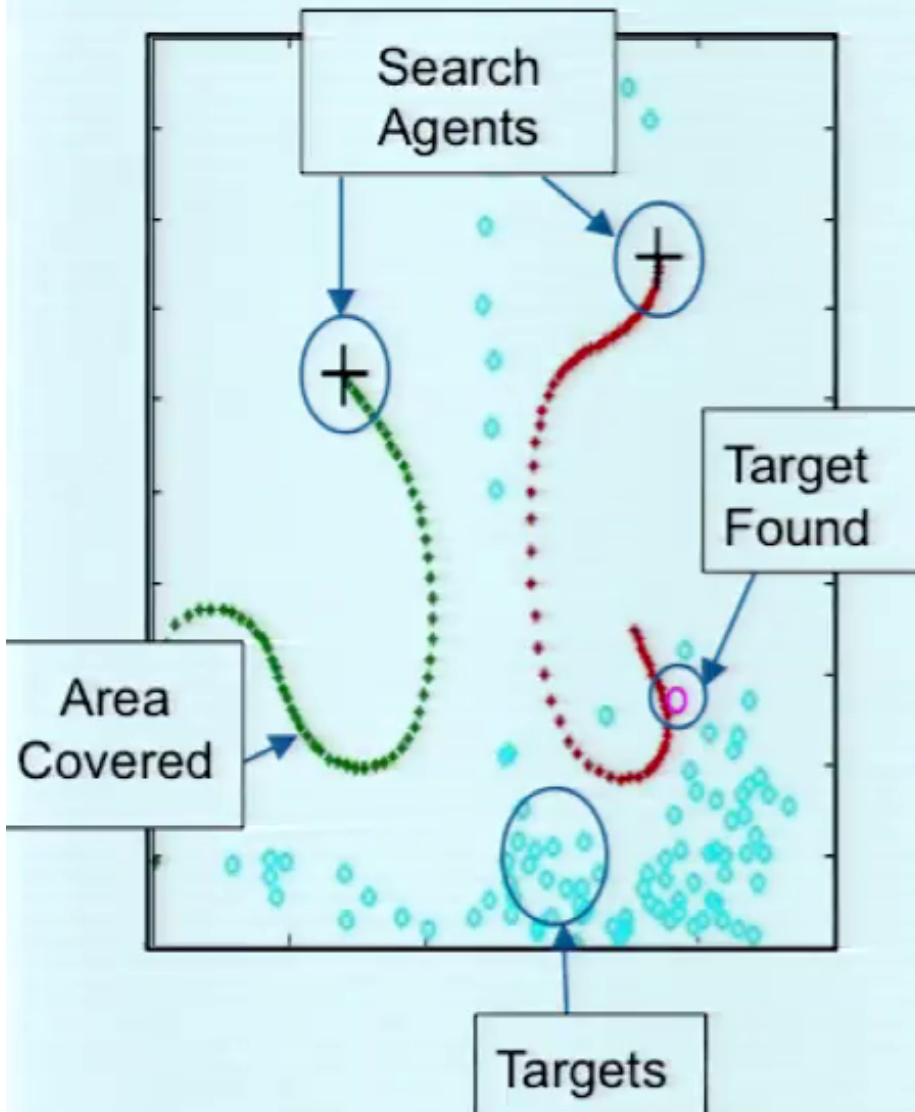
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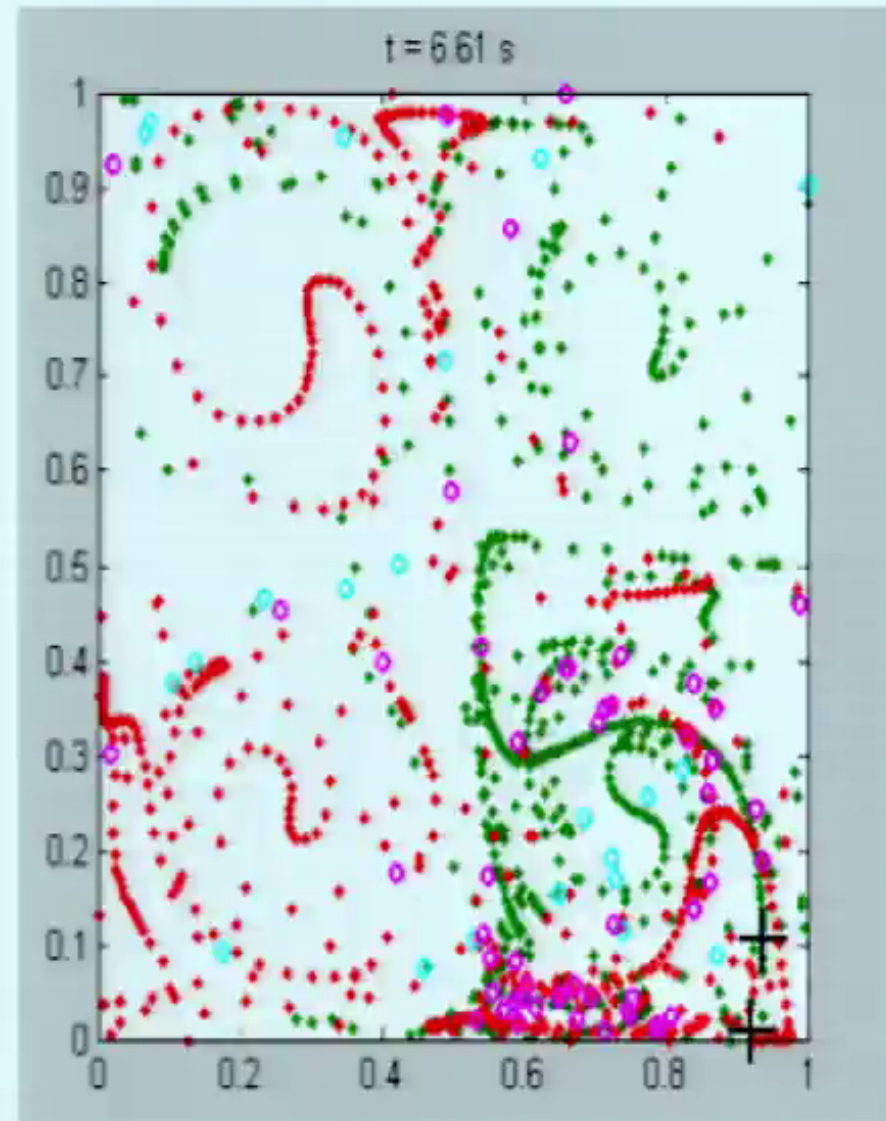
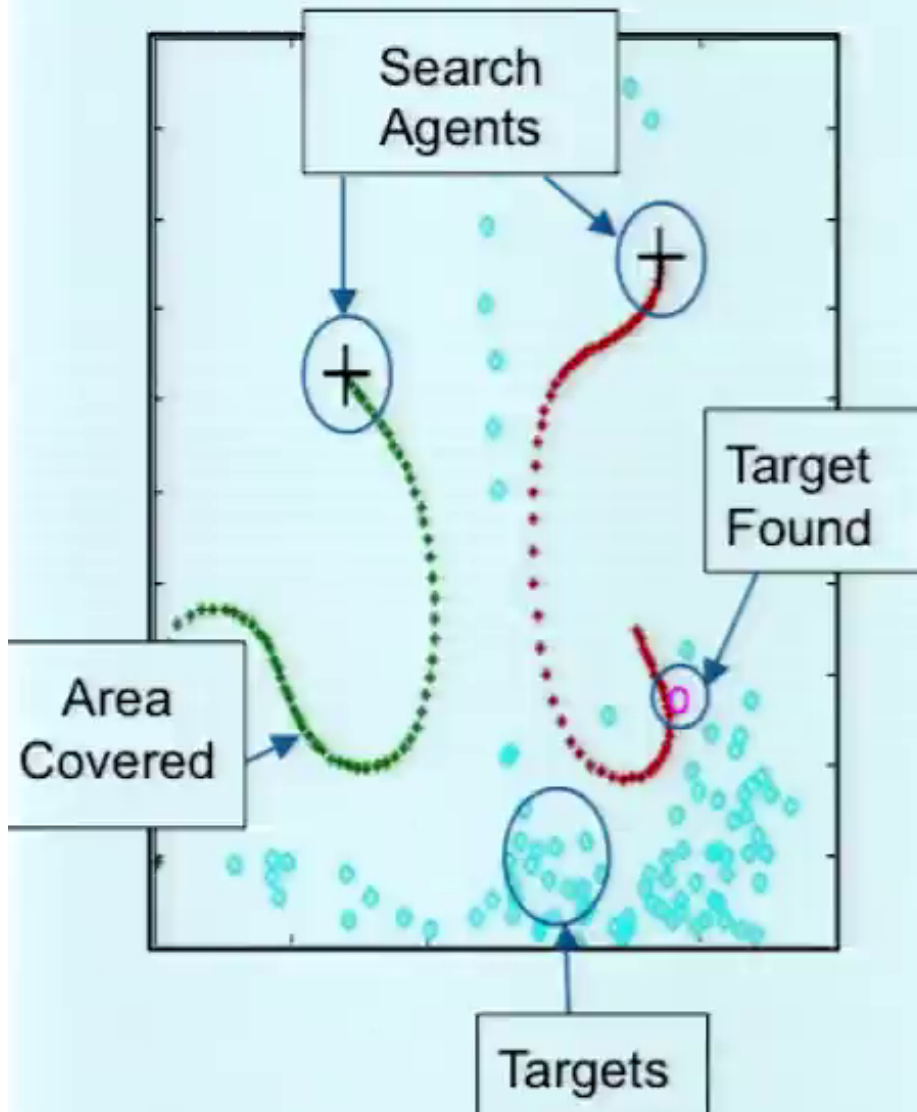
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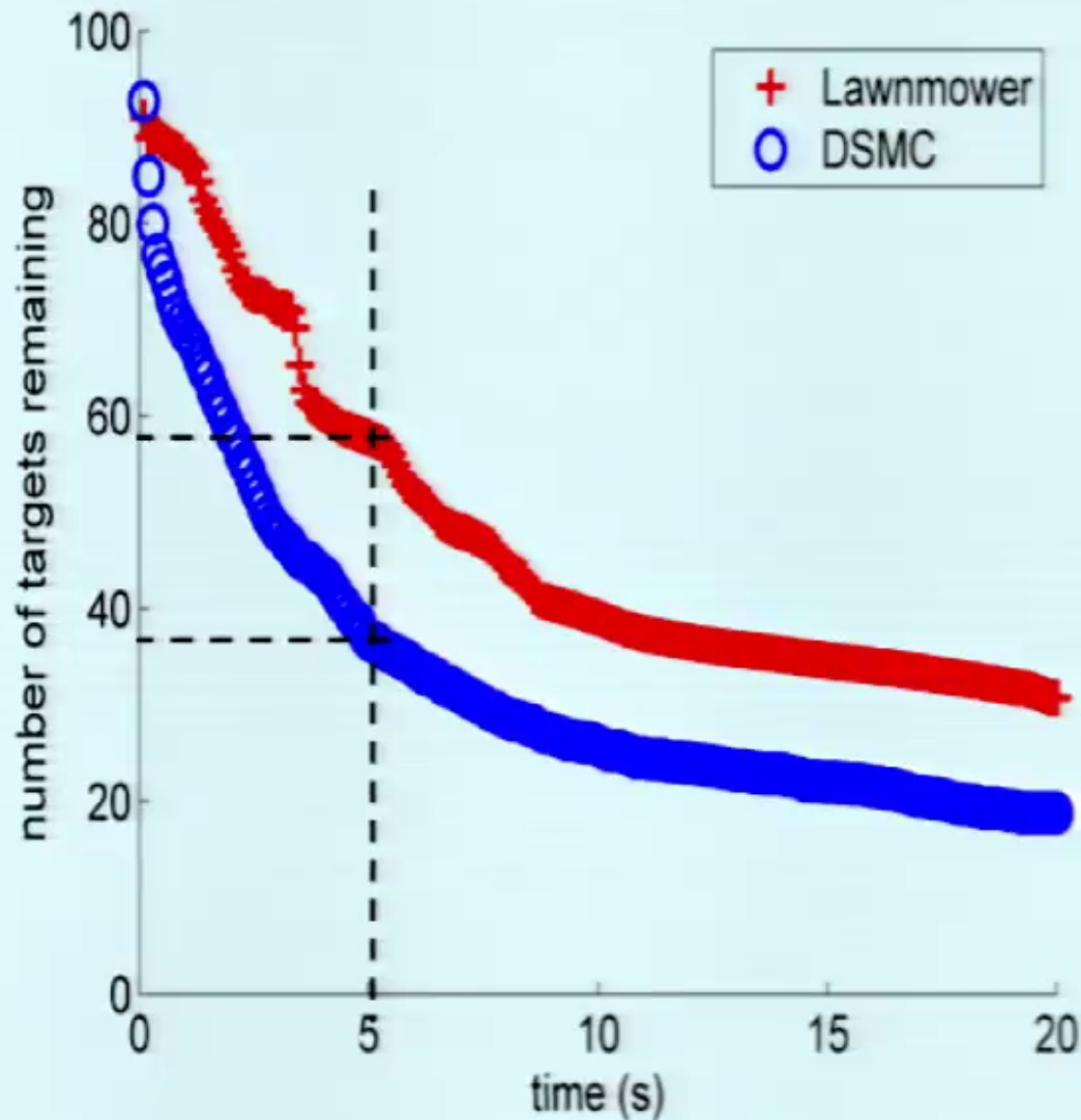


Search of targets moving in a double gyre

DSMC strategy



Efficiency comparison



The Indian Ocean Case and the hunt for MH370 floating objects

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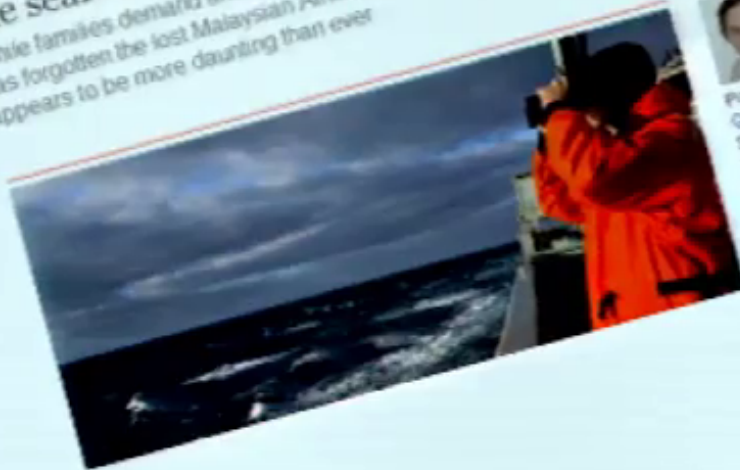
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Flight MH370: out of the headlines, but the search goes on

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Posted by
Gavin To
Sunday
13:30
The

STORY HIGHLIGHTS

(CNN) — When authorities confirmed last month that four “pings”

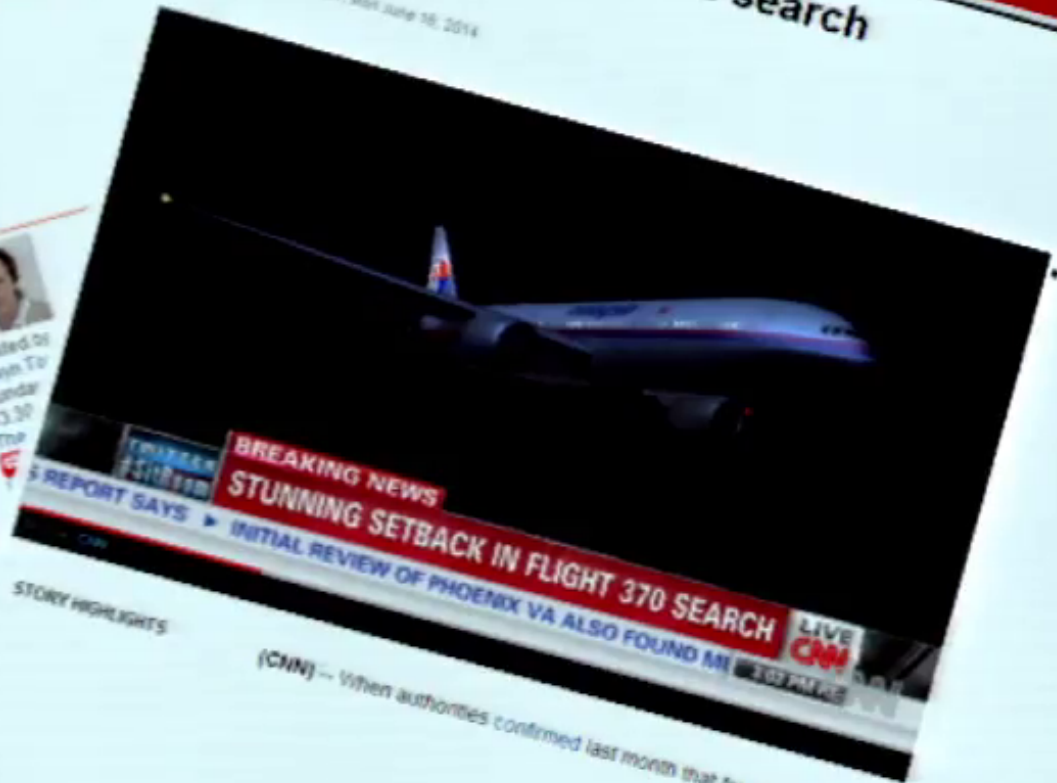
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MH370: How long will the search continue?

By Sophie Brown, CNN
Updated 11:58 AM EDT, Mon July 16, 2014



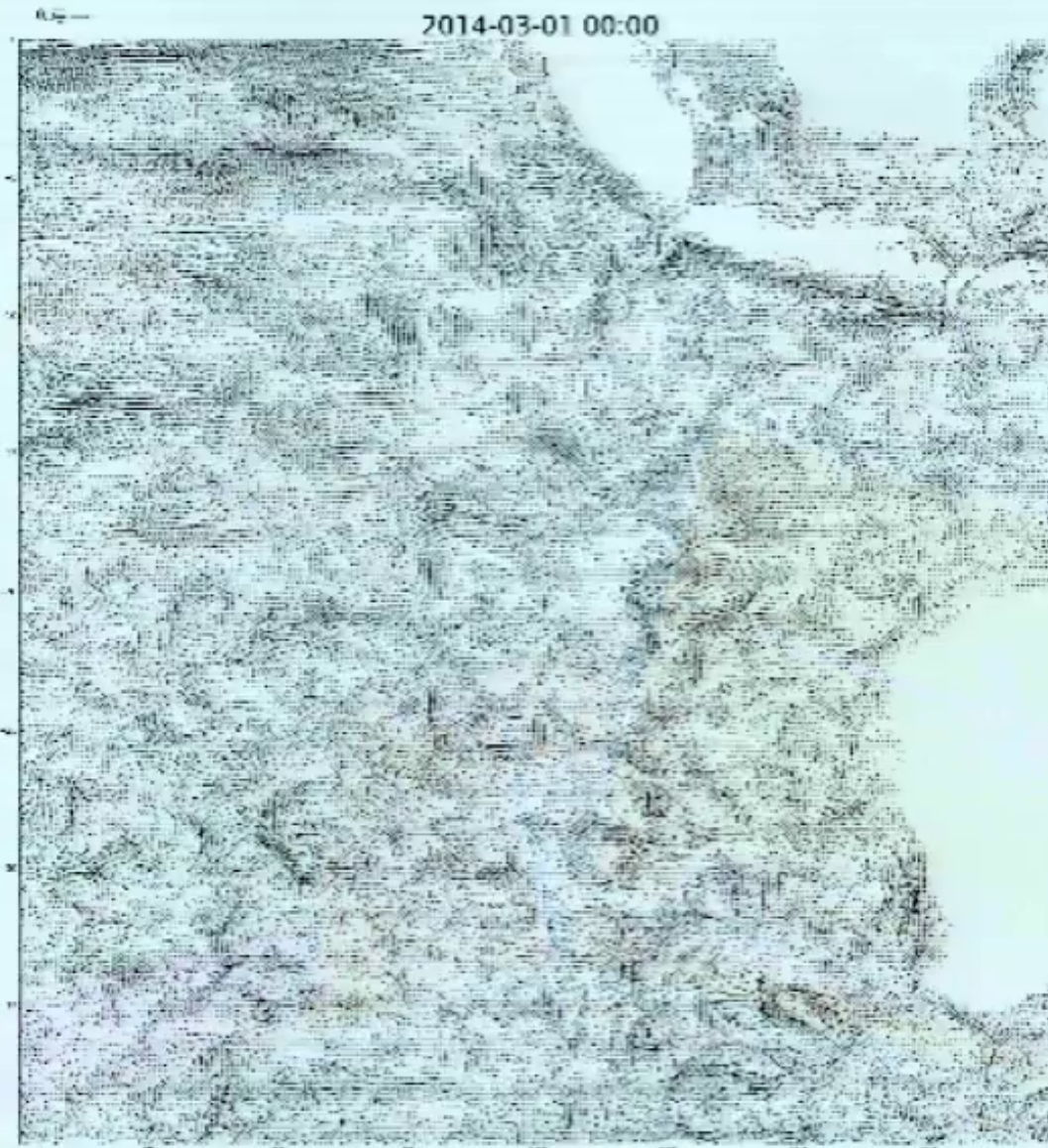
BREAKING NEWS

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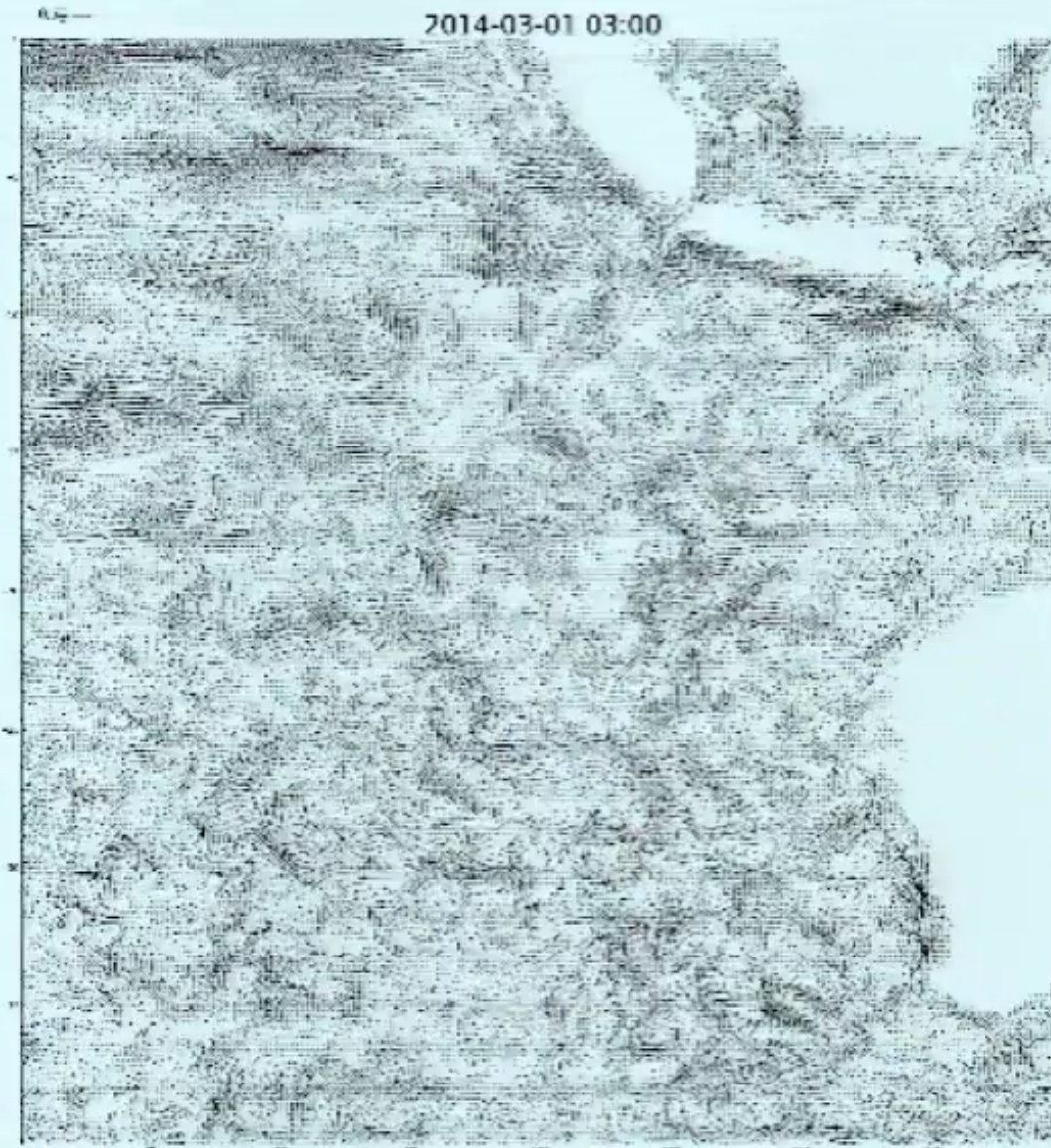
LIVE CNN

First Step: Where to search?



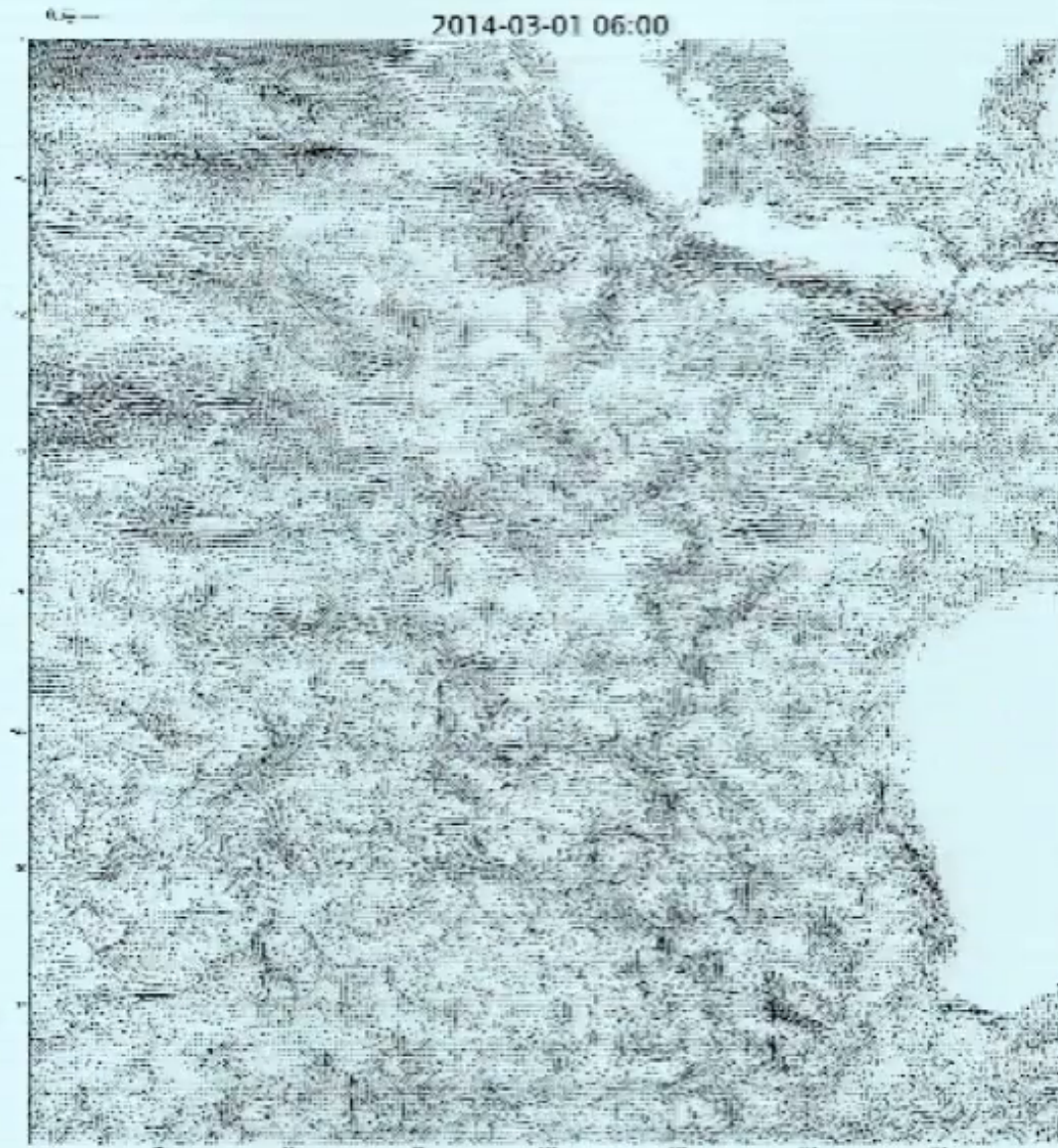
Velocity data provided
by Pat Hogan
& Ole Martin Smedstad
(Navy)

First Step: Where to search?



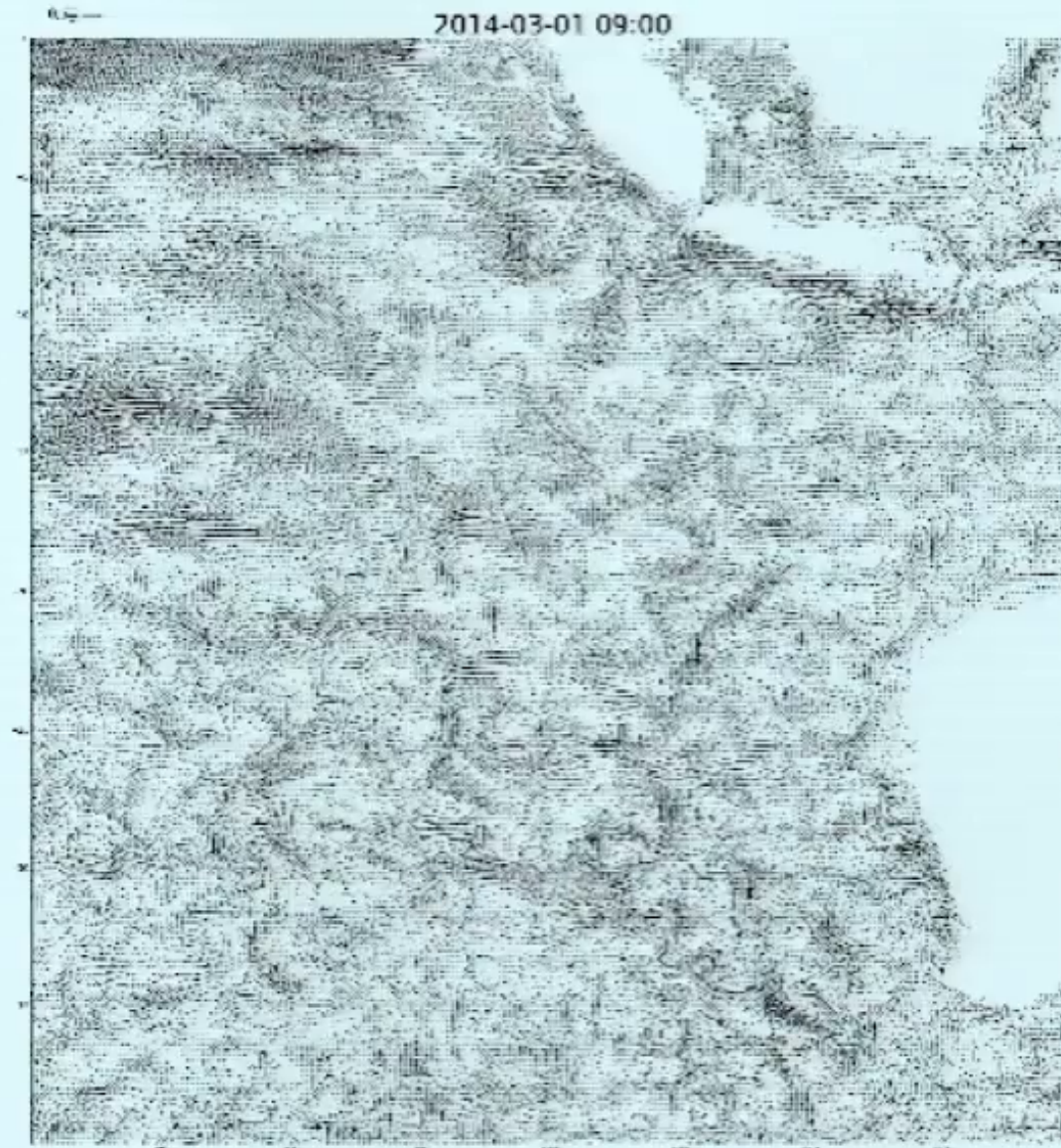
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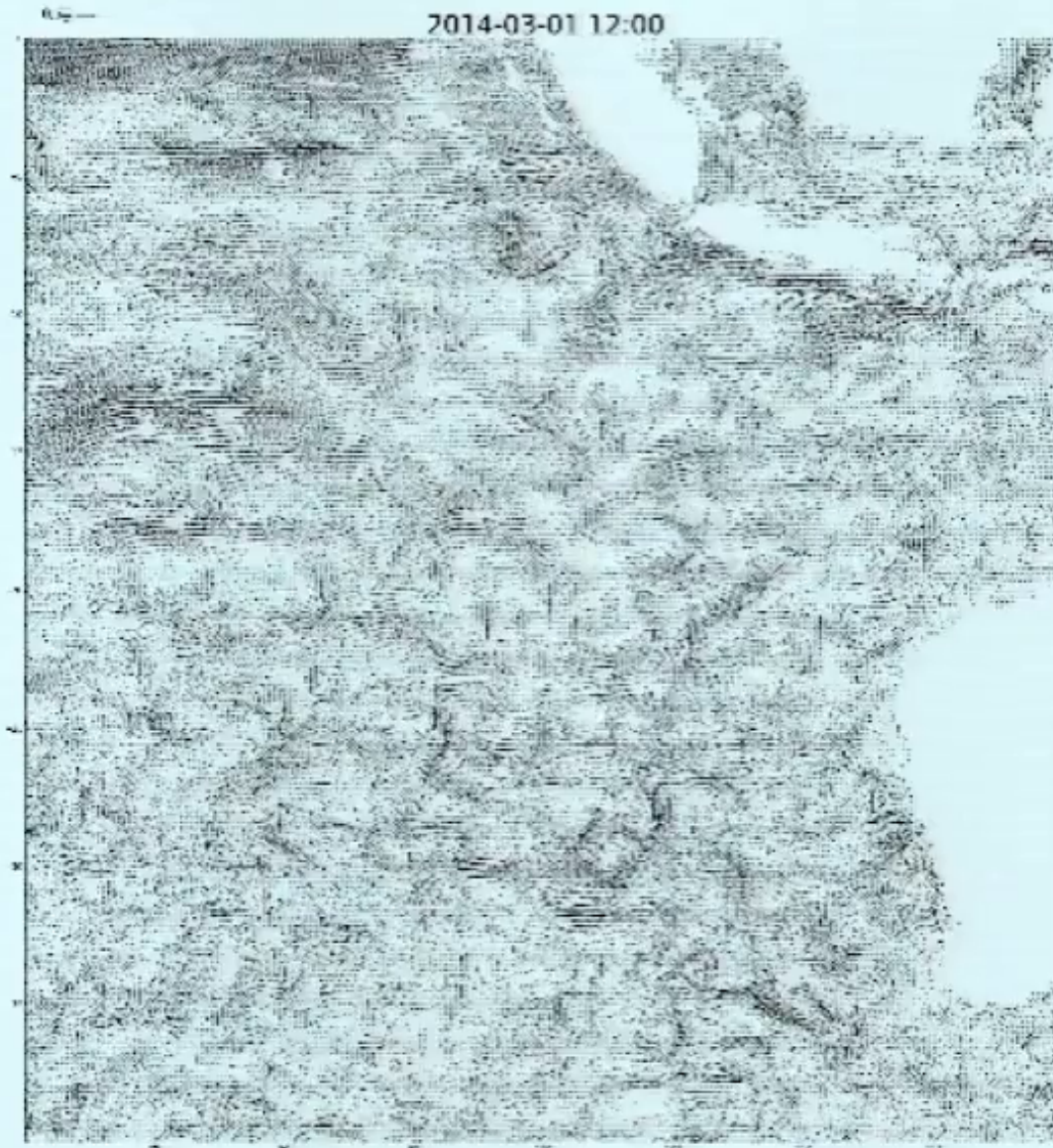
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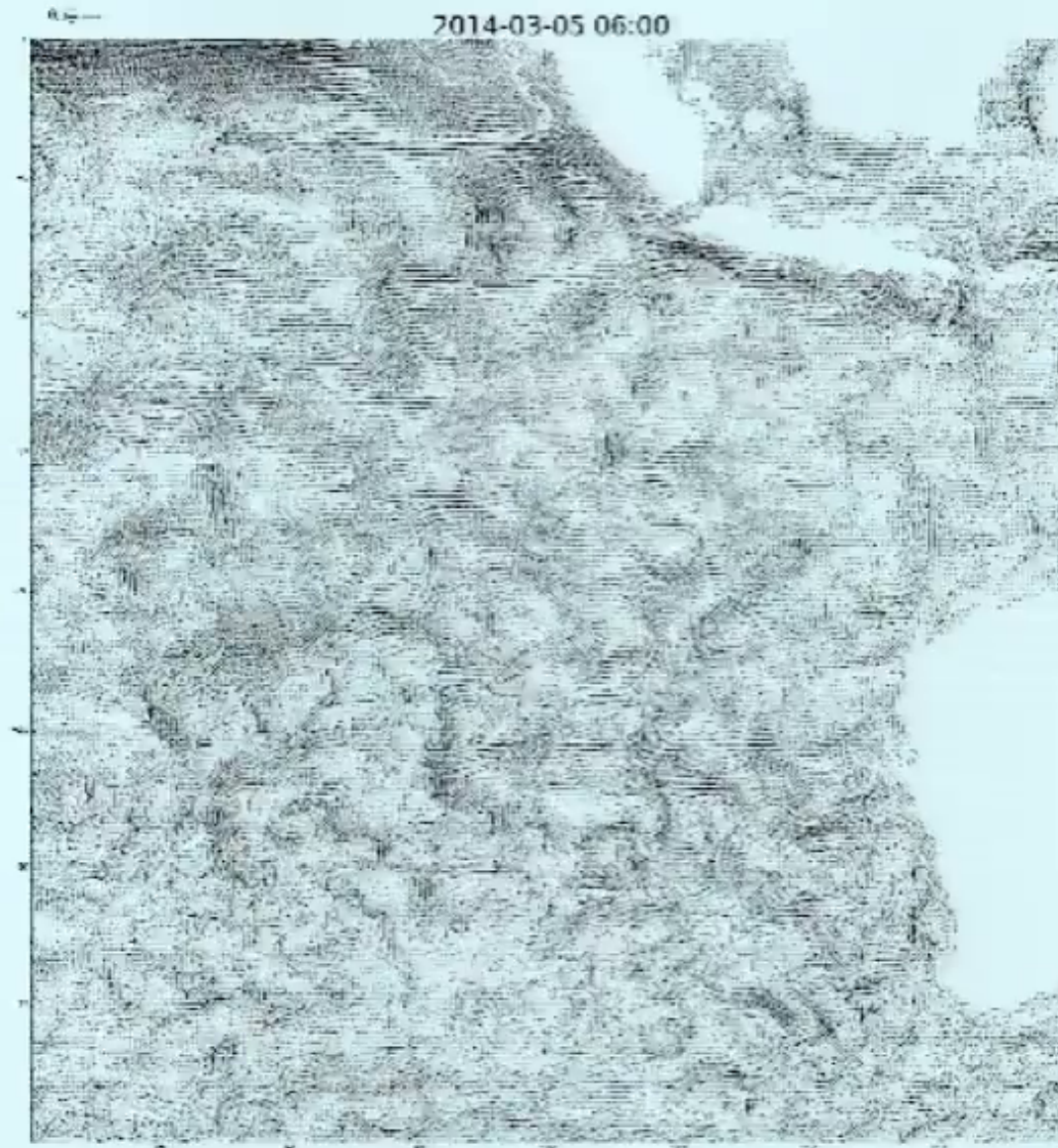
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First Step: Where to search?



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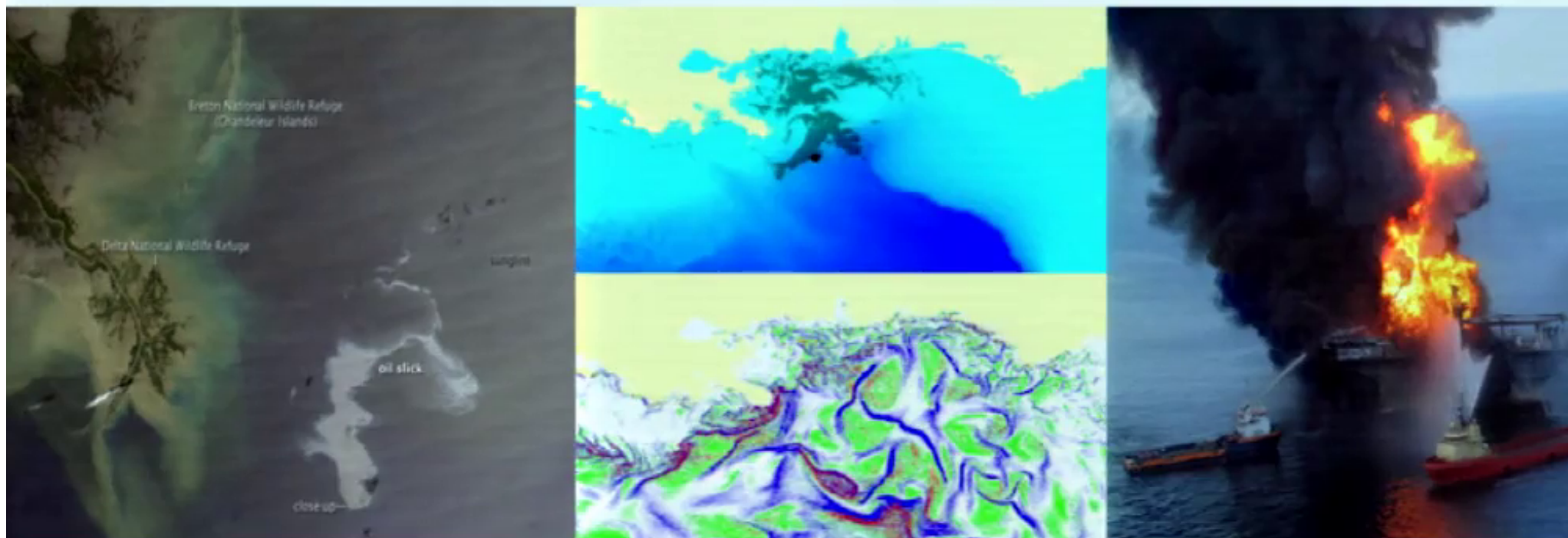


Velocity data provided
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First Step: Where to search?

What about using Hypergraph Maps
to design a strategy for the hunt of MH370?

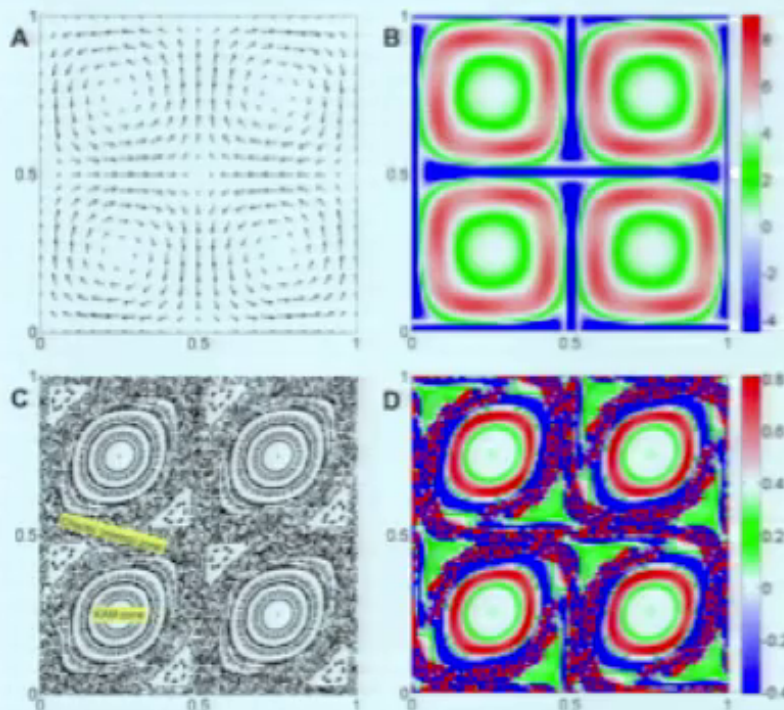
Spread of Deepwater Horizon oil slick predicted using Hypergraph Calculation



A New Mixing Diagnostic and Gulf Oil Spill Movement, Igor Mezić, S. Loire, Vladimir A. Fonoberov, and P. Hogan, Science 22 October 2010

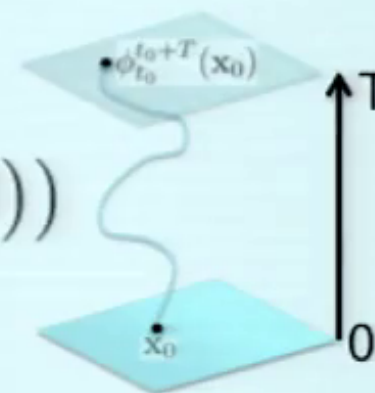
Dynamic autoinoculation and the microbial ecology of a deep water hydrocarbon irruption, David L. Valentine, Igor Mezić, Senka Maćešić, Nelida Črnjarić-Žic, Stefan Ivić, Patrick J. Hogan, Vladimir A. Fonoberov, and Sophie Loire, PNAS 2012

Hypergraph: Meso-hyperbolicity, Meso-ellipticity, Meso-Helical



Average Lagrangian velocity

$$\mathbf{v}^* = \frac{1}{T} (\mathbf{X}(T) - \mathbf{X}(0))$$



Eigenvalues of its gradient

$$\nabla \mathbf{v}^*$$

$\det |\nabla \mathbf{v}^*|$

Meso-Hyperbolic

Meso-Elliptic

Meso-Helical

strain

$$\det |\nabla \mathbf{v}^*| < 0$$



rotation

$$0 < \det |\nabla \mathbf{v}^*| < \frac{4}{T^2}$$



rotation + strain

$$\frac{4}{T^2} < \det |\nabla \mathbf{v}^*|$$



Hypergraph: Meso-hyperbolicity, Meso-ellipticity, Meso-Helical For Complex and dynamic environment:

Classification of regions of

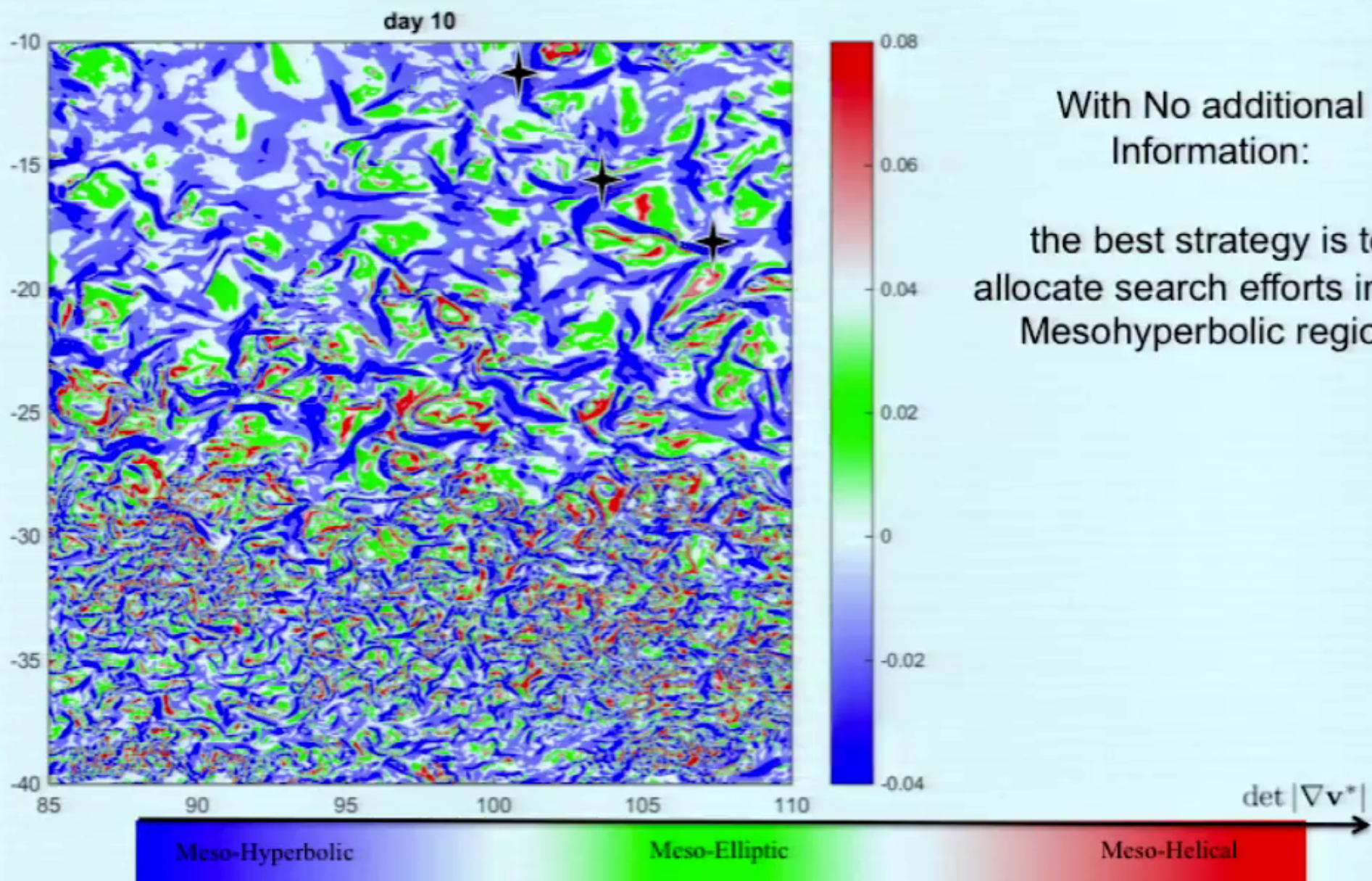
- Mixing
- High shear
- Ellipticity (including location of possible coherent vortices)

For search strategies:

- Predict future location of targets and define search area
- Help define initial target position probability distribution
- Prioritize search in large shear regions

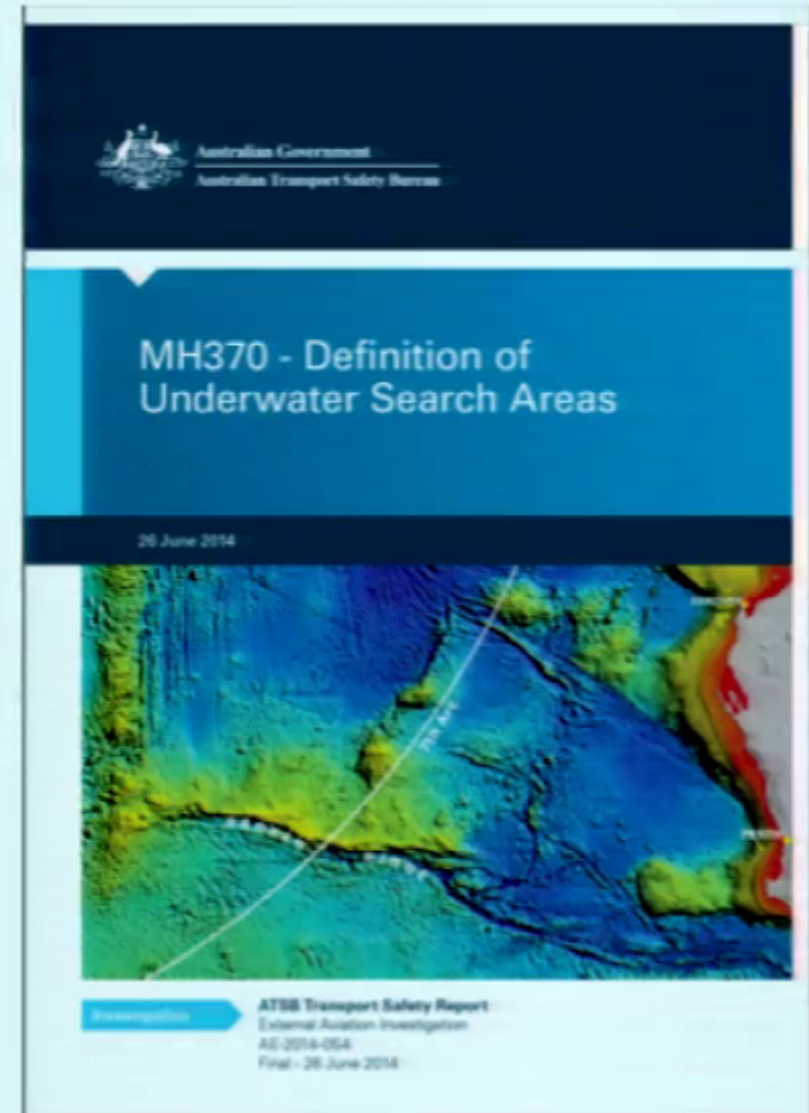
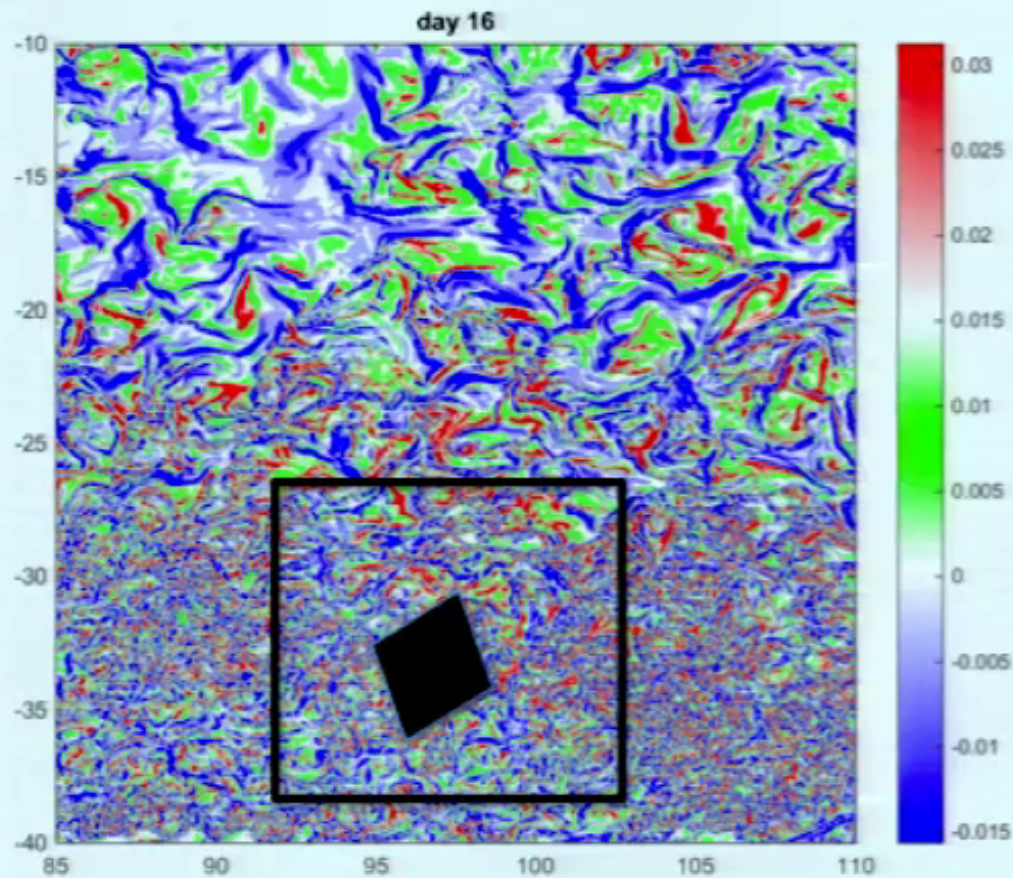
Based on the gradient of the average Lagrangian velocity

First Step: Where to search?



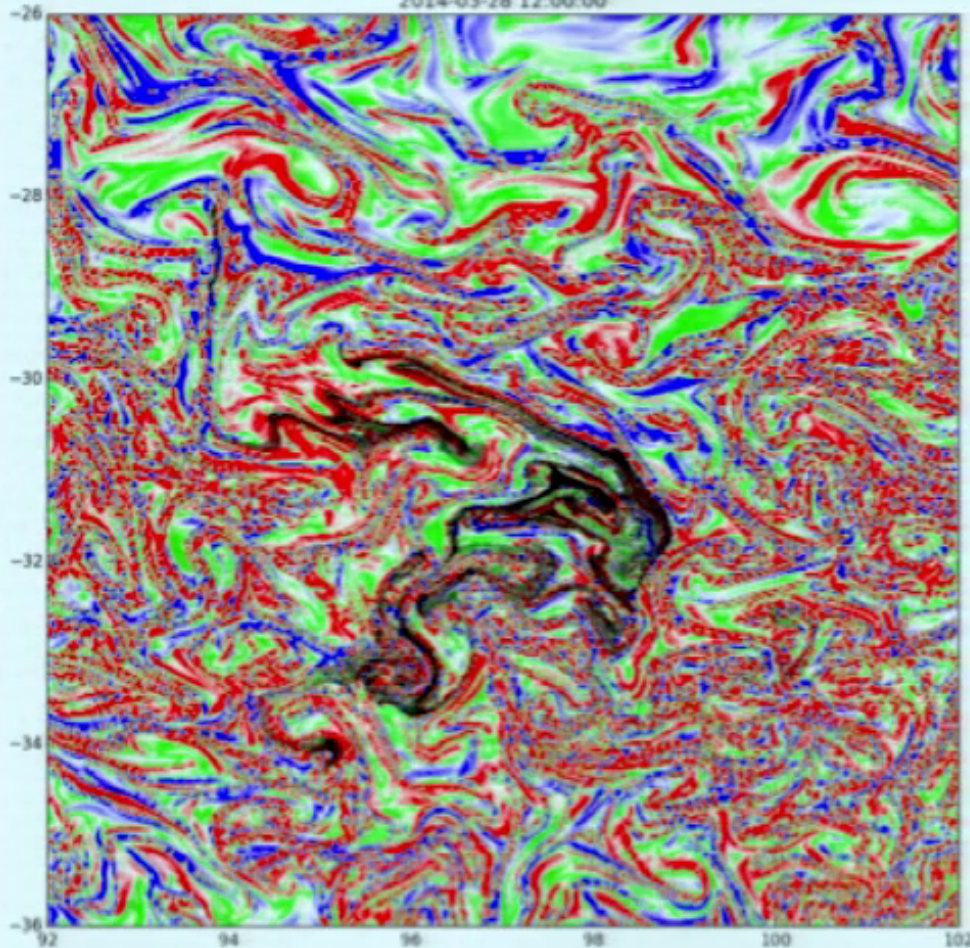
The Indian Ocean Case and the search for MH370 floating objects

Inmarsat satellite: March 24 2014
First Announced Splash Location



The Indian Ocean Case and the search for MH370 floating objects

2014-03-28 12:00:00



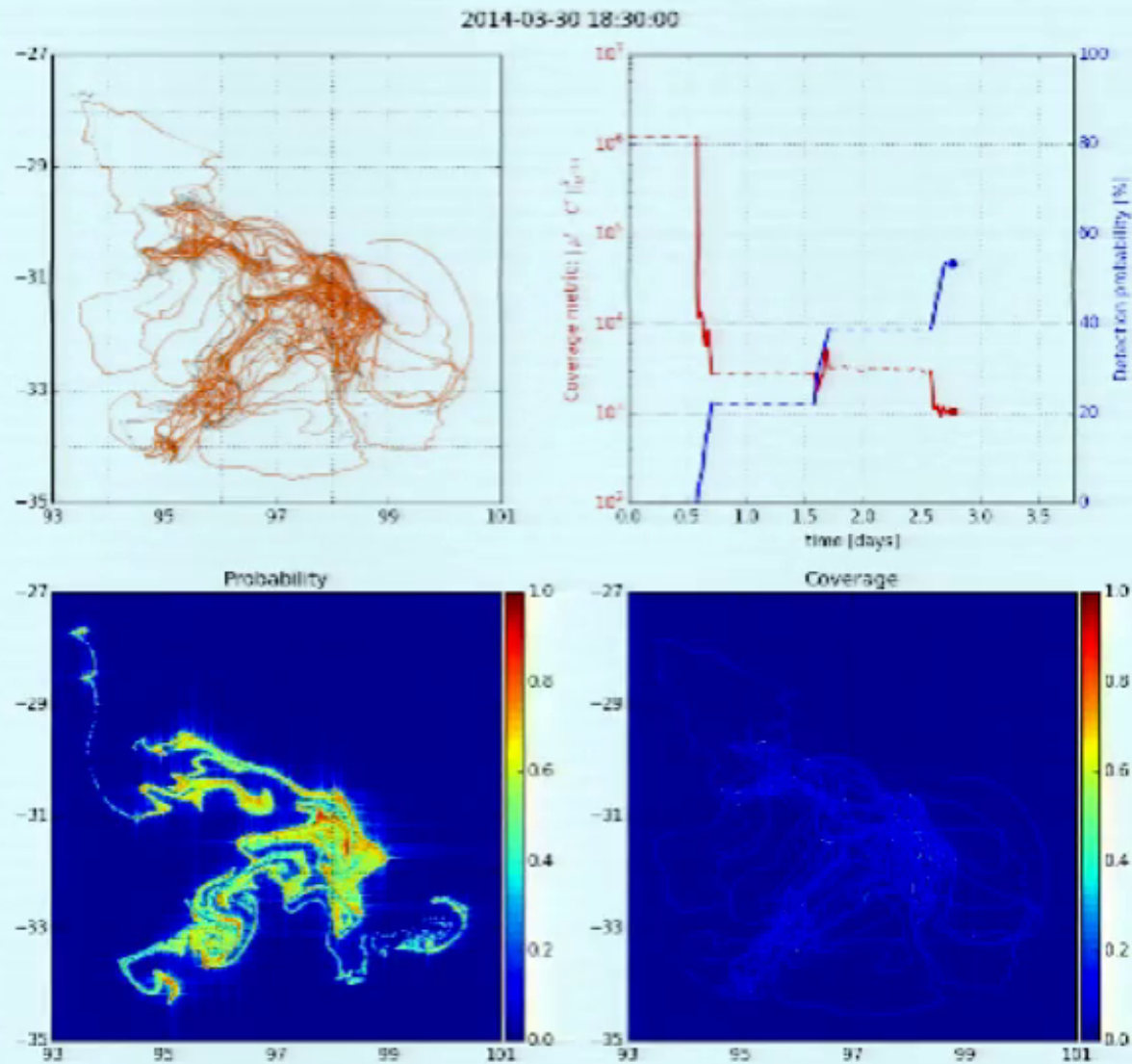
Search area A

2014-03-31 (Search day 4)

Search period: 14:00 to 17:00

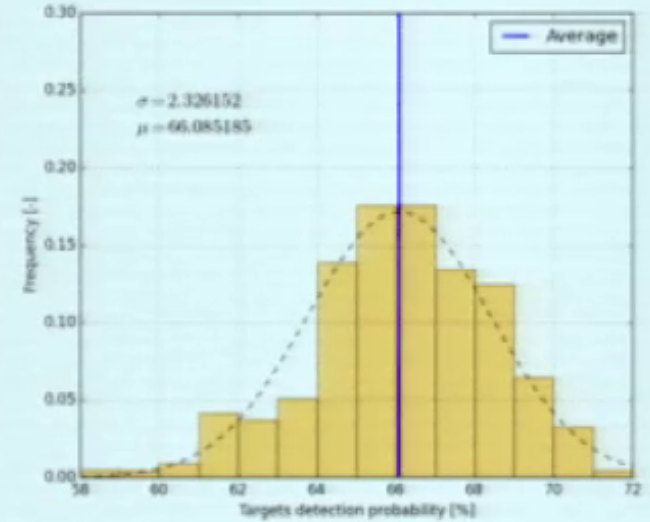
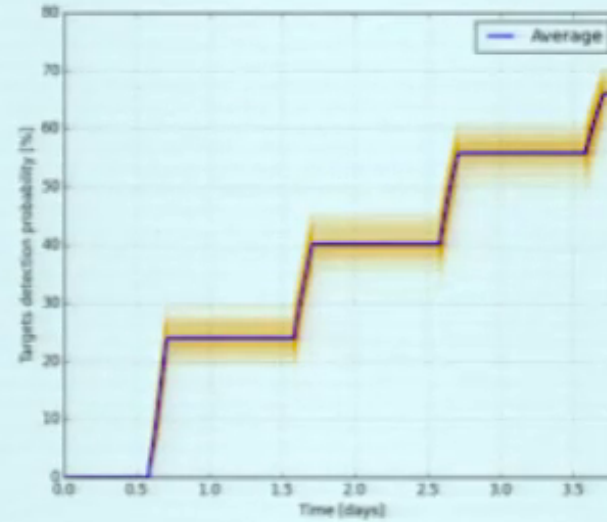
Number of agents: 9

DSMC search area A

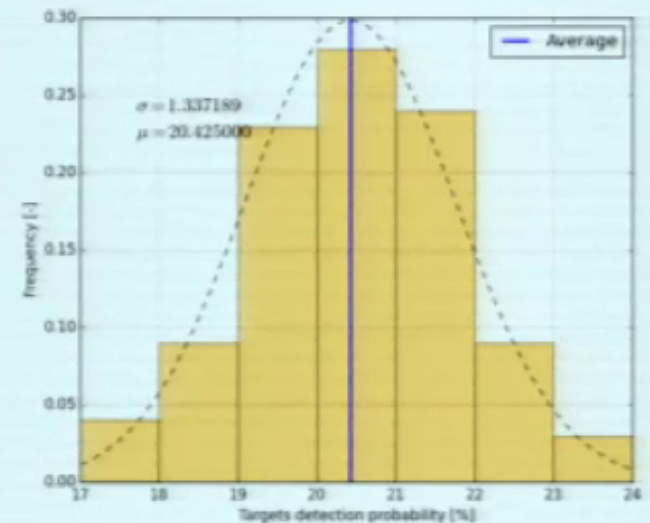
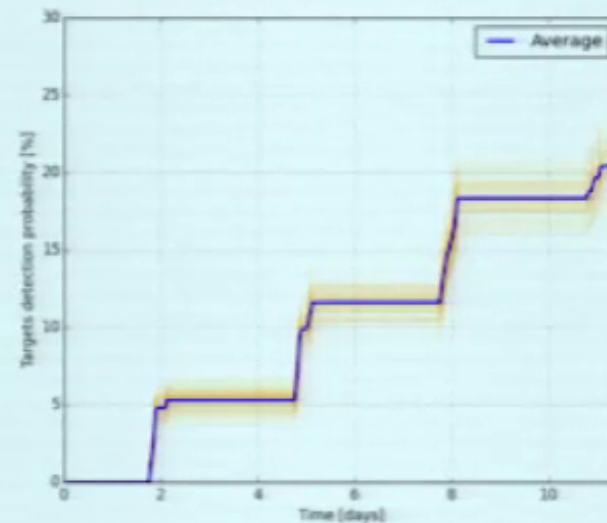


Comparison:

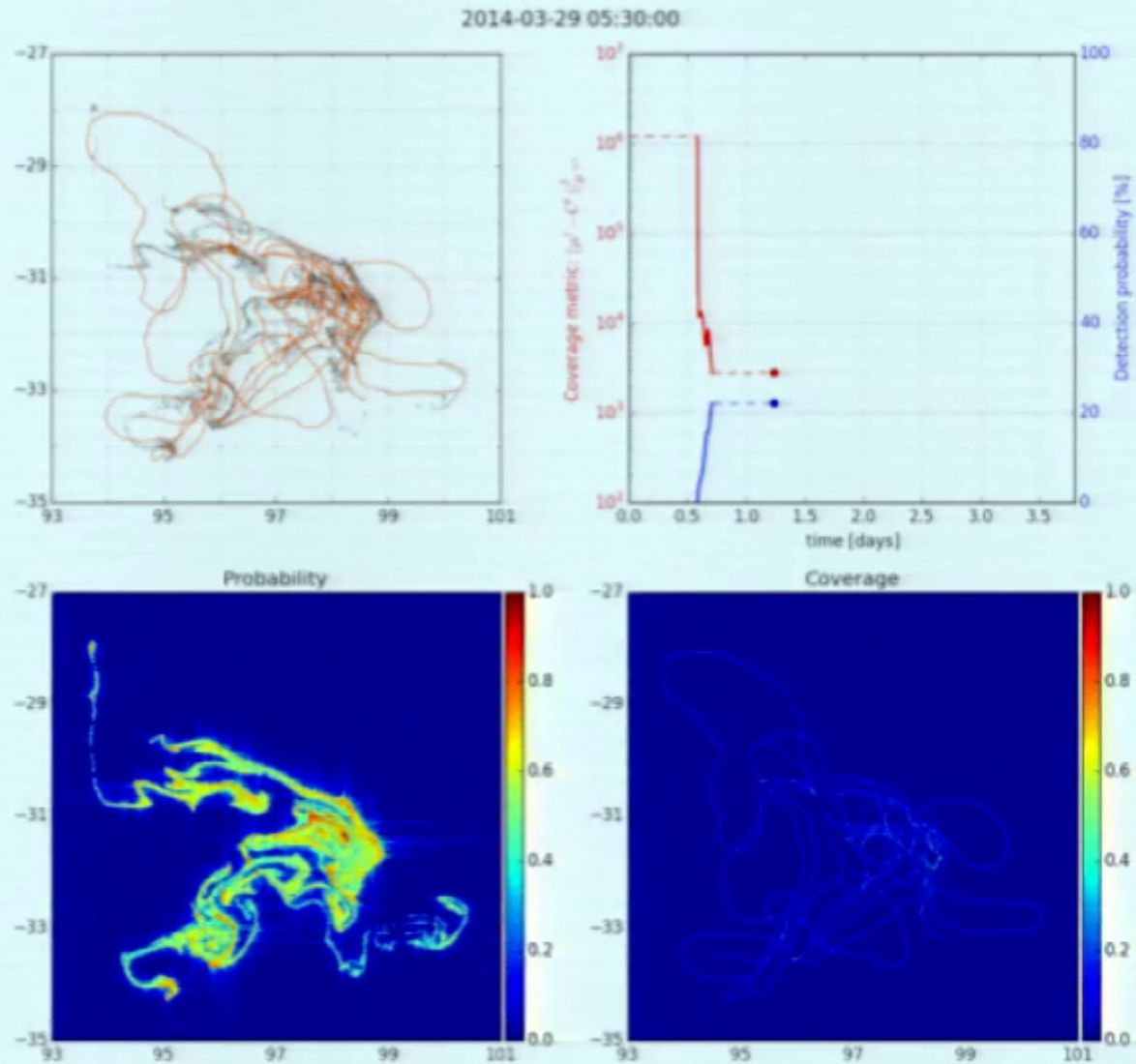
DSMC



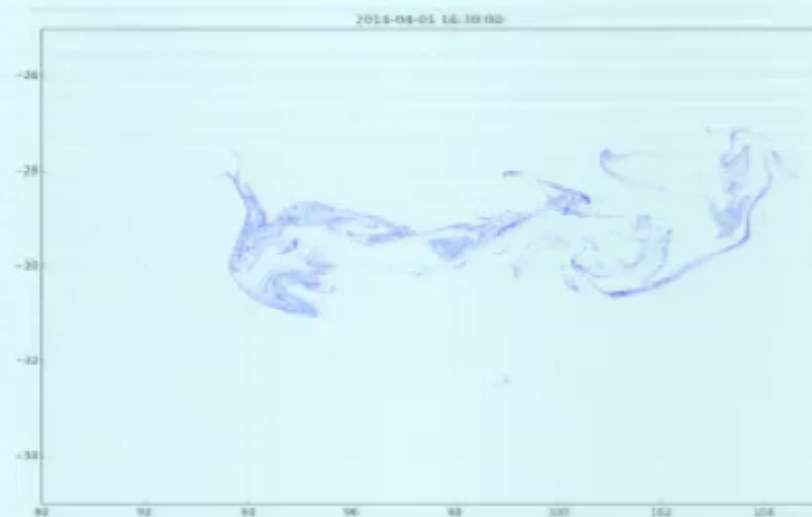
Lawnmower



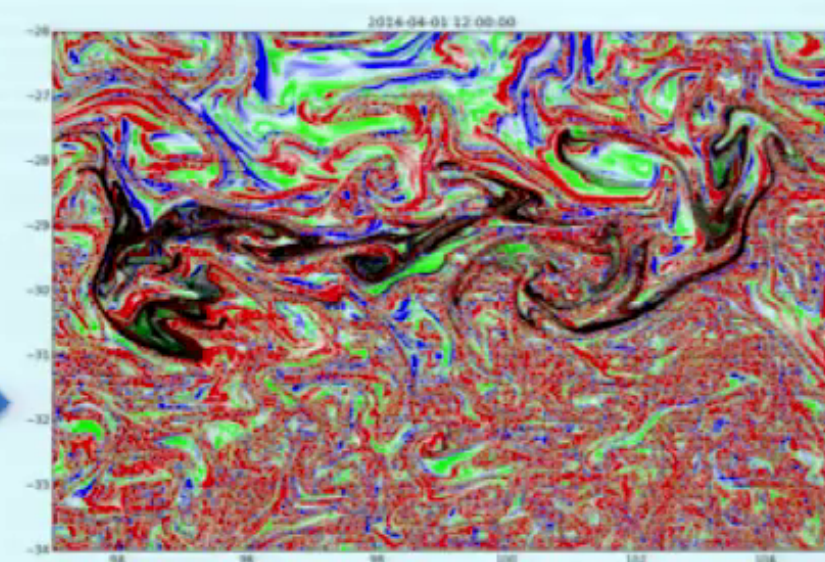
DSMC search area A



Hypergraph: Backward-time hypergraphs from April 01 to March 08 2014



Advection
 Area B



$\det |\nabla \mathbf{v}^*|$



strain



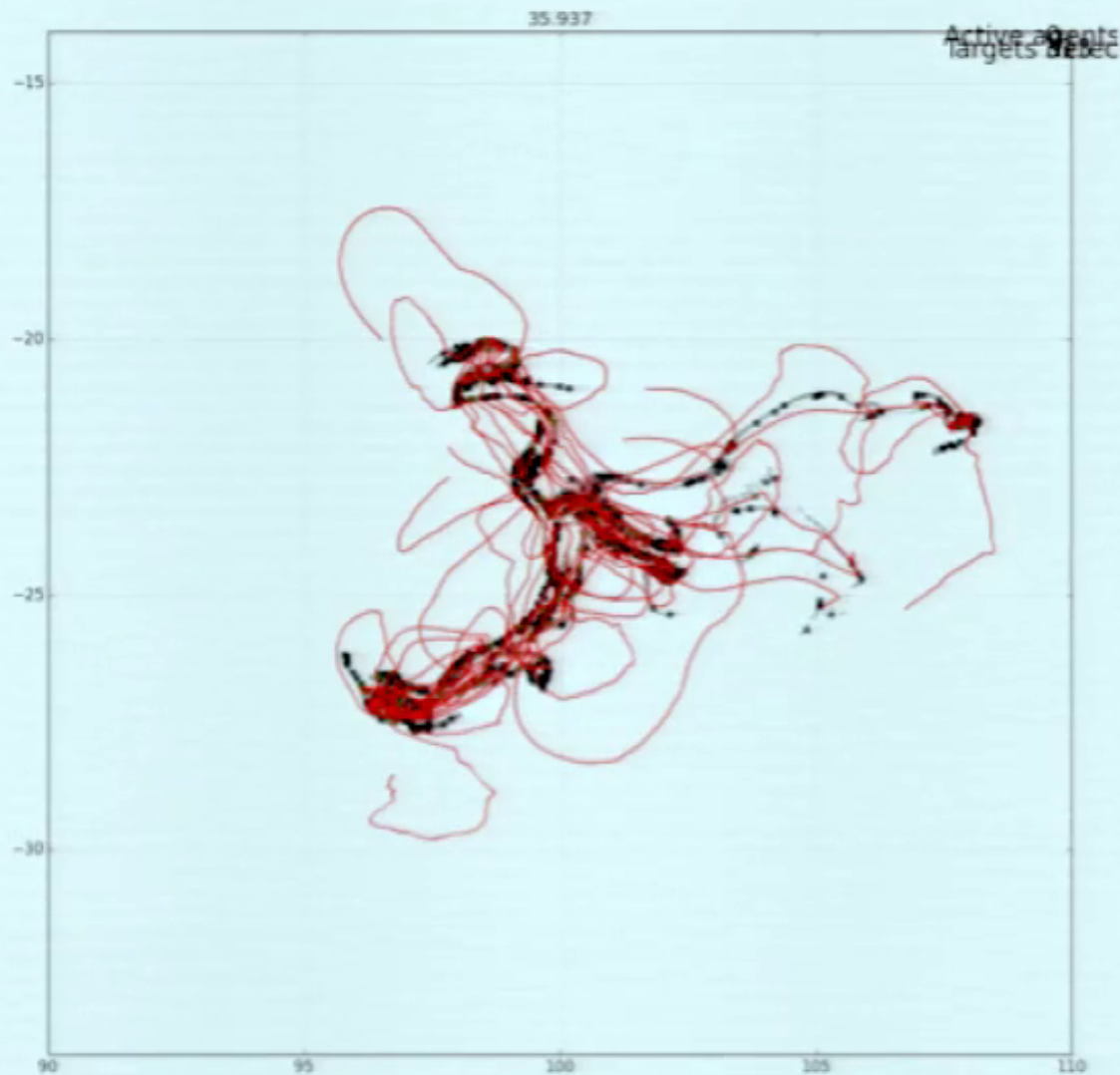
rotation



rotation + strain

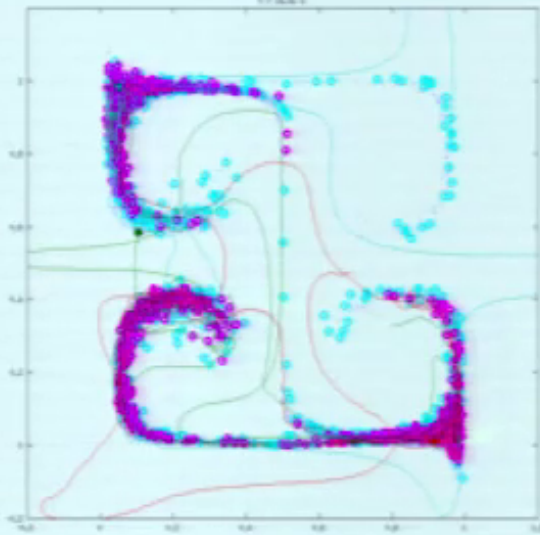


What if the target density is very filamented?

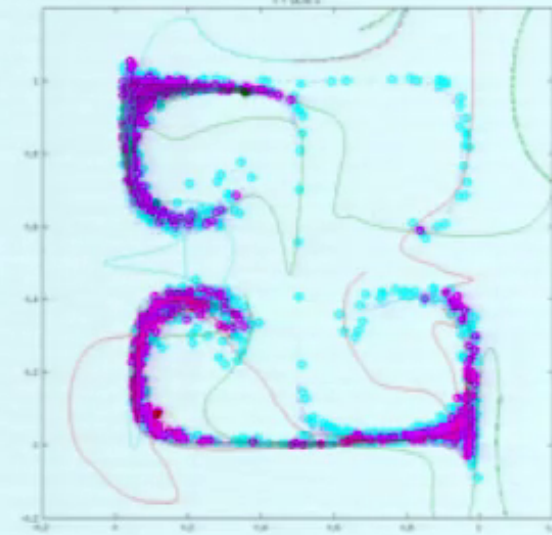


Wavelet vs changing the weight on spatial modes

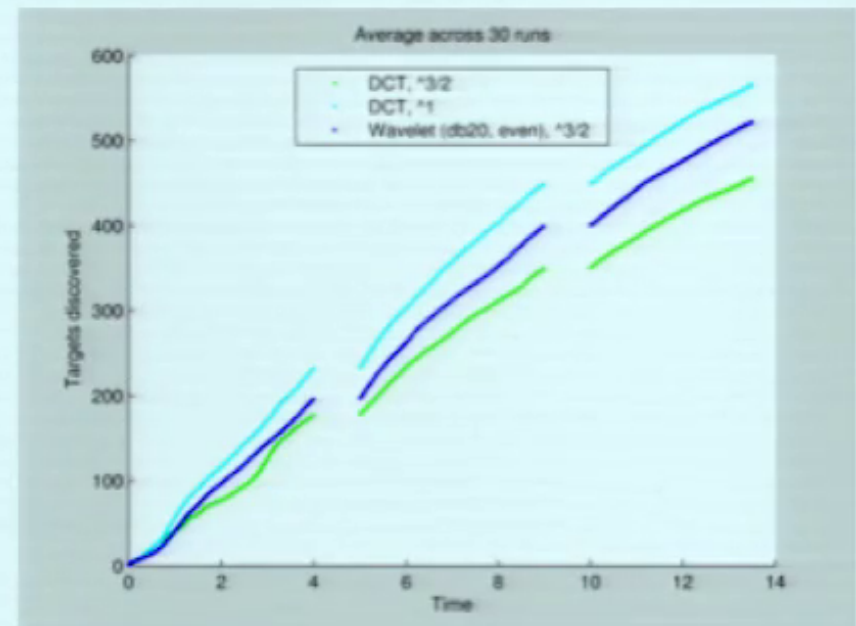
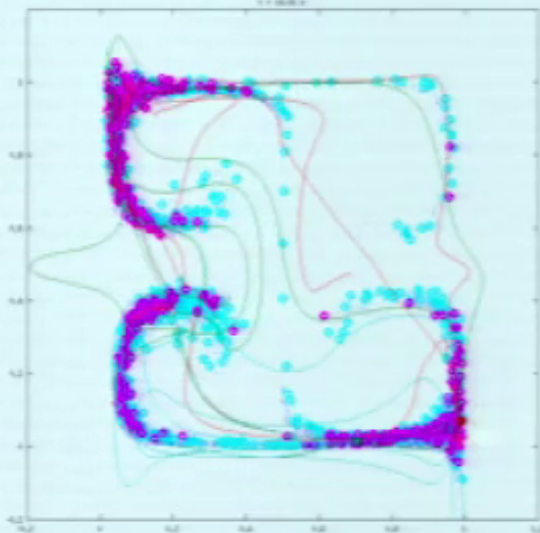
DCT $s=1$



Wavelet



DCT $s=3/2$



Conclusion

- Mesohyperbolicity: Hypergraph maps
prediction of target dynamic
help define a search strategy
give useful information on the dynamic of the targets
- Mix-Norm: DSMC search strategies
searches the region with high probability
searches from large to low spatial scale
large spatial scale can be weighted in the filamented case
- The Hunt for MH370 is still going on!