SIAM Conference on Mathematics of Planet Earth
September 30 - October 2, 2016
DoubleTree by Hilton Hotel Philadelphia Center City
Philadelphia, Pennsylvania, USA

Simulation and analysis of the predator-prey dynamics of dinoflagellates

MICHAEL MAZZOLENI[1], TIMOTHY ANTONELLI[2], KATHRYN COYNE[3], LOUIS ROSSI[3]

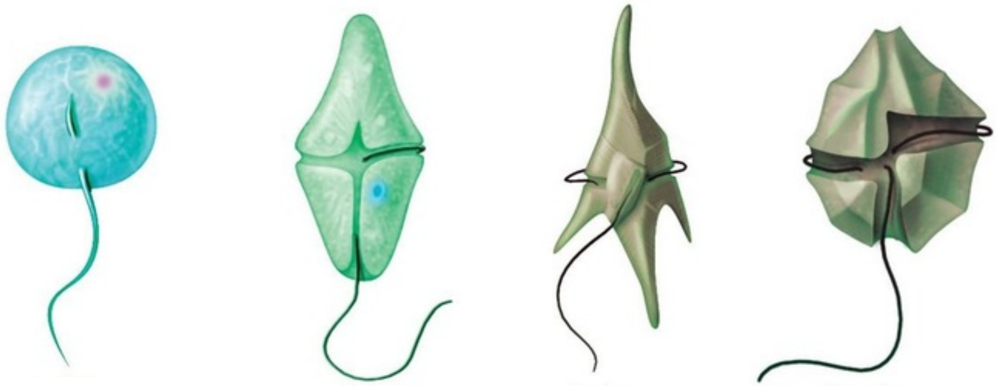
[1]Duke University [2]Worcester State University [3]University of Delaware

Outline

- Introduction to dinoflagellates
- Modeling predator-prey dynamics
  - Calculating encounter rates (based on chemical kinetics)
  - Accounting for inefficient predation
- Simulations (varying levels of complexity)
- Comparison to experimental studies (Sheng 2007, 2010)

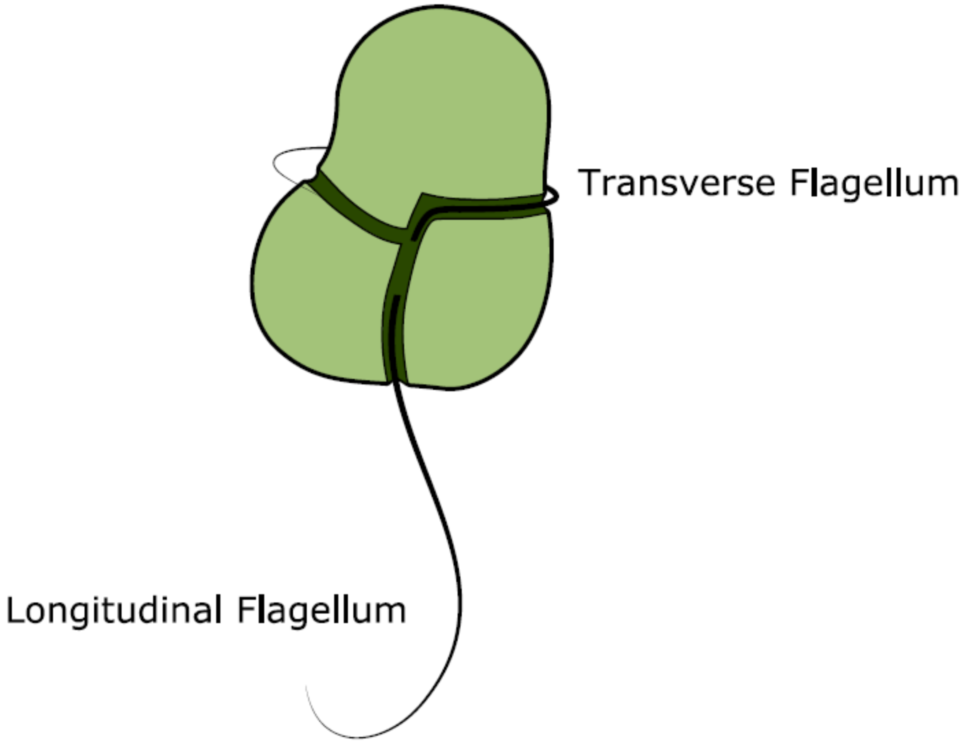
What are dinoflagellates?

- Second largest group of phytoplankton in marine environments
- Can be autotrophic, heterotrophic, or mixotrophic
- Exhibit complex swimming dynamics (helical motion patterns)
- Predatory dinoflagellates have been observed to utilize toxins to enhance predation
- These toxins contribute to harmful algal blooms (HAB)



Dinoflagellate Locomotion

- Swimming is driven by two flagella, which results in a unique helical swimming motion
- The toxins released by predatory dinoflagellates have been observed to effectively immobilize or slow down their prey
- Predatory dinoflagellates have been observed to significantly alter their swimming behavior in the presence of prey



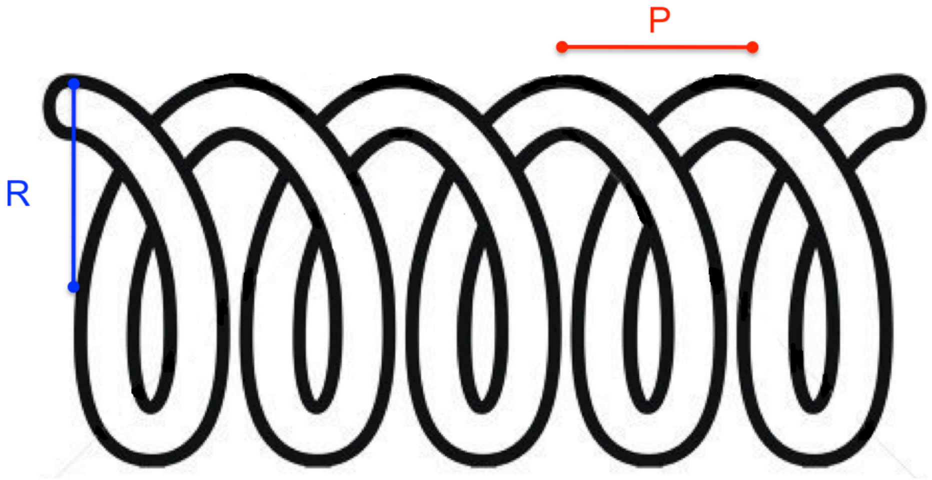
Helical Swimming Dynamics

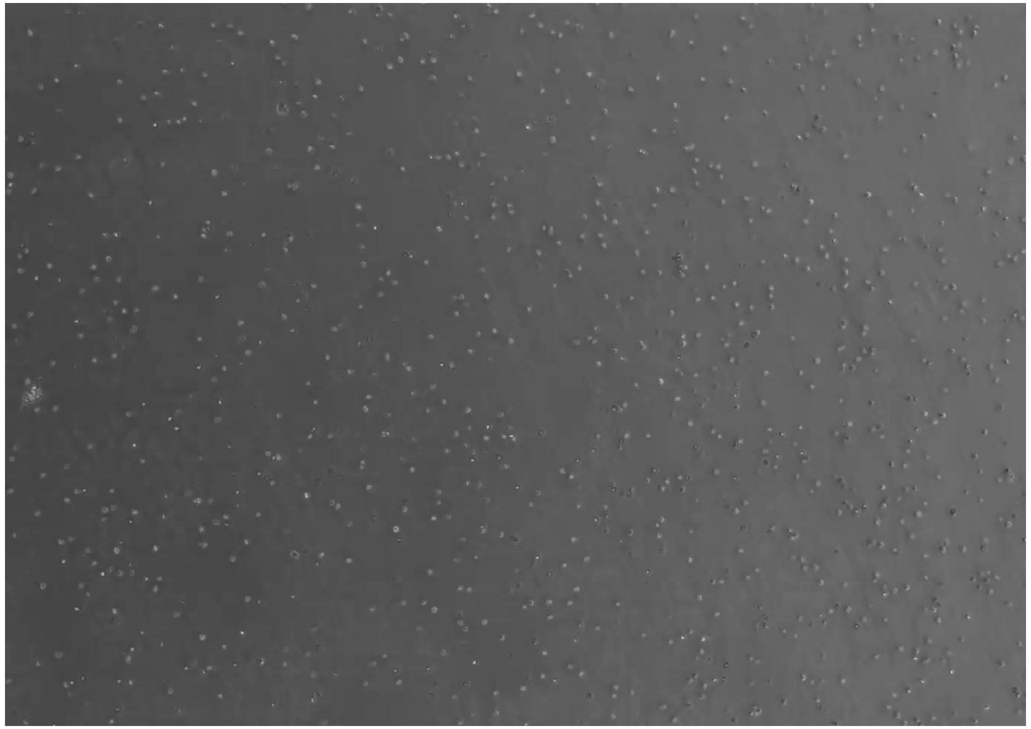
- Helical motion can be parametrically described

$$H(t) = R\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right] + \left(\frac{p\omega t}{2\pi}\right)\hat{k}$$

- The velocity can then be described as a function of the helical motion parameters

$$\mathbf{v} = \omega\sqrt{R^2 + \left(\frac{p}{2\pi}\right)^2}.$$



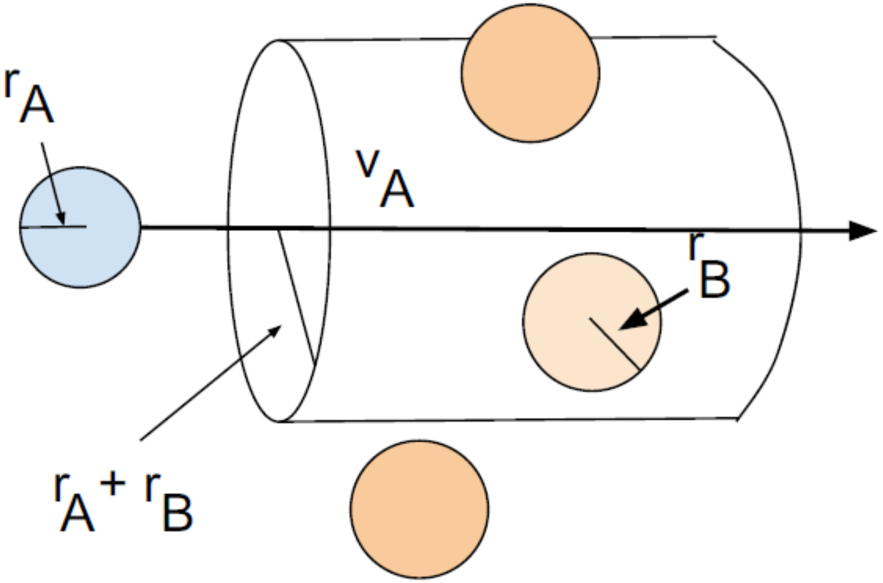


Modeling predator-prey interactions

- Basic predator-prey relationship

$$\frac{d\rho_B}{dt} = -k\rho_A\rho_B$$

- ρ_A : predator density
- ρ_B : prey density
- k : encounter rate
- Interactions can be modeled using analogies from chemical kinetics



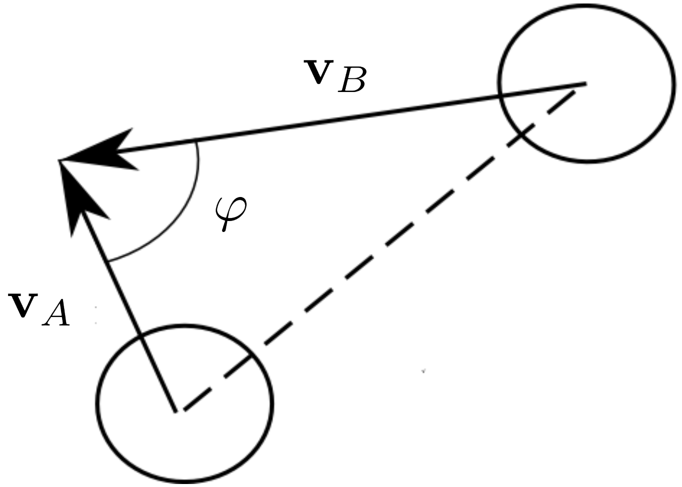
Modeling predator-prey interactions

$$\mathbf{v}_{\text{rel}} = \mathbf{v}_A - \mathbf{v}_B$$

$$\alpha = |\mathbf{v}_A|/|\mathbf{v}_B|$$

$$|\mathbf{v}_{\text{rel}}|^2 = |\mathbf{v}_A|^2 + |\mathbf{v}_B|^2 - 2|\mathbf{v}_A||\mathbf{v}_B|\cos\varphi$$

$$\begin{aligned}\langle|\mathbf{v}_{\text{rel}}|\rangle &\equiv \left[\frac{\int_0^{2\pi}\int_0^{\pi}|\mathbf{v}_A||\mathbf{v}_B|\left(\alpha+\alpha^{-1}-2\cos\varphi\right)\sin\varphi d\varphi d\theta}{\int_0^{2\pi}\int_0^{\pi}\sin\varphi d\varphi d\theta}\right]^{1/2} \\ &= \left[\frac{1}{2}\int_0^{\pi}|\mathbf{v}_A||\mathbf{v}_B|\left(\alpha+\alpha^{-1}-2\cos\varphi\right)\sin\varphi d\varphi\right]^{1/2} \\ &= \left(|\mathbf{v}_A|^2+|\mathbf{v}_B|^2\right)^{1/2}.\end{aligned}$$



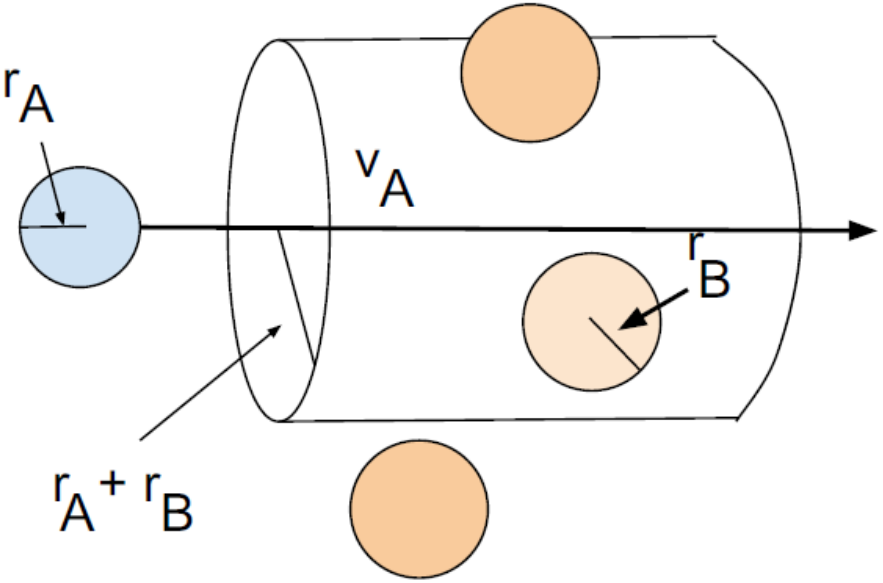
Modeling predator-prey interactions

- Assume perfectly efficient predation (for now)

$$\eta(|\mathbf{v}_{\mathrm{rel}}|) = 1$$

$$k = \pi(r_A + r_B)^2 \langle |\mathbf{v}_{\mathrm{rel}}| \rangle$$

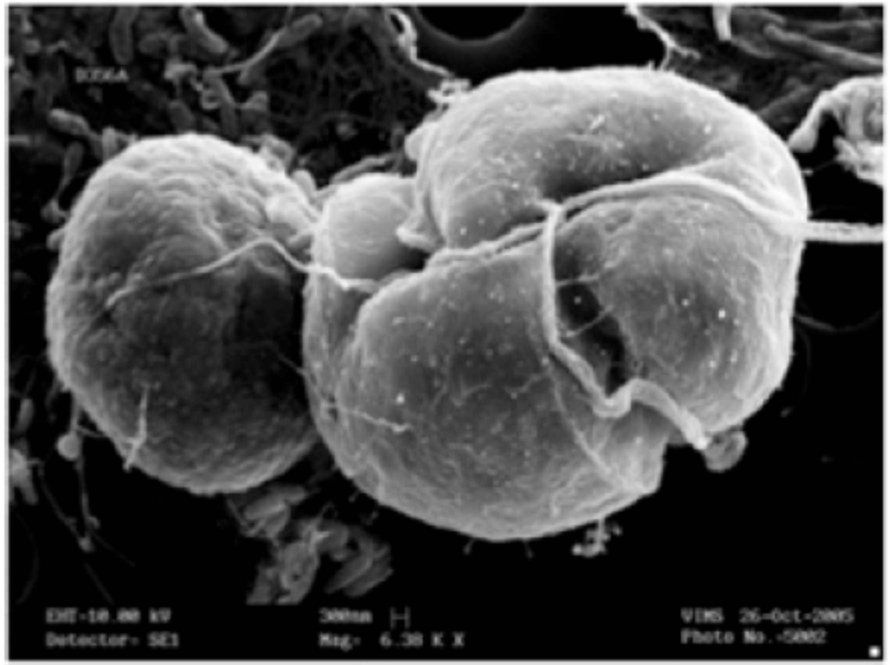
$$k = \pi(r_A + r_B)^2 \sqrt{|\mathbf{v}_A|^2 + |\mathbf{v}_B|^2}$$



- We now have a simple model for predator-prey interactions that is a function of experimentally measurable parameters $(r, \mathbf{v})$

Accounting for inefficient predation

- Not all predator-prey encounters result in successful predation
- The prey is often able to escape encounters by fighting off the predator and swimming away
- It is hypothesized that toxins are released by some predatory dinoflagellates to slow down or completely immobilize their prey to increase predation efficiencies



Modeling inefficiencies with a sharp cutoff based on relative velocity

- Define predation efficiency as a function of relative velocity (simple model: sharp cutoff)

$$\eta(|\mathbf{v}_{\mathrm{rel}}|) = \begin{cases} 1, & |\mathbf{v}_{\mathrm{rel}}| < S_{\mathrm{h}}, \\ 0, & \text{otherwise} \end{cases}$$

- Conceptually, what does this model attempt to account for?
  - If both the predator and prey are moving at high velocities when they bump into each other, they will likely bounce off of each other
  - If the predator is moving at a high velocity, their hydrodynamic profile might alert prey of their approach
  - If the prey has a low velocity and impaired motility due to toxins released by the predator, they will be easier to catch

Modeling inefficiencies with a sharp cutoff based on relative velocity

- Define $\langle|\mathbf{v}_{\mathrm{pred}}|\rangle$ as $\langle|\mathbf{v}_{\mathrm{rel}}|\rangle$ when predation occurs

$$\langle|\mathbf{v}_{\mathrm{pred}}|\rangle \equiv \left[\frac{1}{2}\int_0^\pi |\mathbf{v}_A||\mathbf{v}_B|\left(\alpha+\alpha^{-1}-2\cos\varphi\right)\eta(|\mathbf{v}_A||\mathbf{v}_B|\left(\alpha+\alpha^{-1}-2\cos\varphi\right))\sin\varphi d\varphi\right]^{1/2}$$

$$\frac{\langle|\mathbf{v}_{\mathrm{pred}}|\rangle}{\sqrt{|\mathbf{v}_A||\mathbf{v}_B|}} \equiv \left[\frac{1}{2}\int_0^\pi \left(\alpha+\alpha^{-1}-2\cos\varphi\right)\eta(|\mathbf{v}_A||\mathbf{v}_B|\left(\alpha+\alpha^{-1}-2\cos\varphi\right))\sin\varphi d\varphi\right]^{1/2}$$

- We can determine the maximum angle at which predation occurs

$$\varphi_0 = \arccos w \qquad w = \frac{1}{2}\left(\alpha+\alpha^{-1}-\beta\right) \qquad \beta = S_{\mathrm{h}}^2/(|\mathbf{v}_A||\mathbf{v}_B|)$$

Modeling inefficiencies with a sharp cutoff based on relative velocity

- The domain for partially efficient predation:

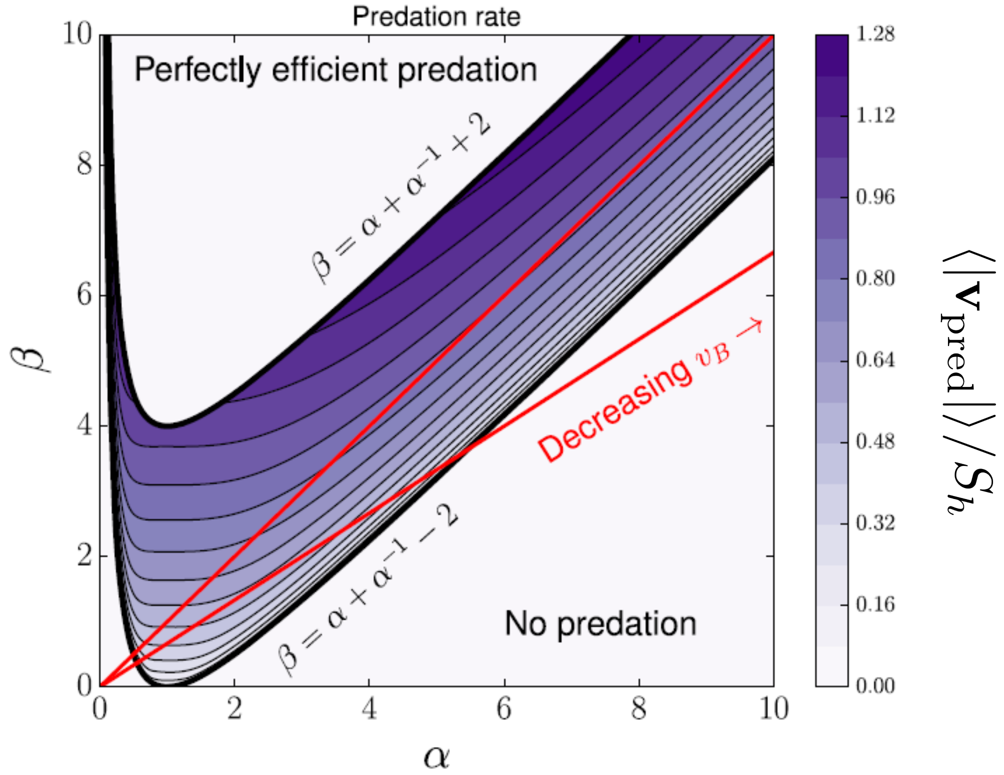
$$\langle|\mathbf{v}_{\text{pred}}|\rangle^2 = \frac{S_{\text{h}}^2}{8}\frac{1}{\beta}\left[\beta^2 - \left(2-(\alpha+\alpha^{-1})\right)^2\right]$$

$$\alpha+\alpha^{-1}-2 \leq \beta \leq \alpha+\alpha^{-1}+2$$

- The domain for perfectly efficient predation:

$$\beta > \alpha+\alpha^{-1}+2$$

$$\langle|\mathbf{v}_{\text{pred}}|\rangle^2 = \frac{S_{\text{h}}^2}{\beta}(\alpha+\alpha^{-1})$$



Modeling inefficiencies with a “filter” based on predator-prey velocities individually

- The prior model for predation inefficiency is simple and illustrates important concepts analytically
- However, a more phenomenologically accurate model would account for the predator-prey velocities individually and would vary continuously instead of having a sharp cutoff
- A 2-D “filter” model is proposed

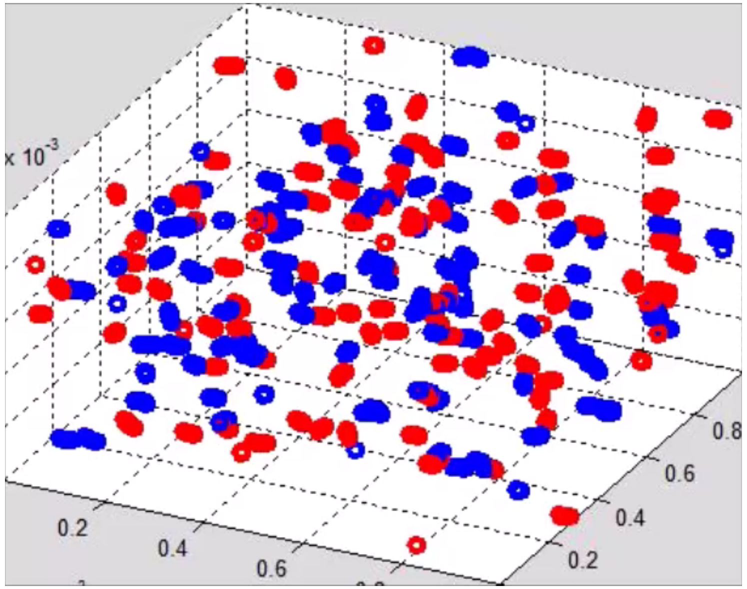
$$\eta(\mathbf{v}_A, \mathbf{v}_B) = \frac{1}{\sqrt{1 + \left(\frac{\mathbf{v}_A}{\lambda_A}\right)^{2\xi_A} + \left(\frac{\mathbf{v}_B}{\lambda_B}\right)^{2\xi_B}}}$$

Simulations

- Basic predator-prey interactions (assume perfectly efficient predation)
- Simple predation efficiency model (sharp cutoff based on relative velocity)
- Comparisons to experimental studies (using 2-D “filter” model)

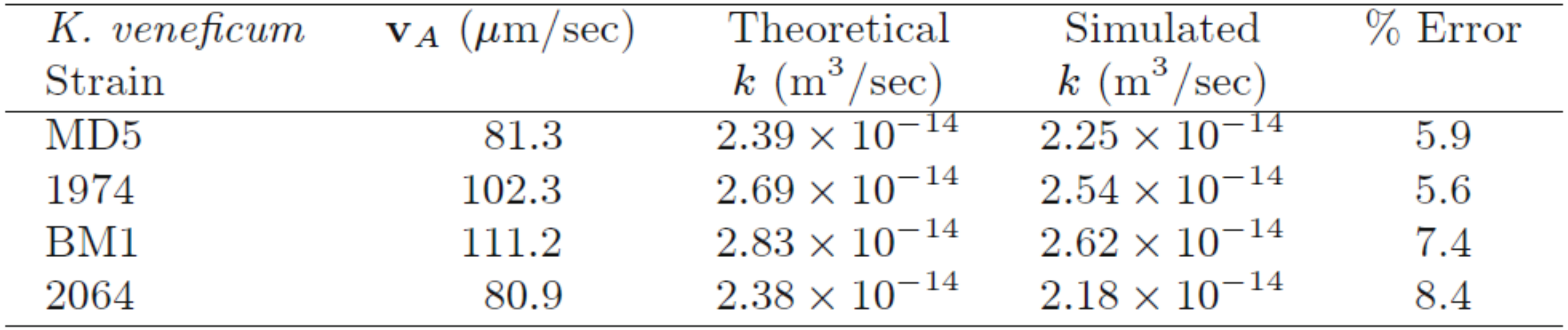
Simulation Mechanics

- Contained in a cubic volume
- Periodic boundary conditions were implemented to eliminate boundary effects
- Predator-prey densities and time durations had to be sufficiently large so as to remove simulation artifacts
- Intelligent collision detection algorithms were implemented to avoid using brute force calculations
- Size and movement parameters were based on experimental studies (Sheng 2007, 2010)



Basic predator-prey interactions

- Assume perfectly efficient predation
- Dinoflagellate parameters were base on experimental data (Sheng 2007, 2010)
- Purpose: Verify simulation mechanics for predicting predator-prey interactions
- Results: Low error (<10%) for all cases



K. veneficum Strain	$\mathbf{v}_A$ (μm/sec)	Theoretical k (m^3/sec)	Simulated k (m^3/sec)	% Error
MD5	81.3	2.39×10^{-14}	2.25×10^{-14}	5.9
1974	102.3	2.69×10^{-14}	2.54×10^{-14}	5.6
BM1	111.2	2.83×10^{-14}	2.62×10^{-14}	7.4
2064	80.9	2.38×10^{-14}	2.18×10^{-14}	8.4

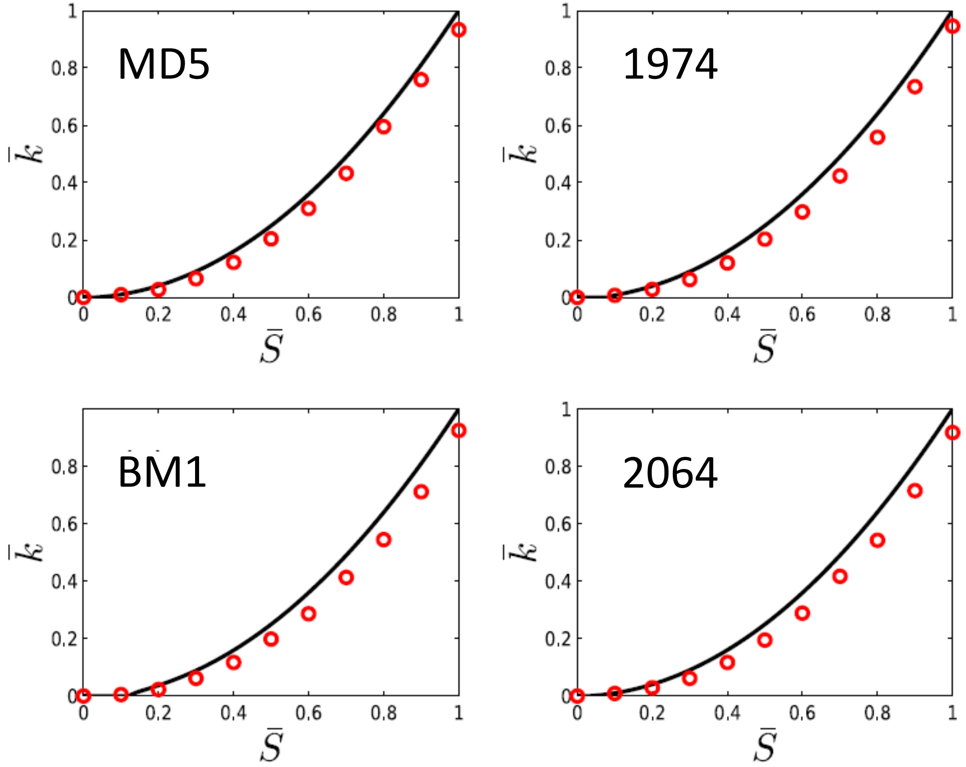
Simple predation efficiency model

- Assume a sharp cutoff based on relative velocity
- Compare normalized predation rate to normalized cutoff velocity

$$\bar{k} = \frac{\left[(1-w)(\alpha+\alpha^{-1}+\beta-2)|\mathbf{v}_A||\mathbf{v}_B|\right]^{1/2}}{2\sqrt{|\mathbf{v}_A|^2+|\mathbf{v}_B|^2}}$$

$$\bar{S} = S_h/(\mathbf{v}_A+\mathbf{v}_B)$$

- Results: Good agreement between simulations and theoretical curves

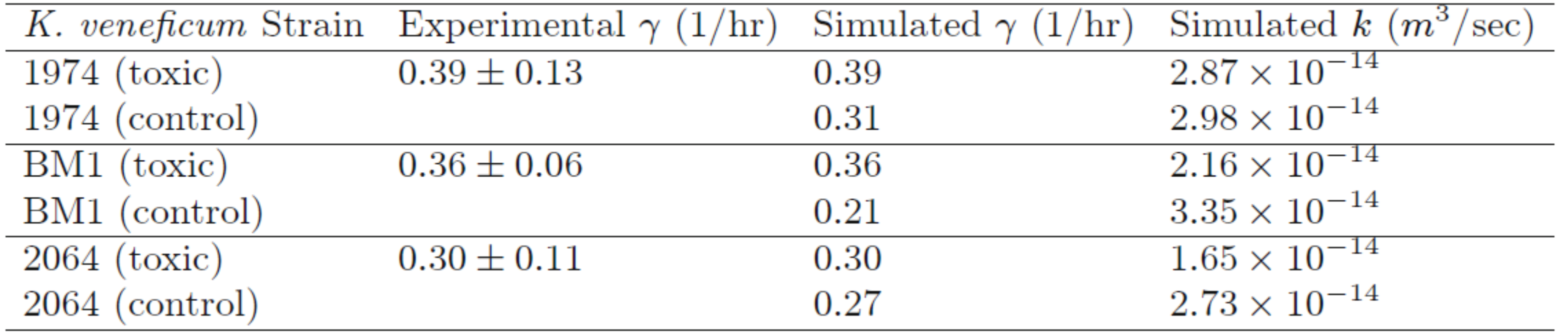


Comparisons to experimental studies (Sheng 2007, 2010)

- Sheng quantified the movement patterns of dinoflagellates and observed that predatory dinoflagellates alter their swimming behavior in the presence of prey (2007)
- A follow-up study by Sheng investigated how some predatory dinoflagellates exploit toxins to immobilize their prey to improve predation (2010)
- We attempted to fit our model to these experimental results to demonstrate how a mathematical model can reveal the positive impact that toxins have on dinoflagellate predation
- Simulations were conducted using dinoflagellate parameters provided by Sheng (2007, 2010) and the predation efficiency model which considered predator and prey velocities individually (2-D “filter” concept)
- The predation efficiency parameters were fit using a genetic algorithm

Comparisons to experimental studies (Sheng 2007, 2010)

- We were able to match the predation rates observed by Sheng (2010)
- Using the predation efficiency parameters calculated during the model fitting process, we then simulated a hypothetical scenario to see how predation rates for these dinoflagellates would be effected in the absence of toxins (control)
- Encounter rates (k) increased in the hypothetical scenario without toxins, but reduced predation efficiencies resulted in lower overall simulated predation rates (γ is the time-averaged predation rate)



K. veneficum Strain	Experimental γ (1/hr)	Simulated γ (1/hr)	Simulated k (m^3/sec)
1974 (toxic)	0.39 ± 0.13	0.39	2.87×10^{-14}
1974 (control)		0.31	2.98×10^{-14}
BM1 (toxic)	0.36 ± 0.06	0.36	2.16×10^{-14}
BM1 (control)		0.21	3.35×10^{-14}
2064 (toxic)	0.30 ± 0.11	0.30	1.65×10^{-14}
2064 (control)		0.27	2.73×10^{-14}

Conclusions

- Mathematical models and simulations demonstrate that toxins can improve dinoflagellate predation by reducing encounter speeds, which increases predation efficiency
- The decline in predator-prey encounters is offset by the increased predation efficiency, resulting in improved overall predation rates
- If you are curious about our work or would like to see all of the details related to the model derivations and simulations, please see our article: **Simulation and analysis of a model dinoflagellate predator-prey system**, **The European Physical Journal Special Topics, Vol. 224, No. 17, pp. 3257-3270, DOI: 10.1140/epjst/e2015-50101-x.**

Acknowledgments

National Science Foundation (NSF)

Dr. Allen Place (University of Maryland Center for Environmental Science)

Dr. Don Schwendeman (Rensselaer Polytechnic Institute Department of Mathematical Sciences)

Seth Cowall, John Cummings, Jennifer Karkoska, Tim Krumwiede, Brittany McCollom, Rashmi Murthy, Thao Nguyen, and Yuzhou Qian (Graduate Student Mathematical Modeling Camp 2014)