Waves in Pulse Coupled Phase Models

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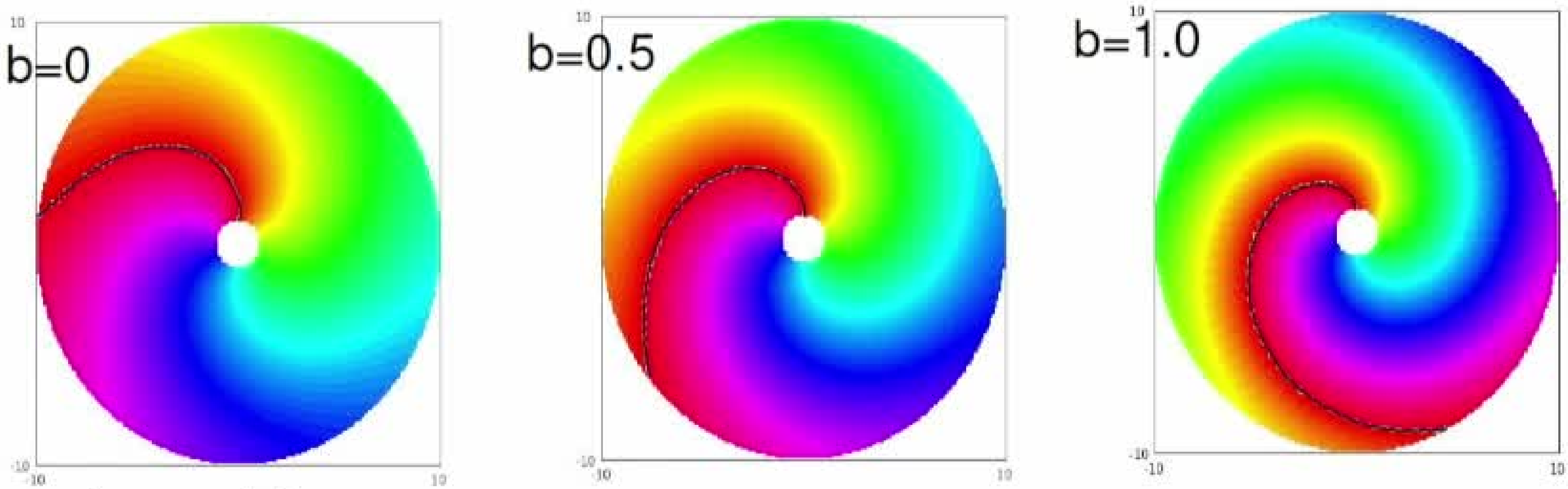
May 19, 2019

Solving this awful equation

- There are probably better ways, but this is what we did
- Use a Gaussian for K then
- Discretize the integral into $N+1$ bins. There are $N+1$ odes for $U_j(y)$ leading to $N+1$ initial conditions for $V_j(y) = U(y, r_j)$, $j = 0 \ldots N$
- There are $2N+2$ boundary conditions.
- There are $N+1$ free parameters, $c, \xi_1, \ldots, \xi_N$ so at least it is well-posed

Example solutions;N=40



1 < r < 10

SMALL Δ, "WEAK COUPLING"

- Then, can apply the Fredholm alternative to get:

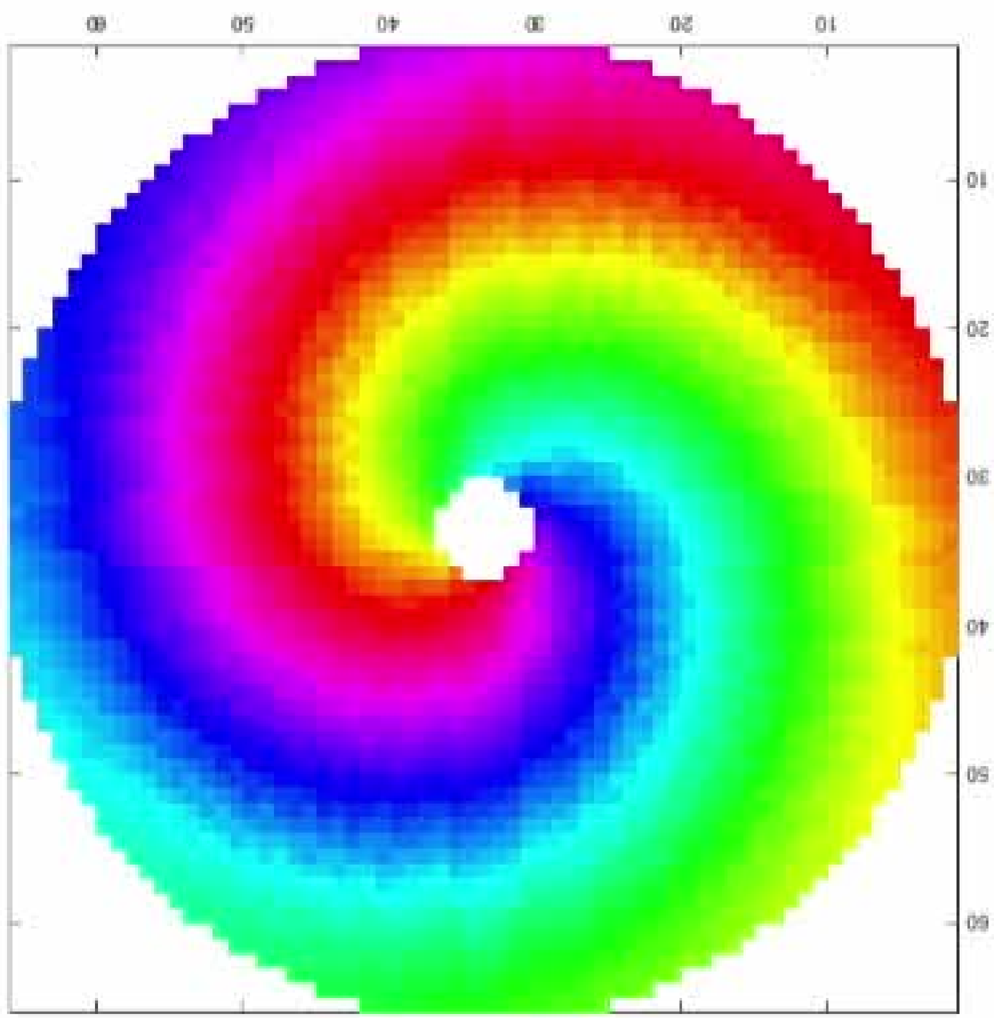
$$c_1 = \int_a^b s ds \int_0^{2\pi} dy\, K(r^2 + s^2 - 2rs\cos(y + \xi(s) - \xi(r)))\Delta(y)$$

- If K is a Gaussian, $\Delta(y) = \sin(b) - \sin(b + y)$ then:

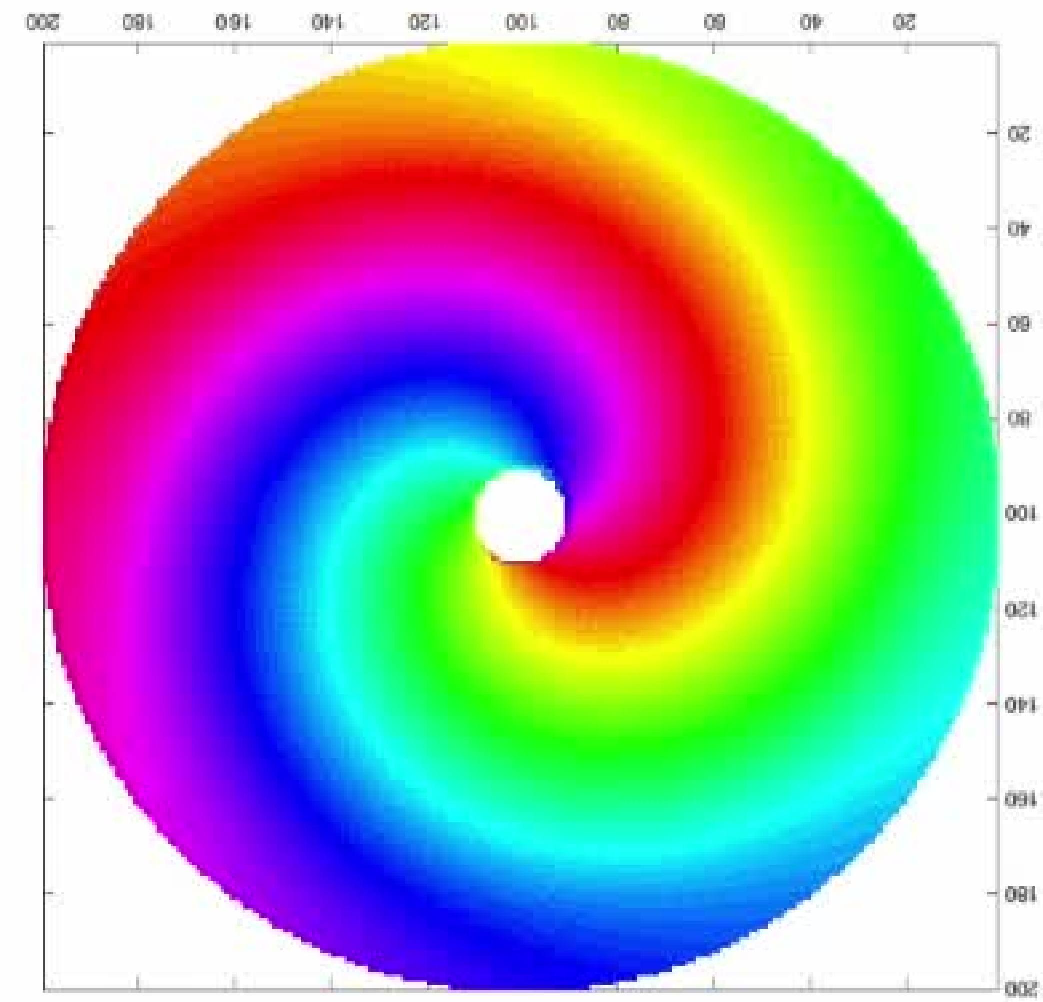
$$c_1 = \int_a^b s ds\, e^{-r^2 - s^2}\left[I_0(2rs)\sin(b) + I_1(2rs)\sin(\xi(s) - \xi(r) - b)\right]$$

- If $b = 0$, then $c_1 = 0$ and $\xi(r) = 0$, straight-armed spiral
- This is the subject of current analysis, for example stability

Example



full BVP (N=30)



approx (N=100)

having to solve the BVP allows for much finer discretization!

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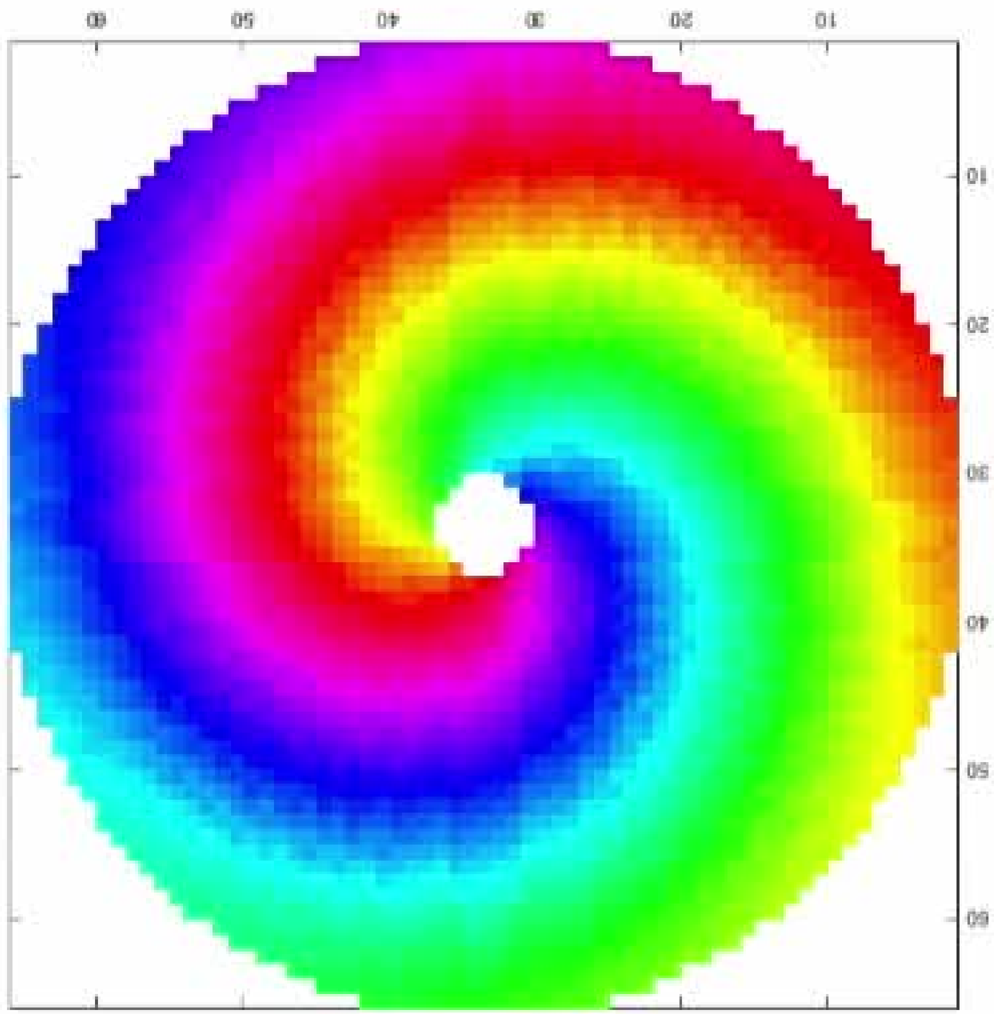

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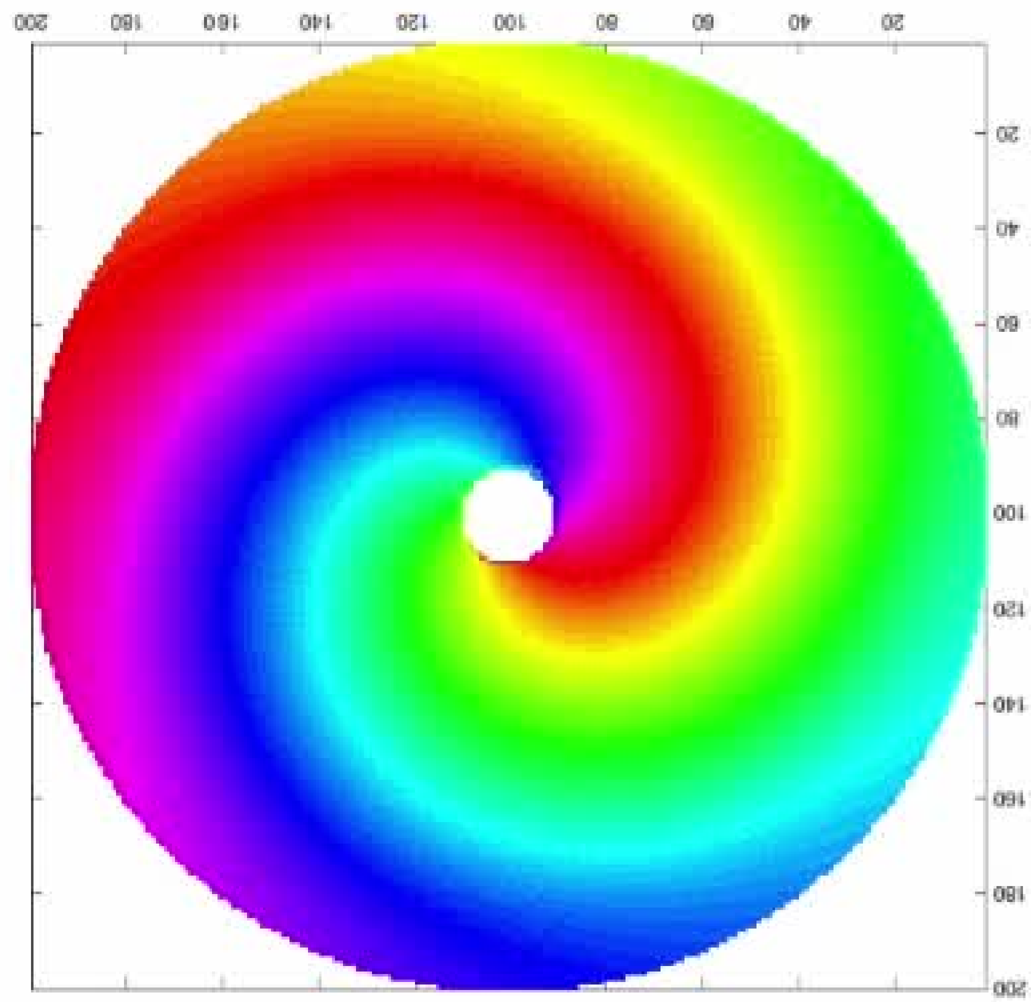

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