TV-regularized Extended LSRTM

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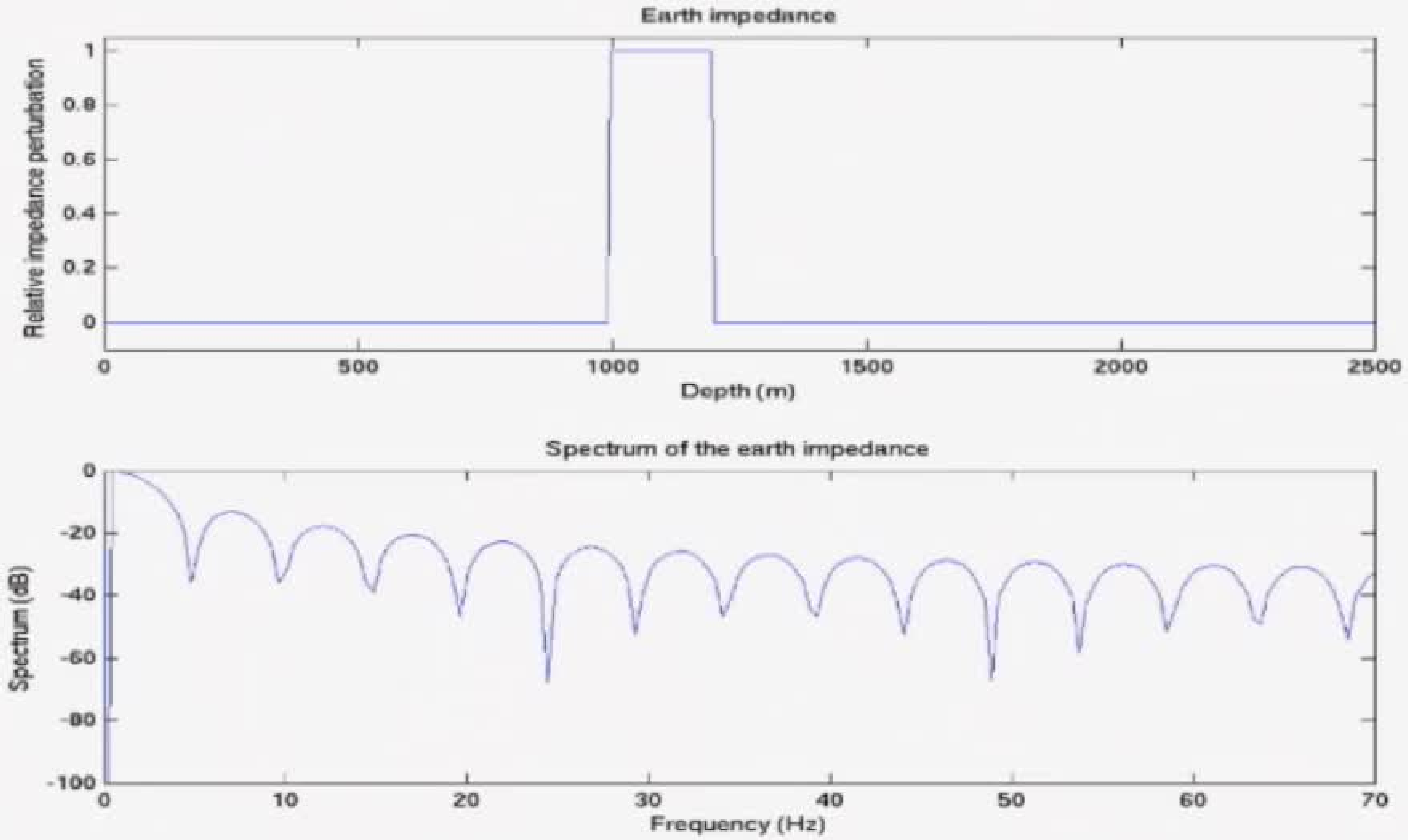
Motivation



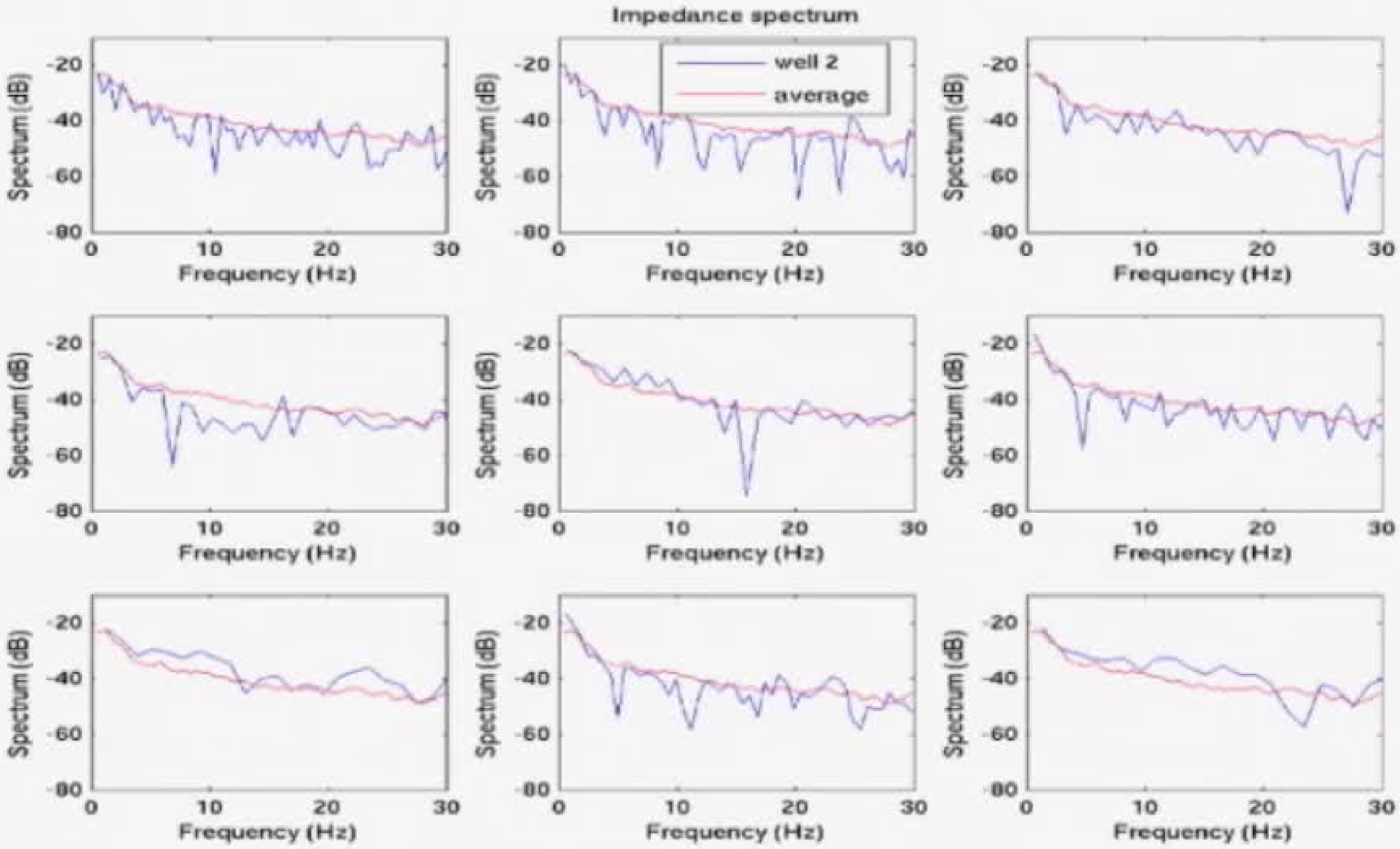
Earth impedance
Relative impedance perturbation
1
0.8
0.6
0.4
0.2
0
0
500
1000
1500
2000
2500
Depth (m)
Spectrum of the earth impedance
Spectrum (dB)
0
-20
-40
-60
-80
-100
0
10
20
30
40
50
60
70
Frequency (Hz)

Motivation

- Quantitative inversion (eg, Becquey et al., 1979; van Riel, 2000; Savic et al., 2000)
  - Work on migrated images/gathers
  - Dip-dependent wavelet stretch
  - Require a highly accurate source signature
- Least-squares migration (eg, Clayton and Stolt, 1981; Nemeth et al., 1999)
  - Target at reflectivity
- Waveform acoustic impedance inversion with spectral shaping (Lazaratos et al., 2011, Li et al., 2012)
  - Resolve impedance
  - Obtain the earth spectrum in the first iteration

What is the shaping filter to apply on the model?

What is the Earth spectrum?



What is the Earth spectrum?

- Dominated by low frequency
- Can be idealized as layered medium

$$\min ||\mathbf{A}m - I||$$

$$\text{s.t.} \quad ||\mathrm{m}||_{\mathrm{TV}} < \epsilon$$

$\mathbf{A}$: Nonstationary convolution operator

I : Migration image

m : Impedance model

Gholami, Geophysics, 2015

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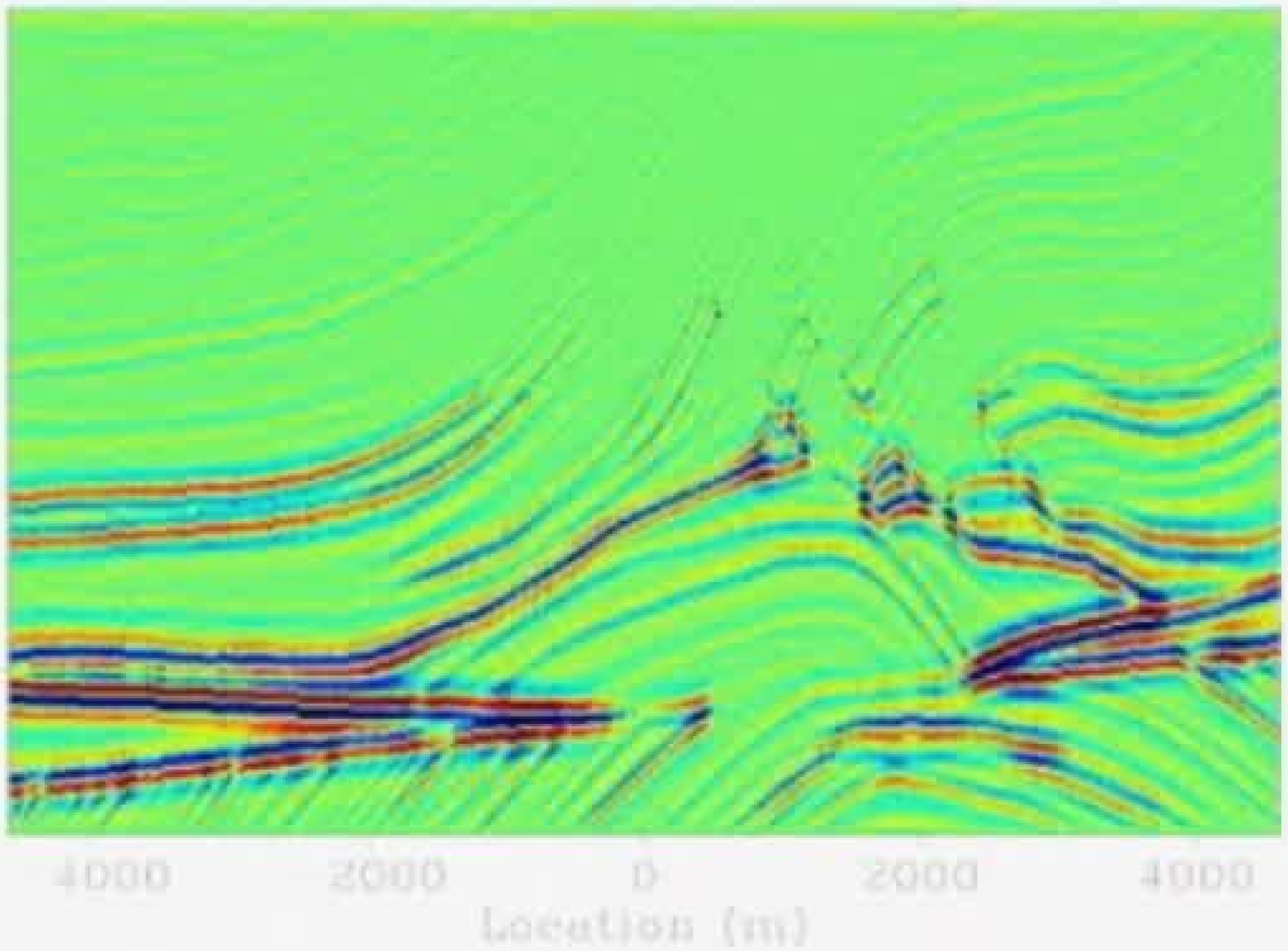
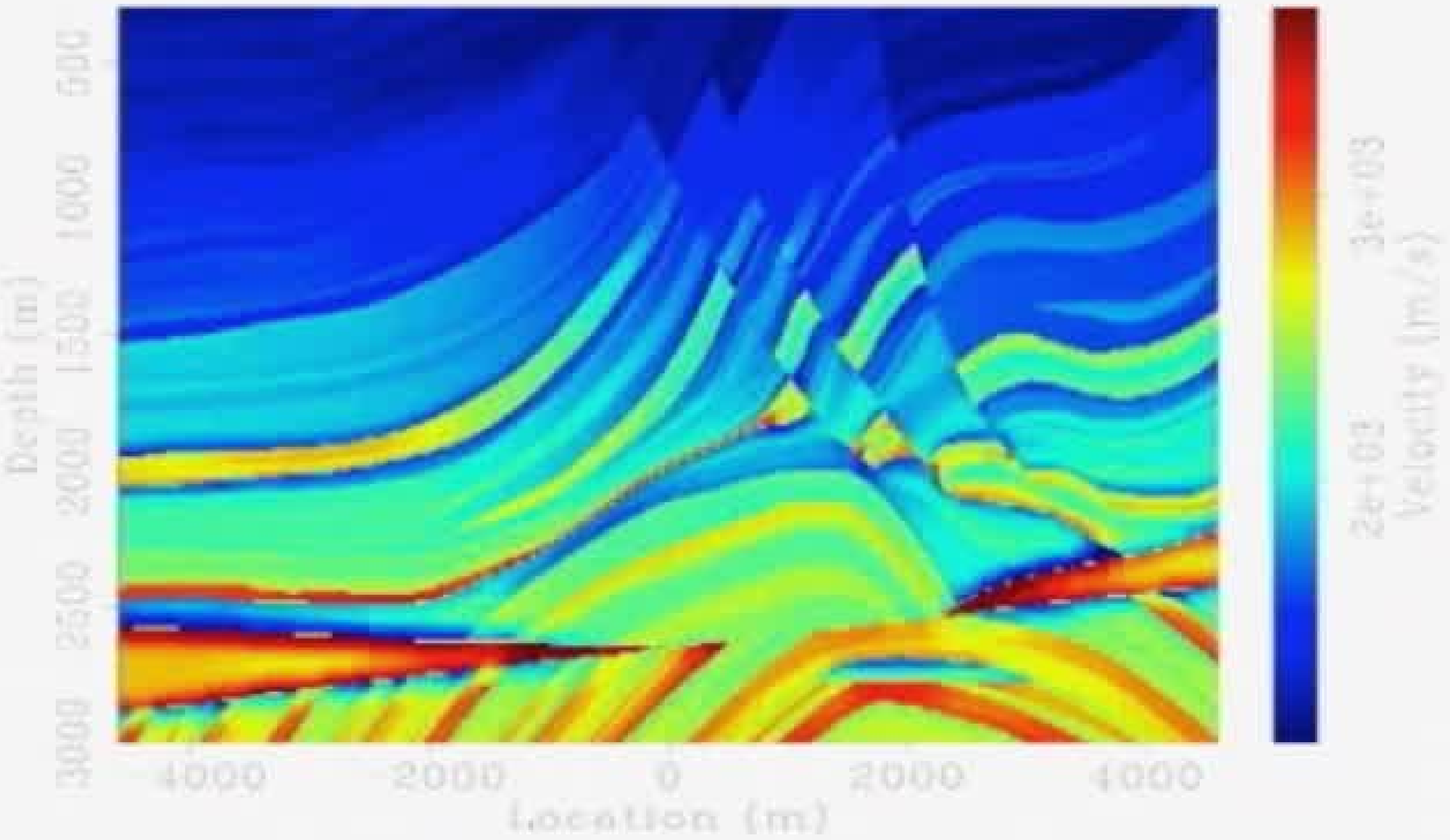




Outline

- Motivation and TV-constrained deconvolution
- TV-constrained extended LSRTM and frequency extrapolation FWI
- Discussions and conclusions

Extended imaging operator

(Sava and fomel, 2009)

$$I(\mathbf{x},\mathbf{h}) = \frac{2}{v_0^3(\mathbf{x})}\int d\mathbf{x}_s d\mathbf{x}_r dt d\tau \, w(t)G(\mathbf{x}_s,\mathbf{x}-\mathbf{h},\tau) G(\mathbf{x}+\mathbf{h},\mathbf{x}_r,t-\tau)\frac{\partial^2}{\partial t^2}\delta d(\mathbf{x}_s,\mathbf{x}_r,t)$$

Source wavefield

Scattered wavefield

$$I(\mathbf{x},\mathbf{h}) = \mathbf{F}_e^*\delta d$$

Extended modeling operator

Band-limited source

Extended image

$$\delta\hat{d}(\mathbf{x}_s,\mathbf{x}_r,t) = \frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\mathbf{h}d\tau\, w(\tau) G(\mathbf{x}_s,\mathbf{x}-\mathbf{h},\tau)\frac{2I(\mathbf{x},\mathbf{h})}{v_0^3(\mathbf{x})}G(\mathbf{x}+\mathbf{h},\mathbf{x}_r,t-\tau)$$

Source propagation

Scattered propagation

Kinematics of data are preserved despite errors in Greens' functions

$$\delta\hat{d} = \mathbf{F}_e I(\mathbf{x},\mathbf{h})$$

Data fitting with extended inversion

$$\min ||\mathbf{F}_e \Delta m(\mathbf{x}, \mathbf{h}) - \delta d||_2^2$$
$$\text{s.t.} \quad ||\Delta m(\mathbf{x}, \mathbf{h})||_{\text{TV}} \le \tau$$

$\mathbf{F}_e$: extended Born modeling operator

$||\cdot||_{\text{TV}}$: total variation norm in $\mathbf{x}$

Least-squares extended reverse time migration (LSERM)

Li and Demanet, 2017, Lin and Huang, 2015

Deconvolution with TV constraints

$$\min ||\mathbf{A}\Delta m(\mathbf{x},\mathbf{h}) - I(\mathbf{x},\mathbf{h})||_2^2$$
$$\text{s.t.} \quad ||\Delta m(\mathbf{x},\mathbf{h})||_{\mathrm{TV}} \leq \tau$$

$\Delta m(\mathbf{x},\mathbf{h}) \longleftarrow I(\mathbf{x},\mathbf{h})$

- Bandwidth extrapolated
- Kinematics preserved w.r.t. m_0

- Bandlimited to the data frequencies
- Kinematics preserved w.r.t. m_0

Synthesize low frequencies

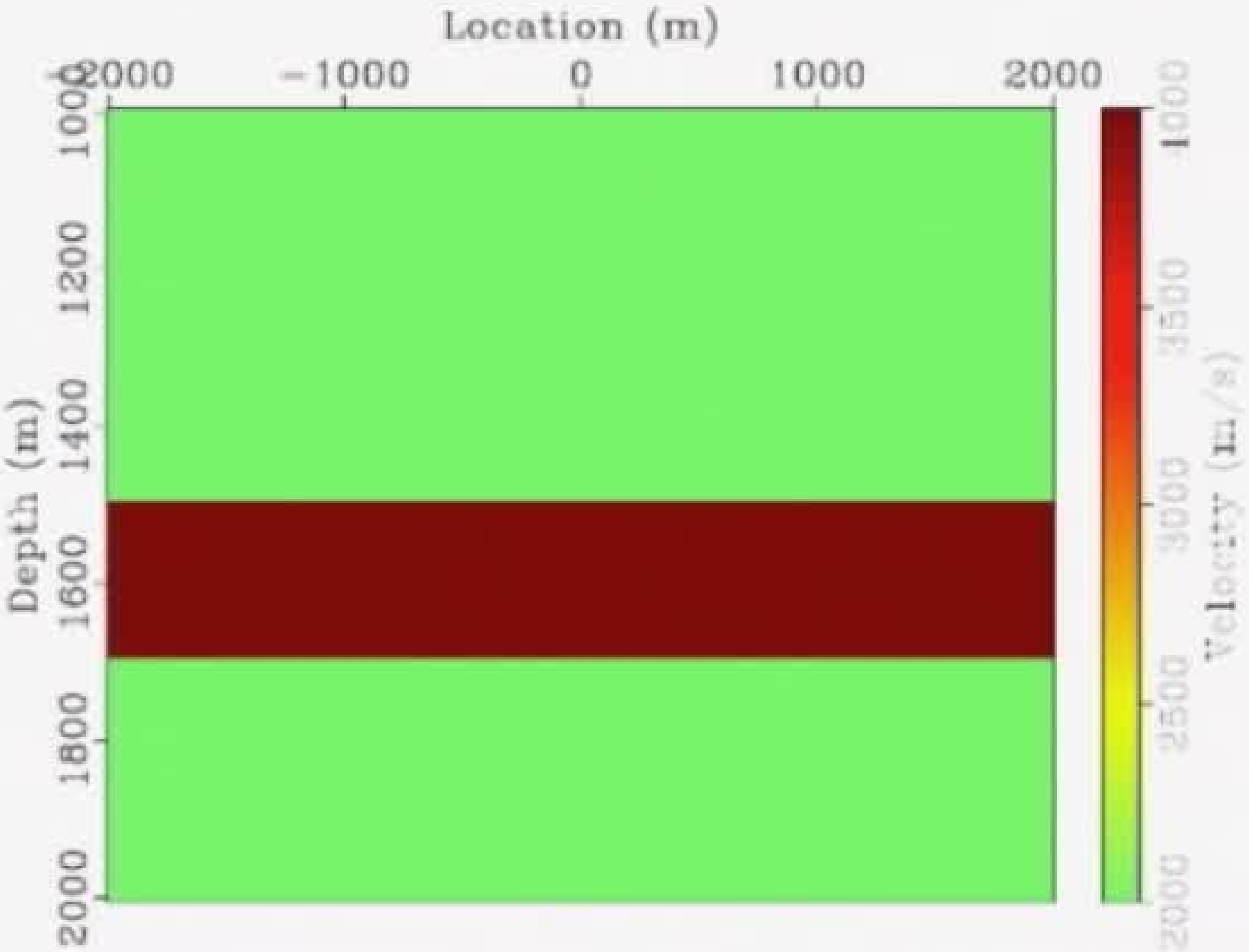
$$\delta\hat{d}_l(\mathbf{x}_s,\mathbf{x}_r,t) = \frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\mathbf{h}d\tau\, w_l(\tau) G(\mathbf{x}_s,\mathbf{x}-\mathbf{h},\tau)\frac{2\Delta m(\mathbf{x},\mathbf{h})}{v_0^3(\mathbf{x})}G(\mathbf{x}+\mathbf{h},\mathbf{x}_r,t-\tau)$$

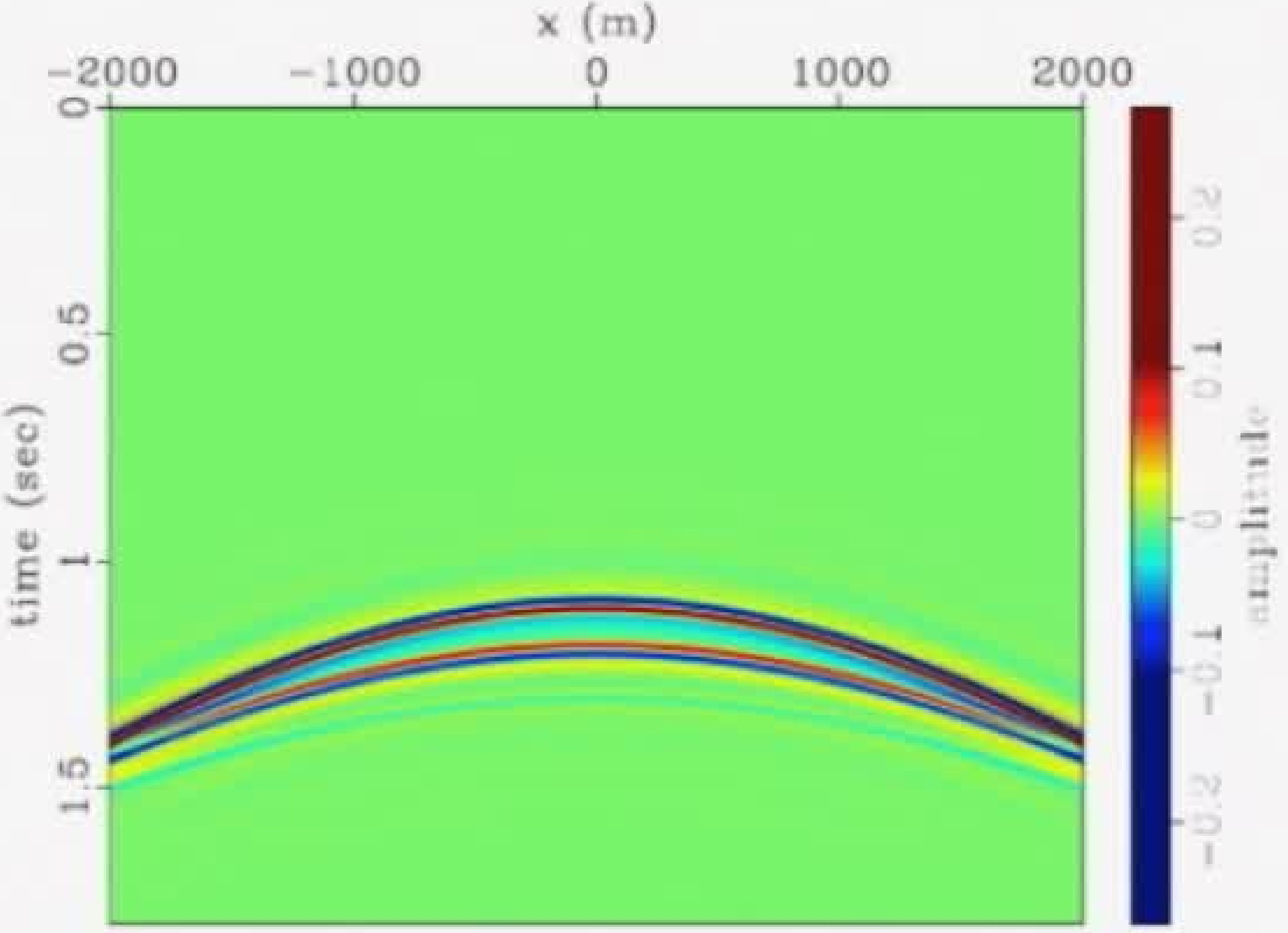
Synthesized low frequency data

Low frequency source wavelet

- G and $\Delta m(x,h)$ preserve kinematics
- w_l and $\Delta m(x,h)$ extend the frequency content

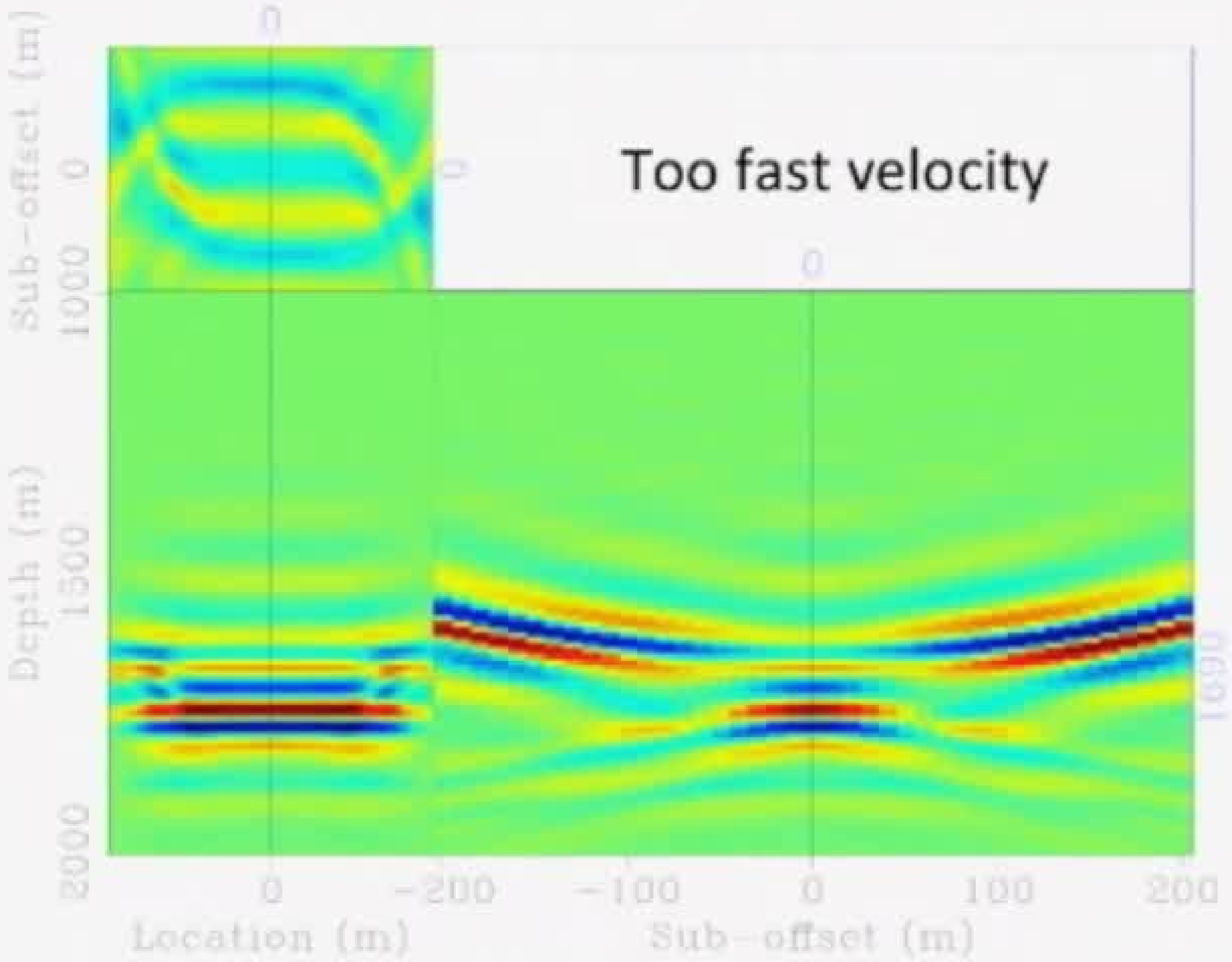
Toy three-layer example

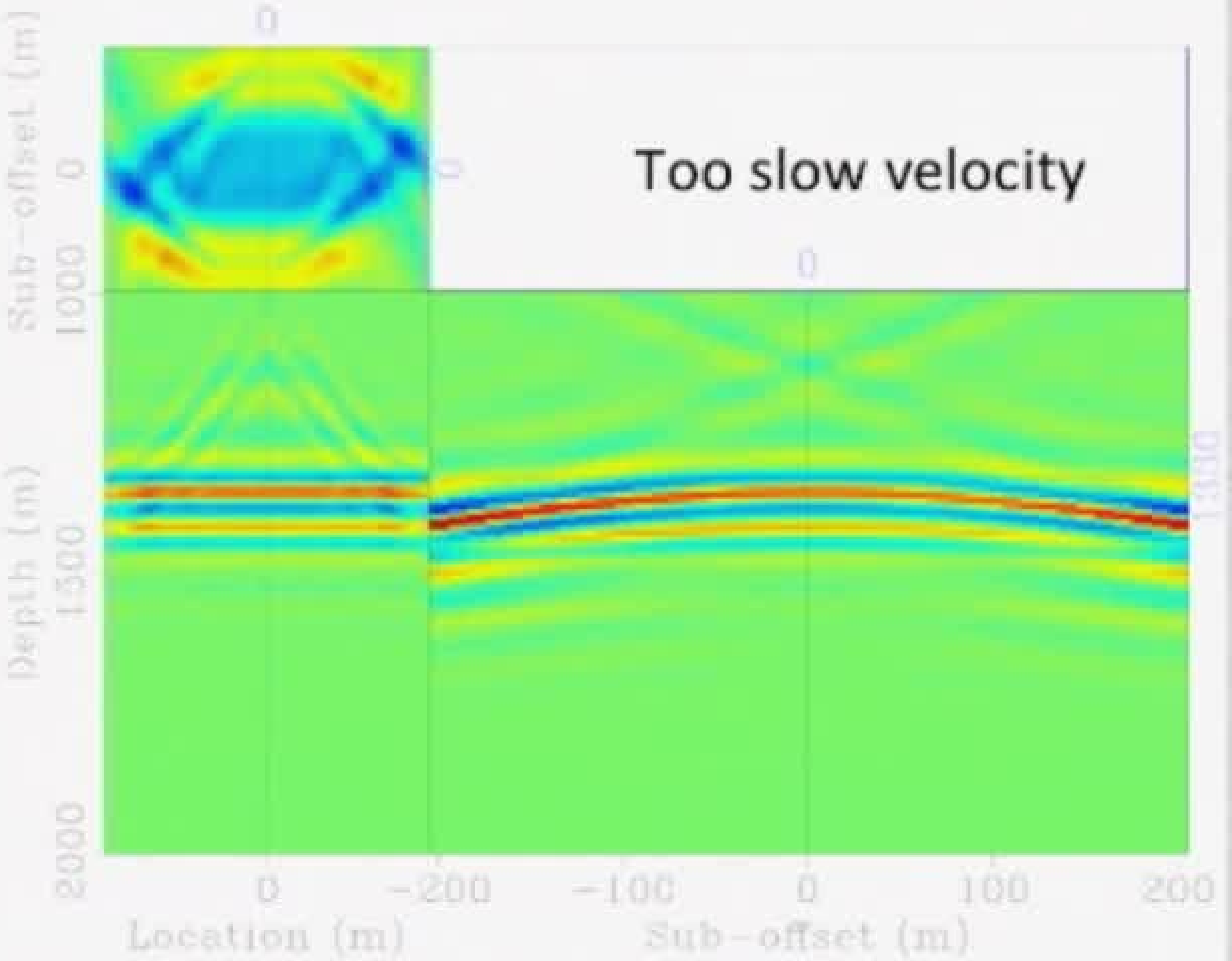




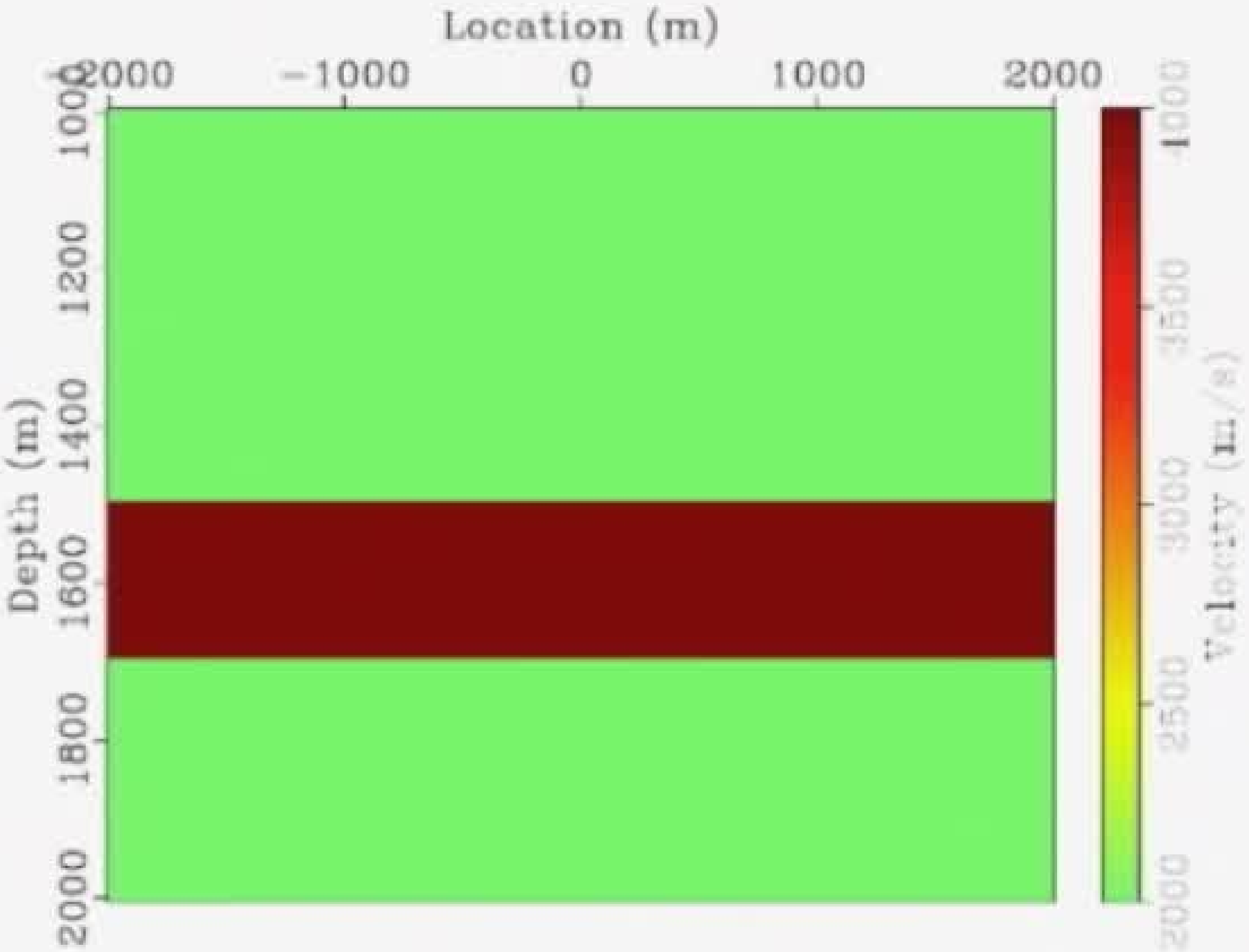
Recorded data 6 – 50 Hz

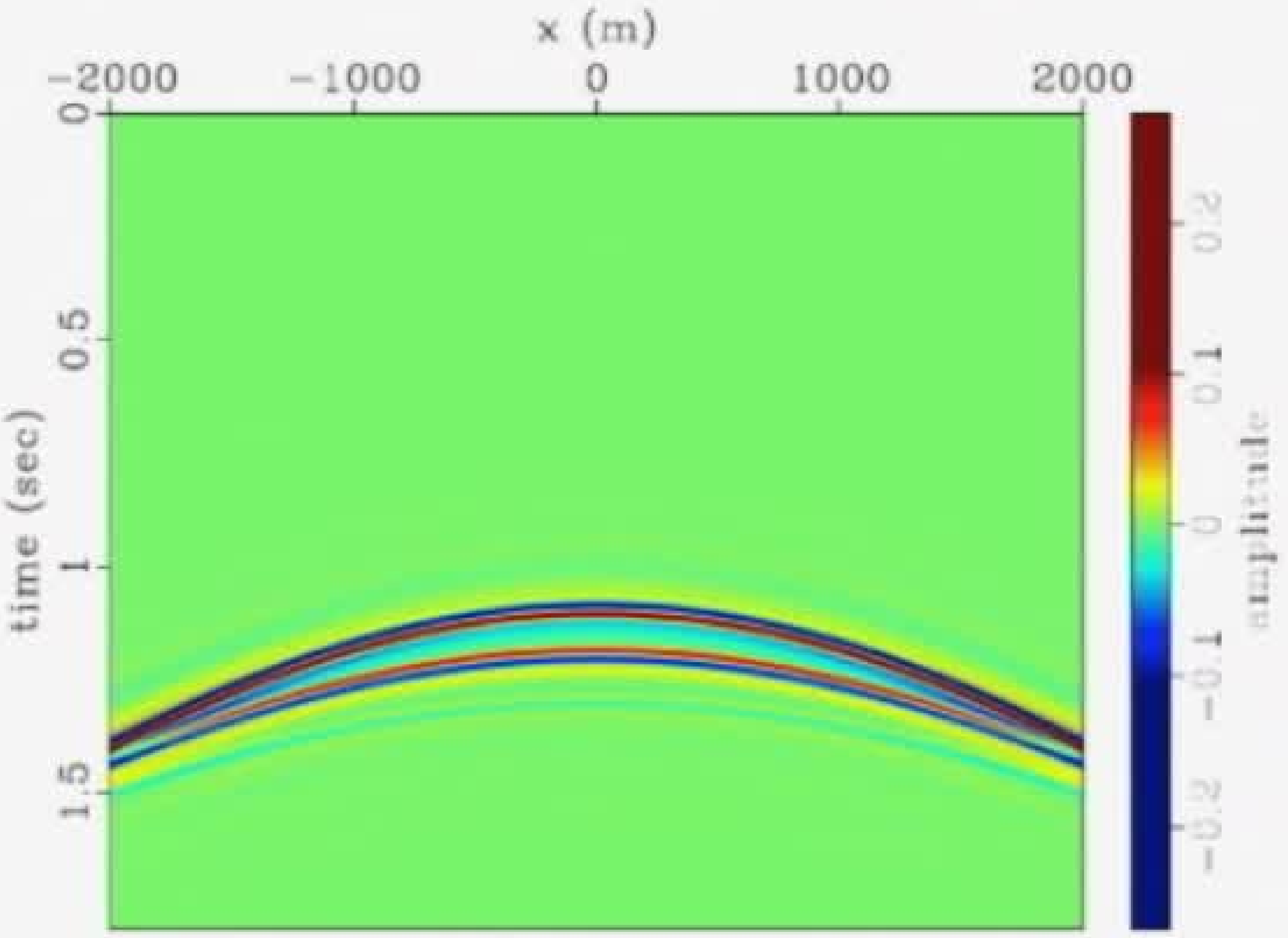
Extended images





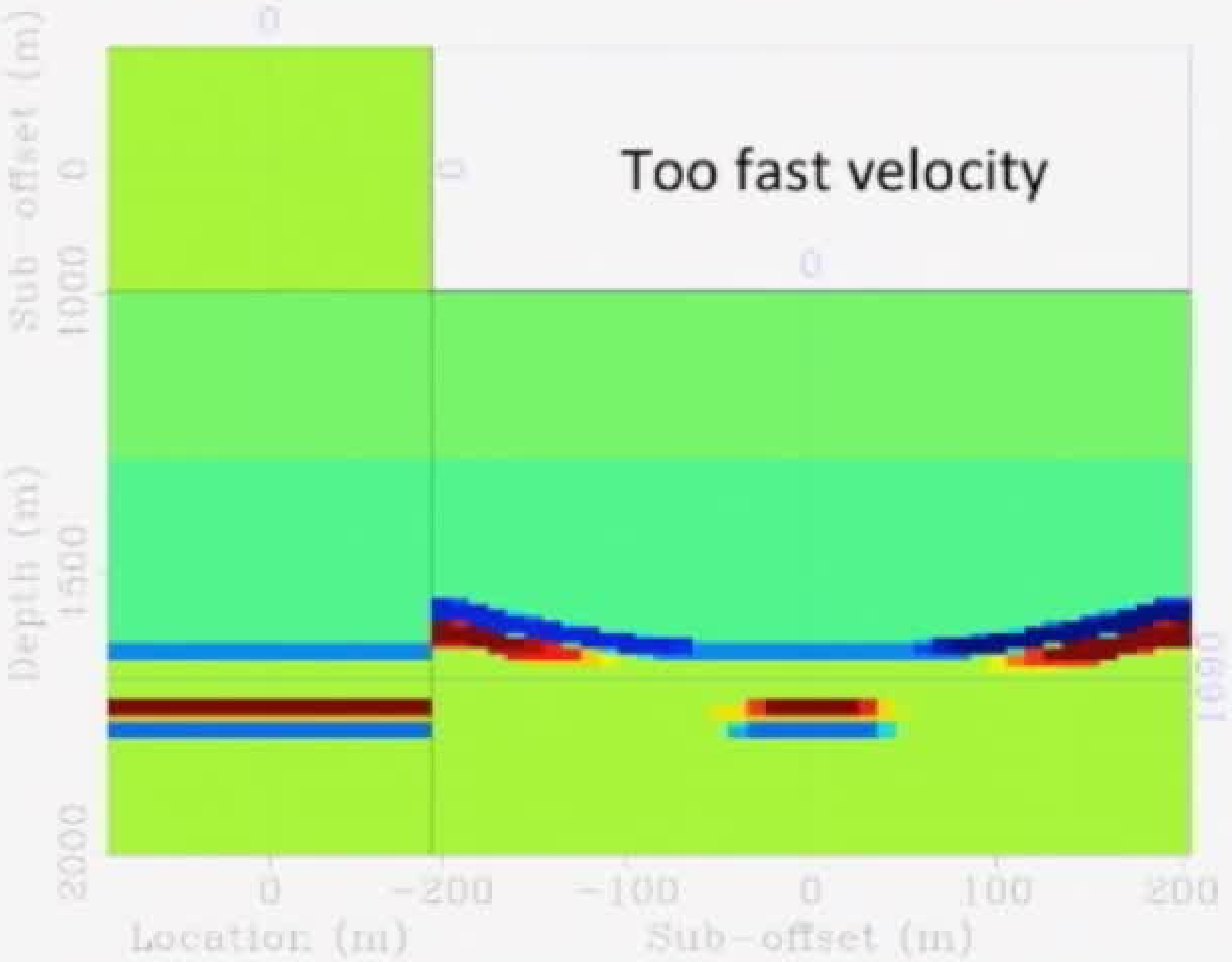
Toy three-layer example



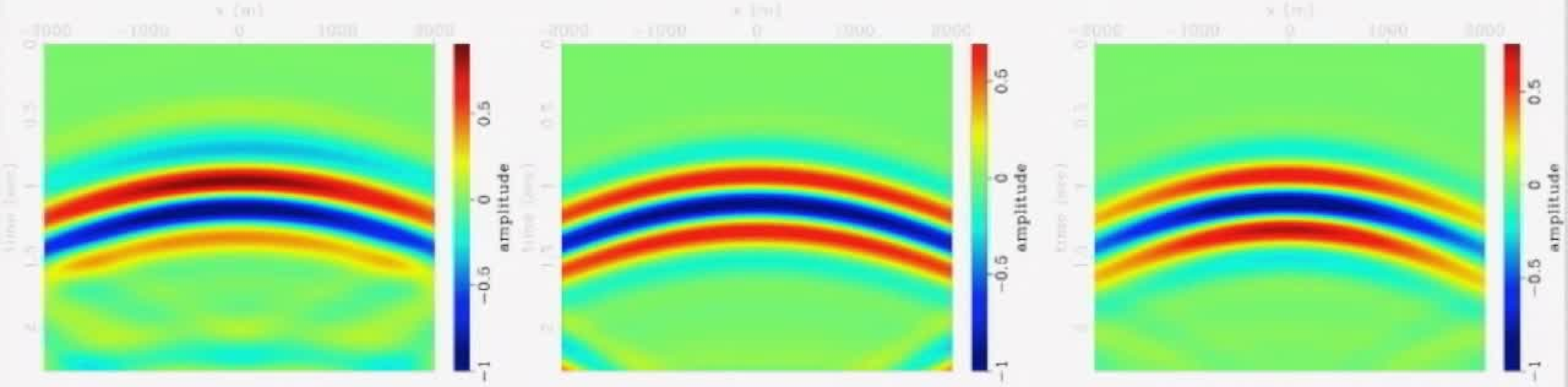


Recorded data 6 – 50 Hz

Extended images with TV constraint



Extrapolated data [0 – 3 Hz]

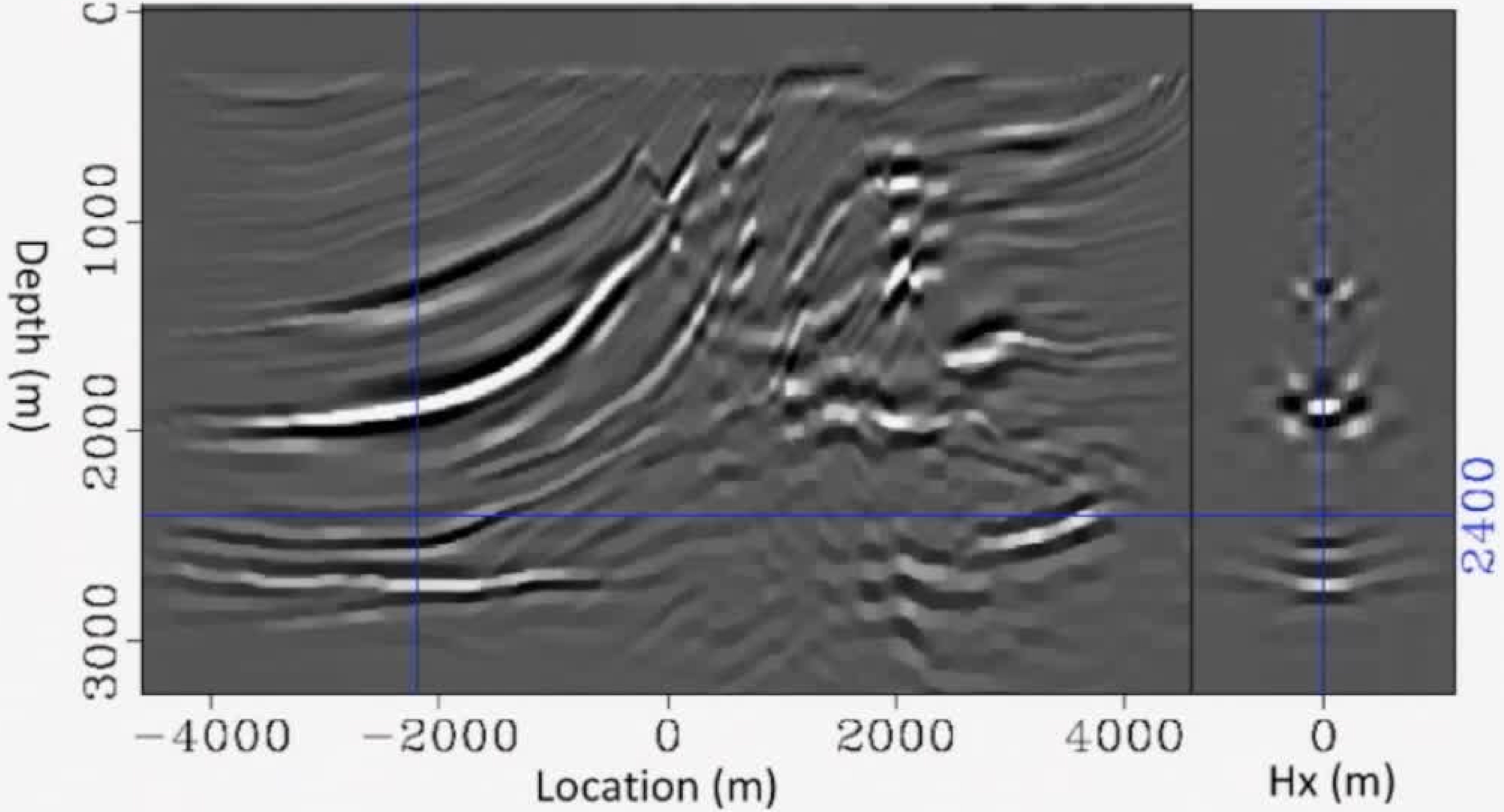


Too fast velocity

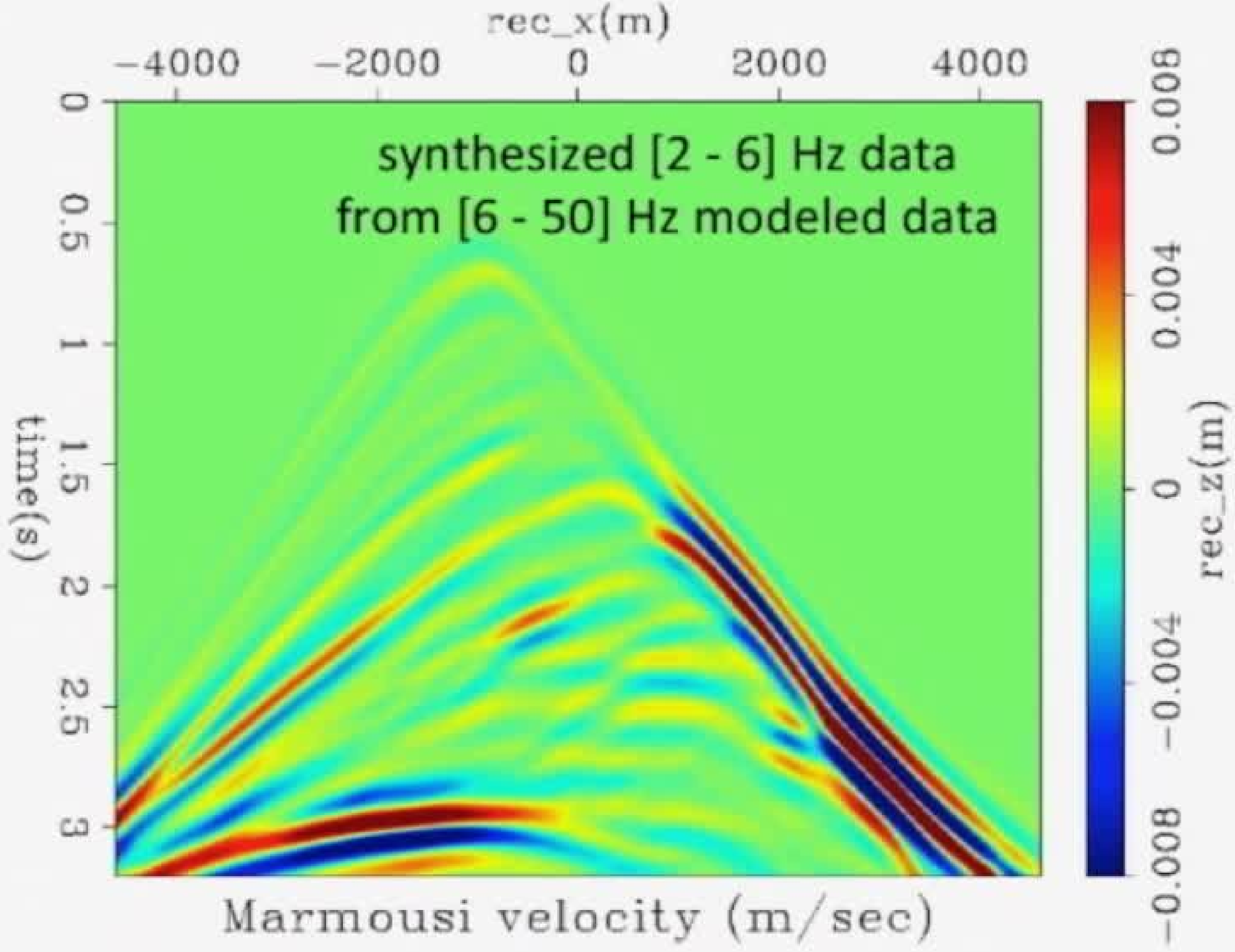
True modeled data

Too slow velocity

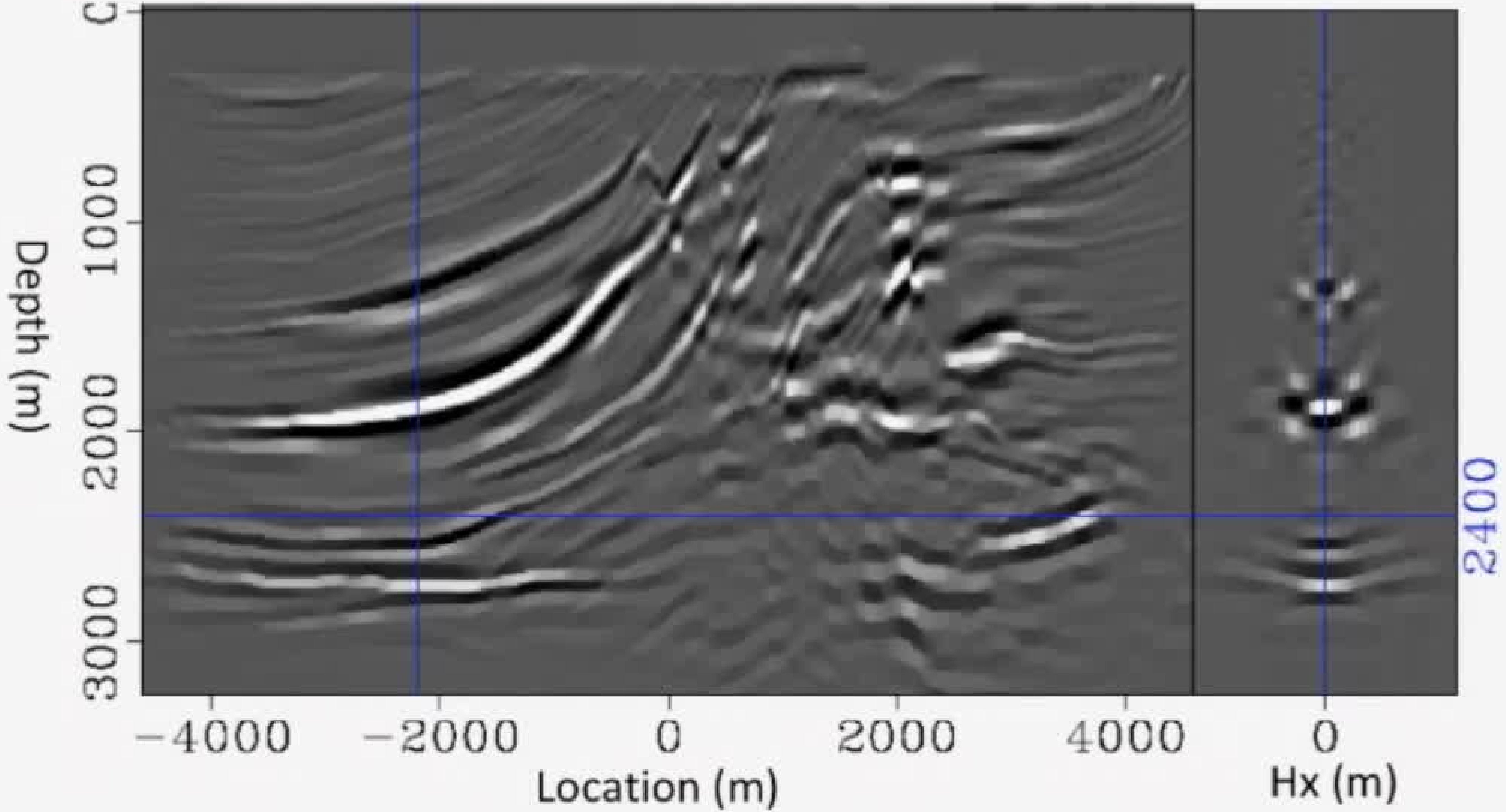
Extended Born imaging + TV constraint



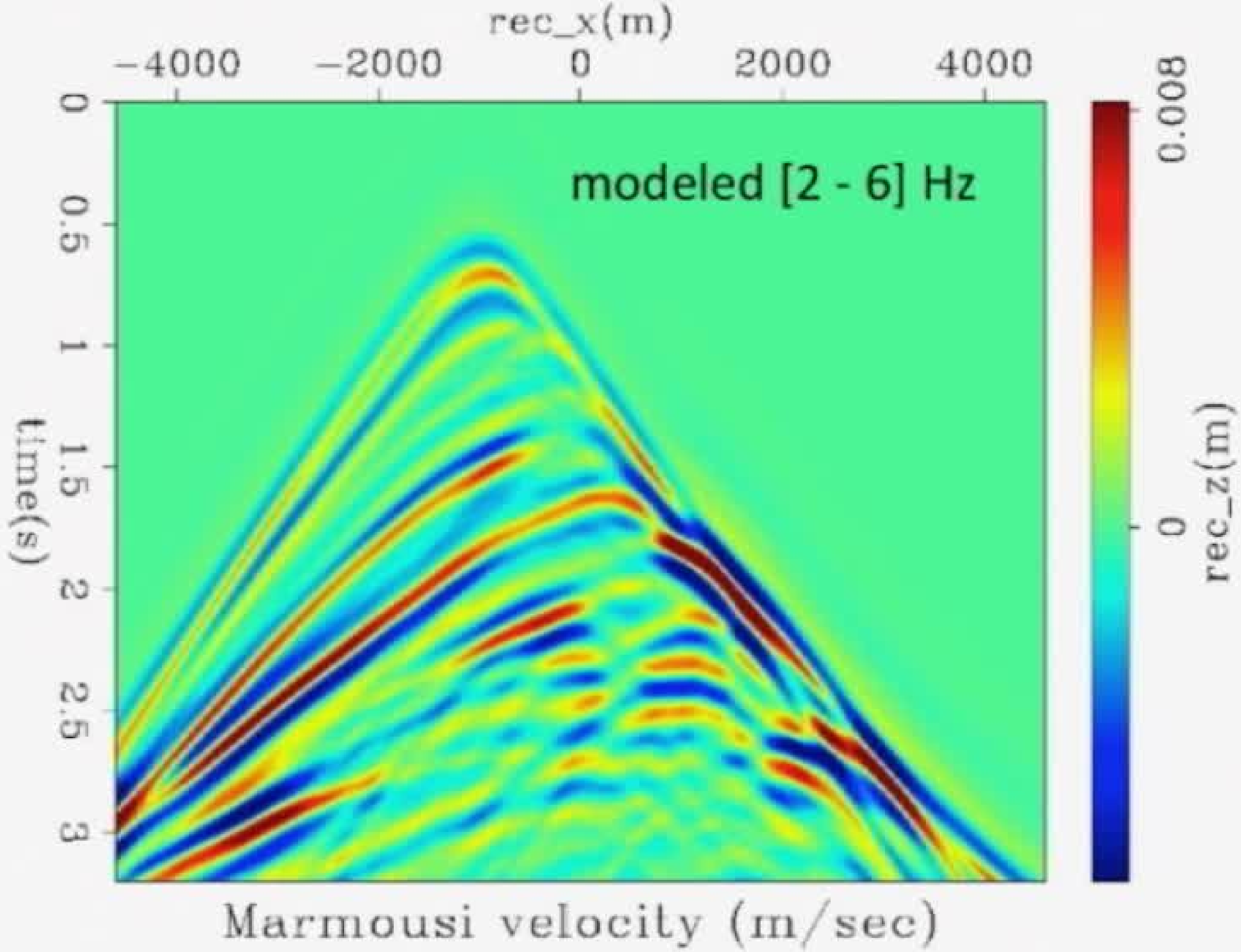
Synthesized low frequencies



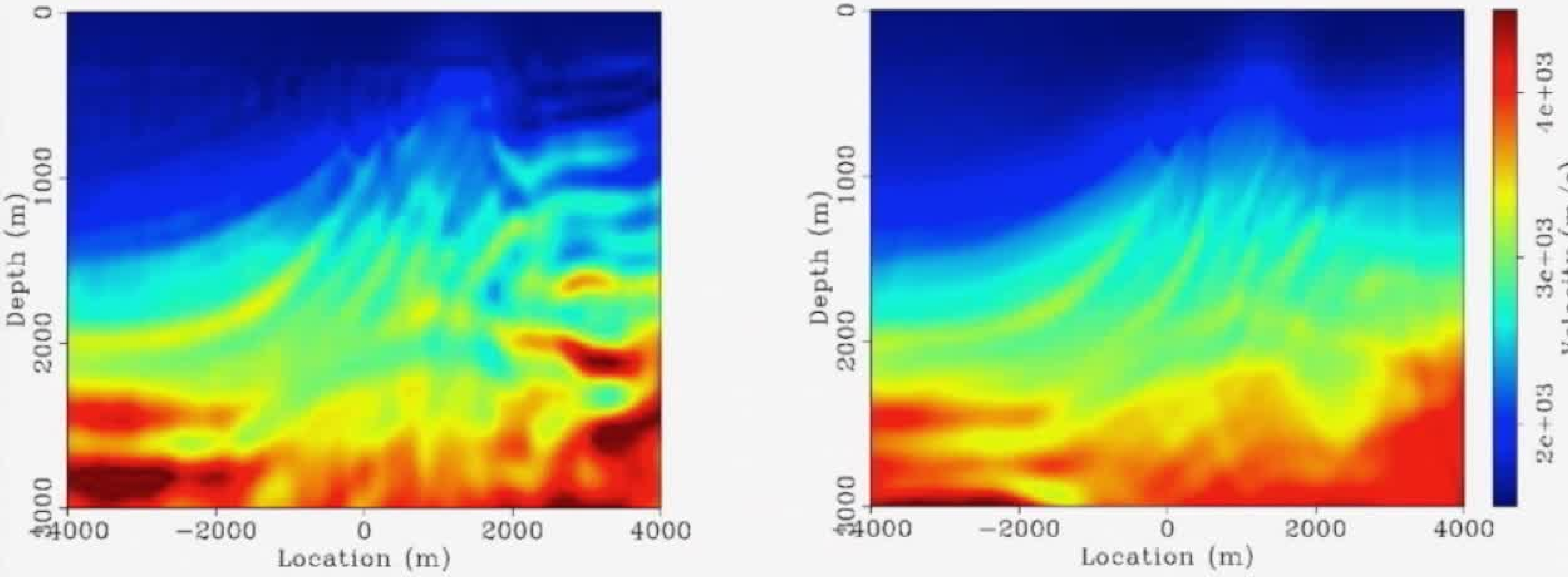
Extended Born imaging + TV constraint



Modeled low frequencies



Low frequency models



FWI with [2-5] Hz modeled data

FWI with [2-5] Hz extrapolated data

Discussions and conclusions

- **Extended imaging extrapolation**

- Fully utilizes existing imaging and waveform inversion algorithms
  - Amplitude and phase extrapolated implicitly by TV regularization
- Requires reasonable initial velocity model
  - TV regularization with initial image becomes unstable when velocity error is too large

- More stable because of higher S/N in the image space
  - Reconstruct data at all offsets

- More reliable (relative) extrapolated amplitude as well as extrapolated phase
  - LSERTM inverts for the true velocity contrasts
  - Nondispersive assumptions are more realistic

Acknowledgement

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