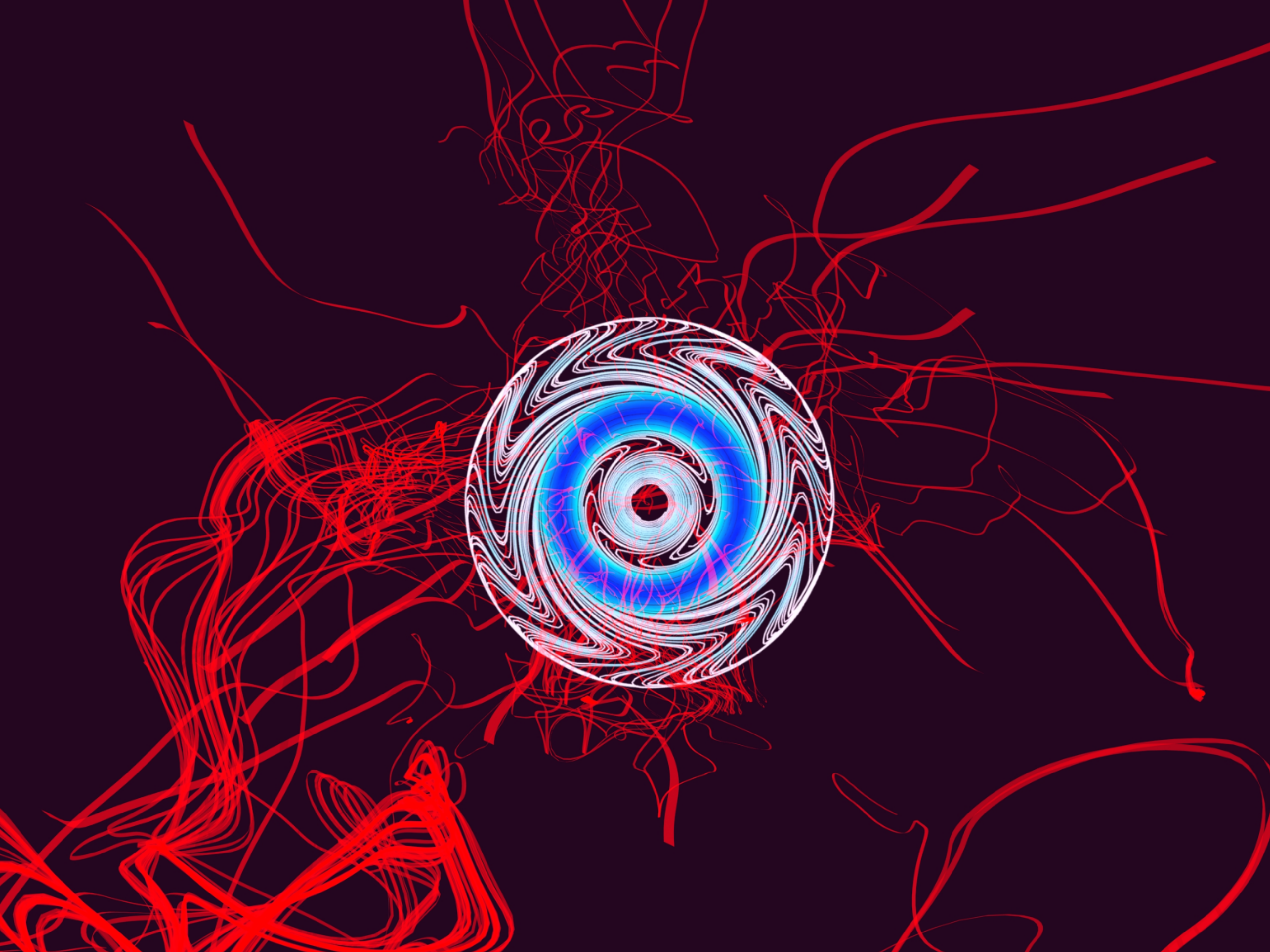
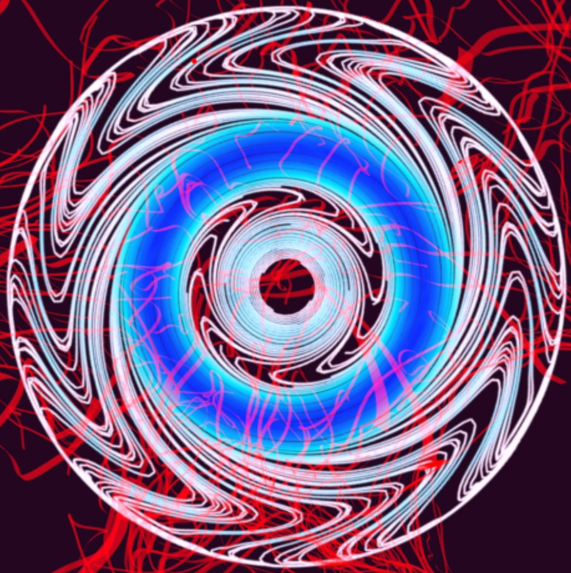


Ecological collapse and the phase transition to turbulence

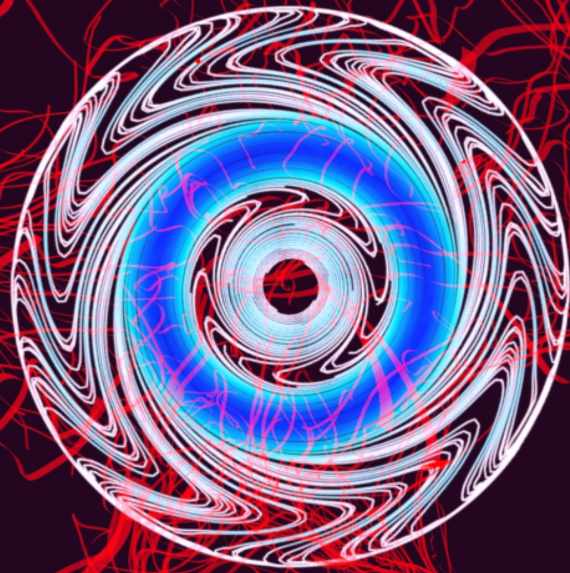


Hong-Yan Shih, Tsung-Lin Hsieh, Nigel Goldenfeld

~~Grudgingly~~ Partially supported by NSF-DMR-1044901

Nature Physics, March 2016: Advance Online Publication 15 Nov 2015

Statistical mechanics of the phase transition to turbulence: zonal flows, ecological collapse and extreme value statistics



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Nature Physics, March 2016: Advance Online Publication 15 Nov 2015

Physics Vol. 2, No. 6, pp. 263-272, 1966. Physics Publishing Co. Printed in Great Britain.

SCALING LAWS FOR ISING MODELS NEAR T_c*

LEO P. KADANOFF†

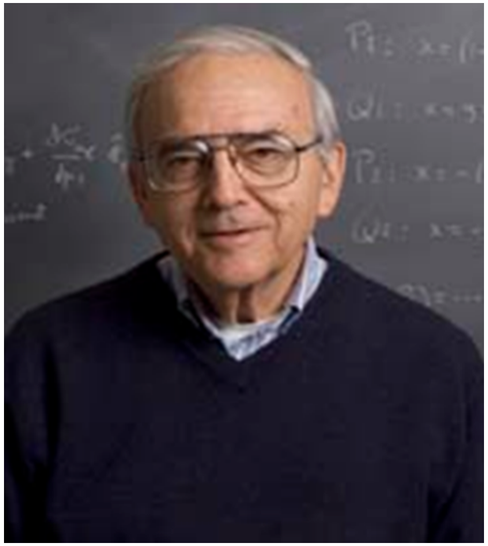
Department of Physics, University of Illinois
Urbana, Illinois

(*Received* 3 *February* 1966)

Abstract

A model for describing the behavior of Ising models very near T_c is introduced. The description is based upon dividing the Ising model into cells which are microscopically large but much smaller than the coherence length and then using the total magnetization within each cell as a collective variable. The resulting calculation serves as a partial justification for Widom's conjecture about the homogeneity of the free energy and at the same time gives his result $s\nu' = \gamma' + 2\beta$.

How was critical phenomena solved?



Ben Widom discovered "data collapse" (1963)

Leo Kadanoff explained data collapse, with scaling concepts (1966)

Ken Wilson developed the RG based on Kadanoff's scaling ideas (1970)

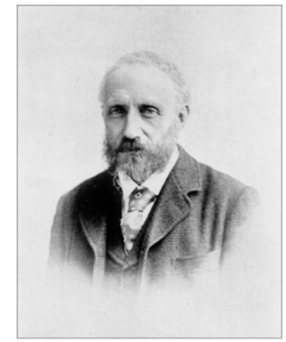
- Common features
  - Strong fluctuations
  - Power law correlations

- Can we solve turbulence by following critical phenomena?
- **Does turbulence exhibit critical phenomena at its onset?**


“EXPLORING
IS BORING”
-NO FURRY
THREE-YEAR OLD
EVER

Transitional turbulence: puffs

- Reynolds' originally pipe turbulence (1883) reports on the transition



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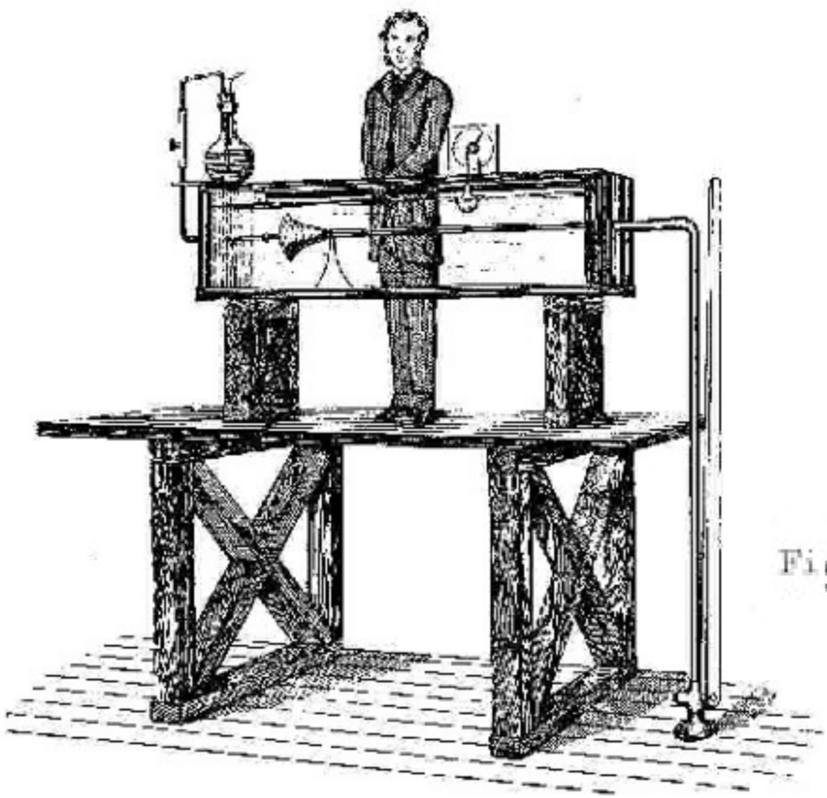


"Flashes" of turbulence:

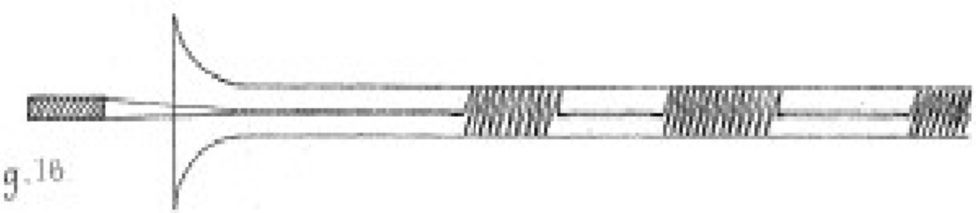
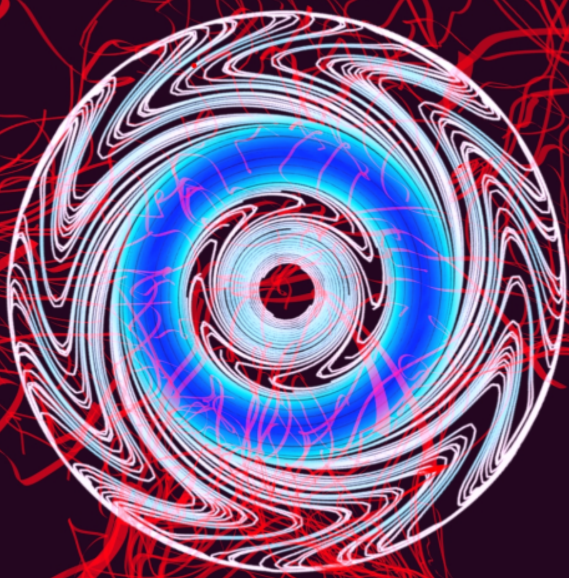


Fig.16

“EXPLORING
IS BORI[illegible]”

-N[illegible]
THREE-YEAR OLD
EVER

Ecological collapse and the phase transition to turbulence



Hong-Yan Shih, Tsung-Lin Hsieh, Nigel Goldenfeld

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Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven *Prize Problems*. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each. During the Millennium Meeting held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled *The Importance of Mathematics*, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

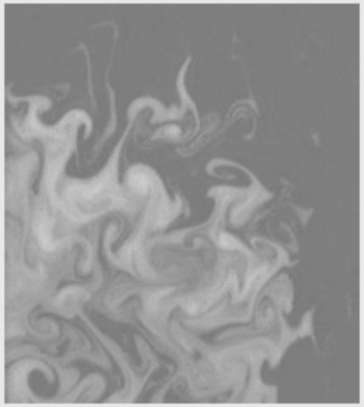
- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

- Rules
- Millennium Meeting Videos

Navier-Stokes Equation

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations.

- The Millennium Problems
- Official Problem Description — Charles Fefferman
- Lecture by Luis Cafarelli (video)



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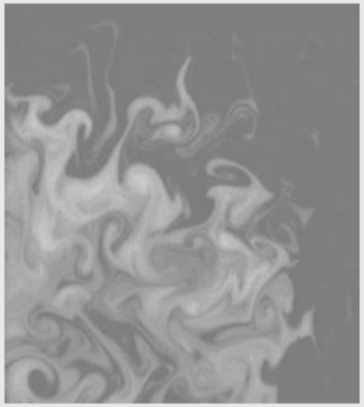

Navier-Stokes Equation

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A fundamental problem in analysis is to decide whether such smooth, physically reasonable solutions exist for the Navier–Stokes equations. To give reasonable leeway to solvers while retaining the heart of the problem, we ask for a proof of one of the following four statements.

(A) Existence and smoothness of Navier–Stokes solutions on $\mathbb{R}^3$. Take $\nu > 0$ and $n = 3$. Let $u^\circ(x)$ be any smooth, divergence-free vector field satisfying (4). Take $f(x,t)$ to be identically zero. Then there exist smooth functions $p(x,t), u_i(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$ that satisfy (1), (2), (3), (6), (7).

(B) Existence and smoothness of Navier–Stokes solutions in $\mathbb{R}^3/\mathbb{Z}^3$. Take $\nu > 0$ and $n = 3$. Let $u^\circ(x)$ be any smooth, divergence-free vector field satisfying (8); we take $f(x,t)$ to be identically zero. Then there exist smooth functions $p(x,t)$, $u_i(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$ that satisfy (1), (2), (3), (10), (11).

(C) Breakdown of Navier–Stokes solutions on $\mathbb{R}^3$. Take $\nu > 0$ and $n = 3$. Then there exist a smooth, divergence-free vector field $u^\circ(x)$ on $\mathbb{R}^3$ and a smooth $f(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$, satisfying (4), (5), for which there exist no solutions (p,u) of (1), (2), (3), (6), (7) on $\mathbb{R}^3 \times [0,\infty)$.

(D) Breakdown of Navier–Stokes Solutions on $\mathbb{R}^3/\mathbb{Z}^3$. Take $\nu > 0$ and $n = 3$. Then there exist a smooth, divergence-free vector field $u^\circ(x)$ on $\mathbb{R}^3$ and a smooth $f(x,t)$ on $\mathbb{R}^3 \times [0,\infty)$, satisfying (8), (9), for which there exist no solutions (p,u) of (1), (2), (3), (10), (11) on $\mathbb{R}^3 \times [0,\infty)$.

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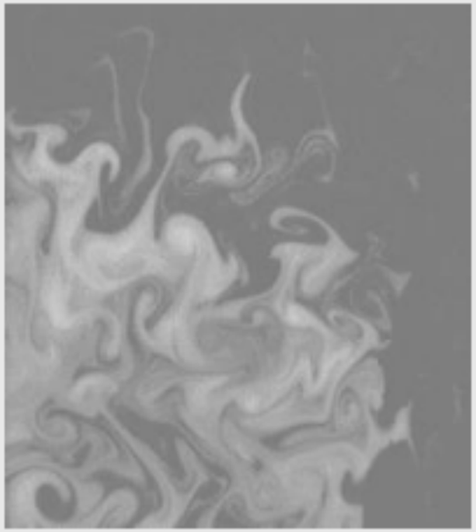
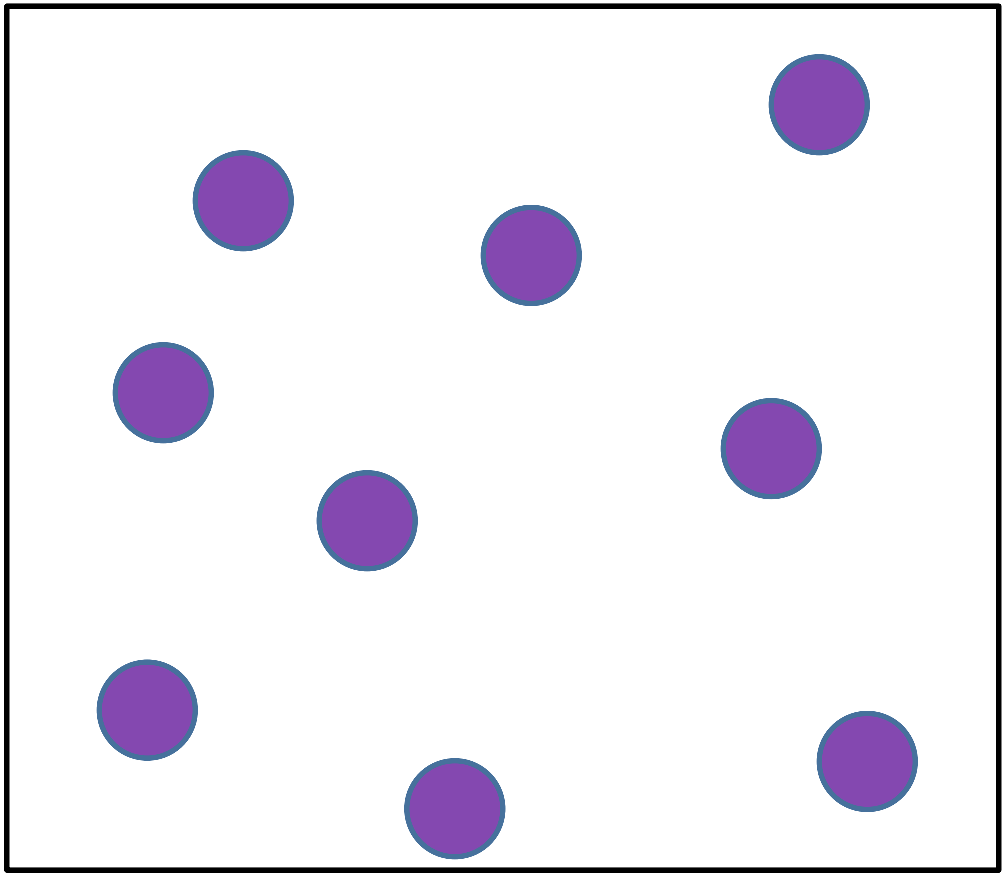
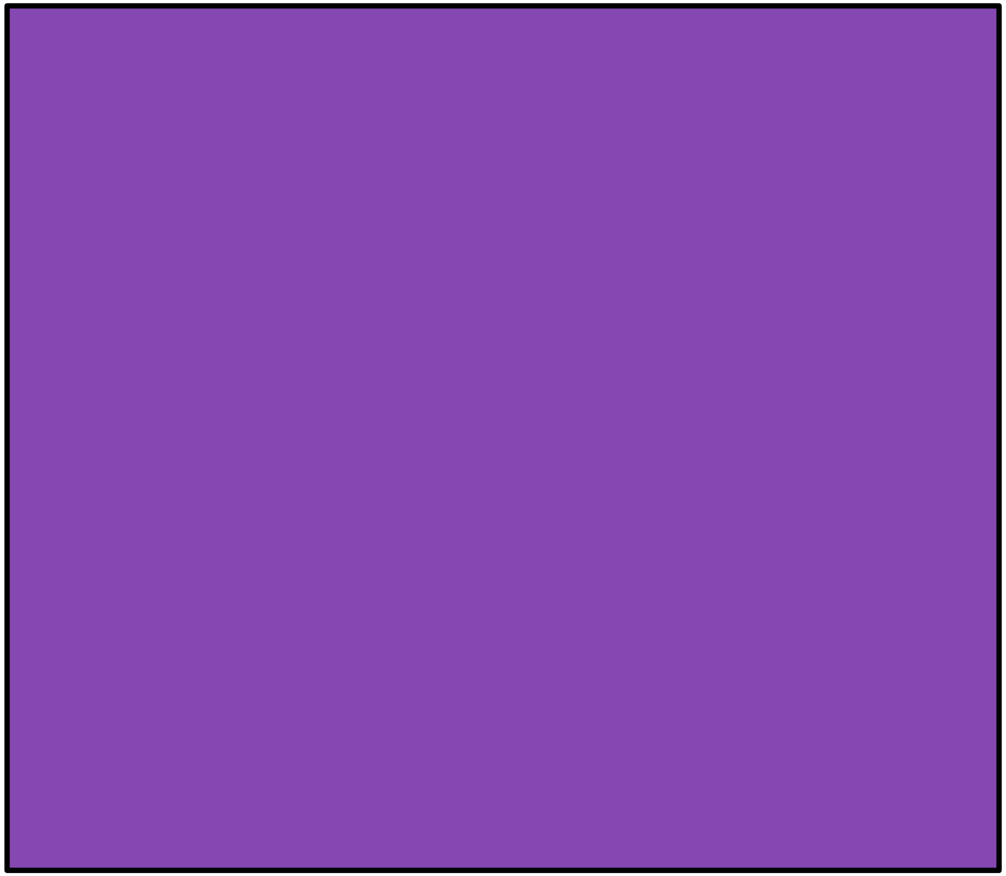




Deterministic classical mechanics of many particles in a box → statistical mechanics



Deterministic classical mechanics of infinite number of particles in a box

= Navier-Stokes equations for a fluid

➔ statistical mechanics

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \mu \nabla^2 \mathbf{u}$$

Deterministic classical mechanics of infinite number of particles in a box

= Navier-Stokes equations for a fluid

➔ statistical mechanics

Turbulence is stochastic and wildly fluctuating

Soap film experiment

M. A. Rutgers, X-l. Wu, and W. I. Goldburg.
"The Onset of 2-D Grid Generated Turbulence in Flowing Soap Films,"
Phys. Fluids 8, S7, (Sep. 1996).

Turbulence generates structure at many scales

Soap film experiment

M. A. Rutgers, X-l. Wu, and W. I. Goldburg.
"The Onset of 2-D Grid Generated Turbulence in Flowing Soap Films,"
Phys. Fluids 8, S7, (Sep. 1996).

Scale invariance in turbulence

- Eddies spin off other eddies in a Hamiltonian process.
  - Does not involve friction!
  - Hypothesis due to Richardson, Kolmogorov, …
- Implication: viscosity will not enter into the equations

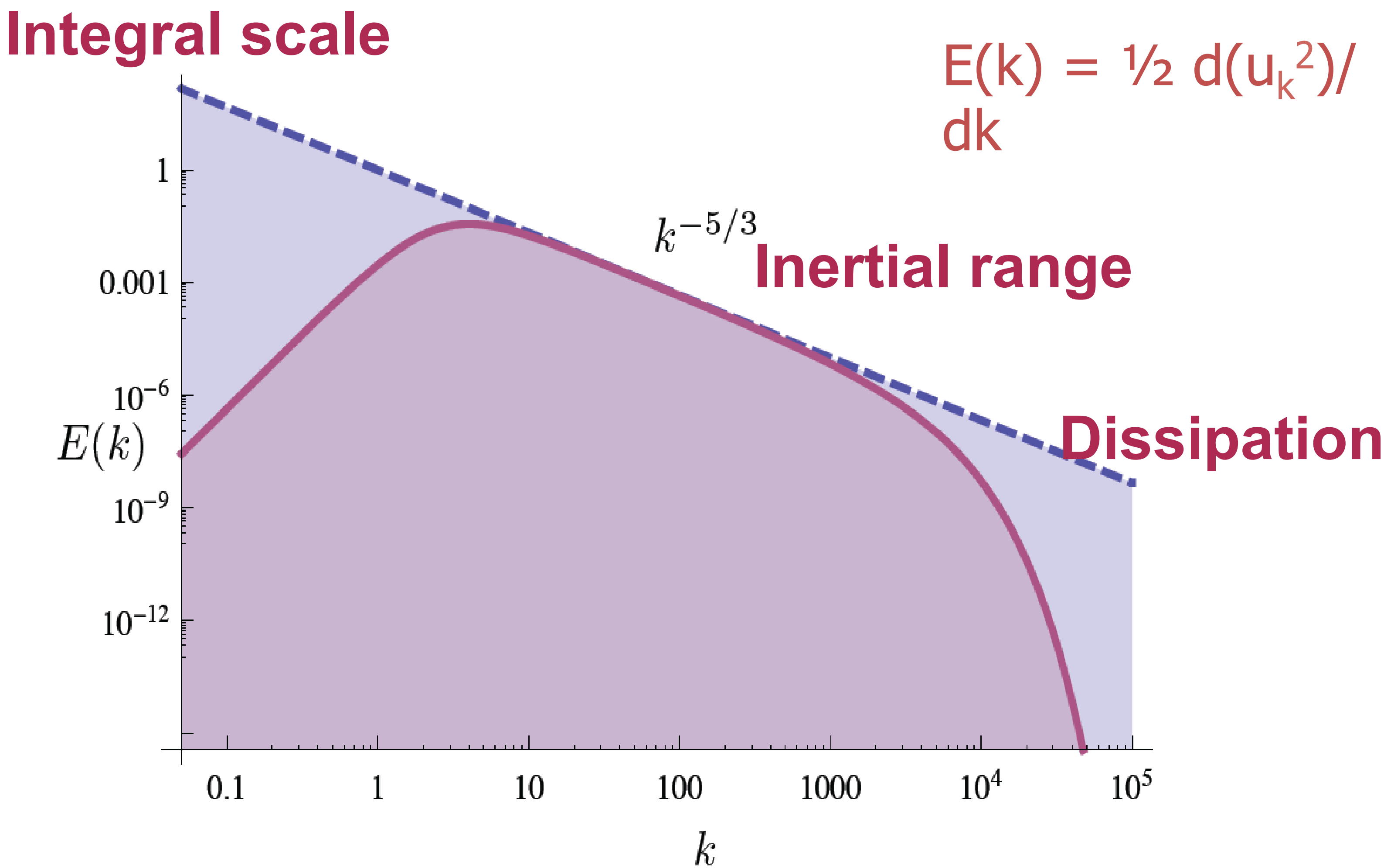
Scale invariance in turbulence

A.N. Kolmogorov

- Compute $E(k)$, turbulent kinetic energy in wave number range k to k+dk
  - $E(k)$ depends on k
  - $E(k)$ will depend on the rate at which energy is transferred between scales: ε
- Dimensional analysis:
  - $E(k) \sim \varepsilon^{2/3} k^{-5/3}$

The energy spectrum

$E(k) = ½ \, d(u_k^2)/ dk$



Turbulent cascades

3D forward cascade

2D inverse cascade

Energy flows to small scales

Energy flows to large scales

Turbulent cascades

3D forward cascade

2D inverse cascade

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Turbulent cascades

2D forward cascade

2D inverse cascade

Vorticity flows to small scales

Energy flows to large scales

Turbulent cascades

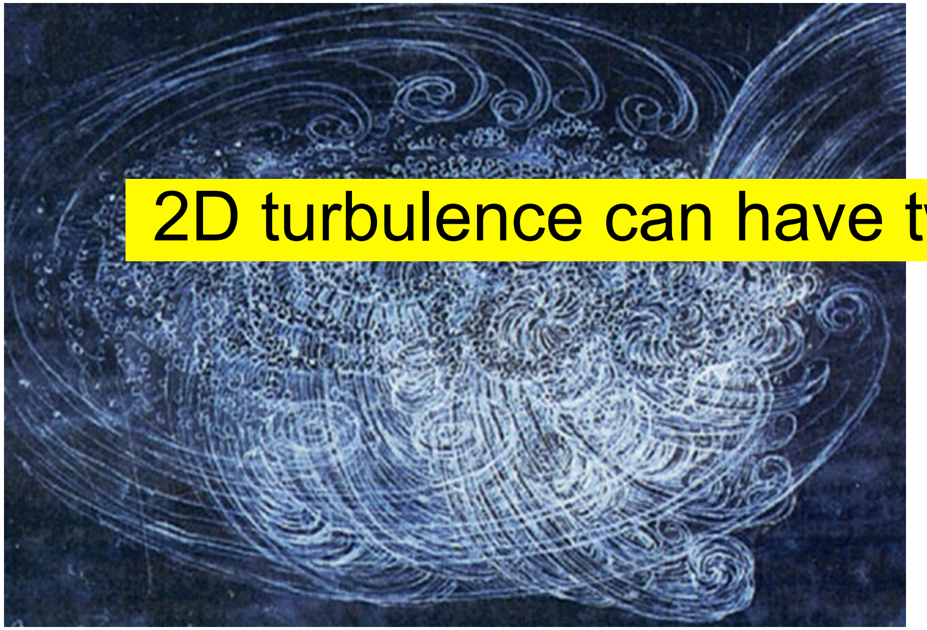
2D forward cascade

$$E(k) \propto \lambda^{2/3} k^{-3}$$

2D inverse cascade

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Turbulent cascades



2D turbulence can have two separate cascades

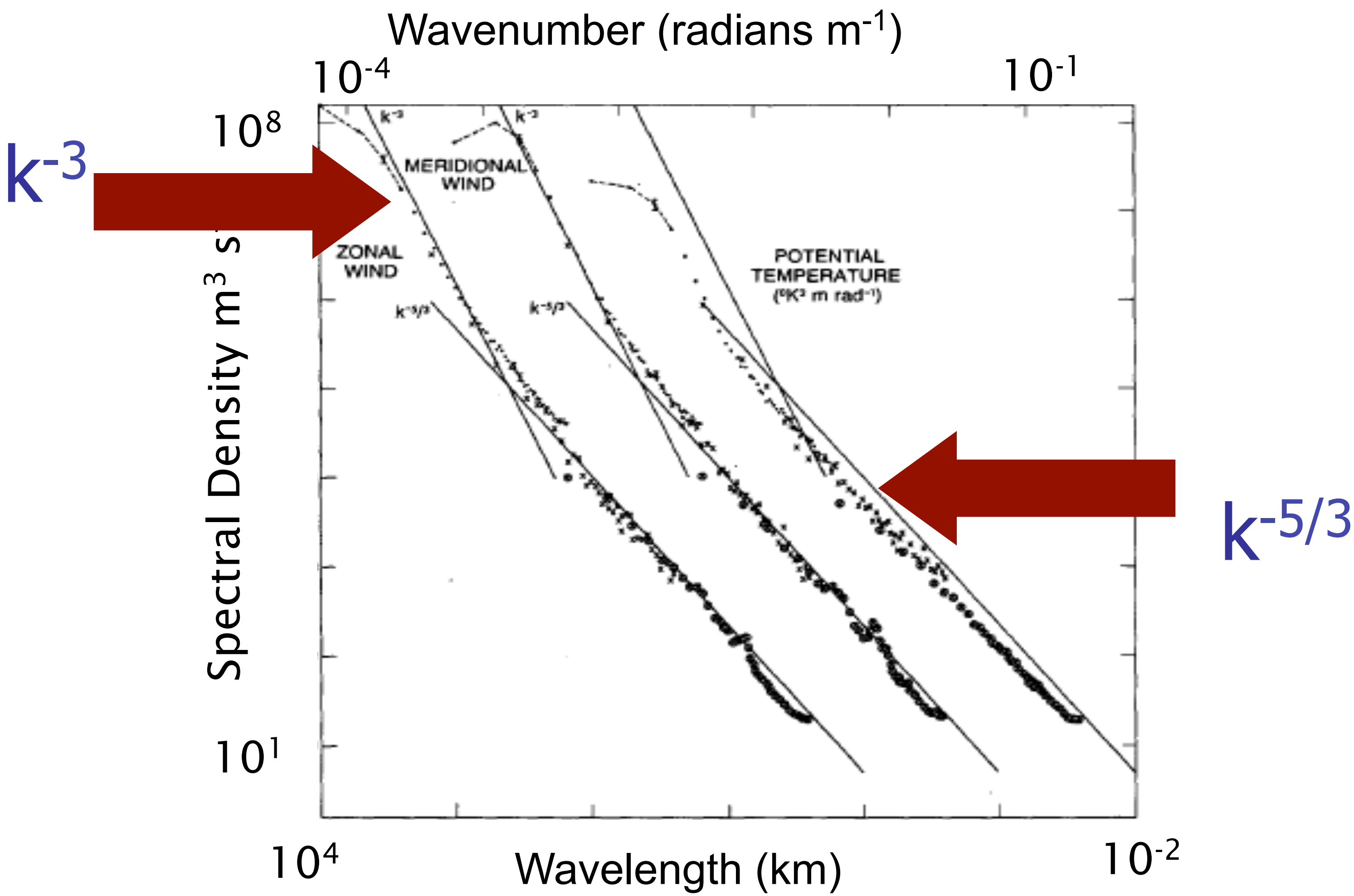
2D forward cascade

$$E(k) \propto \lambda^{2/3} k^{-3}$$

2D inverse cascade

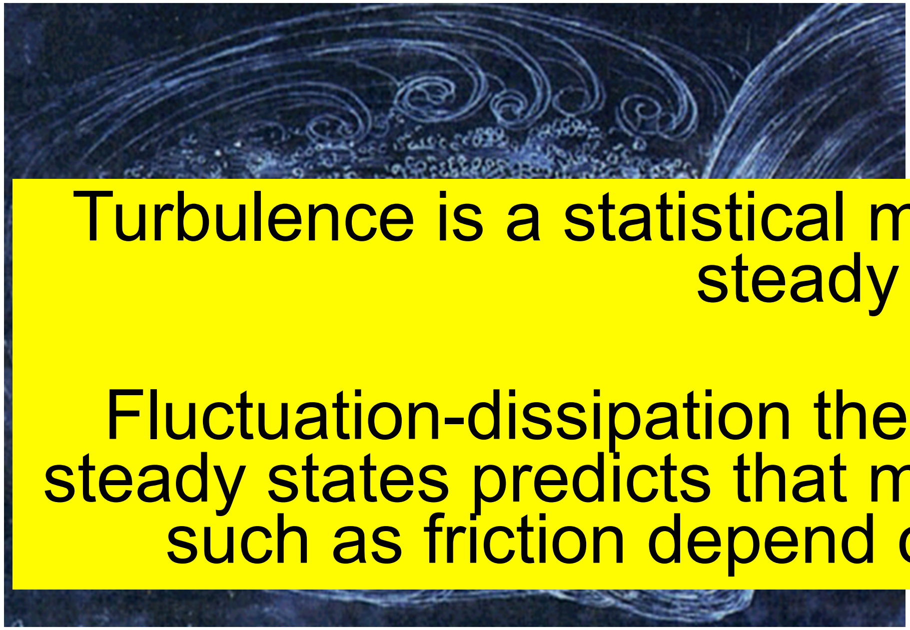
$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

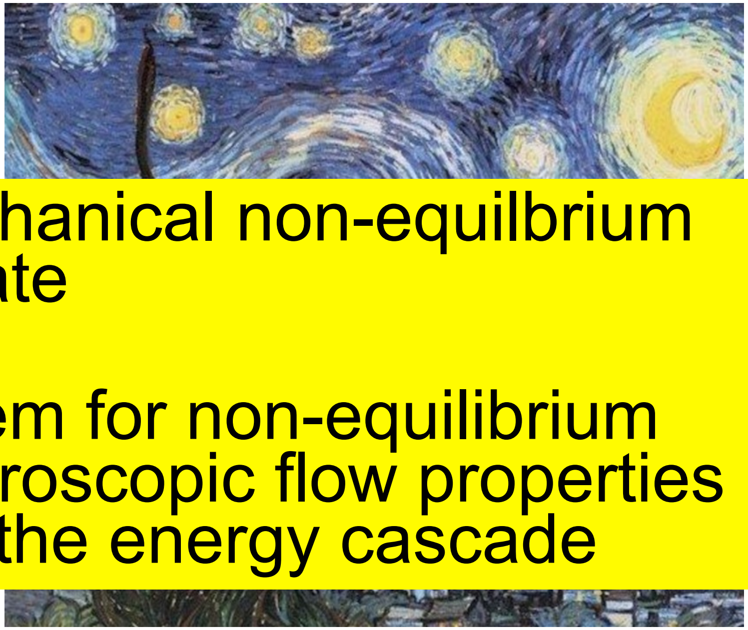
Atmospheric turbulence



G. D. Nastrom and K. S. Gage, “A Climatology of Atmospheric Wavenumber Spectra of Wind and Temperature Observed by Commercial Aircraft”, Jour. Atmos. Sci. vol 42, 1985 p953

Turbulent cascades





Turbulence is a statistical mechanical non-equilbrium steady state

Fluctuation-dissipation theorem for non-equilibrium steady states predicts that macroscopic flow properties such as friction depend on the energy cascade

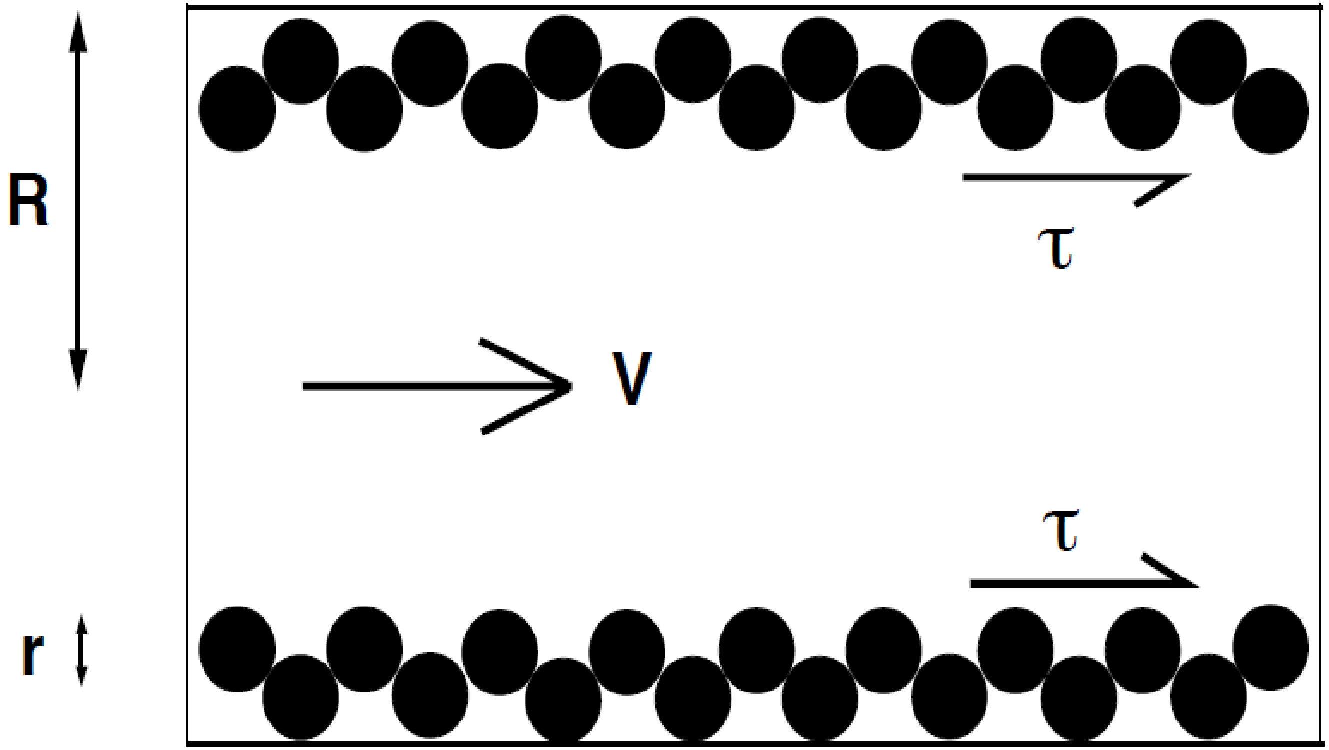
2D forward cascade

$$E(k) \propto \lambda^{2/3} k^{-3}$$

2D inverse cascade

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

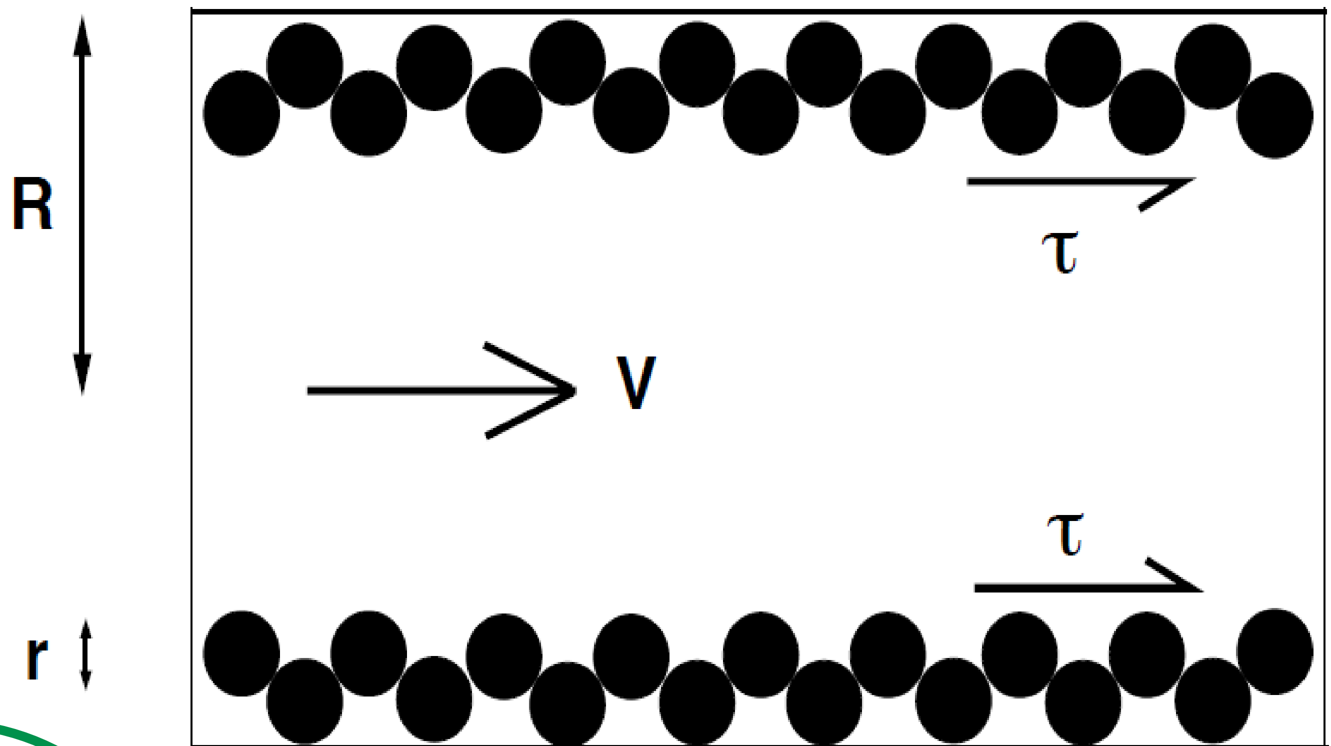
Pipe flow



$$\mathrm{Re} \equiv \frac{\mathrm{VR}}{\nu} \qquad \mathrm{Roughness} \equiv \frac{\mathrm{r}}{\mathrm{R}} \qquad \mathrm{Friction\ Factor, f} \equiv \frac{\tau}{\rho \mathrm{V}^2}$$

Re = Reynolds number **ν = viscosity/density = kinematic viscosity**

Pipe flow



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Re = Reynolds number ν = viscosity/density = kinematic viscosity

Critical behaviour in fully-developed turbulence?

Re ~ infinity

Q. Is there universal scaling behaviour in fully-developed turbulence?

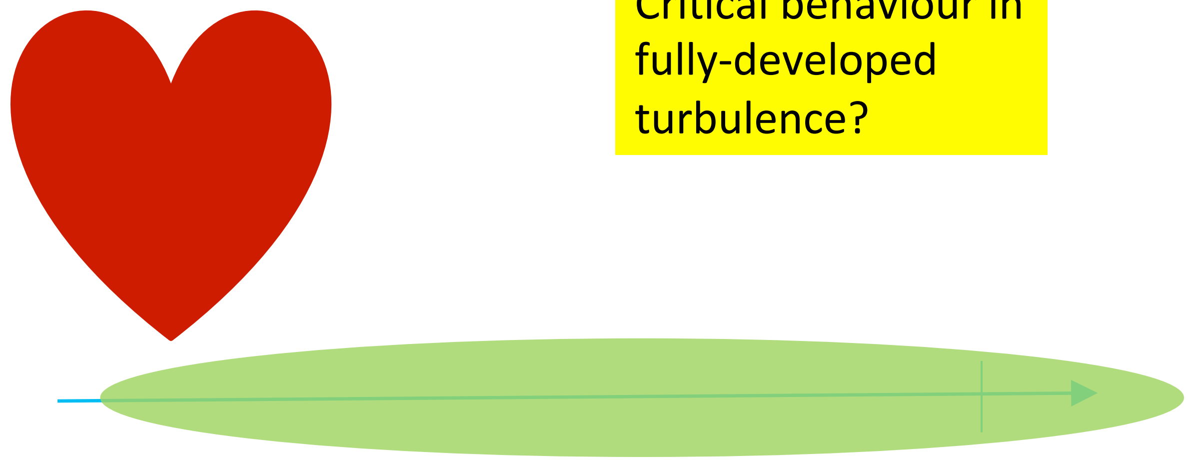
Critical behaviour in fully-developed turbulence?

Q. Is there universal scaling behaviour in fully-developed turbulence?

A. Yes! And regime of influence extends to finite Re and dominates the macroscopic flow behaviour

Critical behaviour in fully-developed turbulence?

You are here



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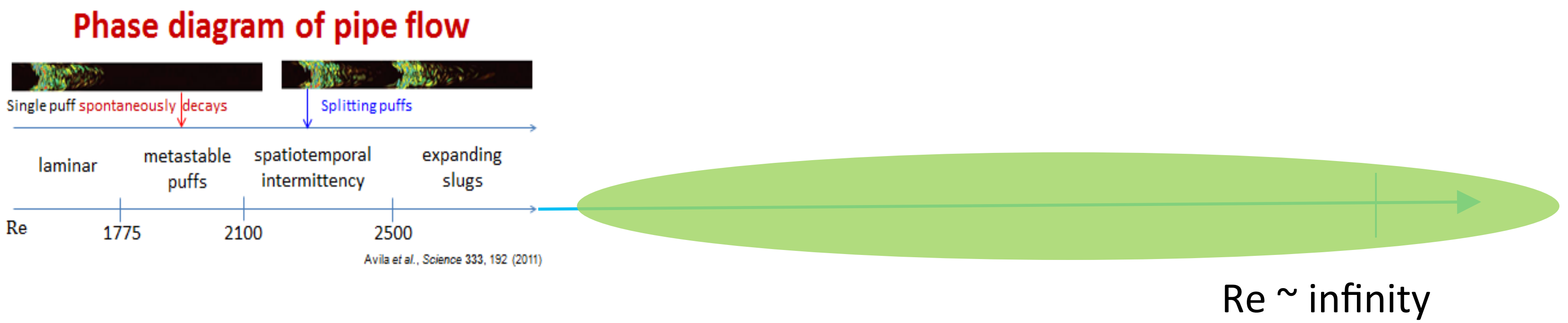
Critical behaviour at laminar-turbulence transition

Critical behaviour in fully-developed turbulence?



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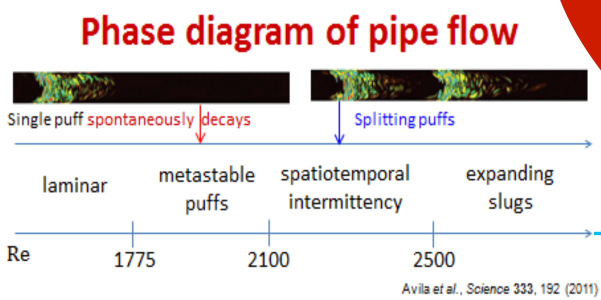

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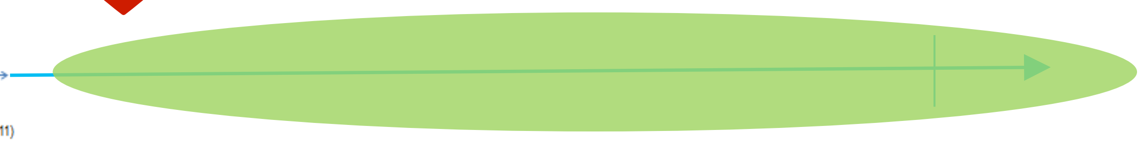

Re ~ infinity

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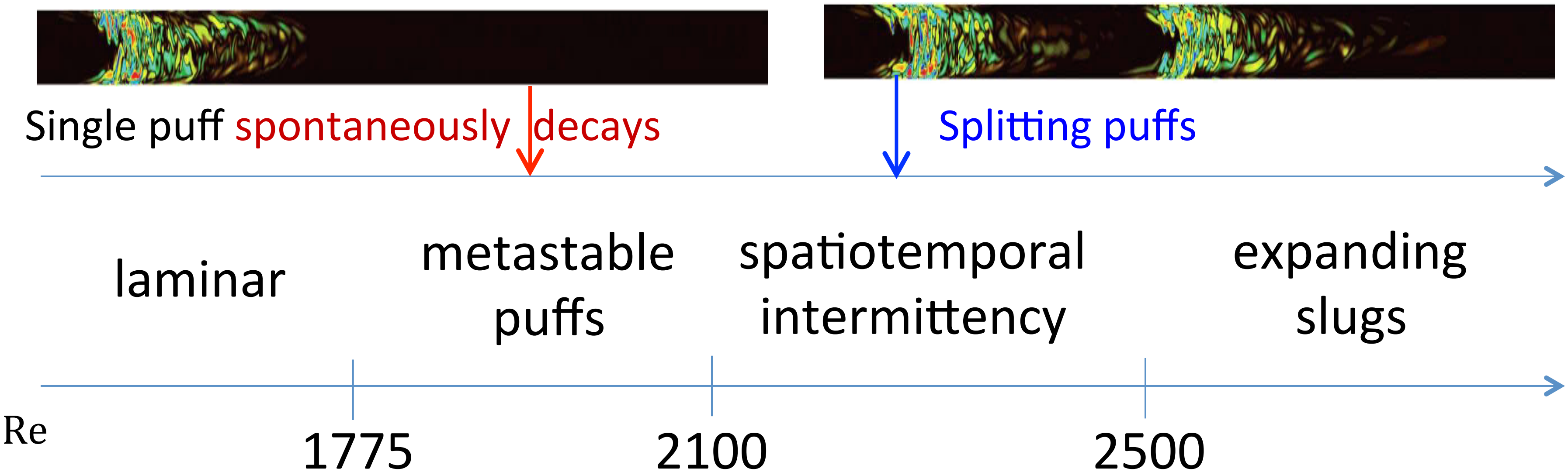
Phase diagram of pipe flow



Avila *et al.*, *Science* **333**, 192 (2011)

Critical behaviour at laminar-turbulence transition

Q. What is the universality class of the transition to turbulence?

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A. Transitional turbulence is controlled by
predator-prey interactions
implying rigorously the
universality class of
directed percolation

Transitional turbulence: puffs

- Reynolds' originally pipe turbulence (1883) reports on the transition

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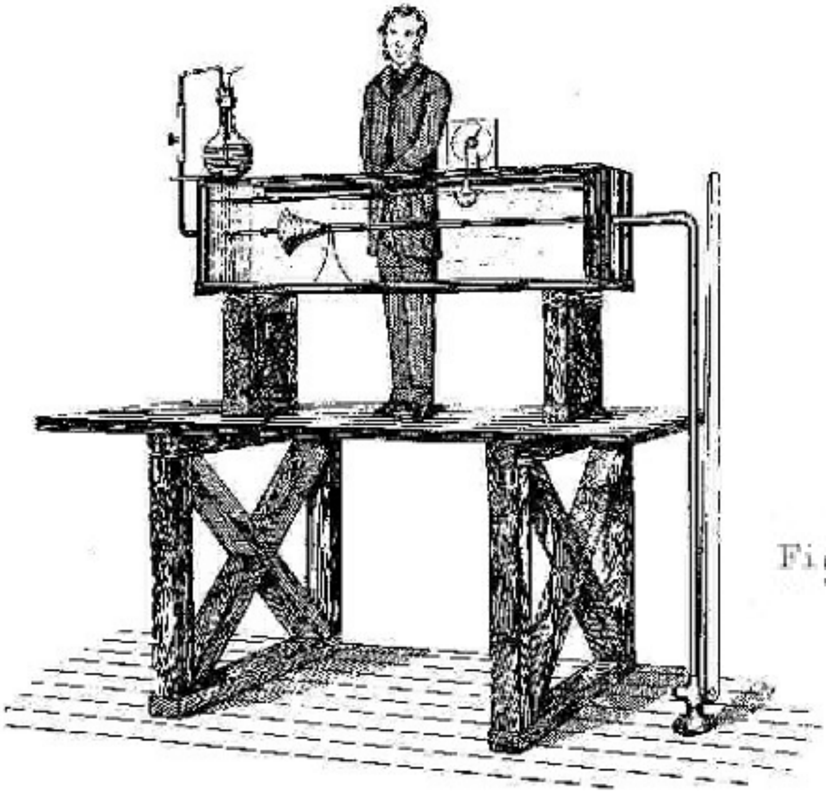


"Flashes" of turbulence:

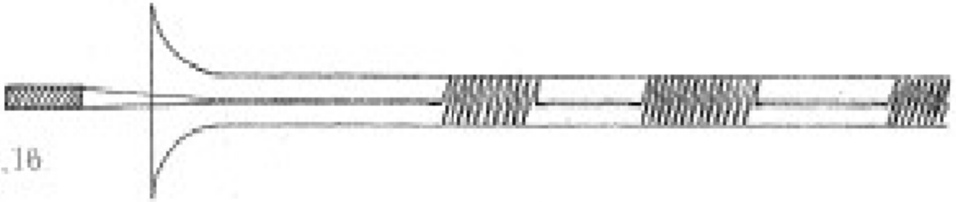
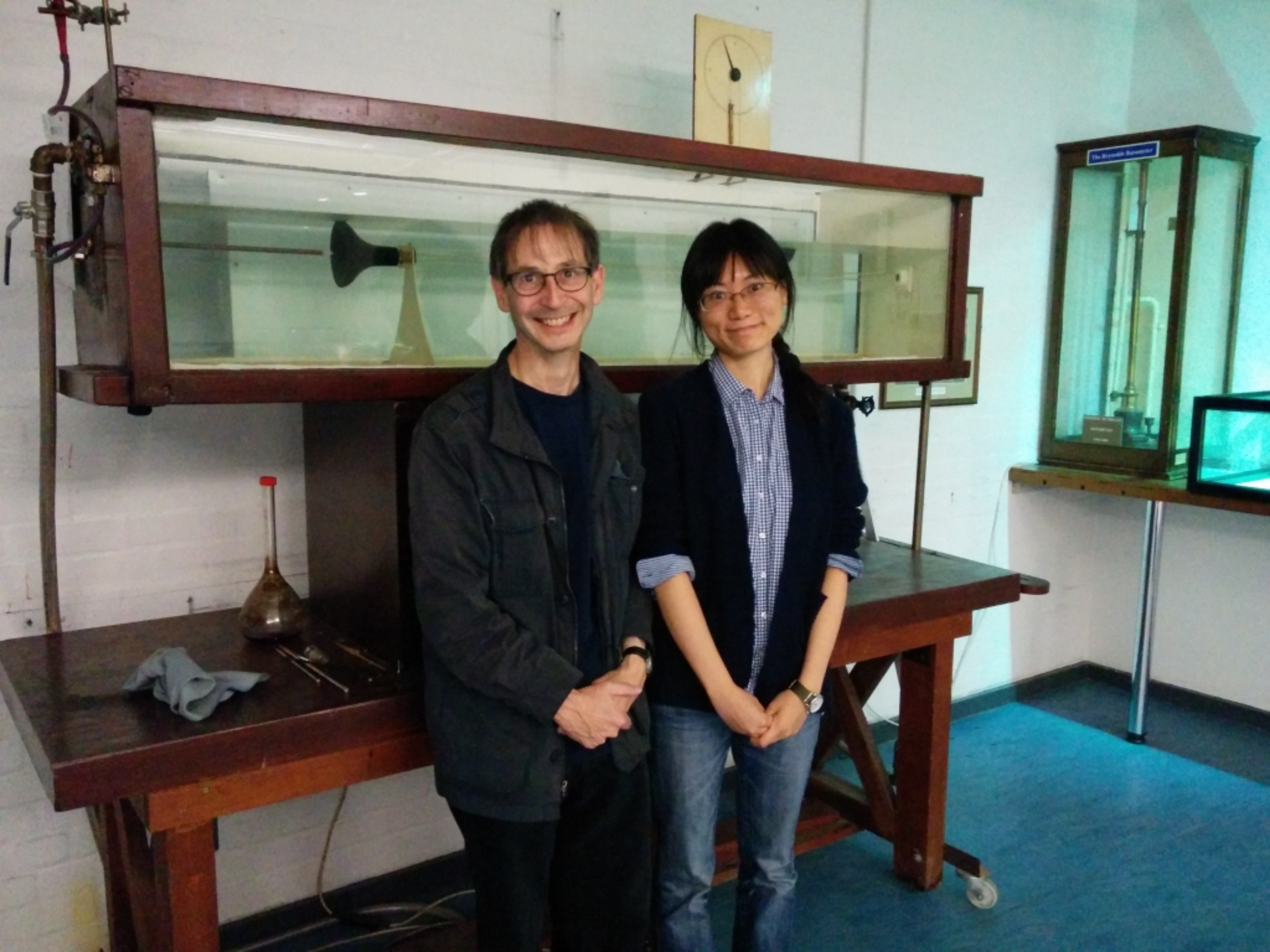
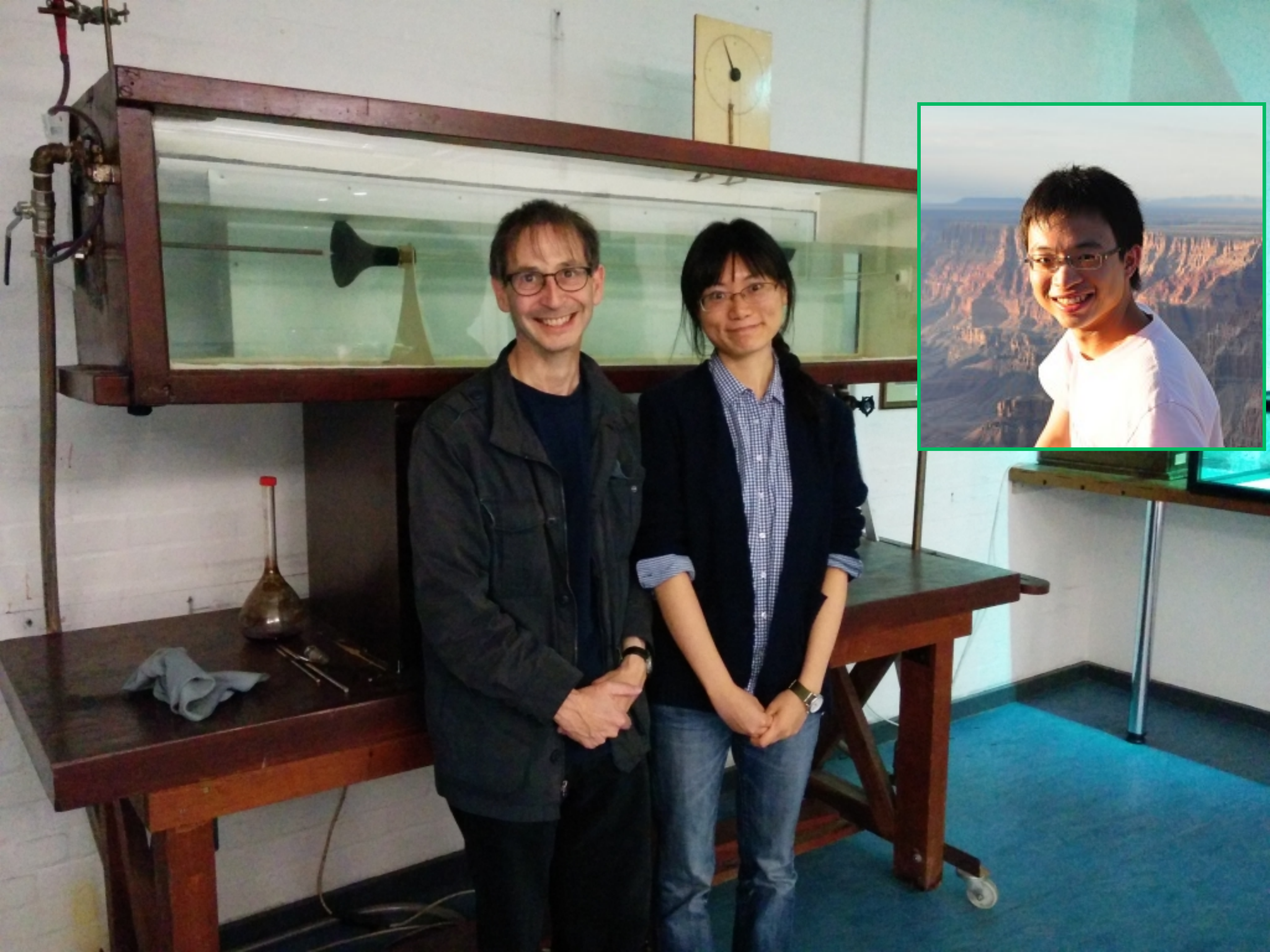


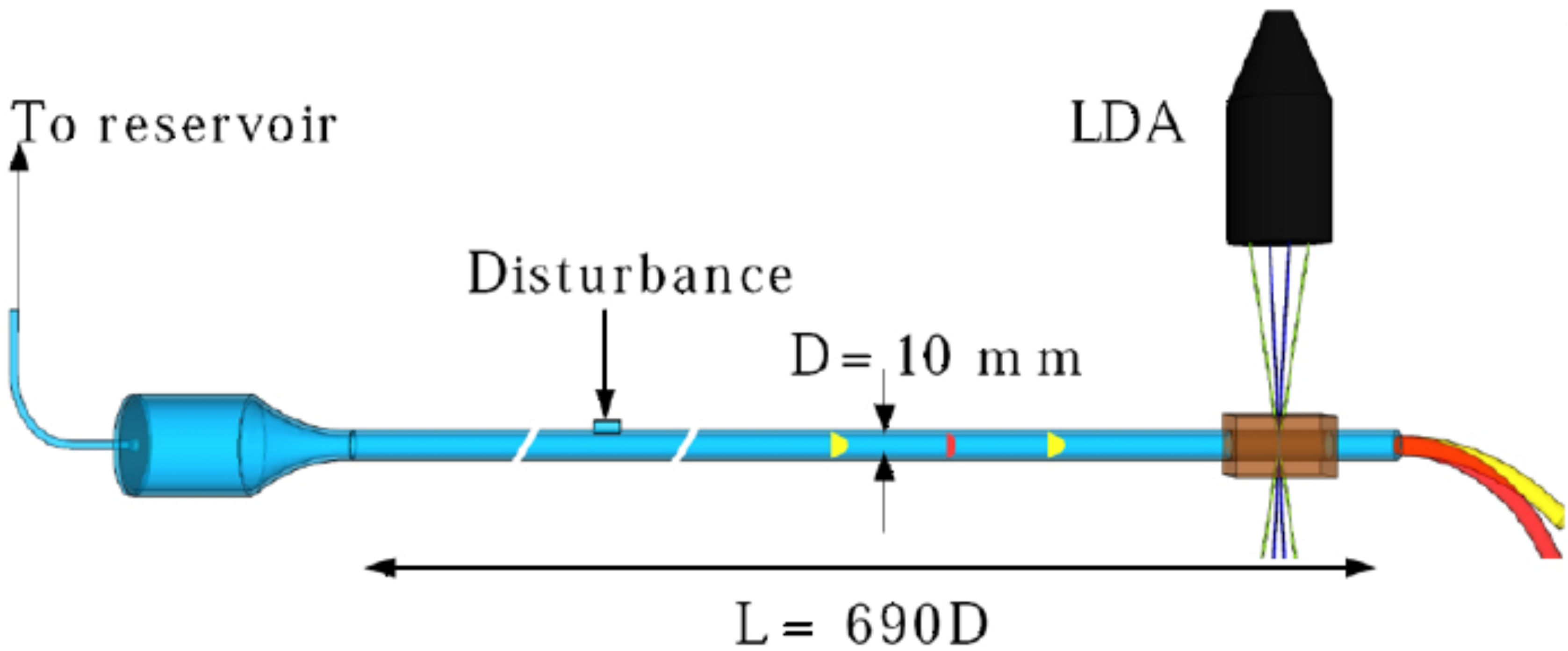
Fig. 18





Precision measurement of turbulent transition

Q: will a puff survive to the end of the pipe?



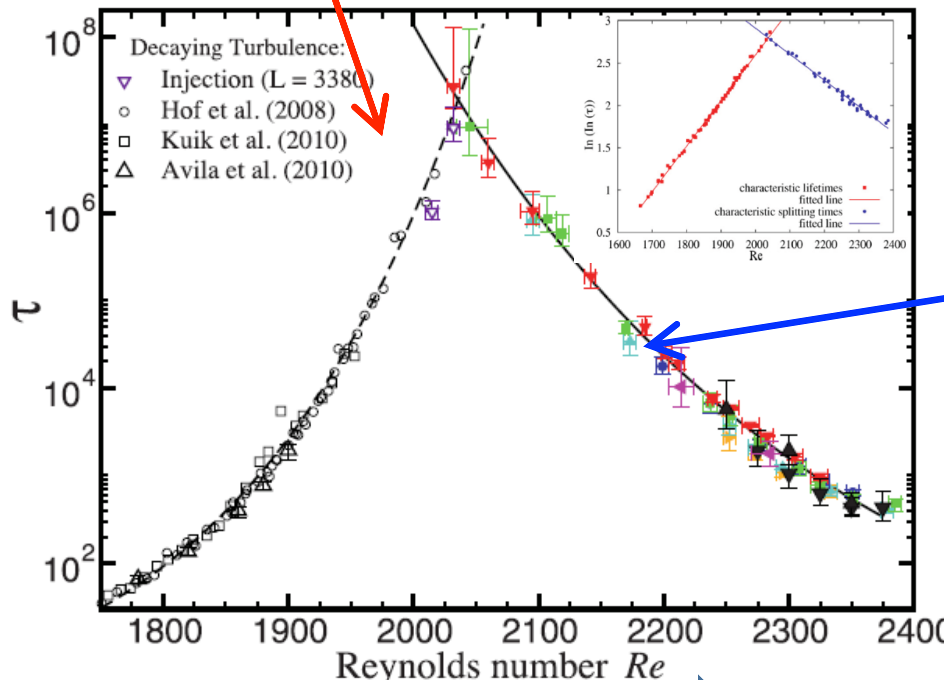
Many repetitions ➔ survival probability P(Re, t)

Hof et al., *PRL* **101**, 214501 (2008)

Fluid in a pipe near onset of turbulence

Turbulent puff lifetime

Log (lifetime)



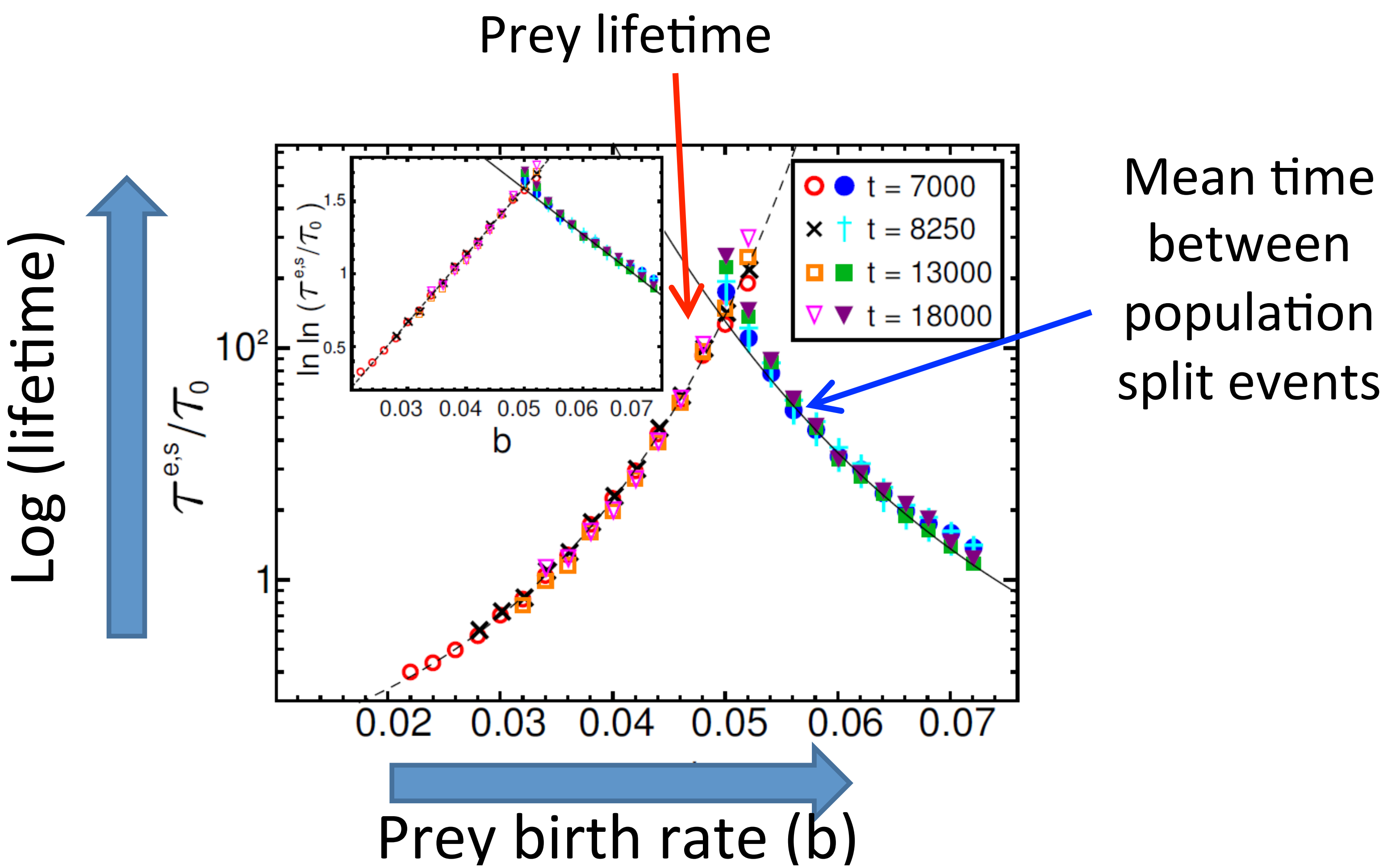
Mean time between puff split events

Reynolds number

Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp Re)$

Avila *et al.*, *Science* **333**, 192 (2011) Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Predator-prey ecosystem in a pipe near extinction

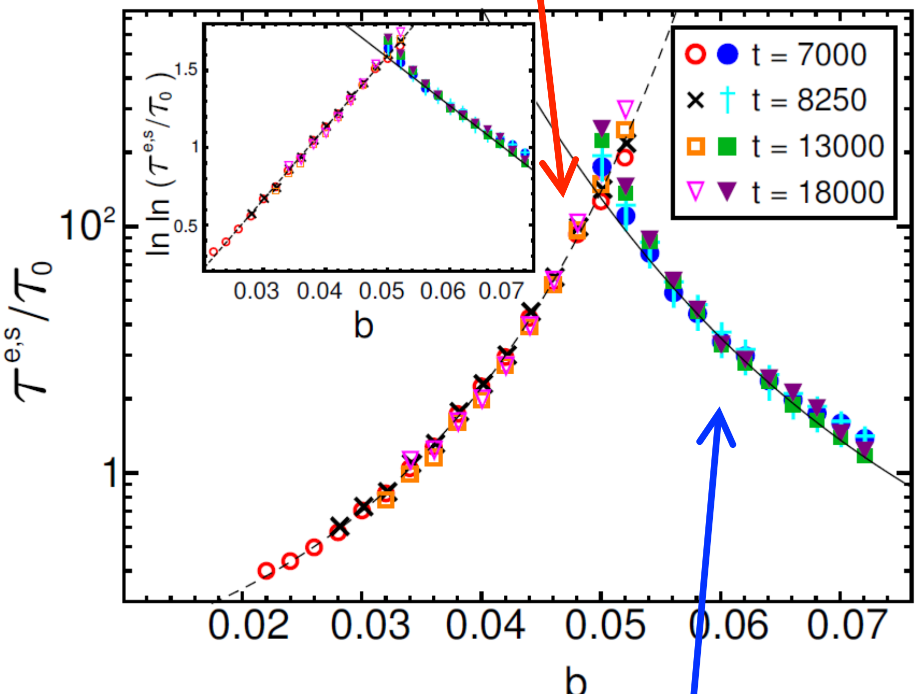


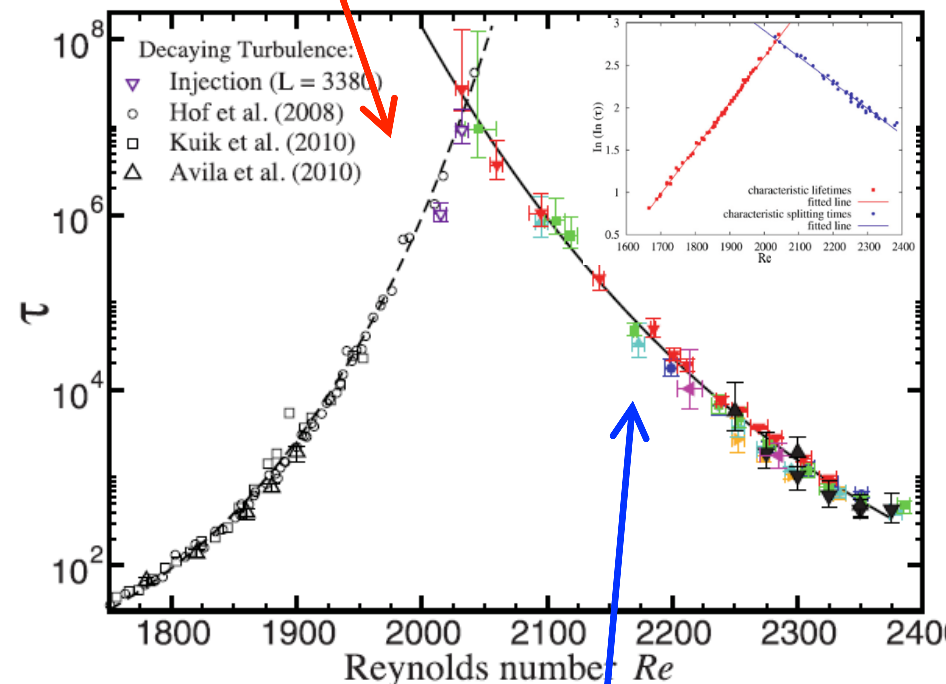
Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp b)$

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

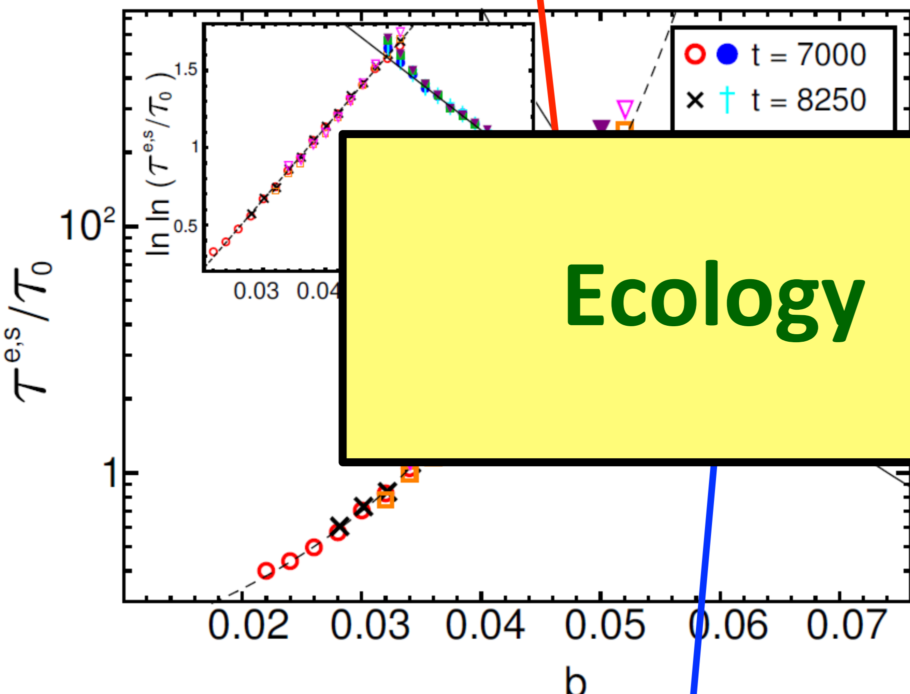
Mean time between puff split events

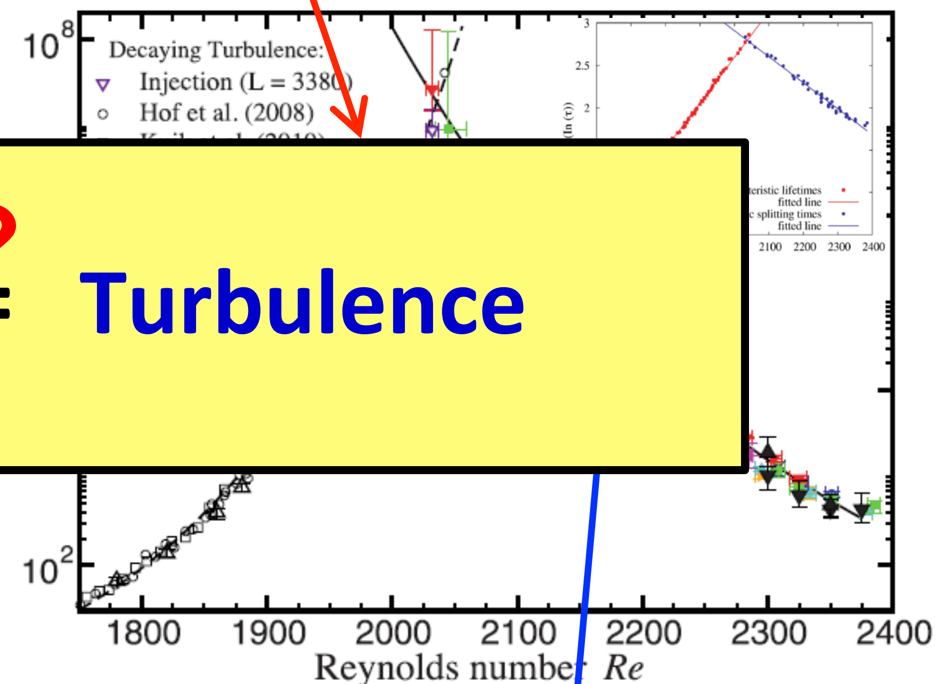
Avila *et al.*, *Science* **333**, 192 (2011)
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Ecology $\overset{?}{=}$ Turbulence

Mean time between population split events

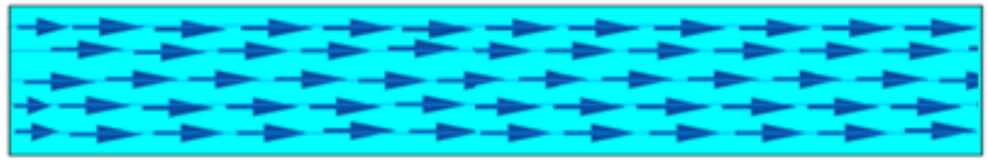
Mean time between puff split events

Avila *et al.*, *Science* **333**, 192 (2011)

Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Flow in a pipe

- Fluid flow can be in 2 regimes:
  - Laminar



  - Turbulent

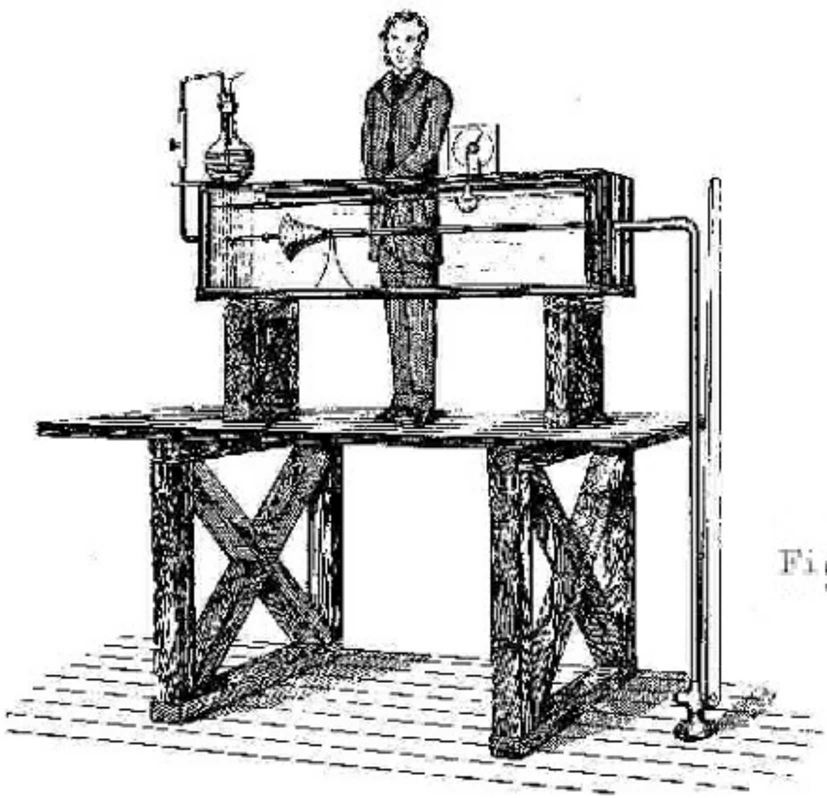
- Phase of the flow is characterized by the dimensionless **Reynolds number**: $\mathrm{Re} = V\rho D/\mu = VD/\nu$

and $V \equiv$ mean velocity, $\rho \equiv$ density, $D \equiv$ pipe diameter,

$\mu \equiv$ dynamic viscosity, $\nu \equiv$ kinematic viscosity

Transitional turbulence: puffs

- Reynolds' originally pipe turbulence (1883) reports on the transition



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"Flashes" of turbulence:

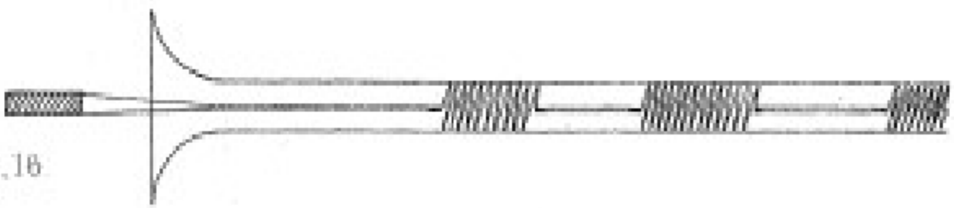
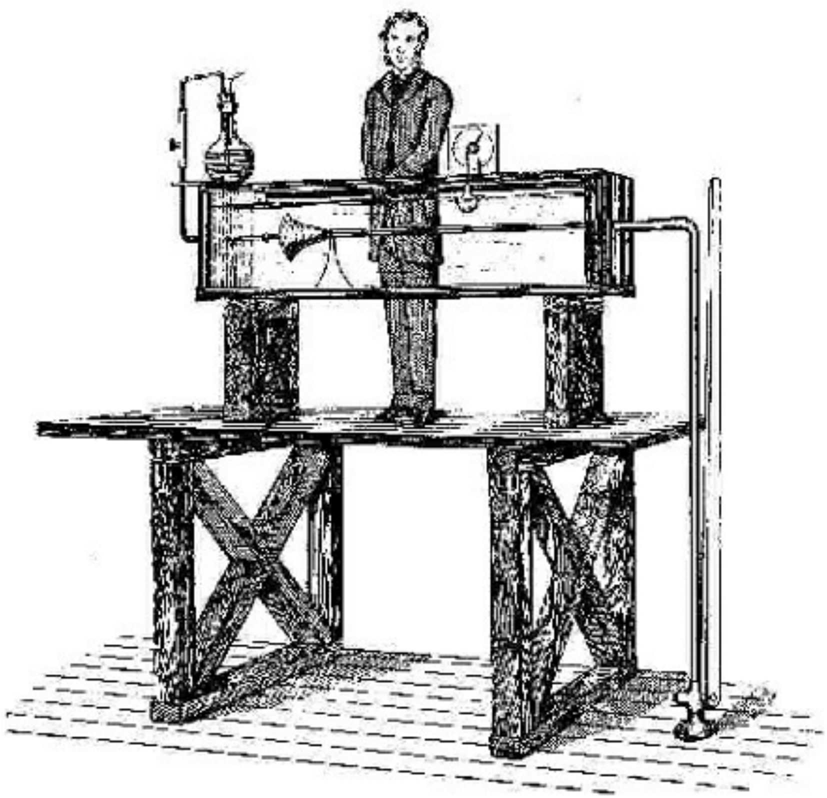
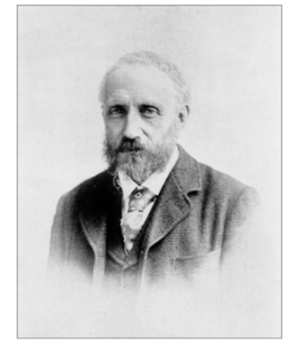


Fig.18

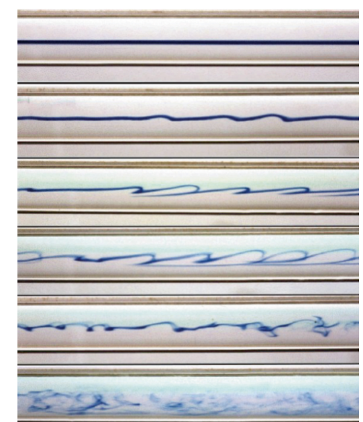
Transitional turbulence: puffs

- Reynolds' originally pipe turbulence (1883) reports on the transition





Univ. of Manchester



http://homepages.warwick.ac.uk/~masax/Research/pipe_models/pipe_models.html#sdfootnote8sym

Laminar-Turbulent Transition

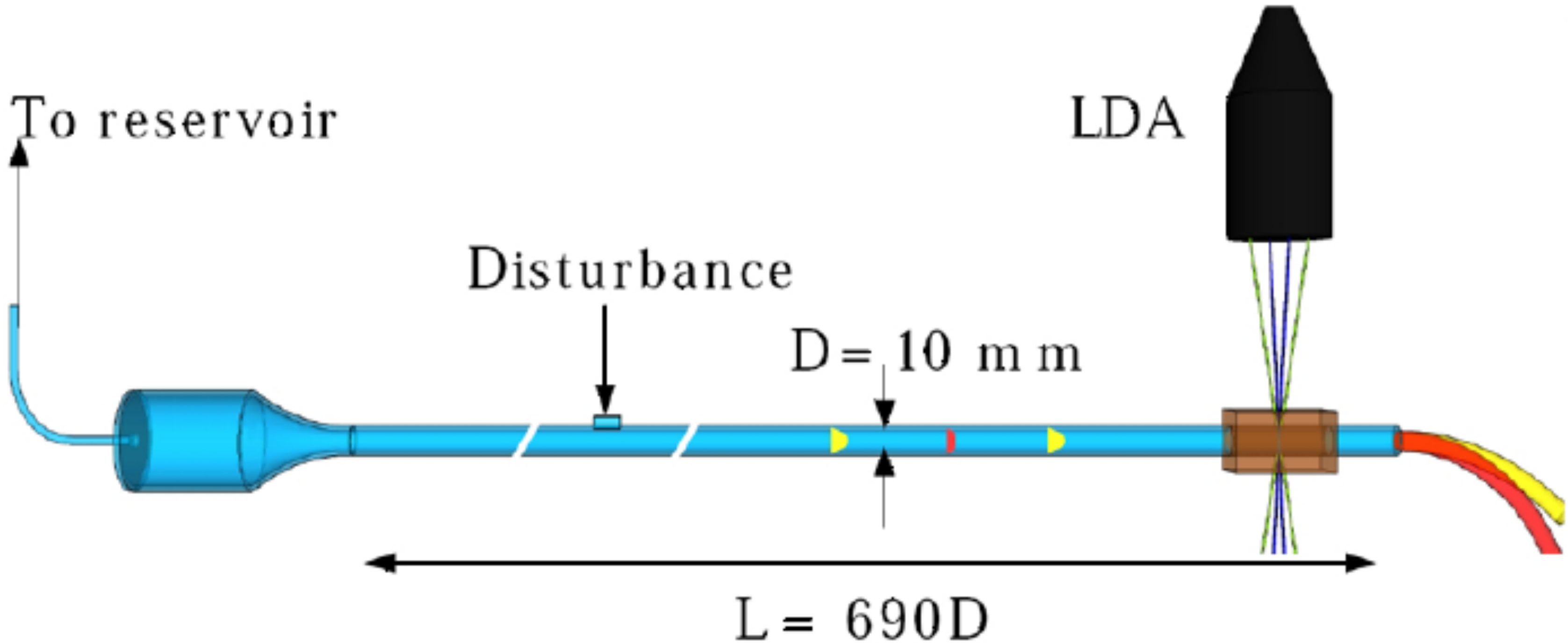
- Laminar state: steady (small *Re*)
- Turbulent state: fluctuating (large *Re*)

Re = Reynolds number = UL/ν

http://homepages.warwick.ac.uk/~masax/Research/pipe_models/pipe_models.html#sdfootnote8sym   Tsinober, An informal conceptual introduction to turbulence (2009)

Precision measurement of turbulent transition

Q: will a puff survive to the end of the pipe?



Many repetitions ➔ survival probability P(Re, t)

Hof et al., *PRL* **101**, 214501 (2008)

Laminar-Turbulent Transition

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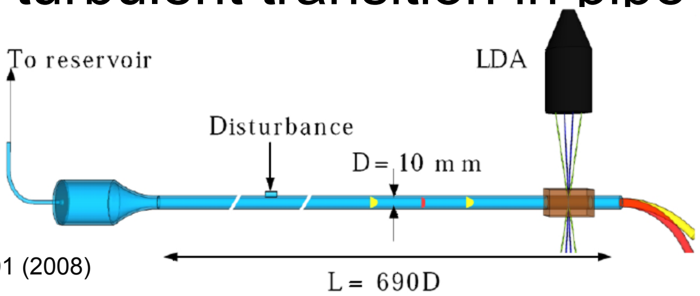

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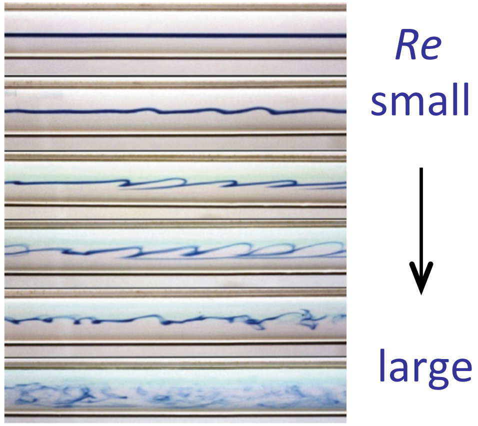

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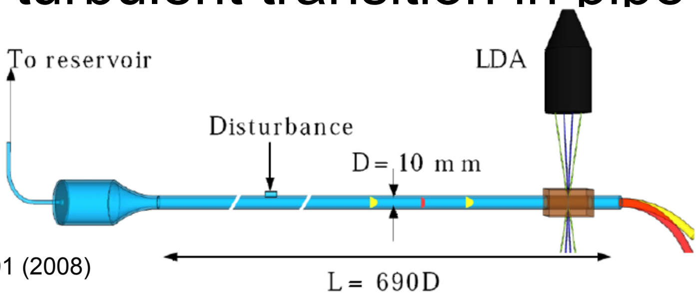

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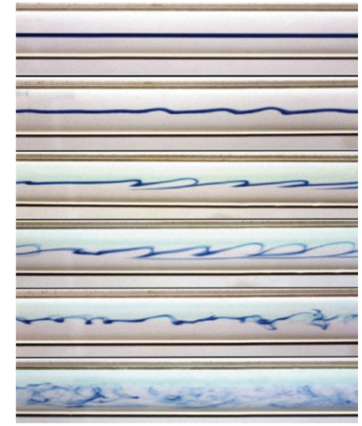


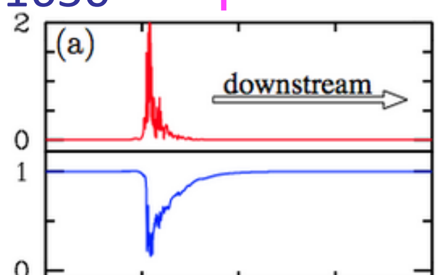
Re small

large

Re — laminar flows — 1650 — metastable puffs — 2050 — 2500

turbulence intensity

mean shear



puff lifetime $\tau \sim \exp(\exp(Re))$

http://homepages.warwick.ac.uk/~masax/Research/pipe_models/pipe_models.html#sdfootnote8sym Tsinober, An informal conceptual introduction to turbulence (2009)

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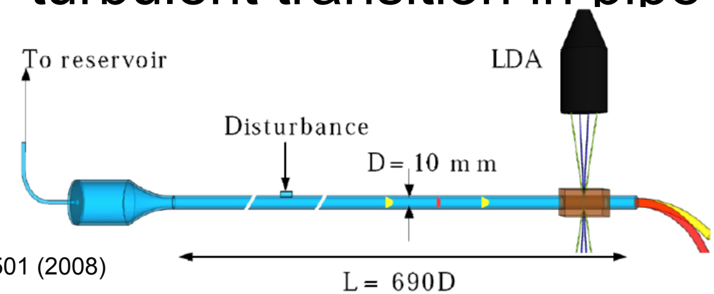

Hof et al., *PRL* **101**, 214501 (2008)

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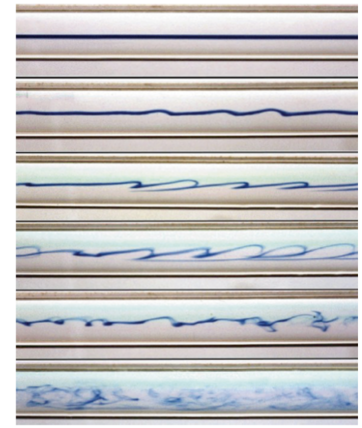

Re small → large

Re: laminar flows | 1650 | metastable puffs | 2050 | splitting puffs/ intermittency | 2500

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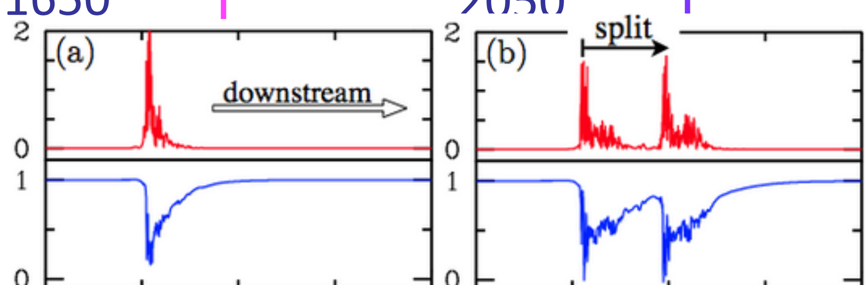
mean shear



pipe axis

puff lifetime $\tau \sim \exp(\exp(Re))$

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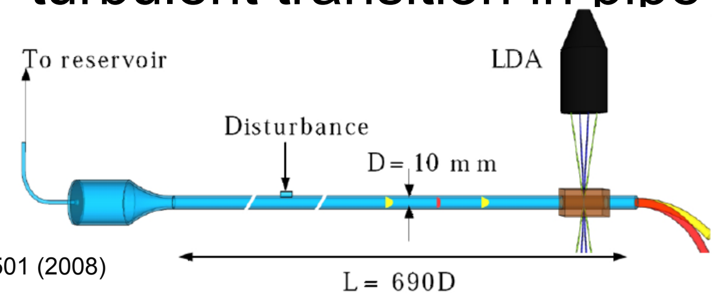

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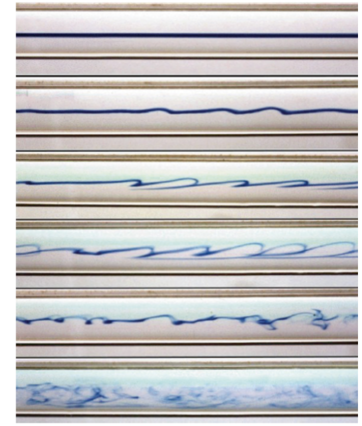

Re small

large

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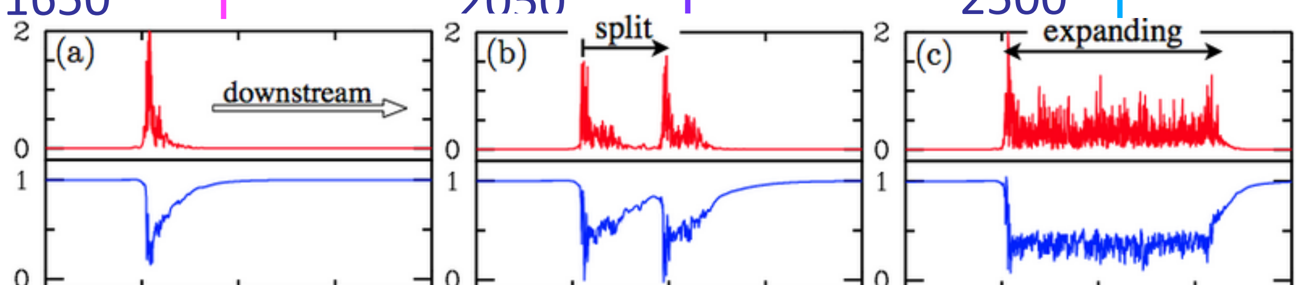
turbulence intensity

mean shear



pipe axis   pipe axis

puff lifetime $\tau \sim \exp(\exp(Re))$

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Directed percolation and the laminar-turbulent transition

- Y. Pomeau (1986) first suggested the universality class of DP:
  - Turbulent regions can spontaneously relaminarize (go into an absorbing state).
  - They can also contaminate their neighbourhood with turbulence.
- Our work:
  - What quantitative aspects of the transitional turbulence phenomenology can be described by such a minimal model?
  - Can we derive such a statistical mechanics, minimal model from fluid dynamics flow equations?

Main messages

- Transition to turbulence is in the universality class of directed percolation
  - Generic absorbing state argument
  - Puff lifetime as a function of Re
  - Extreme value statistics and finite-size scaling
  - Slug spreading rate as a function of Re

- How to derive universality class from hydrodynamics
  - Transitional turbulence maps into predator-prey dynamics
  - Statistical field theory of ecology of turbulence
  - Observational signatures

Phase diagram of pipe flow

Single puff that can spontaneously decay

Regions of turbulence with clear growth rate.

laminar	equilibrium puffs	spatiotemporal intermittency	expanding slugs

Re 1775 2100 2500

Extensive puffs that can interact and split. Regions of turbulence with intermittent laminar patches.

But: laminar state is linearly stable.

Meseguer and Trefethen (J. Comp. Phys (2003)

to N-S

Phase diagram of pipe flow

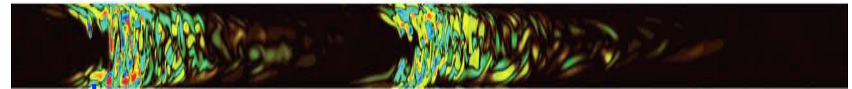
Single puff spontaneously decays

laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re 1775 2100 2500

Avila *et al.*, *Science* **333**, 192 (2011)

Phase diagram of pipe flow



Single puff spontaneously decays

Splitting puffs

laminar

metastable puffs

spatiotemporal intermittency

expanding slugs

Re

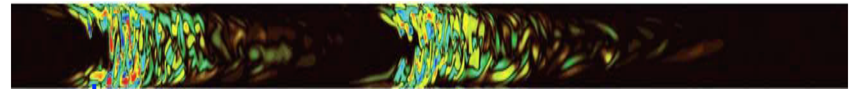
1775

2100

2500

Avila *et al.*, *Science* **333**, 192 (2011)

Phase diagram of pipe flow



Single puff spontaneously decays

Splitting puffs

laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re 1775 2100 2500

Survival *probability* $P(\mathrm{Re},t)=e^{-(t-t_0)/\tau(\mathrm{Re})}$

Avila *et al.*, *Science* **333**, 192 (2011)
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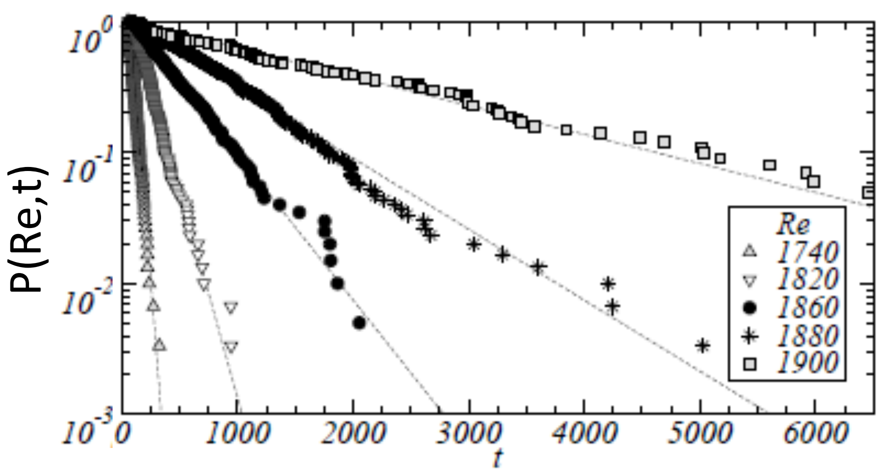

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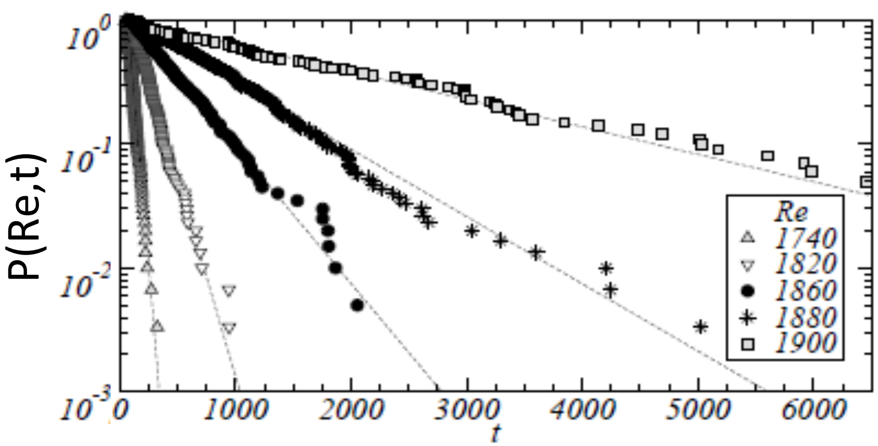
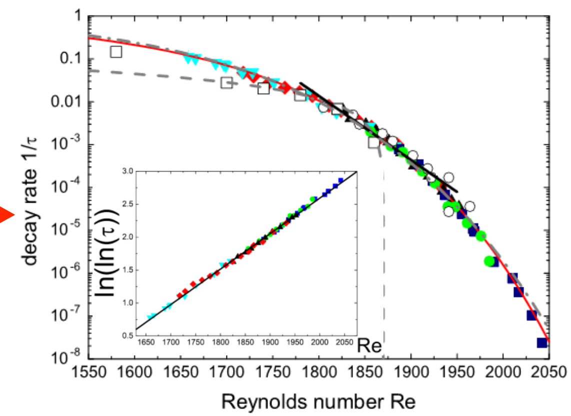
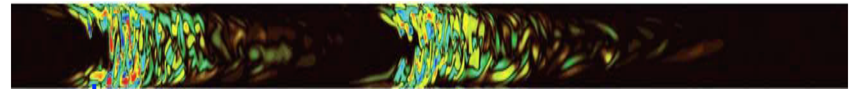




Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp \mathrm{Re})$

Phase diagram of pipe flow



Single puff spontaneously decays

Splitting puffs

laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

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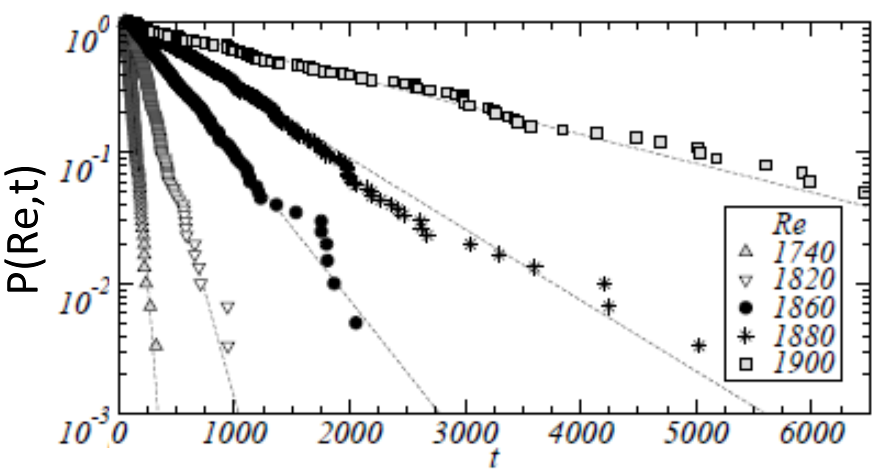

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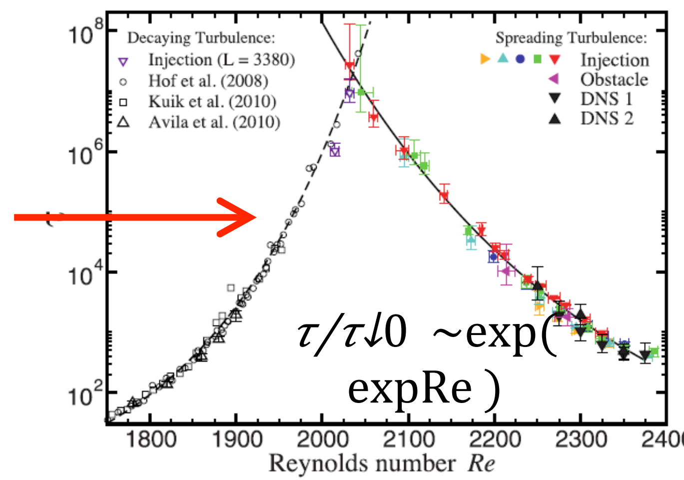
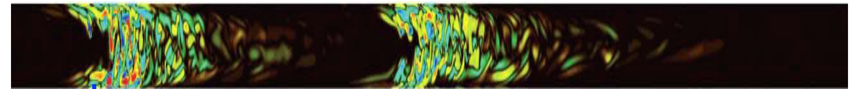


Phase diagram of pipe flow



Single puff spontaneously decays

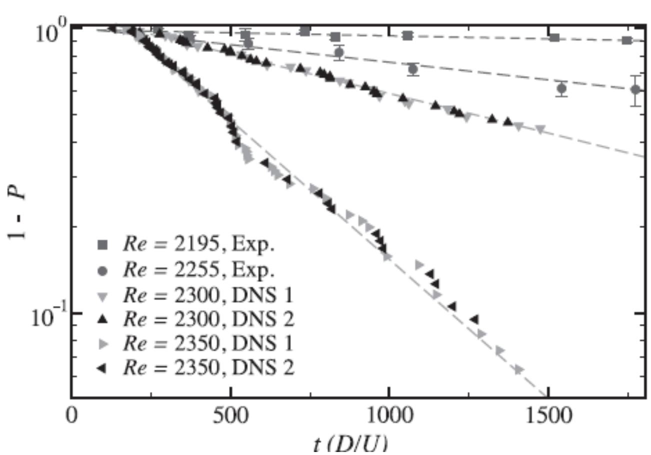
Splitting puffs

laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

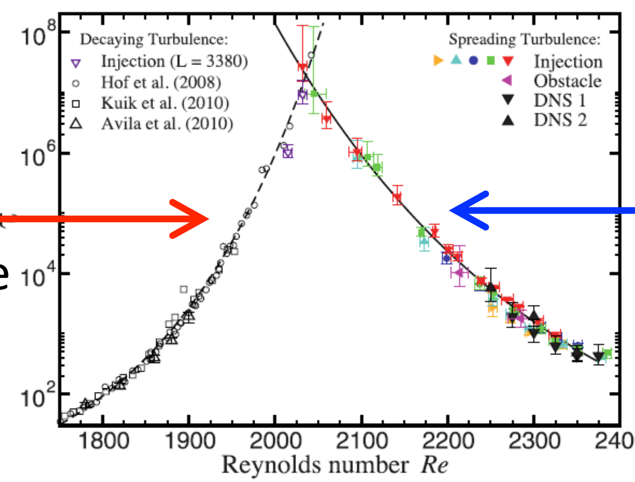
Re 1775 2100 2500

Survival *probability* $1-P(\mathrm{Re},t)=e^{-(t-t_0)/\tau(\mathrm{Re})}$

Avila *et al.*, *Science* **333**, 192 (2011)
Hof *et al.*, *PRL* **101**, 214501 (2008)

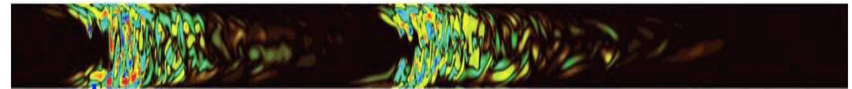


Puff lifetime



Mean time between split events

Phase diagram of pipe flow



Single puff spontaneously decays

Splitting puffs

laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

Re 1775 2100 2500

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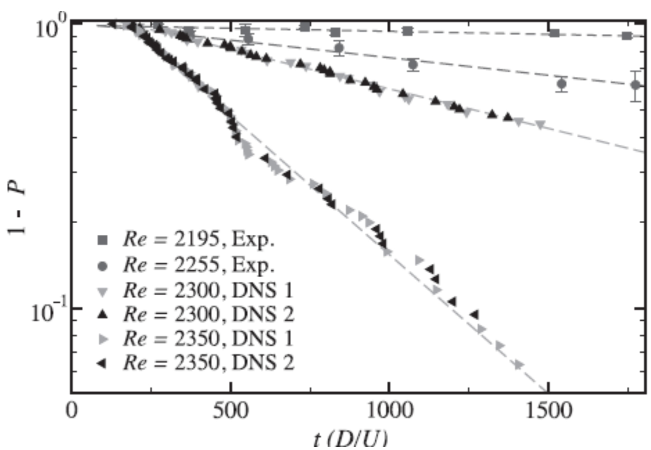
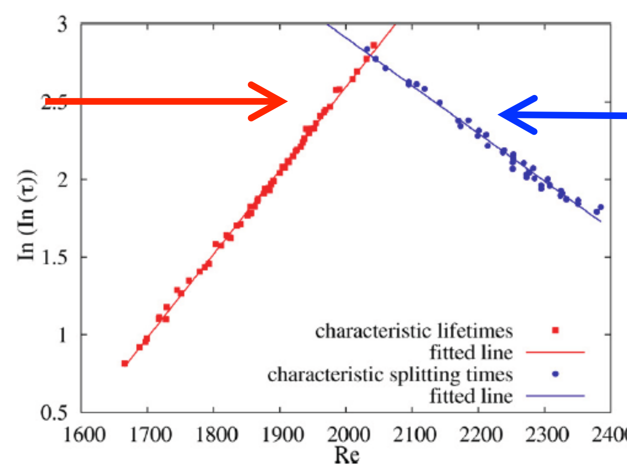
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

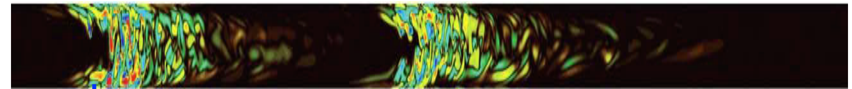


Puff lifetime



Mean time between split events

Phase diagram of pipe flow



Single puff spontaneously decays

Splitting puffs

laminar

metastable

spatiotemporal

expanding

Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp Re)$

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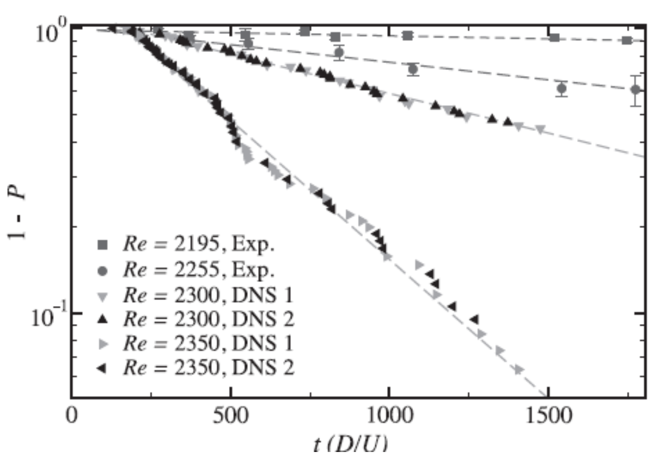
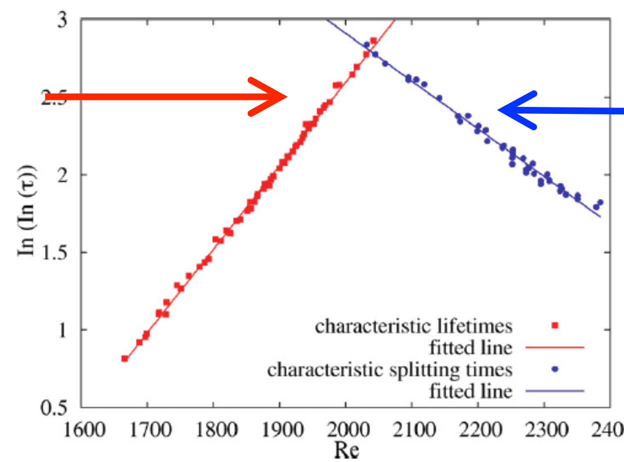
3)

Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

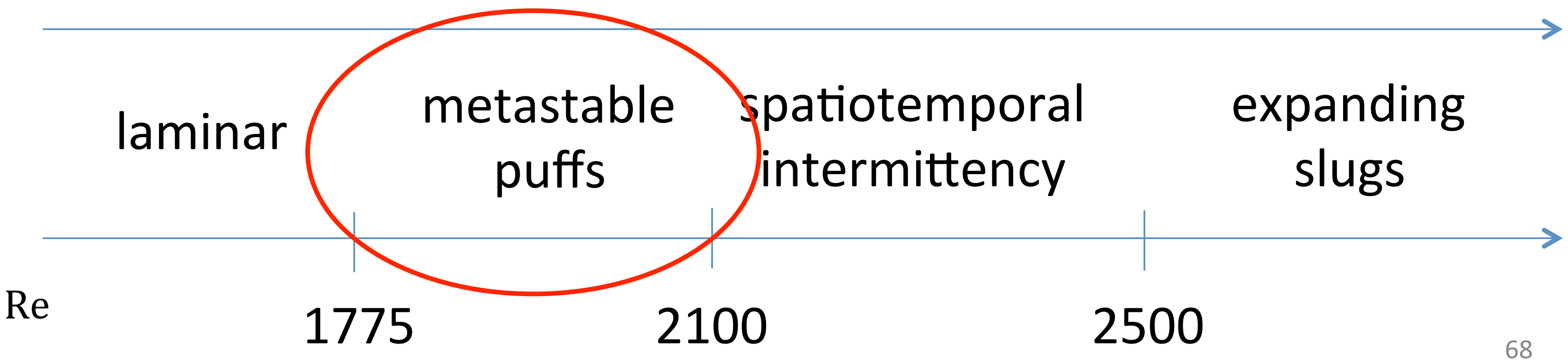


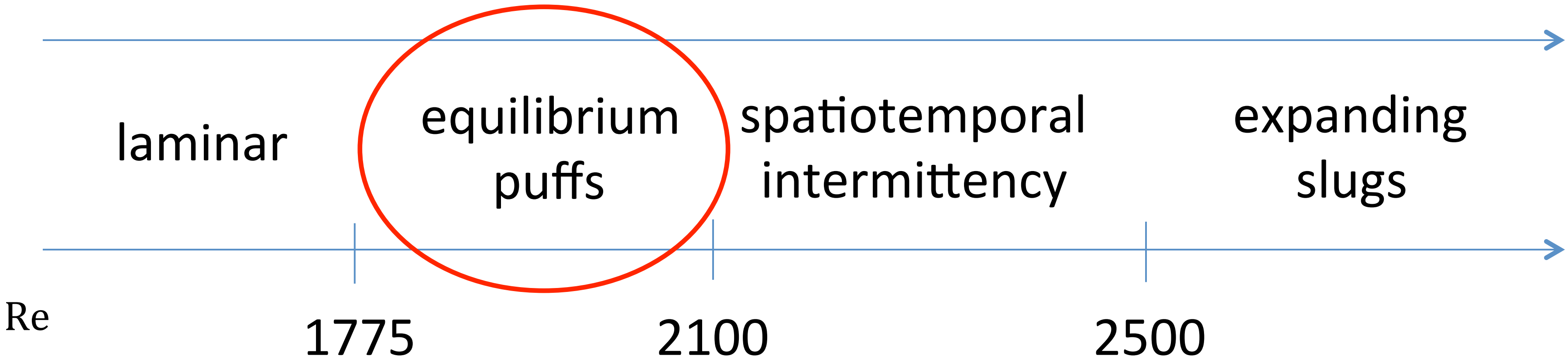
Puff lifetime



Mean time between split events

MODEL FOR METASTABLE TURBULENT PUFFS

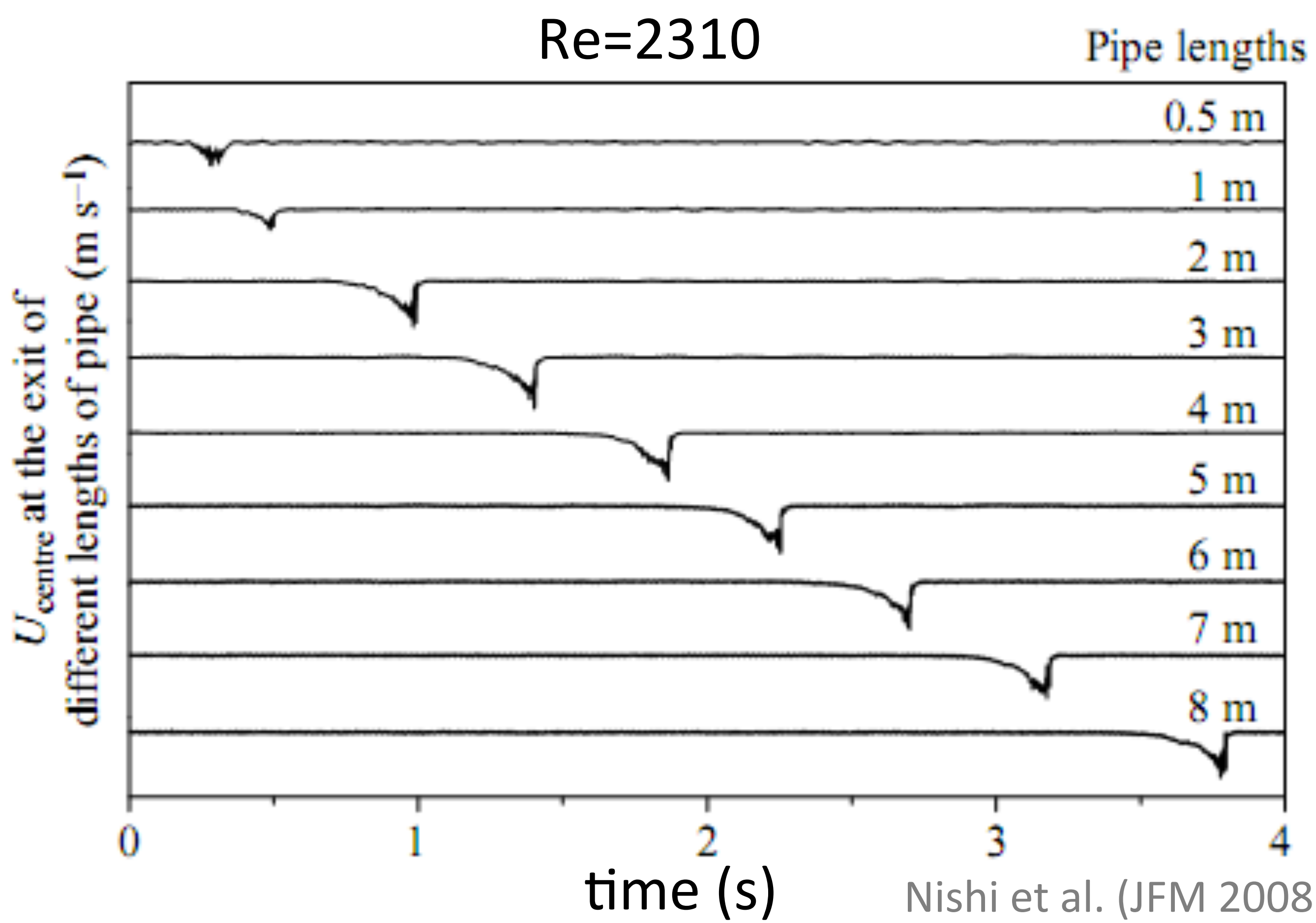




Van Doorne and Westerweel (Phil. Trans. R. Soc. A 2009)

Metastable puff

- Hot wire measurements:

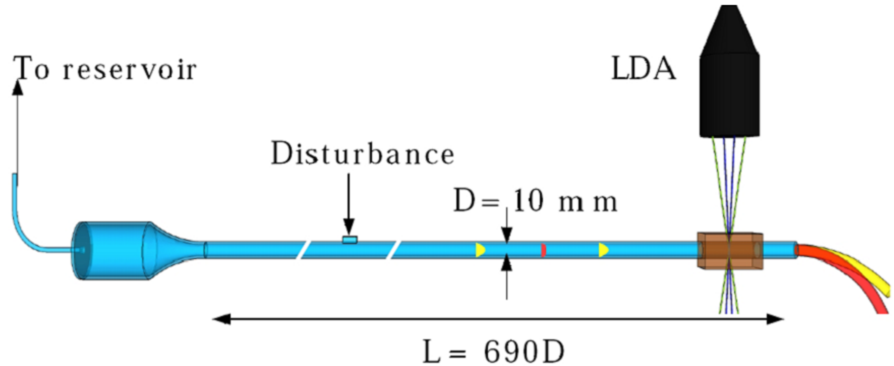


Nishi et al. (JFM 2008)

Metastable puff

- Lifetime of puff decay was measured conclusively by Hof et al. (PRL **101**, 214501 2008).

- Their experimental setup:



Hof *et al.* (PRL 2008)

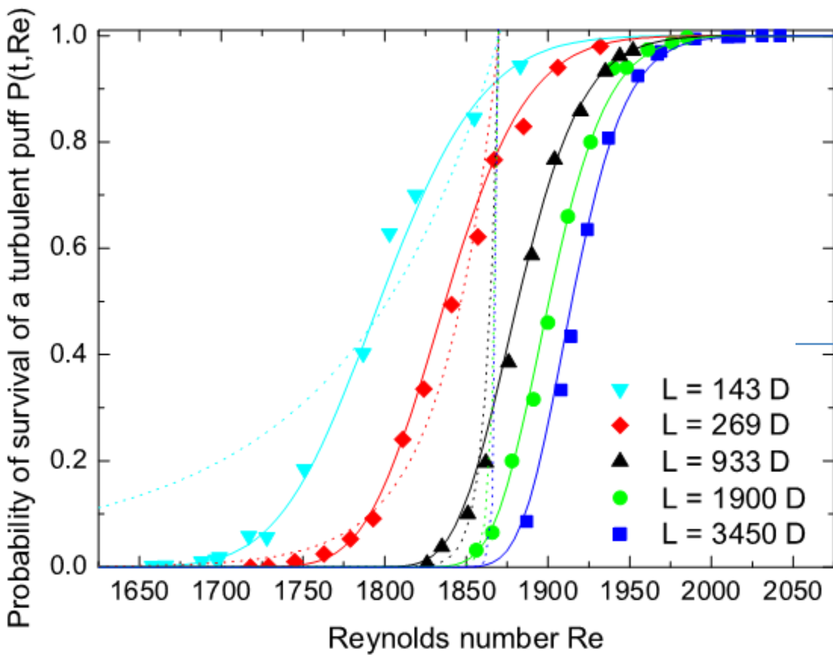
- They measured the survival probability (probability that a puff is alive when it reaches the outlet of the pipe).

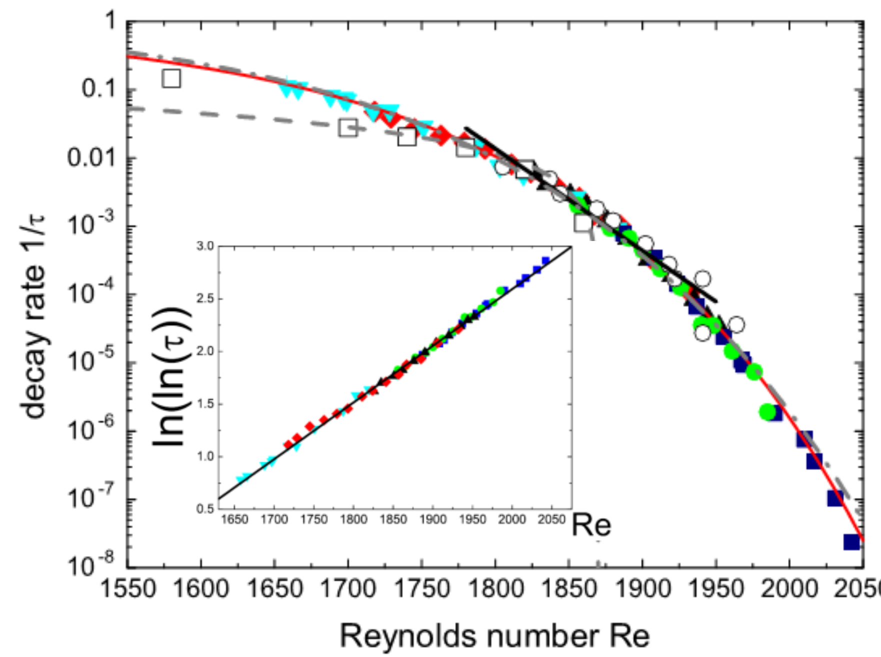
Metastable puff

- S-shaped curves imply that survival probability has the form:

$$P(\mathrm{Re}, t) = e^{-(t - t_0)/\tau(\mathrm{Re})}$$

to extra slide



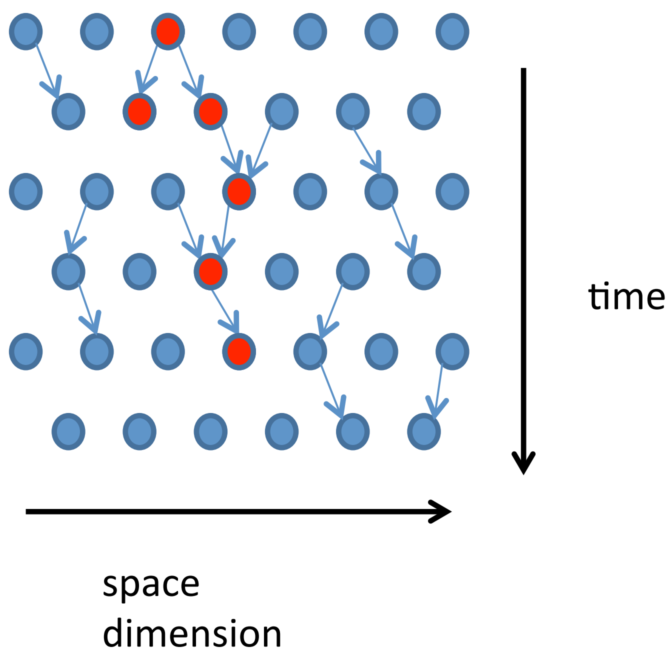


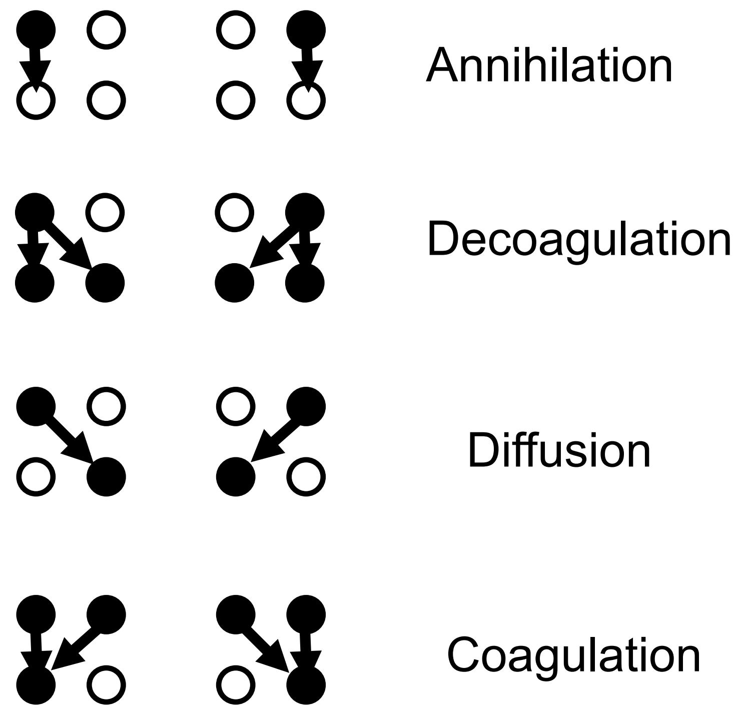
Hof *et al.* (PRL 2008)

Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp \mathrm{Re})$

DP & the laminar-turbulent transition

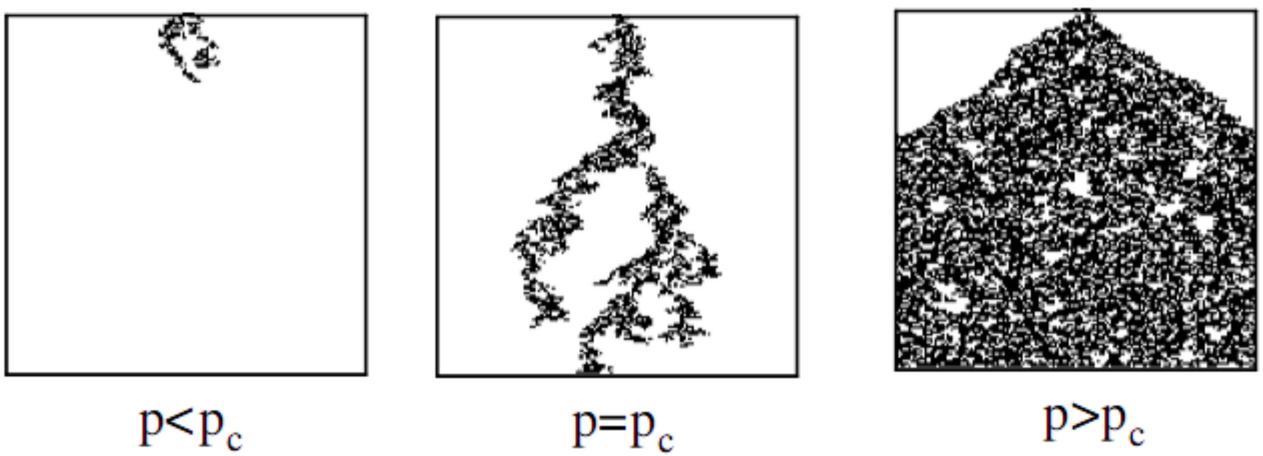
- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)





Directed Percolation Transition

- A continuous phase transition occurs at p_c .



Hinrichsen (Adv. in Physics 2000)

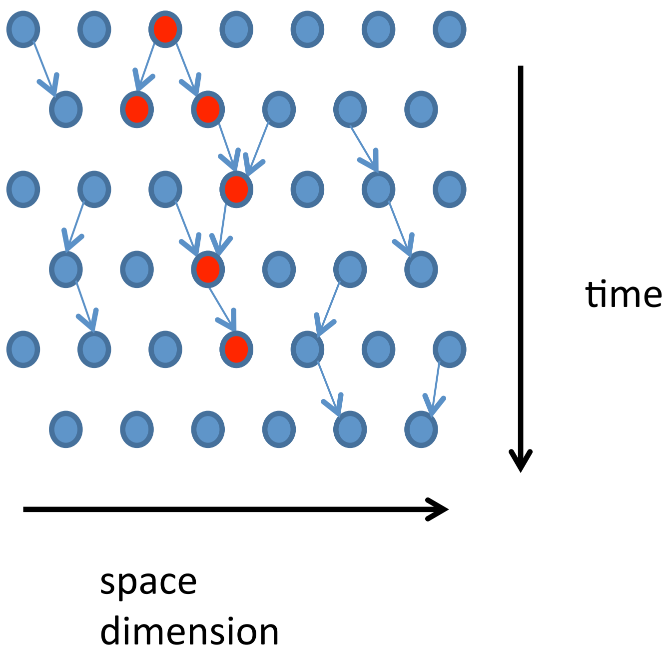
- Phase transition characterized by universal exponents:

$$\rho \sim (p-p_c)^{\beta} \qquad \xi_{\perp} \sim (p-p_c)^{-\nu_{\perp}} \qquad \xi_{\parallel} \sim (p-p_c)^{-\nu_{\parallel}}$$

to DP models

Pomeau's heuristic argument

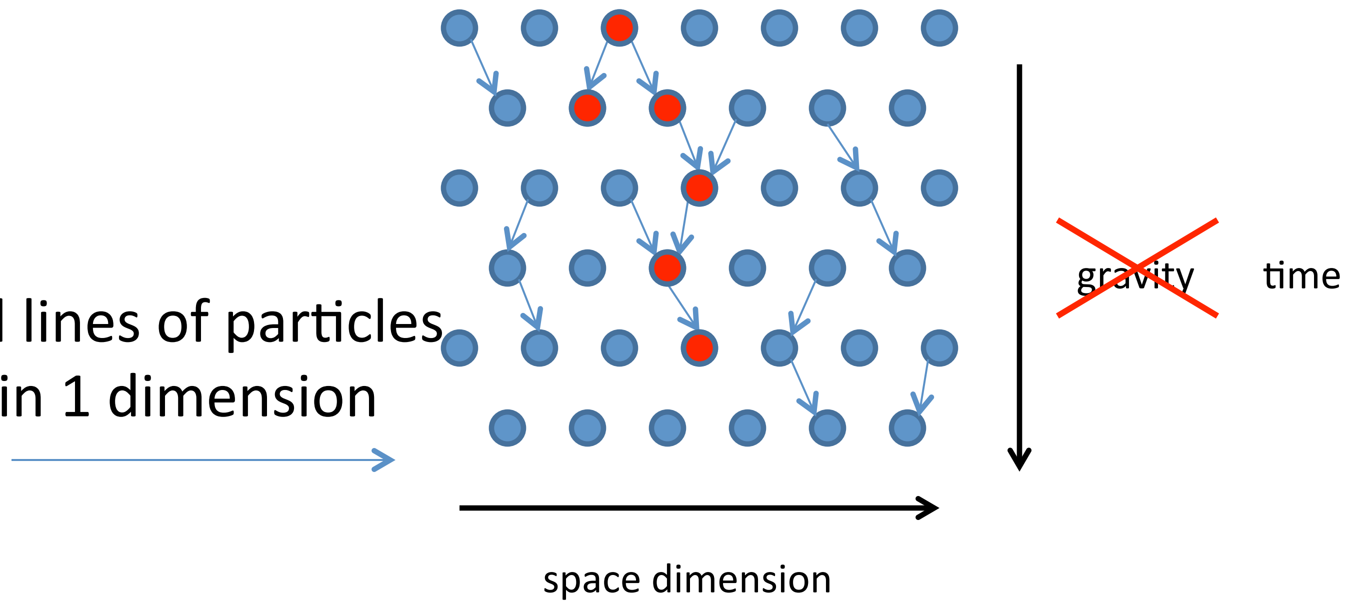
- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)



Directed percolation

- Bond percolation: Diagonal lattice with bonds open with probability p.

World lines of particles living in 1 dimension

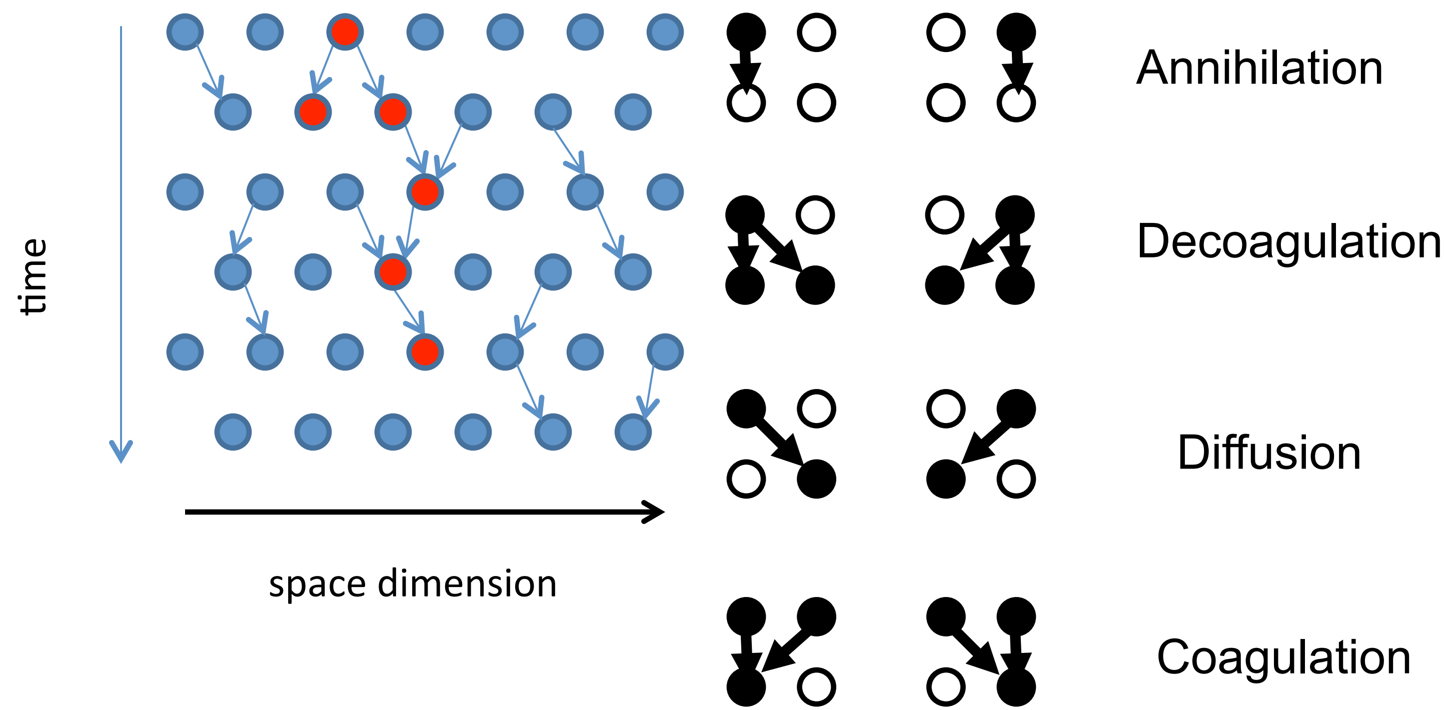


- This would be called 1 + 1-dimensional DP.

to DP models

Directed percolation

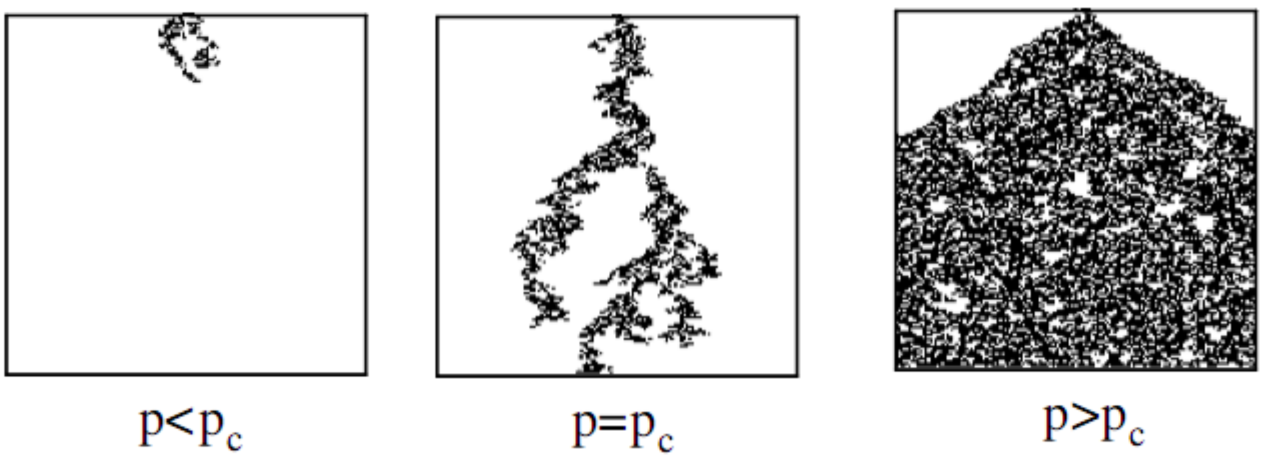
- Bond percolation: Diagonal lattice with bonds open with probability p.



to DP models

Directed Percolation Transition

- Order parameter is the size of the percolating cluster.
- A continuous phase transition occurs at p_c .



Hinrichsen (Adv. in Physics 2000)

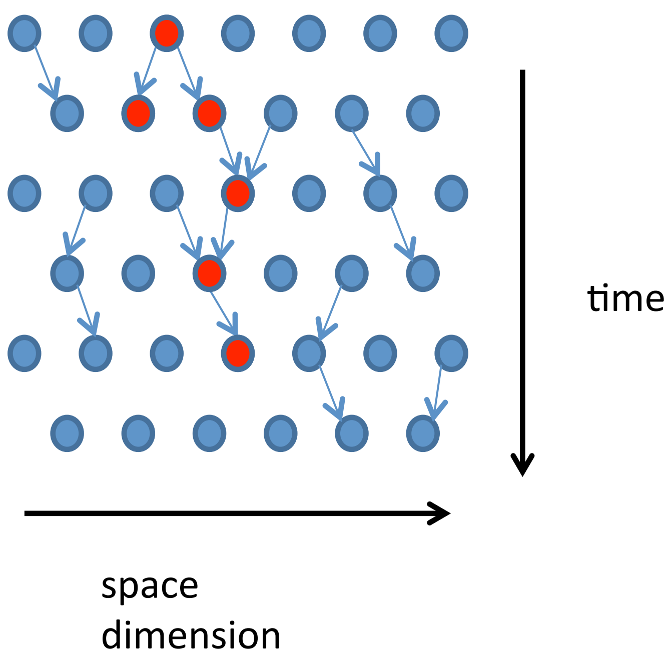
- Phase transition characterized by universal exponents:

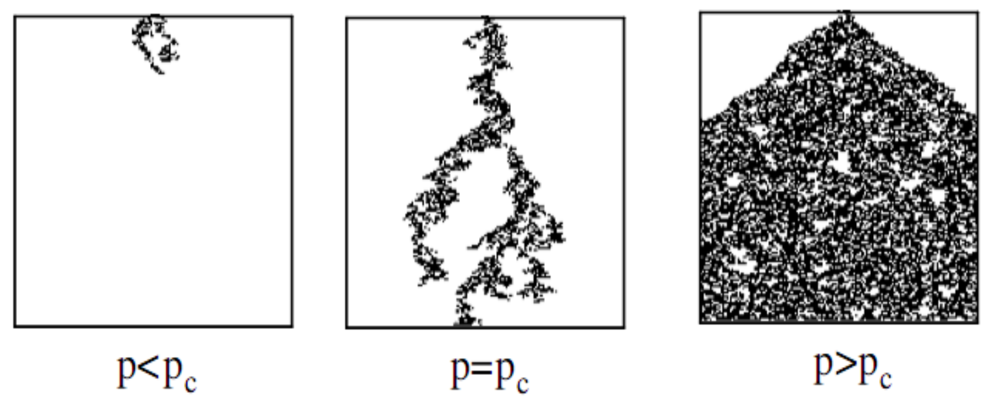
$\rho \sim (p-p_c)^{\beta}$ $\qquad$ $\xi_{\perp} \sim (p-p_c)^{-\nu_{\perp}}$ $\qquad$ $\xi_{\parallel} \sim (p-p_c)^{-\nu_{\parallel}}$

to DP models

Modeling the laminar-turbulent transition

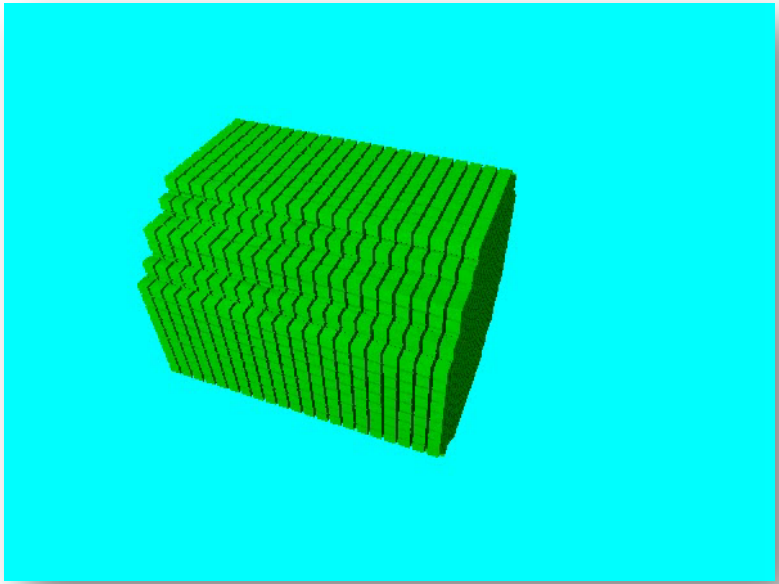
- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)



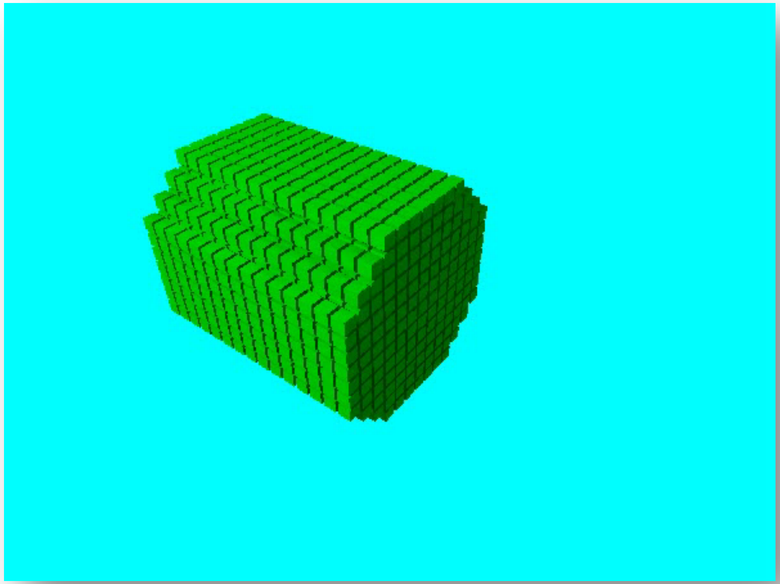


Hinrichsen (Adv. in Physics 2000)

DP in 3 + 1 dimensions in pipe



Puff decay



Slug spreading

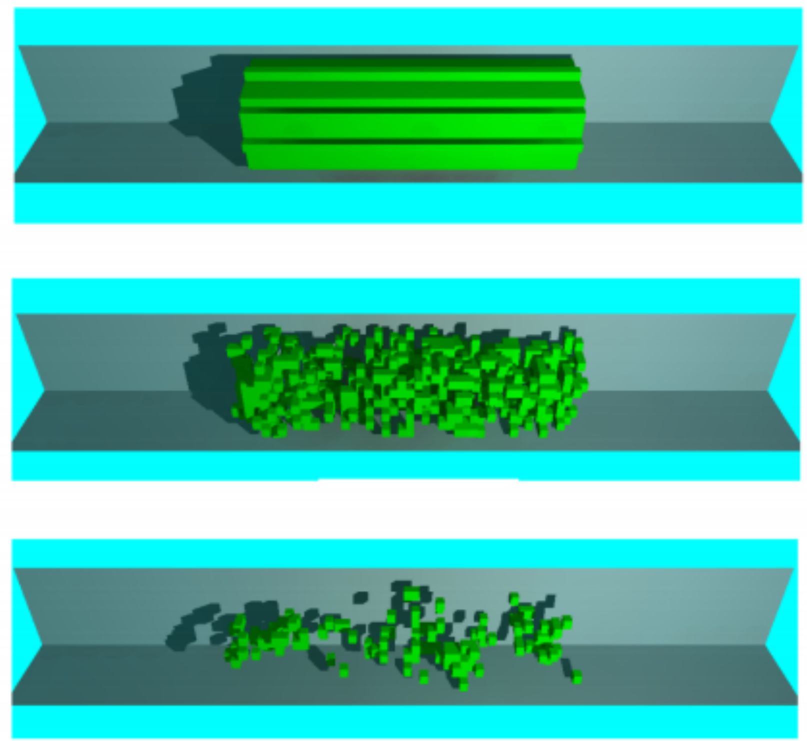
Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

Turbulent transients: Puffs

- Here we consider decay not of a single seed but an initial puff
- Below p_c, DP cluster decays as a memoryless process.

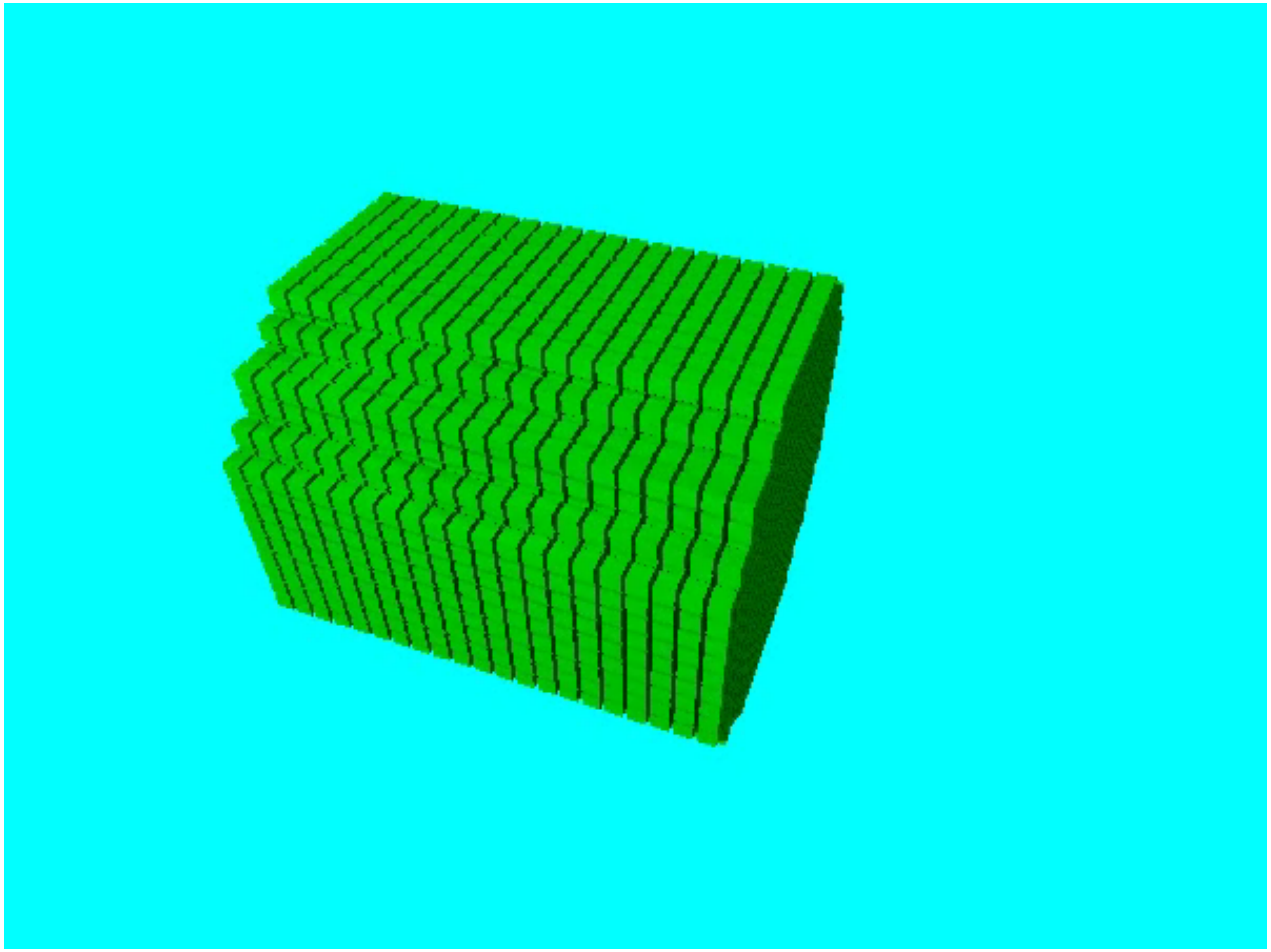
1+1 DP

3+1 DP



M. Sipos and NG, *PRE* **84**, 035304(R) (2011)

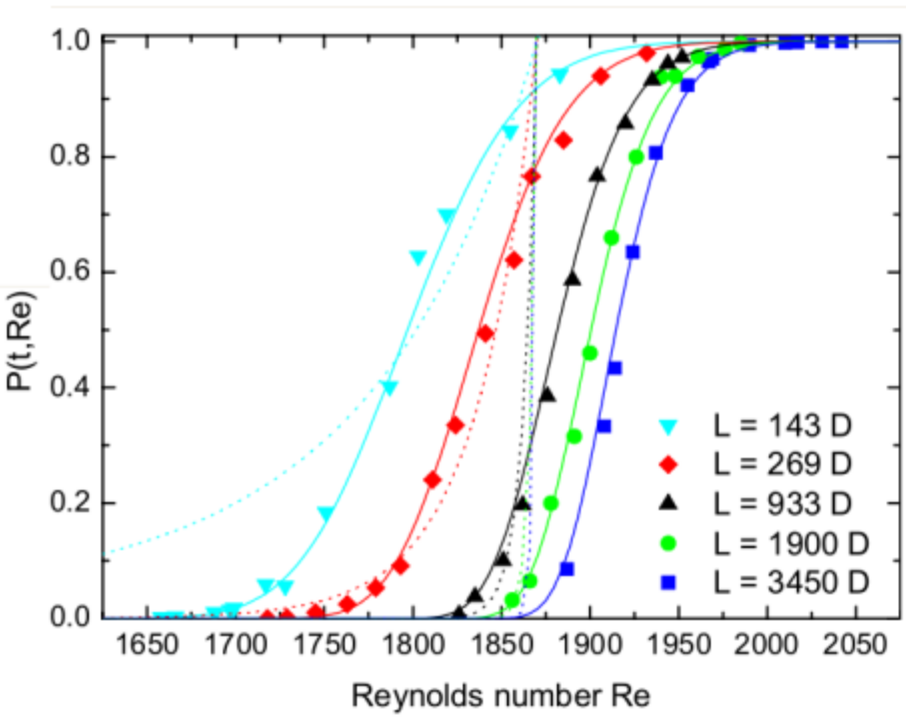
Turbulent transients: Puffs



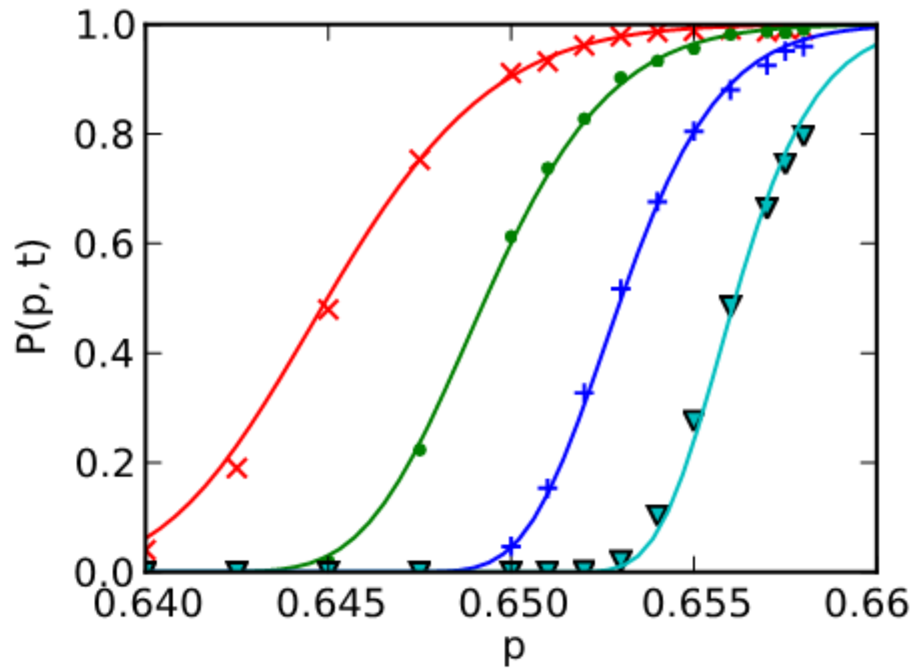
Turbulent transients: Puffs

- We can measure the survival probability of active DP regions, like Hof *et al.* did in pipe experiments:

$$P(\mathrm{Re}, t) = e^{-(t-t_0)/\tau(\mathrm{Re})}$$



Hof *et al.* (PRL 2008)



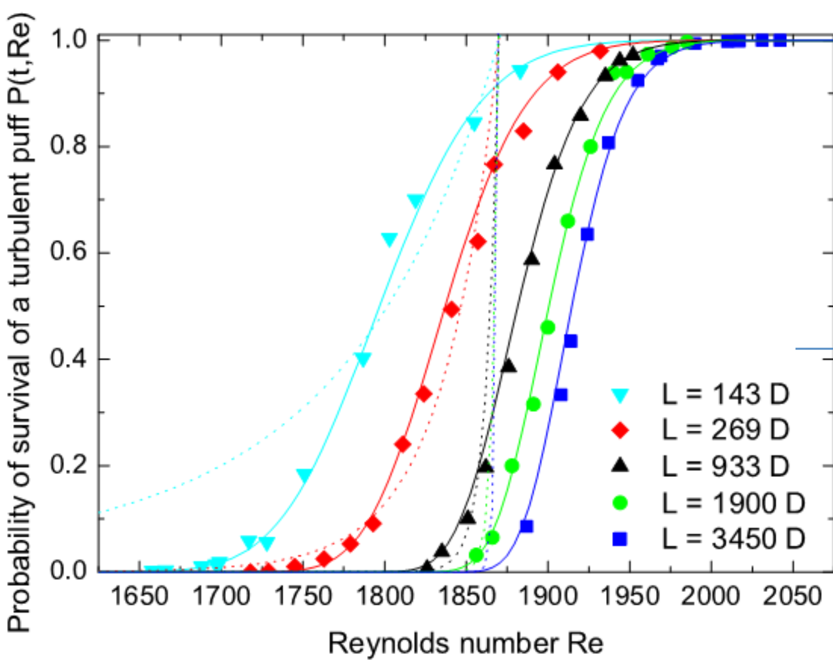
1+1 DP

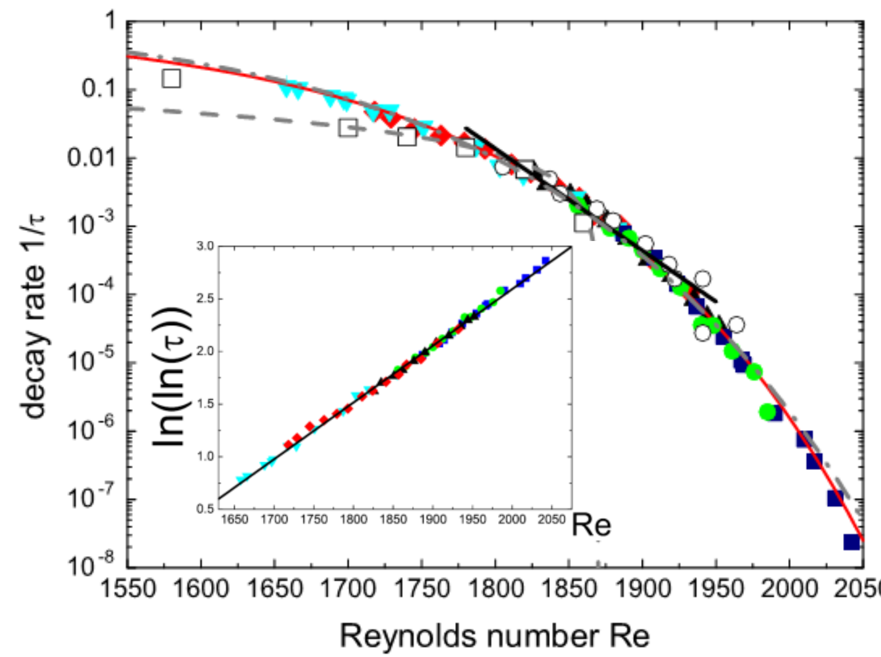
Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

Metastable puff

- S-shaped curves imply that survival probability has the form:

to extra slide



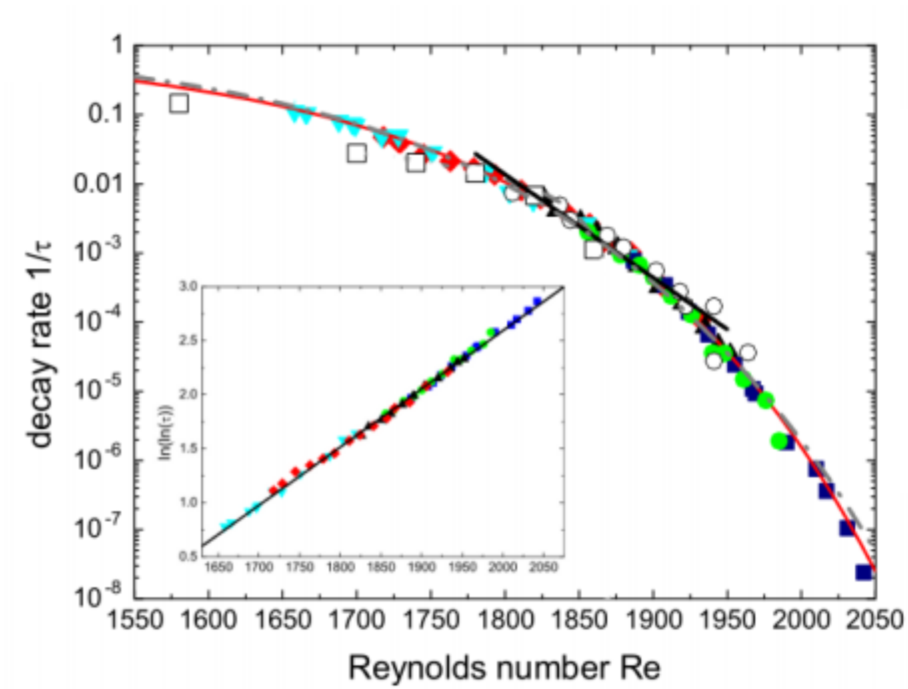


Hof *et al.* (PRL 2008)

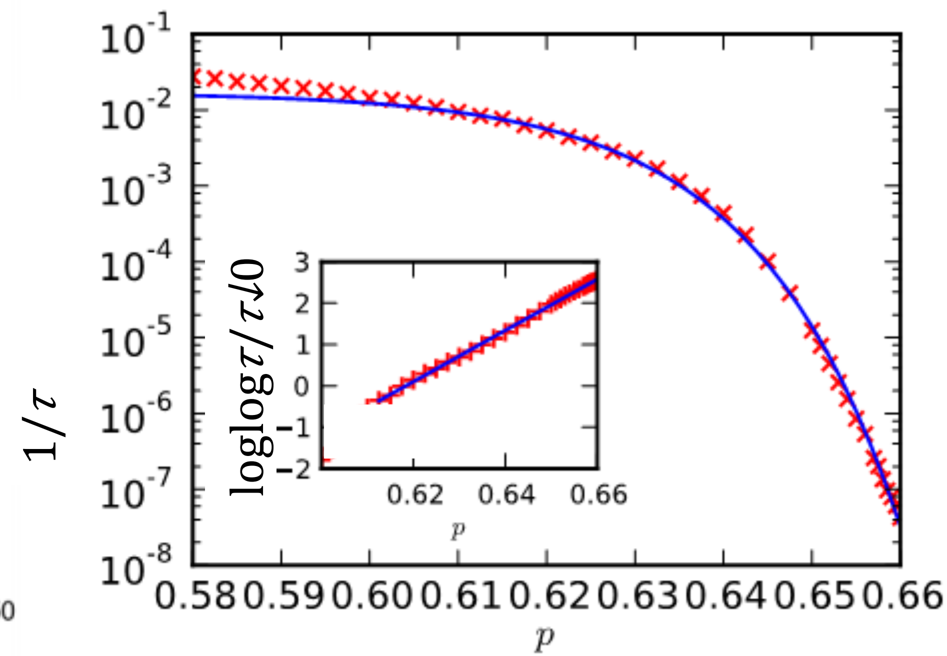
Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp \mathrm{Re})$

Turbulent transients: Puffs

- The lifetime τ fits a super-exponential scaling
- $\tau/\tau_0 \sim \exp(\exp \mathrm{Re})$



Hof *et al.* (PRL 2008)



M. Sipos and NG,
PRE **84**, 035304(R) (2011)

Super-exponential scaling and extreme statistics

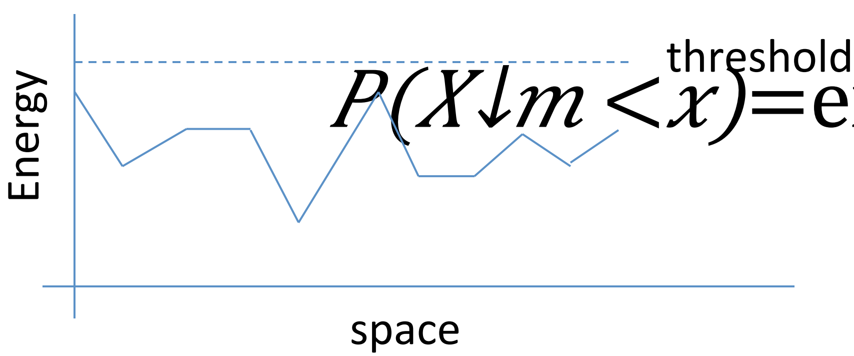
- Consider identical and independently distributed random variables X_i whose distribution decays sufficiently fast at infinity
- Their mean $X \propto \sum_i X_i$ is normally distributed (Central limit theorem).

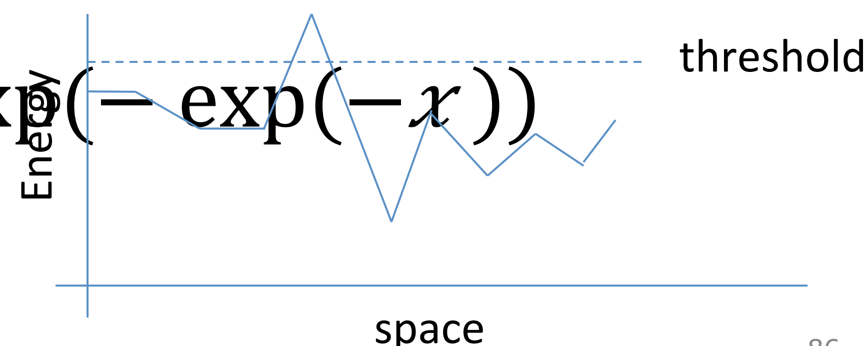
more on FT

- Their maximum $X_m \propto \max X_i$ is distributed according to the Fisher-Tippett type I distribution:

more on FT 2

$$P(X_m < x) = \exp(-\exp(-x))$$





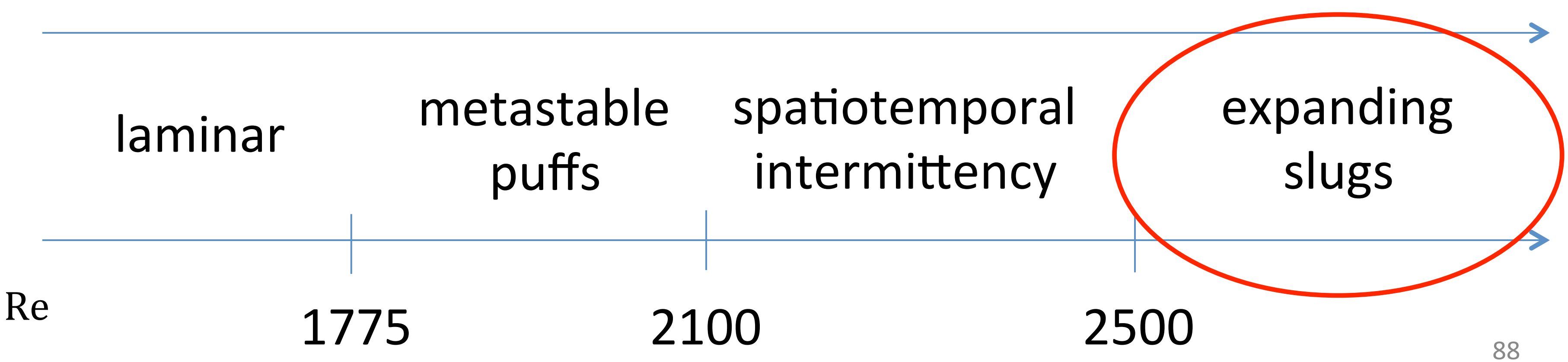
Goldenfeld, Gioia, Guttenberg (2010)

Super-exponential scaling and extreme statistics

- Active state persists until the most long-lived percolating "strands" decay.
  - extreme value statistics
- Why do we not observe the power law divergence of lifetime of DP near transition?
- Close to transition, transverse correlation length diverges, so initial seeds are not independent
  - Crossover to single seed behaviour
  - Asymptotically will see the power law behavior in principle

$$\xi_{\perp} \sim (p-p_c)^{-\nu_{\perp}}$$

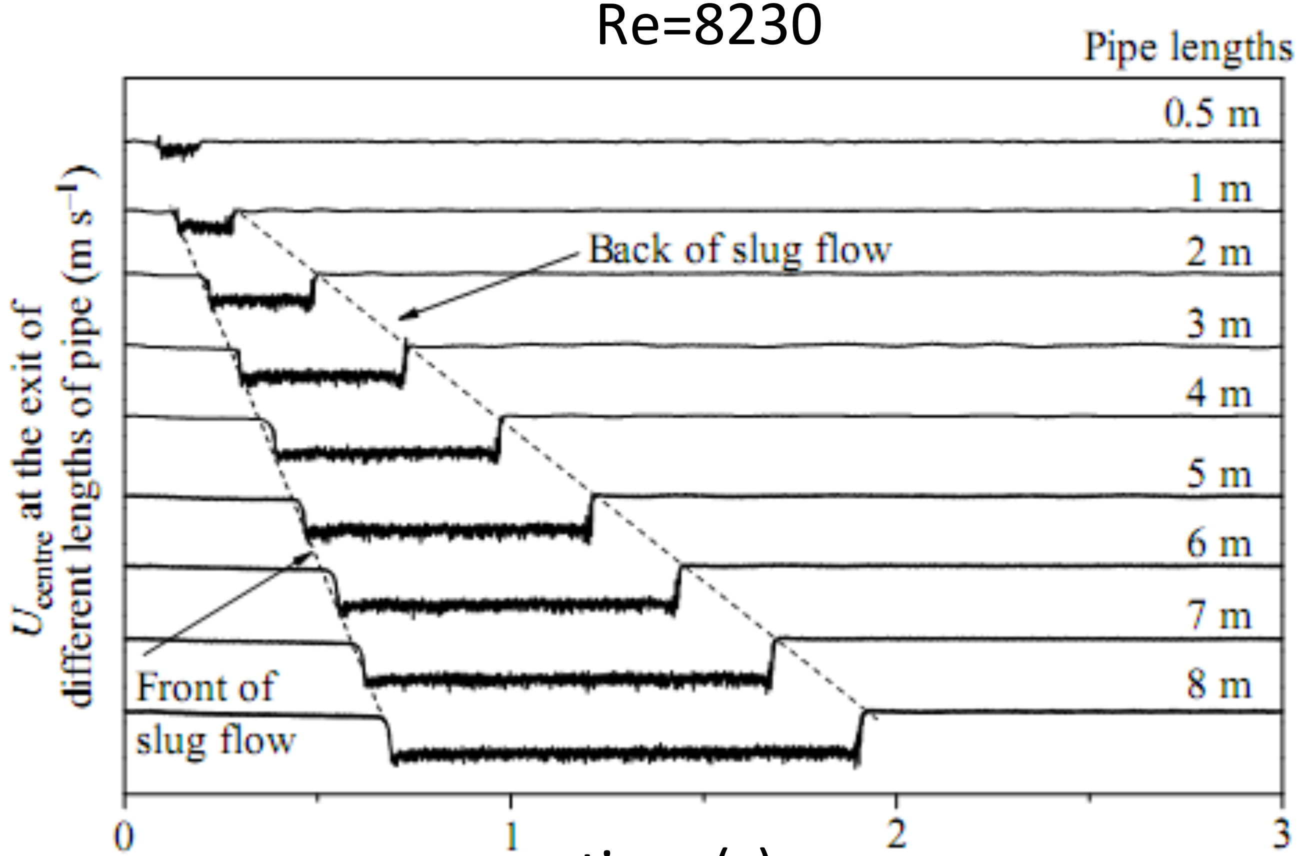
MODEL FOR EXPANDING TURBULENT SLUGS



Turbulent slugs

- Turbulent slugs have well-defined fronts with well-defined expansion

Re=8230



Nishi et al. (JFM 2008)

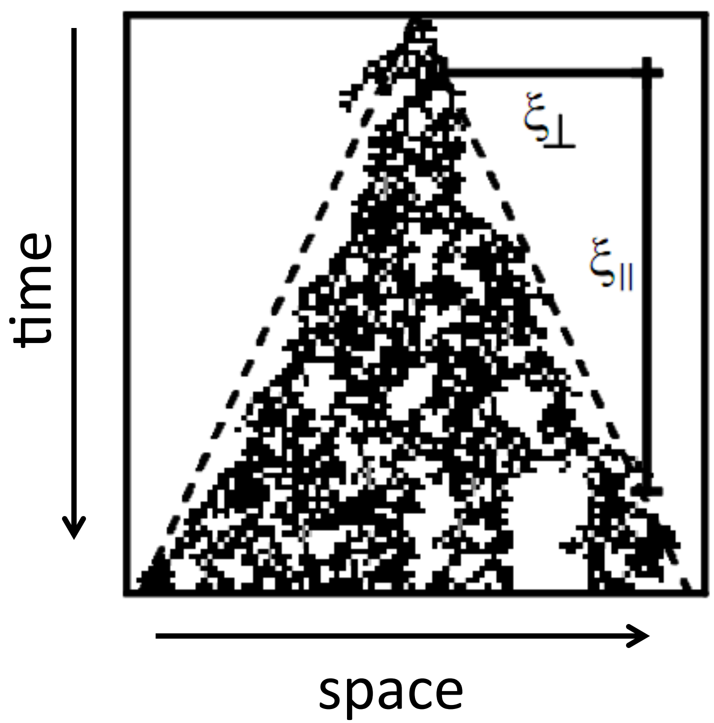
Growing fronts in DP

- Above p_c, percolating clusters grow with front velocity:

$$G \sim \xi_{\perp}/\xi_{\parallel} \sim (p-p_c)^{\nu_{\parallel} -}$$

dim	$\nu_\parallel - \nu_\perp$
1+1	0.637
2+1	0.561
3+1	0.524

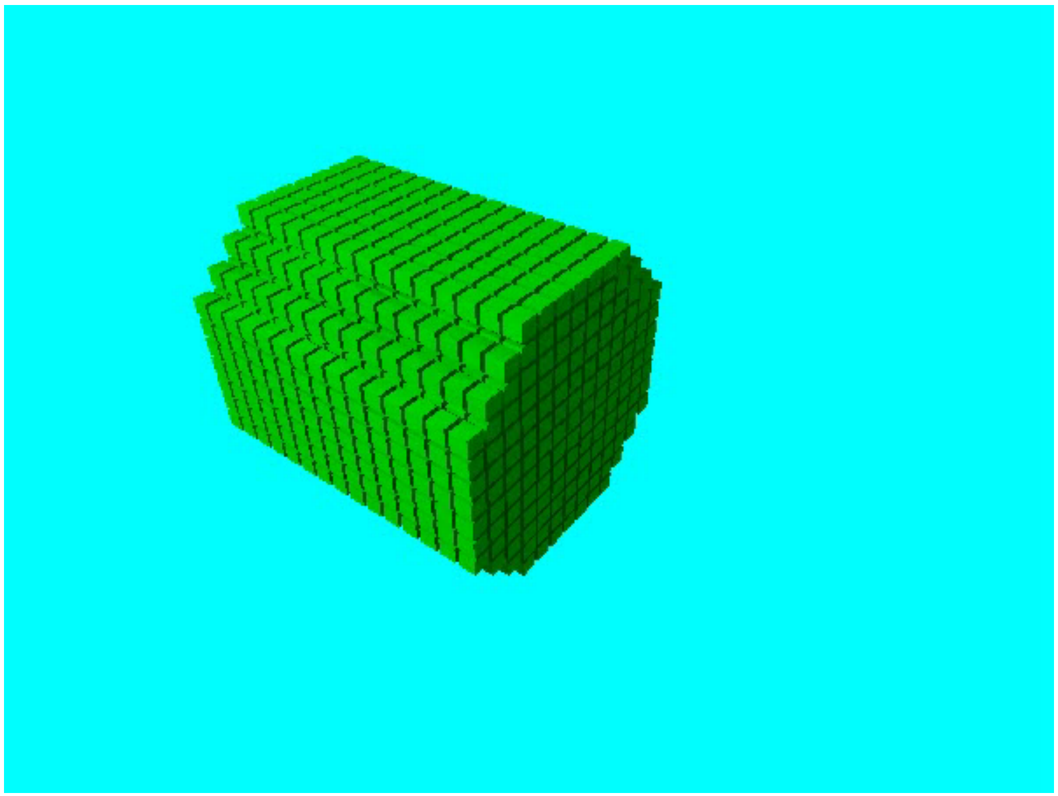
- In 1+1 DP:



Hinrichsen (Adv. in Physics 2000)

Growing fronts in DP

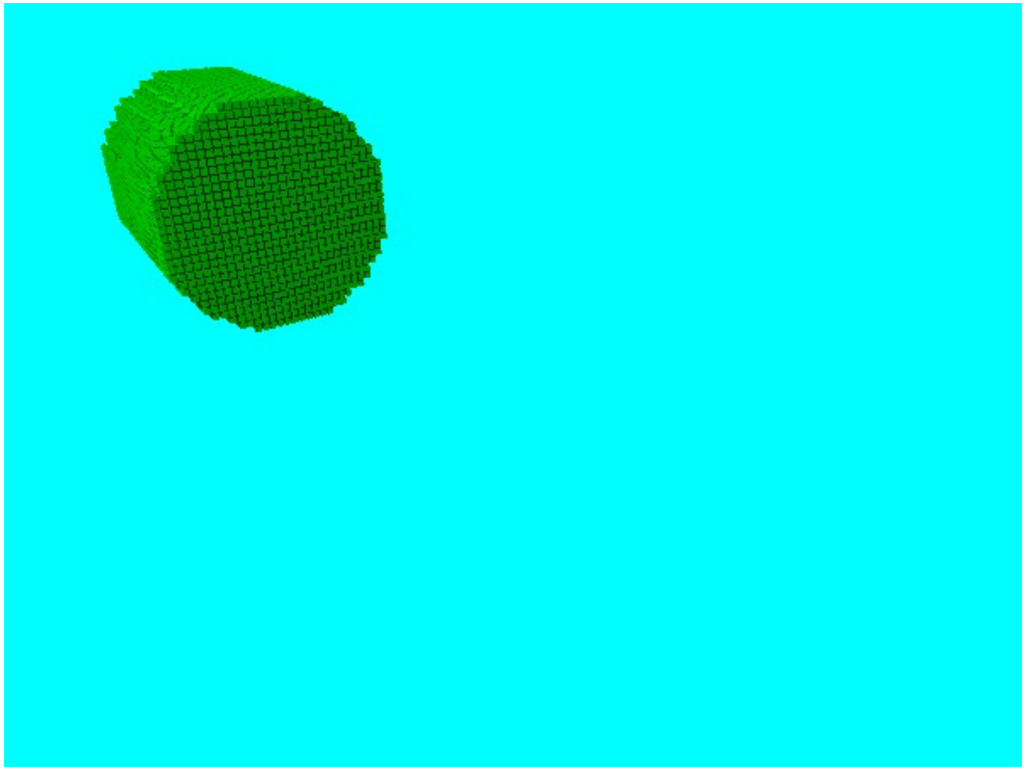
When $p-p_c$ is small:



$(p-p_c)/p_c \sim 0.05$

Growing fronts in DP

When $p - p_c$ is large:

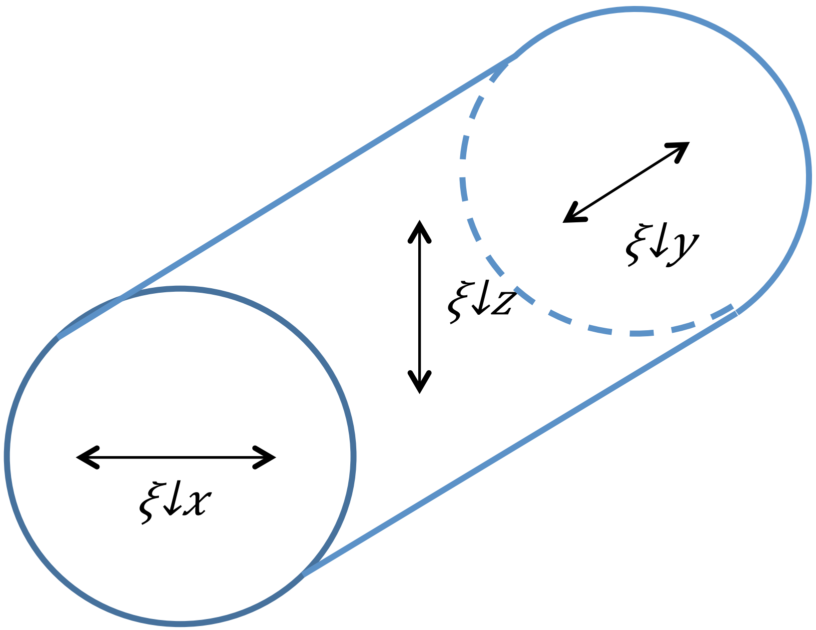


$(p - p_c)/p_c \sim 3$

Crossover in pipe geometry

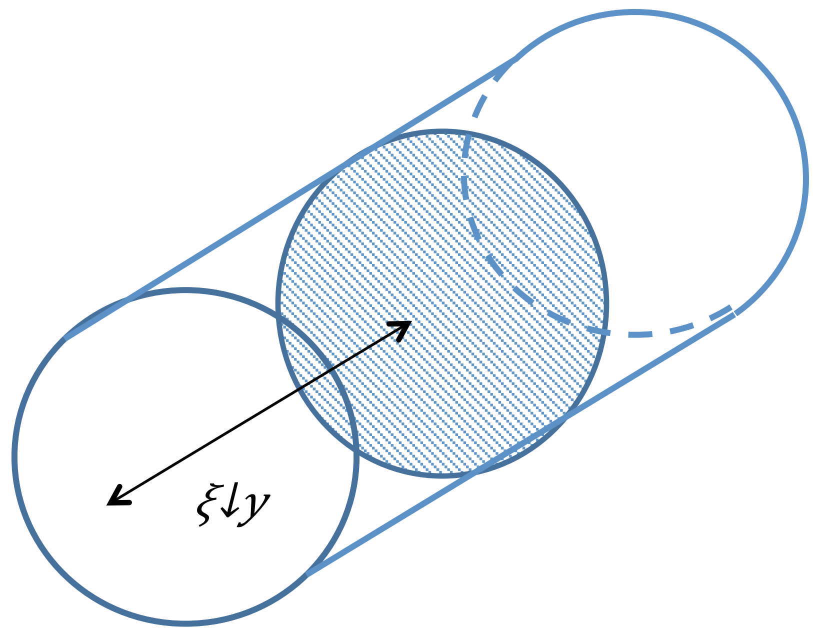
$$\xi_\perp = (p - p_c)^{-\nu_\perp}$$

- When $p - p_c$ is large



$\xi_\perp < D \rightarrow$ 3+1 DP

- When $p - p_c$ is small

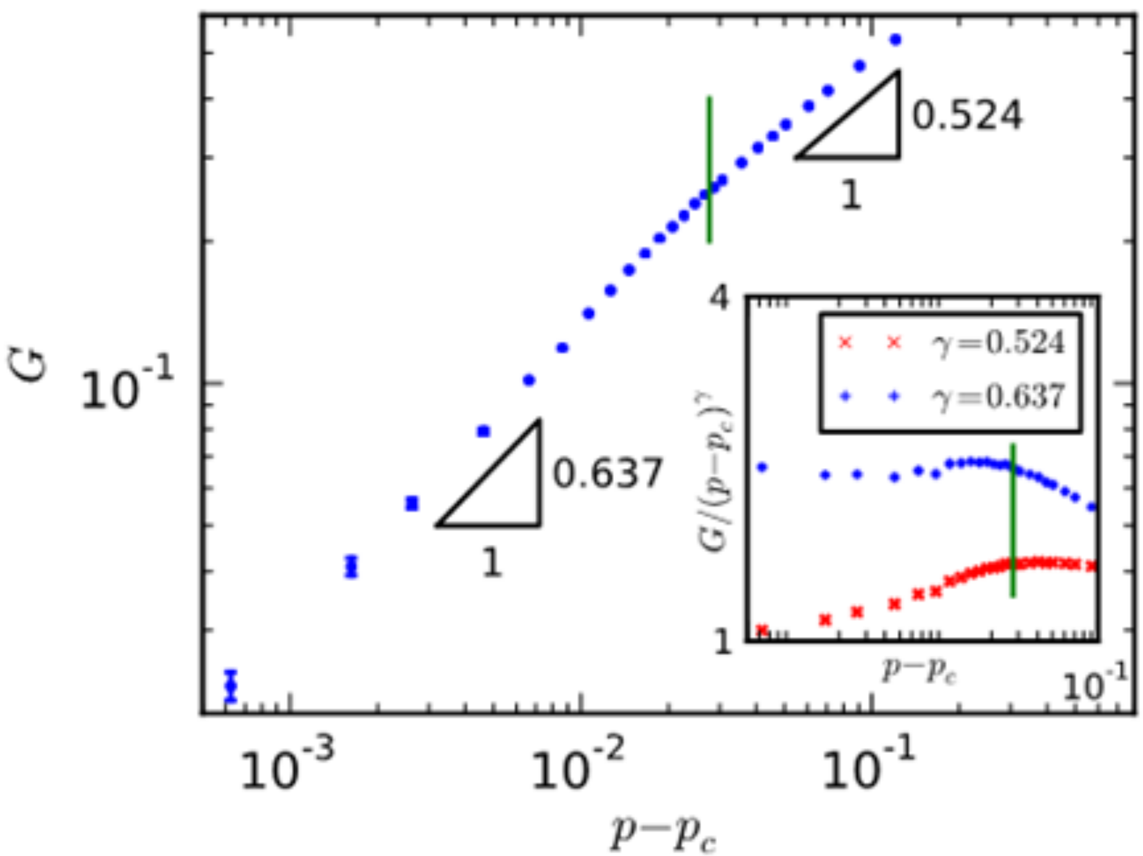


$\xi_\perp > D \rightarrow$ 1+1 DP

Growing fronts in DP

- In 3+1 DP, $G \sim (p-p_c)^{0.524}$

$G \sim (p-p_c)^{0.637}$



dim	$\nu_\parallel - \nu_\perp$
1+1	0.637
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Growing fronts in DP

- In 3+1 DP, $G \sim (p - p_c)^{0.524}$
- In 1+1 DP, $G \sim (p - p_c)^{0.637}$

dim	$\nu_\parallel - \nu_\perp$
1+1	0.637
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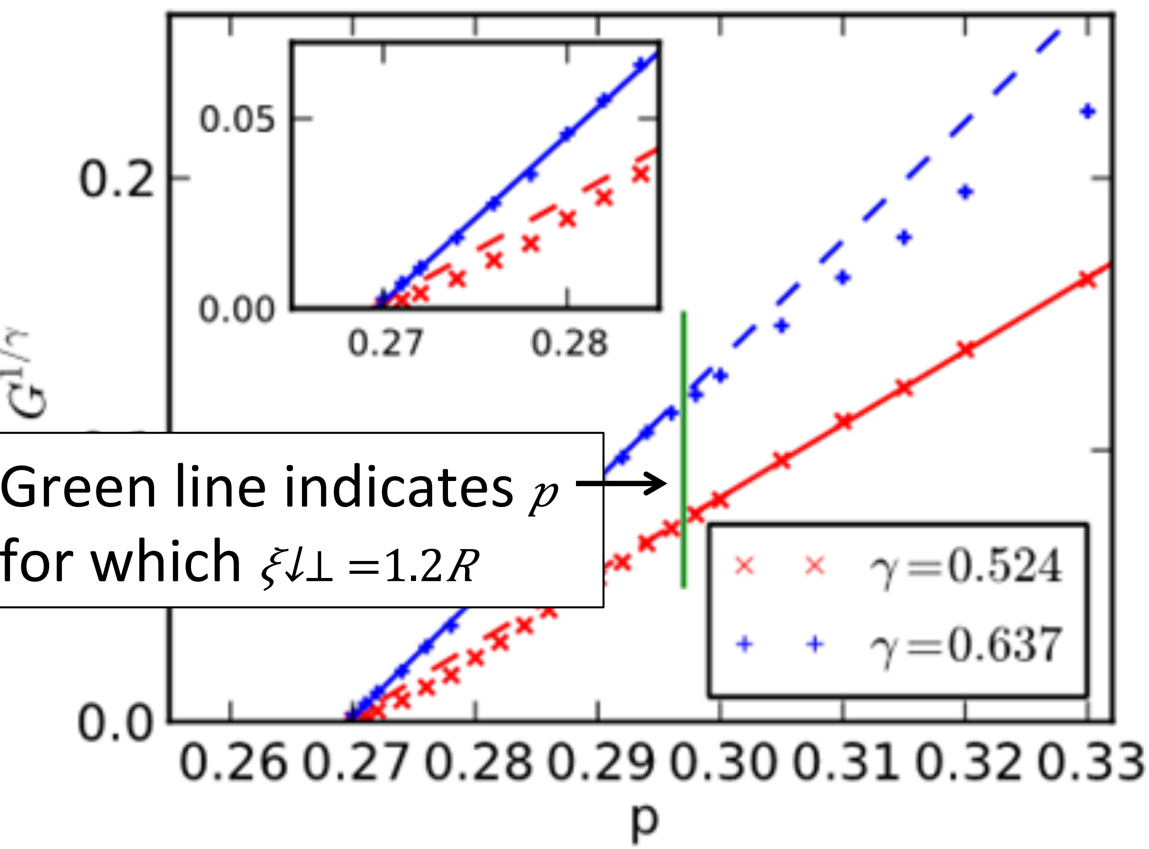


Green line indicates p for which $\xi_\perp = 1.2R$

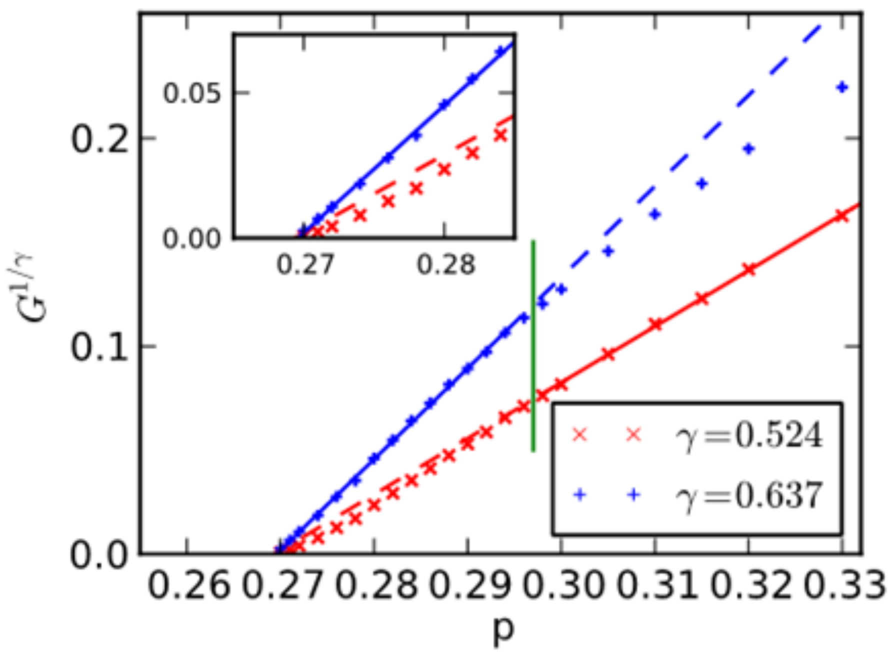
Can’t use p_c from the literature since it depends on the size of the system.

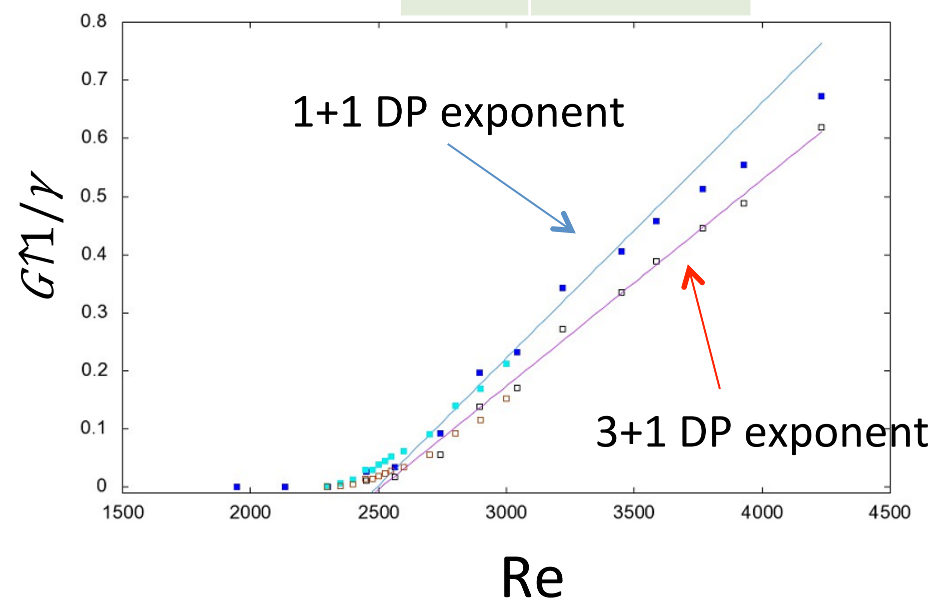
Experimental measurements of slug fronts

- In 3+1 DP, $G \sim (p - p_c)^{0.524}$
- In 1+1 DP, $G \sim (p - p_c)^{0.637}$

dim	$\nu_\parallel - \nu_\perp$
1+1	0.637
2+1	0.561
3+1	0.524

Preliminary data:

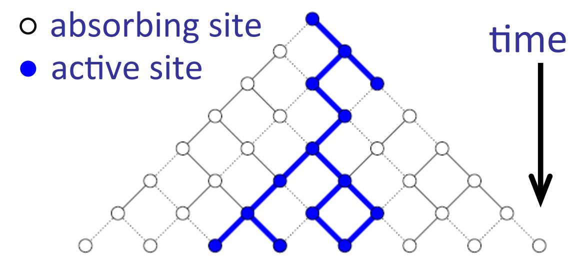




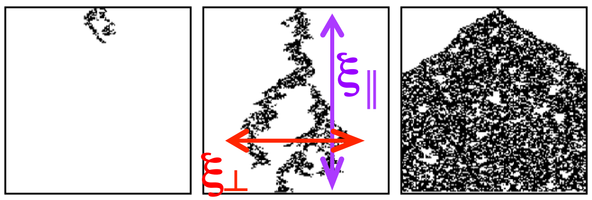
Hof *et al*. (Unpublished, 2010)

Summary: Transitional Turbulence as DP

- Transitional turbulence ~ Directed percolation (Pomeau, 1986)
- Directed percolation (DP)
  - percolating probability ***p*** at each site
  - absorbing state → laminar flows
  - active state → turbulent slugs



- Critical transition threshold p_c:

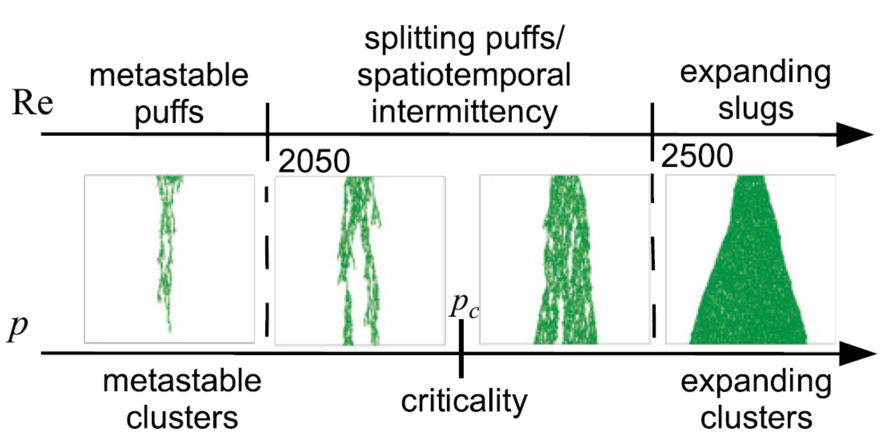


$p < p_c$ $p \approx p_c$ $p > p_c$

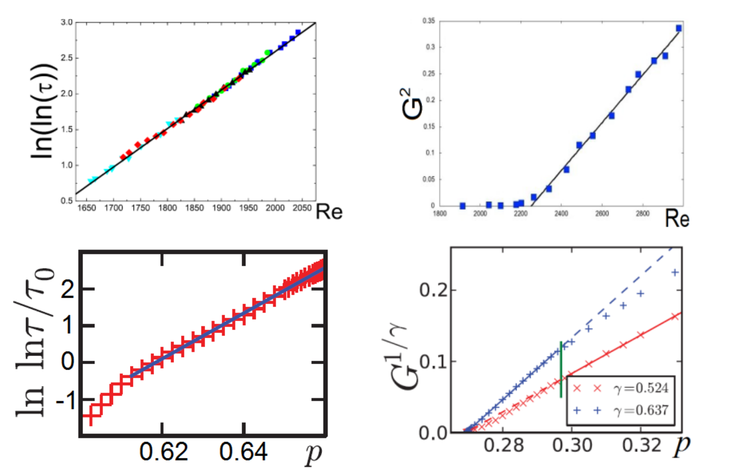
$p \approx p_c,$ correlation length $\xi_\parallel \sim |p - p_c|^{\nu_\parallel}$, $\xi_\perp \sim |p - p_c|^{\nu_\perp}$

growth rate $G \sim \xi_\perp / \xi_\parallel \sim (p - p_c)^{\nu_\parallel - \nu_\perp}$

- Turbulence vs. (3+1) DP in pipe:



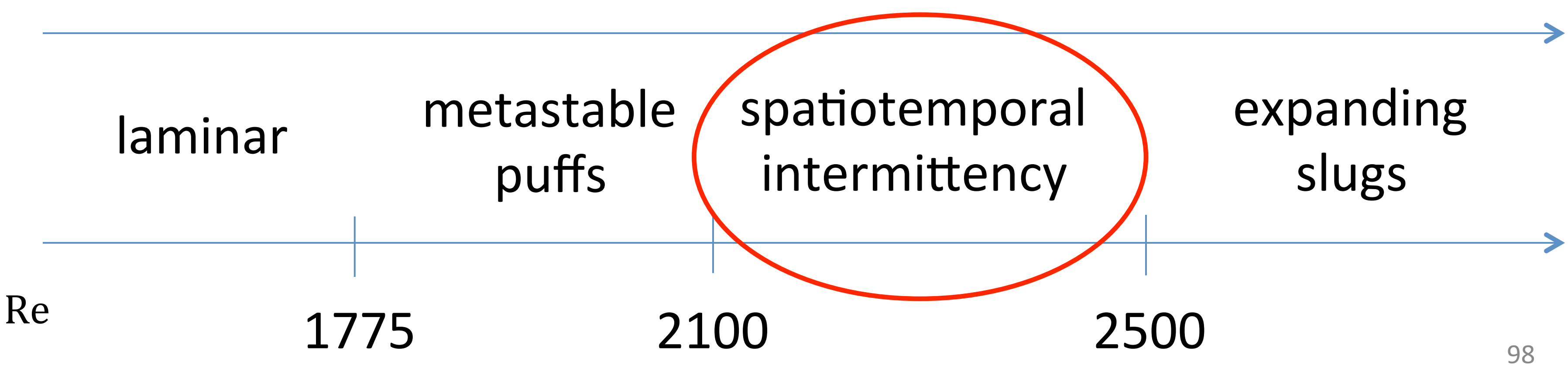
Hof et al., *PRL* **101**, 214501 (2008) De Lozar et al. arXiv:1001.2481 (2010)



Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

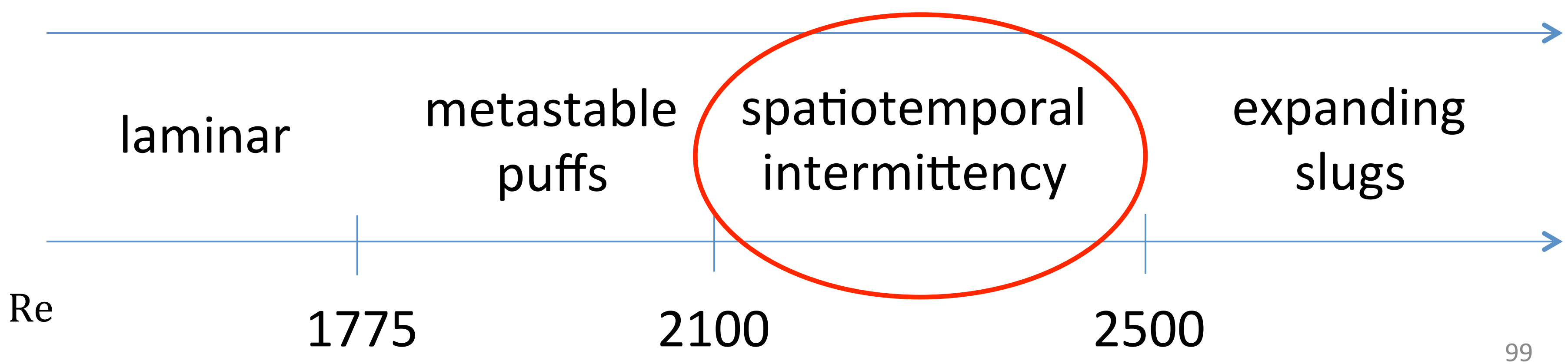
Hinrichsen, *Adv. Phys.* **49**, 815 (2000) Henkel, Non-Equilibrium Phase Transitions vol.1 (2008) skip this section

MODEL FOR SPATIOTEMPORAL INTERMITTENCY



MODEL FOR SPATIOTEMPORAL INTERMITTENCY

Very complex behavior and we need to understand precisely what happens at the transition, and where the DP universality class comes from.



How to model transitional turbulence?

- Statistical description of phase transitions based on effective (Landau) theory for:
  - Order parameter
  - Collective modes
  - Hydrodynamic modes (long-wavelength, long-time)

- Effective theory functional form determined by symmetry, conservation laws
  - Direct derivation from microscopic theory usually not possible
  - Direct derivation from microscopic theory usually not desirable, because technical assumptions restrict the regime of validity of the effective theory

Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

↓

Landau theory

↓

RG universality class

Logic of modeling phase transitions

Magnets	Turbulence
Electronic structure	Kinetic theory
↓	↓
Ising model	Navier-Stokes eqn
↓	↓
Landau theory	**?**
↓	↓
RG universality class	**?**

Logic of modeling phase transitions

Magnets		Turbulence
Electronic structure		Kinetic theory
↓		↓
Ising model		Navier-Stokes eqn
↓		↓
Landau theory	⟷	?
↓		↓
RG universality class	⟷	?

Identification of collective modes at the laminar-turbulent transition

To avoid technical approximations,
we use DNS of Navier-Stokes

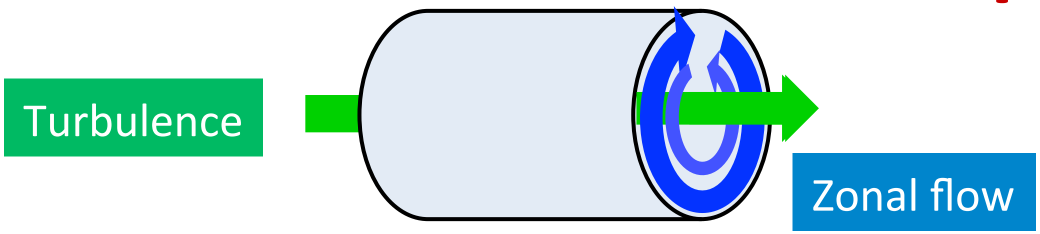
How to model transitional turbulence?

- Pipe flow consists of two regions, turbulence and roughly laminar large scale flow

How to model transitional turbulence?

- Pipe flow consists of two regions, turbulence and roughly laminar large scale flow
- The large scale flow is driven by the turbulent fluctuations
- The large scale flow suppresses the turbulent fluctuations
- Suggests: transitional turbulence = predator-prey ecosystem

Observation of predator-prey oscillations in numerical simulation of pipe flow



Simulation based on the open source code by Ashley Willis: openpipeflow.org

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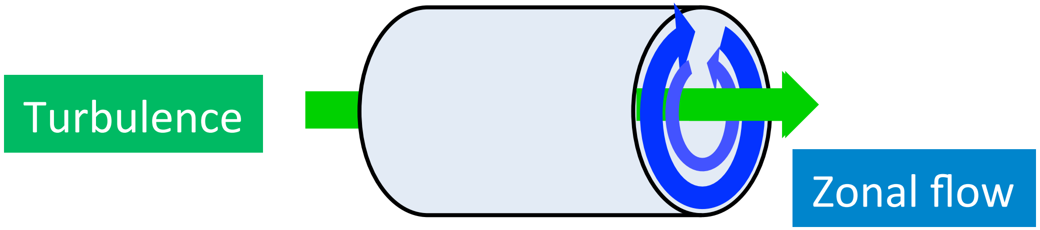

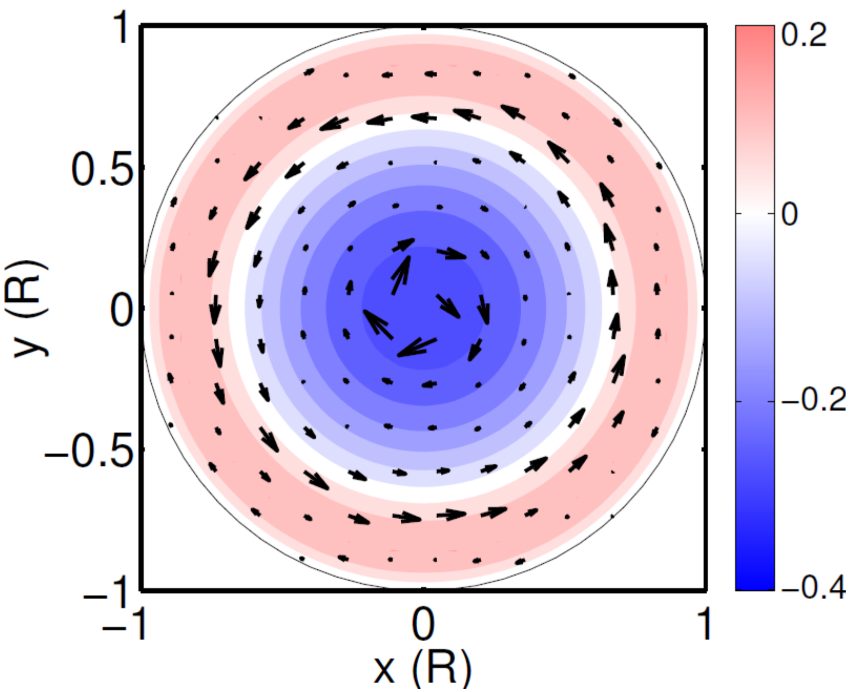

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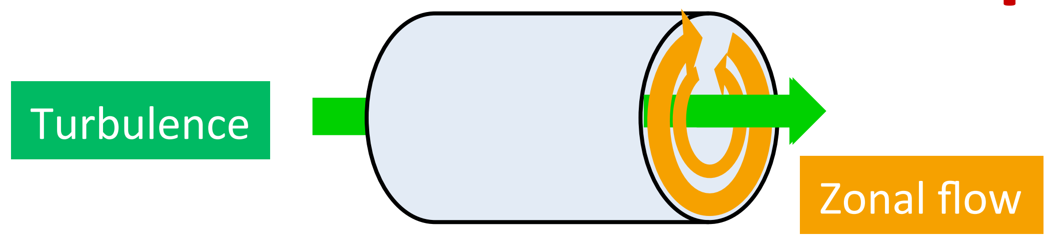

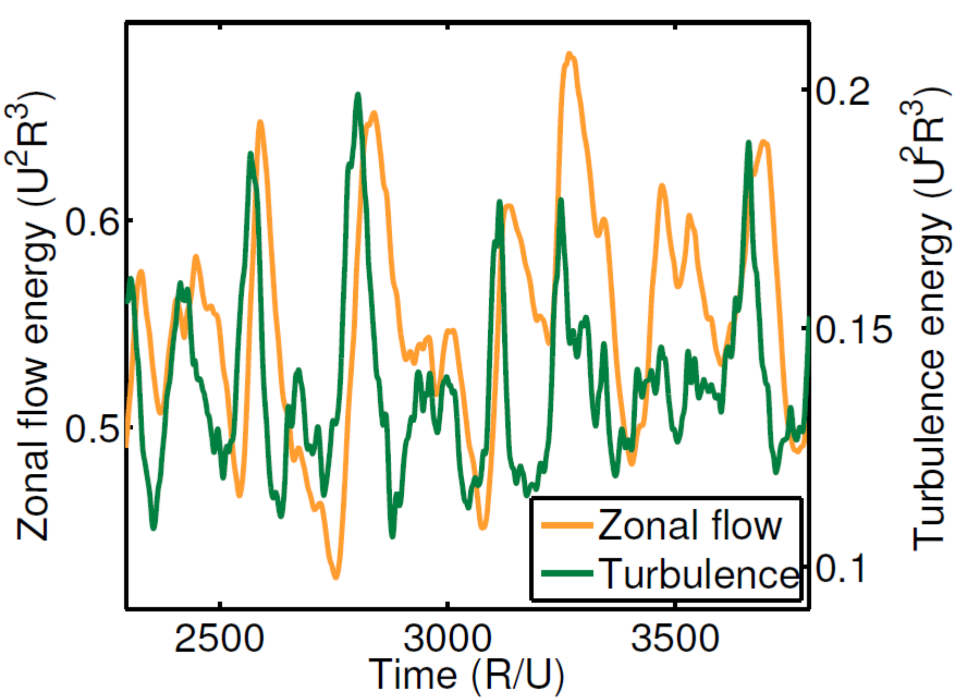

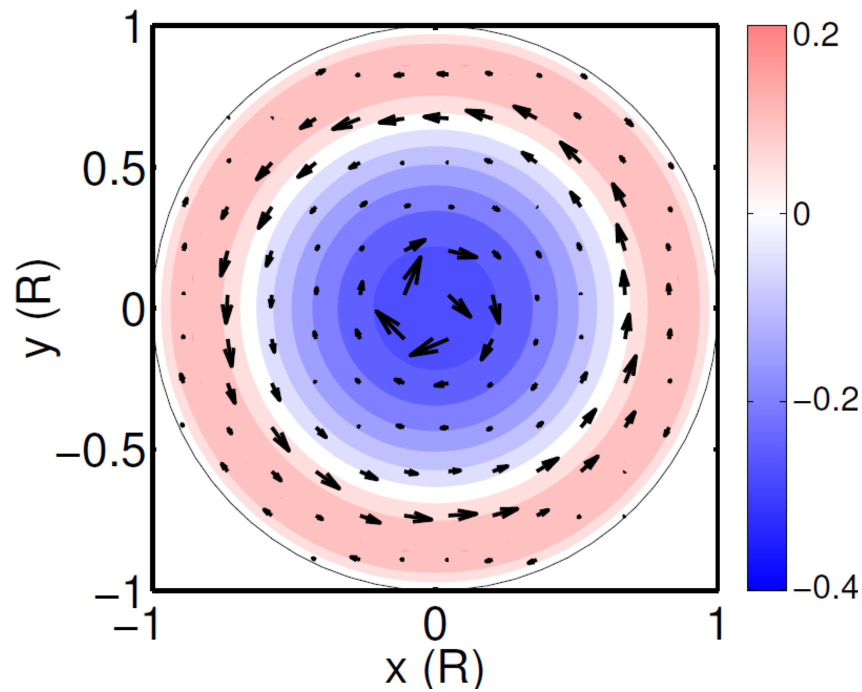

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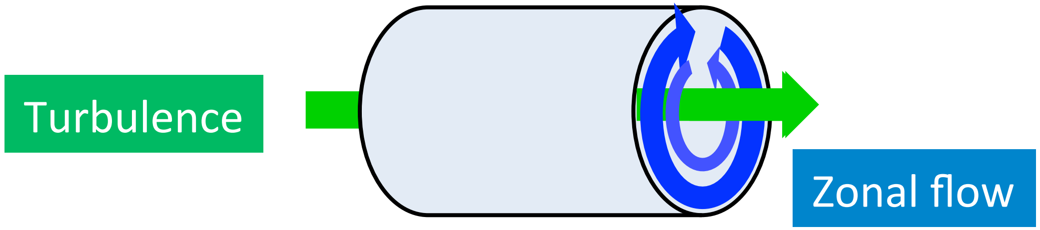

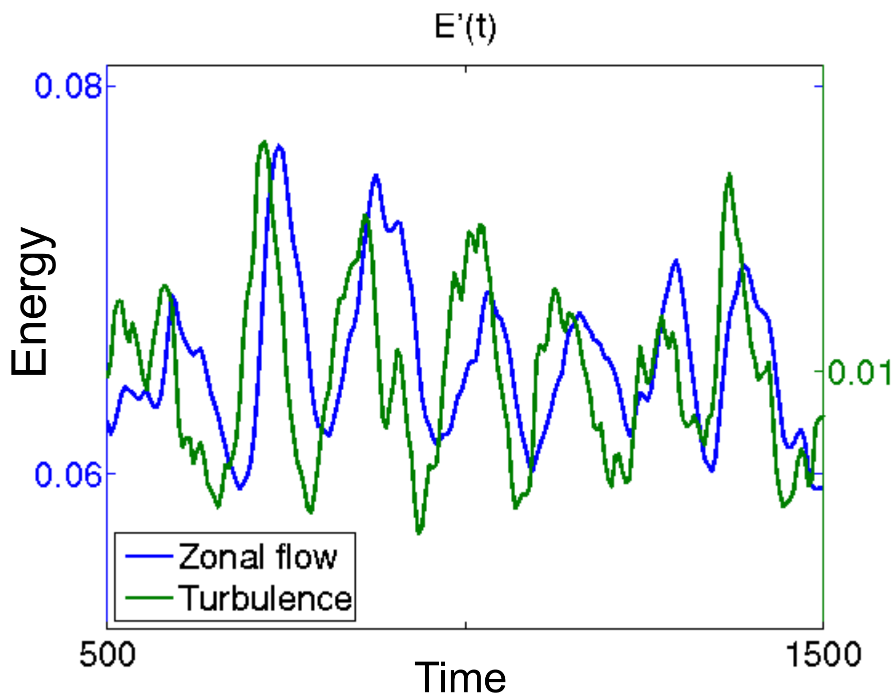

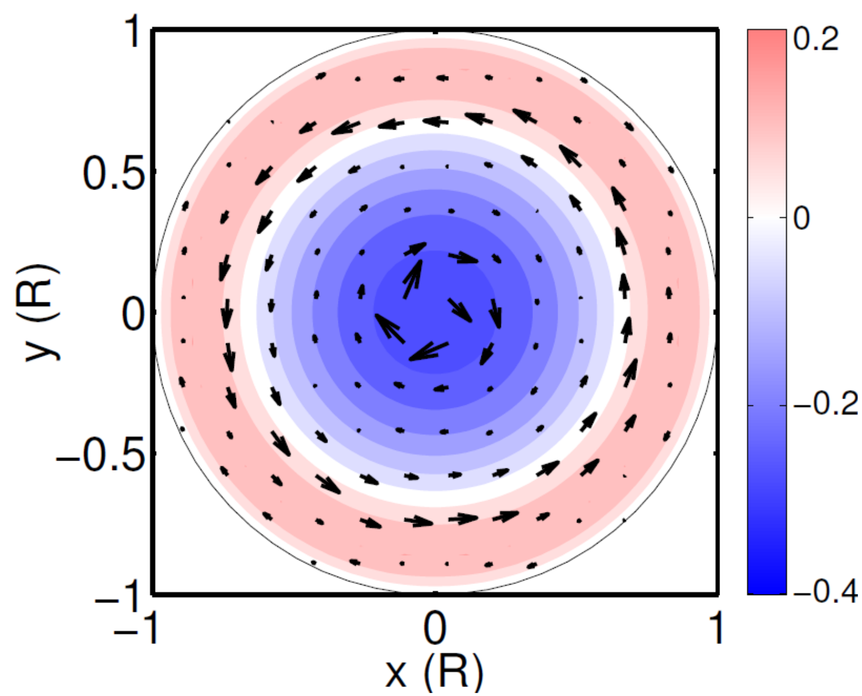

Re = 2600

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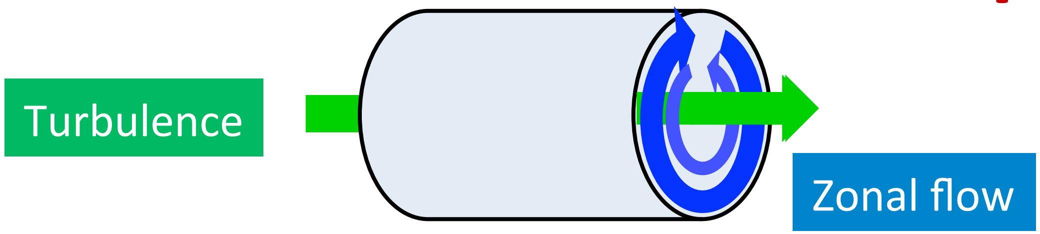

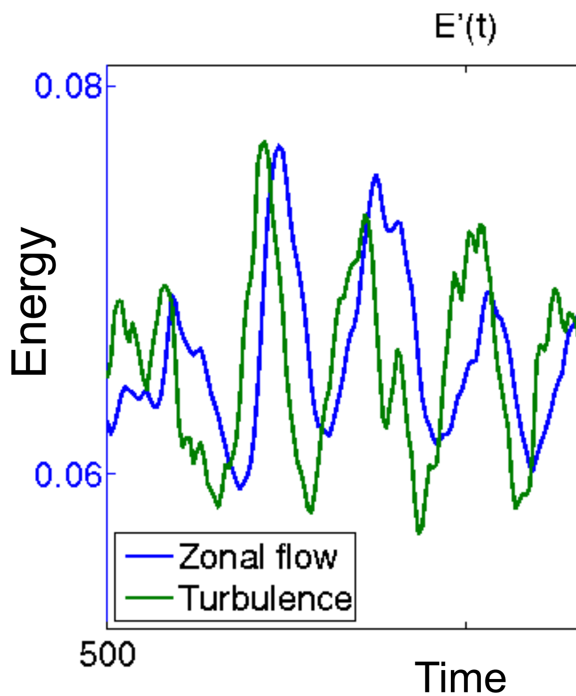

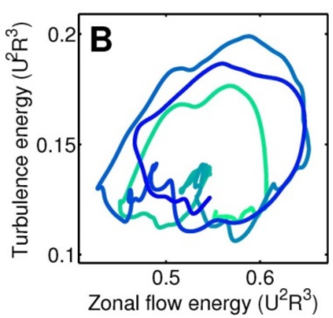

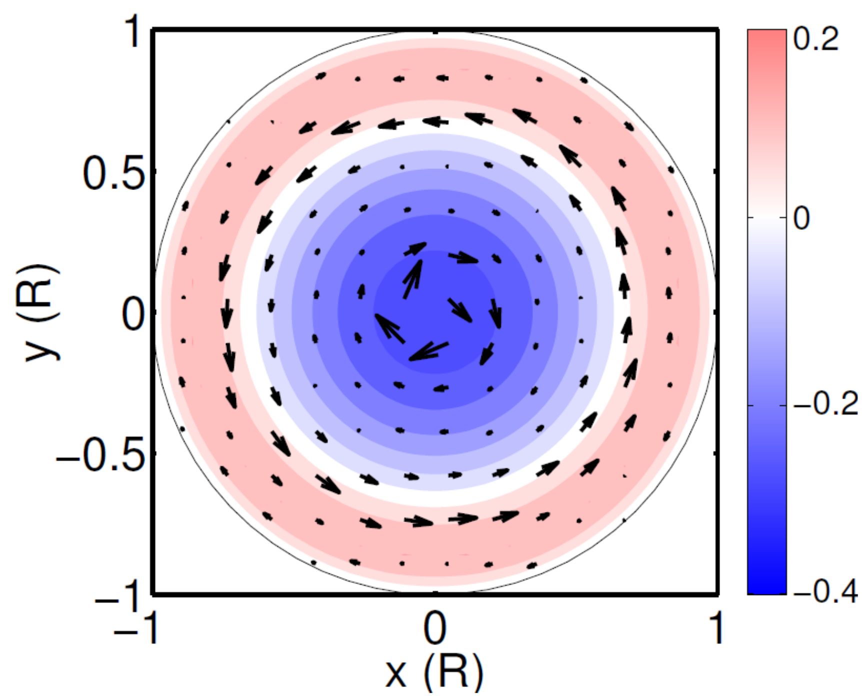

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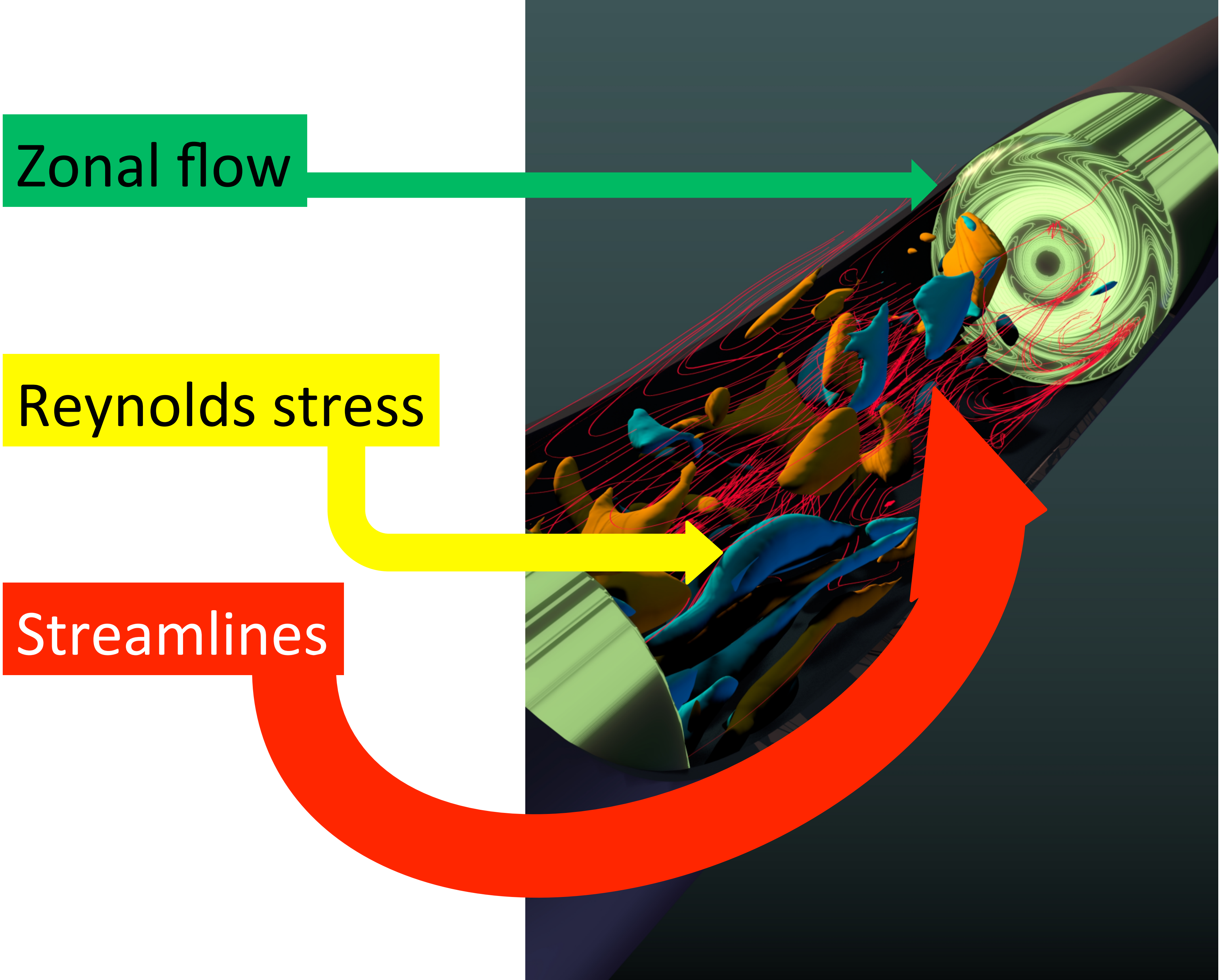
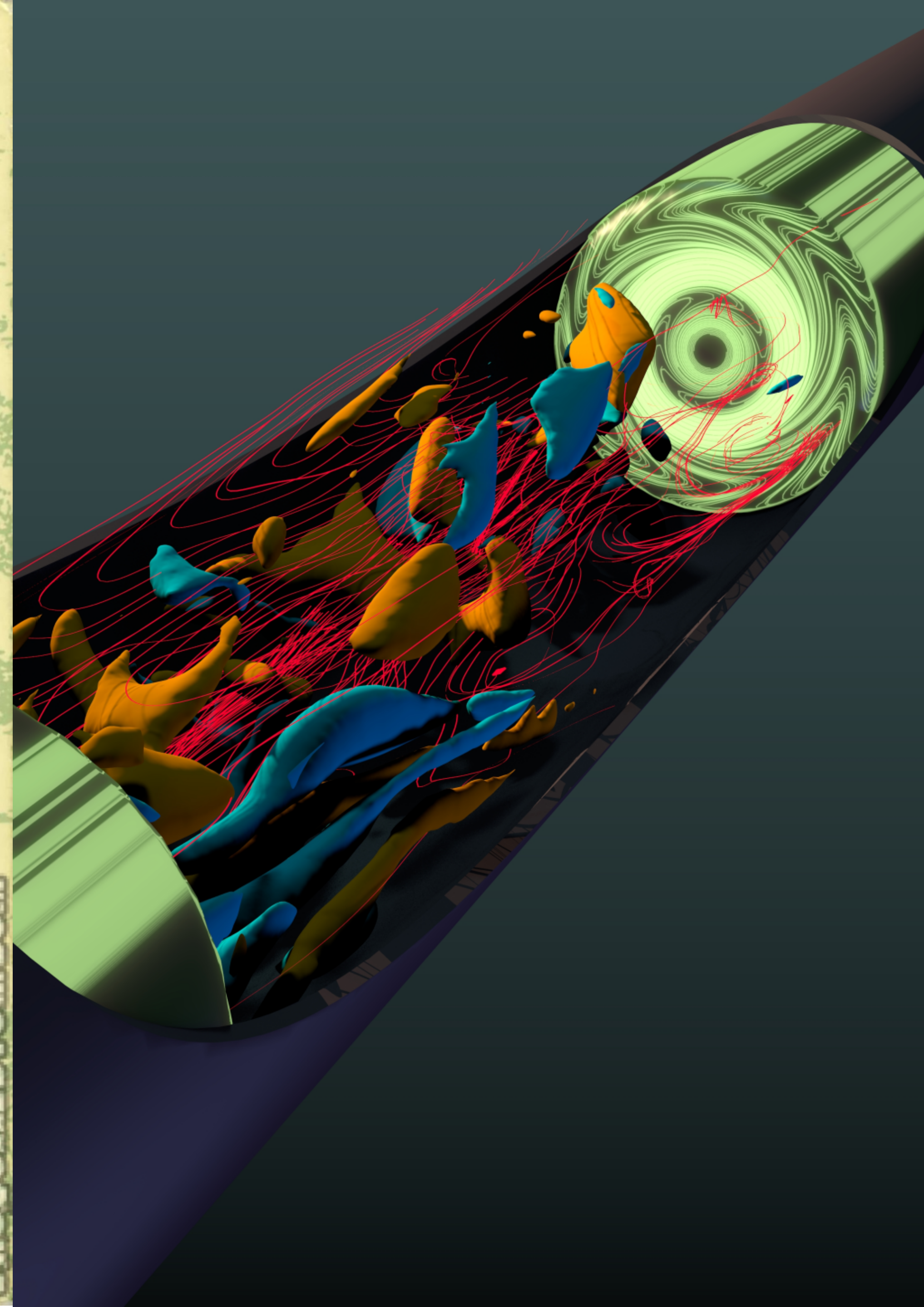
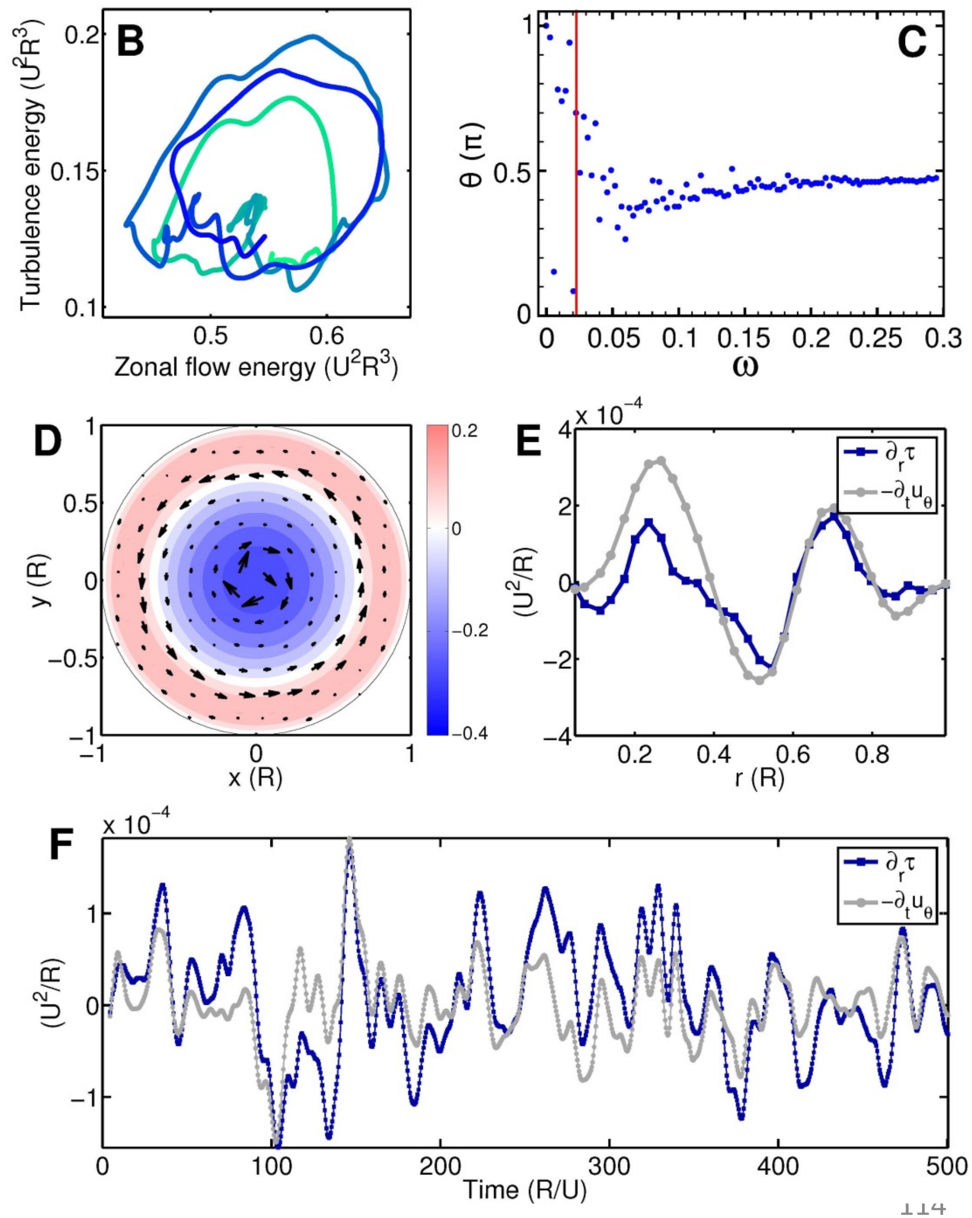

Zonal flow
Reynolds stress
Streamlines


Disney
WINNIE
THE POOH
Winnie The Pooh
AND THE BLUSTERY DAY
Based on A.A. Milne's Classic Tales
VHSCOLLECTOR.COM



Characterizing predator-prey dynamics

- Oscillations phase shifted by $\pi/2$
- Zonal flow is correlated with the radial gradient of the Reynolds stress
  - In space
  - In time



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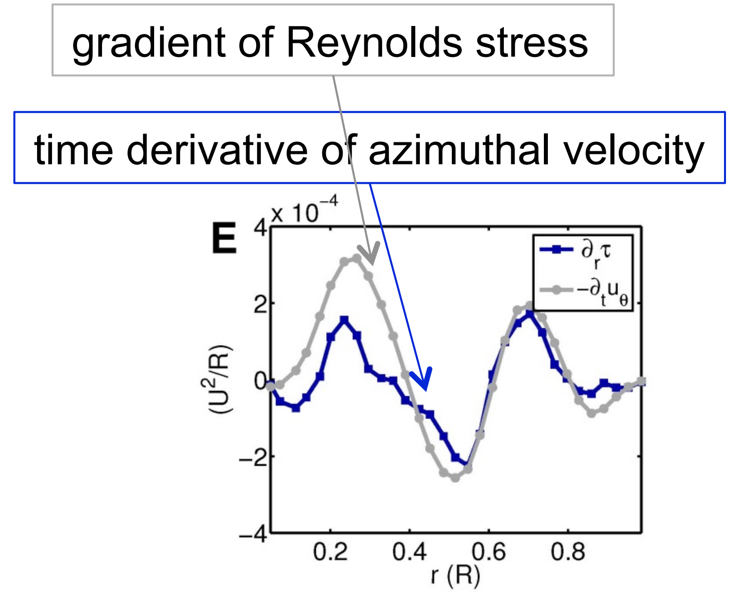
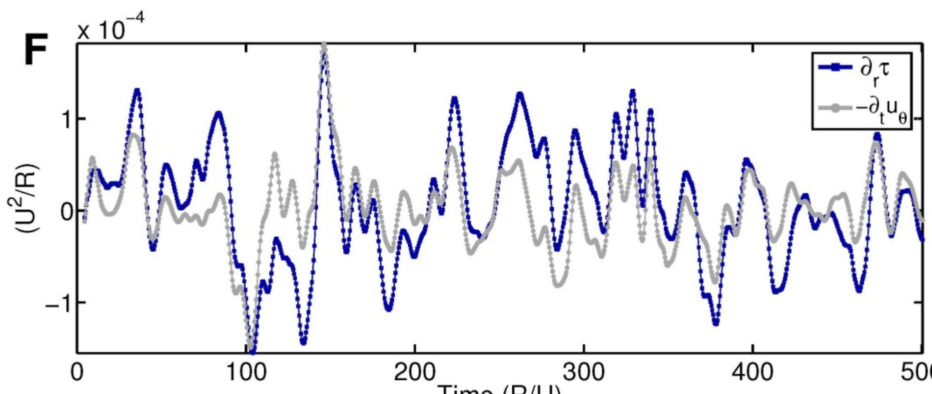
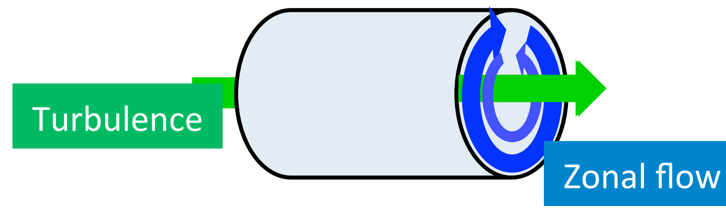




What drives the zonal flow?

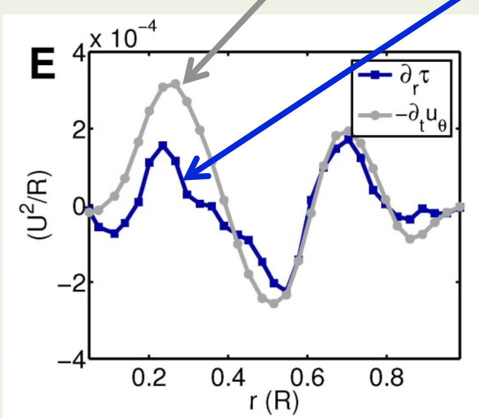
- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence

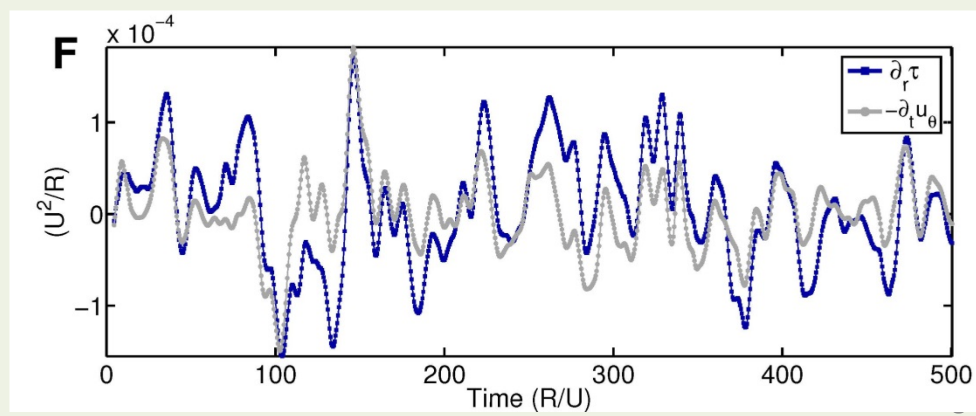


1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence





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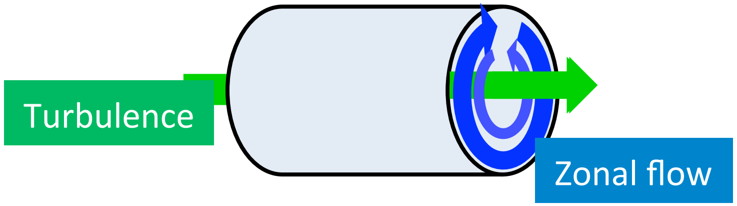

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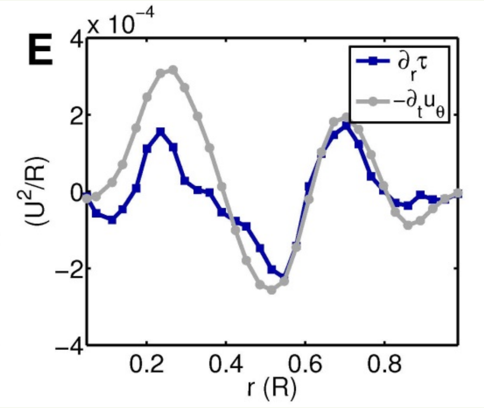

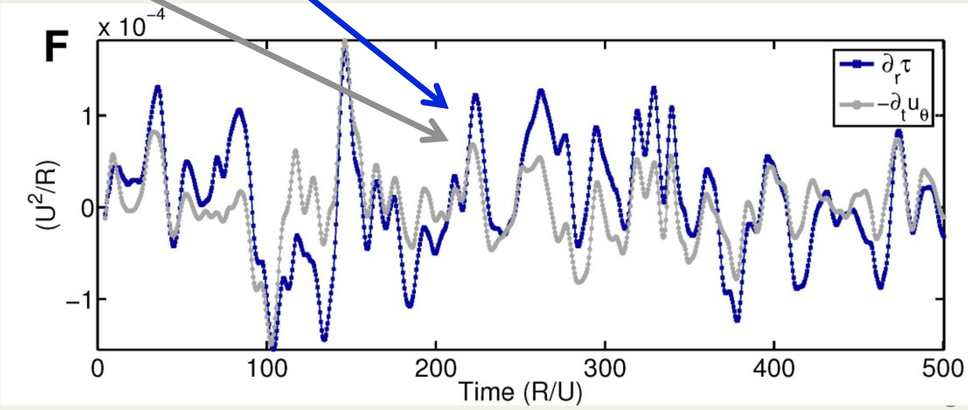

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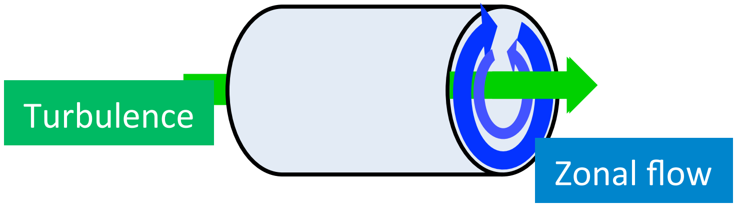

- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence

1) Anisotropy of turbulence creates Reynolds stress which generates the mean strain shear in azimuthal direction

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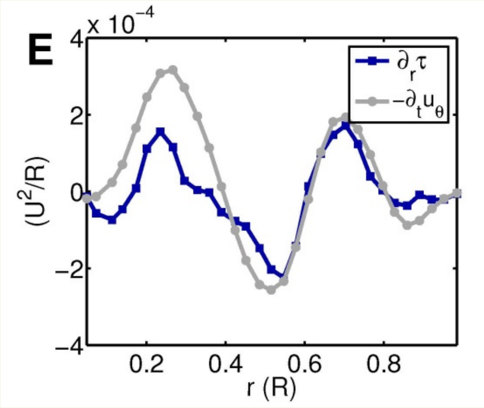
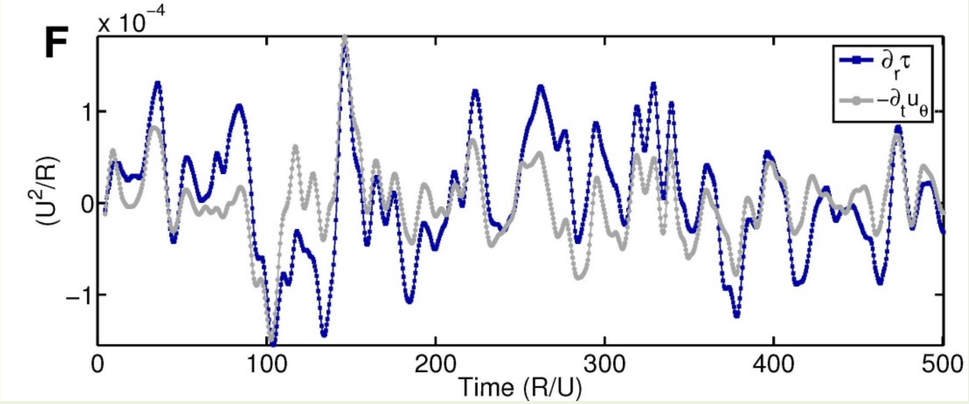
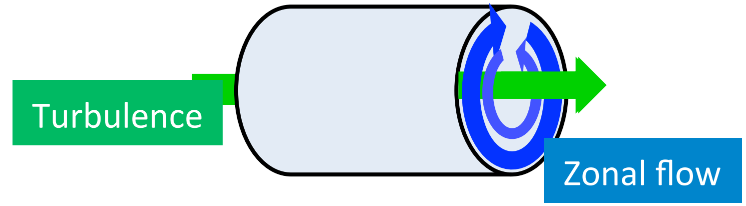




What drives the zonal flow?



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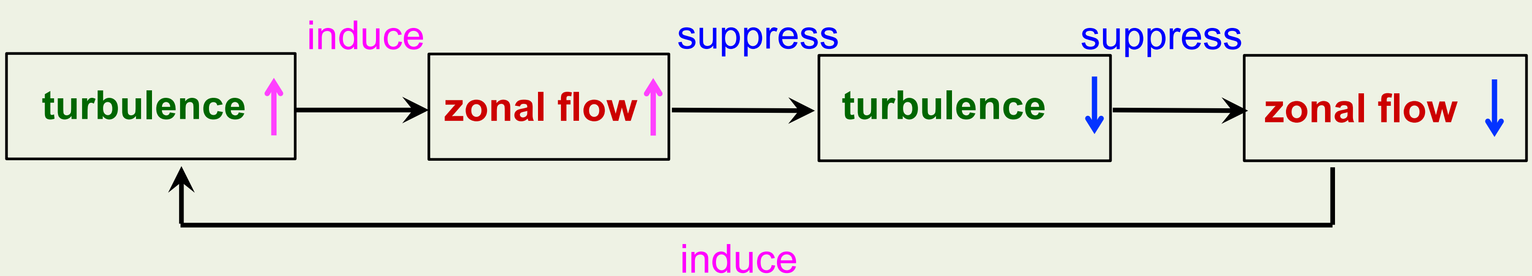

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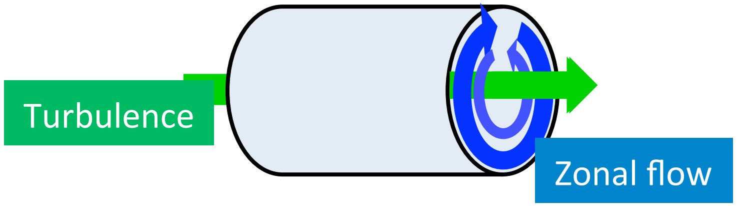

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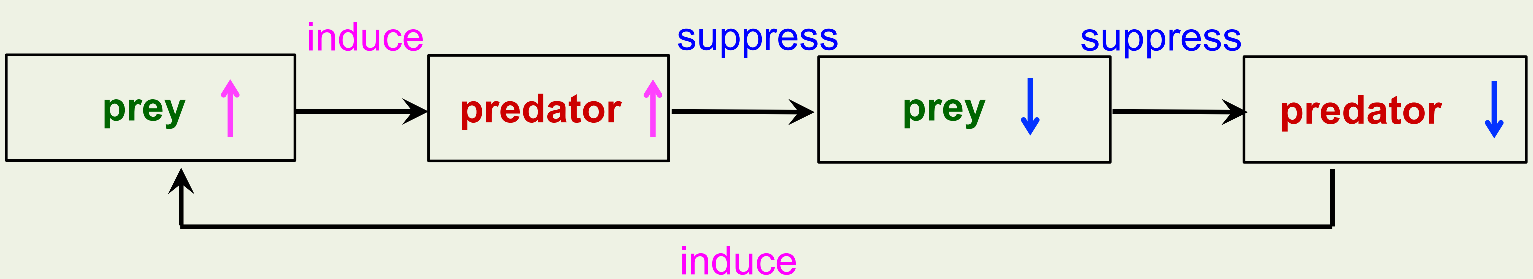

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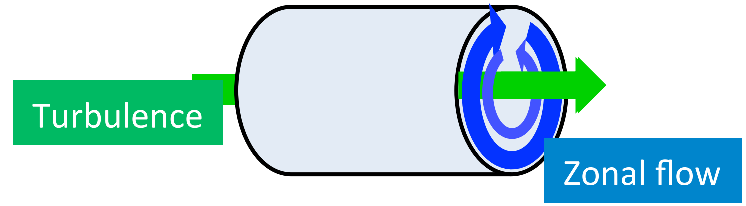

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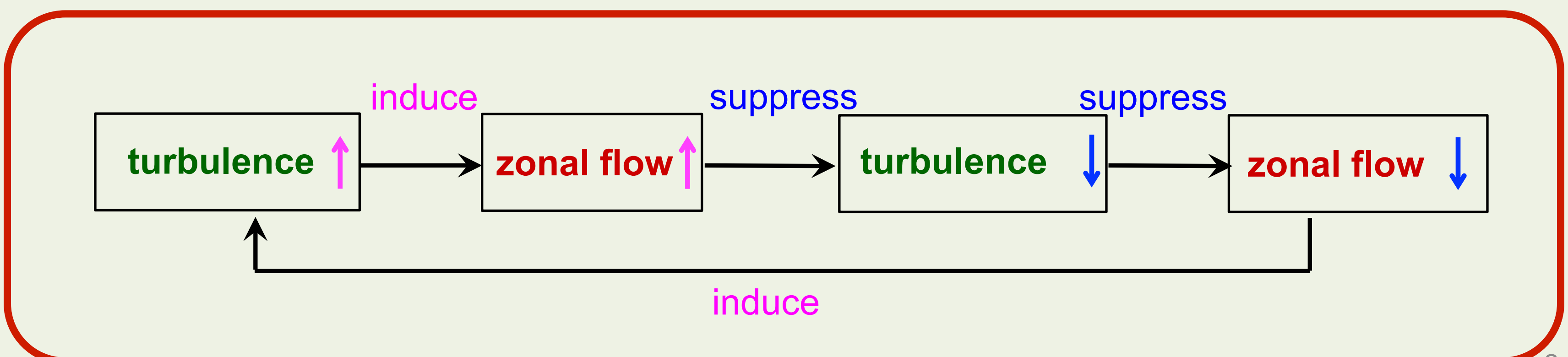

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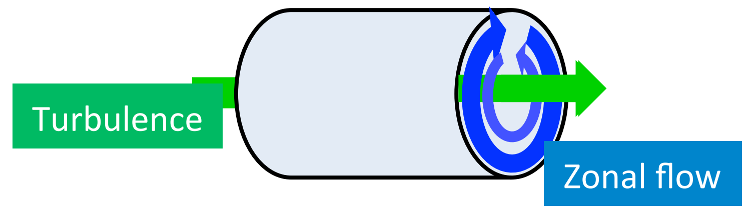

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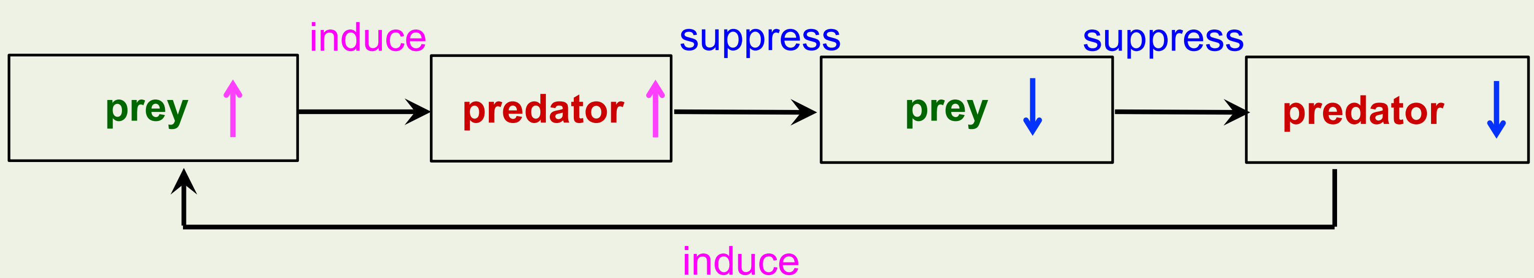

Normal population cycles in a predator-prey system

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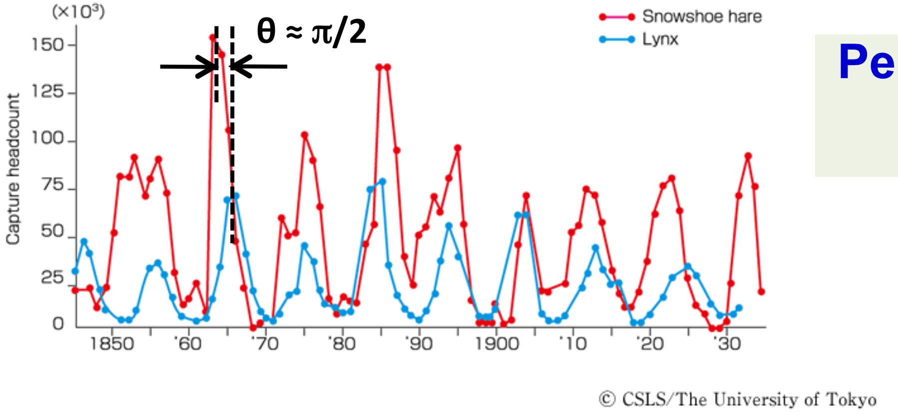
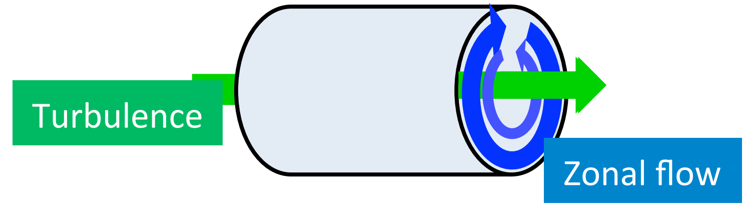


Persistent oscillations
+
Fluctuations

https://interstices.info/jcms/n_49876/des-especes-en-nombre

Ecology of turbulence



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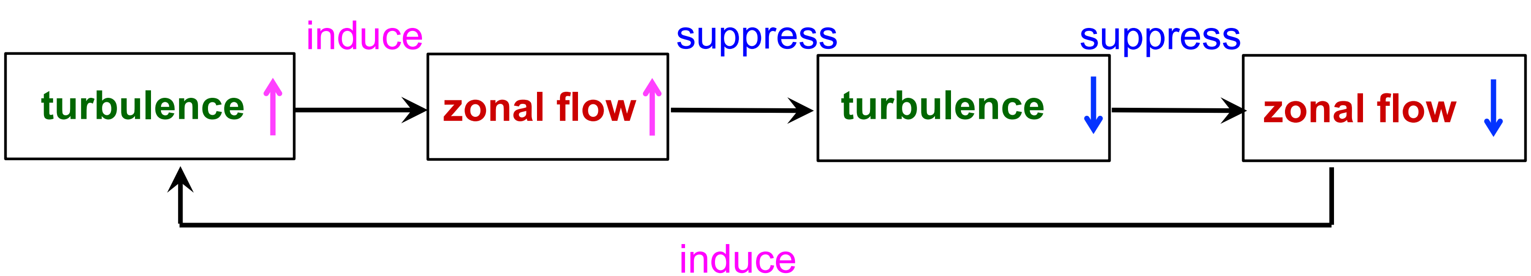

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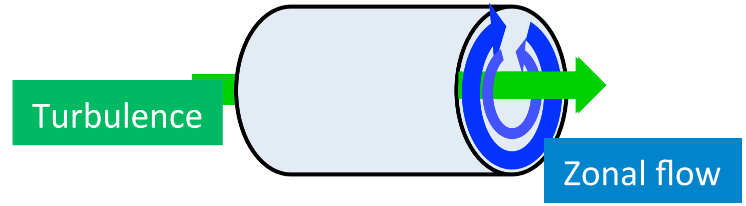

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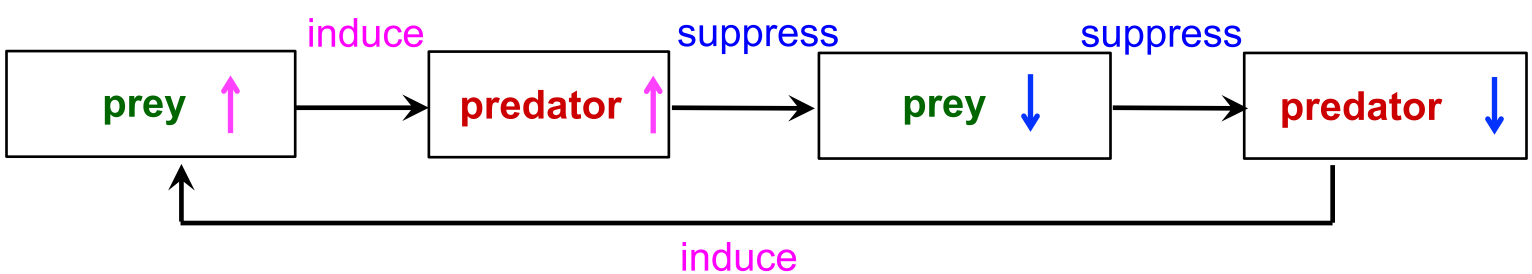

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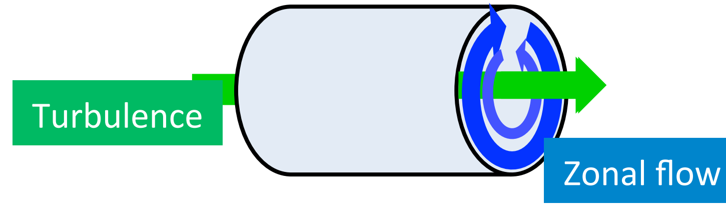

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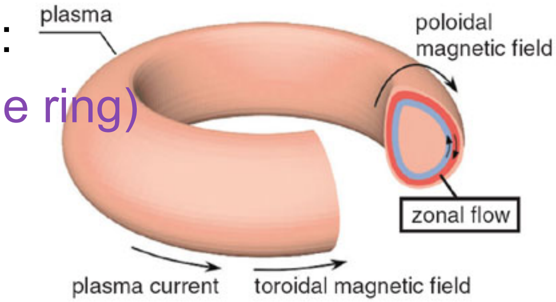
Laminar flow → Turbulenc e → Zonal flow

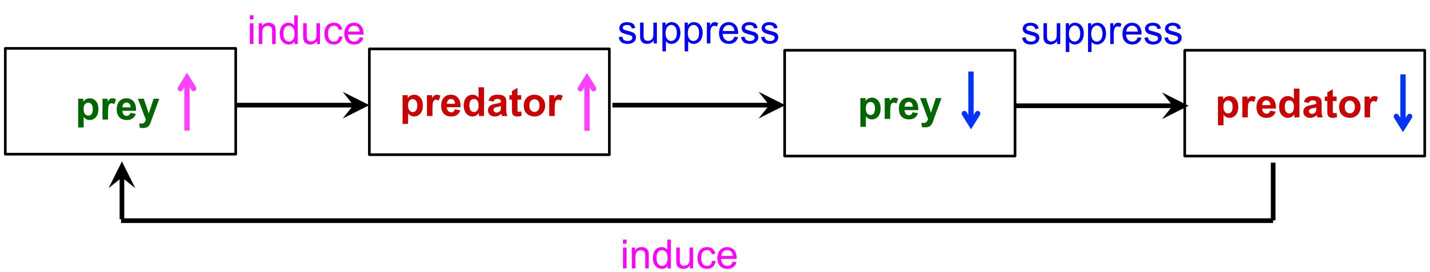
Nutrient → Prey → Predator

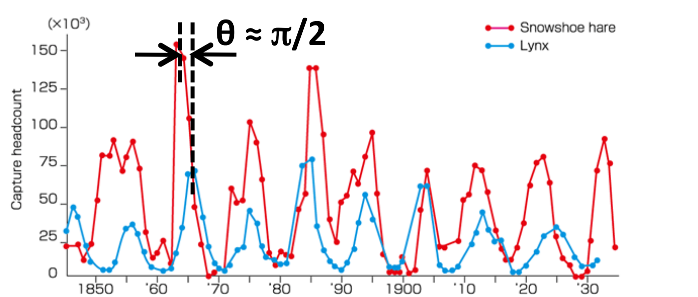
Predator-Prey Dynamics in Tokomaks

- In tokamak (toroidal chamber with axial magnetic field):
  - **turbulent plasma** (small-scale drift waves along the ring)
  - **zonal flows**:
    - E_r x B turbulence-induced flow on small circles
    - cause radial shear to damp turbulent plasma
    - decrease due to dissipation



- Self-organized dynamics in **Ecology**:





https://interstices.info/jcms/n_49876/des-especes-en-nombre

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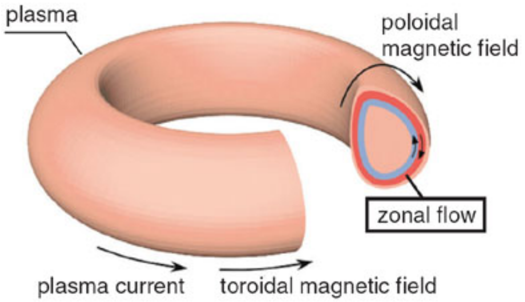
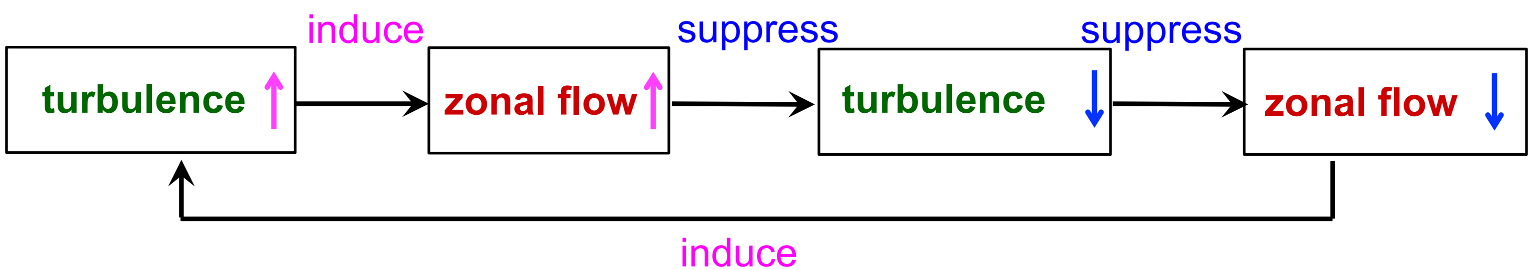
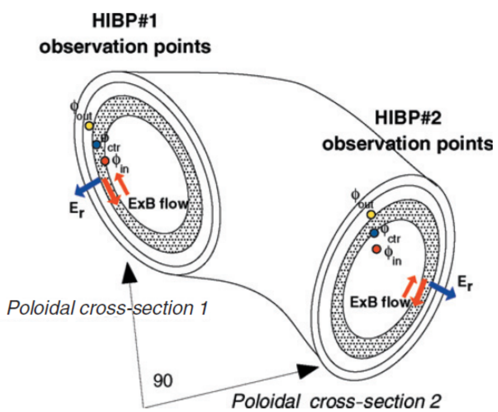
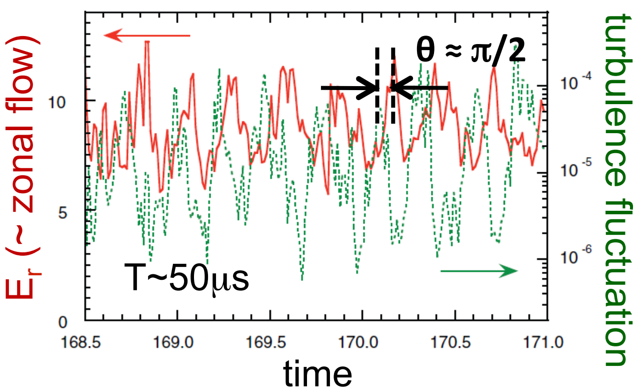


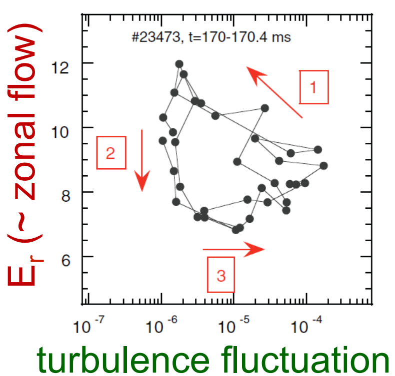
- Self-organized dynamics in **Magneto-hydrodynamics**:



Estrada et al. EPL (2012)







Ecology model for turbulence

Laminar flow → Turbulence → Zonal flow

Nutrient (E) → Prey (B) → Predator (A)

$$B + E \xrightarrow{b} B + B$$

$$A + B \xrightarrow{p} A + A$$

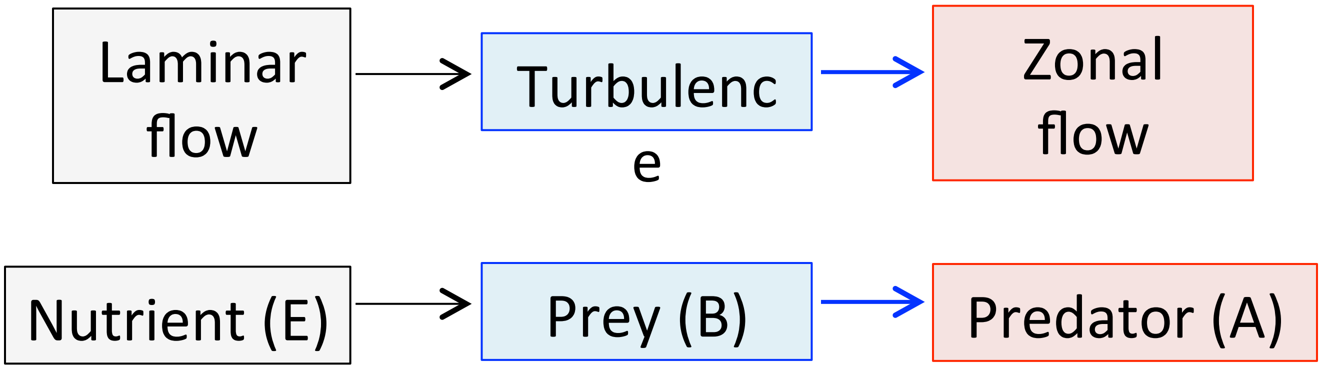
$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

$$B \xrightarrow{m} A$$

mean-field rate equation:

$$\begin{aligned} \frac{dA}{dt} &= pAB - d_A A + mB \\ \frac{dB}{dt} &= b\left(1 - A - B\right) B - pAB \\ &\quad - d_B B - mB \end{aligned}$$

Ecology model for turbulence



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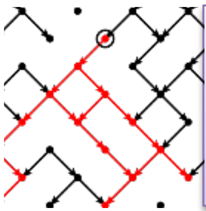
Q. What is the universality class of the transition to turbulence?

Tentative answer: directed percolation … but why?

Strategy: transitional turbulence to directed percolation



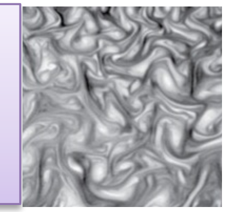
(Wikimedia Commons)

Directed Percolation

?

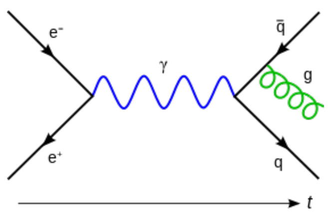
(Classical) Turbulence

(Boffetta and Ecke, 2012)



Reggeon field theory
(Janssen, 1981)

Field Theory



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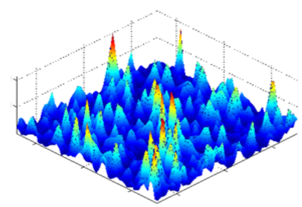


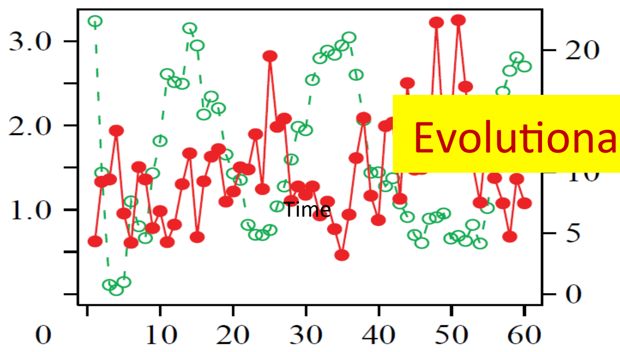
Two-fluid model

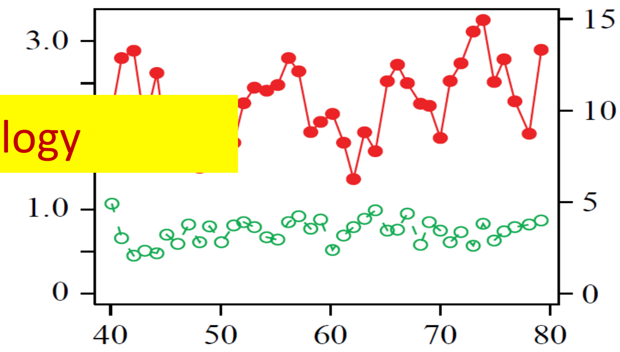
Extinction transition
(Mobilia et al., 2007)

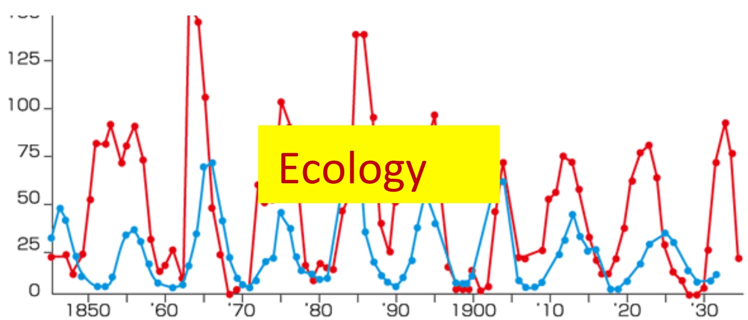
Predator-Prey

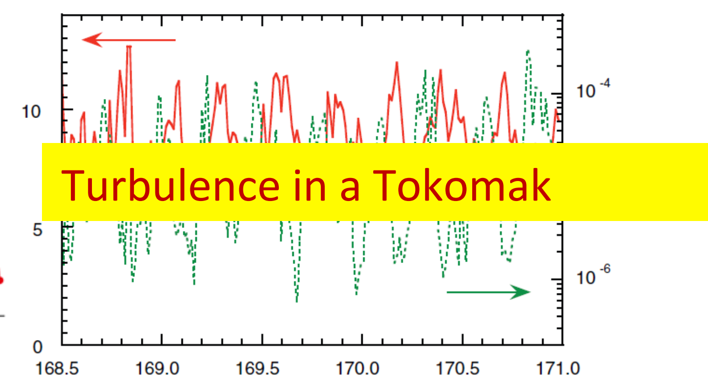
(Pearson Education, Inc., 2009)

Introduction to stochastic predator-prey systems


Evolutionary Ecology
Time
3.0
2.0
1.0
0
10
20
30
40
50
60
20
5
0


3.0
1.0
0
40
50
60
70
80
15
10
5
0

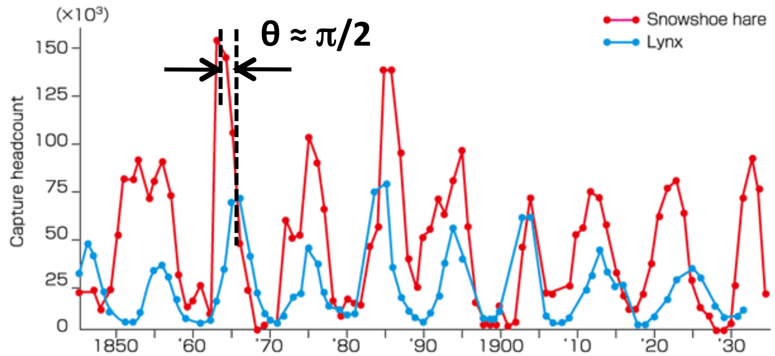

Ecology
125
100
75
50
25
0
1850
'60
'70
'80
'90
1900
'10
'20
'30


Turbulence in a Tokomak
10
5
0
10^{-4}
10^{-6}
168.5
169.0
169.5
170.0
170.5
171.0

Normal population cycles in a predator-prey system

Resource → Prey → Predator

$\pi/2$ phase shift between prey and predator population



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Persistent oscillations
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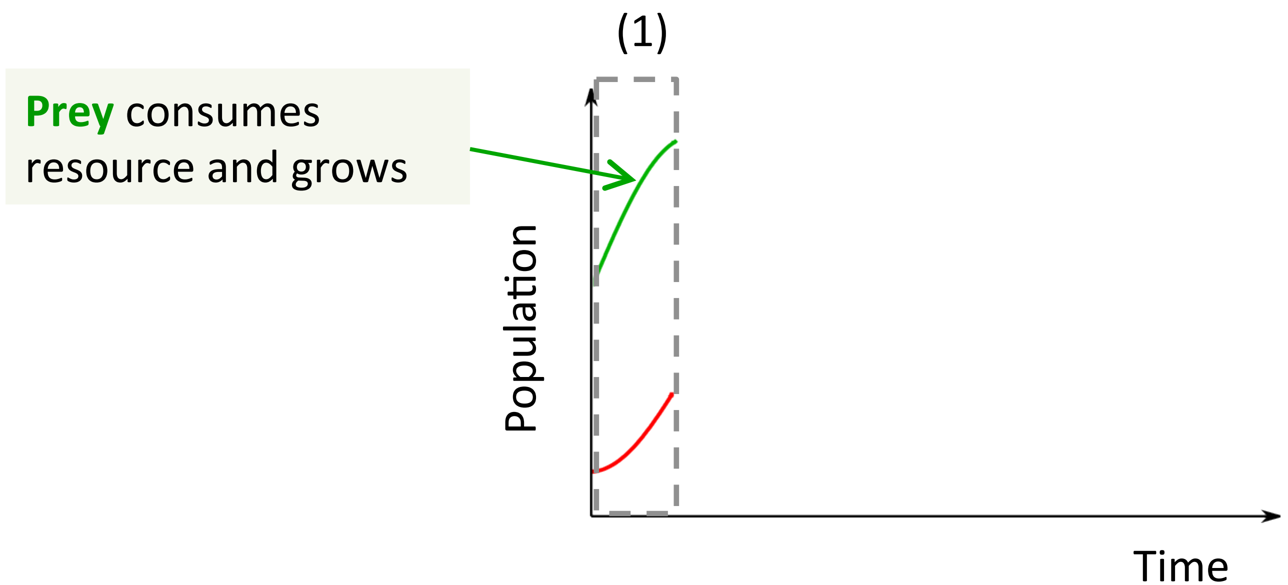
Cartoon picture for normal cycles (π/2 phase shift)



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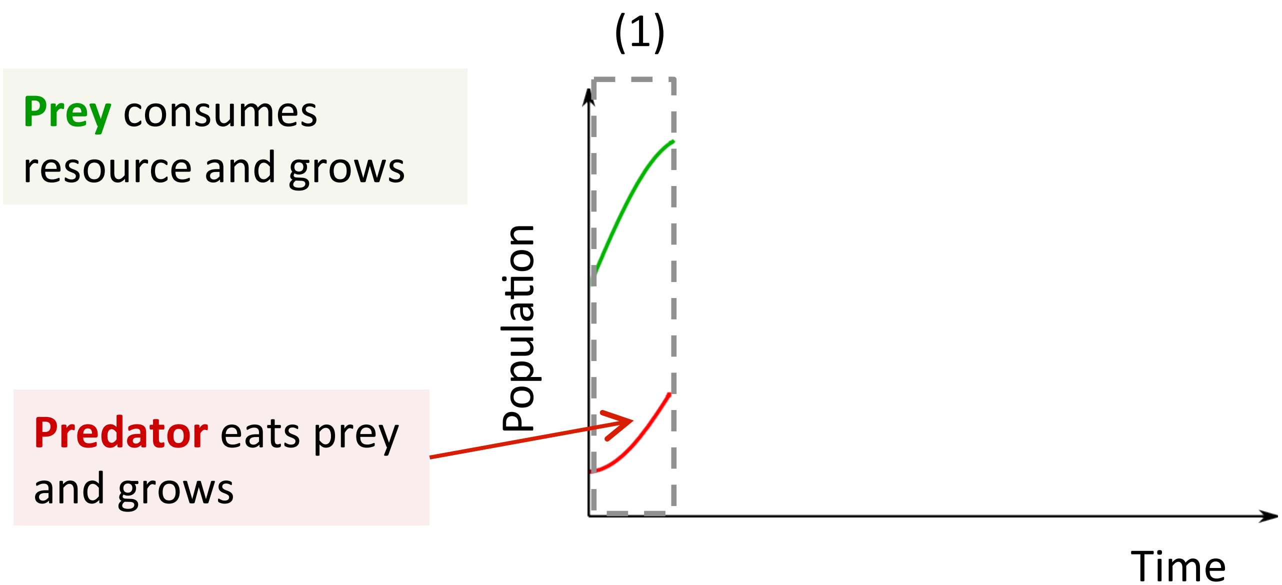

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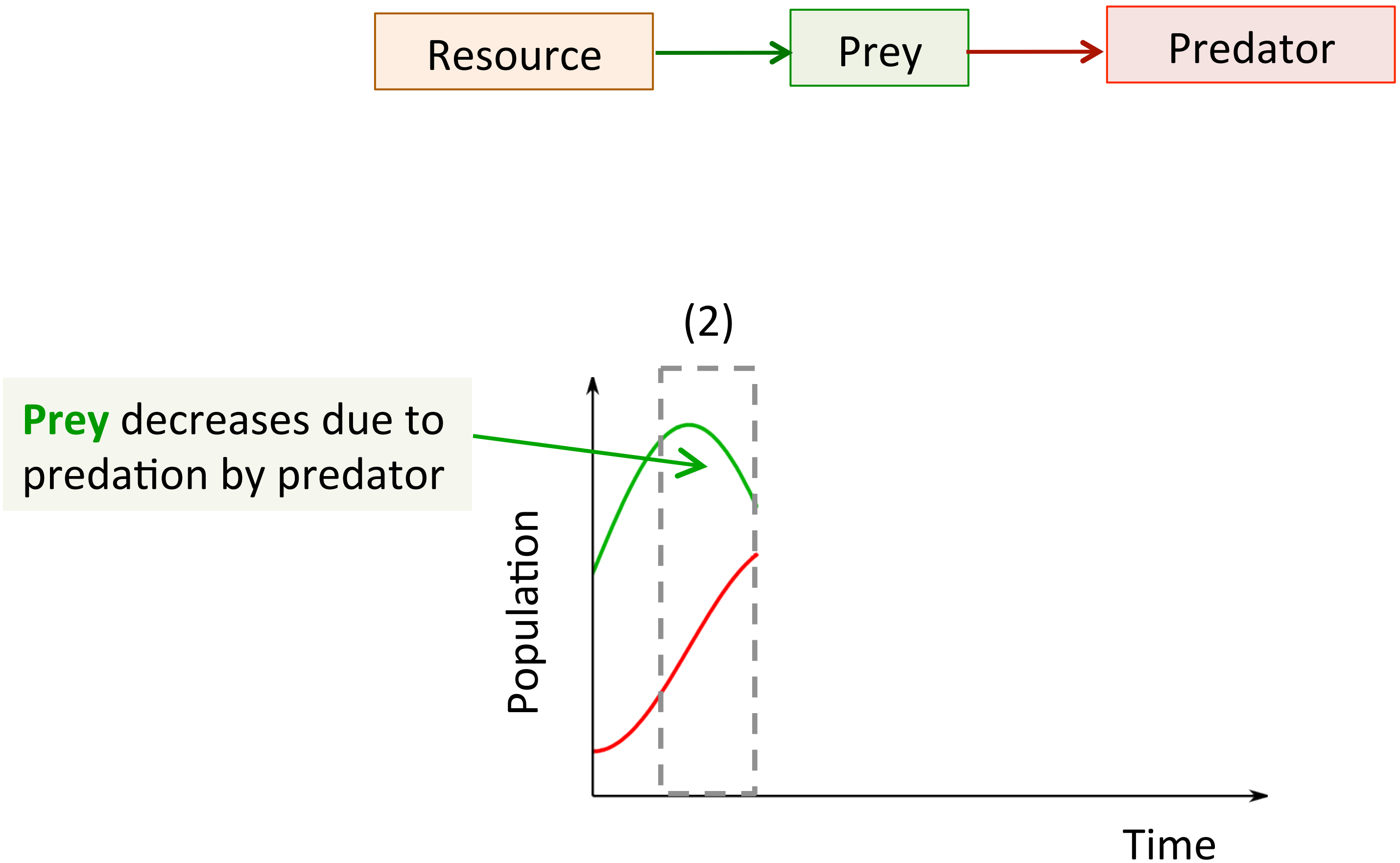
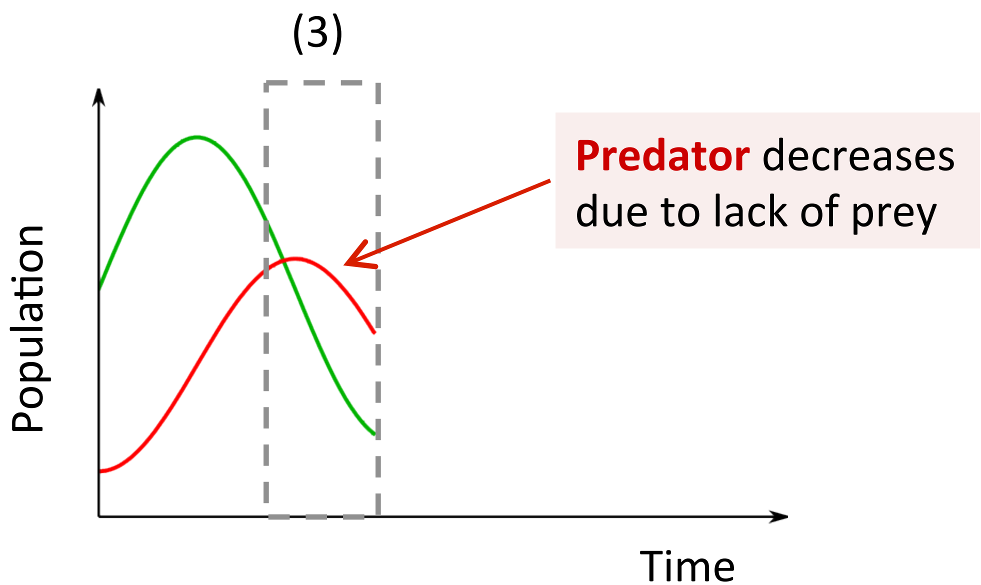


Cartoon picture for normal cycles ($\pi/2$ phase shift)



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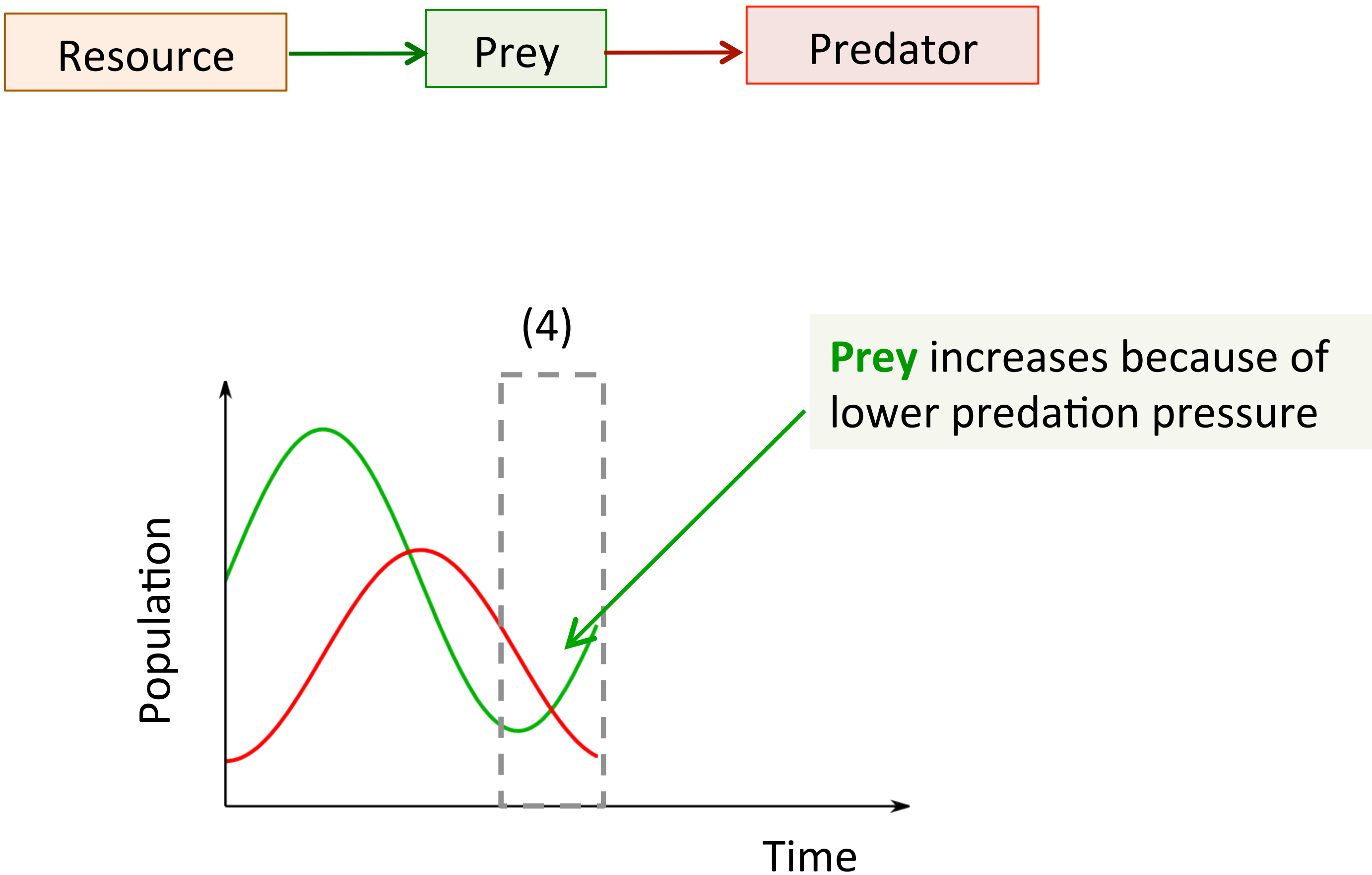

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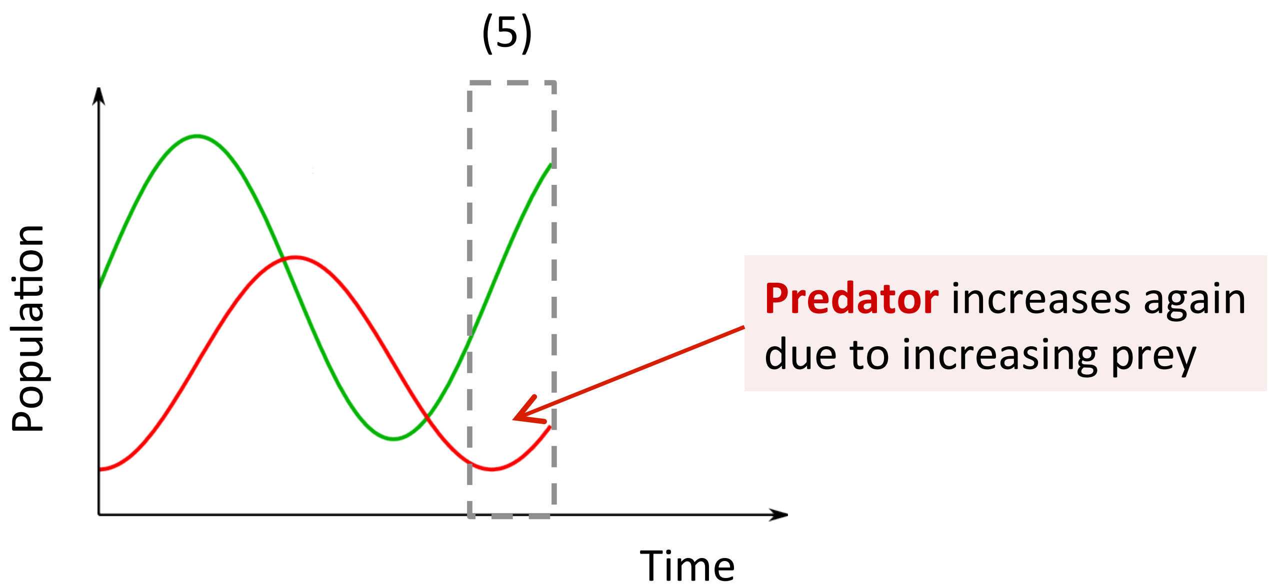

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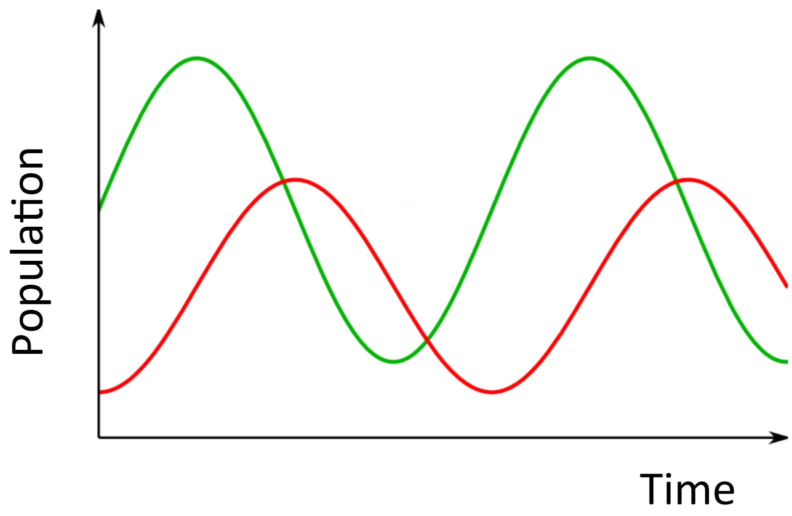
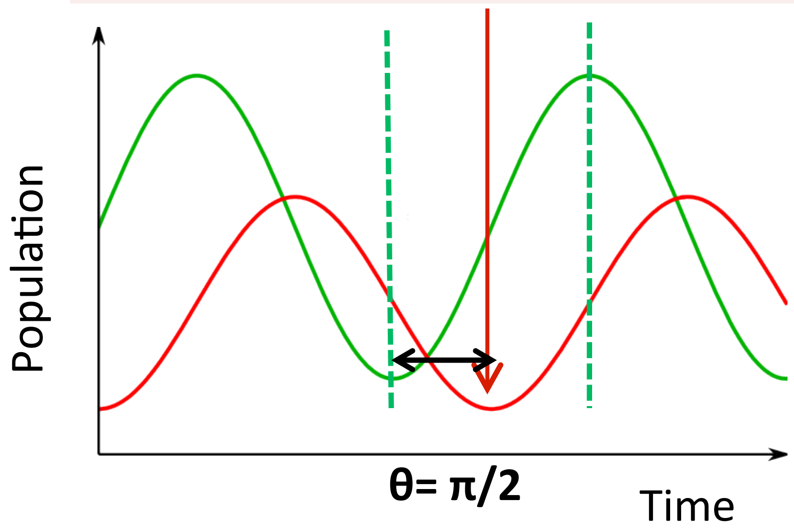


Cartoon picture for normal cycles (π/2 phase shift)

Resource → Prey → Predator

Predator can only start to grow after **prey** grows and before **prey** declines



Phase shift is a quarter period

Questions

1. The turbulence of ecology

   A. What is the role of intrinsic noise in spatially-extended ecosystems with predator-prey interactions?

   B. What happens when ecological and evolutionary timescales are comparable?

2. The ecology of turbulence

   C. What is the universality class of the transition from laminar fluid flow to turbulence?

Answers

1. The turbulence of ecology

    A. Demographic stochasticity can generate quasi-patterns in ecosystems

    B. Rapid evolution can emerge from demographic stochasticity

2. The ecology of turbulence

    C. Transitional turbulence is controlled by predator-prey interactions and is in the universality class of directed percolation

Lotka-Volterra equations for predator-prey dynamics

- Lotka-Volterra eqn: conventional model for population dynamics
- L-V for prey-predator system:

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$$\frac{du}{dt} = bu$$

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$$\frac{du}{dt} = bu(1 - \frac{u}{K_u})$$

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$$\frac{du}{dt} = bu(1 - \frac{u}{K_u}) - puv$$

$$\frac{dv}{dt} = puv - dv$$

u: prey v: predator b: prey metabolic rate
K_u: prey carrying capacity
p: predation rate d: predator death rate

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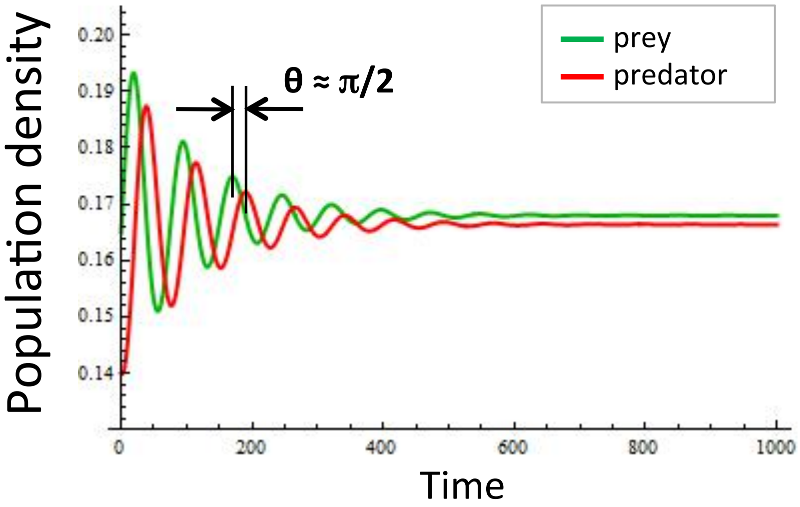

- Predicts π/2 phase shift between prey and predator
- **Problems**: No oscillations → **Contrary to experiments!**

Satiation model

- Add Michaelis-Menten kinetics to rewrite predation term
- Satiation effects as the **additional mechanism**; introduce **additional parameter K_s** : half saturation constant

$$\frac{du}{dt} = bu\left(1 - \frac{u}{K_u}\right) - \boxed{p\frac{uv}{K_s + u}}$$

$$\frac{dv}{dt} = \boxed{p\frac{uv}{K_s + u}} - dv$$

u: prey   v: predator   b: prey metabolic rate
K_u: prey carrying capacity   p: predation rate
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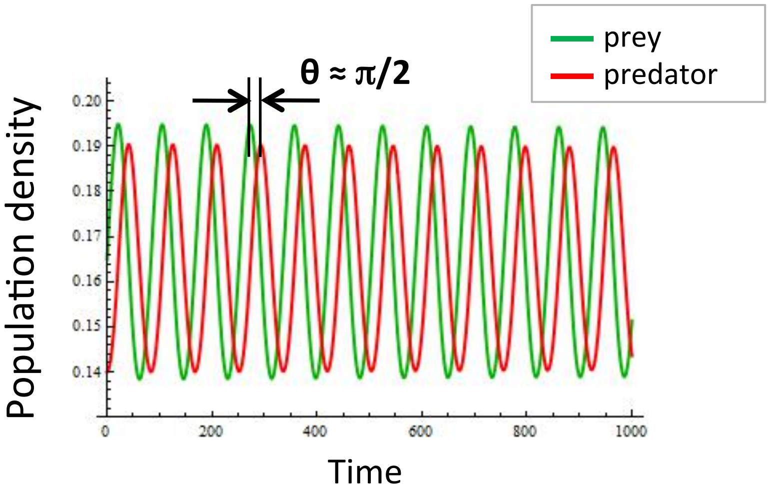

Satiation model

- Add Michaelis-Menten kinetics to rewrite predation term
- Satiation effects as the **additional mechanism**; introduce **additional parameter K_s** : half saturation constant

$$\frac{du}{dt} = bu\left(1 - \frac{u}{K_u}\right) - \boxed{p\frac{uv}{K_s + u}}$$
$$\frac{dv}{dt} = \boxed{p\frac{uv}{K_s + u}} - dv$$

u: prey v: predator b: prey metabolic rate
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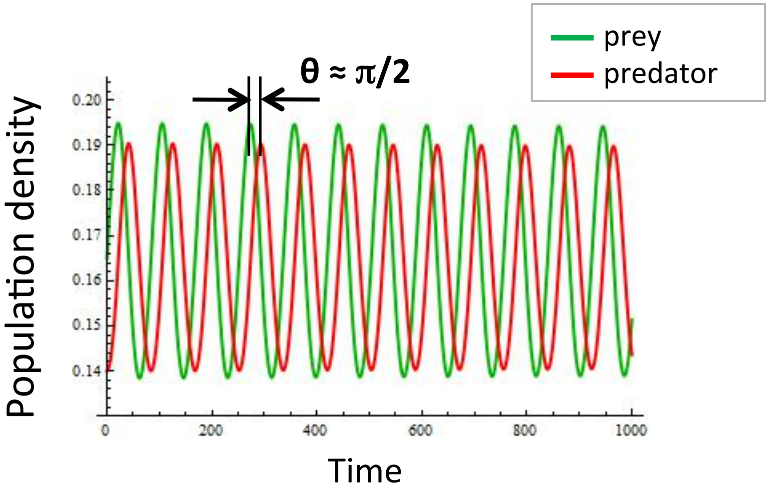

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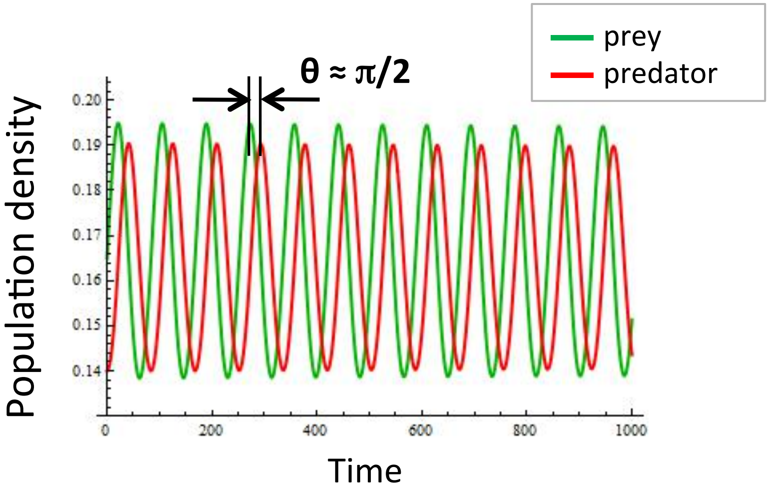

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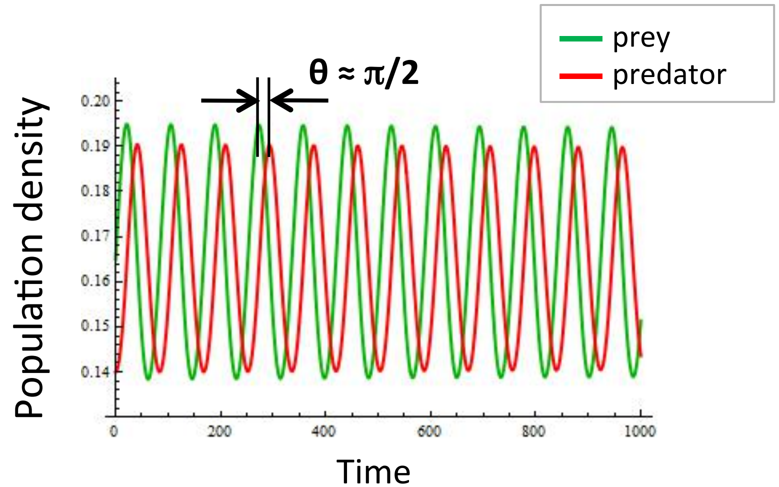
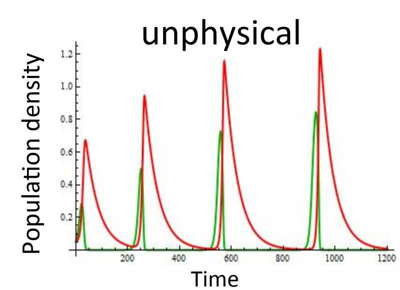


- Predicts: (1) π/2 phase shift (2) undamped oscillations
- **Problems**:
  - No fluctuations → **Contrary to experiments!**
  - Parameters can be rather sensitive to achieve coexistence without high likelihood of extinction → **Not generic!**



Population level vs. individual level

Deterministic population-level model:

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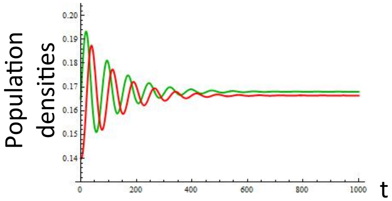

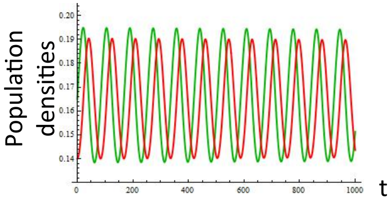

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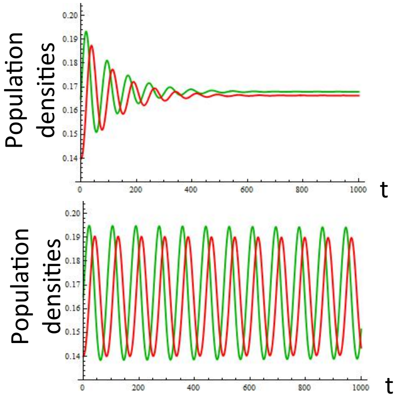

Stochastic individual-level model (Newman & McKane PRL 2005):

- Based on **individual processes** of species (e.g. reproduce process: **A**→2**A**)
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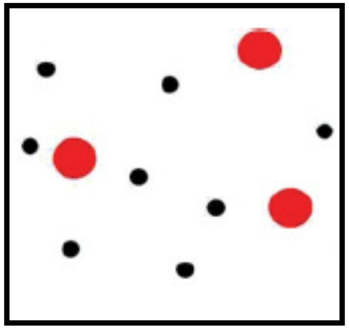

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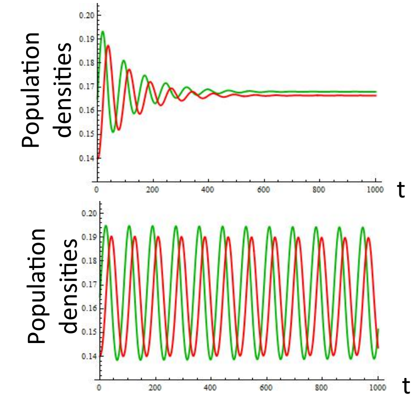

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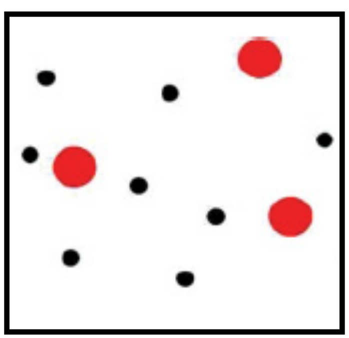
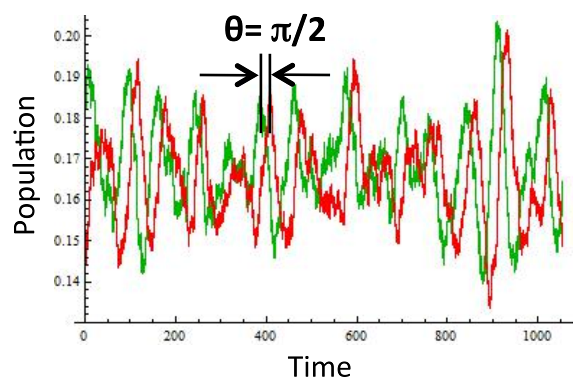


Statistical Mechanics →



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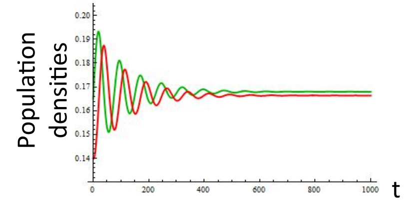

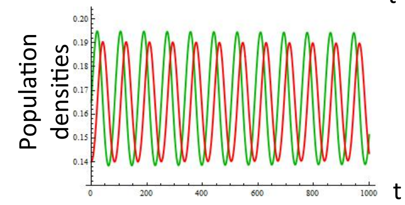

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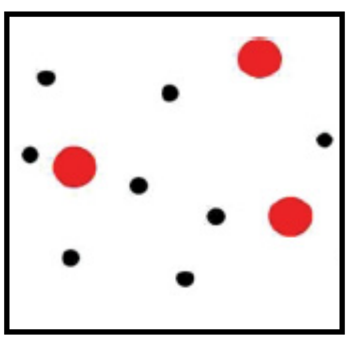

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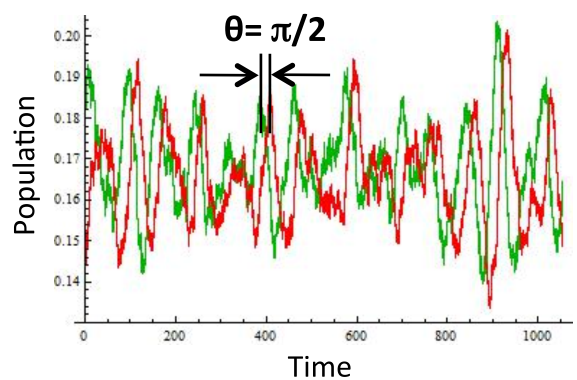

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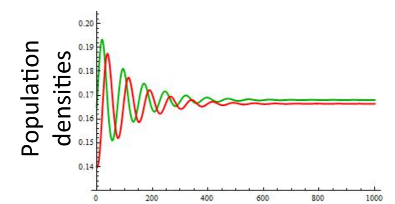

+

internal demographic noise

- **No additional mechanism**
- **Quasicycles emerge from intrinsic demographic stochasticity**

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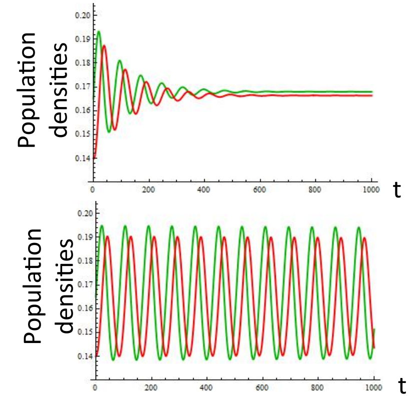

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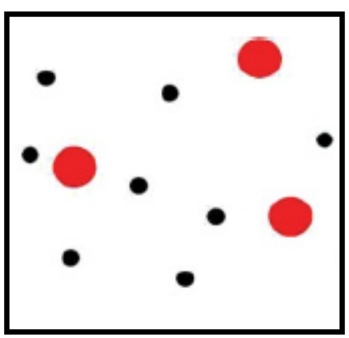

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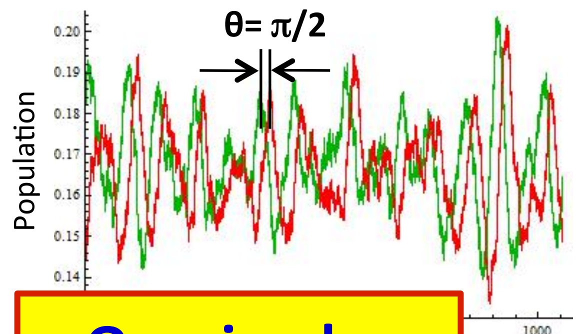
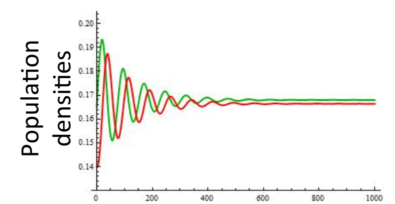


Quasicycles

=



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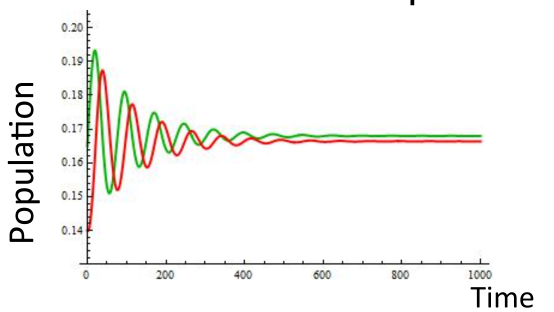
Models for predator-prey ecosystem

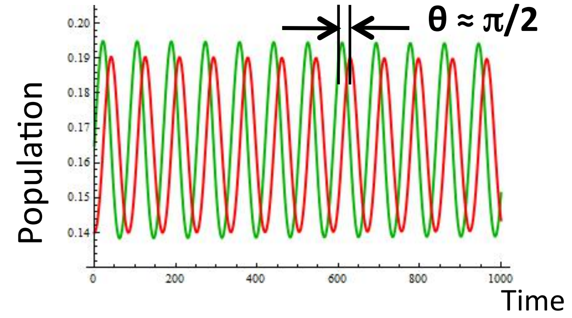
- **Deterministic models**

Lotka-Volterra equations



No persistent oscillations

Satiation model (Holling type II function)



No fluctuations

Models for predator-prey ecosystem

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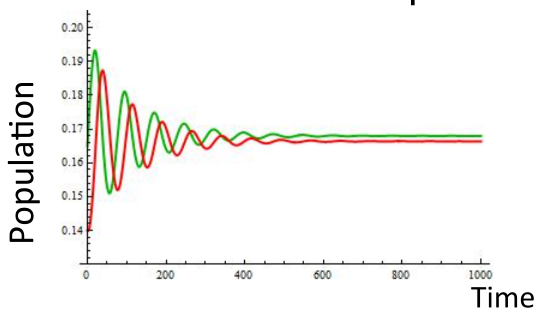

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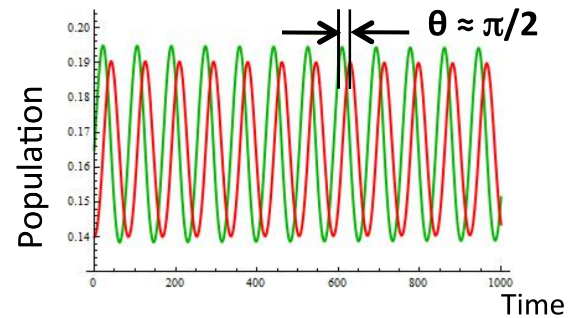
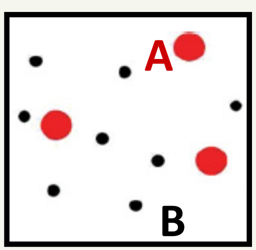


No fluctuations

- **Stochastic individual level model**
  fluctuations in number → demographic stochasticity that induces **quasi-cycles**



$$A \to \emptyset$$
$$A + B \to A + A$$
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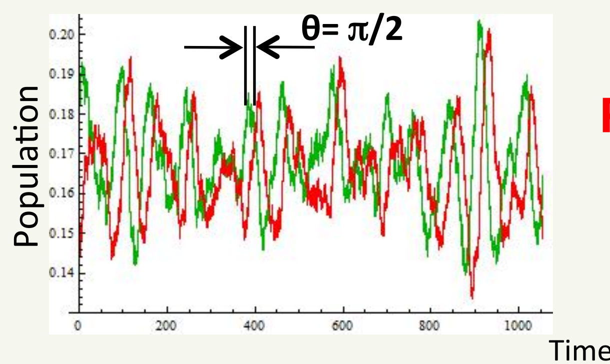

Persistent oscillations + Fluctuations

McKane & Newman. *PRL* **94**, 218102 (2005).

Models for predator-prey ecosystem

- **Deterministic models**

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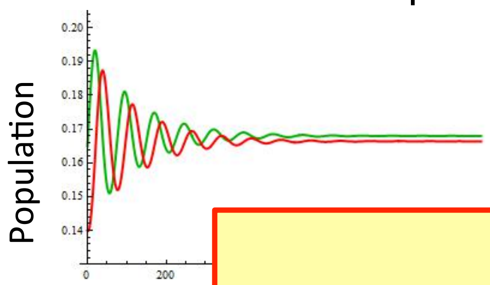

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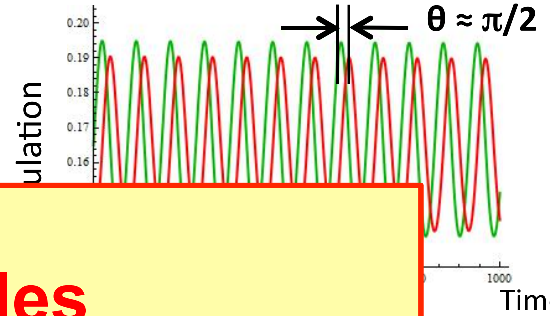
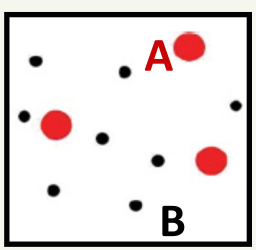


No pers ... ns

Quasicycles
emerge from intrinsic demographic stochasticity

- **Stochastic** ...
  fluctuations ... es **quasi-cycles**



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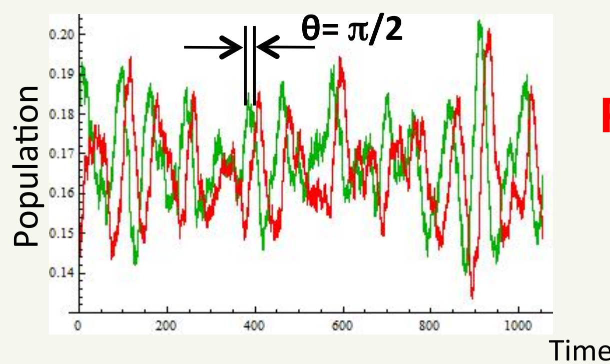

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Individual-level stochastic model of predator-prey dynamics

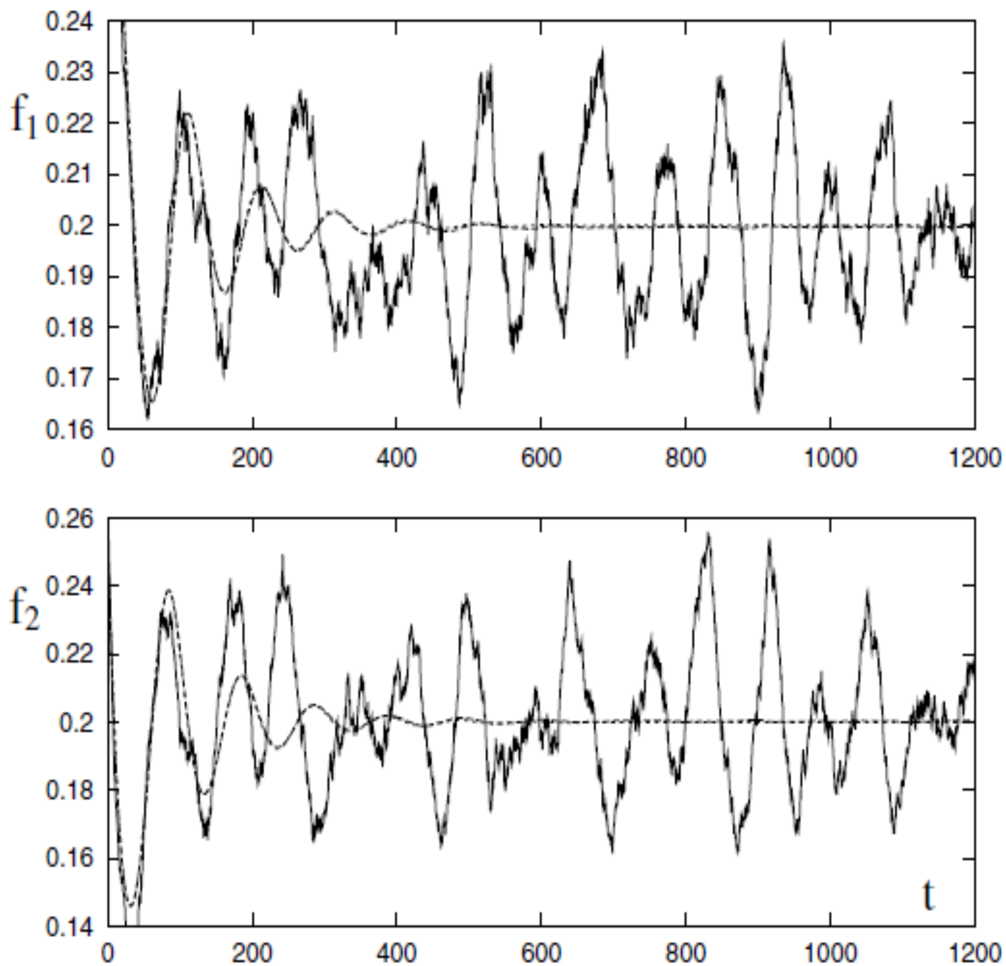
$$A \xrightarrow{d_1} E$$ **Predators**

$$B \xrightarrow{d_2} E$$ **Prey**

$$BE \xrightarrow{b} BB$$

$$AB \xrightarrow{p_1} AA$$

$$AB \xrightarrow{p_2} AE$$



A.J. McKane and T. Newman. Predator-Prey Cycles from Resonant Amplification of Demographic Stochasticity. Phys. Rev. Lett. 94, 218102 (2005)

Master equation for predator-prey model

Basic individual processes in predator (A) and prey (B) system:

Death $B_i \xrightarrow{d_B} E_i$ $\qquad$ $A_i \xrightarrow{d_A} E_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\qquad$ $A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

$$\boxed{\partial_t P(m,n) = \mathrm{stuff}(P(m,n), P(m \pm 1, n \pm 1), \mathrm{etc...})}$$

Master equation for individual-level model

$$
\begin{aligned}
\partial_t P(m,n) = {} & d_1(-nP(m,n) + (n+1)P(m,n+1)) \\
& + c(-n^2 P(m,n) + (n+1)^2 P(m,n+1)) \\
& + b_1(-nP(m,n) + (n-1)P(m,n-1)) \\
& + p_1(-mnP(m,n) + (n+1)mP(m,n+1)) \\
& + p_2(-mnP(m,n) + (m-1)(n+1)P(m-1,n+1)) \\
& + d_2(-mP(m,n) + (m+1)P(m+1,n))
\end{aligned}
$$

m=predators   n=prey

Master equation as a quantum field theory

- Individuals in a population are quantized, so use annihilation and creation operators to count them and describe their interactions
  - When adding a new individual to the system, there is only one to chose
  - When removing an individual from the system there are many to chose
- Result: even classical identical particles obey commutation relations familiar from quantum field theory

Doi 1976; Grassberger & Scheunert 1980; Cardy & Sugar 1980; Mikhalov 1981; Goldenfeld 1982, 1984; Peliti 1985

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$$A \xrightarrow{d_1} E$$ **Predators**

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$$\begin{aligned}
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\end{aligned}$$

Master equation as a quantum field theory

- Individuals in a population are quantized, so use annihilation and creation operators to count them and describe their interactions
- Time evolution given by Liouville equation

$$a|m,n\rangle = m|m-1,n\rangle$$

$$\hat{a}|m,n\rangle = |m+1,n\rangle$$

$$[a,\hat{a}] = 1$$

$$b|m,n\rangle = n|m,n-1\rangle$$

$$\hat{b}|m,n\rangle = |m,n+1\rangle$$

$$\left[b,\hat{b}\right] = 1$$

$$|\psi\rangle = \sum P(m,n)|m,n\rangle$$

$$\partial_t|\psi\rangle = -\hat{H}(a,\hat{a},b,\hat{b})|\psi\rangle$$

$$\hat{H} = b_1(\hat{b}b - \hat{b}^2 b) + d_1(\hat{b}b - b) + \frac{c}{V}(\hat{b}^2 b^2 - \hat{b}b^2)$$
$$+ \frac{p_1}{V}(\hat{a}a\hat{b}b - \hat{a}ab) + \frac{p_2}{V}(\hat{a}a\hat{b}b - \hat{a}^2 ab)$$
$$+ d_2(\hat{a}a - a)$$

Doi 1976; Grassberger & Scheunert 1980; Cardy & Sugar 1980; Mikhalov 1981; Goldenfeld 1982, 1984; Peliti 1985

Resonance from demographic noise

- Expand the number of predators and prey about average values in $\sqrt{N}$/ expansion

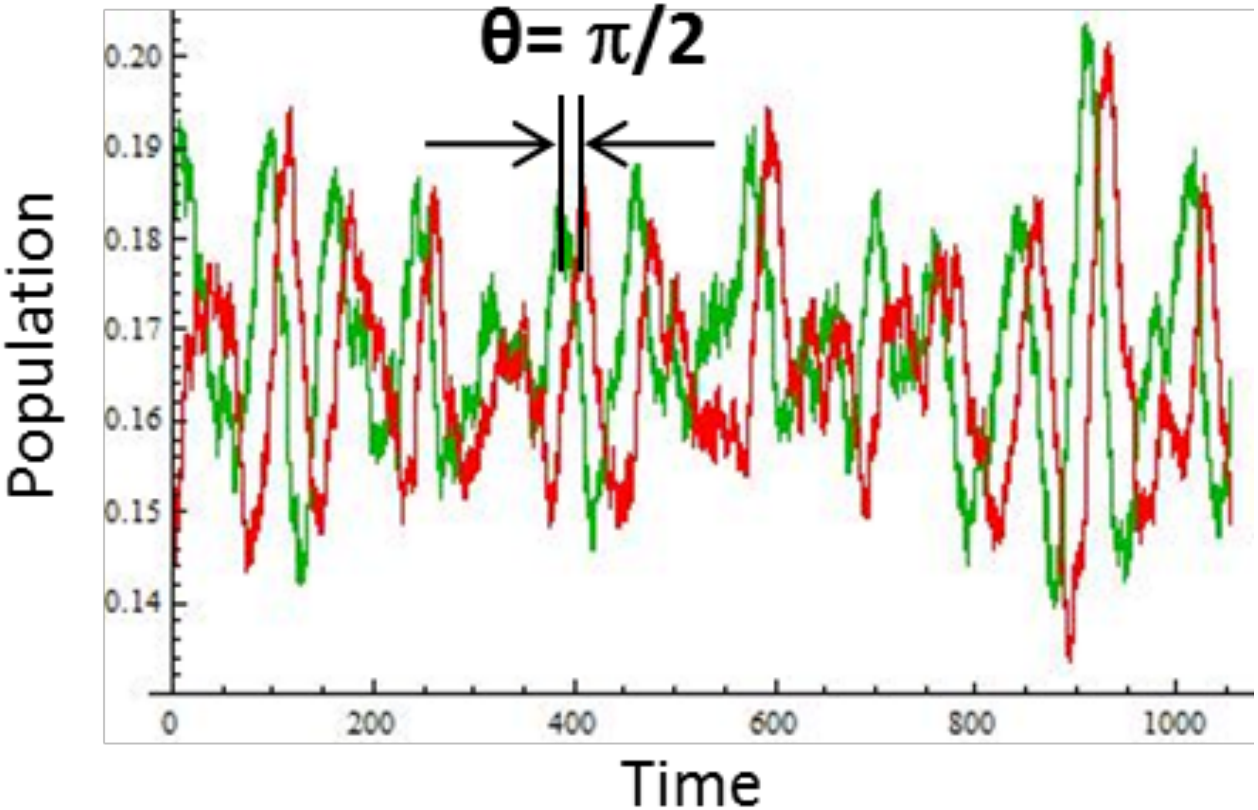
$$n/N = f_1 + x/\sqrt{N}$$

$$m/N = f_2 + y/\sqrt{N}$$

- Resulting equation is a linear stochastic equation in x, y with Langevin noise and power spectrum, sharply peaked about an internally-generated natural frequency

$$\boxed{P(\omega) = \frac{\alpha + \beta\omega^2}{[(\omega^2 - \Omega_0^2)^2 + \Gamma^2\omega^2]}}$$

Quasi-cycles



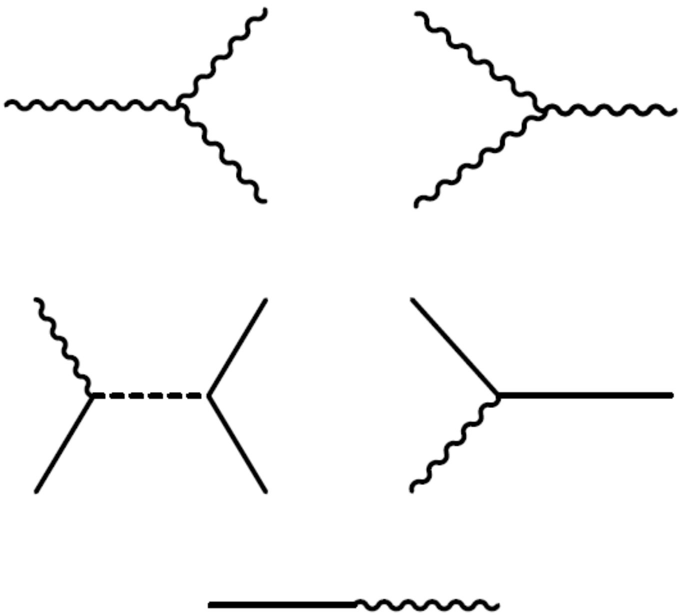
Extinction/decay statistics for stochastic predator-prey systems

Derivation of predator-prey equations

Predator/Zonal flow

Prey/Turbulence

Zonal flow-turbulence



Predator-prey

$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

$$A + B \xrightarrow{p} A + A$$

$$A + B \xrightarrow{p'} A + E$$

$$B \xrightarrow{m} A$$

$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

Ecology model for turbulence

Laminar flow → Turbulence → Zonal flow

Nutrient (E) → Prey (B) → Predator (A)

$$B + E \xrightarrow{b} B + B$$

$$A + B \xrightarrow{p} A + A$$

$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

$$B \xrightarrow{m} A$$

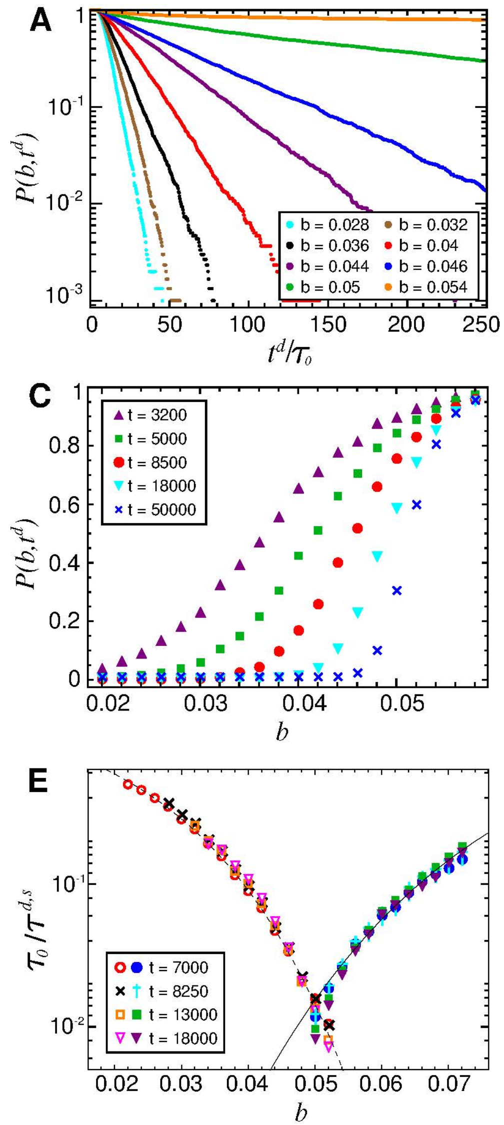
mean-field rate equation:

$$\frac{dA}{dt} = pAB - d_A A + mB$$

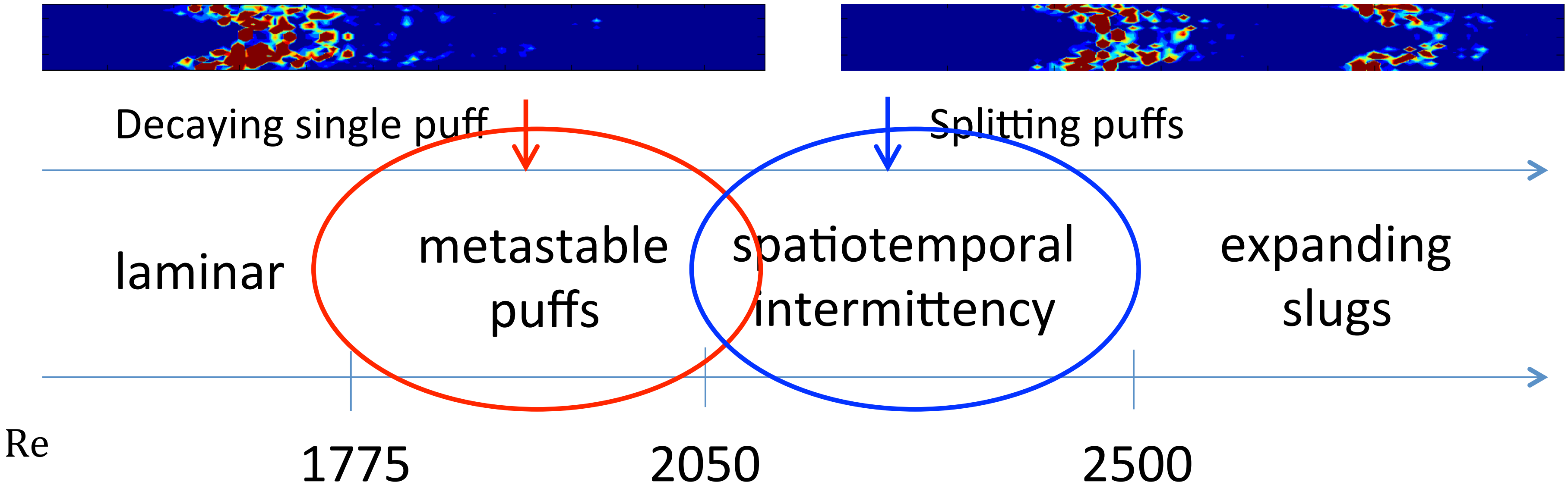
$$\frac{dB}{dt} = b\left(1 - A - B\right)B - pAB - d_B B - mB$$

Survival probability near extinction

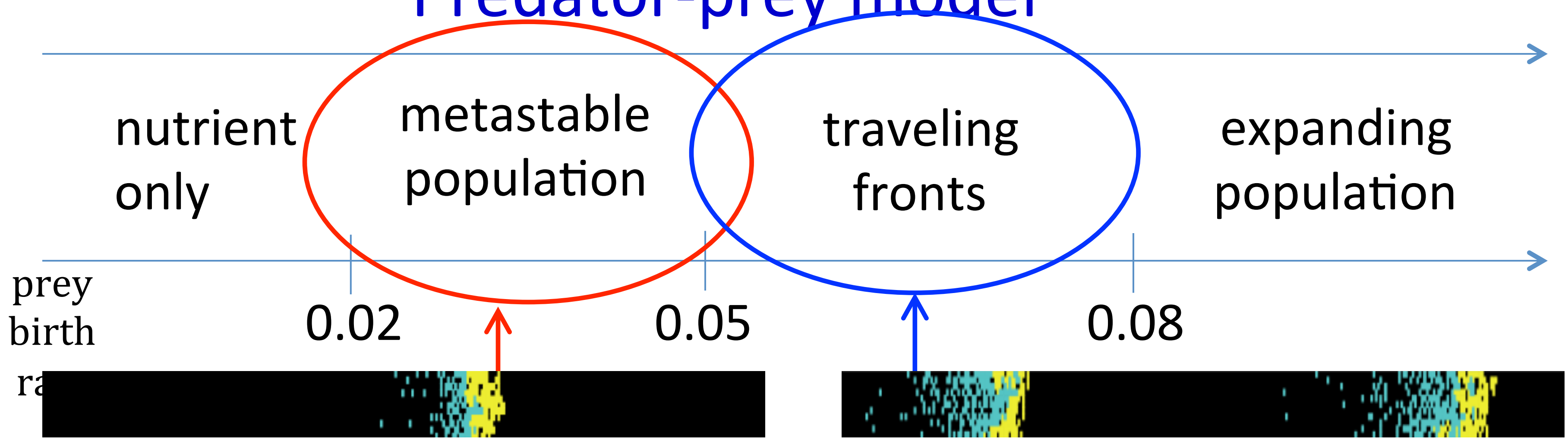
- Decay of population is a memoryless process
  - Extract lifetime in both decay and splitting modes
- Log-linear plot of lifetime shows curvature
  - superexponential dependence on prey birth rate



Pipe flow turbulence



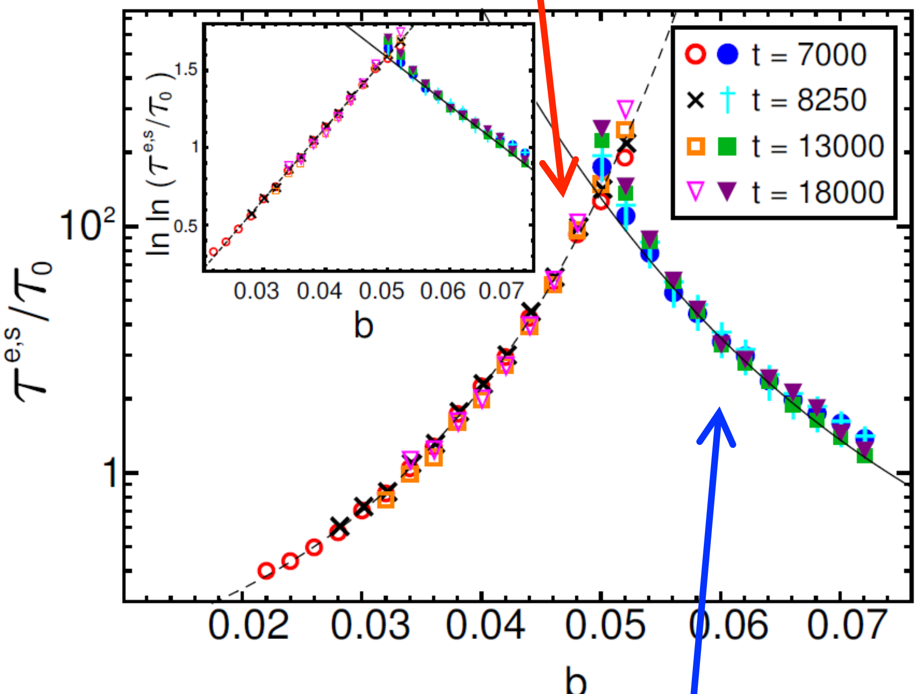
Predator-prey model

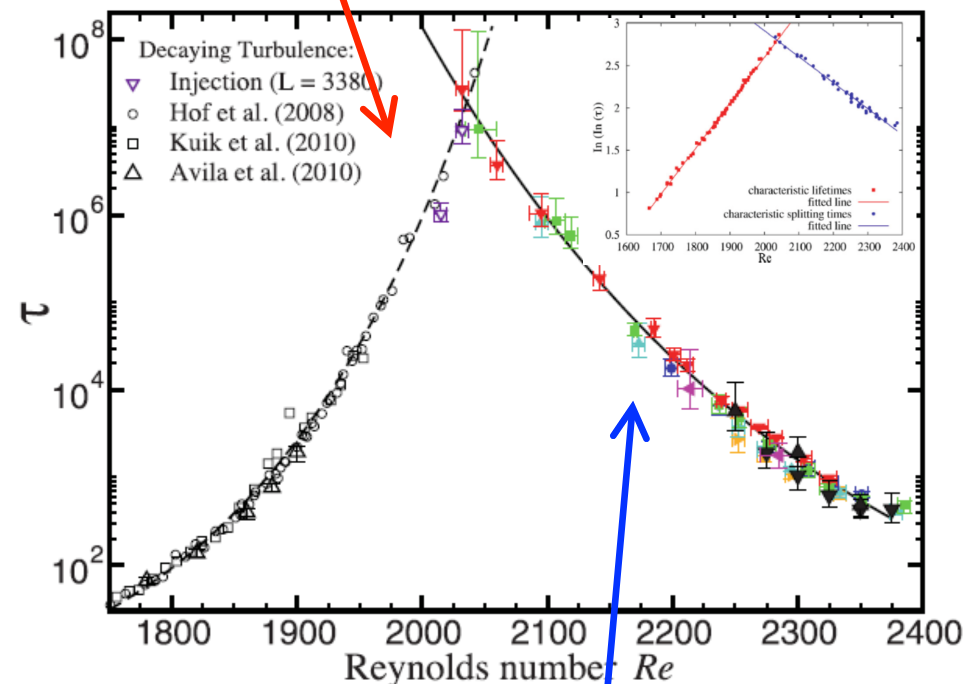


Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

Avila *et al.*, *Science* **333**, 192 (2011)

Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

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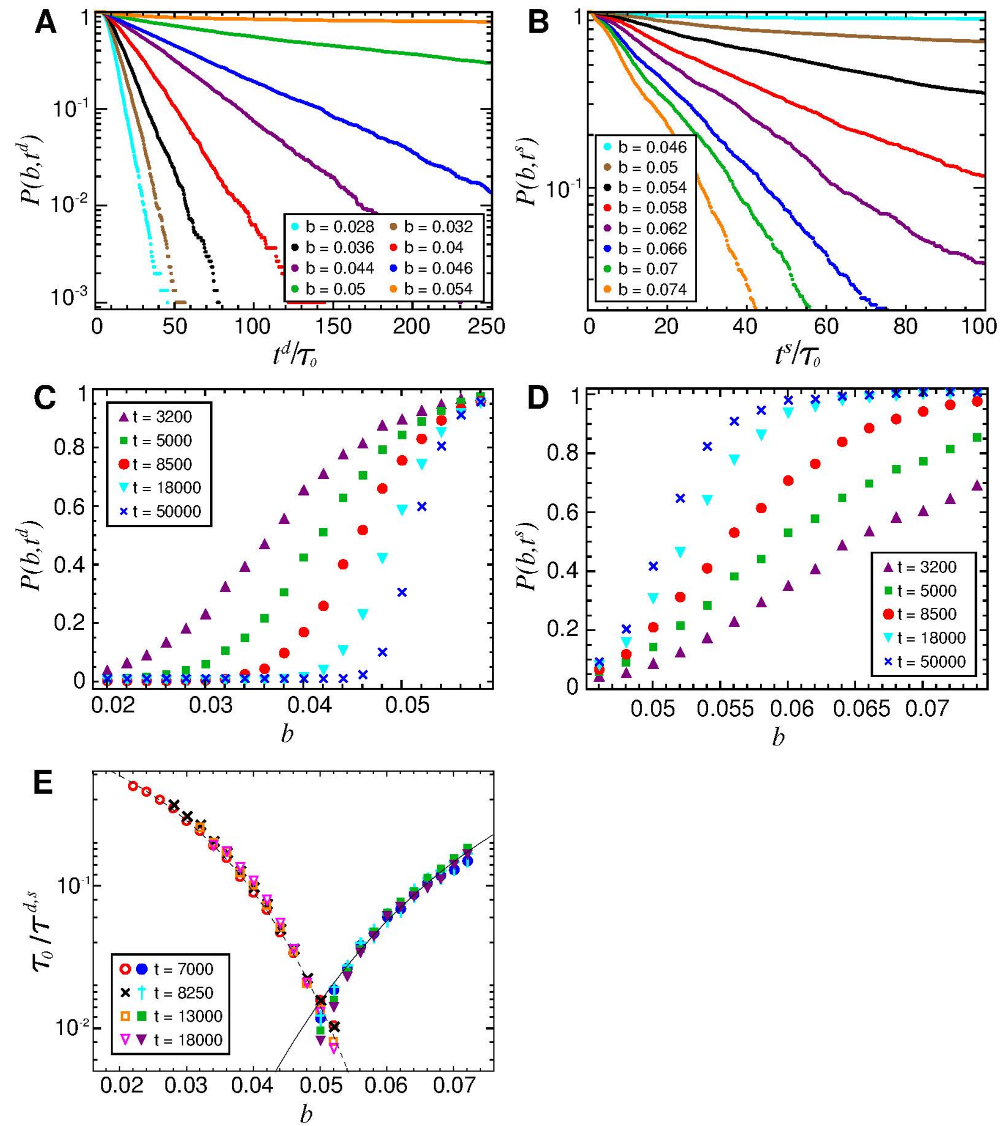
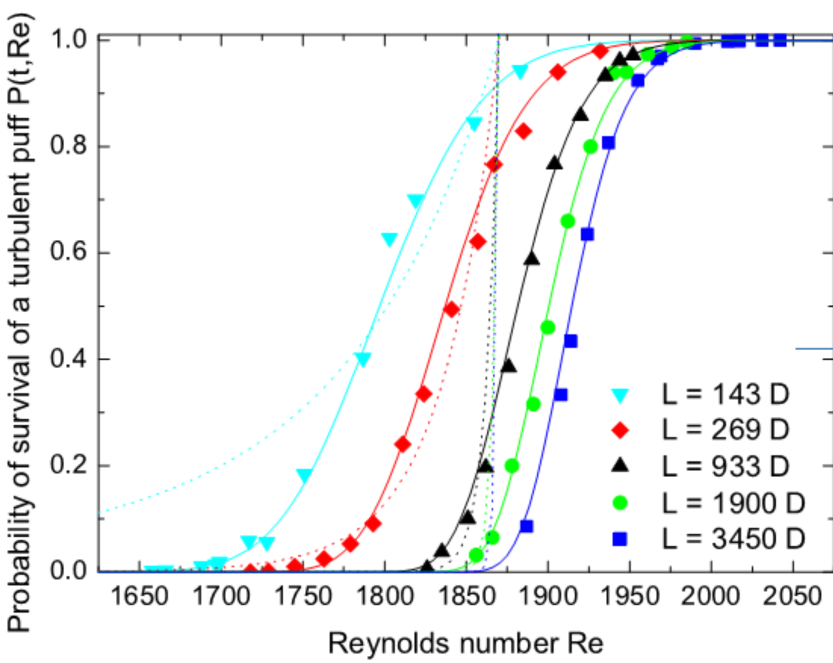


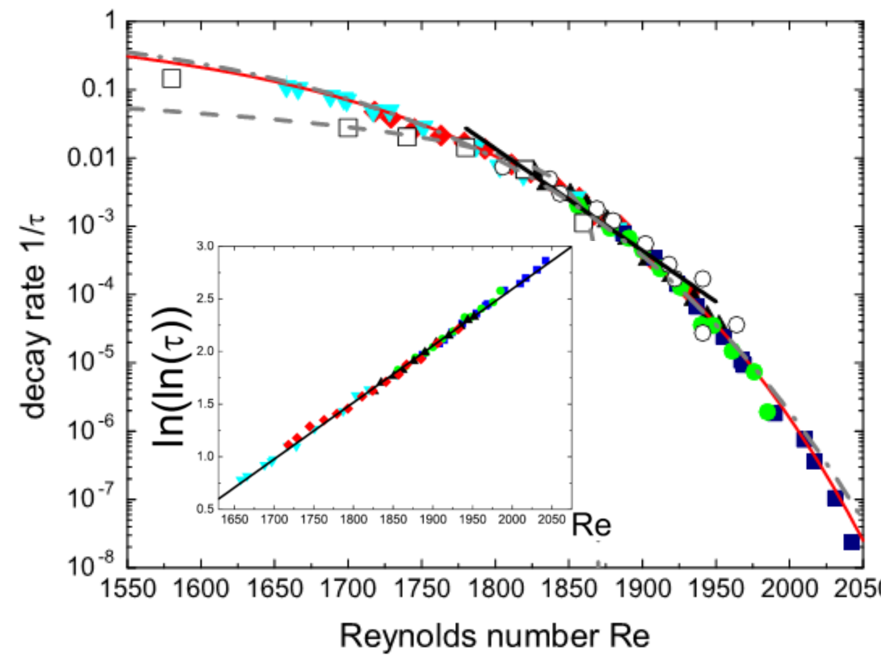
Metastable turbulent puff

- S-shaped curves imply that survival probability has the form:

$$P(\mathrm{Re}, t) = e^{-(t-t_0)/\tau(\mathrm{Re})}$$

to extra slide





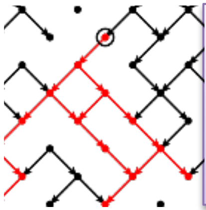
Hof *et al.* (PRL 2008)

Super-exponential scaling: $\tau/\tau_0 \sim \exp(\exp \mathrm{Re})$

Universality class of the transition

Strategy: transitional turbulence to directed percolation

(Boffetta and Ecke, 2012)

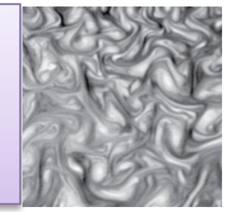


Directed Percolation

(Wikimedia Commons)

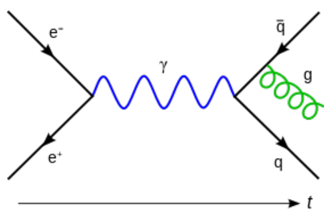
?

(Classical) Turbulence

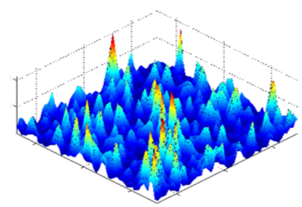


Reggeon field theory
(Janssen, 1981)

Field Theory



(Wikimedia Commons)



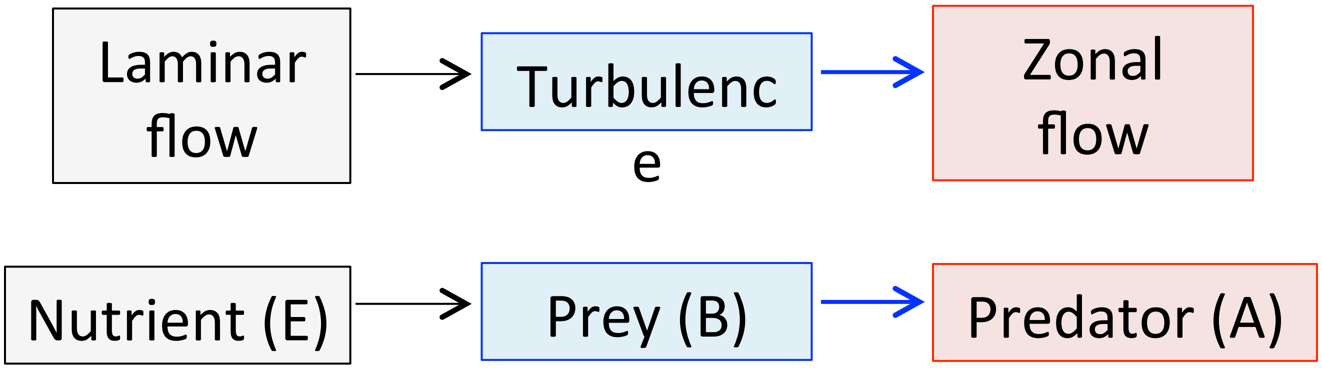
Two-fluid model

Extinction transition
(Mobilia et al., 2007)

Predator-Prey

(Pearson Education, Inc., 2009)

Ecology model for turbulence



Universality class of predator-prey system near extinction

Basic individual processes in predator (A) and prey (B) system:

Death $B_i \xrightarrow{d_B} E_i$ $A_i \xrightarrow{d_A} E_i$ $B_i \xrightarrow{m} A_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $B_i \xrightarrow{d_B} E_i$ $\cancel{A_i \xrightarrow{d_A} E_i}$ $\cancel{B_i \xrightarrow{m} A_i}$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\cancel{A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j}$

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ $\cancel{A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j}$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $$B_i \xrightarrow{d_B} E_i$$

Birth $$B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$$

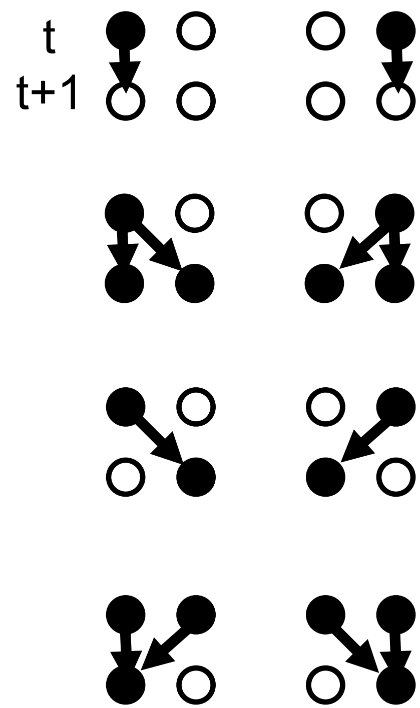
Diffusion $$B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$$

Carrying capacity $$B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death	$B_i \xrightarrow{d_B} E_i$	Annihilation
Birth	$B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$	Decoagulation
Diffusion	$B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$	Diffusion
Carrying capacity	$B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$	Coagulation



Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

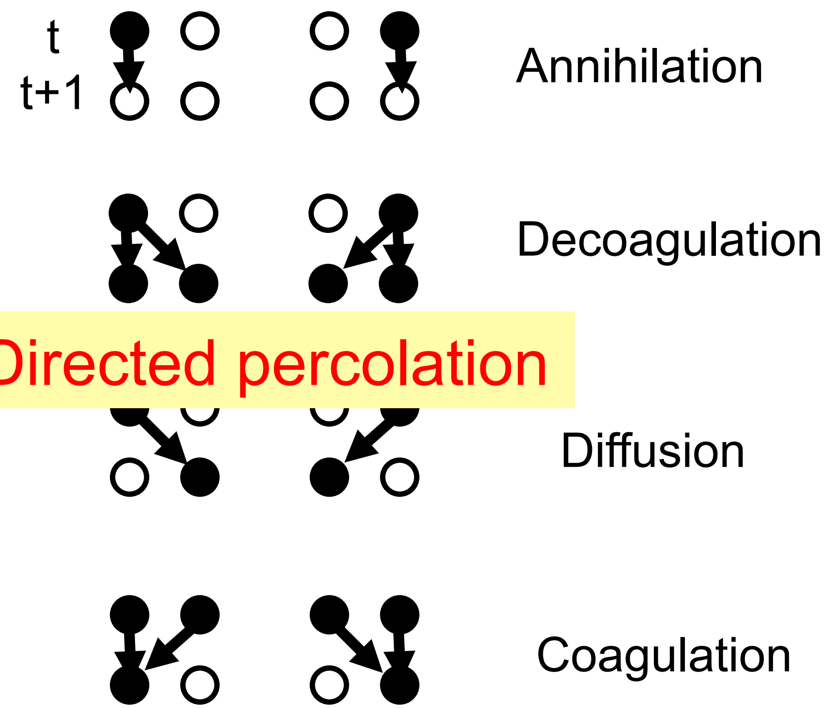
Death $B_i \xrightarrow{d_B} E_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$

Predator-prey = Directed percolation

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$



Master equation for predator-prey model

Basic individual processes in predator (A) and prey (B) system:

Death $B_i \xrightarrow{d_B} E_i$ $\qquad$ $A_i \xrightarrow{d_A} E_i$

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ $\qquad$ $A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$

$$\boxed{\partial_t P(m,n) = \mathrm{stuff}(P(m,n), P(m \pm 1, n \pm 1), \mathrm{etc...})}$$

Master equation as a quantum field theory

- Individuals in a population are quantized, so use annihilation and creation operators to count them and describe their interactions

  - When adding a new individual to the system, there is only one to chose

  - When removing an individual from the system there are many to chose

- Result: even classical identical particles obey commutation relations familiar from quantum field theory

Doi 1976; Grassberger & Scheunert 1980; Cardy & Sugar 1980; Mikhalov 1981; Goldenfeld 1982, 1984; Peliti 1985

Master equation as a quantum field theory

- Individuals in a population are quantized, so use annihilation and creation operators to count them and describe their interactions
- Time evolution given by Liouville equation

$$|\psi\rangle = \sum P(m,n)|m,n\rangle$$

$$\partial_t|\psi\rangle = -\hat{H}(a,\hat{a},b,\hat{b})|\psi\rangle$$

$$a|m,n\rangle = m|m-1,n\rangle$$

$$\hat{a}|m,n\rangle = |m+1,n\rangle$$

$$[a,\hat{a}] = 1$$

$$b|m,n\rangle = n|m,n-1\rangle$$

$$\hat{b}|m,n\rangle = |m,n+1\rangle$$

$$\left[b,\hat{b}\right] = 1$$

$$\hat{H} = b_1(\hat{b}b - \hat{b}^2 b) + d_1(\hat{b}b - b) + \frac{c}{V}(\hat{b}^2 b^2 - \hat{b}b^2)$$
$$+ \frac{p_1}{V}(\hat{a}a\hat{b}b - \hat{a}ab) + \frac{p_2}{V}(\hat{a}a\hat{b}b - \hat{a}^2 ab)$$
$$+ d_2(\hat{a}a - a)$$

Doi 1976; Grassberger & Scheunert 1980; Cardy & Sugar 1980; Mikhalov 1981; Goldenfeld 1982, 1984; Peliti 1985

Field theory for predator-prey model

- Near extinction model reduces to simpler system

$$
\begin{aligned}
A &\to \emptyset && \text{with rate } \mu, \\
A + B &\to A + A && \text{with rate } \lambda', \\
B &\to B + B && \text{with rate } \sigma.
\end{aligned}
$$

- Express as Hamiltonian

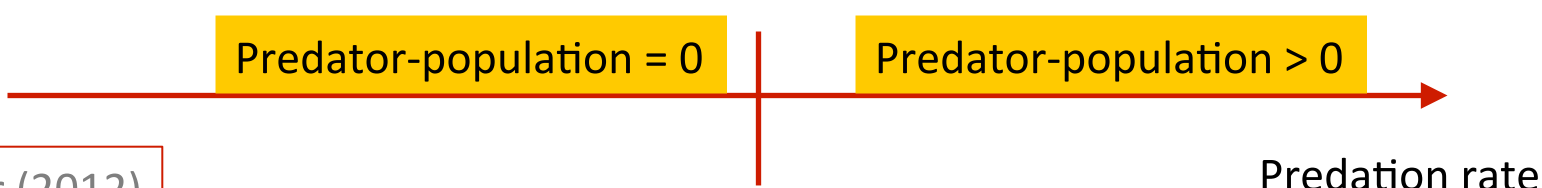
$$H_{\mathrm{reac}} = -\sum [\mu (1 - a_i^\dagger) a_i + \sigma (b_i^\dagger - 1)\, b_i^\dagger\, b_i + \lambda' (a_i^\dagger - b_i^\dagger) a_i^\dagger\, a_i\, b_i]$$

- Map into a coherent state path integral

$$S[\hat{a}, \hat{b}; a, b] = \int \mathrm{d}^d x \int \mathrm{d}t \left[\hat{a} \left(\frac{\partial}{\partial t} - D_A \nabla^2 \right) a + \hat{b} \left(\frac{\partial}{\partial t} - D_B \nabla^2 \right) b \right.$$

$$\left. + \mu (\hat{a} - 1)\, a - \sigma (\hat{b} - 1) \hat{b}\, b\, e^{-a_0^d \hat{b} b} + \nu (\hat{b} - 1)\, \hat{b}\, b^2 - \lambda (\hat{a} - \hat{b}) \hat{a}\, a\, b \right]$$

- Phase diagram



See Tauber (2012)

Extinction in predator-prey systems

- This field theory can be reduced to

$$A \rightarrow \emptyset$$
$$A + B \rightarrow A + A$$
$$B \rightarrow B + B$$

$$S'_{\infty}[\widetilde{\psi}, \psi] = \int \mathrm{d}^d x \int \mathrm{d}t \left[\widetilde{\psi} \left(\frac{\partial}{\partial t} + D_A (r_A - \nabla^2) \right) \psi - u \, \widetilde{\psi} (\widetilde{\psi} - \psi) \psi + \tau \widetilde{\psi}^2 \psi^2 \right]$$

Action of Reggeon field theory and universality class of directed percolation (Mobilia et al (2007)

- **Summary: ecological model of transitional turbulence predicts the DP universality class**

Extinction in predator-prey systems

- This field theory can be reduced to

$$A \to \emptyset$$
$$A + B \to A + A$$
$$B \to B + B$$

$$S'_\infty[\widetilde{\psi}, \psi] = \int \mathrm{d}^d x \int \mathrm{d}t \left[\widetilde{\psi} \left(\frac{\partial}{\partial t} + D_A (r_A - \nabla^2) \right) \psi - u \, \widetilde{\psi} (\widetilde{\psi} - \psi) \psi + \tau \widetilde{\psi}^2 \psi^2 \right]$$

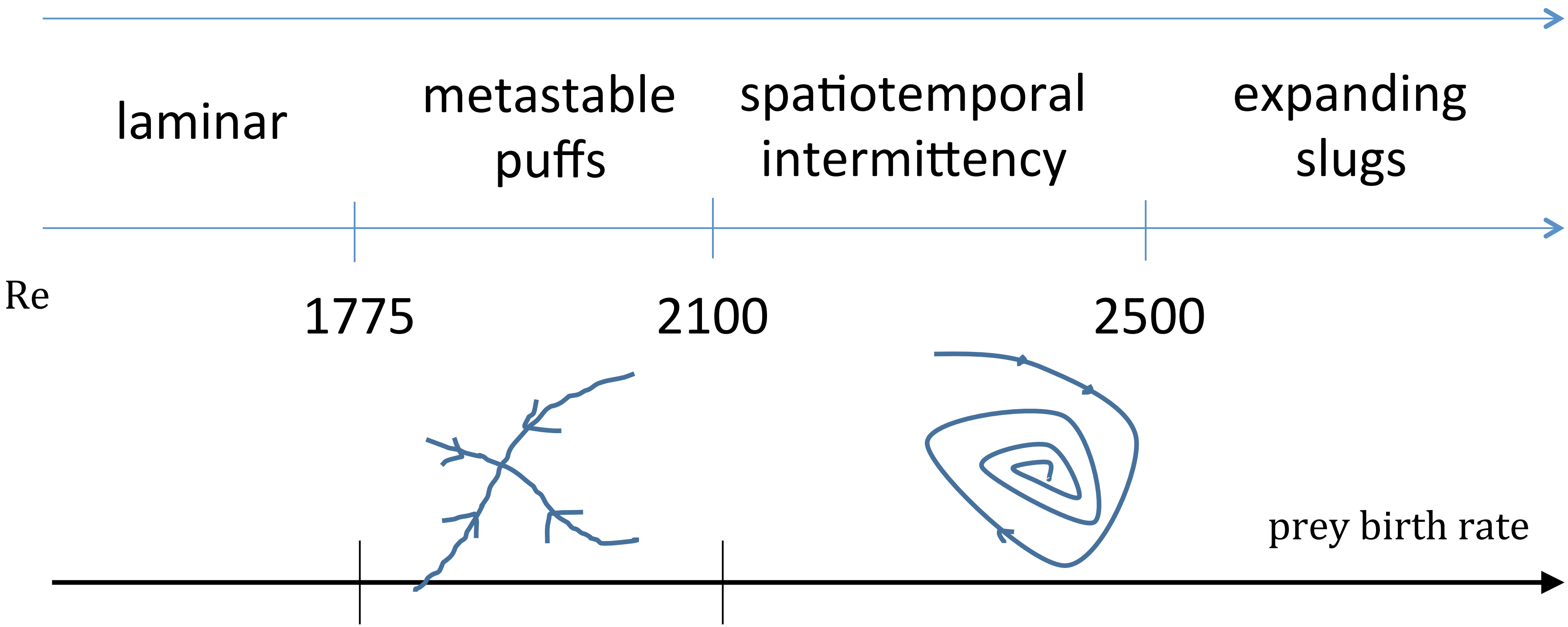
- Reggeon field theory ↔ Extinction transition in predator-prey model (Mobilia et al (2007)
- Reggeon field theory ↔ DP universality class: non-equilibrium critical dynamics with absorbing state

- **Summary: ecological model of transitional turbulence predicts the DP universality class**

Puff splitting in ecology model

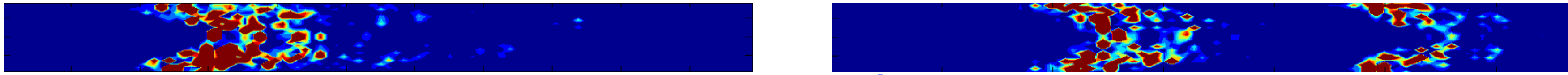
Driven by emerging traveling waves
of populations

Puff splitting in predator-prey systems



- Stability of predator-prey mean field theory has a transition between stable node and spiral
  - Near transition, no oscillations
  - Away from the transitions, oscillations begin

Pipe flow turbulence

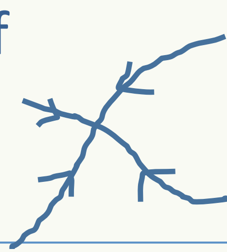


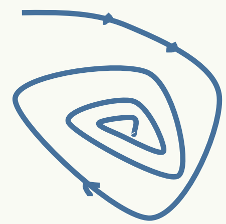
laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re 1775 2050 2500

Predator-prey model

linear stability of mean-field solutions

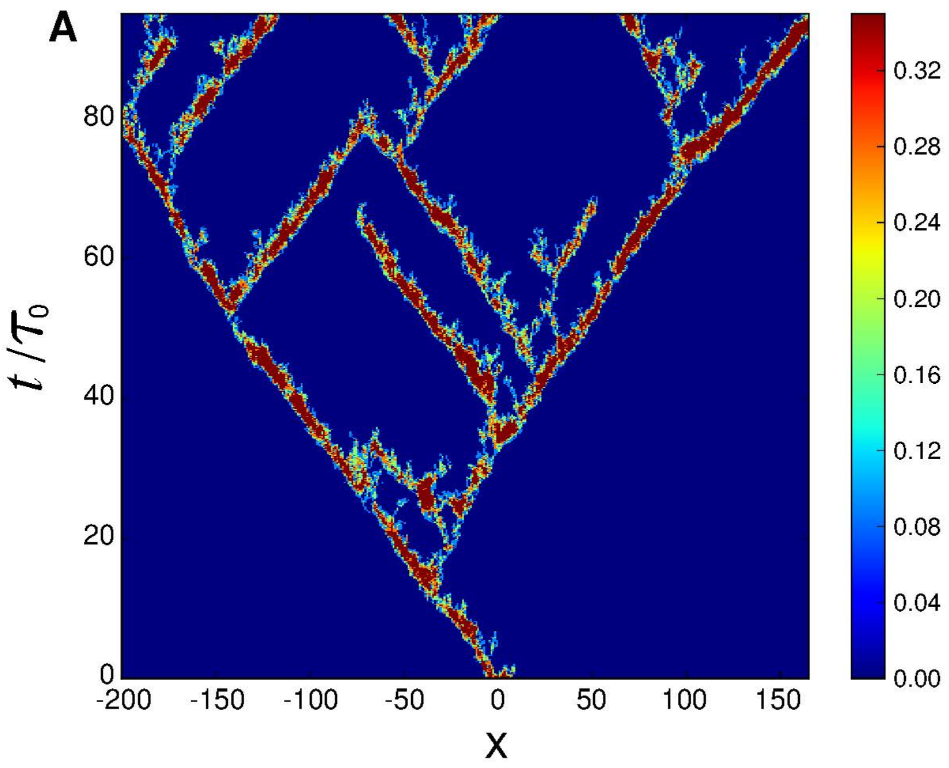


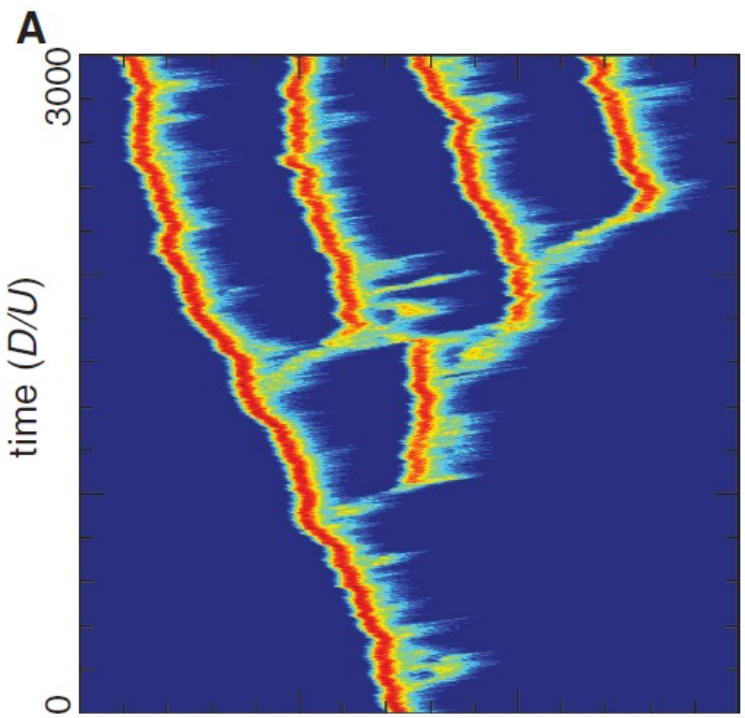


nutrient only | metastable population | traveling fronts | expanding population

prey birth rate

Puff splitting in predator-prey systems



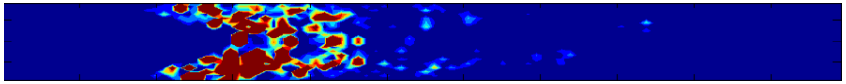


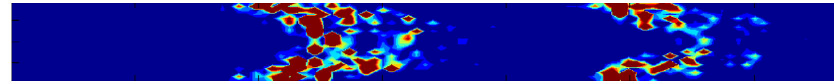
Puff-splitting in predator-prey ecosystem in a pipe geometry

Puff-splitting in pipe turbulence

Avila et al., Science (2011)

Pipe flow turbulence





Decaying single puff

Splitting puffs

laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re: 1775, 2050, 2500

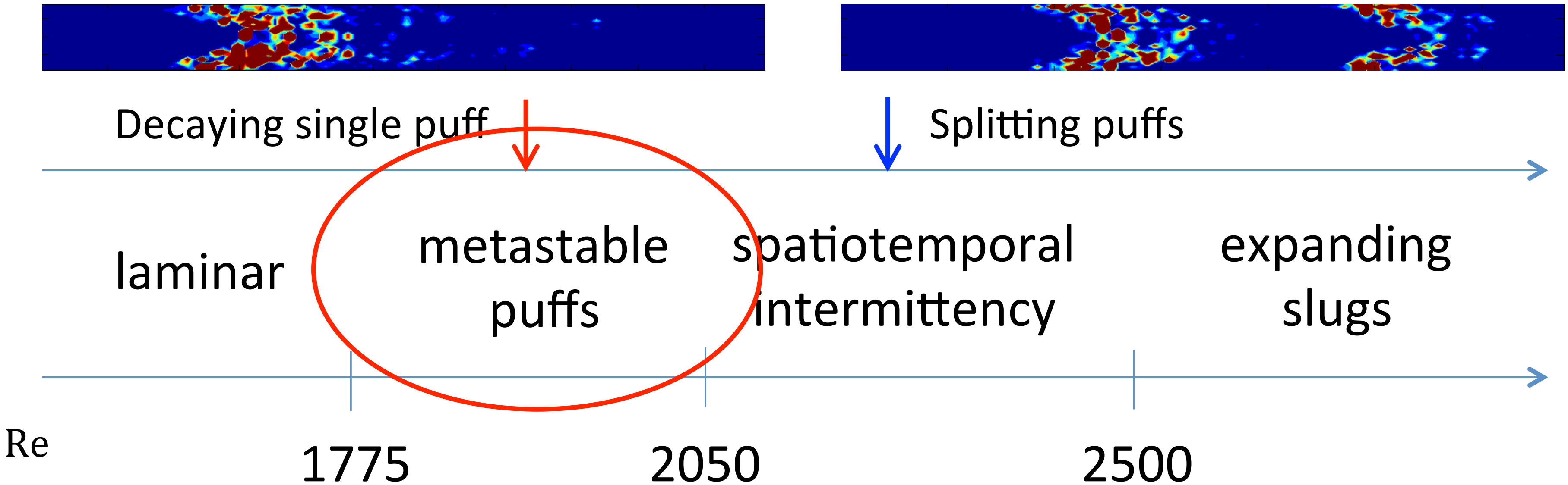
Predator-prey model

nutrient only | metastable population | traveling fronts | expanding population

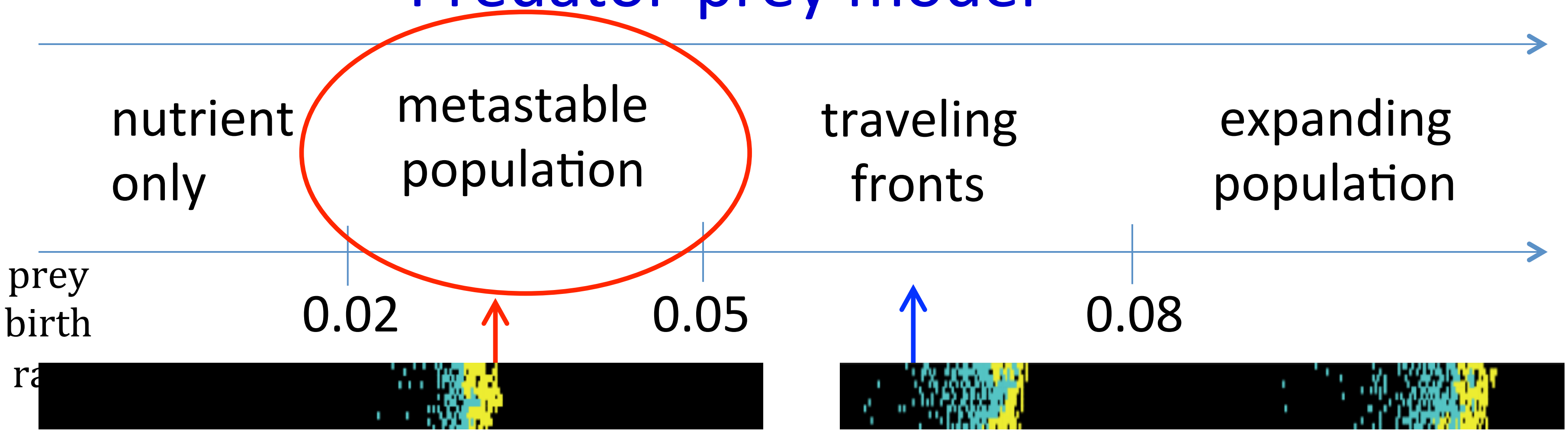
prey birth ra

0.02, 0.05, 0.08

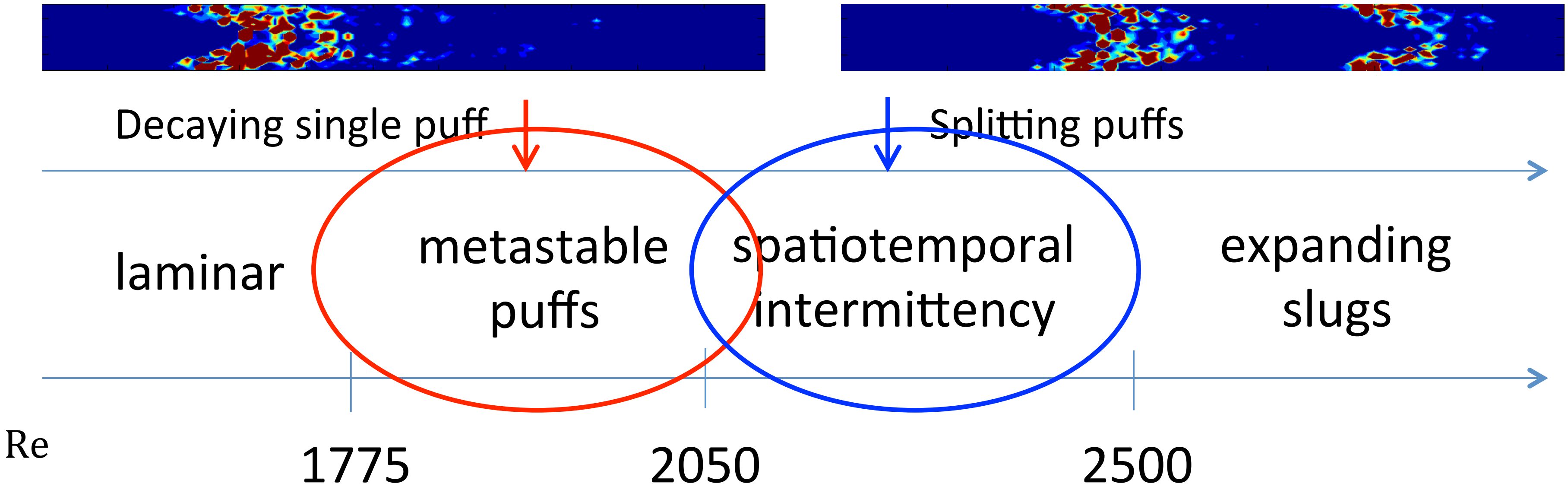
Pipe flow turbulence



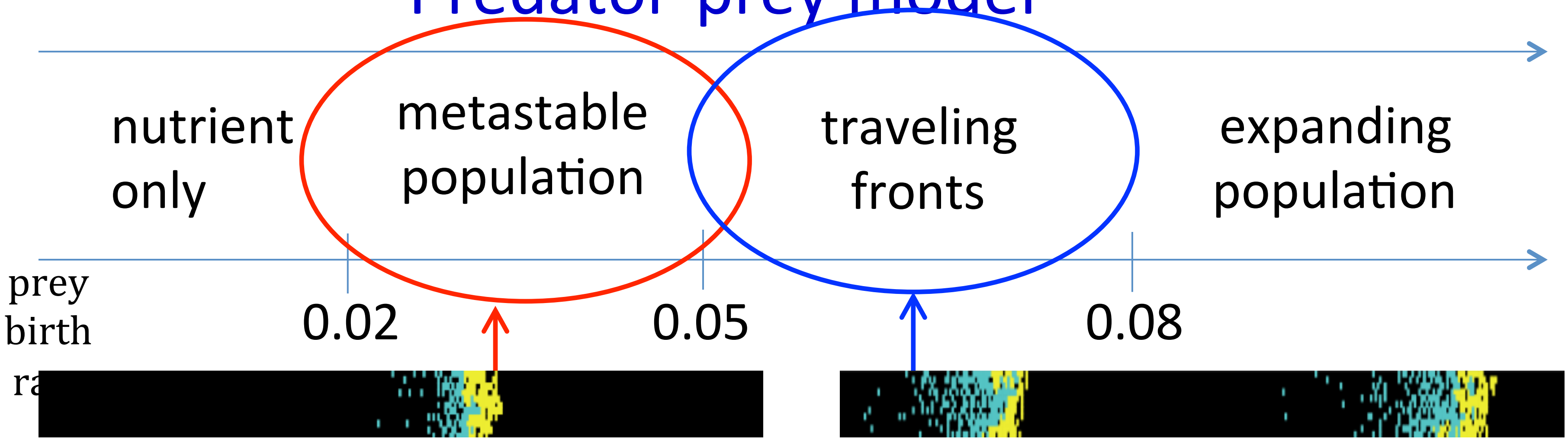
Predator-prey model



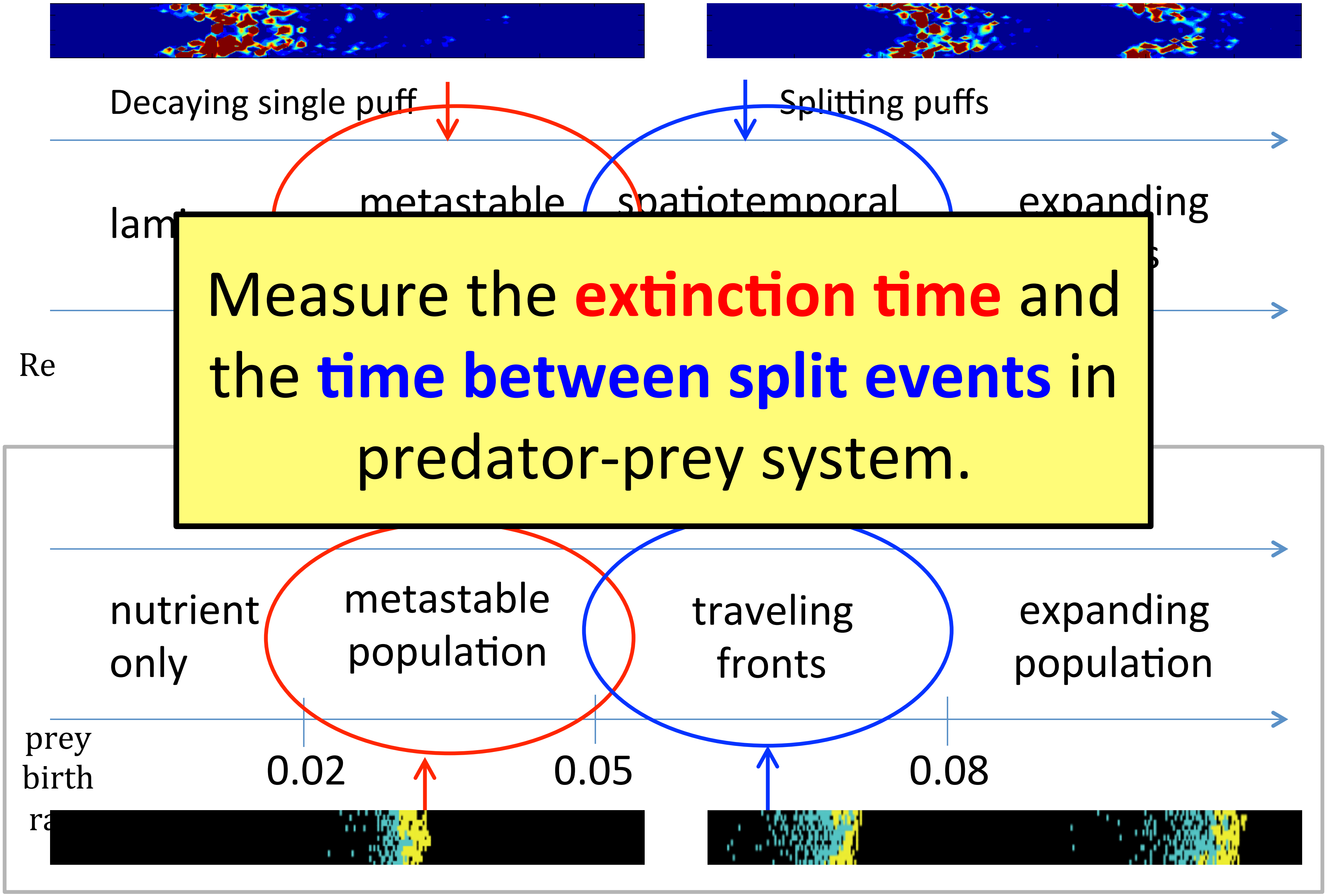
Pipe flow turbulence



Predator-prey model



Pipe flow turbulence

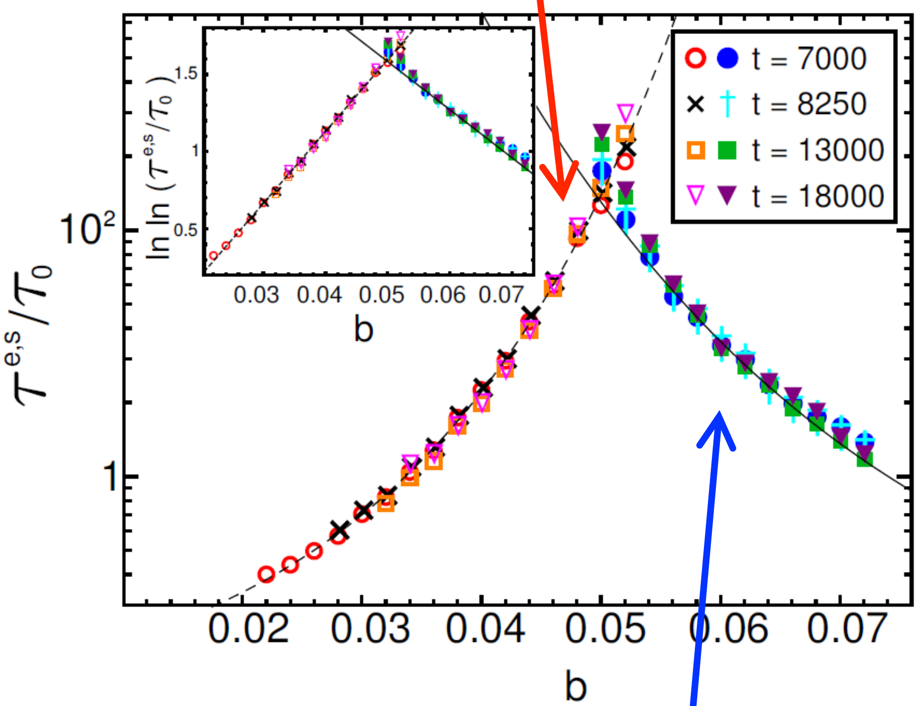


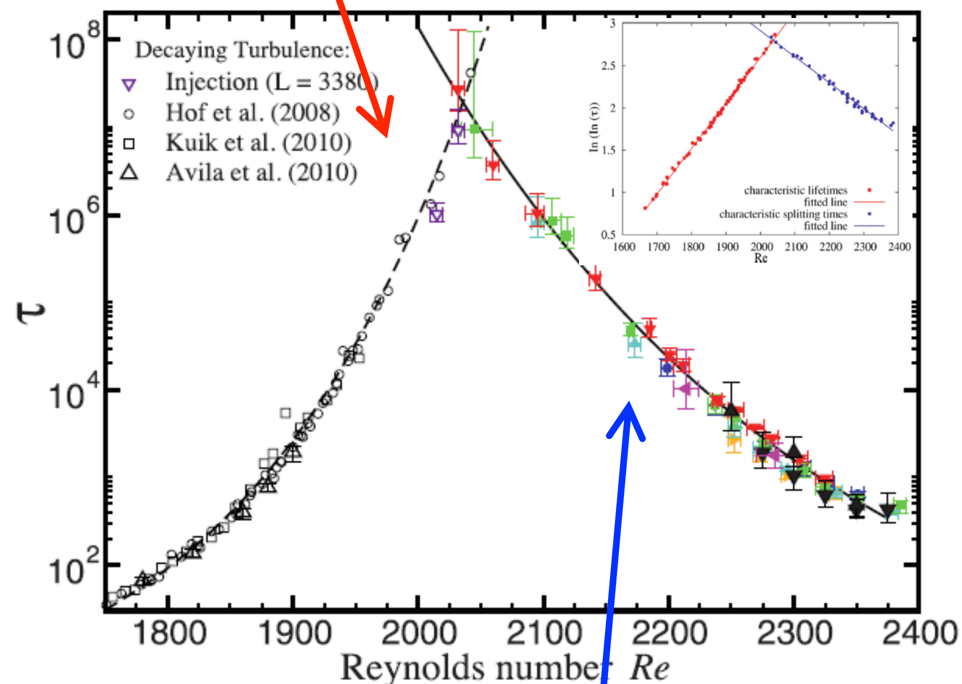
Measure the **extinction time** and the **time between split events** in predator-prey system.

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

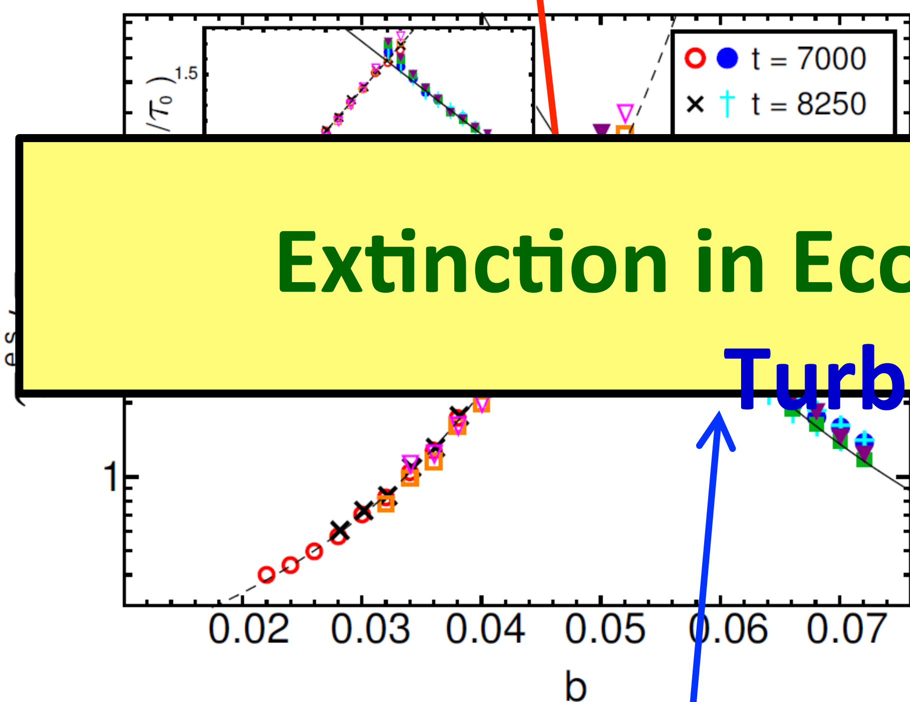
Avila *et al.*, *Science* **333**, 192 (2011)

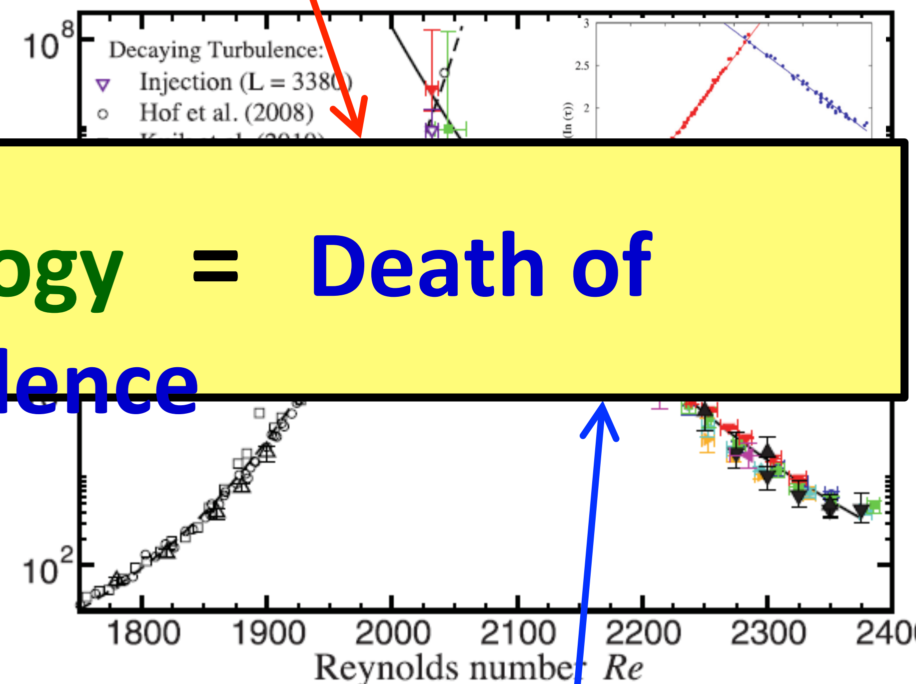
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





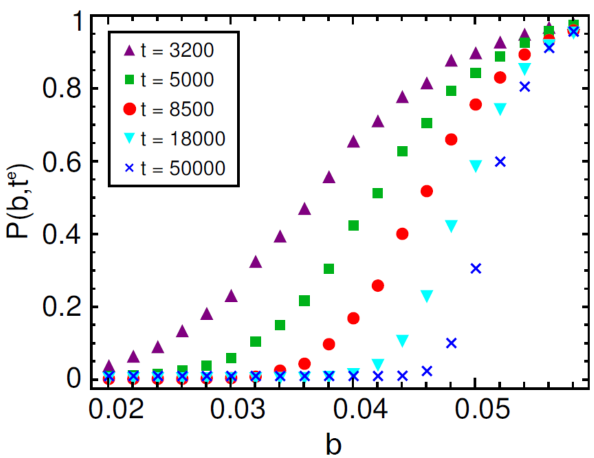
Extinction in Ecology = Death of Turbulence

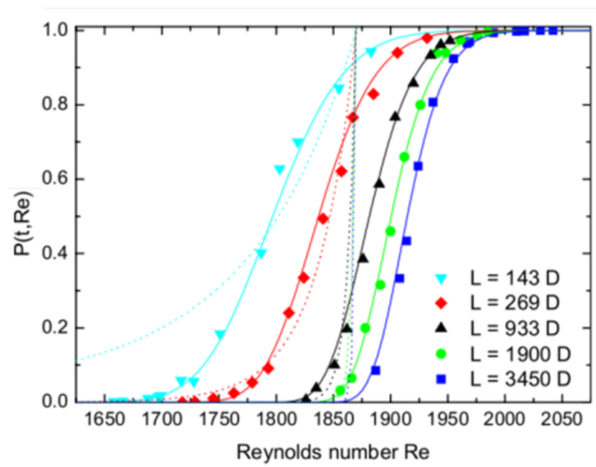
Mean time between population split events

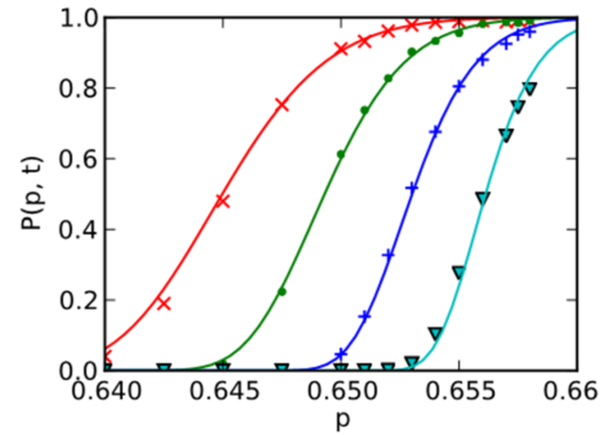
Mean time between puff split events

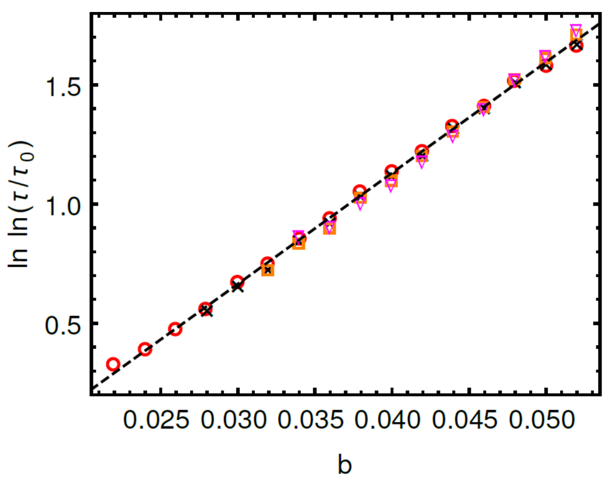
Avila *et al.*, *Science* **333**, 192 (2011)
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

Ecology = turbulence = DP

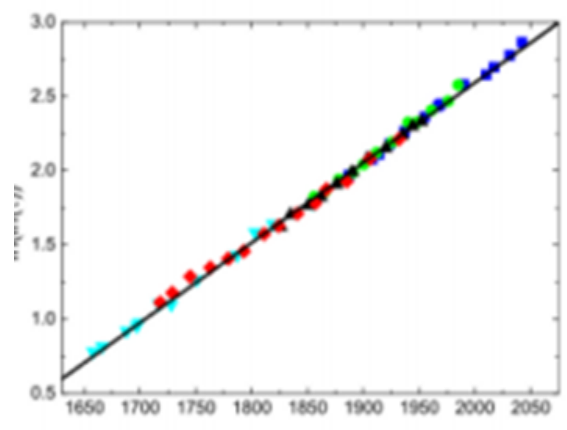




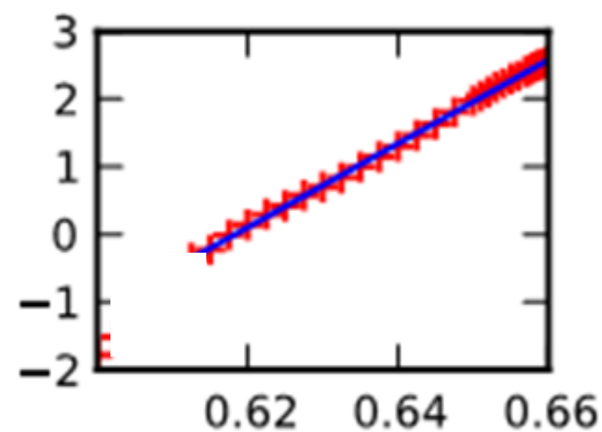




Shih, Hsieh, NG (2015)

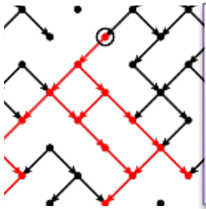


Hof et al., *PRL* **101**, 214501 (2008)



Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

Summary: universality class of transitional turbulence

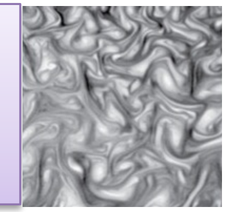


(Wikimedia Commons)

Directed Percolation

(Boffetta and Ecke, 2012)

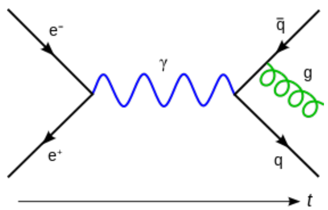
(Classical) Turbulence



Reggeon field theory (Janssen, 1981)

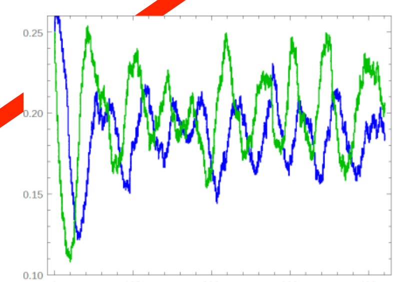
Direct Numerical Simulations of Navier-Stokes

Field Theory



(Wikimedia Commons)

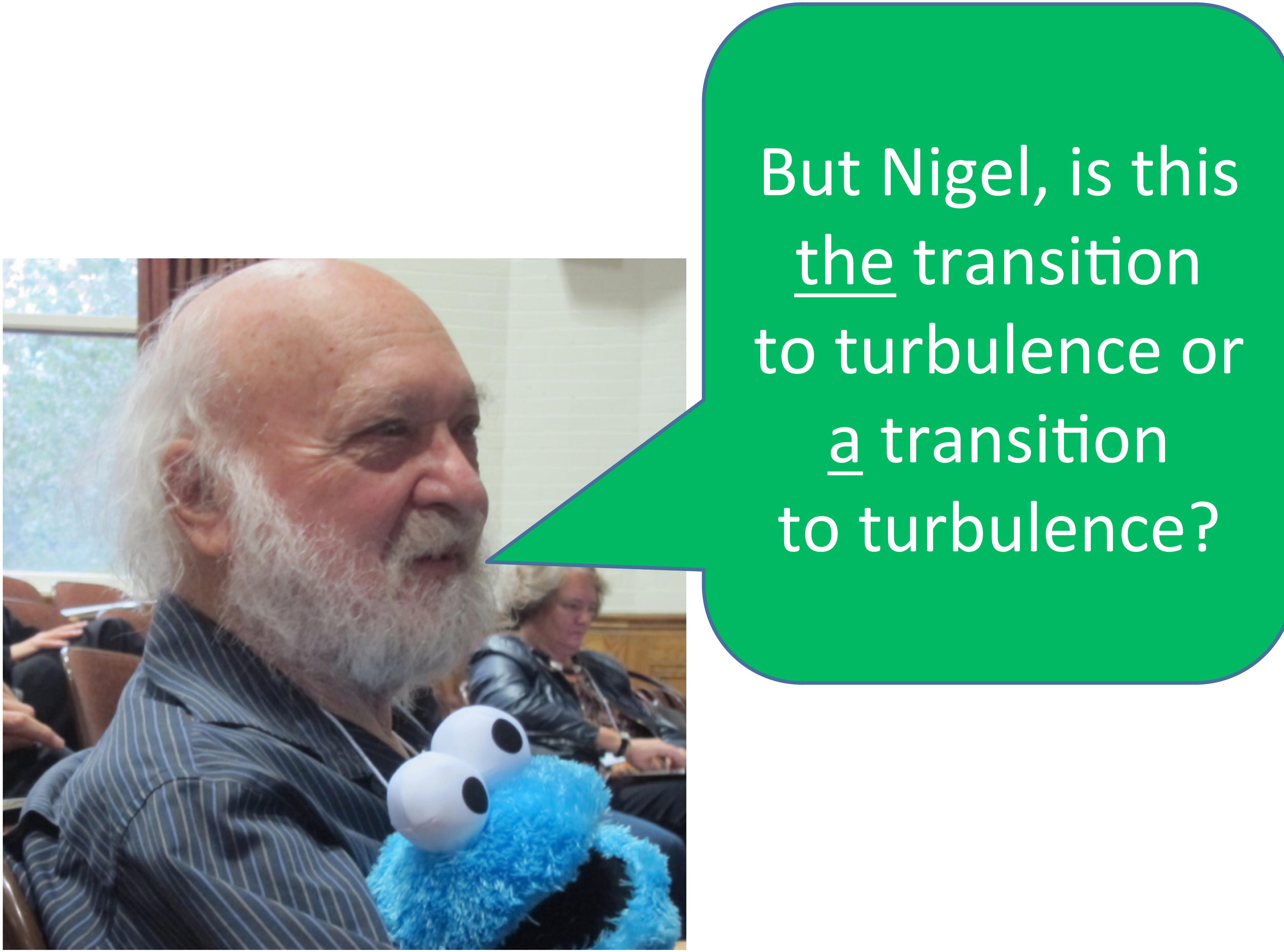
Two-fluid model



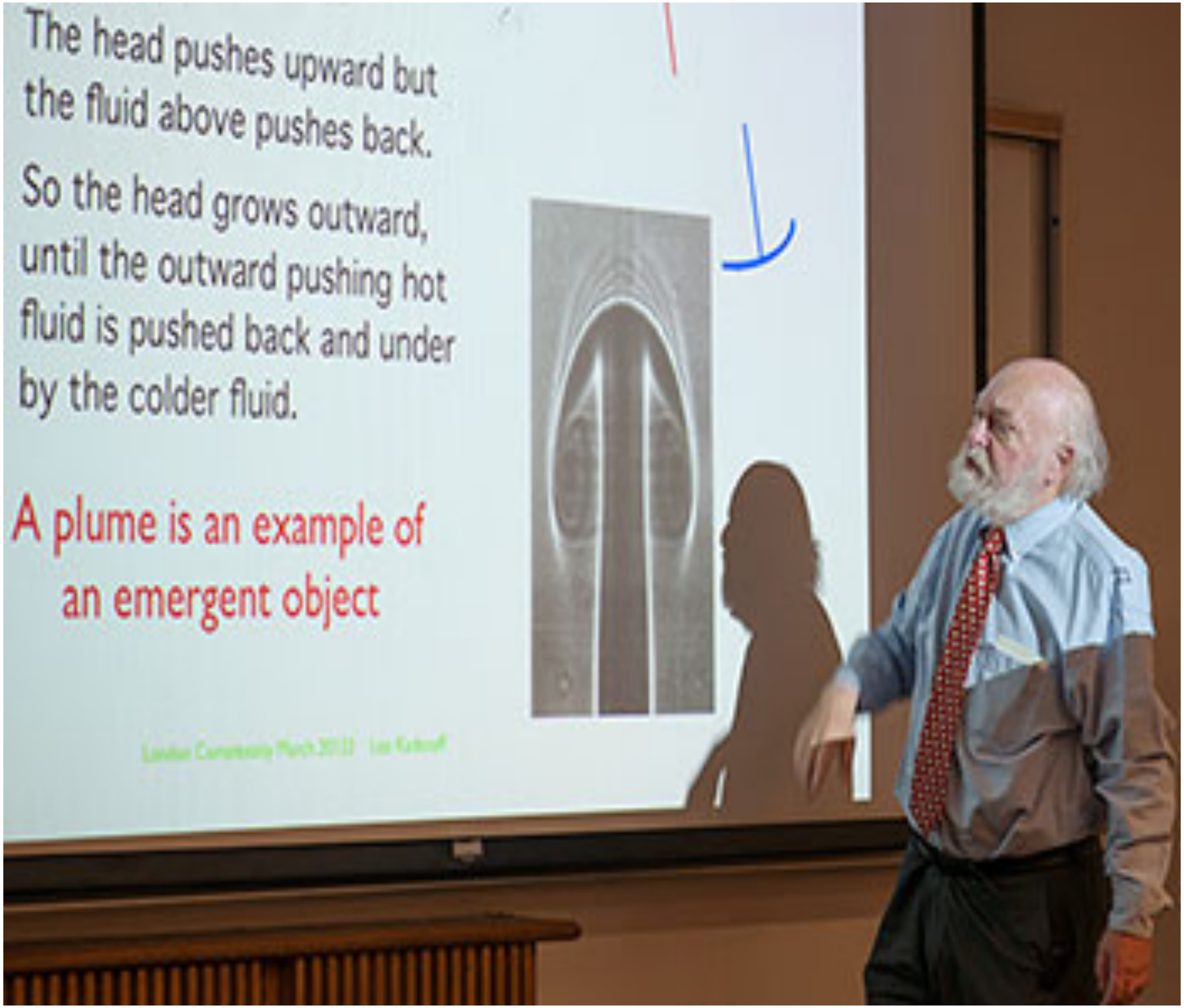
Extinction transition (Mobilia et al., 2007)

Predator-Prey

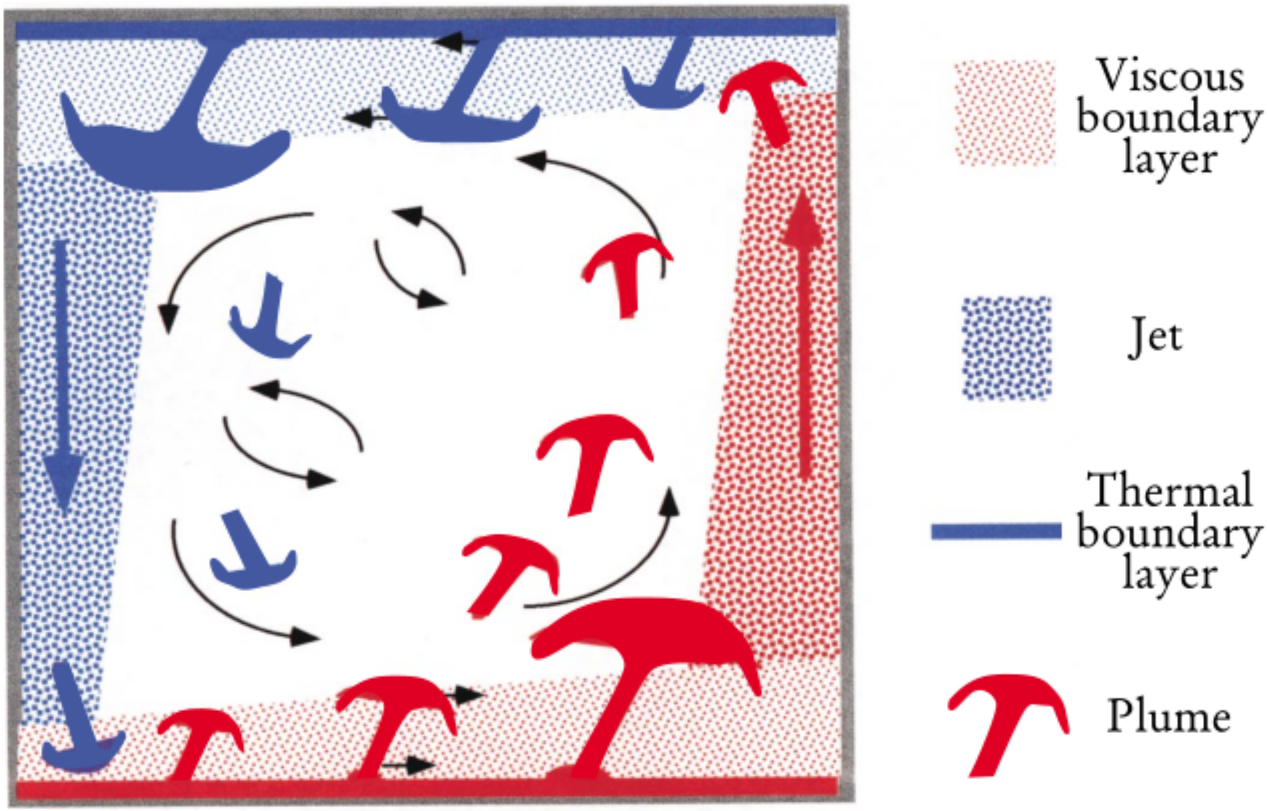
(Pearson Education, Inc., 2009)


But Nigel, is this the transition to turbulence or a transition to turbulence?

Predator-prey oscillations in convection

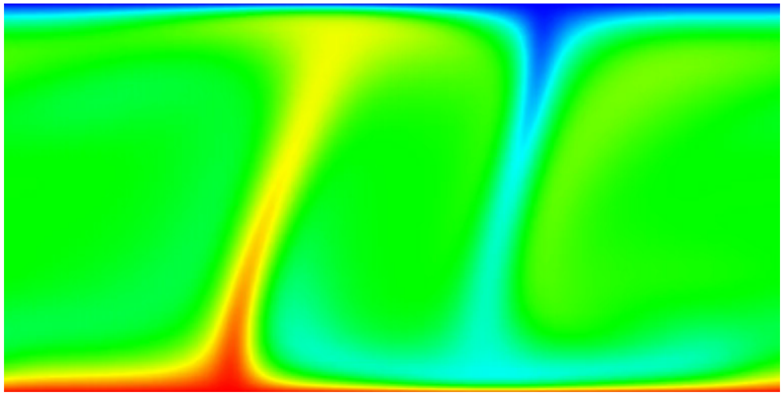


Predator-prey oscillations in convection



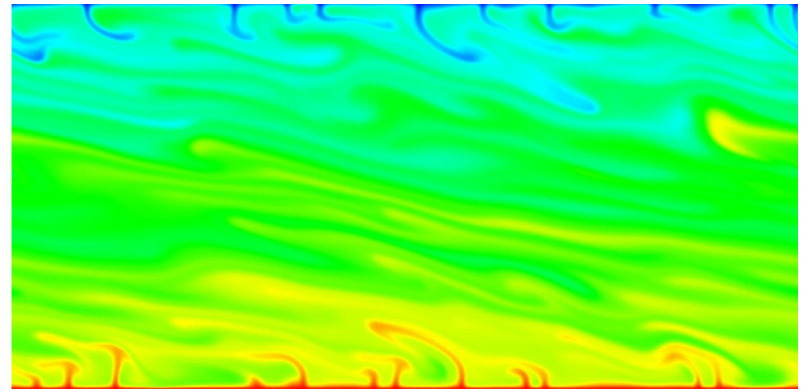
L.P. Kadanoff, Physics Today (2001).

Predator-prey oscillations in convection



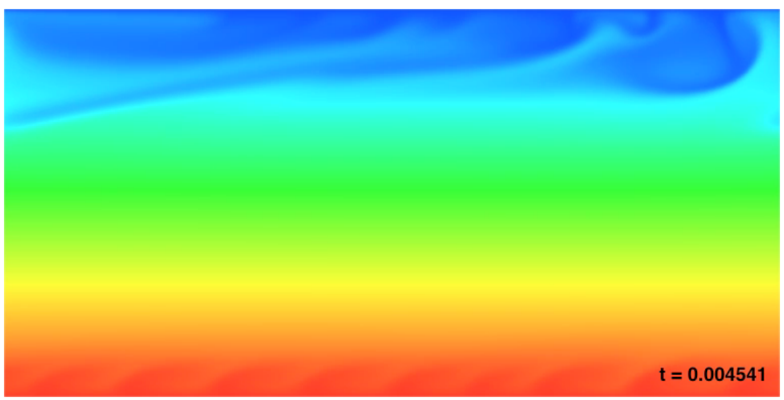
Pr=10 Ra=2 x 10^5

Sustained shearing convection



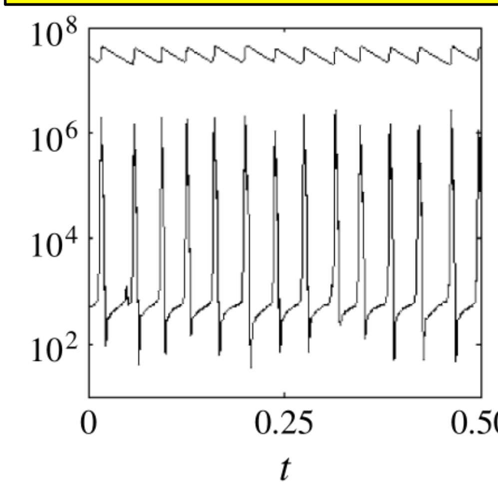
Pr=10 Ra=2 x 10^8

E_x = Horizontal component of KE



Pr = 1 Ra=2 x 10^8

Bursty shearing convection



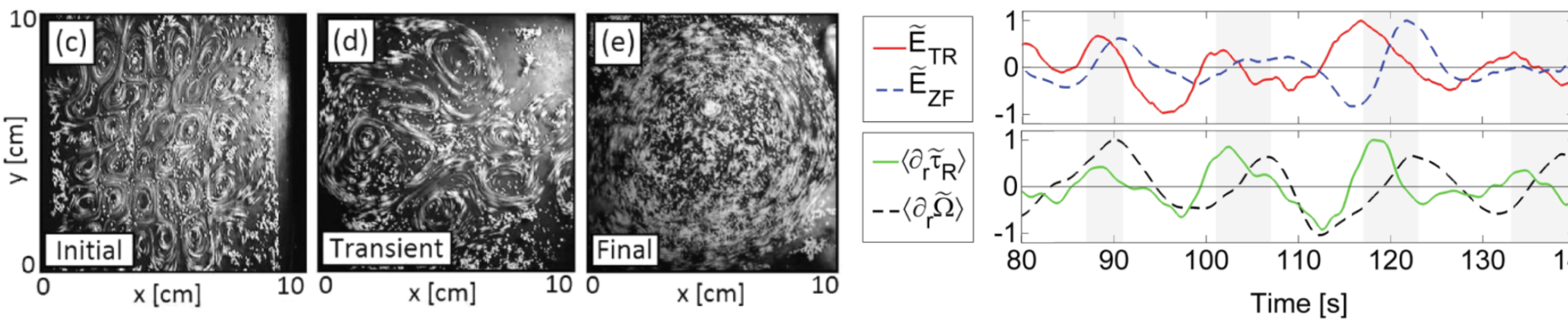
E_z = Vertical component of KE

Energy in zonal flow and vertical plumes shows predator-prey oscillations

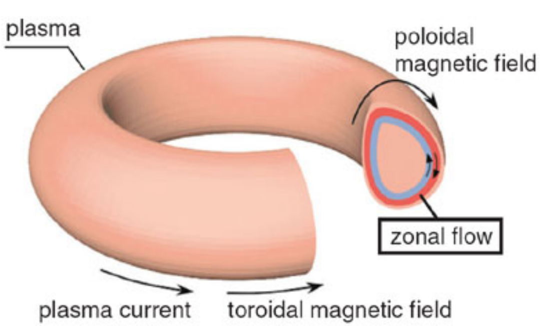
D. Goluskin et al. JFM (2014)

Universal predator-prey behavior in transitional turbulence

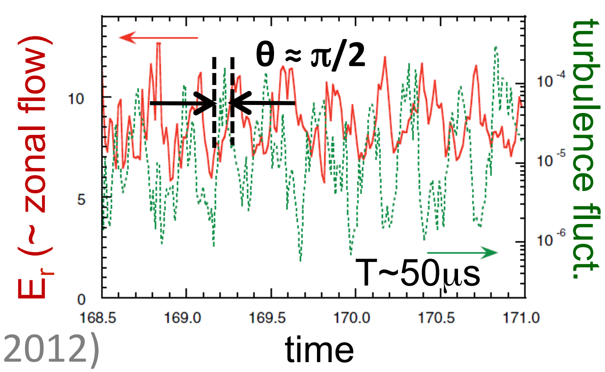
- Experimental observations
  - L-H mode transition in fusion plasmas in tokamak
  - 2D magnetized electroconvection

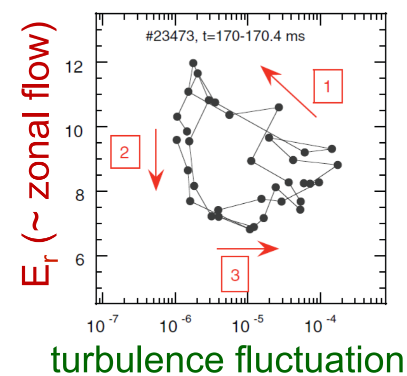


Bardoczi et al. Phys. Rev E (2012)



Estrada et al. EPL (2012)





Transition to turbulence in Taylor-Couette flow

PHYSICAL REVIEW E **81**, 025301(R) (2010)

Transient turbulence in Taylor-Couette flow

Daniel Borrero-Echeverry and Michael F. Schatz
Center for Nonlinear Science and School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430, USA

Randall Tagg
Department of Physics, University of Colorado, Denver, Colorado 80217-3364, USA

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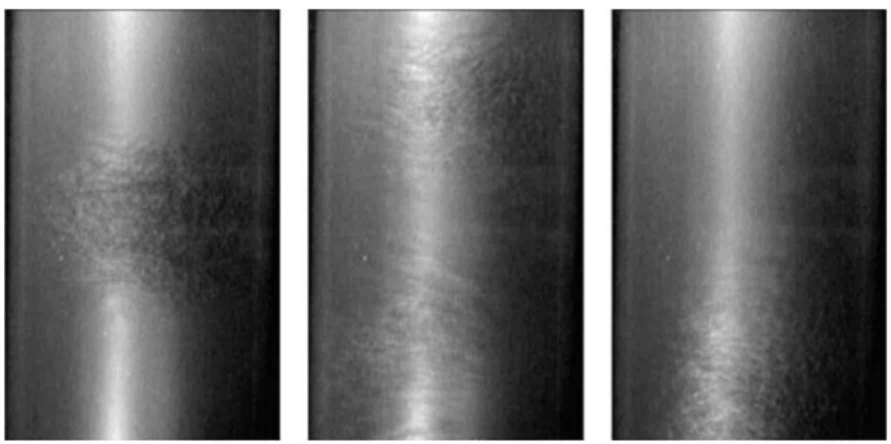
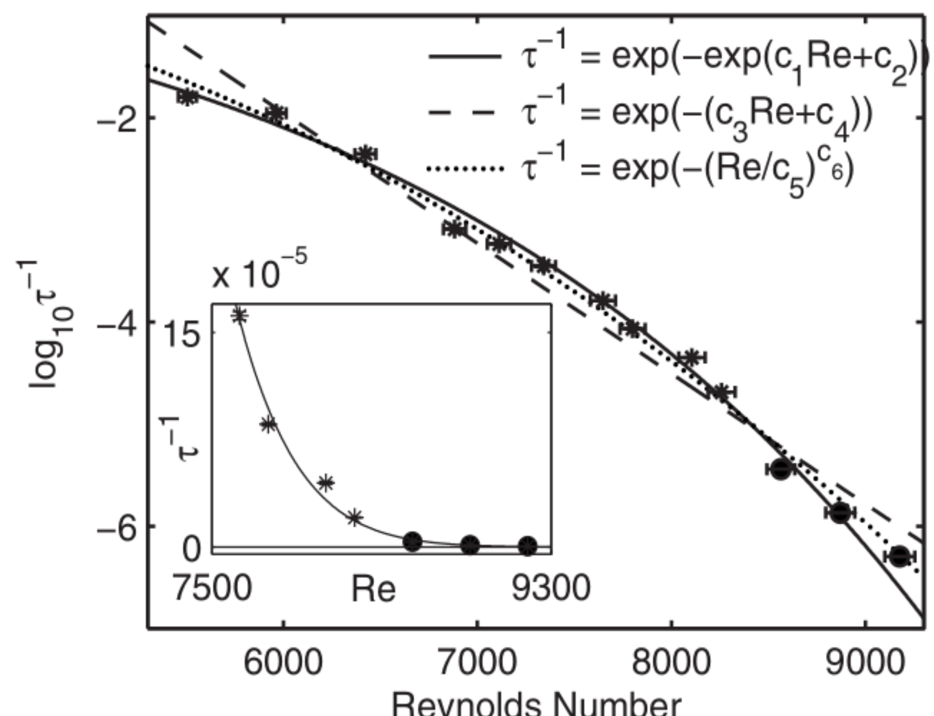
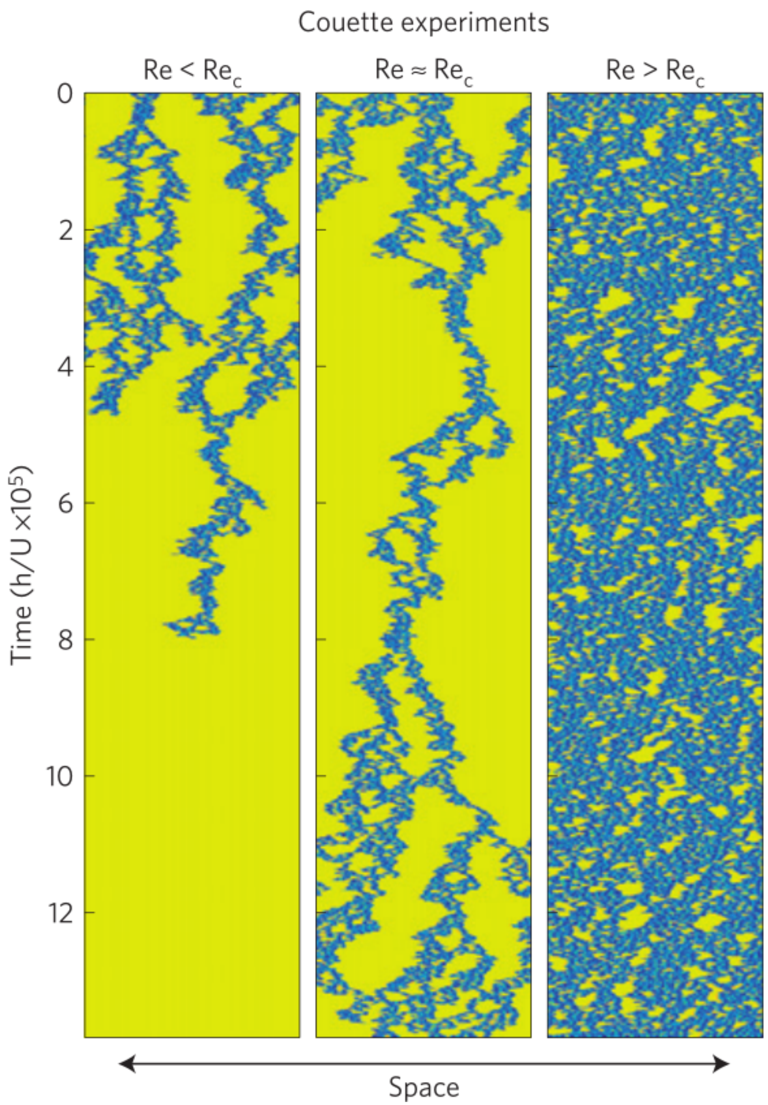


FIG. 1. Photographs of turbulent patches in TCF at Re=7500 with only the outer cylinder rotating. In this regime, turbulent patches coexist with the laminar flow and evolve in space and time. For all Re studied, these patches decay away in a probabilistic manner with a characteristic time scale τ dependent on Re. The photographs show a 25 cm high region of the flow.



Measurement of DP exponents



Lemoult et al., Nature Physics (2016)

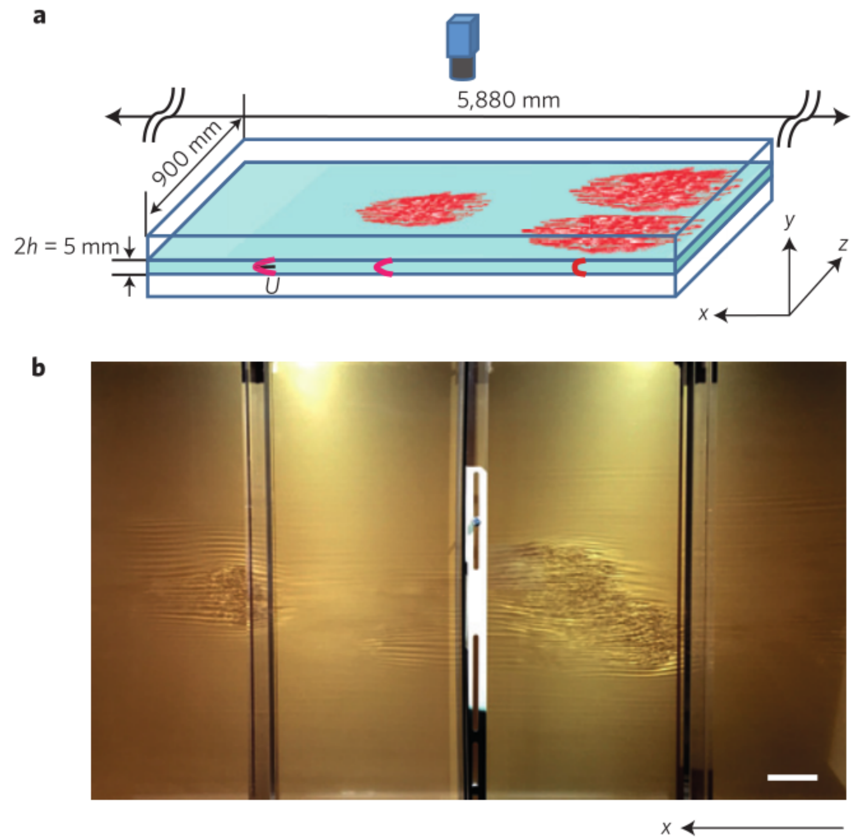


Figure 1 | Apparatus and snapshot of turbulent spots. a, Schematic of the apparatus. The aspect ratio of the channel is $2{,}352h \times 2h \times 360h$, where the depth $2h$ is 5 mm. **b**, Turbulent spots are visualized near the middle ($x = 3$ m) downstream location of the channel at Re = 810. The turbulent flows are injected by using a grid at the inlet ($x = 0$) of the channel. Visualization was assisted by means of micro-platelets and grazing angle illumination. Scale bar, 100 mm.

Sano & Tamai, Nature Physics (2016)

DP in large aspect ratio Taylor-Couette

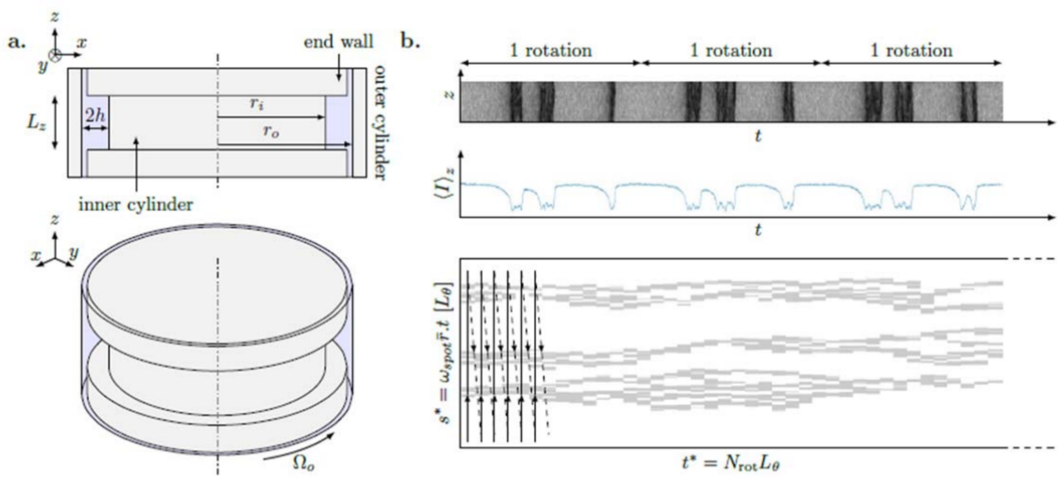
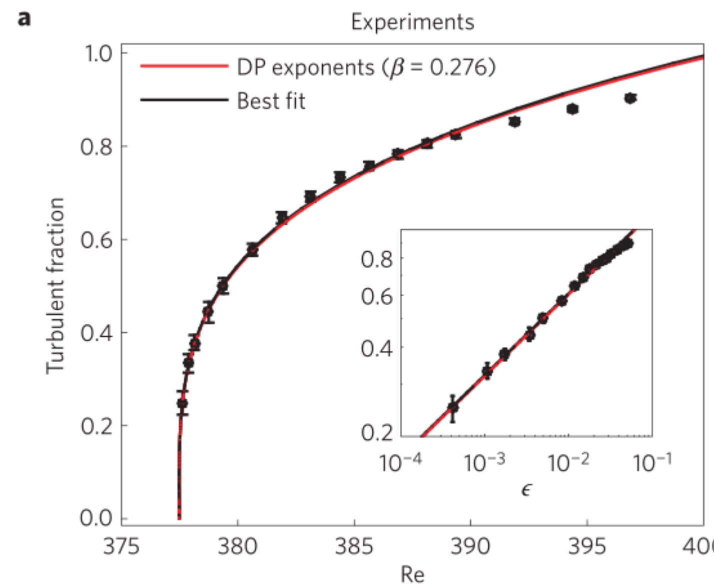
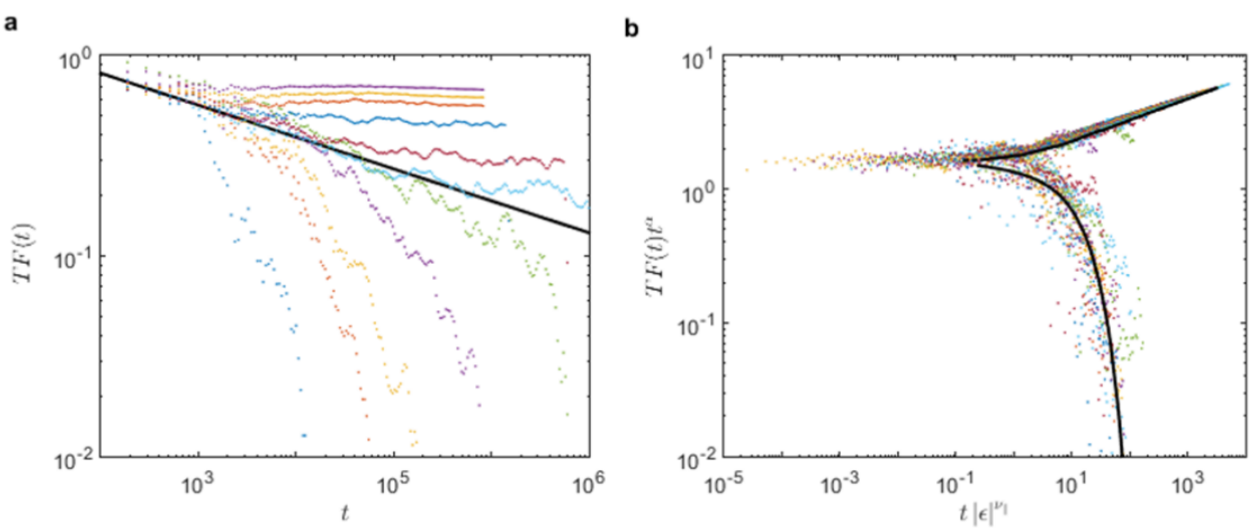


Figure S1: **a** Schematic of experimental set up (not to scale) see text for details. **b** Image of turbulent spots and the conversion to a intensity time series (see text for details).





Dynamic scaling of turbulent fraction following a critical quench from Re > Re_c

$$TF(t) \simeq t^{-\alpha} f(\varepsilon t^{1/\nu_\parallel})$$

Lemoult et al., Nature Physics (2016)

Summary

- Pipe flow consists of two regions, turbulence and roughly large scale flow
  - These behave as prey and predator in an ecosystem
- We report first observation of predator-prey oscillations in pipe turbulence
  - Turbulence is the prey
  - Zonal (azimuthal) flow is the predator
- Predator-prey in a pipe gives
  - lifetime and population splitting exhibit superexponential behavior with reproduction rate
  - The predator-prey transition is already known to be directed percolation (Mobilia et al. 2007) and reproduces observational phenomenology (Sipos & NG 2011)

Universal predator-prey behavior in transitional turbulence

- Experimental observations
  - L-H mode transition in fusion plasmas in tokamak
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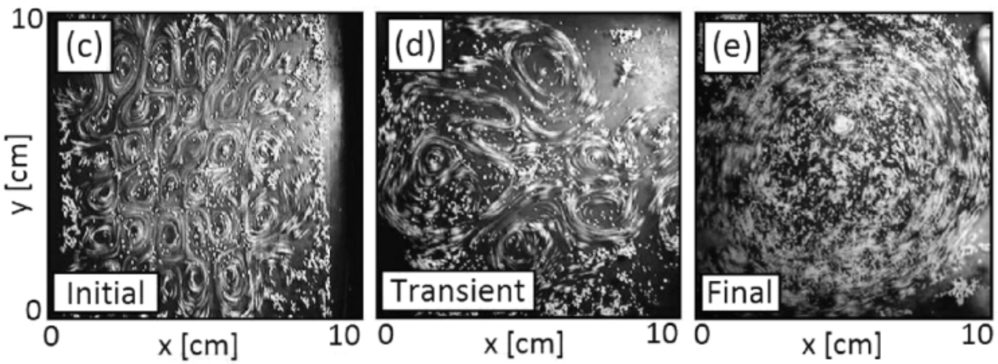
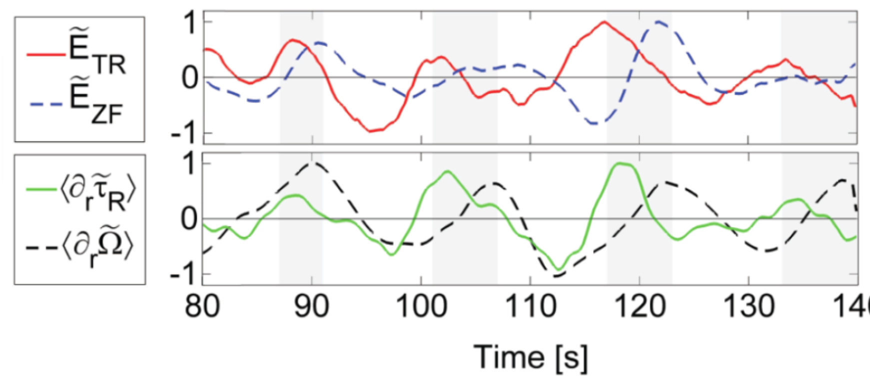
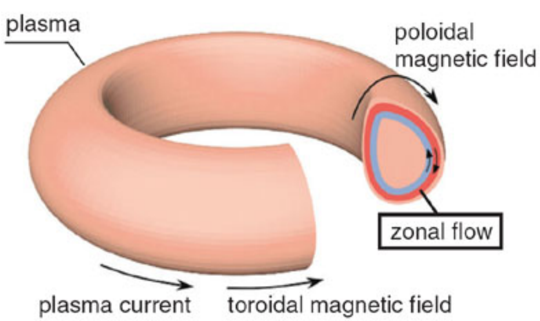
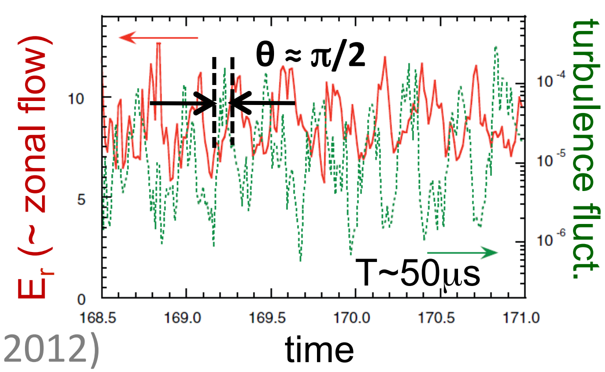
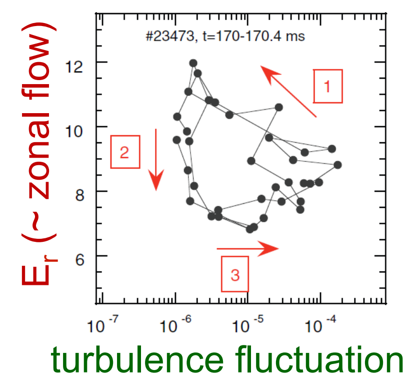




Bardoczi et al. Phys. Rev E (2012)







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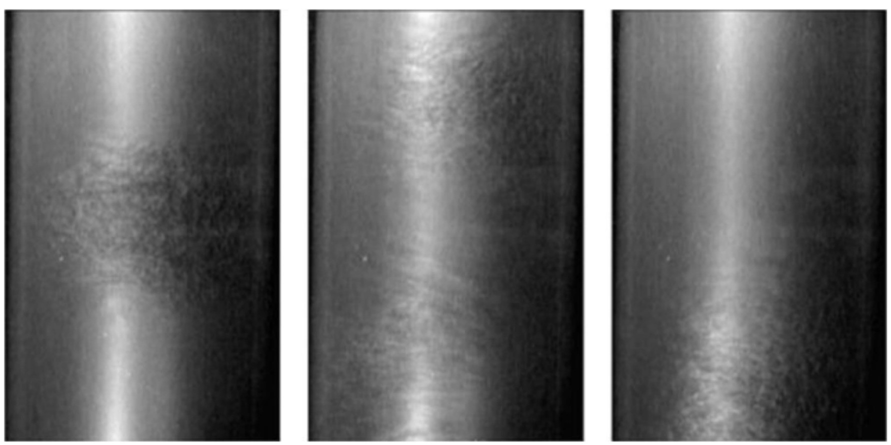

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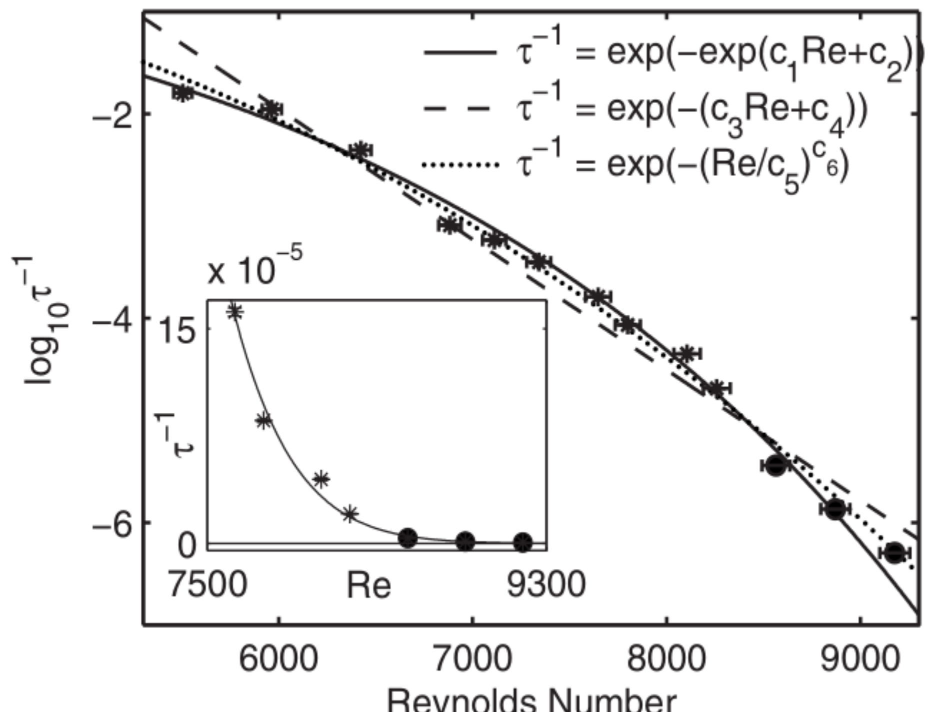

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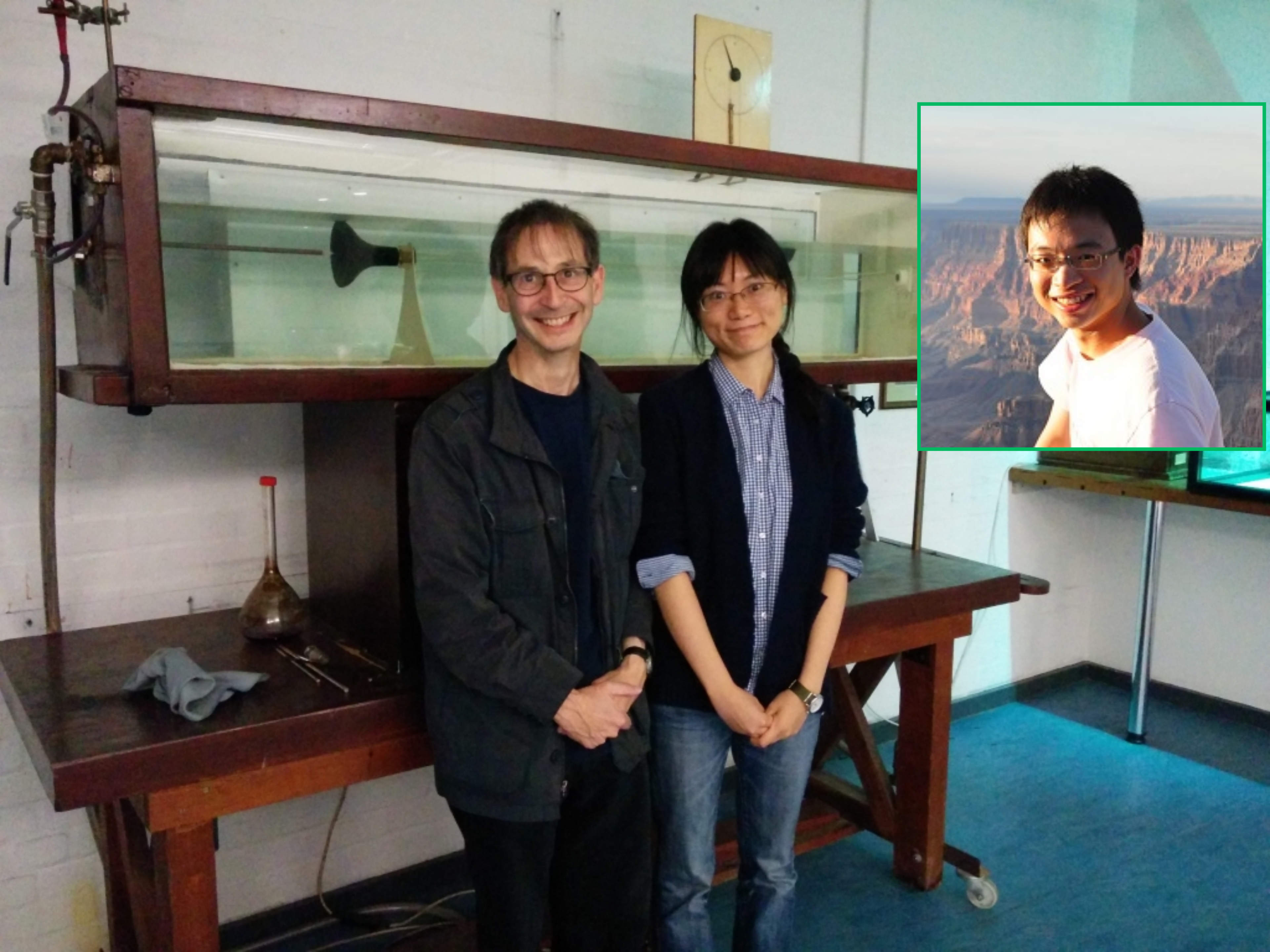
  - The predator-prey transition is already known to be directed percolation (Mobilia et al. 2007) and reproduces observational phenomenology (Sipos & NG 2011)

Conclusion

- Transition to pipe turbulence is in the universality class of directed percolation, evidenced by:
  - Puff lifetime as a function of Re
  - Extreme value statistics and finite-size scaling
  - Slug spreading rate as a function of Re
- How to derive universality class from hydrodynamics
  - Small-scale turbulence activates large-scale zonal flow which suppresses small-scale turbulence
  - Effective theory ("Landau theory") is stochastic predator-prey ecosystem
  - Exact mapping: fluctuating predator-prey = Reggeon field theory = DP near extinction
- Observational signatures
  - Predator-prey near extinction shows superexponential lifetime scaling for decay and splitting of puffs



Turbulence is a life force. It is opportunity.
Let's love turbulence and use it for change.

Lucky Numbers 34, 15, 28, 4, 19, 20

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TRANSITIONAL TURBULENCE

- Nigel Goldenfeld, N. Guttenberg and G. Gioia. Extreme fluctuations and the finite lifetime of the turbulent state. *Phys. Rev. E Rapid Communications* **81**, 035304 (R): 1-3 (2010)
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LETTERS

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Ecological collapse and the emergence of travelling waves at the onset of shear turbulence

Hong-Yan Shih, Tsung-Lin Hsieh and Nigel Goldenfeld*

The mechanisms and universality class underlying the remarkable phenomena at the transition to turbulence remain a puzzle 130 years after their discovery[1]. Near the onset to turbulence in pipes[1], plane Poiseuille flow[2] and Taylor–Couette flow[3], transient turbulent regions decay either directly[4] or through splitting[5-8], with characteristic timescales that exhibit a super-exponential dependence on Reynolds number[9,10]. The statistical behaviour is thought to be related to directed percolation (DP; refs 6,11–13). Attempts to understand transitional turbulence dynamically invoke periodic orbits and streamwise vortices[14-19], the dynamics of long-lived chaotic transients[20], and model equations based on analogies to excitable media[21]. Here we report direct numerical simulations of transitional pipe flow, showing that a zonal flow emerges at large scales, activated by anisotropic turbulent fluctuations; in turn, the zonal flow suppresses the small-scale turbulence leading to stochastic predator–prey dynamics. We show that this ecological model of transitional turbulence, which is asymptotically equivalent to DP at the transition[22], reproduces the lifetime statistics and phenomenology of pipe flow experiments. Our work demonstrates that a fluid on the edge of turbulence exhibits the same transitional scaling behaviour as a predator–prey ecosystem on the edge of extinction, and establishes a precise connection with the DP universality class.

References

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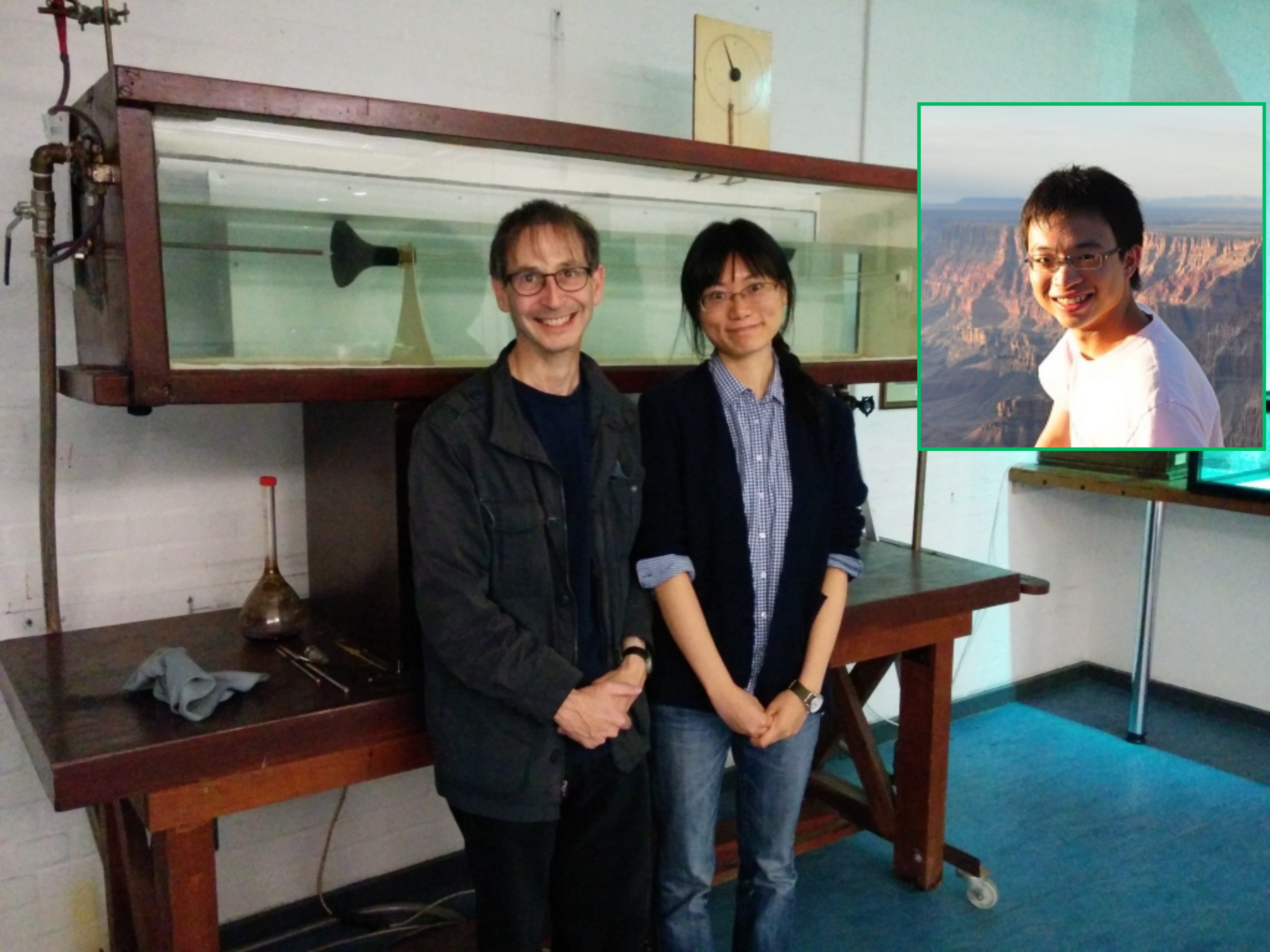


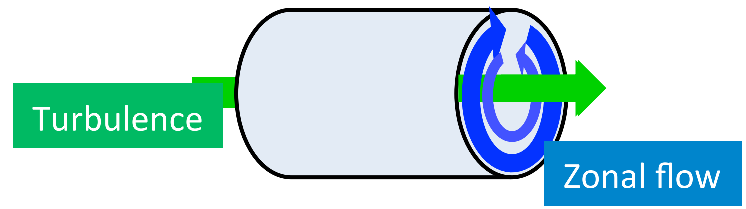
Wishing you many more happy birthdays, Jim! And thanks for putting us into a happy, fruitful and long-lived metastable state!

Super-exponential scaling and extreme statistics

- Active state persists until the most long-lived percolating "strands" decay.
  - extreme value statistics
- Why do we not observe the power law divergence of lifetime of DP near transition?
- Close to transition, transverse correlation length diverges, so initial seeds are not independent
  - Crossover to single seed behaviour
  - Asymptotically will see the power law behavior in principle

$$\xi_\perp \sim (p-p_c)^{-\nu_\perp}$$

Ecology of turbulence



- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0)
  - Anisotropy of turbulence creates Reynolds stress
  - The radial gradient of Reynolds stress generates the large scale fluctuations in azimuthal direction (zonal flow)

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

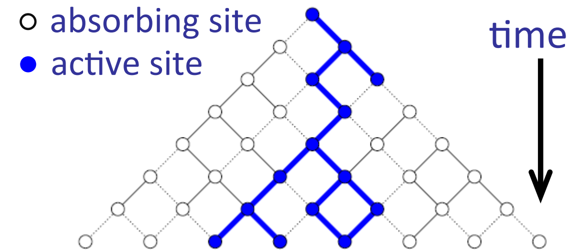
  - Zonal flow creates shear to turbulence and decreases the anisotropy of turbulence and thus suppress turbulence

Laminar flow → Turbulenc e → Zonal flow

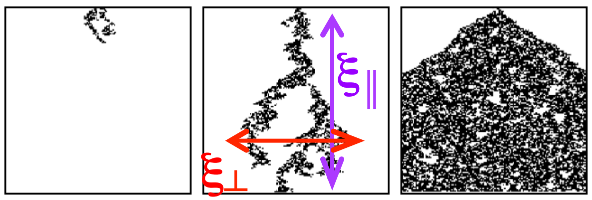
Nutrient → Prey → Predator

Summary: Transitional Turbulence as DP

- Transitional turbulence ~ Directed percolation (Pomeau, 1986)
- Directed percolation (DP)
  - percolating probability ***p*** at each site
  - absorbing state → laminar flows
  - active state → turbulent slugs



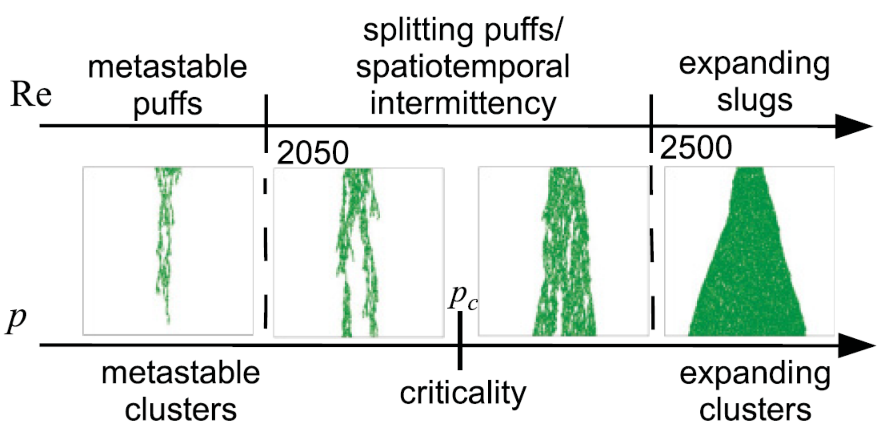
- Critical transition threshold p_c:



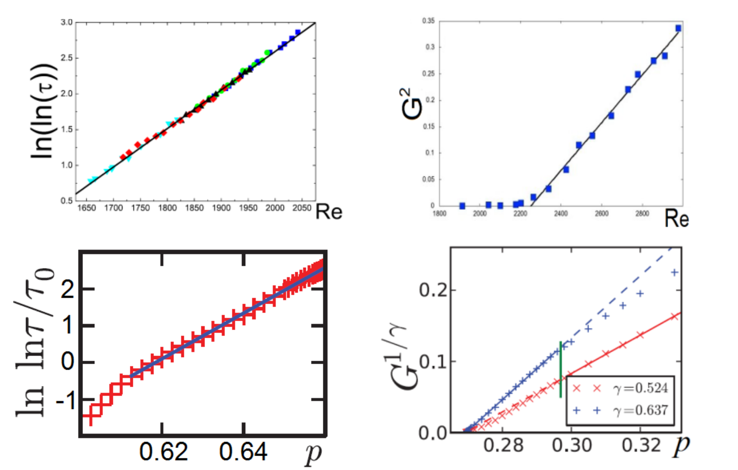
$p < p_c$ $\quad$ $p \approx p_c$ $\quad$ $p > p_c$

$p \approx p_c$,
correlation length $\xi_\parallel \sim |p - p_c|^{\nu_\parallel}$, $\xi_\perp \sim |p - p_c|^{\nu_\perp}$
growth rate $G \sim \xi_\perp / \xi_\parallel \sim (p - p_c)^{\nu_\parallel - \nu_\perp}$

- Turbulence vs. (3+1) DP in pipe:



Hof et al., *PRL* **101**, 214501 (2008) De Lozar et al. arXiv:1001.2481 (2010)

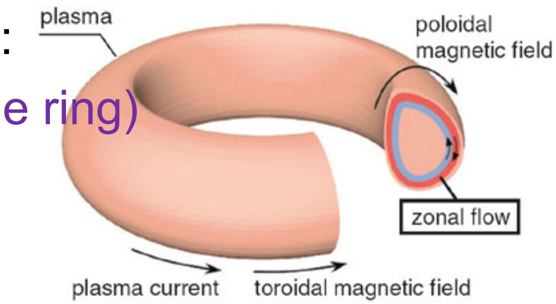


Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

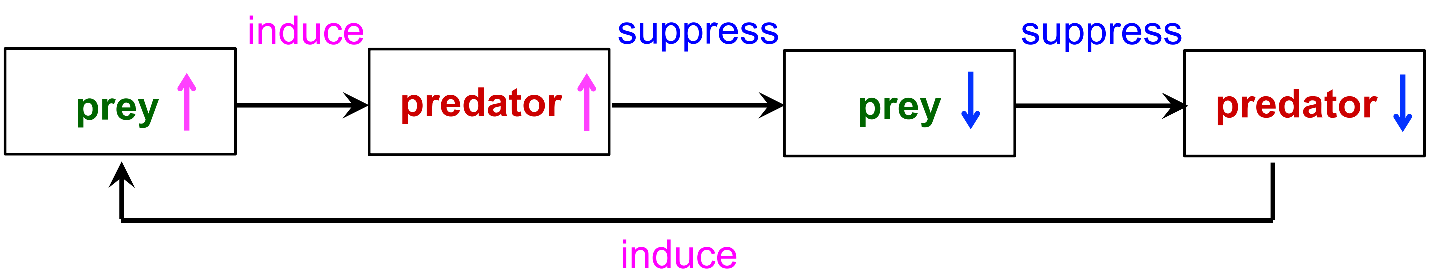
Hinrichsen, *Adv. Phys.* **49**, 815 (2000) Henkel, Non-Equilibrium Phase Transitions vol.1 (2008)

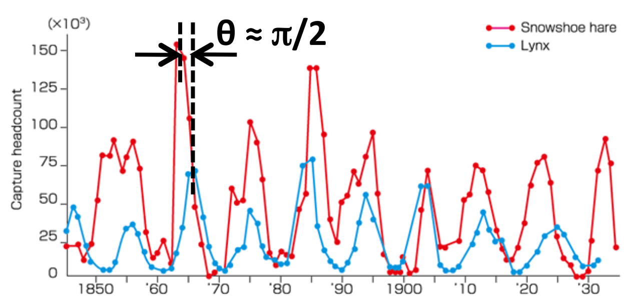
Predator-Prey Dynamics in Tokomaks

- In tokamak (toroidal chamber with axial magnetic field):
  - **turbulent plasma** (small-scale drift waves along the ring)
  - **zonal flows**:
    - E_r x B turbulence-induced flow on small circles
    - cause radial shear to damp turbulent plasma
    - decrease due to dissipation



- Self-organized dynamics in **Ecology**:





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Predator-Prey Dynamics in Tokomaks

- In tokamak (toroidal chamber with axial magnetic field):
  - turbulent plasma (prey)
  - zonal flows (predator):
    - E_r x B turbulence-induced flow on small circles
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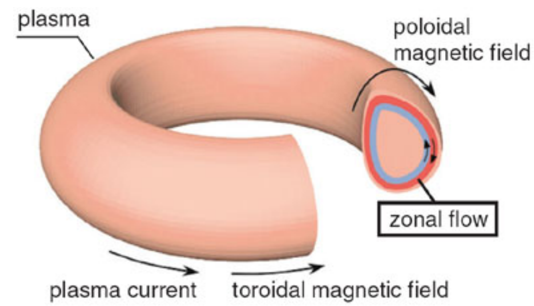
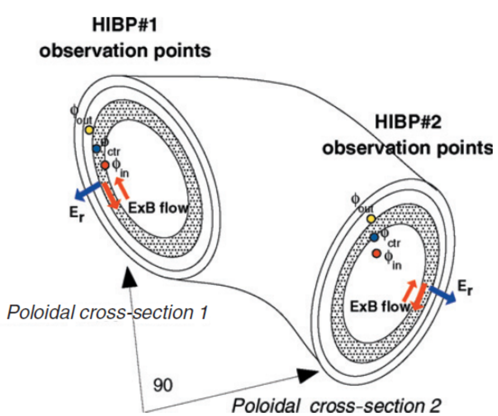
    - decrease due to dissipation

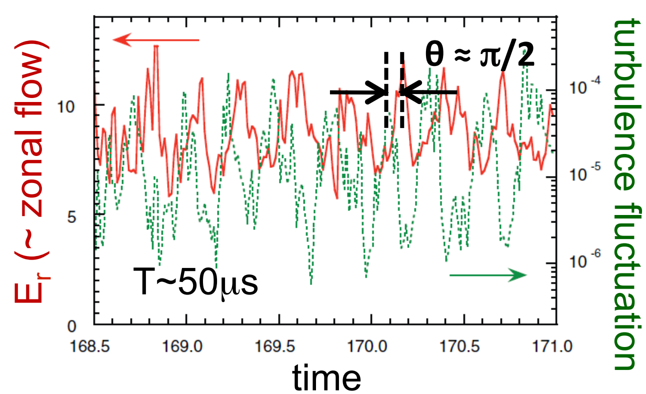


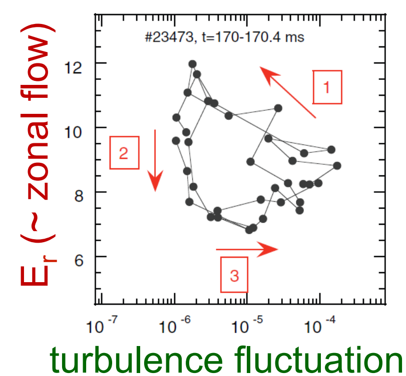
- Self-organized dynamics in **Magneto-hydrodynamics**:

turbulent plasma ↑ —induce→ **zonal flow ↑** —suppress→ **turbulent plasma ↓** —suppress→ **zonal flow ↓** —induce→ **turbulent plasma ↑**

Estrada et al. EPL (2012)







Two-fluid predator-prey model for transitional turbulence

Can we observe predator-prey oscillations?

Predator-Prey Dynamics in Tokomaks

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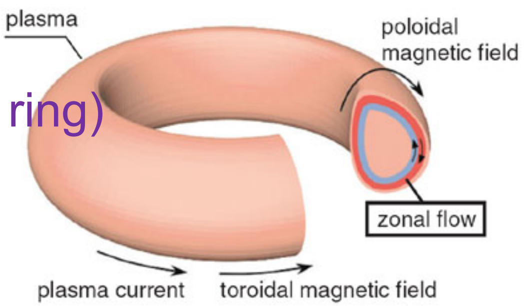

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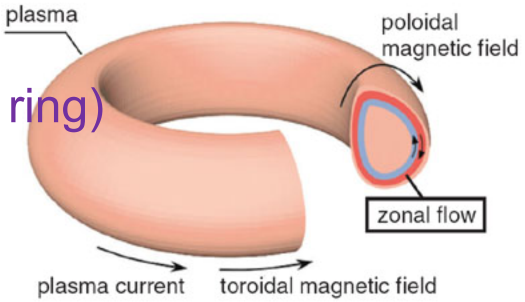

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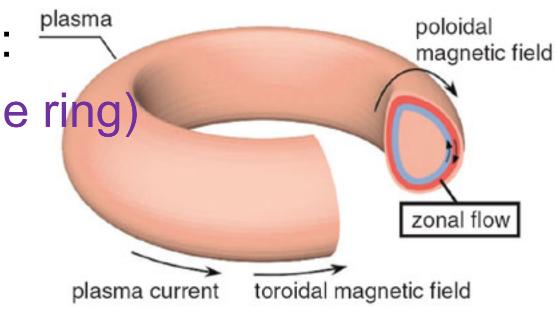
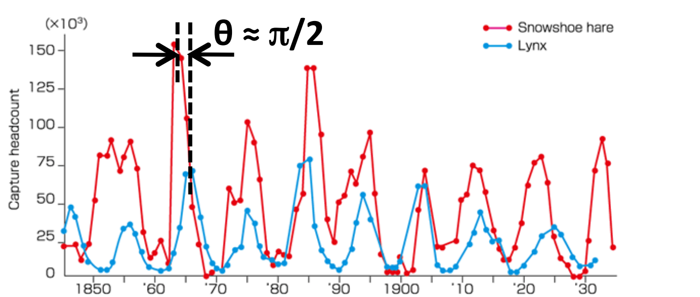


- Self-organized dynamics in **Ecology**:

prey ↑ —induce→ predator ↑ —suppress→ prey ↓ —suppress→ predator ↓

predator ↓ —induce→ prey ↑



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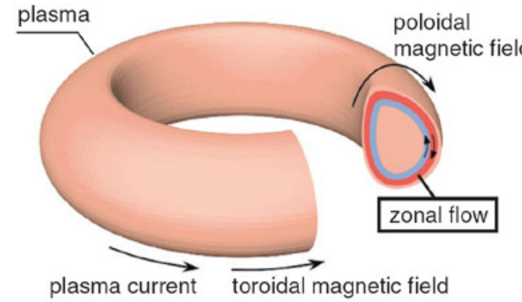

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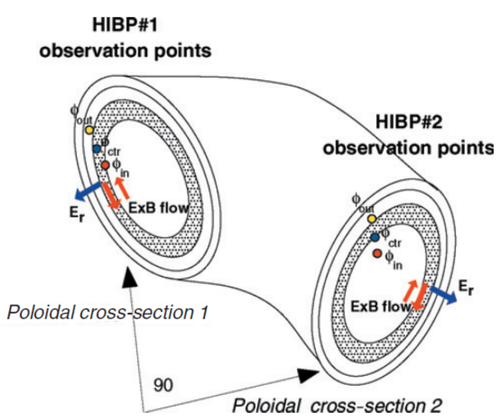
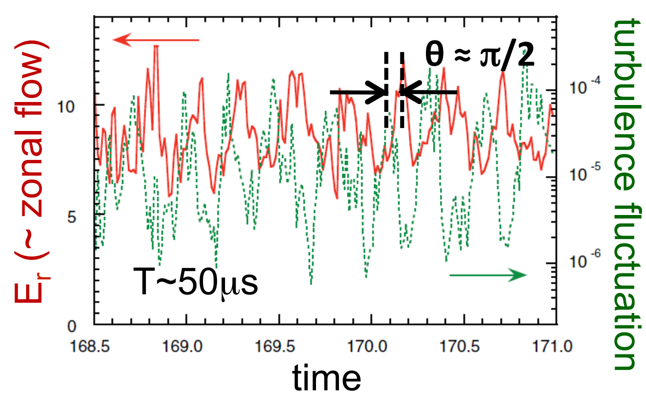
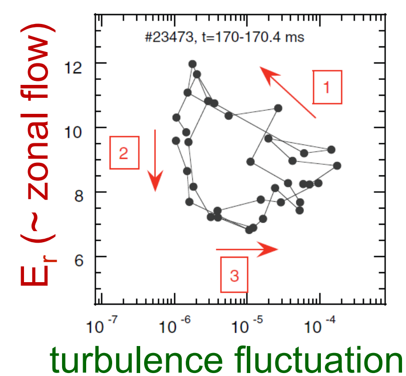
turbulent plasma ↑ $\xrightarrow{\text{induce}}$ zonal flow ↑ $\xrightarrow{\text{suppress}}$ turbulent plasma ↓ $\xrightarrow{\text{suppress}}$ zonal flow ↓ $\xrightarrow{\text{induce}}$ turbulent plasma ↑

Estrada et al. EPL (2012)

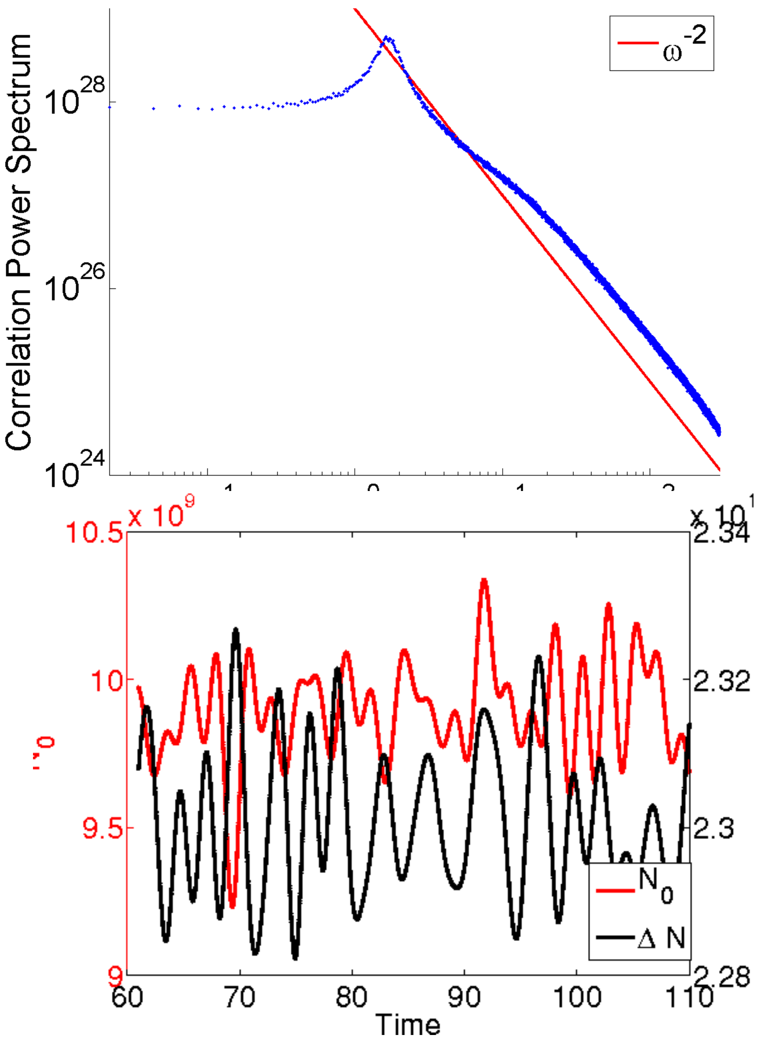






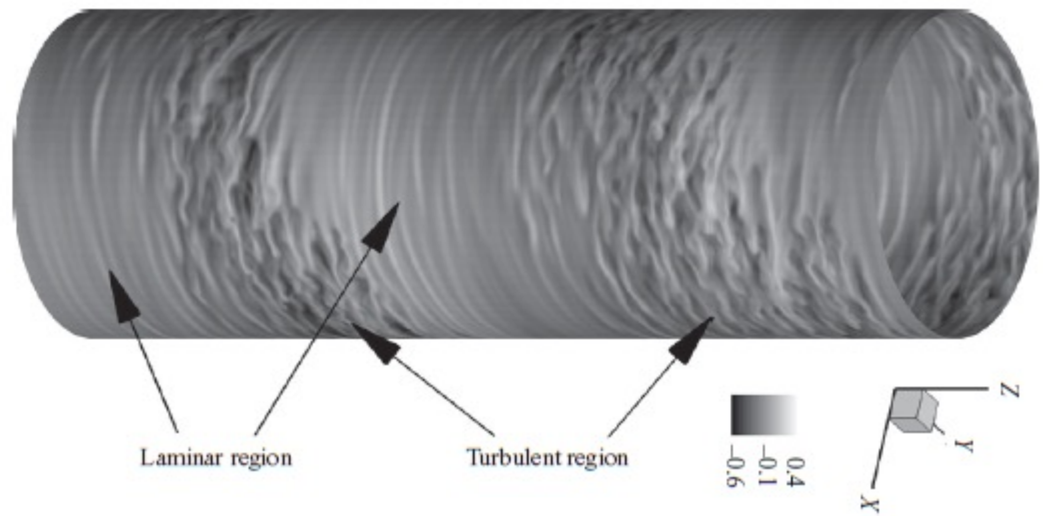
Testing the ecology of turbulence

- Quasi-cycles in ecology are driven by number fluctuations, ie. discreteness
- Quasi-cycles exhibit f^{-2} power spectrum, not f^{-4} expected for noisy limit cycle
  - What sets discreteness in turbulence number fluctuations of large-scale modes (predator) and small-scale turbulence (prey)?
  - Nonlinearity and locality → thresholds for scattering of modes
- Quasi-cycles seen in pumped nonlinear Schrodinger equation
  - Dyachenko et al. (1992) first proposed existence of predator-prey oscillations in NLSE



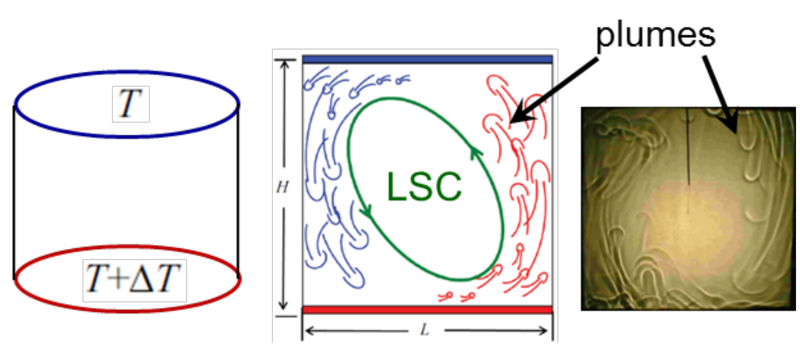
Generic two-fluid behavior in transitional turbulence

- **Spiral turbulence**



Dong and Zheng 2011

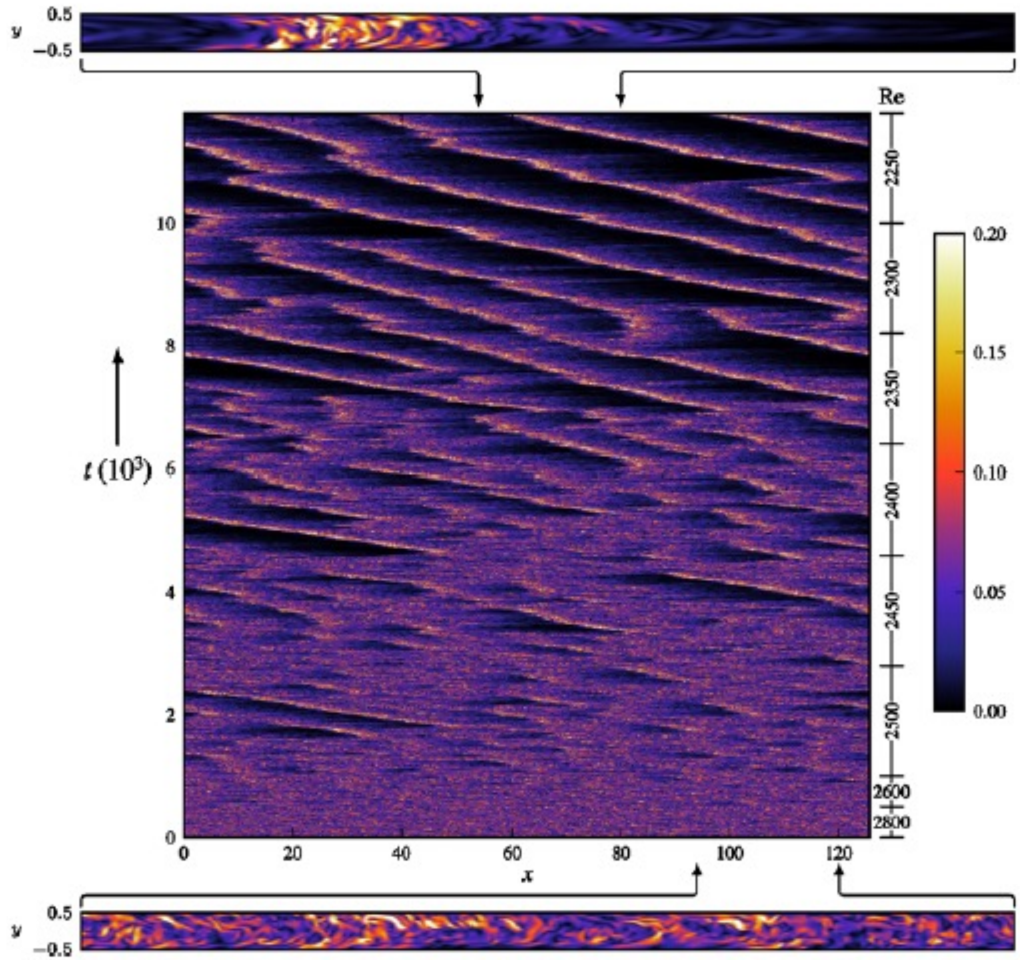
- **Large-scale circulation in turbulent convection**



Xia, *Theor. App. Mech. Lett.* **3**, 052001 (2013)   Ahlers et al., *RMP* **81**, 503 (2009)

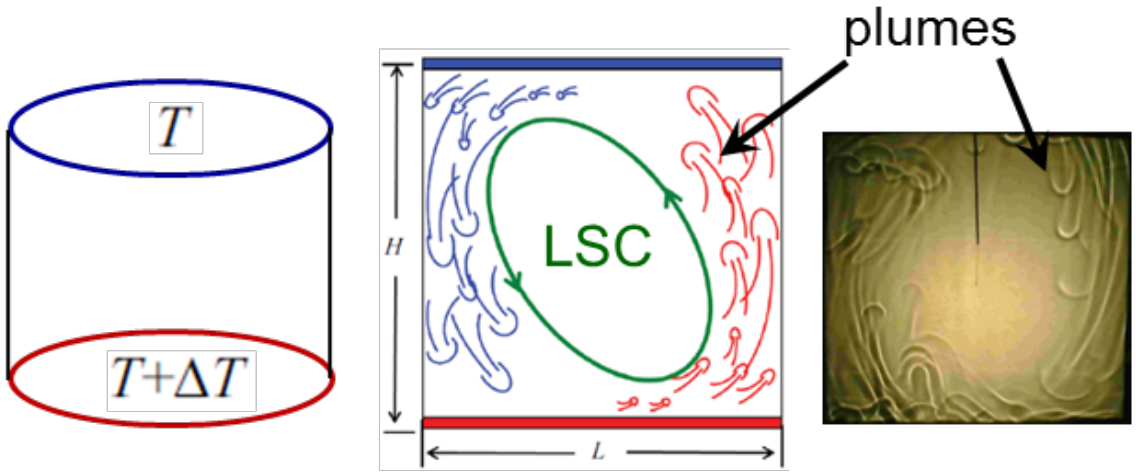
Generic two-fluid behavior in transitional turbulence

- Pipe flow exhibits both laminar and turbulent regions
- The turbulence moves slower than mean flow
- There is an induced or emergent large-scale flow



Moxley and Barkley PNAS 2010

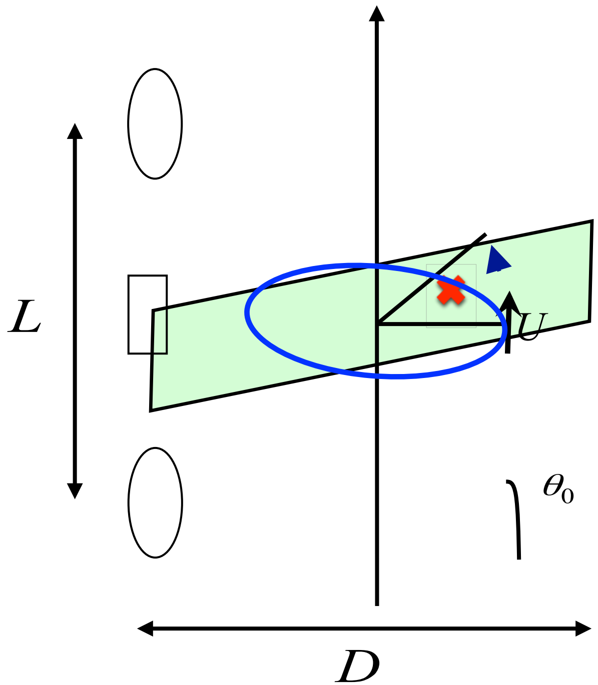
Turbulent convection transition



Xia, *Theor. App. Mech. Lett.* **3**, 052001 (2013) Ahlers et al., *RMP* **81**, 503 (2009)

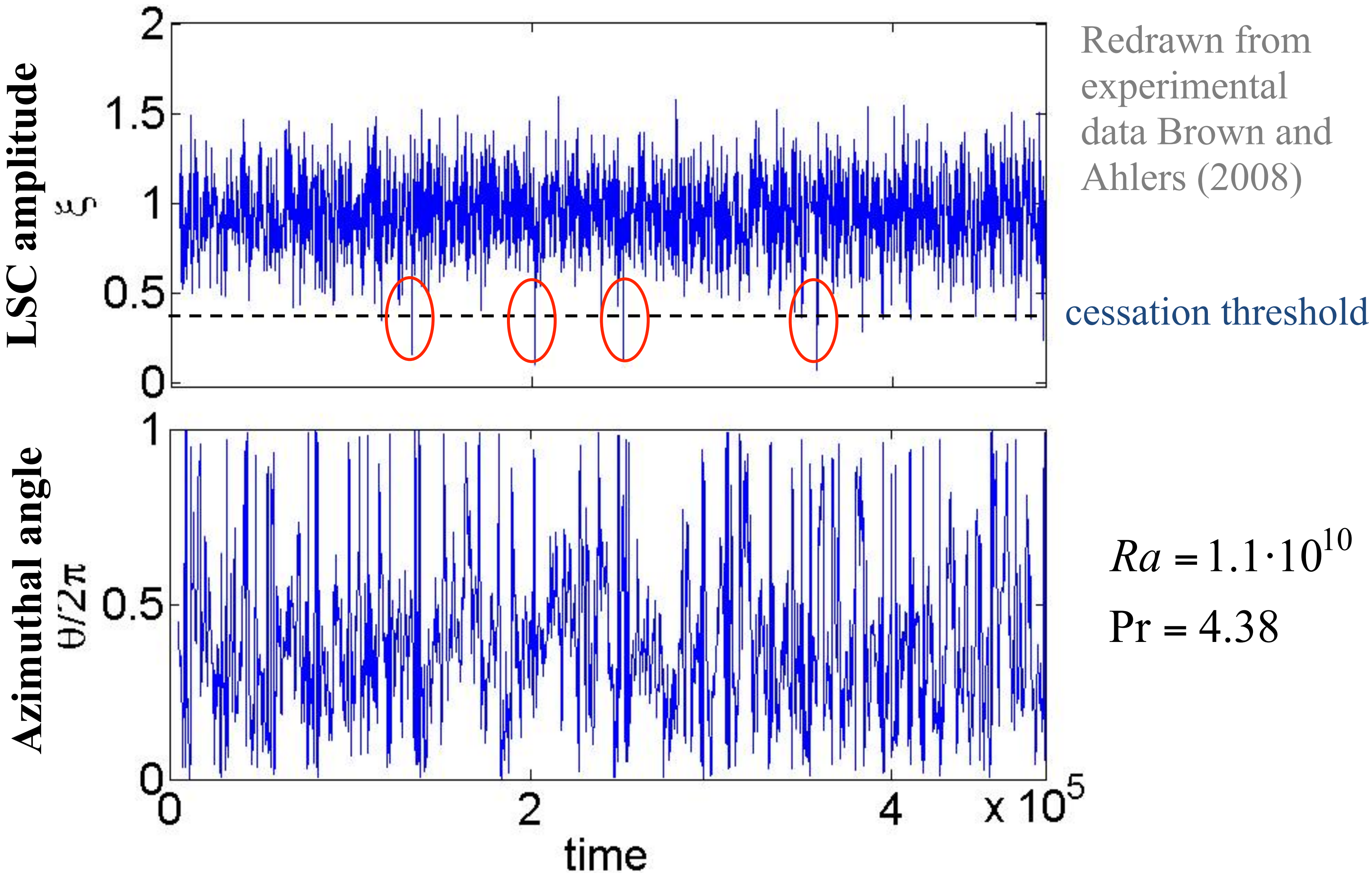
- Next step: are there predator-prey oscillations between the LSC and the turbulent fluctuations?
- Can test this with Brown and Ahlers data
- Come back for GA 90!

Large-scale circulation



Experiment with water, Ra=$3.7 \cdot 10^8$:
Du and Tong, JFM (2000)

- **Coherent LSC**
  - carries warm fluid from the bottom plate up one side of the sample; cools when passes the top plate and goes down on opposite side of the sample
- **Cessations and reorientations**



Redrawn from experimental data Brown and Ahlers (2008)

$Ra = 1.1 \cdot 10^{10}$

$\Pr = 4.38$

Cessations are rare events. However due to their importance, we want to accurately estimate how rare is rare!

Large-scale Circulation (LSC)

- Microscopic turbulence (plumes) + mesoscopic mean flow (LSC) → predator-prey relations **?**
- Ocean and atmospheric flow
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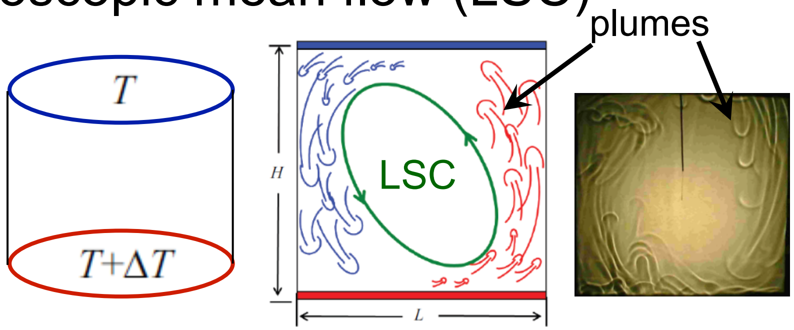
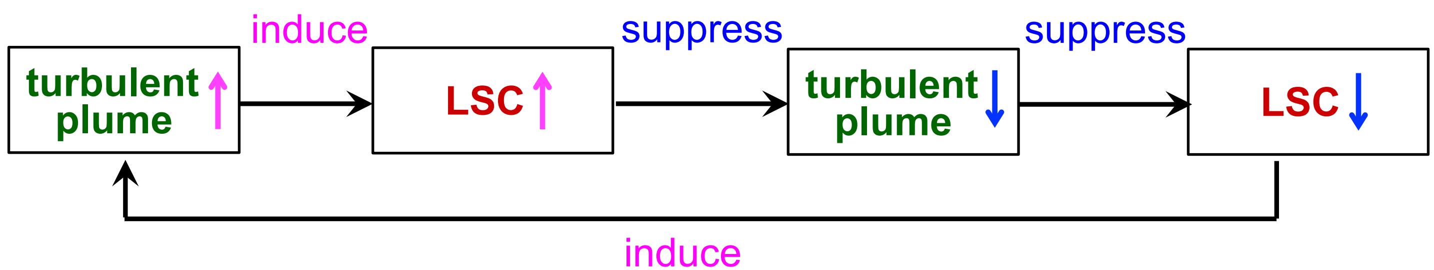
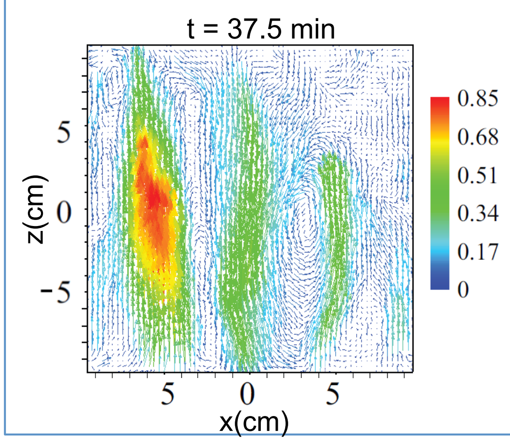
- Rayleigh number $Ra = \dfrac{\alpha g \Delta T H^3}{\nu\kappa} \gtrsim 10^6$



- Self-organized dynamics in **Large-scale circulation**:





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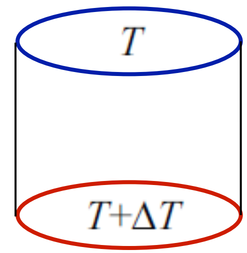

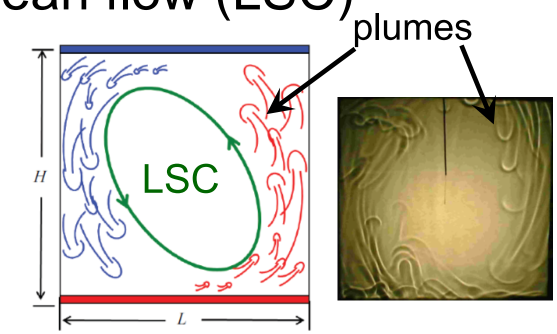

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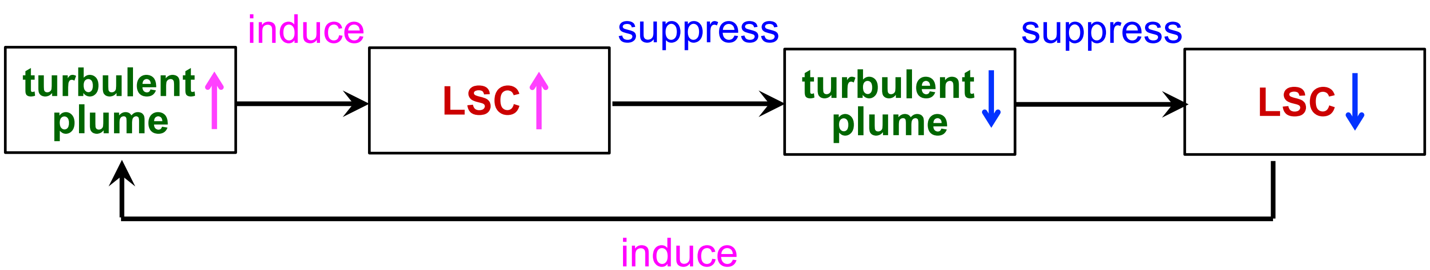

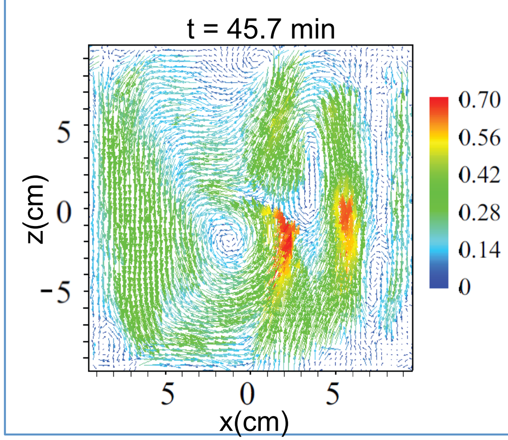

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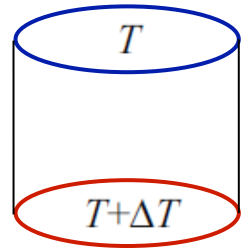

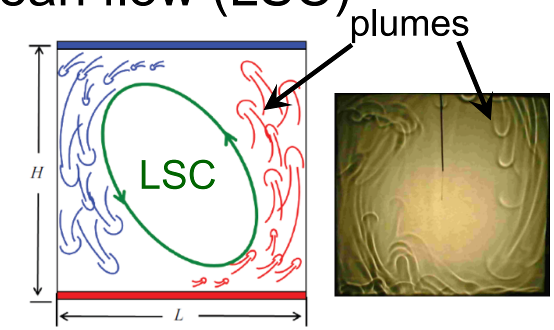

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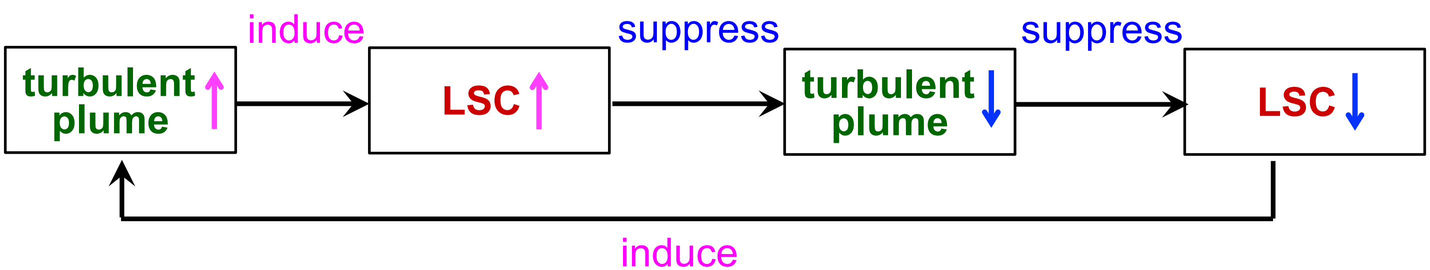

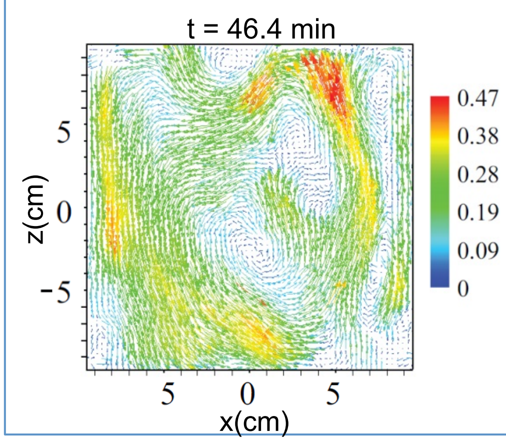

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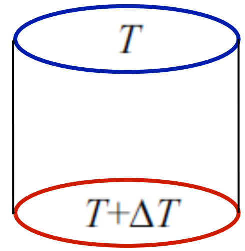

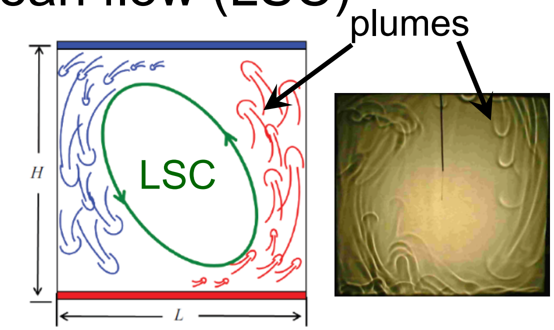

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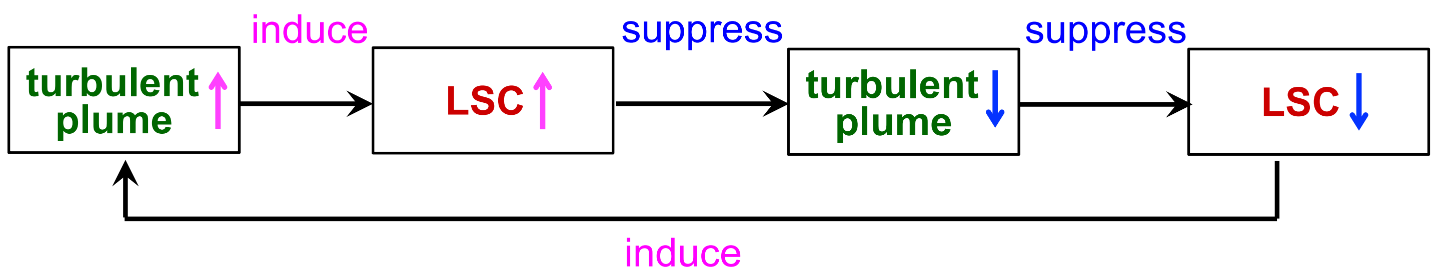

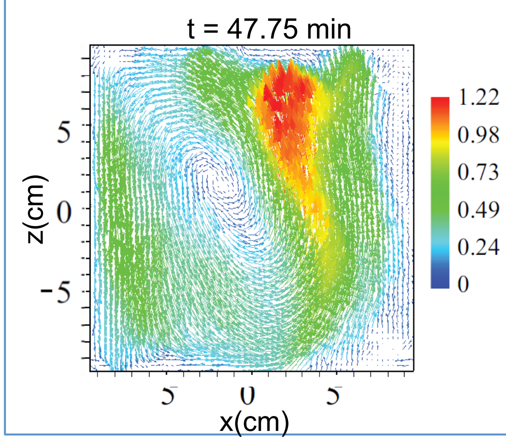

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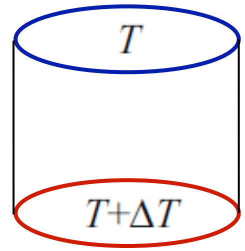

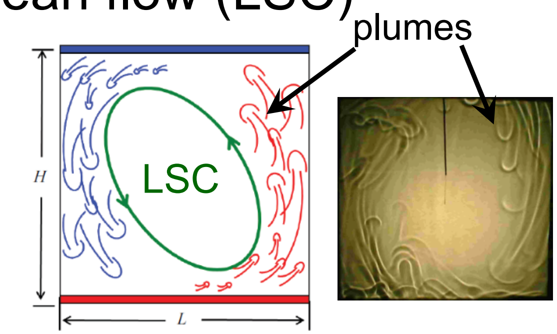

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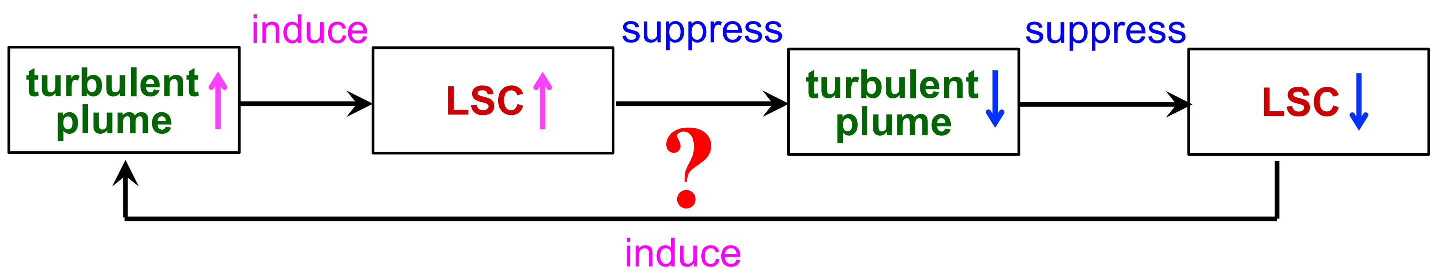

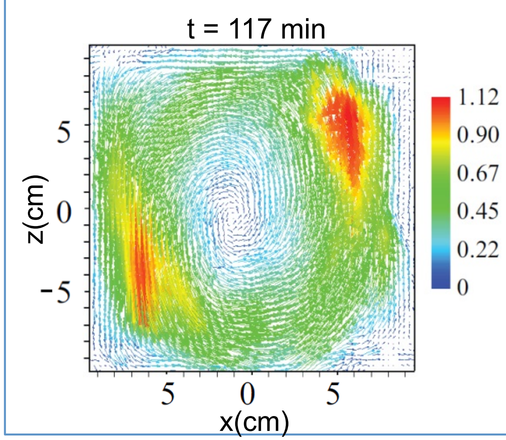

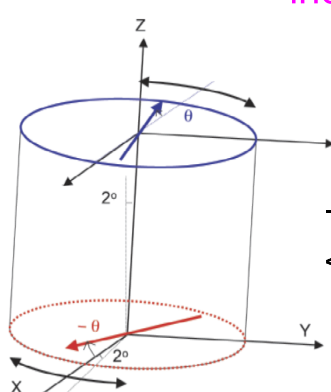

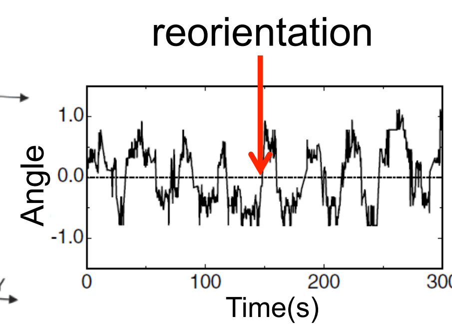

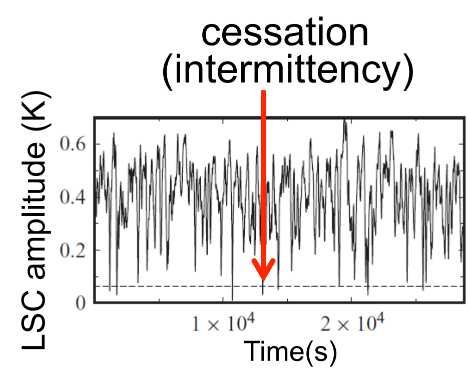

Funfschilling et al., *PRL* **87**, 194502 (2004) Zhong et al., *J. Fluid Mech.* **665**, 300 (2010)

Xia, *Theor. App. Mech. Lett.* **3**, 052001 (2013) Ahlers et al., *RMP* **81**, 503 (2009)

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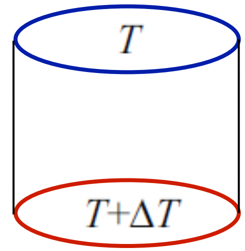

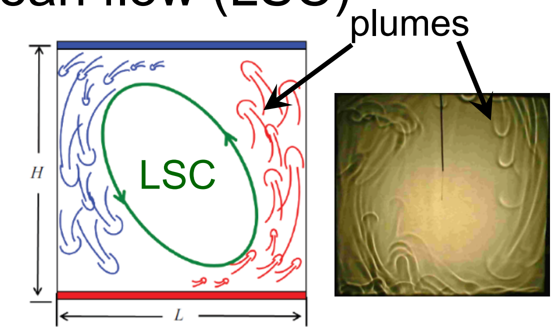

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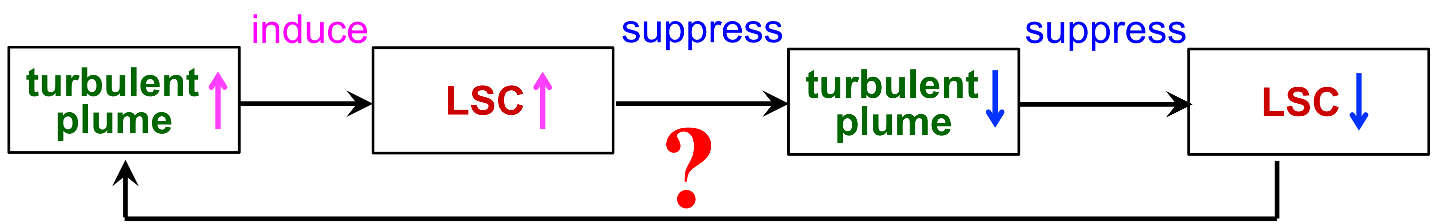
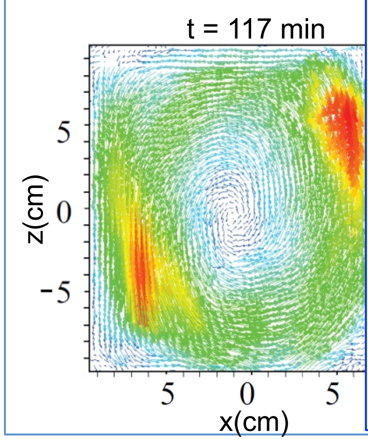
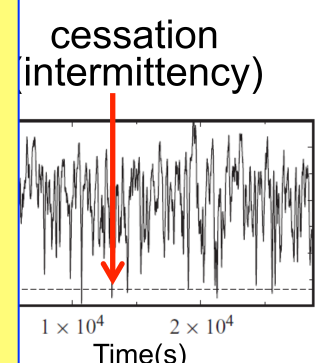




Future work:

1. Are there heuristic predator-prey equations for LSC?
2. If so, investigate predator-prey dynamics and phase shift in experimental data
3. individual level model & quasicycle theory



Fluid Mech. **665**, 300 (2010)

Xia, *Theor. App. Mech. Lett.* **3**, 052001 (2013)   Ahlers et al., *RMP* **81**, 503 (2009)

Navier-Stokes

- Incompressible NS:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla}\mathrm{p} + \mathrm{Re}^{-1}\,\nabla^2\,\mathbf{u},$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0.$$

Linear stability

- Express $\boldsymbol{u}$ in cylindrical coordinates.
- Linearize around laminar solution

$$\boldsymbol{u} = \boldsymbol{u}_{\boldsymbol{laminar}} + \boldsymbol{\delta u}$$

- Write as

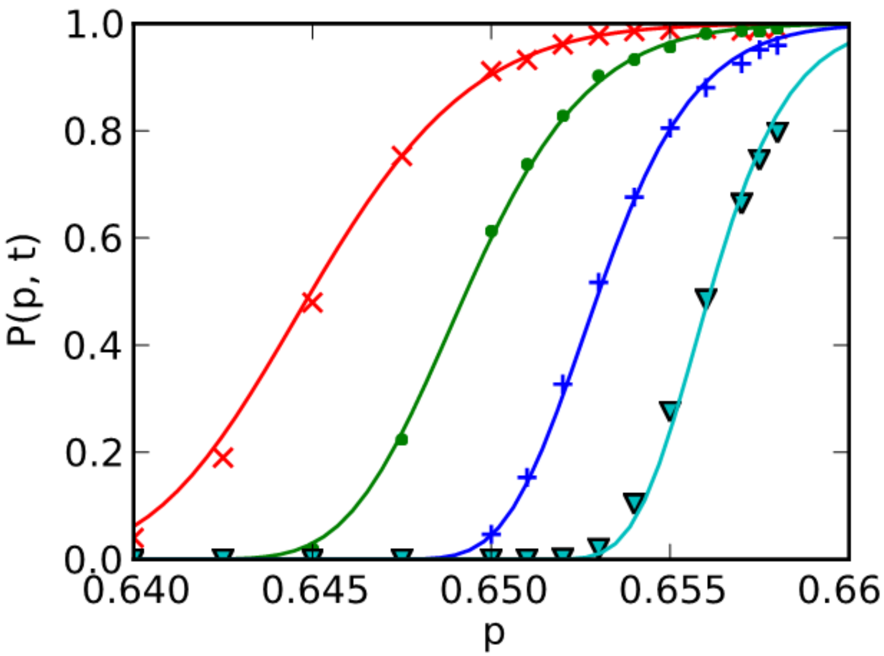
back

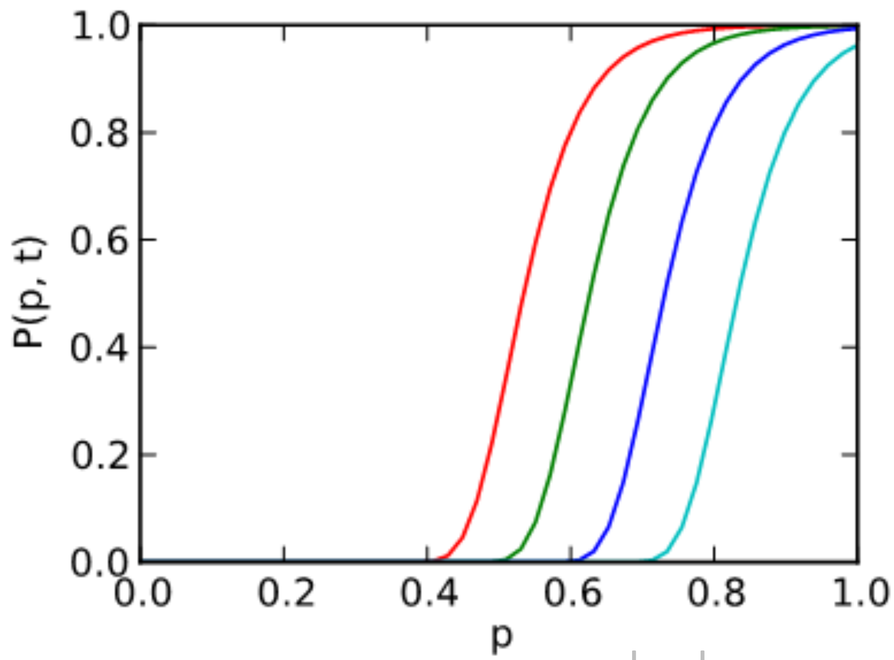
$$\partial_t\ \boldsymbol{\delta u} = \mathcal{L}\ \boldsymbol{\delta u}$$

Super-exponential scaling of lifetimes

- Slopes become steeper: τ grows faster than exponential.

- We plot survival curves at same times, but with assumed τ exponential.





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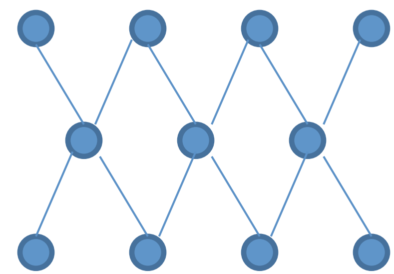
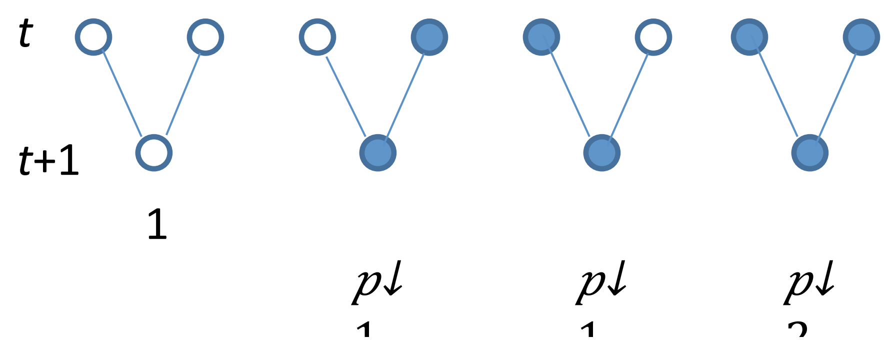
Simulating DP Models

Diagonal lattice models:

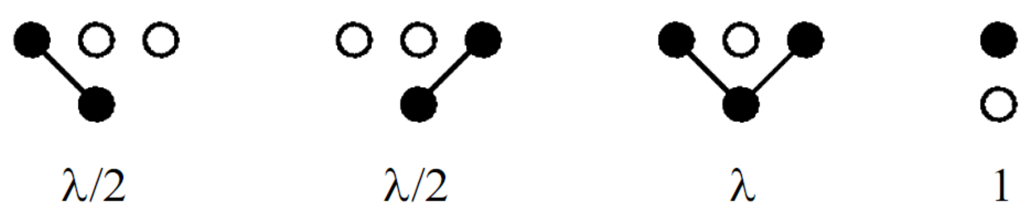


- Bond percolation
  - each bond open with probability p
- Site percolation
  - each site passable with probability p
- Domany-Kinzel
  - 2 probabilities



Contact process:

- Continuous time
- Contact rate λ



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Fisher-Tippett

- Distribution of the extremum depends on the tail of the source distribution $P(X_i)$

- If $P(X_i) \lesssim e^{-X_i}$ then one uses Fisher-Tippett type I or Gumbel distribution

$$F(x) = e^{-e^{-(x-\mu)/\sigma}}$$

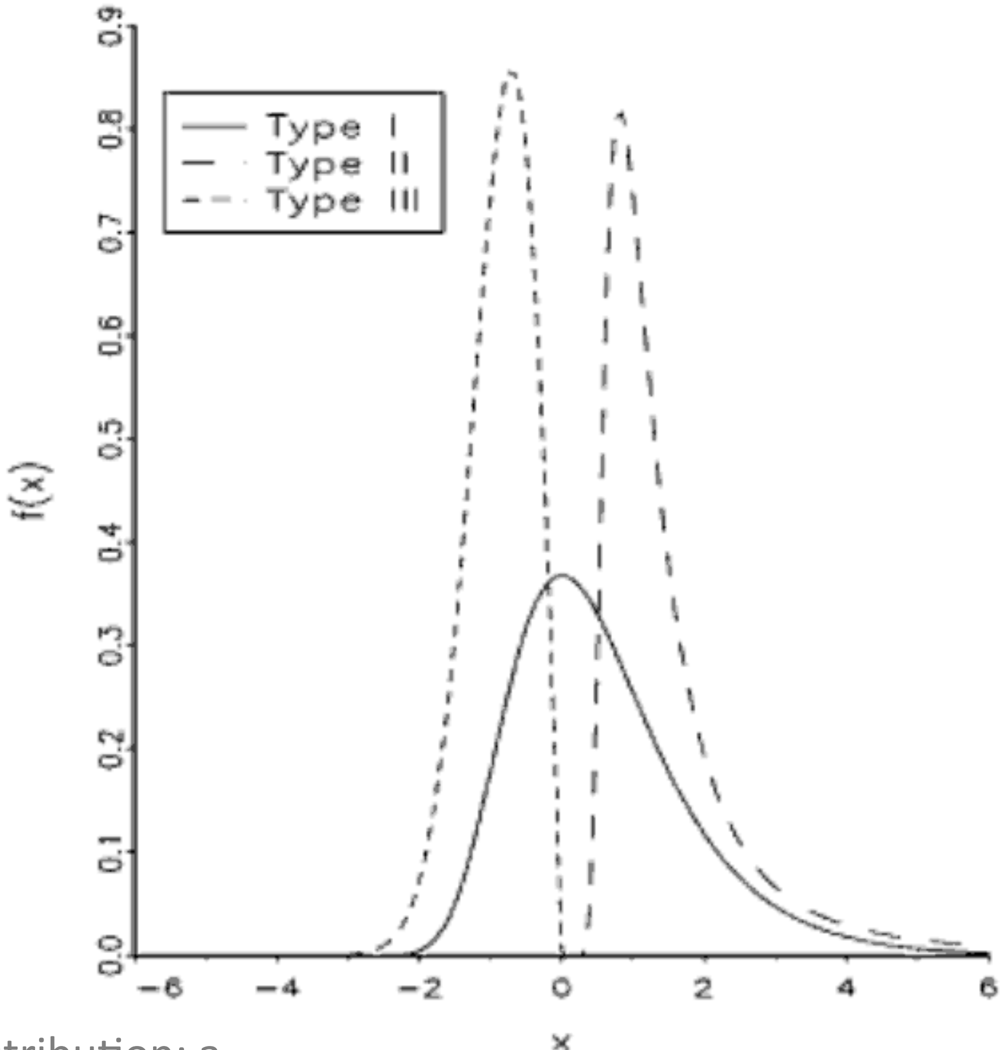
- Otherwise, one uses the Fisher-Tippett type II and III (Frechet and Weibull) distributions

$$F(x) = \exp\{-[1+\xi(x-\mu/\sigma)]^{-1/\xi}\}$$

where the shape parameter $\xi > 0$ for Frechet and $\xi < 0$ for Weibull.

back

Fisher-Tippett 2



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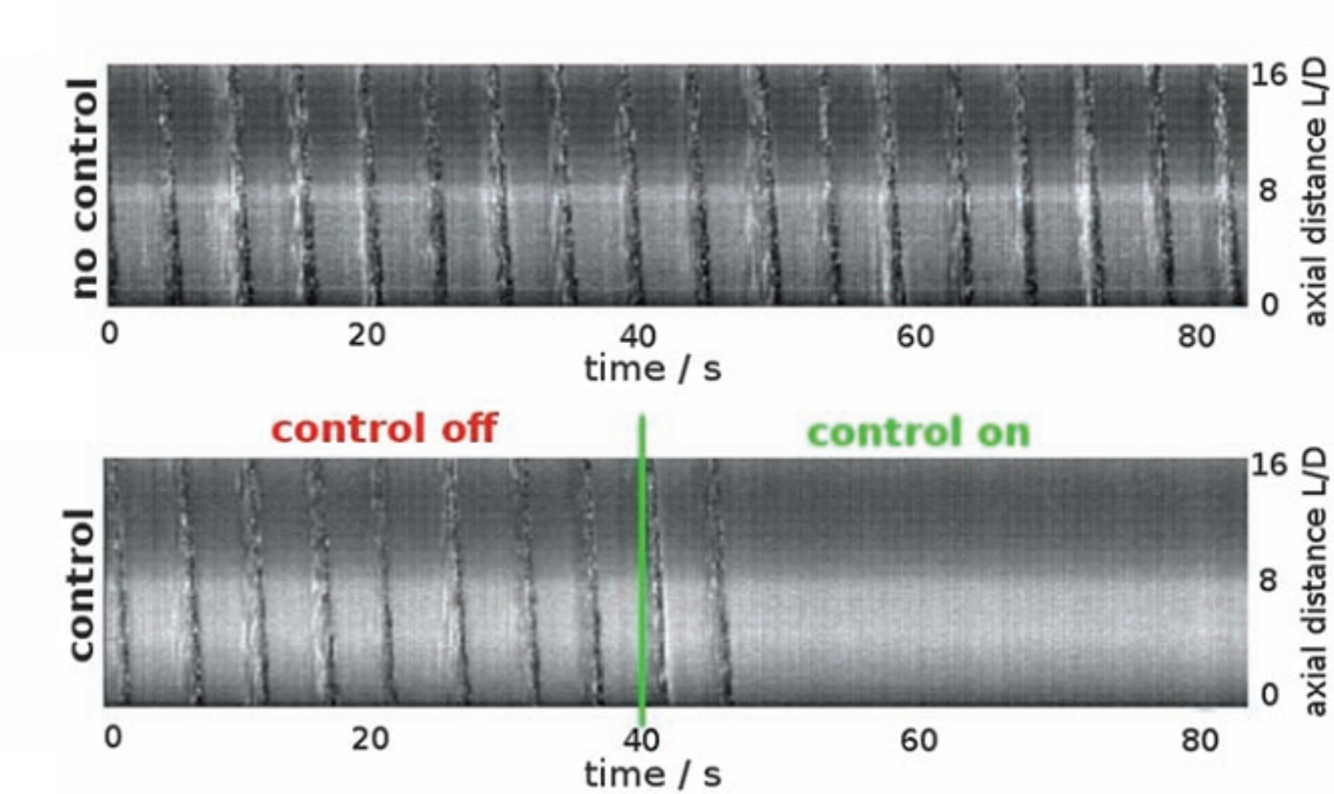
The Weibull distribution: a handbook by H Rinne (2009)

Hydrodynamic Phenomena

- Interaction of puffs (puffs that are close by can annihilate each other).



Hof et al. (Science 2010)

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Laminar patch size and fractal dimension

Size of laminar patches will follow

$$P(A) = A^{d_f}$$

where d_f can be calculated by noting that

$$\xi^{d_f} = (p - p_c)^{\beta} (\xi_{\perp})^{d-1} \xi_{\parallel}$$

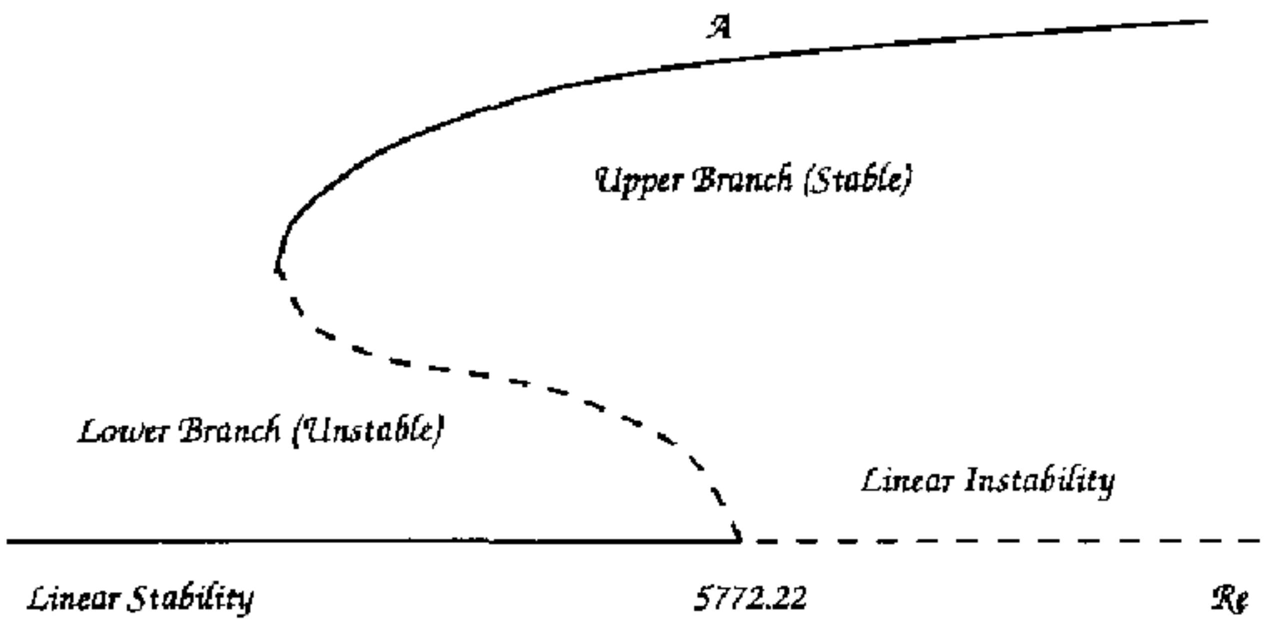
and using

$$\xi = (p - p_c)^{\nu}$$

back

2D Poiseuille Flow

- Not linearly stable for all Re.



Fortin *et al*. (J. Comp. Phys **115**, 455-469, 1994)

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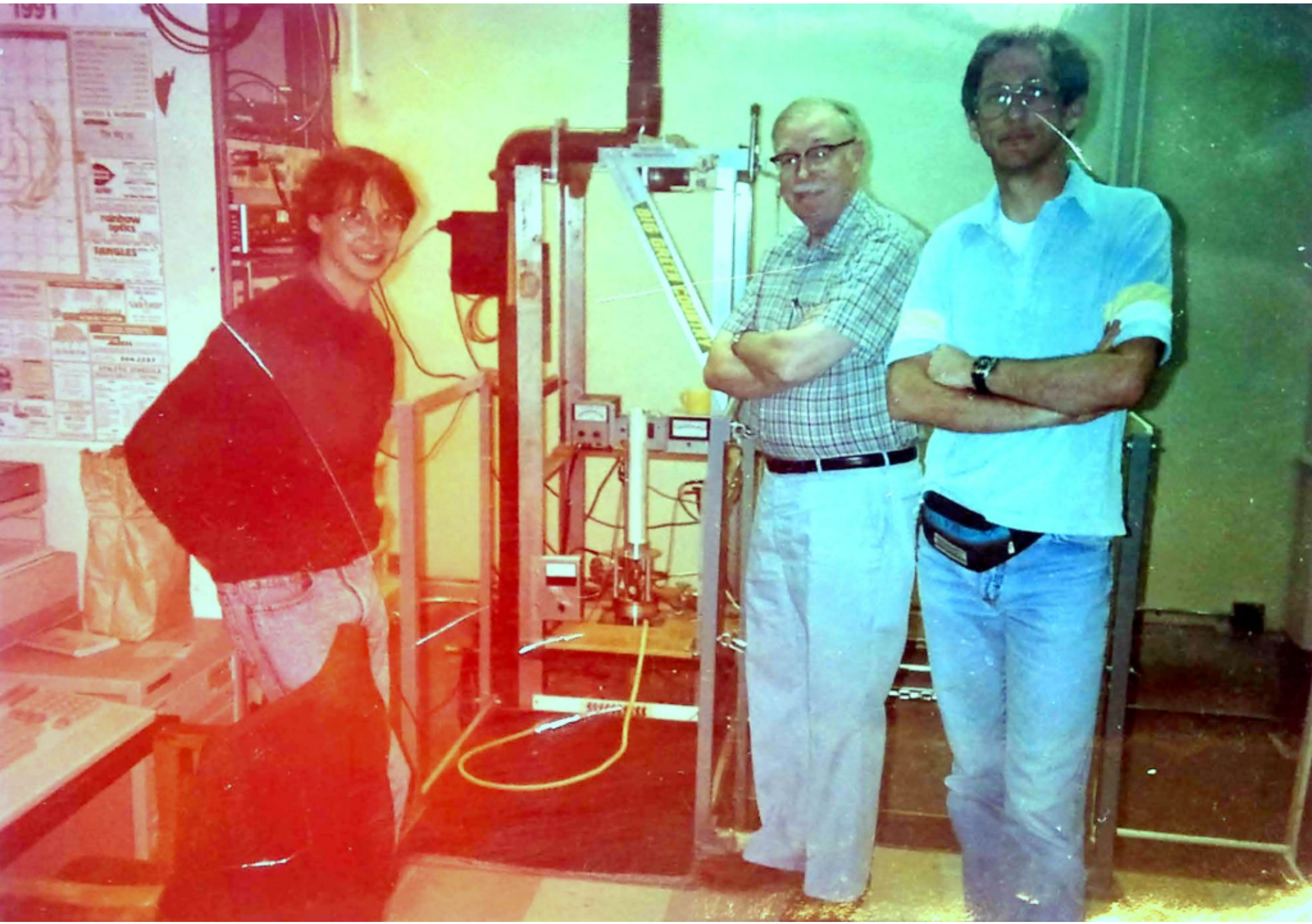
Decay of turbulence to rest



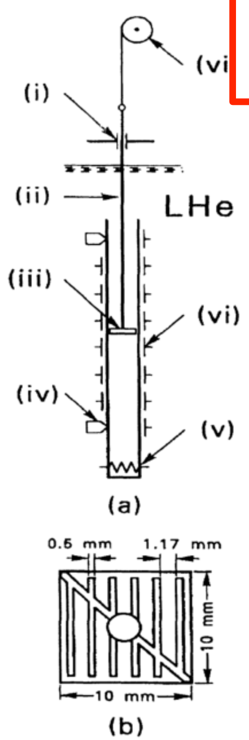
Eugene, Oregon 1993

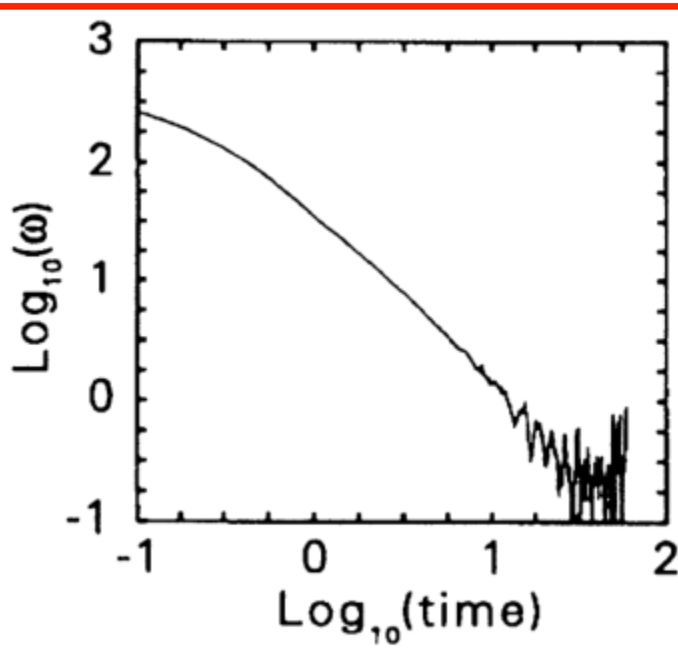
Decay of Vorticity in Homogeneous Turbulence

Michael R. Smith,[1] Russell J. Donnelly,[1] Nigel Goldenfeld,[2] and W. F. Vinen[3]
[1]*Department of Physics, University of Oregon, Eugene, Oregon 97403*
[2]*Department of Physics and Beckman Institute, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801*
[3]*School of Physics and Space Research, University of Birmingham, Birmingham B15 2TT, United Kingdom*
(Received 13 April 1993)

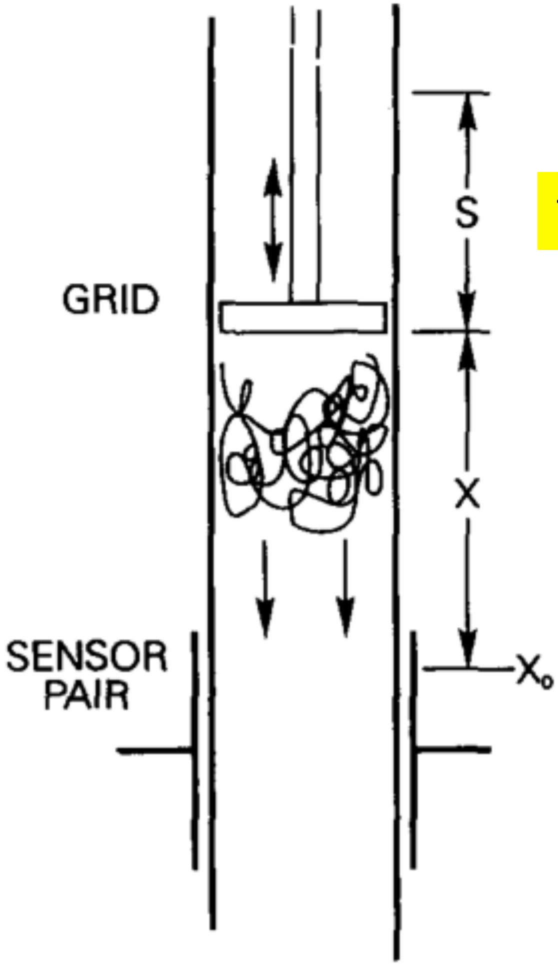
We report on observations of turbulent behavior made without requiring the use of Taylor's "frozen turbulence" hypothesis. Initially, a towed grid generates homogeneous turbulence of grid Reynolds number of order 10^5 within a stationary channel filled with helium II. The subsequent decay in time t of the line density of quantum vortices is measured by second sound attenuation, and the associated rms vorticity ω follows the behavior expected of a *classical fluid* with $\omega \sim t^{-3/2}$, consistent with the notion of a coupled turbulent state of helium II. This technique also yields the time dependence of the Kolmogorov microscale.

$$\omega = \omega_0 \left[1 + \frac{\nu t}{2}\left(\frac{2}{3}\frac{\omega_0 \varepsilon_0}{l_e \nu}\right)^{2/3}\right]^{-3/2} \underset{t\to\infty}{\approx} \left(\frac{2}{\nu}\right)^{1/2} \frac{3 l_e}{\varepsilon_0} t^{-3/2}$$





Propagation of turbulence



$$\partial_t q = \partial_z[\alpha h(t)\sqrt{q}\,\partial_z q] - \frac{\epsilon q^{3/2}}{\alpha h(t)}$$

Turbulence energy density

Size of turbulent burst

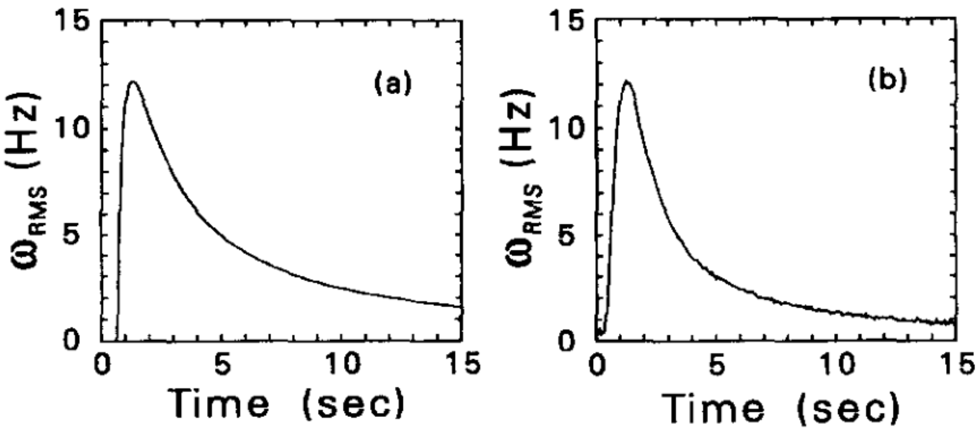


Fig. 5. Theoretical and observed turbulent bursts. Panel (a) shows the theoretical pulse proposed from Chen and Goldenfeld [15]. A good example of an experimentally observed burst after signal averaging is shown in (b). In both cases, $x = 1.337$ cm and $V = 50$ cm/s.

G.I. Barenblatt (1983); Chen & Goldenfeld, Phys. Rev. A (1992); M. Smith, Physica B (1994)

Decay of turbulence to laminar flow