A Phase-field for Fluid-Structure Interaction with Application to Cell Biology

Sebastian Aland[1,2], Helmut Abels[3], Dominic Mokbel[2]

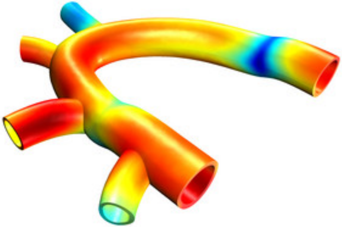
[1]HTW Dresden - University of Applied Sciences, Germany

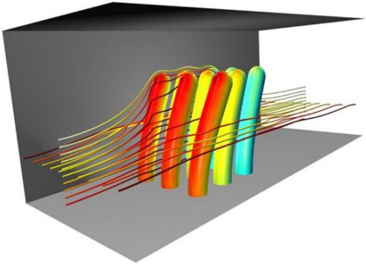
[2]TU Dresden, Germany

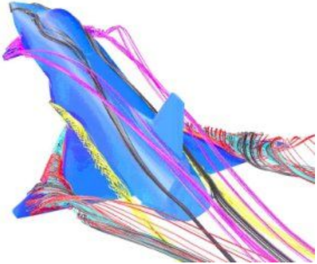
[3]Universtität Regensburg

SIAM Life Science, Minneapolis, 2018

Fluid-Structure Interaction

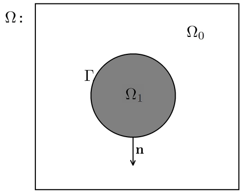






Classical two-phase setting

- Ω_0 ... incompressible fluid
- Ω_1 ... incompressible elasic solid



Classical two-phase model

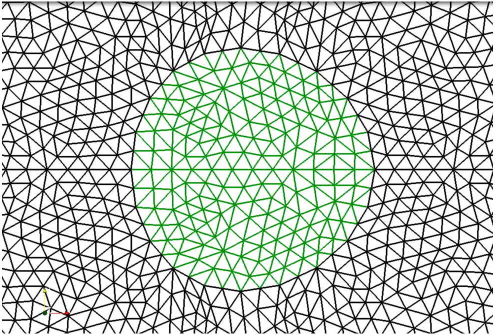
Balance of momentum and mass

$$\begin{aligned} \rho_i \partial_t^{\bullet} \mathbf{v} - \nabla \cdot \mathbb{S}_i + \nabla p_i &= 0 && \text{in } \Omega_i(t), i = 0, 1 \\ \nabla \cdot \mathbf{v} &= 0 && \text{in } \Omega_i(t), i = 0, 1 \end{aligned}$$

Force balance at the interface Γ

$$\mathbf{n} \cdot [\mathbb{S}_i - p_i \mathbb{I}]_0^1 = 0 \qquad \text{on } \Gamma(t)$$

The standard method: ALE



- Solid grid points (green) move with the velocity field $\mathbf{v}_1$
- Fluid grid points (black) move with auxiliary velocity $\tilde{\mathbf{v}}$ resulting from following Laplace problem

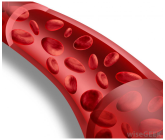
$$\begin{aligned} \Delta\tilde{\mathbf{v}} &= 0 && \text{in } \Omega_0, \\ \tilde{\mathbf{v}} &= 0 && \text{on } \delta\Omega_0 \backslash \Gamma, \\ \tilde{\mathbf{v}} &= \mathbf{v}_1 && \text{on } \Gamma, \end{aligned}$$

  to harmonically extend the interface movement into Ω_0
- Problem: this fails for larger deformations/translations/rotations (re-triangulation necessary)

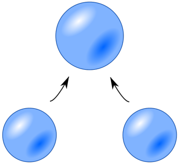
Challenges

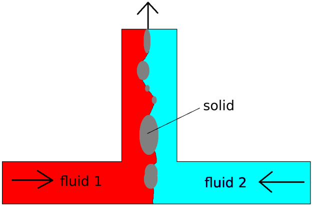
- complex deformations that cannot be tackled by simple grid movement

- interactions of fluid, elastic and viscoelastic phases



- topological changes, solidification, fluidization





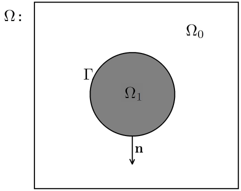
Phase field modeling

Benchmark for cells in flow

Summary

Classical two-phase setting

- Ω_0 ... incompressible fluid
- Ω_1 ... incompressible elasic solid



Classical two-phase model

Balance of momentum and mass

$$\begin{aligned} \rho_i \partial_t^\bullet \mathbf{v} - \nabla \cdot \mathbb{S}_i + \nabla p_i &= 0 && \text{in } \Omega_i(t), i = 0, 1 \\ \nabla \cdot \mathbf{v} &= 0 && \text{in } \Omega_i(t), i = 0, 1 \end{aligned}$$

Force balance at the interface Γ

$$\mathbf{n} \cdot [\mathbb{S}_i - p_i \mathbb{I}]_0^1 = 0 \qquad \text{on } \Gamma(t)$$

Distribution form

momentum equation:

$$\begin{aligned} \rho_i \partial_t^\bullet \mathbf{v} - \nabla \cdot \mathbb{S}_i + \nabla p_i &= 0 && \text{in } \Omega_i(t) \\ \mathbf{n} \cdot [\mathbb{S}_i - p_i \mathbb{I}]_0^1 &= 0 && \text{on } \Gamma(t) \end{aligned}$$

Weak form

$$\int_{\Omega_0} \rho_0 \partial_t^\bullet \mathbf{v}\xi + \mathbb{S}_0 : \nabla\xi - p_0 \nabla \cdot \xi + \int_{\Omega_1} \rho_1 \partial_t^\bullet \mathbf{v}\xi + \mathbb{S}_1 : \nabla\xi - p_1 \nabla \cdot \xi = 0$$

Use characteristic functions χ_i for Ω_i, resp., such that: $\int_{\Omega_i} \psi = \int_\Omega \chi_i \psi$

$$\int_\Omega (\rho_0\chi_0 + \rho_1\chi_1)\partial_t^\bullet \mathbf{v}\xi + (\chi_0\mathbb{S}_0 + \chi_1\mathbb{S}_1) : \nabla\xi - (\chi_0 p_0 + \chi_1 p_1)\nabla \cdot \xi = 0$$

strong form

$$(\rho_0\chi_0 + \rho_1\chi_1)\partial_t^\bullet \mathbf{v} - \nabla \cdot (\mathbb{S}_0\chi_0 + \mathbb{S}_1\chi_1) + \nabla p = 0 \qquad \text{in } \Omega$$

where $p = \chi_0 p_0 + \chi_1 p_1$

Distribution form

momentum equation:

$$\begin{aligned} \rho_i \partial_t^\bullet \mathbf{v} - \nabla \cdot \mathbb{S}_i + \nabla p_i &= 0 && \text{in } \Omega_i(t) \\ \mathbf{n} \cdot [\mathbb{S}_i - p_i \mathbb{I}]_0^1 &= 0 && \text{on } \Gamma(t) \end{aligned}$$

Weak form

$$\int_{\Omega_0} \rho_0 \partial_t^\bullet \mathbf{v}\xi + \mathbb{S}_0 : \nabla\xi - p_0 \nabla \cdot \xi + \int_{\Omega_1} \rho_1 \partial_t^\bullet \mathbf{v}\xi + \mathbb{S}_1 : \nabla\xi - p_1 \nabla \cdot \xi = 0$$

Use characteristic functions χ_i for Ω_i, resp., such that: $\int_{\Omega_i} \psi = \int_\Omega \chi_i \psi$

$$\int_\Omega (\rho_0\chi_0 + \rho_1\chi_1) \partial_t^\bullet \mathbf{v}\xi + (\chi_0 \mathbb{S}_0 + \chi_1 \mathbb{S}_1) : \nabla\xi - (\chi_0 p_0 + \chi_1 p_1) \nabla \cdot \xi = 0$$

strong form

$$(\rho_0\chi_0 + \rho_1\chi_1) \partial_t^\bullet \mathbf{v} - \nabla \cdot (\mathbb{S}_0 \chi_0 + \mathbb{S}_1 \chi_1) + \nabla p = 0 \qquad \text{in } \Omega$$

$$\text{where } p = \chi_0 p_0 + \chi_1 p_1$$

Distribution form

momentum equation:

$$\begin{aligned} \rho_i \partial_t^\bullet \mathbf{v} - \nabla \cdot \mathbb{S}_i + \nabla p_i &= 0 && \text{in } \Omega_i(t) \\ \mathbf{n} \cdot [\mathbb{S}_i - p_i \mathbb{I}]_0^1 &= 0 && \text{on } \Gamma(t) \end{aligned}$$

Weak form

$$\int_{\Omega_0} \rho_0 \partial_t^\bullet \mathbf{v} \xi + \mathbb{S}_0 : \nabla \xi - p_0 \nabla \cdot \xi + \int_{\Omega_1} \rho_1 \partial_t^\bullet \mathbf{v} \xi + \mathbb{S}_1 : \nabla \xi - p_1 \nabla \cdot \xi = 0$$

Use characteristic functions χ_i for Ω_i, resp., such that: $\int_{\Omega_i} \psi = \int_\Omega \chi_i \psi$

$$\int_\Omega (\rho_0 \chi_0 + \rho_1 \chi_1) \partial_t^\bullet \mathbf{v} \xi + (\chi_0 \mathbb{S}_0 + \chi_1 \mathbb{S}_1) : \nabla \xi - (\chi_0 p_0 + \chi_1 p_1) \nabla \cdot \xi = 0$$

strong form

$$(\rho_0 \chi_0 + \rho_1 \chi_1) \partial_t^\bullet \mathbf{v} - \nabla \cdot (\mathbb{S}_0 \chi_0 + \mathbb{S}_1 \chi_1) + \nabla p = 0 \qquad \text{in } \Omega$$

$$\text{where } p = \chi_0 p_0 + \chi_1 p_1$$

Distribution form

momentum equation:

$$\begin{aligned} \rho_i \partial_t^\bullet \mathbf{v} - \nabla \cdot \mathbb{S}_i + \nabla p_i &= 0 && \text{in } \Omega_i(t) \\ \mathbf{n} \cdot [\mathbb{S}_i - p_i \mathbb{I}]_0^1 &= 0 && \text{on } \Gamma(t) \end{aligned}$$

Weak form

$$\int_{\Omega_0} \rho_0 \partial_t^\bullet \mathbf{v} \xi + \mathbb{S}_0 : \nabla \xi - p_0 \nabla \cdot \xi + \int_{\Omega_1} \rho_1 \partial_t^\bullet \mathbf{v} \xi + \mathbb{S}_1 : \nabla \xi - p_1 \nabla \cdot \xi = 0$$

Use characteristic functions χ_i for Ω_i, resp., such that: $\int_{\Omega_i} \psi = \int_\Omega \chi_i \psi$

$$\int_\Omega (\rho_0 \chi_0 + \rho_1 \chi_1) \partial_t^\bullet \mathbf{v} \xi + (\chi_0 \mathbb{S}_0 + \chi_1 \mathbb{S}_1) : \nabla \xi - (\chi_0 p_0 + \chi_1 p_1) \nabla \cdot \xi = 0$$

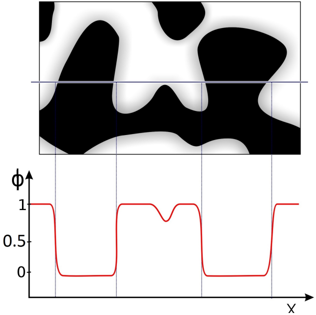
strong form

$$(\rho_0 \chi_0 + \rho_1 \chi_1) \partial_t^\bullet \mathbf{v} - \nabla \cdot (\mathbb{S}_0 \chi_0 + \mathbb{S}_1 \chi_1) + \nabla p = 0 \qquad \text{in } \Omega$$

$$\text{where } p = \chi_0 p_0 + \chi_1 p_1$$

Phase field model

- Introduce a phase field ϕ to represent the different domains ($\phi = 1$, $\phi = 0$)
- Interface given by $\Gamma = \{x | \phi(x) = 0.5\}$
- Allow for a partial mixing: $\phi = \frac{m_1}{m_0 + m_1}$ mass ratio
- Smooth transition at the interface of thickness ϵ



Useful properties:

- interface normal

$$\mathbf{n} = \frac{\nabla \phi}{|\nabla \phi|}$$

- phase field approximates characteristic functions

$$\chi_1 \approx \phi$$
$$\chi_0 \approx 1 - \phi$$
$$\delta_\Gamma \approx |\nabla \phi|$$

Coupled model for viscous/elastic phases

We obtain the general phase field approximation

Multiphase balance equations

$$
\begin{aligned}
\rho(\phi)\partial_t^\bullet \mathbf{v} - \nabla\cdot(\phi\mathbb{S}_0 + (1-\phi)\mathbb{S}_1) + \nabla p &= 0 && \text{in } \Omega \\
\nabla\cdot\mathbf{v} &= 0 && \text{in } \Omega
\end{aligned}
$$

where

$$
\rho(\phi) = \phi\rho_1 + (1-\phi)\rho_0
$$

- can in principle be used for any material stress
- grid movement replaced by transport of phase field
- no re-triangulation necessary!
- converges to the sharp interface equation for $\epsilon \to 0$

Open question: How to define the stress $\mathbb{S}$ for an elastic material

Coupled model for viscous/elastic phases

We obtain the general phase field approximation

Multiphase balance equations

$$\rho(\phi)\partial_t^\bullet \mathbf{v} - \nabla \cdot (\phi \mathbb{S}_0 + (1-\phi)\mathbb{S}_1) + \nabla p = 0 \qquad \text{in } \Omega$$
$$\nabla \cdot \mathbf{v} = 0 \qquad \text{in } \Omega$$

where

$$\rho(\phi) = \phi\rho_1 + (1-\phi)\rho_0$$

- can in principle be used for any material stress
- grid movement replaced by transport of phase field
- no re-triangulation necessary!
- converges to the sharp interface equation for $\epsilon \to 0$

Open question: How to define the stress $\mathbb{S}$ for an elastic material

The left Cauchy-Green strain tensor

- left Cauchy-Green strain tensor

$$\mathbb{B} = \frac{\partial X}{\partial \bar{X}} \frac{\partial X}{\partial \bar{X}}^T$$

where $\bar{X}, X(\bar{X}, t)$ are material point positions in initial and current configuration, resp.

- stress free (initial) configuration: $\mathbb{B} = \mathbb{I}$
- stress in single (neo-Hookean) elastic material

$$\mathbb{S} = \mu(\mathbb{B} - \mathbb{I})$$

where μ is the materials shear modulus

- define upper-convected time derivative: stretches and rotates $\mathbb{B}$ with the flow

$$\overset{\triangledown}{\mathbb{B}} := \partial_t^{\bullet} \mathbb{B} - \nabla \mathbf{v}^T \cdot \mathbb{B} - \mathbb{B} \cdot \nabla \mathbf{v}$$

- for a purely elastic material

$$\overset{\triangledown}{\mathbb{B}} = 0 \qquad \text{where initially } \mathbb{B} = \mathbb{I}$$

Combined strain model

Elastic strains in the two materials

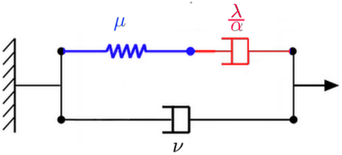
$$\overset{\triangledown}{\mathbb{B}} = 0 \qquad \text{in } \Omega_1$$
$$\mathbb{B} = \mathbb{I} \qquad \text{in } \Omega_0$$

can be combined to

Combined strain model

$$\lambda(\phi)\overset{\triangledown}{\mathbb{B}} + \alpha(\phi)(\mathbb{B} - \mathbb{I}) = 0 \qquad \text{(strain)}$$
$$\mathbb{S}(\phi) = \nu(\phi)(\nabla\mathbf{v} + \nabla\mathbf{v}^T) + \mu(\phi)(\mathbb{B} - \mathbb{I}) \qquad \text{(stress)}$$

where the only free parameter is the relaxation time λ/α



viscosity $\nu(\phi)$	shear modulus $\mu(\phi)$	$\lambda(\phi)$	$\alpha(\phi)$	material
$*$	0	0	1	viscous fluid
0	$*$	T	0	elastic solid
$*$	$*$	T	0	Kelvin-Voigt
0	$*$	$*$	1	Maxwell

Energy dissipation

Define the total energy

$$E = \int_\Omega \underbrace{\frac{\rho(\phi)}{2}|\mathbf{v}|^2}_{\text{kinetic energy}} + \underbrace{\frac{\mu(\phi)}{2}\mathrm{tr}\left(\mathbb{B} - \ln\mathbb{B} - \mathbb{I}\right)}_{\text{elastic energy}} + \underbrace{\tilde{\gamma}\left(\frac{\epsilon}{2}|\nabla\phi|^2 + \frac{1}{\epsilon}W(\phi)\right)}_{\text{surface energy}}\,\mathrm{d}x$$

Insert the balance laws

Phase field Oldroyd-B model (unspecified)

$$\rho(\phi)\partial_t^\bullet \mathbf{v} = -\nabla p + \nabla\cdot\Big(\nu(\phi)(\nabla\mathbf{v} + \nabla\mathbf{v}^T)\Big) + \nabla\cdot(\mu(\phi)(\mathbb{B} - \mathbb{I})) + \mathbf{F} \qquad \text{in } \Omega$$

$$\nabla\cdot\mathbf{v} = 0 \qquad \text{in } \Omega$$

$$\partial_t^\bullet \phi = -\nabla\cdot\mathbf{J} \qquad \text{in } \Omega$$

$$\lambda(\phi)\overset{\triangledown}{\mathbb{B}} = -\alpha(\phi)(\mathbb{B} - \mathbb{I}) \qquad \text{in } \Omega$$

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Energy dissipation II

Obtain non-increasing energy,

$$d_t E = -\int_\Omega \frac{\nu}{2}\left|\nabla \mathbf{v} + \nabla \mathbf{v}^T\right| + \frac{1}{m(\phi)}|\mathbf{J}|^2 \,\mathrm{d}x - \int_{\Omega\setminus\{\lambda=0\}} \frac{\mu\alpha}{2\lambda}\mathrm{tr}(\mathbb{B} + \mathbb{B}^{-1} - 2\mathbb{I})\,\mathrm{d}x$$
$$\leq 0.$$

for the choice

$$\mathbf{F} = -\nabla\cdot(\rho'(\phi)\mathbf{v}\otimes\mathbf{J}) - \epsilon\tilde{\gamma}\nabla\cdot(\nabla\phi\otimes\nabla\phi),$$
$$\mathbf{J} = -m(\phi)\nabla\left[\frac{\mu'(\phi)}{2}\mathrm{tr}(\mathbb{B} - \ln\mathbb{B} - \mathbb{I}) + \tilde{\gamma}\left(\frac{1}{\epsilon}W'(\phi) - \epsilon\Delta\phi\right)\right]$$

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Governing equations

Phase field Oldroyd-B model

$$
\begin{aligned}
\rho(\phi)\partial_t^{\bullet}\mathbf{v} =& -\nabla p + \nabla\cdot\Big(\nu(\phi)(\nabla\mathbf{v}+\nabla\mathbf{v}^T)\Big) + \nabla\cdot(\mu(\phi)(\mathbb{B}-\mathbb{I})) \\
& -\nabla\cdot(\rho'(\phi)\mathbf{v}\otimes\mathbf{J}) - \epsilon\tilde{\gamma}\nabla\cdot(\nabla\phi\otimes\nabla\phi) && \text{in } \Omega \\
\nabla\cdot\mathbf{v} =& 0 && \text{in } \Omega \\
\partial_t^{\bullet}\phi =& -\nabla\cdot\left(-m(\phi)\nabla\left[\frac{\mu'(\phi)}{2}\mathrm{tr}(\mathbb{B}-\ln\mathbb{B}-\mathbb{I}) + \tilde{\gamma}\left(\frac{1}{\epsilon}W'(\phi)-\epsilon\Delta\phi\right)\right]\right) && \text{in } \Omega \\
\lambda(\phi)\overset{\triangledown}{\mathbb{B}} =& -\alpha(\phi)(\mathbb{B}-\mathbb{I}) && \text{in } \Omega
\end{aligned}
$$

- thermodynamically consistent (energy decreasing) model
- stability problems due to strain contribution in phase field flux
- red term can be omitted, resulting error in thermodynamical consistency is higher order

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Asymptotic analysis

Outer expansion at leading order

$$\begin{aligned} \partial_t^\bullet(\rho_i \mathbf{v}_0) - \nabla\cdot\left(\nu_i\left(\nabla\mathbf{v}_0 + \nabla\mathbf{v}_0^T\right) + \mu_i(\mathbb{B}_0 - \mathbb{I})\right) + \nabla p_0 &= 0 \quad \text{in } \Omega_i, \\ \nabla\cdot\mathbf{v}_0 &= 0 \quad \text{in } \Omega_i, \\ \mathbb{B}_0 &= \mathbb{I} \quad \text{in } \Omega_0, \\ \overset{\triangledown}{\mathbb{B}}_0 &= 0 \quad \text{in } \Omega_1, \end{aligned}$$

Inner expansion at leading order

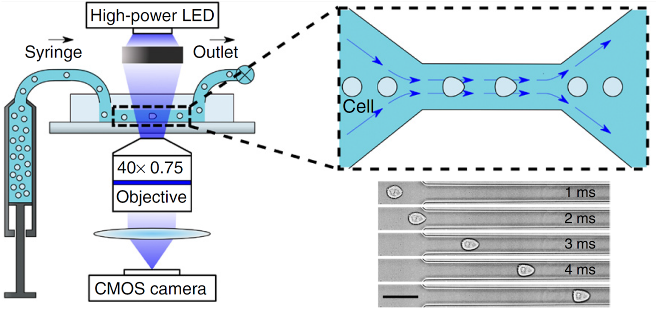
$$\begin{aligned} [\mathbf{v}_0]_0^1 &= 0 \\ -2[\nu D\mathbf{v}_0]_0^1\mathbf{n} + [p_0]_0^1\mathbf{n} - [\mu(\phi_0)(\mathbb{B}_0 - \mathbb{I})]_0^1\mathbf{n} &= \tilde{\gamma}\kappa\mathbf{n}. \end{aligned}$$

Hence, convergence to sharp interface limit for $\epsilon \to 0$ **with and without** the 'red' term

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Test case: biological cells in flow channel

- Biological cells behave similar to elastic solids (Youngs modulus $\approx$ 1kPa)
- Elastic properties change with cell phenotype and morbidness
- Elasticity measurements used to identify cells and their medical condition
- A flow scenario to measure cell elasticity proposed [5]:

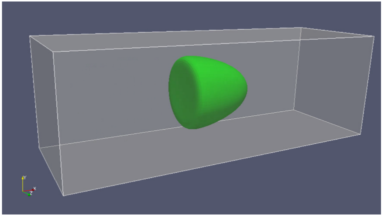


- cell deformation measured by camera image analysis
- offers extremely fast cell screening (1000 cells per second)
- comparison to simulations needed to determine elastic modulus

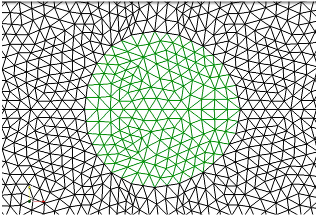
[5] Otto et al. Nature Methods (2015)

Simulation of a single elastic cell

- can be simulated with our phase field approach (cell moving, periodic boundaries) and ALE method (cell centered)



Phase-field simulation (cell moving)

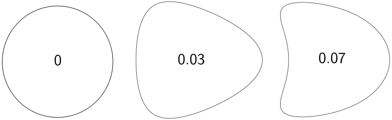


ALE simulation (cell centered)

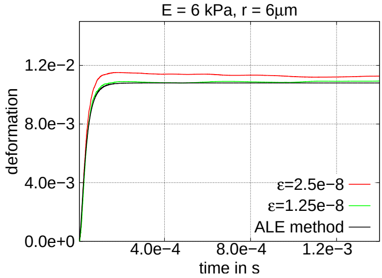
- introduce measure for deformation

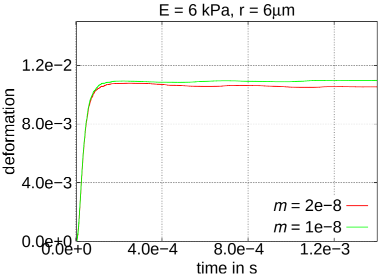
$$\text{deformation} = 1 - \text{circularity} = 1 - \frac{2\sqrt{\pi A}}{P}$$

- example shapes



Influence of interface thickness and mobility



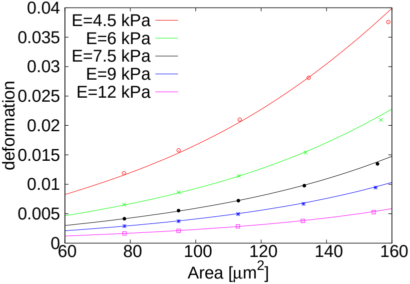


- convergence to reference solution for interface thickness $\epsilon \to 0$
- almost no influence of mobility

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Validation with ALE model

- compare (almost) stationary shapes

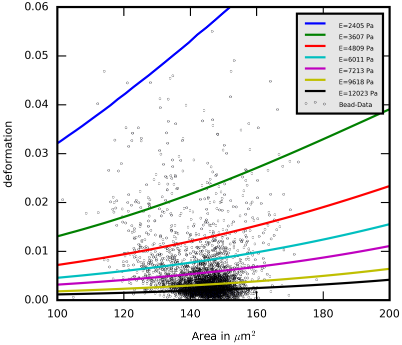


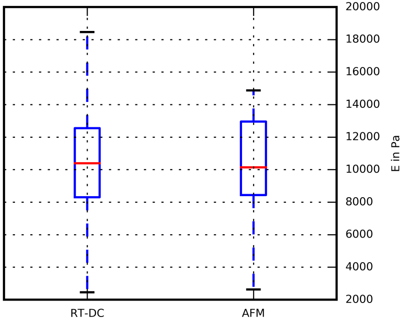
lines: ALE results, circles: phase field results

- very good agreement over a range of cell areas and E-moduli

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Validation with elastic beads

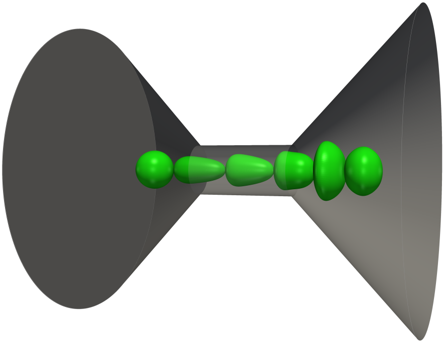




- measuerd elasticity from simulations (RT-DC) matches perfectly with reference values (AFM) [1]
- highly promising method for cell diagnostics
- Phase field approach can be used to extract additional cell parameters by simulating inflow/outflow, membrane tension, bending stiffness, ...

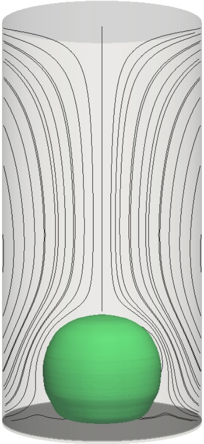
[1] Mokbel, Mietke, Otto, Guck, Aland; ACS Biomat. Sci. Eng. (2017)

Beyond AFM simulations: large translations



[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

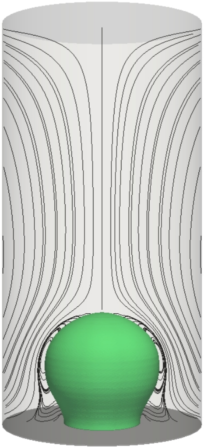
Simulation of a bouncing ball



video with streamlines

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Simulation of a rubber ball with adhesion



video with streamlines

[Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

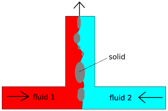
Summary

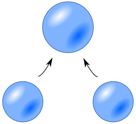
Phase field model for FSI

- viscous, elastic and viscoelastic domains
- based on energy variation
- sharp interface limit shown
- rotations and translations without re-triangulation
- publication[1]

Outlook

- complex deformations, topological changes
- simulation of solidification/fluidization
- elasticity + adhesion + contact line dynamics
- apply to describe mechanics of adhesive biological cells





[1] [Mokbel, Abels, Aland; 'A phase-field model for fluid-structure interaction'; J. Comp. Phys. (2018)]

Advantages

w.r.t. ALE method

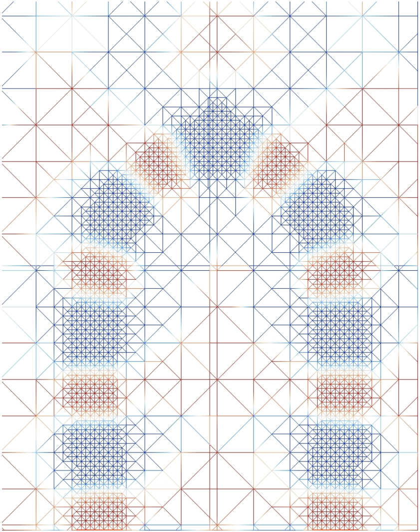
- inflow and outflow (in RT-DC case)
- interaction of multiple cells
- larger deformations, topological changes
- viscoelastic domains
- fluidization, solidification

w.r.t. IB method

- adaptivity is easier
- parallelization is easier

Numerical treatment

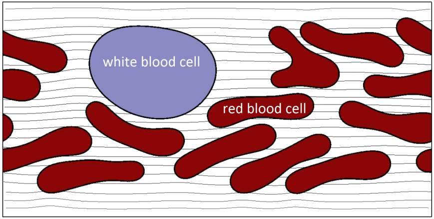
- C++ FEM library AMDiS
- adaptive mesh
- P2 elements except for P1 for pressure
- 2D: direct solver UMFPACK
- 3D: Petsc based parallelization with problem-adapted preconditioners
- semi-implicit time discretization



Simulation of blood flow

- Blood components

  55% Plasma (viscous fluid)
  44% red blood cells (RBC)
  1% white blood cells (RBC)
- multiple viscous fluid phases realized by multiple phase fields (for each cell)
- different viscosities in RBC, WBC and plasma [1]



- faster margination for
  - higher WBC viscosity
  - higher RBC ratio
- agreement with approximative solutions[2] and experiments

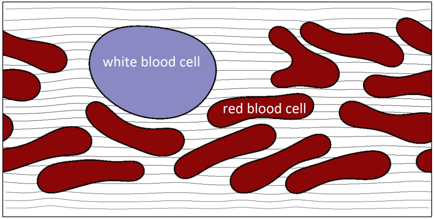
[1] Marth, Aland, Voigt; JFM (2016)

[2] Fedosov & Gompper (2012, 2014)

Simulation of blood flow

- Blood components

  55% Plasma (viscous fluid)
  44% red blood cells (RBC)
  1% white blood cells (RBC)
- multiple viscous fluid phases realized by multiple phase fields (for each cell)
- different viscosities in RBC, WBC and plasma [1]



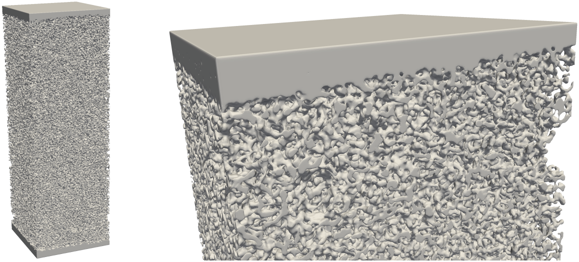
- faster margination for
  - higher WBC viscosity
  - higher RBC ratio
- agreement with approximative solutions[2] and experiments

[1] Marth, Aland, Voigt; JFM (2016)
[2] Fedosov & Gompper (2012, 2014)

elastic/stressless coupling

Polymer foam[1]

- Phase 1: elastic solid material
- Phase 0: ambient air ($\mathbb{S}_0 = 0$)

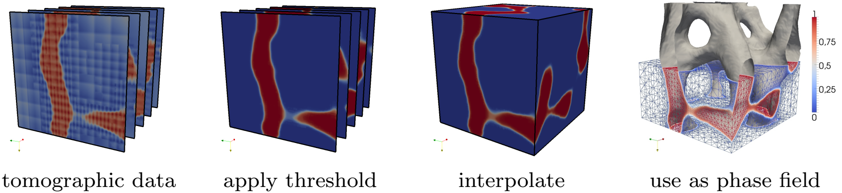


- How does the microstructure deform at a given outer force ?
- How does the elasticity of solid material determine the elastic response of the whole foam ?

[1] Weissenborn et al. ECCM-17 (2016)

Phasefield from tomography data

Tomographic / microscopic imaging data can be directly transferred into a phase field



- use for foam simulation
- calculate local stresses and strains to investigate foam stability