Data Assimilation for State Estimation of Fault Behavior

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Overview

- Introduction: Data assimilation in a seismo-thermo-mechanical model
- Methodology: Use of a time-lag particle filter
- Results: ensemble generation and state updates
- Preliminary conclusions
- Recommendations/ongoing work

Building on work of van Dinther et al. (GJI, 2019)

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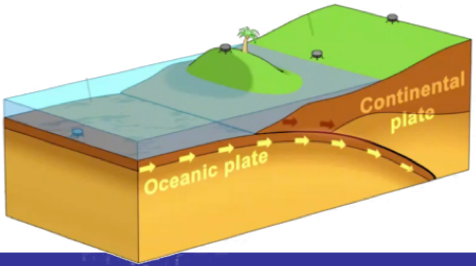
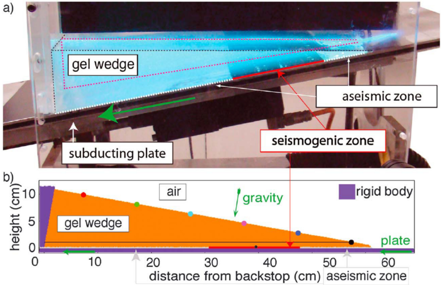
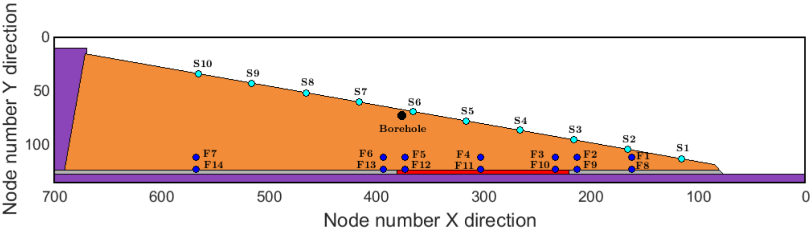


Seismo-Thermo-Mechanical (STM) model

- Seismic cycle simulator that can be configured on a range of scales
- 2D domain for laboratory experiment, as described in van Dinther et al. (2013a,b)
- Conservation equations and rheological constitutive equations
- Continuum-mechanics-based approach for visco-elasto-plastic material
- Rate-dependent friction coefficient, adaptive time-stepping
- Characteristic-based Lagrangian marker-in-cell method of Gerya & Yuen (2007)



Seismo-Thermo-Mechanical (STM) model setup



- Setup representing laboratory scale
- 701x136 nodes
- Air (white), Gelatin (orange), Fault (red- seismic and gray- aseismic zones), backstop wall (magenta)
- Marker-in-cell of interest: surface (GPS) markers (light blue) borehole location (black) and fault markers (blue)
- Assuming observations available at borehole location

Data assimilation concept (Ensemble Kalman Filtering)

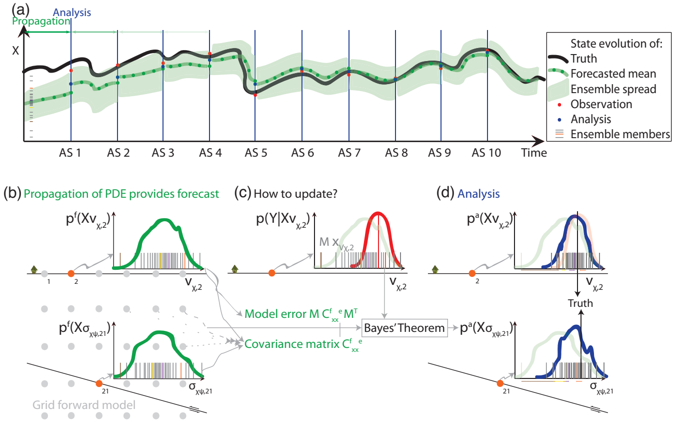
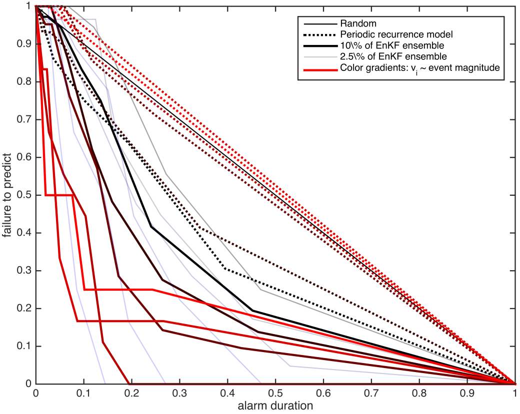


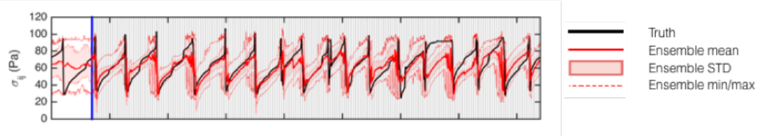
Figure taken from van Dinther et al. (2019)

Data assimilation improves forecasting skills



Results with EnKF

- Assimilated data obtained from the borehole location at intervals of 30 time steps (time step $\approx$ 60 ms; one cycle varies around 20-25 s)
- using a straightforward EnKF implementation with a limited ensemble size (20 members)
- correcting all nodal values for five physical variables (i.e. v_χ, v_ψ, $\sigma_{\chi'\chi}$, $\sigma_{\chi'\psi}$ and P)



- In velocity, seismic events well captured. Dynamic stress increase due to approaching rupture front not well captured. Pressure remains uncertain.
- When dynamics are strongly nonlinear, the use of a particle filter may be more appropriate

Particle Filter in a Seismo-Thermo-Mechanical model

Estimate the dynamic state variable ψ given data d use Bayes' theory to find the posterior:

$$p(\psi_{0:t}|d_{1:t}) = \frac{p(d_{1:t}|\psi_{0:t})p(\psi_{0:t})}{p(d_{1:t})}, \tag{1}$$

Representing prior pdf by particles x_i (dropping subscript t):

$$p(\psi) = \Sigma_{i=1}^{N}\frac{1}{N}\delta(\psi - \psi_i), \tag{2}$$

and using (1) gives

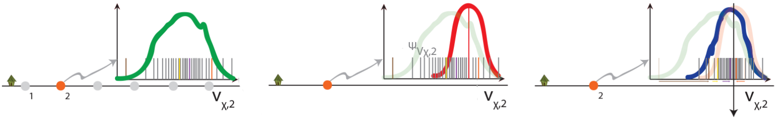
$$p(\psi|d) = \Sigma_{i=1}^{N} w_i \delta(\psi - \psi_i), \tag{3}$$

with w_i given by

$$w_i = \frac{p(d|\psi_i)}{\Sigma_{j=1}^{N} p(d|\psi_j)}. \tag{4}$$

Particle Filter in a Seismo-Thermo-Mechanical model

For variable $v_{\chi,2}$ the prior, likelihood and posterior could look like this:



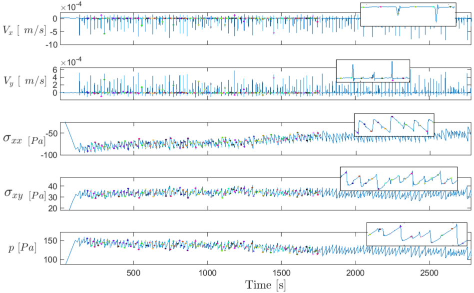
Setup of the perfect model test

- simulate "true" model evolution and sample synthetic observations at the borehole; add noise
- generate 300 initial conditions to simulate ensemble members for the particle filter
- at each assimilation step, assimilate the synthetic observations
- in the particle filter, this is done by multiplying likelihood with prior, in effect: calculating weight for each realisation
- misfit of ensemble mean with truth is an indicator of the performance of the data-assimilation approach

Time-lagged sampling for ensemble generation

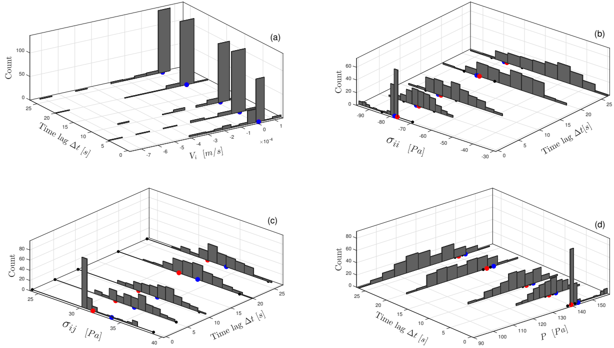
Particles are generated by sampling the model evolution at different times. This approach is similar to the "lagged average forecasting" approach of Hoffman & Kalnay (1983):

$$p(\psi|d) = \sum_{i=1}^{N_s} w_i \delta(\psi - \psi_{t+i\Delta t}).$$



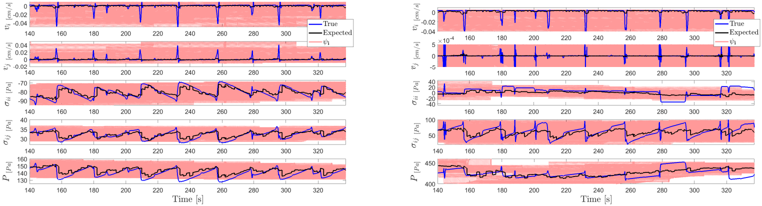
Results

- Ensemble of 300 particles,
- Varying time lag Δt
- Increasing Δt results in a larger ensemble spread for σ_{ii}, σ_{ij} and P, but not for v
- red: synthetic observations for a specific state variable (without noise), blue: observations with noise and black: variance of the observational error



Results

- Present implementation of the PF results in an ensemble spread with sufficient variability
- Generally speaking, fit to the data is less good than in the EnKF implementation
- At borehole location, stresses and pressure are reasonably well captured
- In seismogenic zone, stresses and pressure are poorly captured, velocity somewhat better



a) **b)**

Figure: Analysis at a) the borehole b) the middle of the seismogenic zone. Light background is a collection of 300 particles' paths.

Preliminary conclusions

- Use of particle filter for data assimilation in STM requires further investigation.
- Sampling an evolution of an STM model at lagged time intervals is an effective way to generate an ensemble for particle filtering.
- The assimilation of noisy observations into a perfect model suggests that the particle filter is able to reconstruct the state space of the STM model.
- Strong correlation in variables likely limits solution space.
- Further refinement of the ensemble generation approach should lead to a better ensemble coverage of the state space.

Recommendations/ongoing work

- Explore methodology
  - Use time-lag approach in combination with additional perturbances (e.g. perturb location of Gauss-point markers) for ensemble generation
  - Increase filter efficiency: investigate use of proposal density function in particle filter
  - Investigate parameter updates with particle filter
- Different model setup
  - Different laboratory experiments
  - Induced seismicity due to gas extraction

Interested in this project? We have a number of PhD positions in Delft and Utrecht; please email me at f.c.vossepoel@tudelft.nl

Thank you!