

# Data-driven modeling of complex systems with control

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## Presentation Outline

- Infectious disease spread as a complex system
- Challenges of current modeling efforts
- A data-driven approach, utilizing dynamic mode decomposition (DMD) for infectious disease data
- DMD with control



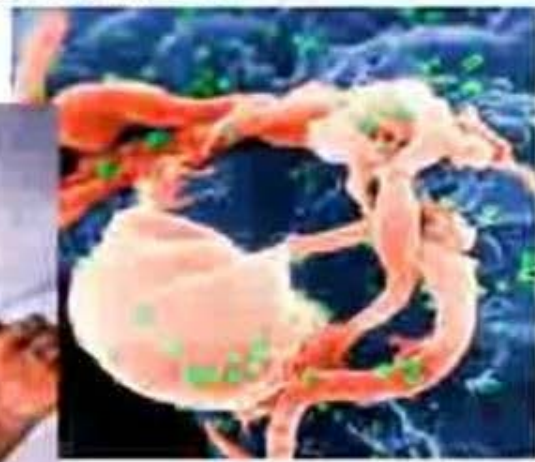
## Global Burden of Infectious Diseases

**~10 million people/year** die from an infectious disease, many deaths are preventable

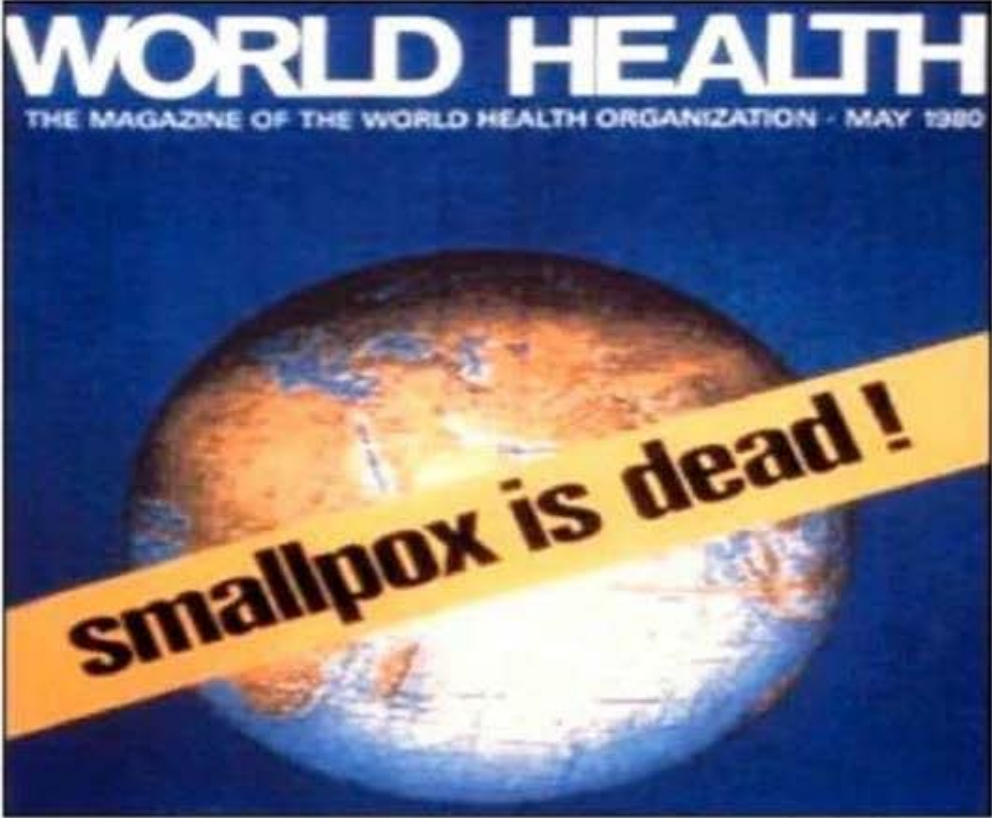
Malaria, Polio, Tuberculosis, HIV/AIDS, Influenza, Measles, etc



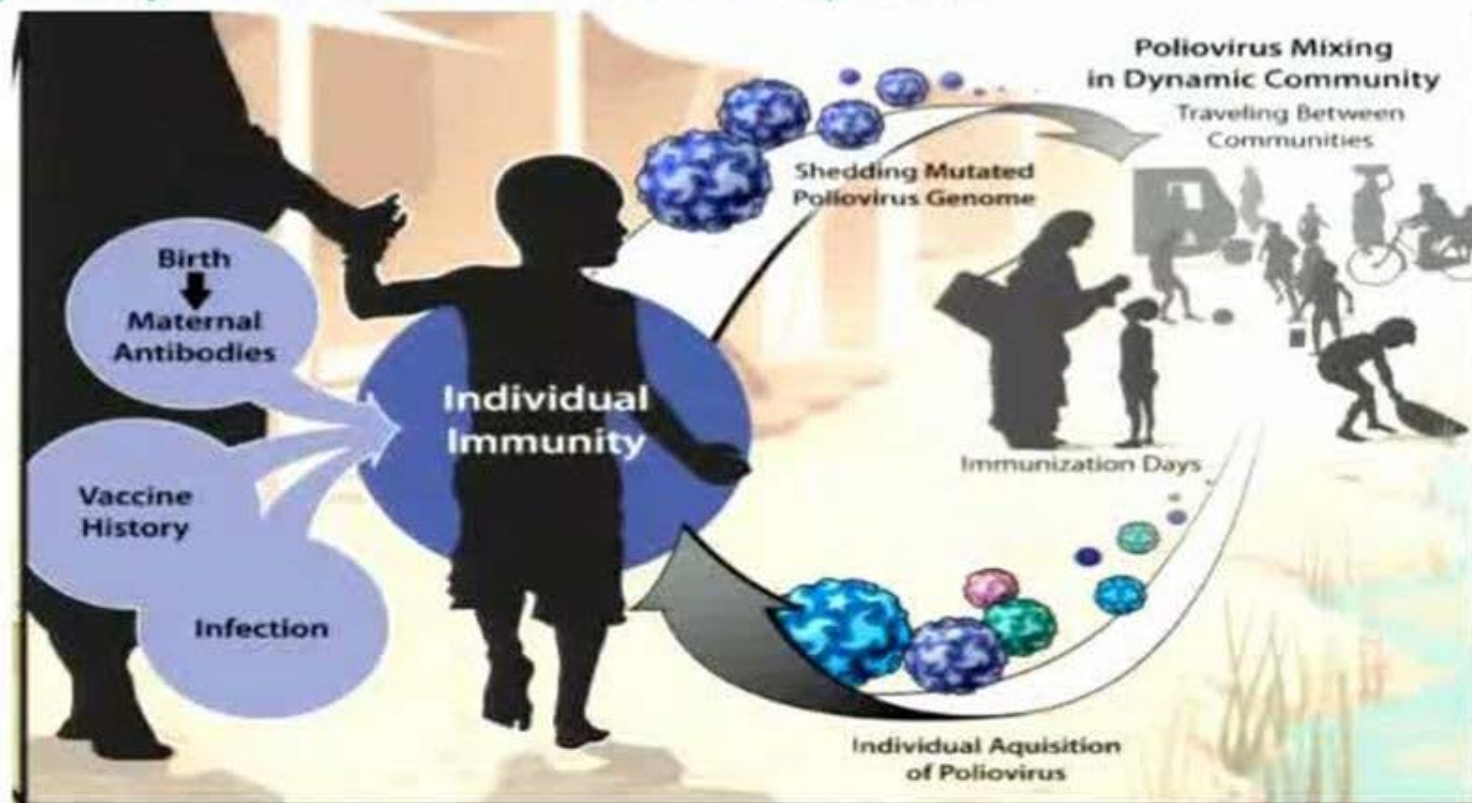
Scanning Electron Micrograph of Mycobacterium tuberculosis



# Smallpox Eradication



## Complexity of infectious disease spread



# Complexity of infectious disease spread

## Complex system

- Multi-Scale
- Spatial Heterogeneity
- Nonlinear effects
- Stochastic Effects
- Temporal dynamics
- ...

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THE EUROPEAN  
PHYSICAL JOURNAL  
SPECIAL TOPICS

Review

### Exploiting sparsity and equation-free architectures in complex systems

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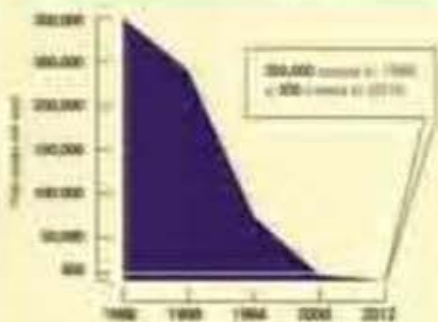
 globalgood

— INTELLECTUAL VENTURES



## WE ARE THIS CLOSE TO ENDING POLIO

### PROGRESS SINCE 1988



Rotary members have devoted their time and money to help eradicate polio since 1988.

**2 BILLION** ↑  
children in 122 countries

The Global Polio Eradication campaign has reduced polio cases by

**99%** ↓  
worldwide

### PROGRESS IN 2012



**INDIA IS  
POLIO FREE**



In February 2012, World Health Organization (WHO) announced India free from the polio endemic list.



**60%** ↓

Reduction in polio cases in 2012 compared to 2011

## POLIO ERADICATION STATUS

**AFGHANISTAN, GUINEA and POLIO** – the only three countries where the wild poliovirus has never been stopped.



## FUND THE FIGHT: GLOBAL

**1.2 MILLION** Rotarians

**\$1.2 BILLION** in contributions

**\$40-50 BILLION**

estimated amount polio eradication will save the world over the next 20 years



Ending polio will be one of the biggest accomplishments in global health.



Rotary and its partners are committed to fighting polio until every child is safe from this devastating disease.



globalgood

1000 1000 1000 1000 1000 1000 1000 1000 1000 1000



# Global Eradication



Google Images

## Complexity of infectious disease spread

### Control Input

- Vaccinations
- Nonlinear effects
- BedNets
- Vector Control
- Accessibility
- ...



Complex system



## Case versus infection ratio



Case Detection  
through paralysis



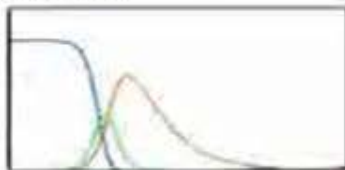
- Sparse data – paralytic cases, not infections detected (few environmental samples)

Case ID	Age	Location	Year	Sex	Paralysis Type	Case Status	Other Metadata
100-1000-1	1000	1000	100	100	1000	1000	1000
100-1000-2	1000	1000	100	100	1000	1000	1000
100-1000-3	1000	1000	100	100	1000	1000	1000
100-1000-4	1000	1000	100	100	1000	1000	1000
100-1000-5	1000	1000	100	100	1000	1000	1000
100-1000-6	1000	1000	100	100	1000	1000	1000
100-1000-7	1000	1000	100	100	1000	1000	1000
100-1000-8	1000	1000	100	100	1000	1000	1000
100-1000-9	1000	1000	100	100	1000	1000	1000
100-1000-10	1000	1000	100	100	1000	1000	1000

History of case data with  
some metadata (age,  
location, etc)

## Experiments

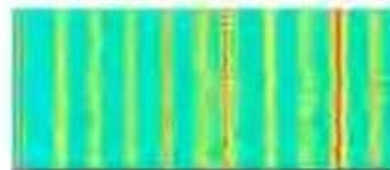
Numerical



Laboratory

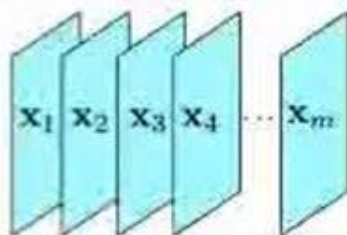


Historical



## State Snapshots in Time

State Measurements



## Data Matrices

$$\mathbf{X} = \begin{bmatrix} | & | & | & \cdots & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_{m-1} \\ | & | & | & \cdots & | \end{bmatrix}$$
$$\mathbf{X}' = \begin{bmatrix} | & | & | & \cdots & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \mathbf{x}_1 & \cdots & \mathbf{x}_m \\ | & | & | & \cdots & | \end{bmatrix}$$

What we really want is a method for understanding how these data snapshots are related in time:

$$\mathbf{X}' \approx \mathbf{A}\mathbf{X}$$

The **Dynamic Mode Decomposition** of the measurement pair  $X$  and  $X'$  is the eigendecomposition of the operator  $A$  defined by the following

$$A = X'X^\dagger$$

where  $\dagger$  is the pseudo inverse of  $X$ . The dynamic modes and eigenvalues are the eigenvalues and vectors of  $A$ .

- High-dimensional, complex systems<sup>1,2,7,5,6</sup>
- Equation-free operating on solely snapshot data<sup>1,2,7,5,6</sup>
- Actively being developed in the fluid dynamics community<sup>1,2,7,5,6</sup>
- A method for analyzing data from nonlinear systems (Koopman Operator theory)<sup>8,7,6</sup>
- The architecture easily lends itself to “sparsifying” techniques (more to come later)<sup>3,4</sup>

1. P. J. Schmid and J. L. Sesterhenn. Dynamic mode decomposition of numerical and experimental data. In *Bull. Amer. Phys. Soc.*, 61st APS meeting, page 208, 2008.
2. P. J. Schmid, K. S. Meier, and G. P. T. Dynamic mode decomposition and an orthogonal decomposition of flow field measurements. *Proc. 4th Int. Symp. on Particle Image Velocimetry*, August 2009.
3. S. L. Brunton, J. L. Proctor, and J. N. Kutz. Compressive sampling dynamic mode decomposition. *arXiv*, 1212.5196, 2012.
4. Mihailo R. Jovanović, Peter J. Schmid, and Joseph W. Nichols. Sparsity-promoting dynamic mode decomposition. *Physics of Fluids* (1994-present), 26(2):024101, 2014.
5. P. J. Schmid. Dynamic mode decomposition of numerical and experimental data. *Journal of Fluid Mechanics*, 656-5(28), August 2010.
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7. C. W. Rowley, I. Mezić, S. Bagheri, P. Schlatter, and D. S. Henningsen. Spectral analysis of nonlinear flows. *Journal of Fluid Mechanics*, 641:115-137, 2008.
8. Mezić. Spectral properties of dynamical systems, model reduction and decompositions. *J. Nonlin. Dyn.*, 41:305-325, 2005.

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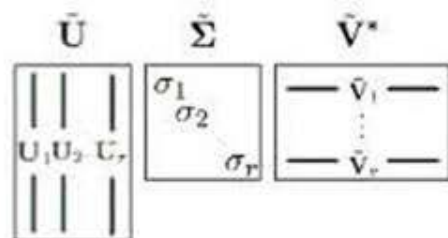
# DMD for infectious disease data

## Dynamic Mode Decomposition (DMD)

Find the dynamic properties of  $\mathbf{A}$  by solving:  $\mathbf{A} = \mathbf{X}'\mathbf{X}^\dagger$

### SVD and Truncation

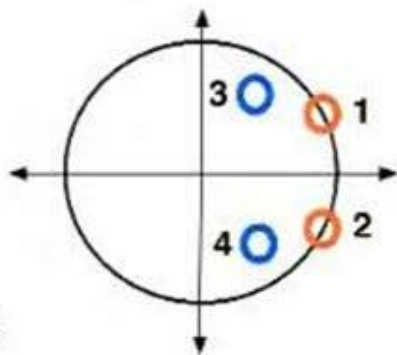
$$\mathbf{X} \approx \tilde{\mathbf{U}}\tilde{\Sigma}\tilde{\mathbf{V}}^*$$



### Eigenvalue Spectrum

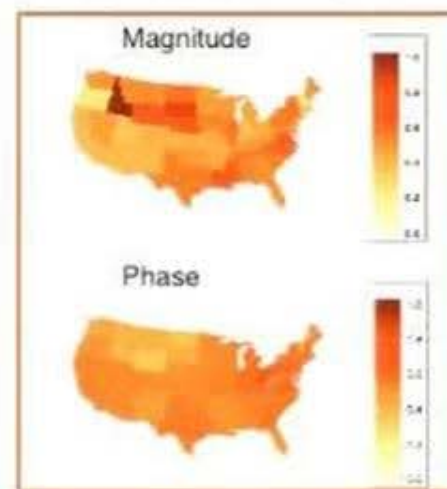
$$\tilde{\mathbf{A}} = \tilde{\mathbf{U}}\mathbf{X}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}$$

$$\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W}\Lambda$$



### Dynamic Modes

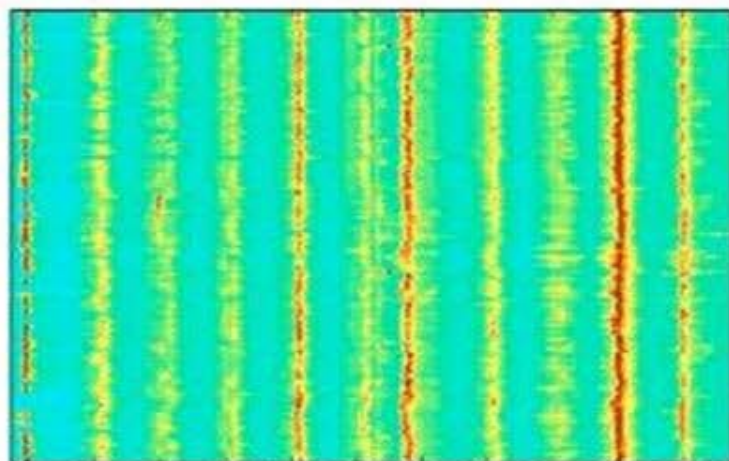
$$\Phi = \mathbf{X}'\tilde{\mathbf{V}}\tilde{\Sigma}^{-1}\mathbf{W}$$



Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. *Int. Health* (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009

# DMD on infectious disease data

## Google Flu Trend data in the United States



2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014

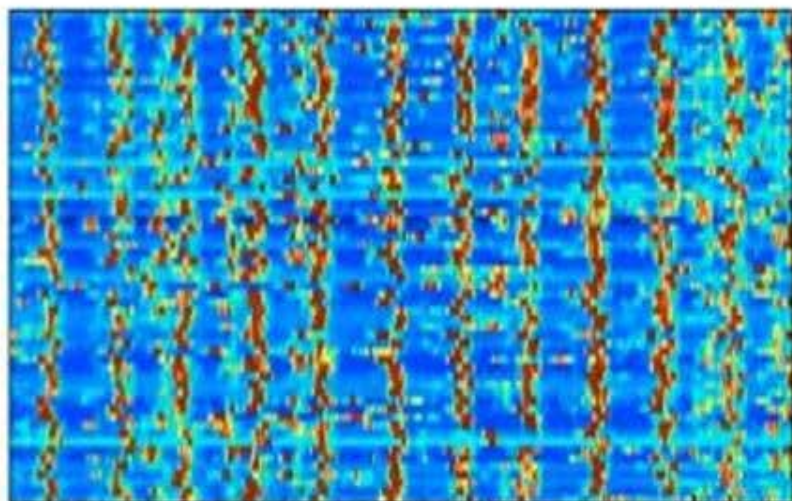
## Phase of Dynamic Mode for yearly eigenvalue



1. Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. *Int. Health* (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009
2. J. Ginsberg, Mohebb MH, Patel RS, Brammer L, Smolinski MS, Brilliant L. Detecting influenza epidemics using search engine query data. *Nature*, 457(7232):1012-1014, 02 2009.
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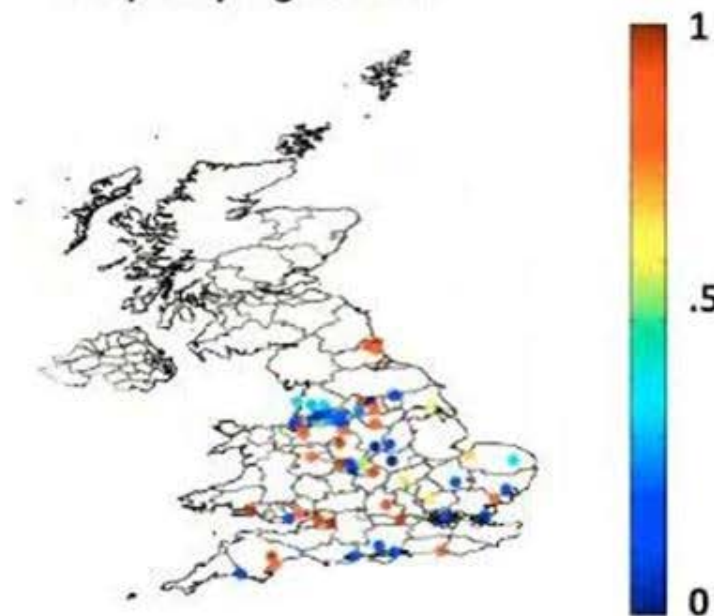
## DMD on infectious disease data

Measles cases from pre-vaccination United Kingdom



1944 1946 1948 1950 1952 1954 1956 1958 1960 1962 1964 1966

Phase of Dynamic Mode  
for yearly eigenvalue



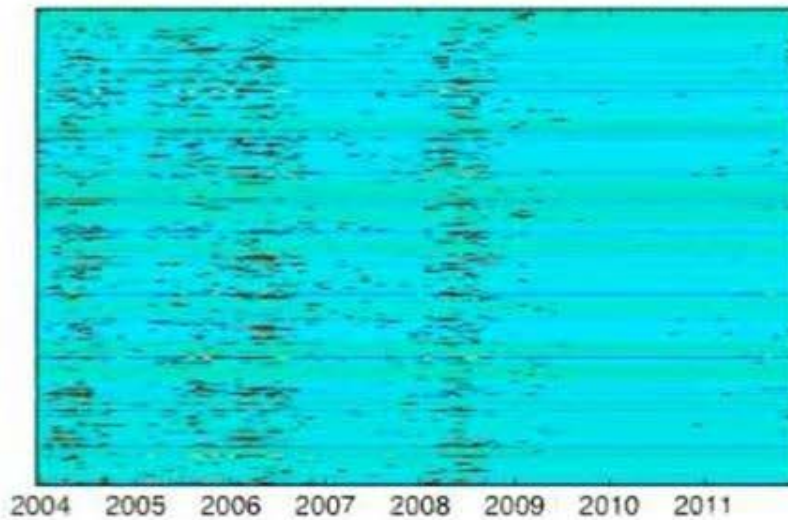
1. Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition *Int. Health* (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009

2. Keeling MJ, Grenfell BT. Disease extinction and community size: Modeling the persistence of measles. *Science*. 275(5296):63-67, 1997.

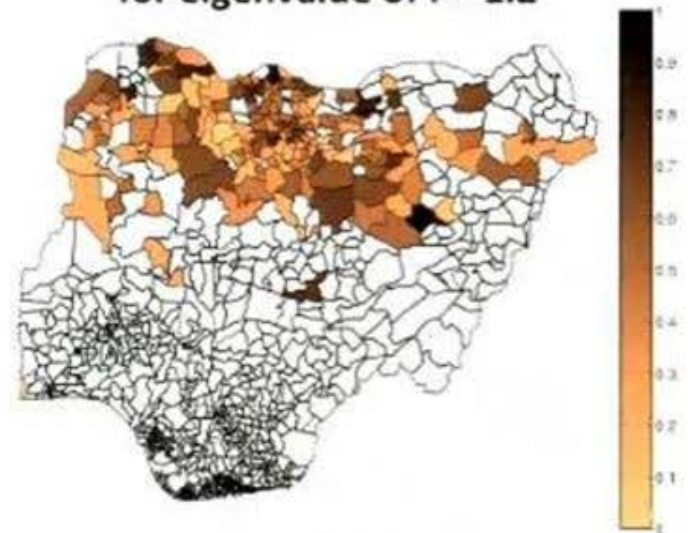


# DMD on infectious disease data

## Type 1 Polio cases from Nigeria



## Magnitude of Dynamic Mode for eigenvalue of $f = 1.2$



1. Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. *Int. Health* (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009
2. Ugfil-Brown AM, Lyons HM, Pate M, Shuaib F, Balg S, Hu H, Eckhoff PA, Chabot-Couture G. Predictive spatial risk model of poliovirus to aid prioritization and hasten eradication in Nigeria. *BMC Medicine*. 12(52):879-887. 2014

## Lessons learned from dynamic modes

- Phase information of individual frequency components
  - useful for planning annual resource allocation of vaccines,
  - surveillance and monitoring teams,
  - timing of interventions
- Epidemiological connectedness of spatial locations
  - helps with design of mop-up campaigns
  - surveillance planning
  - decreases complexity of numerical models

## DMD with control – the method

The **Dynamic Mode Decomposition with control** of the measurement trio  $X$ ,  $Y$ , and  $X'$  is the eigendecomposition of the operator  $\mathbf{A}$  defined by the following

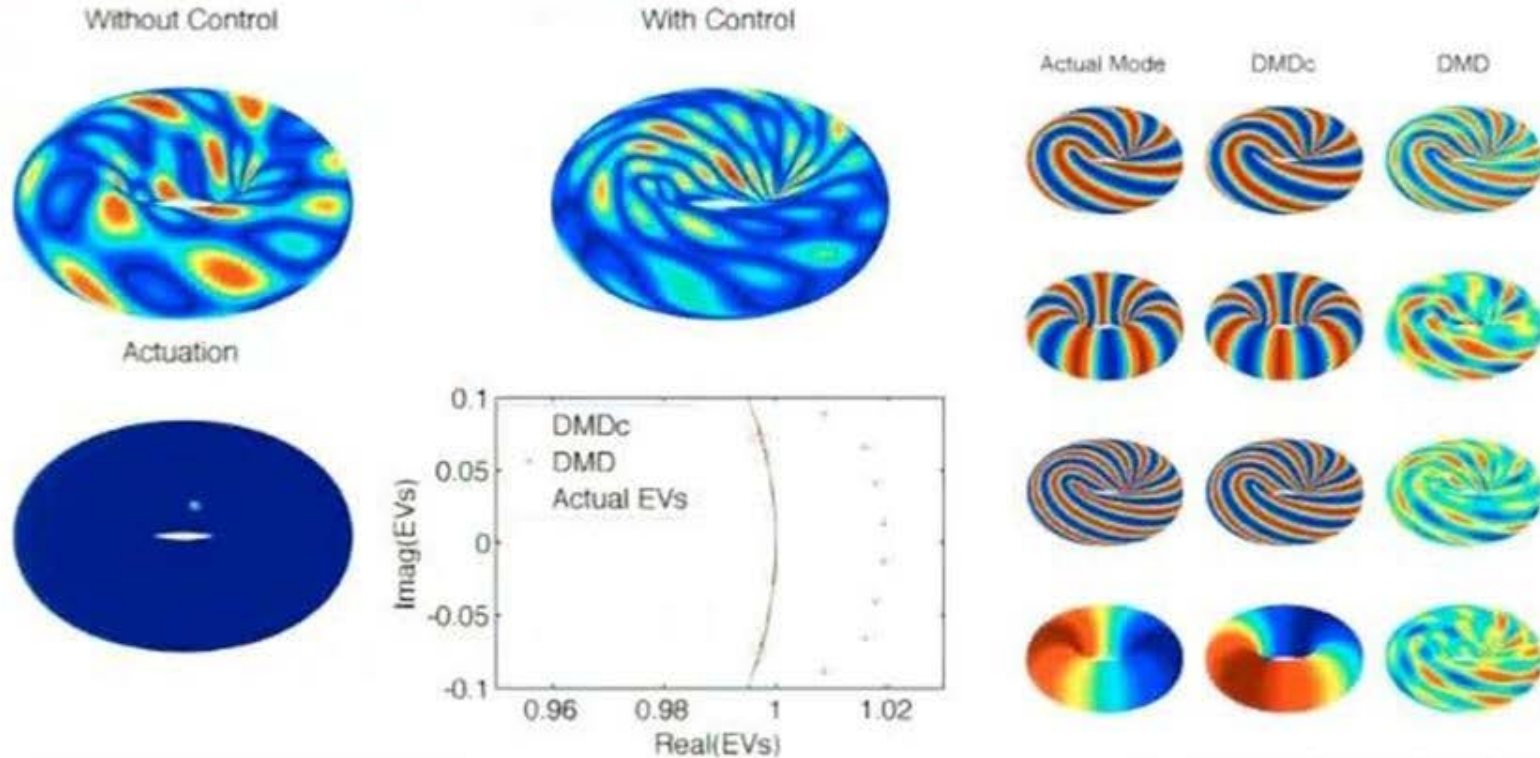
$$\mathbf{G} = \mathbf{X}'\mathbf{\Omega}^{\dagger},$$
$$[\mathbf{A} \ \mathbf{B}] = \mathbf{X}' \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix}^{\dagger}$$

where  $\dagger$  is the pseudo inverse of  $\mathbf{X}$ . The dynamic modes and eigenvalues are the eigenvalues and vectors of  $\mathbf{A}$ . The impact of control on the system is discovered by the operator  $\mathbf{B}$ .

- Analysis tool for systems with exogenous input.
- Disambiguates the inherent dynamics from the effect of input
- Defines a notion of input and output space
- Discovers a “best-fit” operator  $\mathbf{B}$  describing the impact of input on the system
- The operator  $\mathbf{B}$  will now offer an interesting perspective on these complex systems

1. Joshua L. Proctor, Steven L. Brunton, & Nathan Kutz, Dynamic Mode Decomposition with control (submitted) [arXiv:1809.03509](https://arxiv.org/abs/1809.03509)

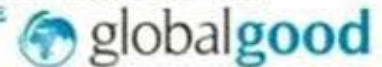
# A numerical Example



A numerical example of a five mode system chosen in the fourier domain with oscillatory dynamics. Spatial domain shown on the Torus.

1. Joshua L. Proctor, Steven L. Brunton, J. Nathan Kutz. Dynamic Mode Decomposition with control submitted.

[arXiv:1310.0308](https://arxiv.org/abs/1310.0308)



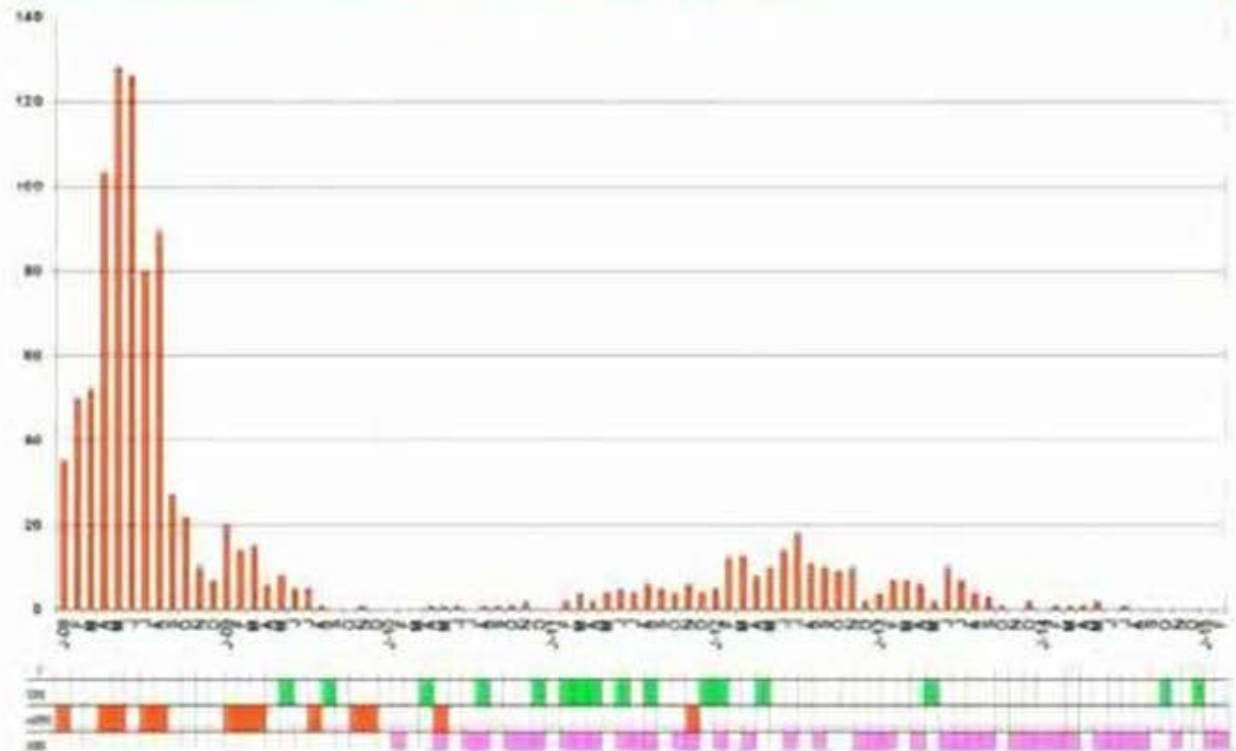
INTELLECTUAL VENTURES

## Next Steps:

- Apply DMD with control on data from Nigeria and Pakistan
- Supplementary Immunization Activities (SIAs)

How will the B matrix  
Inform future control efforts

### WPV 1 monthly onset, with targeted SIAs 2008 – 2015 as at week 8, 2015



## Connections to system identification methods

- In limiting cases, DMD is similar to the Eigensystem realization algorithm<sup>1,2,3</sup>
- In limiting cases, DMD with control is similar to the Observer Kalman Identification (OKID)<sup>2,4</sup>

### System Identification

Specify input control sequence

$$\Upsilon = \begin{bmatrix} \text{[Input sequence matrix]} \end{bmatrix}$$

Run experiment using  $\Upsilon$

$$\rightarrow X, X'$$

Use DMDC to find state-space model

$$x_{k+1} = Ax_k + Bu_k$$

1. T. W. Hill, D. M. Lichtenberg, C. W. Rowley, S. L. Brunton, and J. N. Kutz, "On dynamic mode decomposition: theory and applications," *Journal of Computational Dynamics*, 11(2):391–421, 2014.
2. Joshua L. Proctor, Steven L. Brunton, J. Nathan Kutz, "Dynamic Mode Decomposition with control," submitted: <https://arxiv.org/abs/1409.0009>
3. E. N. Jang and R. S. Pappa, "An eigensystem realization algorithm for modal parameter identification and model reduction," *Journal of Guidance, Control, and Dynamics*, 8(5):620–627, 1985.
4. J. W. Juang, M. Ples, I. G. Hoffa, and R. W. Longman, "Identification of observer/kalman filter markov parameters: Theory and experiments," Technical Memorandum 104069 NASA, 1981.