Chimera states in continuous media

Zachary G. Nicolaou, Hermann Riecke, & Adilson E. Motter

Northwestern University

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Coupled oscillators

- Coupled phase oscillators like Kuramoto model

$$\frac{\mathrm{d}\theta_n}{\mathrm{d}t} = \omega_n + \sum_m \epsilon_{m,n} \sin(\theta_m - \theta_n)$$

- Mean field order parameter for synchronization

$$R\mathrm{e}^{i\Theta} = \frac{1}{N}\sum_n \mathrm{e}^{i\theta_n}, \quad R > 0 \text{ solution for large coupling}$$

- Generalizes to amplitude cases like Stuart-Landau

$$\frac{\mathrm{d}z_n}{\mathrm{d}t} = i\omega_n z_n - \beta z_n |z_n|^2 + \text{coupling}, \;\; z_n = r_n \mathrm{e}^{i\theta_n}$$

Original: Kuramoto in *International symposium on mathematical problems in theoretical physics* (1975)
Kuramoto model Review: Acebrón et. al. Rev. Mod. Phys. **77** (2005)

Chimera states

- Kuramoto & Battogtokh demonstrated concurrence of synchrony and asynchrony in identical oscillators

$$\frac{\mathrm{d}\theta_n}{\mathrm{d}t} = \omega + \sum_m G(|n-m|)\sin(\theta_n - \theta_m - \alpha)$$

- Abrams & Strogatz termed these chimera states and presented a solvable model

- Many other network examples have been found

First report: Kuramoto & Battogtokh, *Nonlinear Phenomena in Complex Science* **5** (2002)
Chimera term: Abrams & Strogatz, Phys. Rev. Lett. **93** (2004)
Solvable model: Abrams et. al. Phys. Rev. Lett. 101 (2008)
Review: Panaggio & Abrams, Nonlinearity **28** (2015)

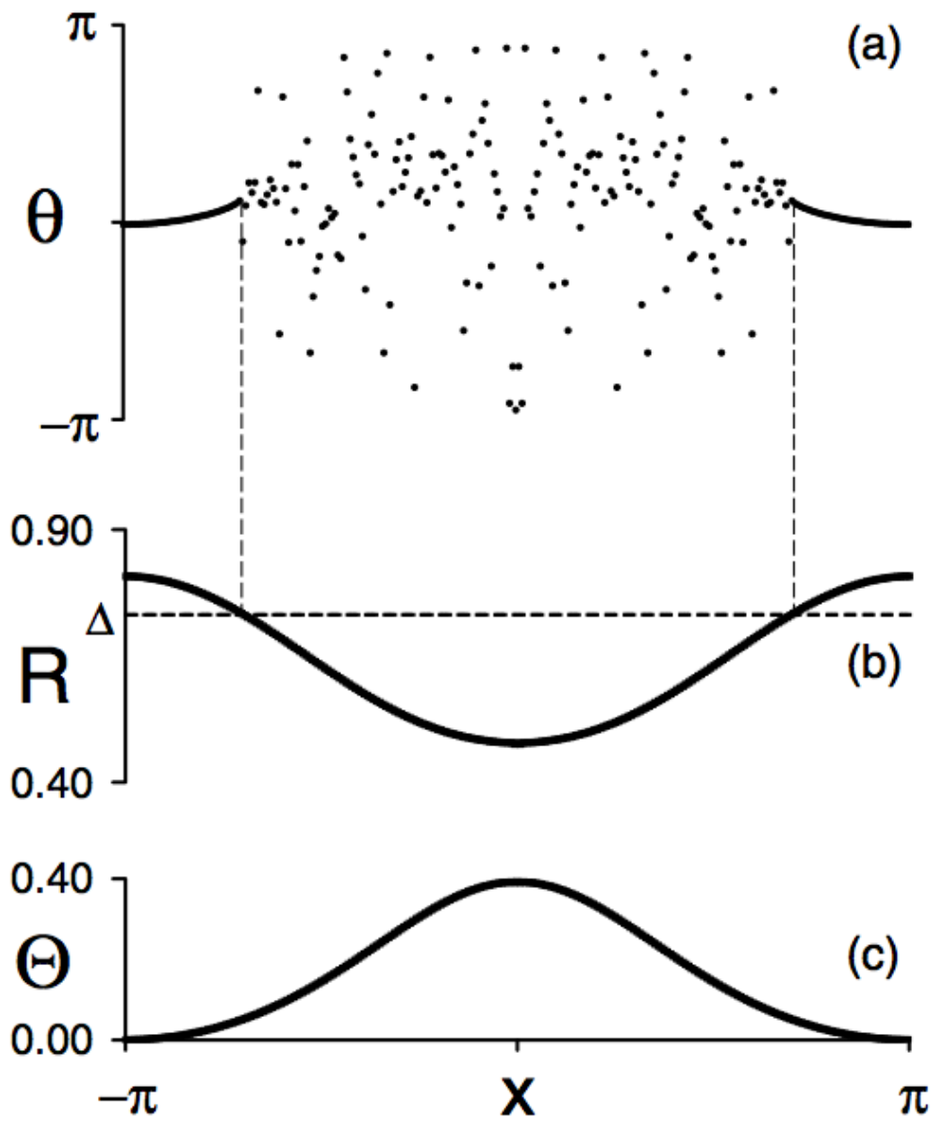
Mean field approach

- Define a local order parameter

$$R_n \mathrm{e}^{i\Theta_n} \equiv \sum_m G(|m-n|)\mathrm{e}^{i\theta_m}$$

- Assume a mean field which is harmonic in time, and find self-consistent equations

- Whether oscillators synchronize depends if this local mean field strength is sufficiently high

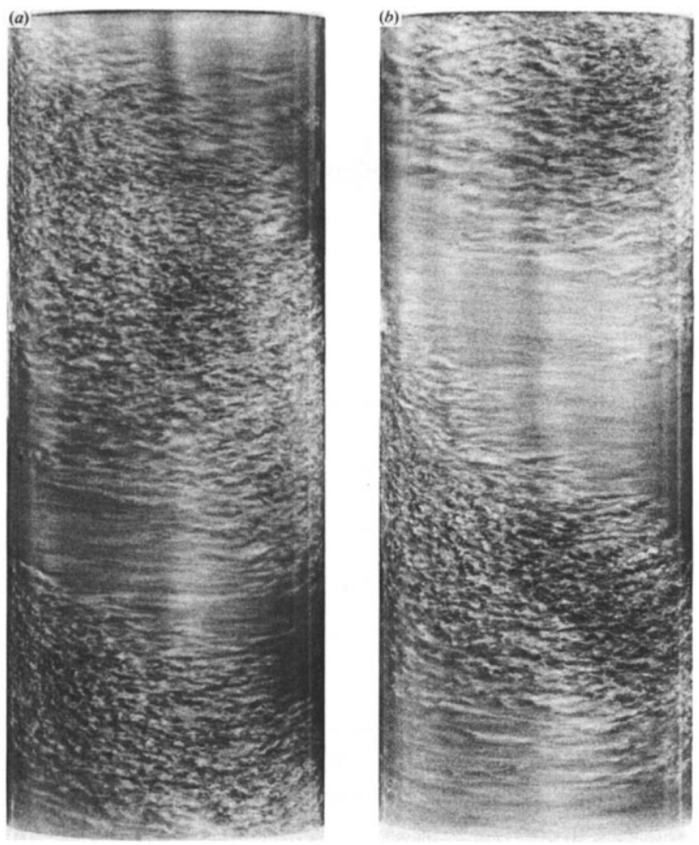


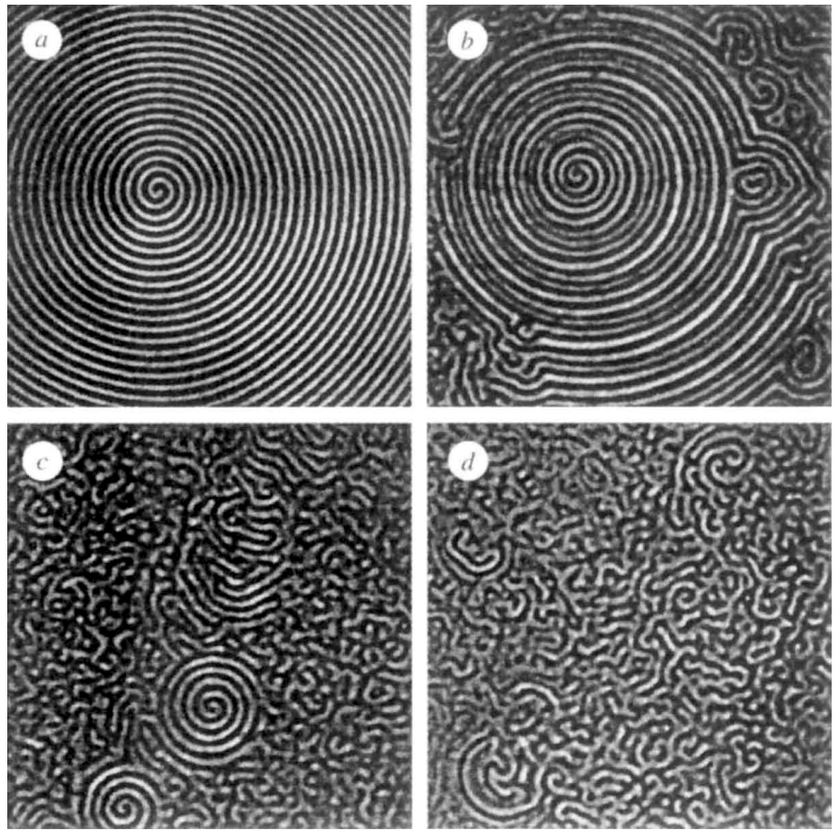
Kuramoto & Battogtokh, *Nonlinear Phenomena in Complex Science* **5** (2002)
Abrams & Strogatz, Phys. Rev. Lett. **93** (2004)

Coexisting order and disorder in continuous media

- Spiral turbulence in Taylor-Couette flow

- Chemical spirals in reaction diffusion systems





Taylor-Couette states: Andereck et. al. J. Fluid Mech **164** (1986)
Reaction diffusion experiments: Ouyang & Flesselles, Nature **379** (1996)

Complex Ginzburg-Landau

- A homogeneous medium in the vicinity of a type III (zero critical wavenumber) supercritical Hopf bifurcation evolves by a universal equation

$$\mathbf{u} = \mathbf{u}_0 + A(\mathbf{x}, t)\mathrm{e}^{i\omega_c t}\mathbf{u}_L(z) + \mathrm{c.c}$$

$$\frac{\partial A}{\partial t} = A + (1 + ic_1)\nabla^2 A - (1 - ic_3)|A|^2 A$$

- We can interpret each point as a Stuart-Landau oscillator with differential local order parameter

$$A \equiv r\mathrm{e}^{i\theta}$$

$$R\mathrm{e}^{i\Theta} \equiv \frac{\sqrt{1 + c_1^2}}{r}\nabla^2 A$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = c_3 r^2 + R\sin(\Theta - \theta + \alpha)$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = r(1 - r^2) + Rr\cos(\Theta - \theta + \alpha)$$

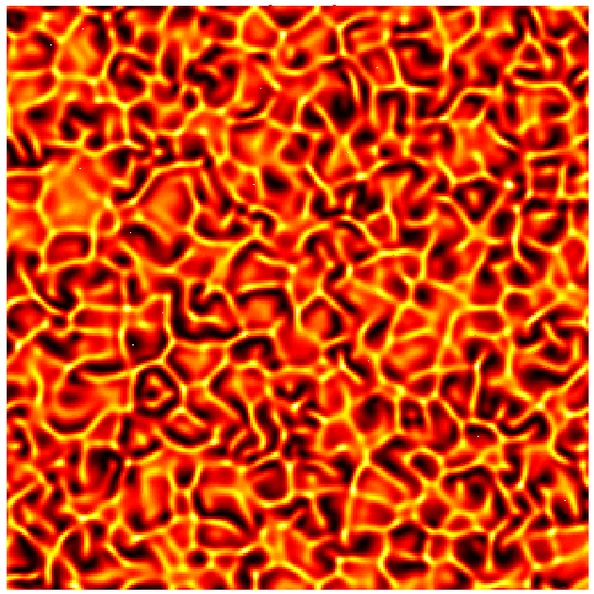
Cross & Hohenberg, Rev. Mod. Phys. **65** (1993)

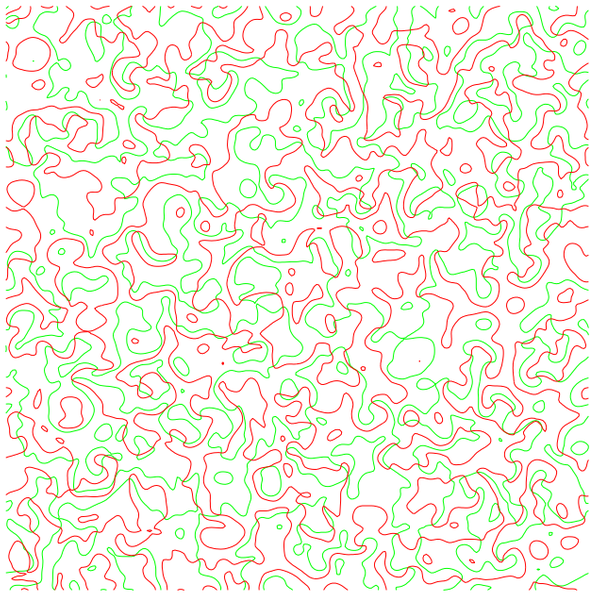
Dynamical Phases of CGLE

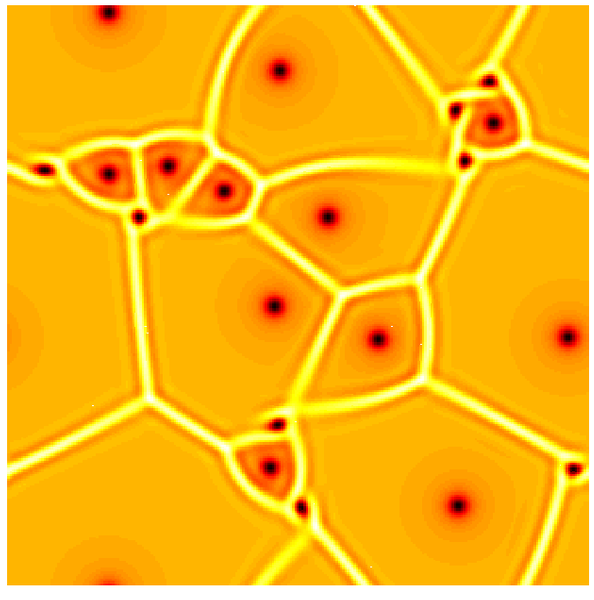
- Amplitude turbulence consists of finite disordered density of point defects where |A|=0

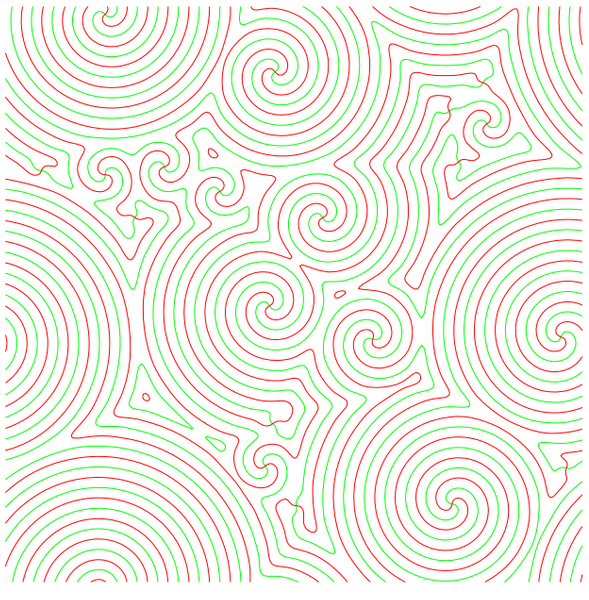
- Frozen vortices are coherent structures with nearly time-independent |A| - they form vortex glasses

- We identified an intermediate state of *frozen vortex chimeras* between these known phases







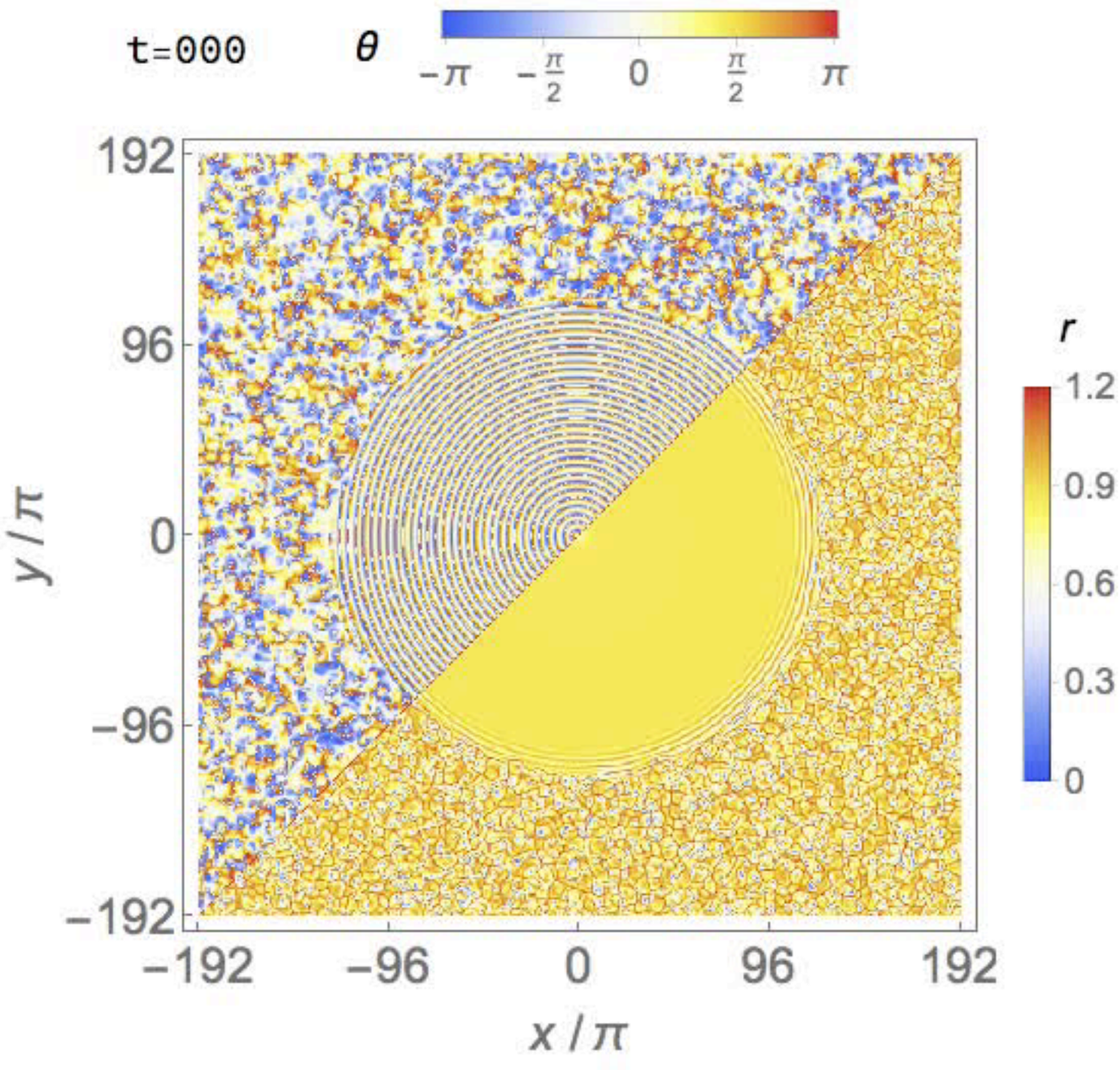


Chaté & Manneville, Physica A **224** (1996)
Aranson & Kramer, Rev. Mod. Phys. **74** (2002)

Frozen vortex chimeras

- Frozen vortex chimeras consist of a coherent frozen spiral surrounded by amplitude turbulence

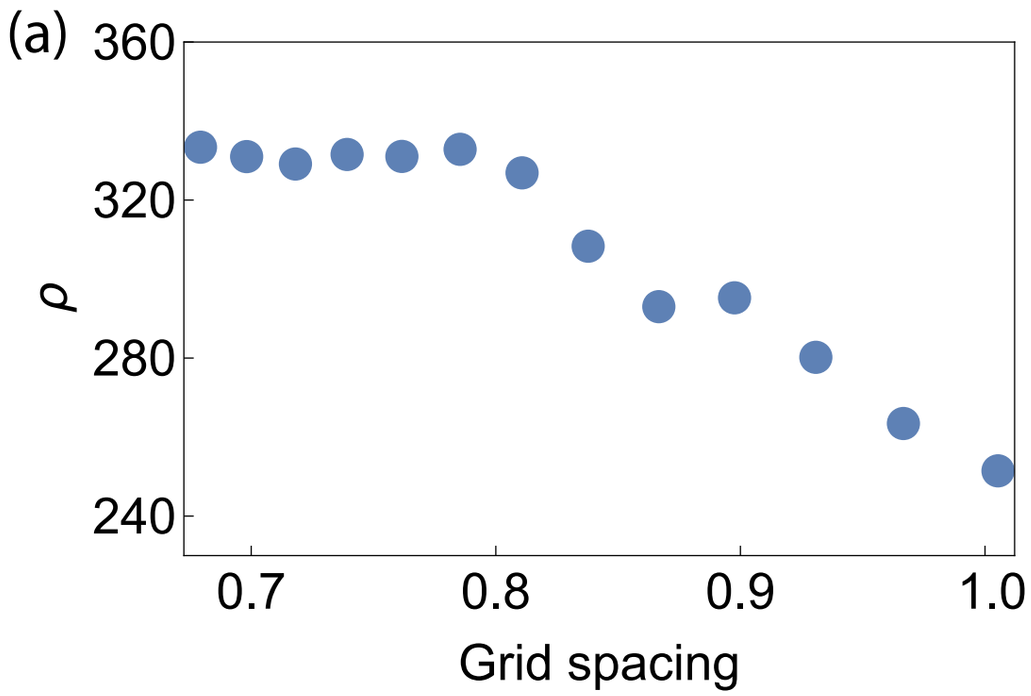
ρ - spiral radius
Ω - spiral oscillation period

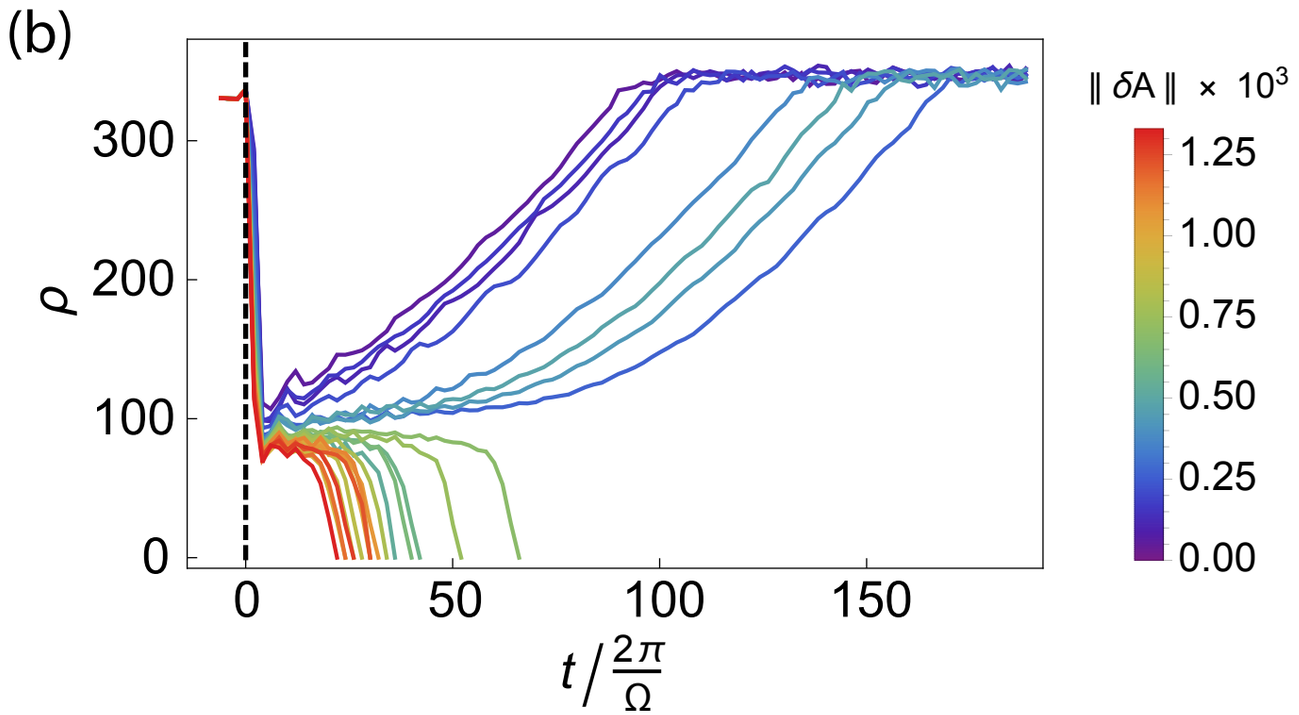


Continuity & Persistance

- Grid and timestep refinement shows convergence to continuum limit

- Small perturbations do not destroy the chimera



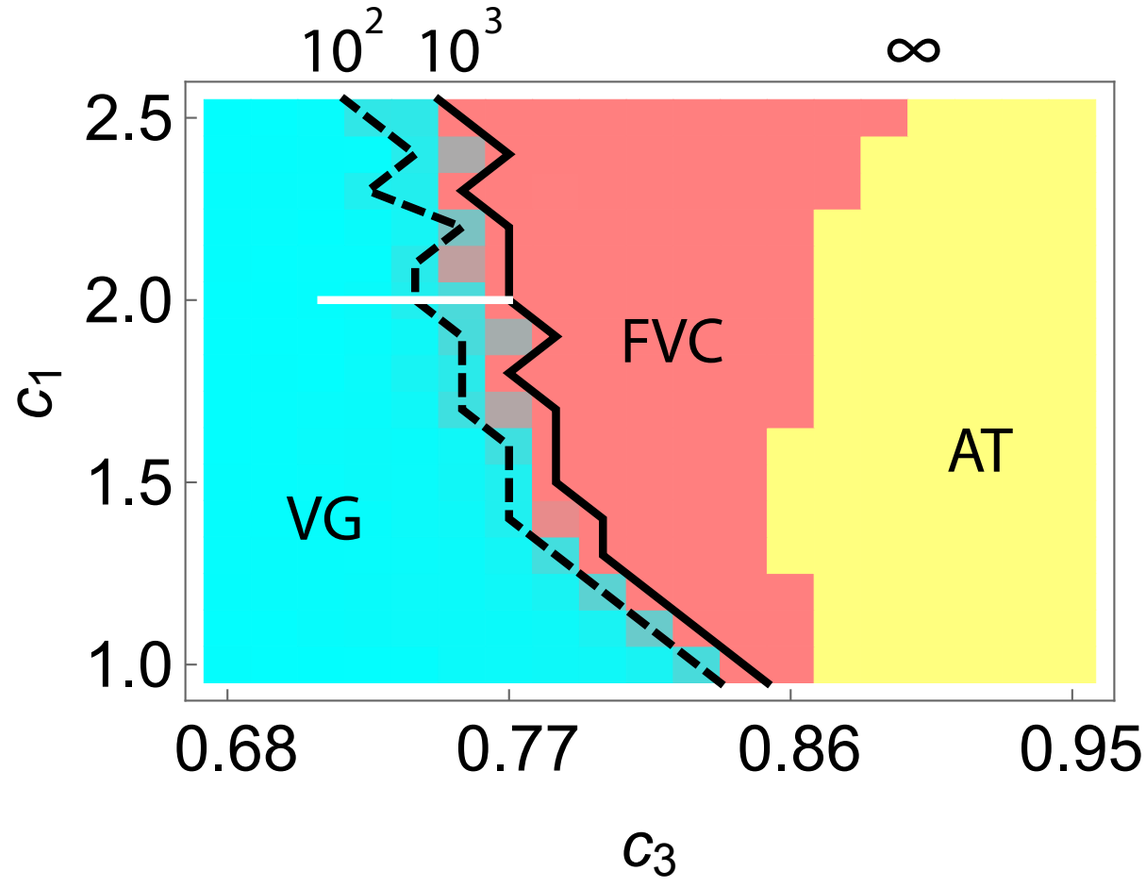


Phase diagram

- We defined a metric based on spiral nucleation rate out of amplitude turbulence

$$\eta \equiv \frac{L^2}{\pi\rho^2}\frac{T_{\mathrm{nuc}}}{2\pi/\Omega}$$

- A phase diagram was obtained though systematic numerics



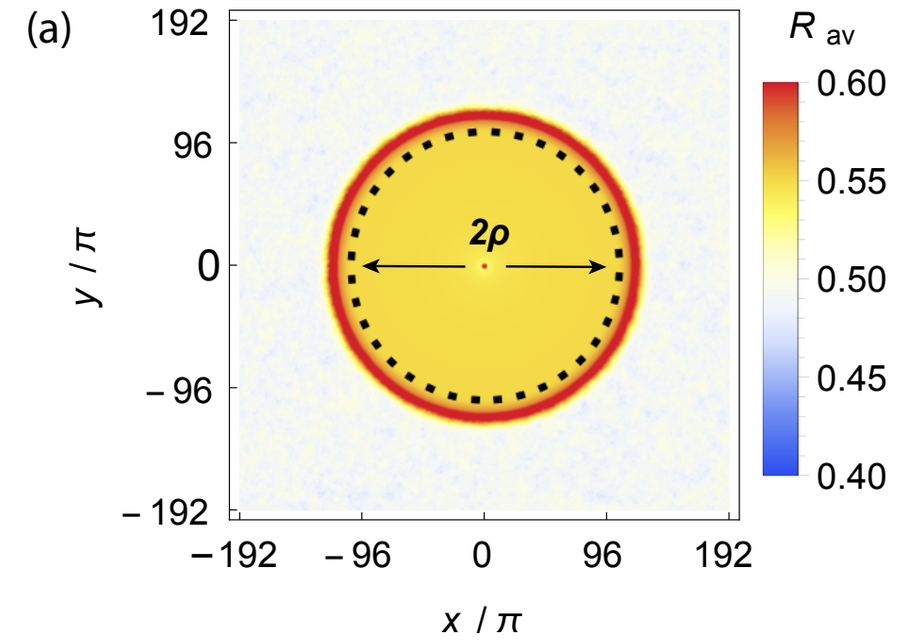
L - domain size
T_{nuc} - spiral nucleation time

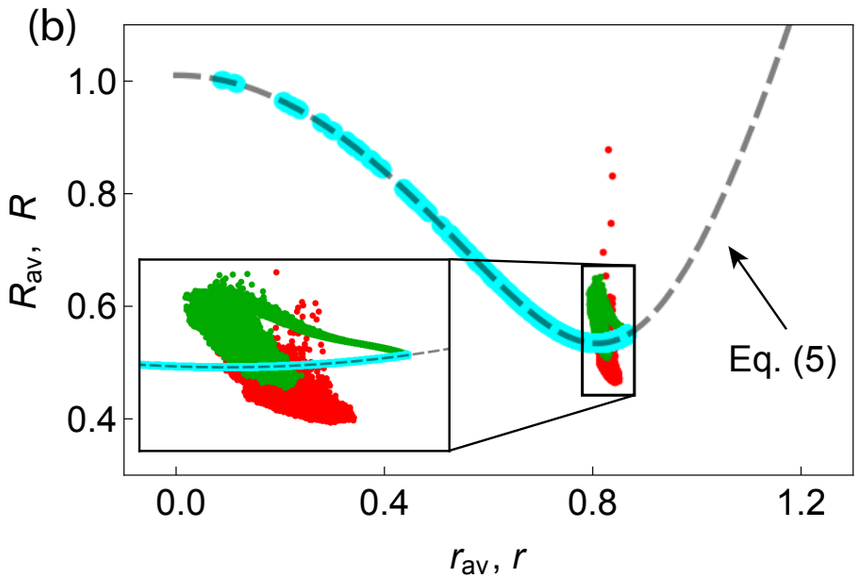
Order parameter & fluctuations

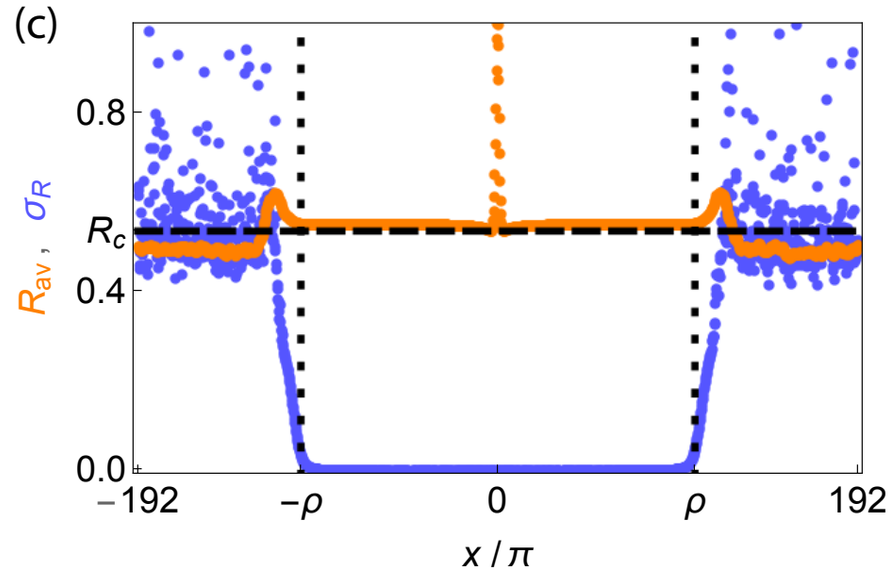
- Kuramoto's local mean field theory approach has analog

- Fluctuations in the order parameter are not negligible and limit the coherent domain

- Such fluctuations are a consequence of local coupling

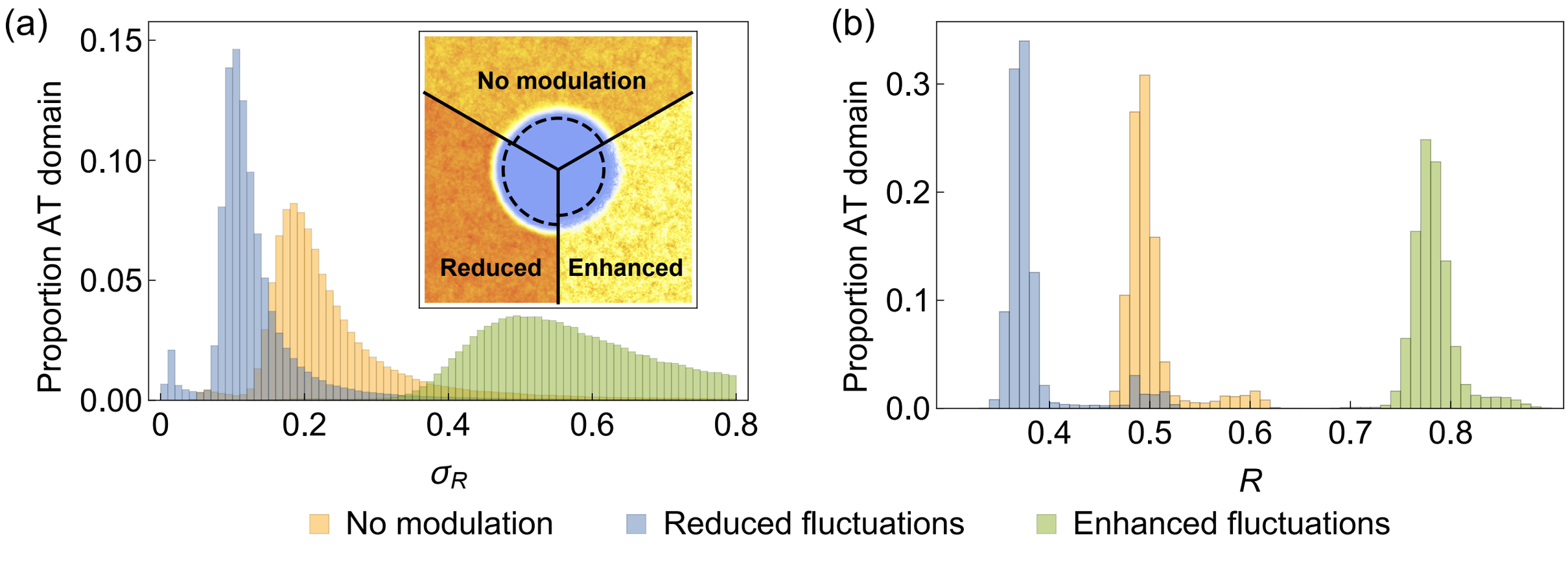






Mechanism validation

- We modified the AT domain with a force $\pm 1/2 \nabla^2 A$
- Reduced (enhanced) fluctuations caused spiral grow (shrink)

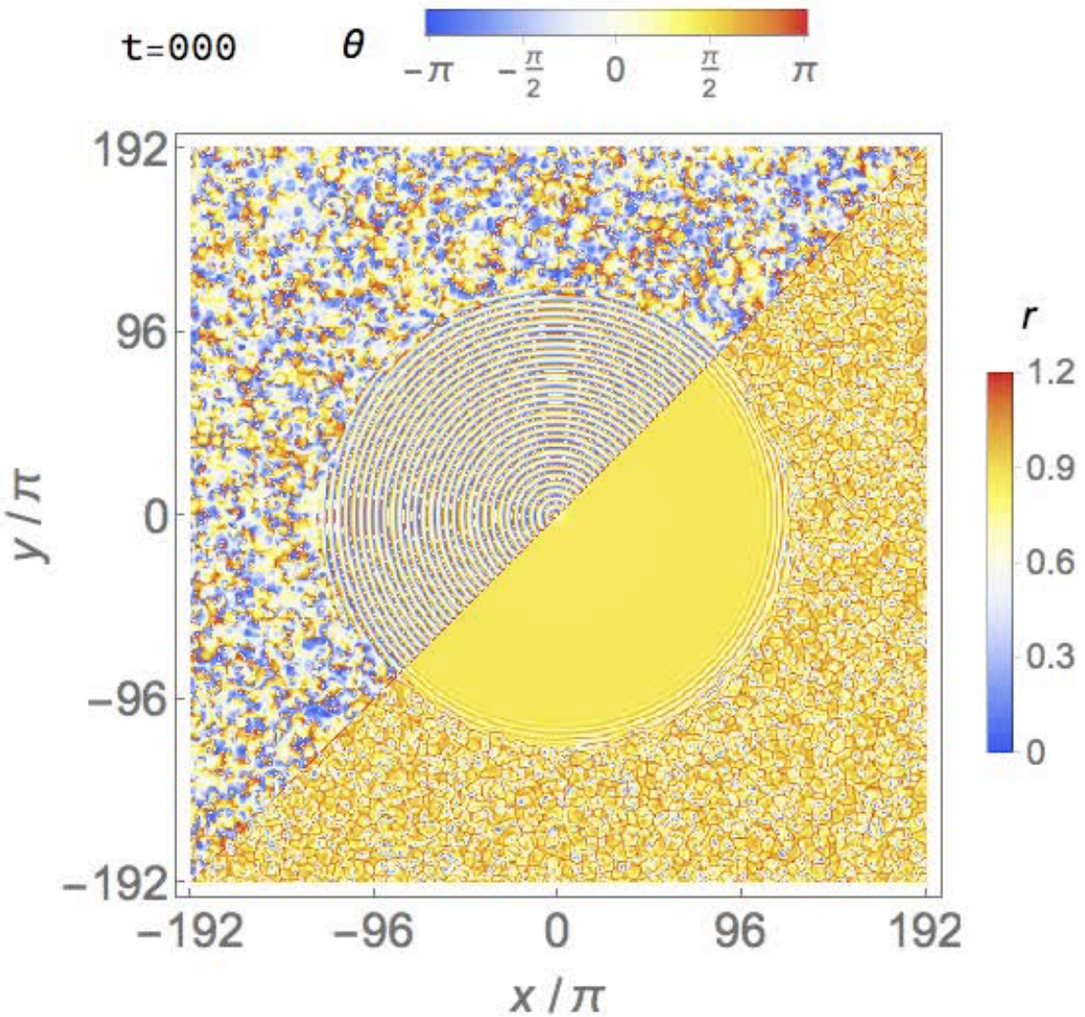


Closing remarks

- Frozen vortex chimeras are continuous chimera states in an under-explored parameter regime of complex Ginzburg-Landau equation

- These states may be experimentally accessible in reaction-diffusion systems

- Local order parameter fluctuations are essential in chimera mechanism



Chimera States in Continuous Media: Existence and Distinctness
Currently under review
zachary.nicolaou@northwestern.edu