Data-driven modeling of complex systems with control

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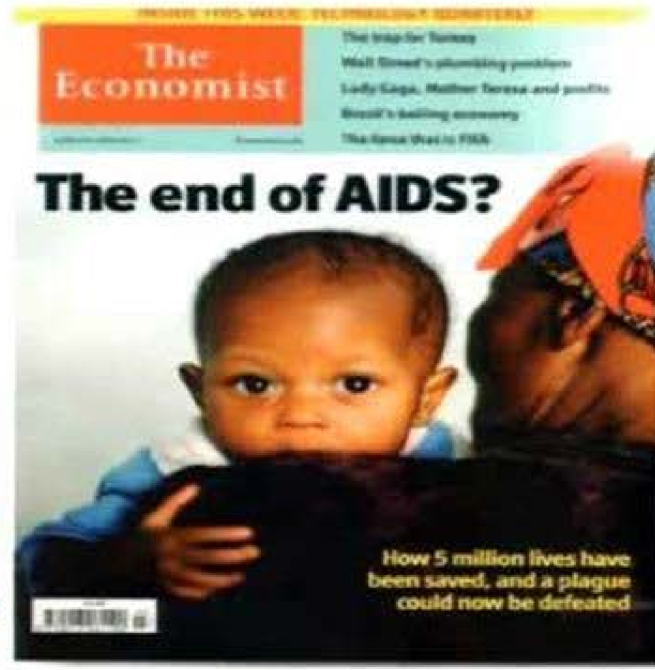
Presentation Outline

- **Infectious disease spread as a complex system**
- **Challenges of current modeling efforts**
- **A data-driven approach, utilizing dynamic mode decomposition (DMD) for infectious disease data**
- **DMD with control**



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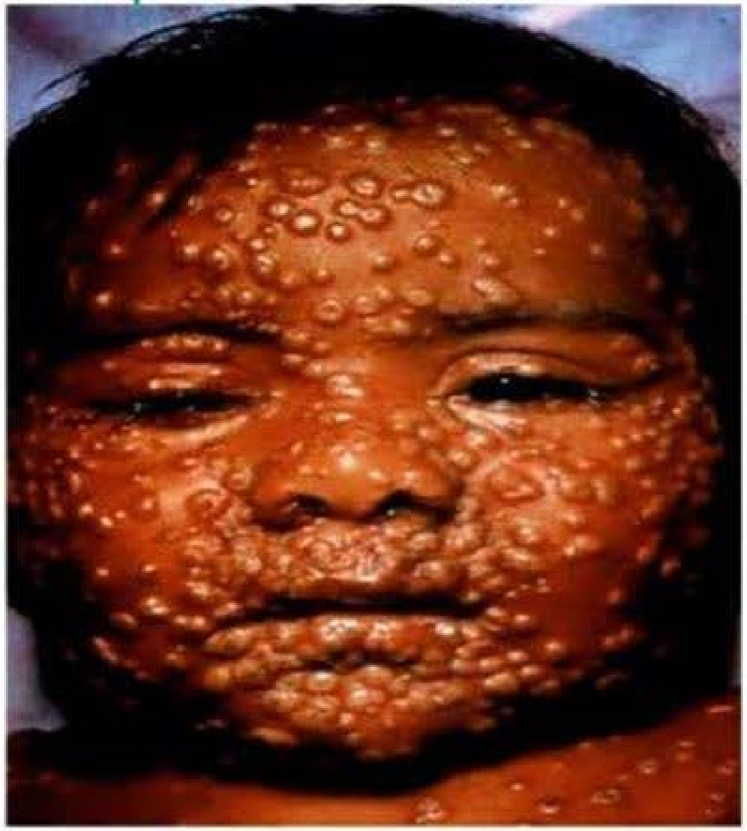
Global Burden of Infectious Diseases

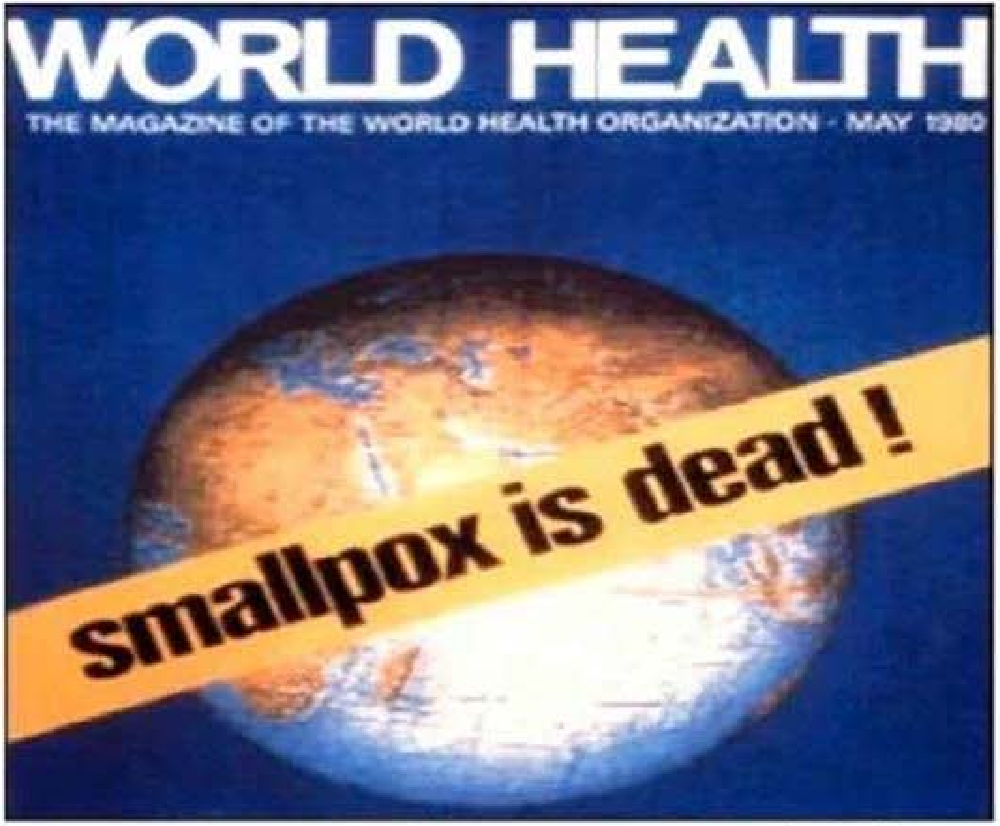
~10 million people/year die from an infectious disease, many deaths are preventable

Malaria, Polio, Tuberculosis, HIV/AIDS, Influenza, Measles, etc

Google images

Smallpox Eradication





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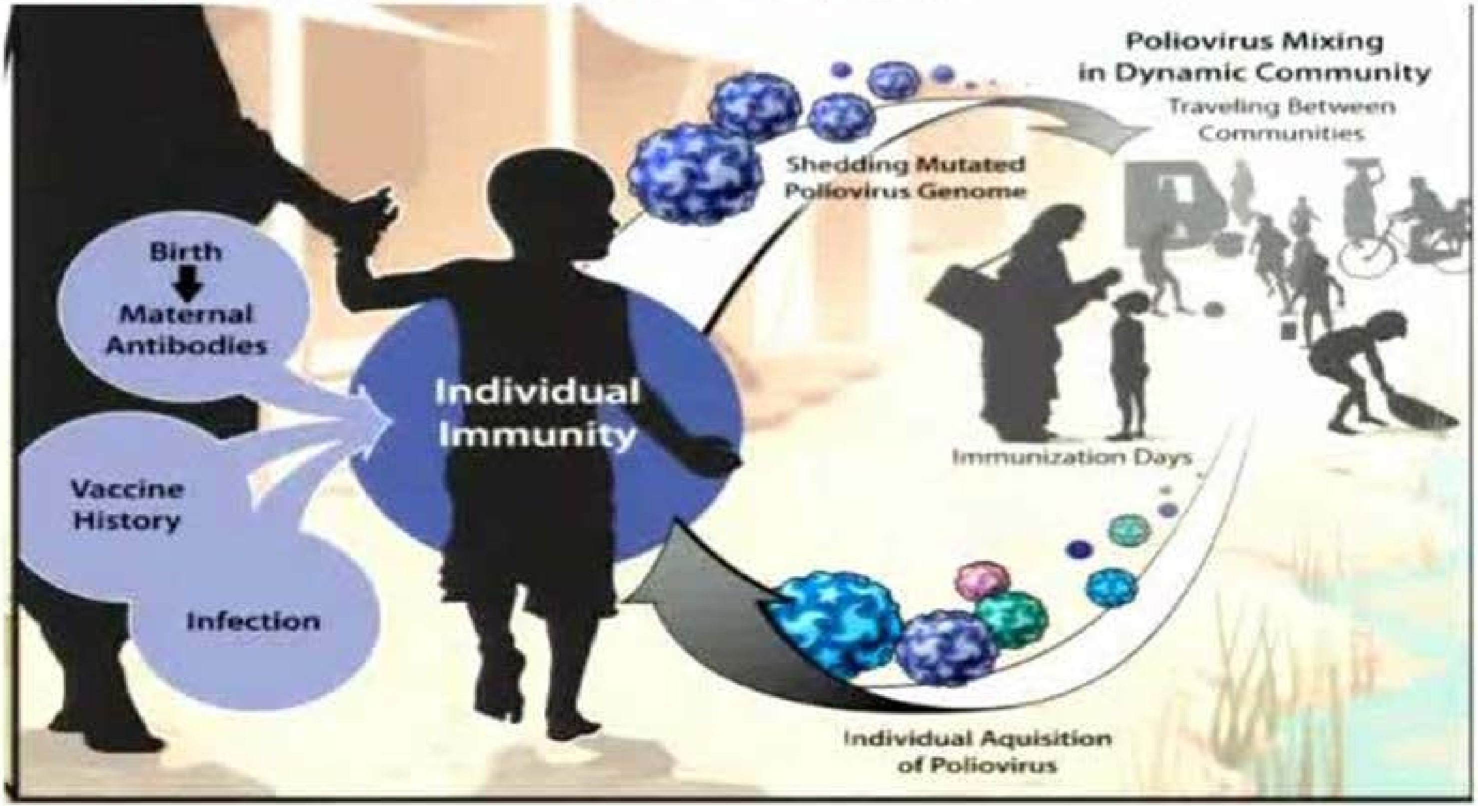
Complexity of infectious disease spread



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Complexity of infectious disease spread

Complex system

- Multi-Scale
- Spatial Heterogeneity
- Nonlinear effects
- Stochastic Effects
- Temporal dynamics
- ...


THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Exploiting sparsity and equation-free architectures in complex systems



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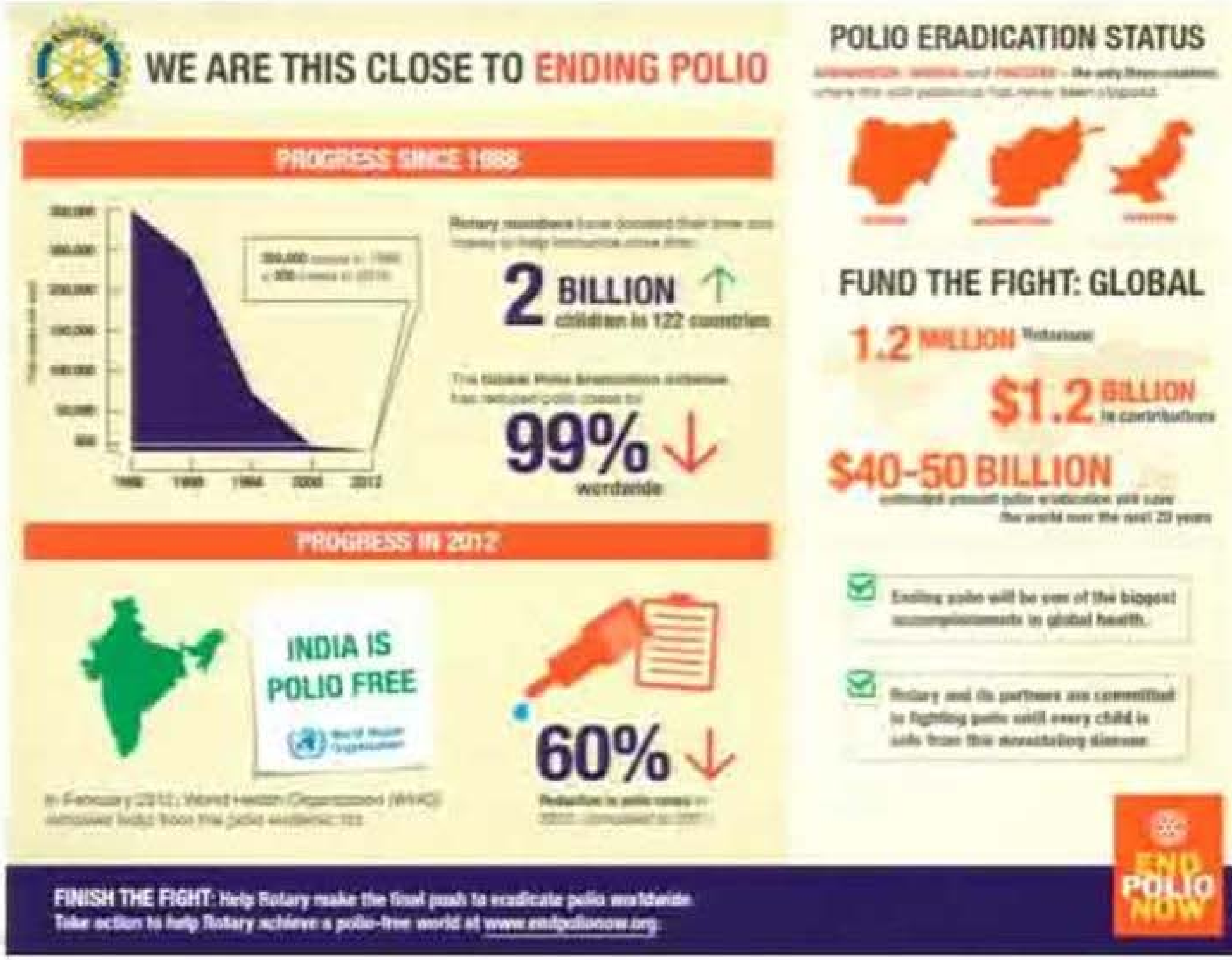

Rotary International

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Global Eradication

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Complexity of infectious disease spread

Control Input

- Vaccinations
- Nonlinear effects
- BedNets
- Vector Control
- Accessibility
- ...

→ **Complex system**

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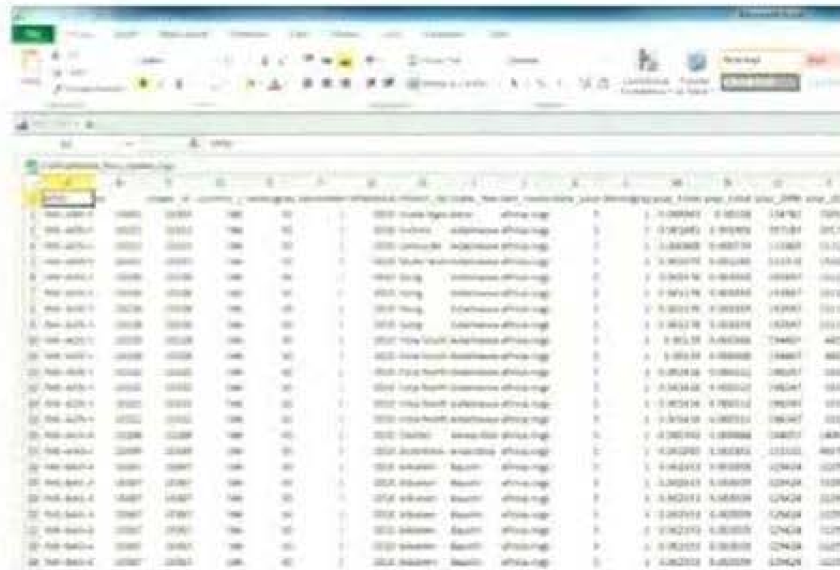
Case versus infection ratio

Case Detection
through paralysis



History of case data with some metadata (age, location, etc)

- Sparse data – paralytic cases, not infections detected (few environmental samples)

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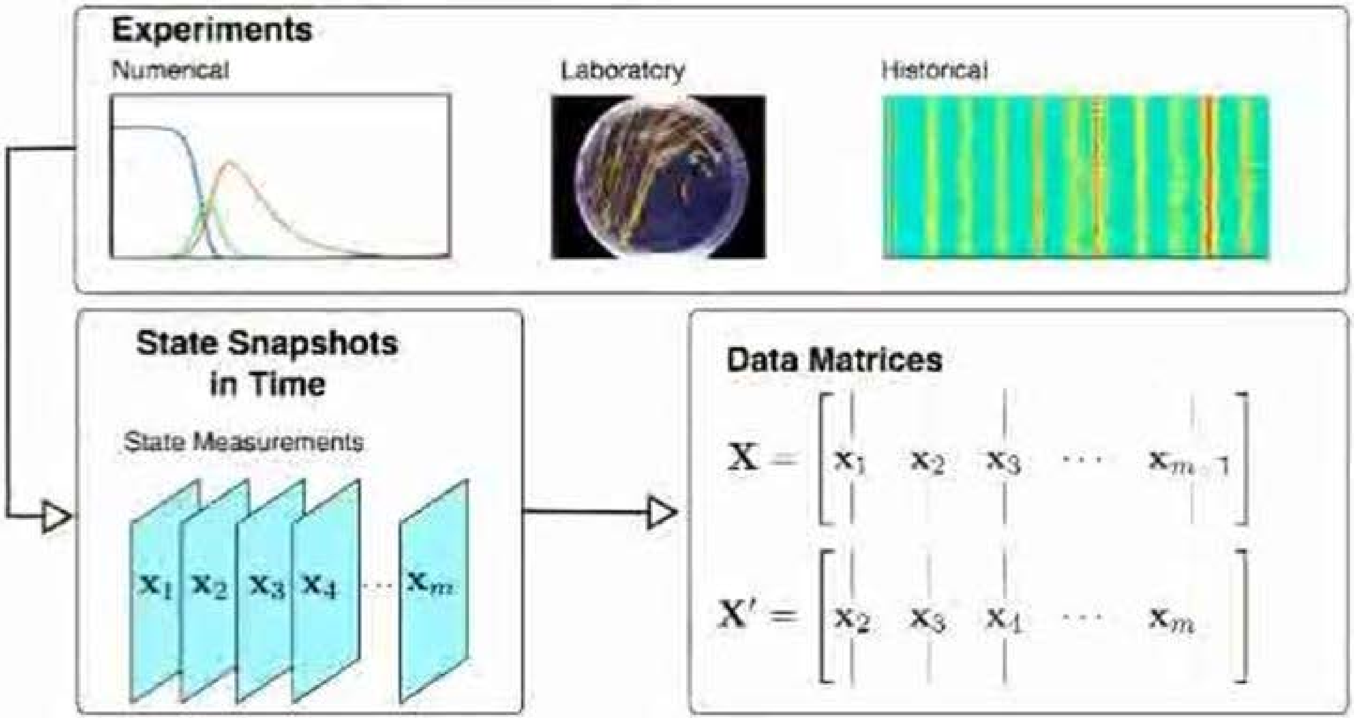

What we really want is a method for understanding how these data snapshots are related in time:

$$\mathbf{X}' \approx \mathbf{A}\mathbf{X}$$

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The **Dynamic Mode Decomposition** of the measurement pair X and X' is the eigendecomposition of the operator A defined by the following

$$\mathbf{A} = \mathbf{X}'\mathbf{X}^{\dagger}$$

where $\dagger$ is the pseudo inverse of **X**. The dynamic modes and eigenvalues are the eigenvalues and vectors of **A**.

- High-dimensional, complex systems[1,2,7,5,6]
- Equation-free operating on solely snapshot data[1,2,7,5,6]
- Actively being developed in the fluid dynamics community[1,2,7,5,6]
- A method for analyzing data from nonlinear systems (Koopman Operator theory)[8,7,6]
- The architecture easily lends itself to "sparsifying" techniques (more to come later) [3,4]

1. P. J. Schmid and J. L. Sesterhenn. Dynamic mode decomposition of numerical and experimental data. In Bull. Amer. Phys. Soc., 61st APS meeting, page 208, 2008.
2. P. J. Schmid, K. E. Meyer and O. Pust. Dynamic mode decomposition and proper orthogonal decomposition of flow in a lid-driven cylindrical cavity. PIV09-0186. 8th International Symposium on Particle Image Velocimetry, August 2009.
3. S. L. Brunton, J. L. Proctor, and J. N. Kutz. Compressive sampling dynamic mode decomposition. arXiv, 1312.5186, 2013.
4. Mihailo R. Jovanovic, Peter J Schmid, and Joseph W Nichols. Sparsity-promoting dynamic mode decomposition. Physics of Fluids (1994-present), 26(2):024103, 2014.
5. P. J. Schmid. Dynamic mode decomposition of numerical and experimental data. Journal of Fluid Mechanics, 656:5-28, August 2010.
6. J. H. Tu, D. M. Luchtenburg, C. W. Rowley, S. L. Brunton, and J. N. Kutz. On dynamic mode decomposition: theory and applications. Journal of Computational Dynamics, 1(2):391-421, 2014.
7. C. W. Rowley, I. Mezic, S. Bagheri, P. Schlatter, and D. S. Henningson. Spectral analysis of nonlinear flows. Journal of Fluid Mechanics, 641:115-127, 2009.
8. Mezic. Spectral properties of dynamical systems, model reduction and decompositions. J. Nonlin. Dyn., 41:309-325, 2005.

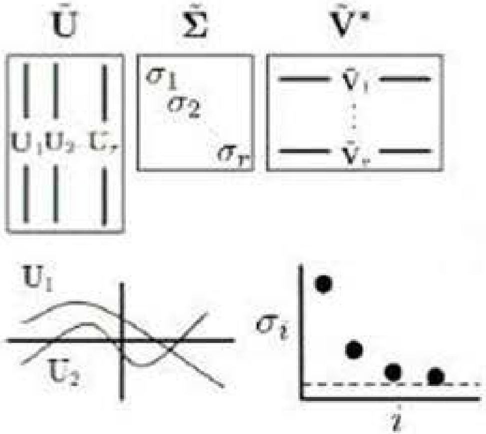
DMD for infectious disease data

Dynamic Mode Decomposition (DMD)

Find the dynamic properties of $\mathbf{A}$ by solving: $\mathbf{A} = \mathbf{X}'\mathbf{X}^{\dagger}$

SVD and Truncation

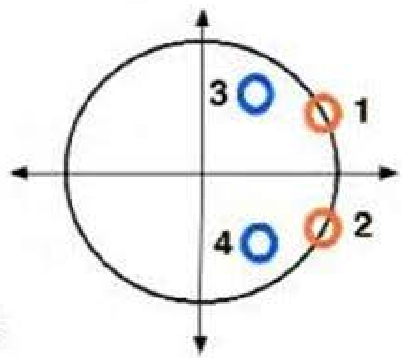
$$\mathbf{X} \approx \tilde{\mathbf{U}}\tilde{\mathbf{\Sigma}}\tilde{\mathbf{V}}^*$$



Eigenvalue Spectrum

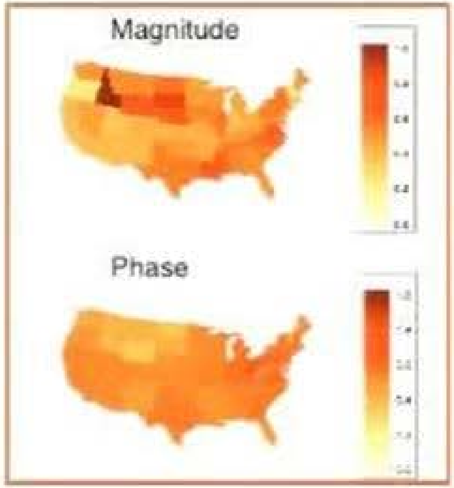
$$\tilde{\mathbf{A}} = \tilde{\mathbf{U}}\mathbf{X}'\tilde{\mathbf{V}}\tilde{\mathbf{\Sigma}}^{-1}$$

$$\tilde{\mathbf{A}}\mathbf{W} = \mathbf{W}\mathbf{\Lambda}$$



Dynamic Modes

$$\mathbf{\Phi} = \mathbf{X}'\mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{W}$$



Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. Int. Health (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009

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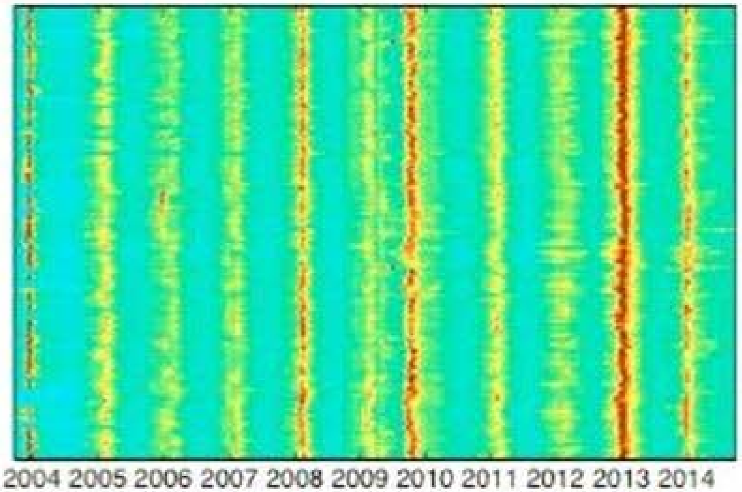
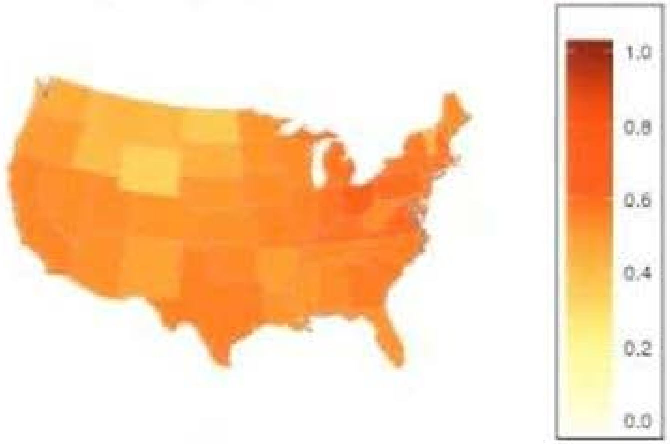
INTELLECTUAL VENTURES

DMD on infectious disease data

Google Flu Trend data in the United States



Phase of Dynamic Mode for yearly eigenvalue



1. Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. Int. Health (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009
2. J. Ginsberg, Mohebb MH, Patel RS, Brammer L, SmolinskiMS, Brilliant L. Detecting influenza epidemics using search engine query data. Nature, 457(7232):1012–1014, 02 2009.
3. David Lazer, Ryan Kennedy, Gary King, and Alessandro Vespignani. The parable of google flu: Traps inbigdata analysis. Science, 343(6176):1203–1205, 2014.

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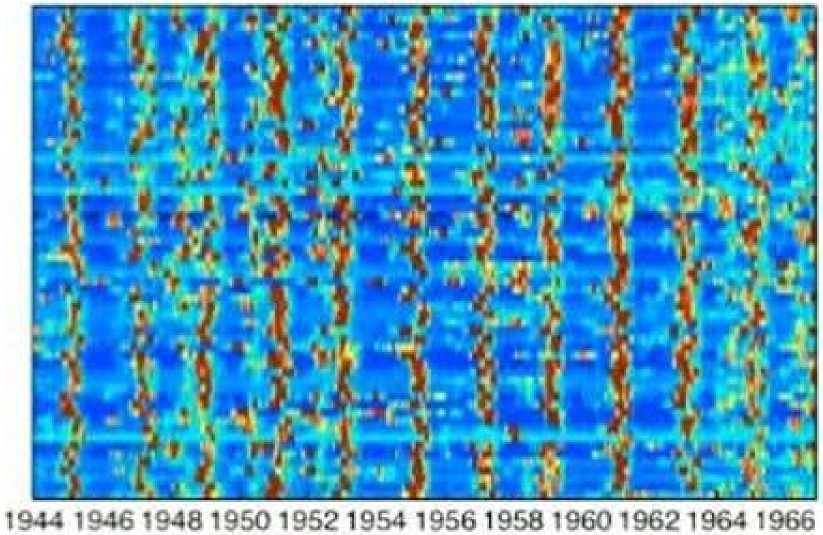
DMD on infectious disease data

Measles cases from pre-vaccination United Kingdom



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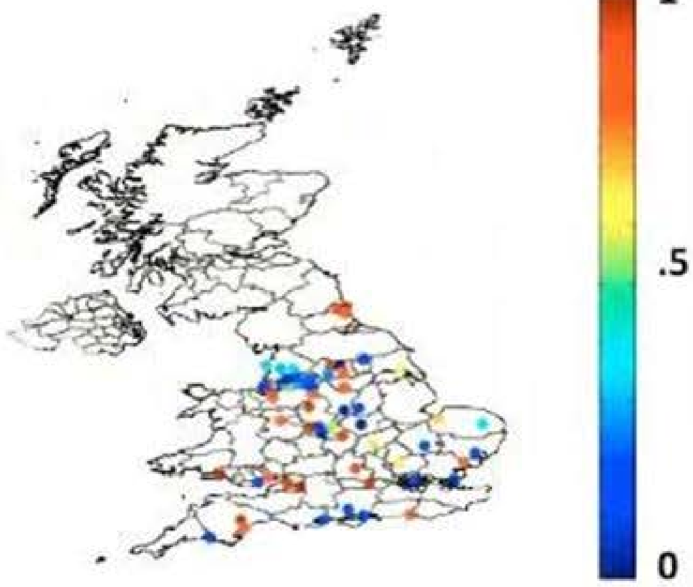

1. Joshua L Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. Int. Health (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009
2. Keeling MJ, Grenfell BT. Disease extinction and community size: Modeling the persistence of measles. Science. 275(5296):65–67, 1997.

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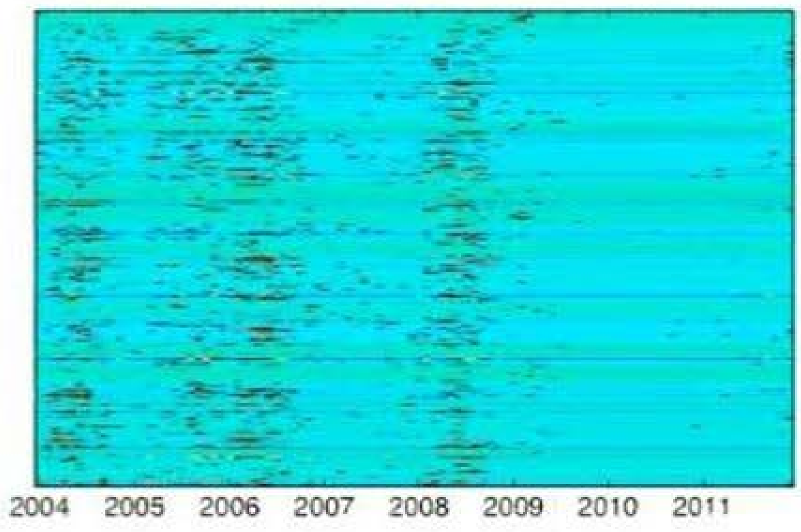
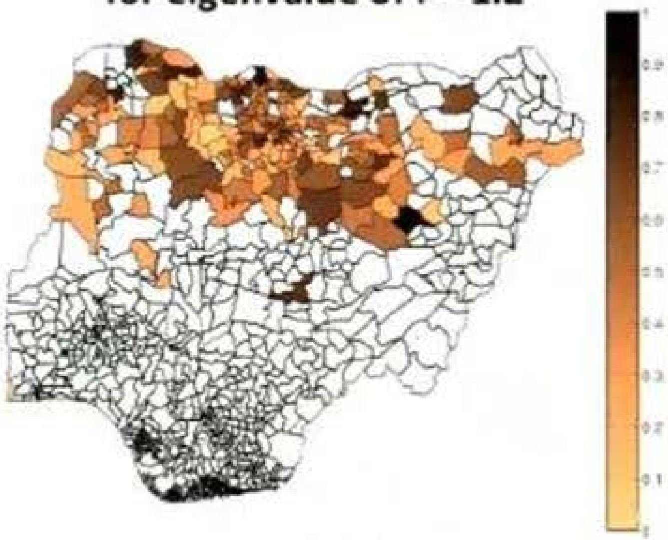
DMD on infectious disease data

Type 1 Polio cases from Nigeria



Magnitude of Dynamic Mode for eigenvalue of f = 1.2



1. Joshua L. Proctor and Philip A. Eckhoff Discovering dynamic patterns from infectious disease data using dynamic mode decomposition. Int. Health (2015) 7 (2): 139-145 doi:10.1093/inthealth/ihv009
2. Upfill-Brown AM, Lyons HM, Pate M, Shuaib F, Baig S, Hu H, Eckhoff PA, Chabot-Couture G. Predictive spatial risk model of poliovirus to aid prioritization and hasten eradication in nigeria. BMC Medicine. 12(92):879–887. 2014

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Lessons learned from dynamic modes

- Phase information of individual frequency components
  - useful for planning annual resource allocation of vaccines,
  - surveillance and monitoring teams,
  - timing of interventions

- Epidemiological connectedness of spatial locations
  - helps with design of mop-up campaigns
  - surveillance planning
  - decreases complexity of numerical models

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DMD with control – the method

The **Dynamic Mode Decomposition with control** of the measurement trio X, Υ, and X' is the eigendecomposition of the operator **A** defined by the following

$$\mathbf{G} = \mathbf{X}'\Omega^{\dagger},$$

$$[\mathbf{A} \;\; \mathbf{B}] = \mathbf{X}' \begin{bmatrix} \mathbf{X} \\ \Upsilon \end{bmatrix}^{\dagger}$$

where † is the pseudo inverse of **X**. The dynamic modes and eigenvalues are the eigenvalues and vectors of **A**. The impact of control on the system is discovered by the operator **B**.

- Analysis tool for systems with exogenous input.
- Disambiguates the inherent dynamics from the effect of input
- Defines a notion of input and output space
- Discovers a "best-fit" operator **B** describing the impact of input on the system
- The operator **B** will now offer an interesting perspective on these complex systems

1. Joshua L. Proctor, Steven L. Brunton, J. Nathan Kutz, Dynamic Mode Decomposition with control, submitted

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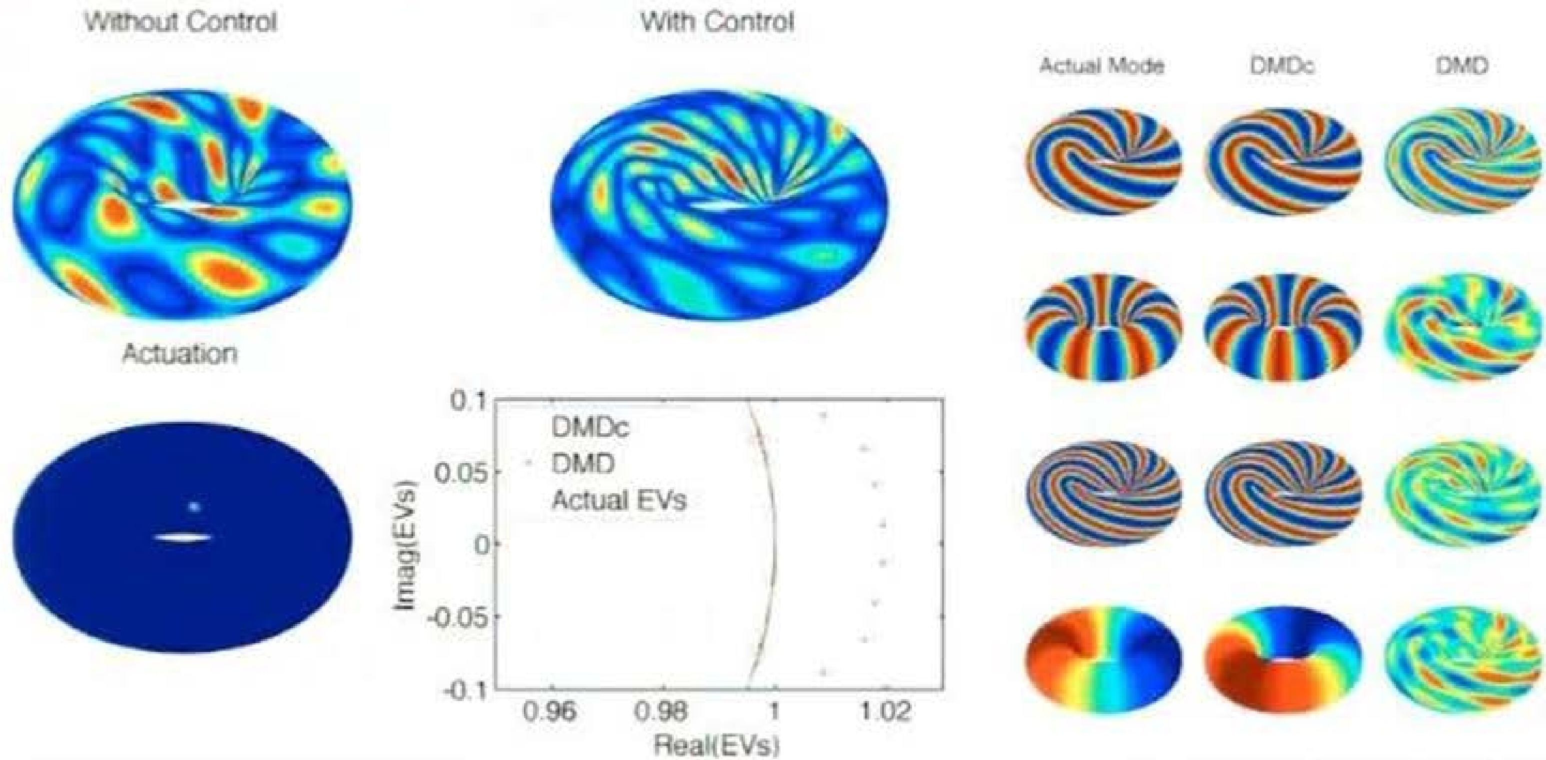
A numerical Example



A numerical example of a five mode system chosen in the fourier domain with oscillatory dynamics. Spatial domain shown on the Torus.

1. Joshua L. Proctor, Steven L. Brunton, J. Nathan Kutz. Dynamic Mode Decomposition with control submitted. arXiv 1409.6358

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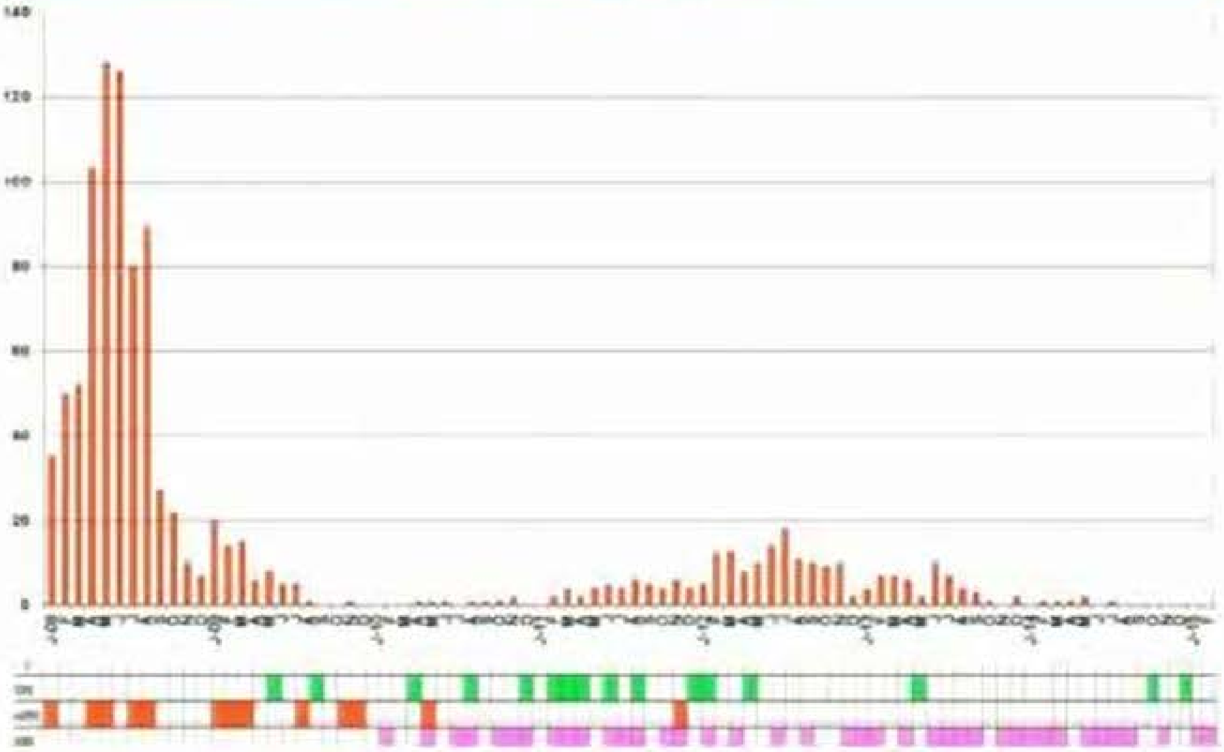
INTELLECTUAL VENTURES

Next Steps:

- Apply DMD with control on data from Nigeria and Pakistan
- Supplementary Immunization Activities (SIAs)

How will the B matrix
Inform future control efforts

WPV 1 monthly onset, with targeted SIAs 2008 – 2015
as at week 8, 2015



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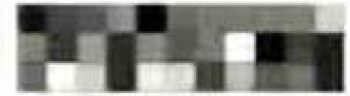
Connections to system identification methods

- In limiting cases, DMD is similar to the Eigensystem realization algorithm[1,2,3]
- In limiting cases, DMD with control is similar to the Observer Kalman Identification (OKID)[2,4]

System Identication

Specify input control sequence

$\Upsilon =$ 

Run experiment using Υ

$\rightarrow \mathbf{X}, \ \mathbf{X}'$

Use DMDc to find state-space model

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

1. J. H. Tu, D. M. Luchtenburg, C. W. Rowley, S. L. Brunton, and J. N. Kutz. On dynamic mode decomposition: theory and applications. Journal of Computational Dynamics, 1(2):391-421, 2014
2. Joshua L. Proctor, Steven L. Brunton, J. Nathan Kutz. Dynamic Mode Decomposition with control. submitted. [illegible]
3. J. N. Juang and R. S. Pappa. An eigensystem realization algorithm for modal parameter identification and model reduction. Journal of Guidance, Control, and Dynamics, 8(5):620–627, 1985
4. J. N. Juang, M. Phan, L. G. Horta, and R. W. Longman. Identification of observer/kalman filter markov parameters: Theory and experiments. Technical Memorandum 104069 NASA, 1991

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