Two extensions of the Immersed Boundary Method and their applications

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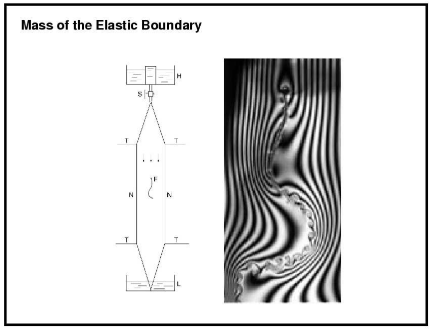
and Yunchang Seol (National Chiao Tung Univ. Taiwan)

Motivation

- Using the immersed boundary (IB) method,
- Investigate the interaction between a thin elastic material and fluid,
- Immersed boundaries are massive or porous.
- Examples: Flapping Filament, Mapleseed, Parachute, Foam



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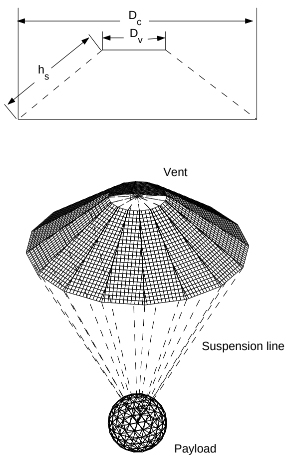

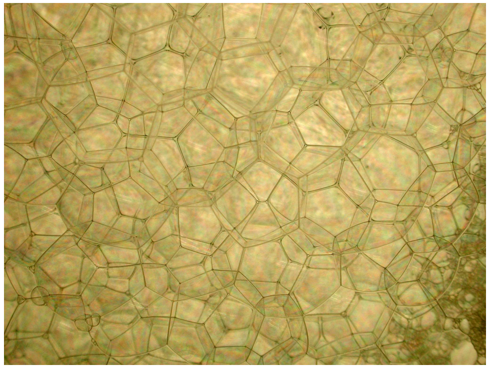

Immersed Boundary Method

- Two types of systems of equations:
  - Incompressible viscous flow (Eulerian).
  - Thin elastic material (Lagrangian).
- Interaction equations
  - Using the Dirac delta function.
  - Elastic force in Lagrangian $\longrightarrow$ Body force in Eulerian.
  - Elastic boundary moves at a local fluid velocity (no slip condition).

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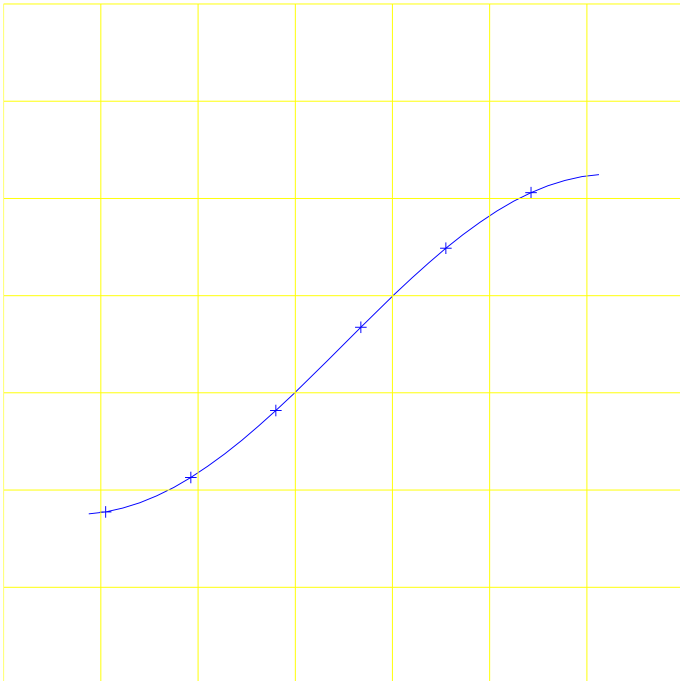

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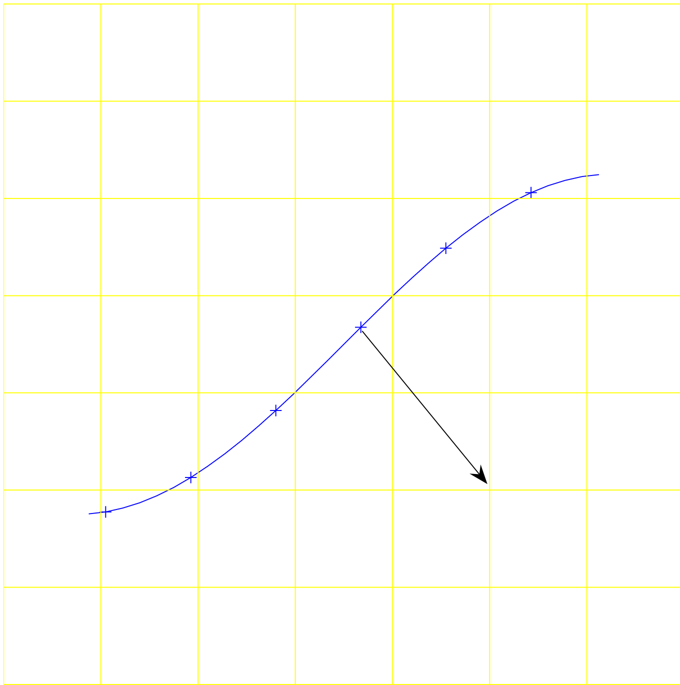

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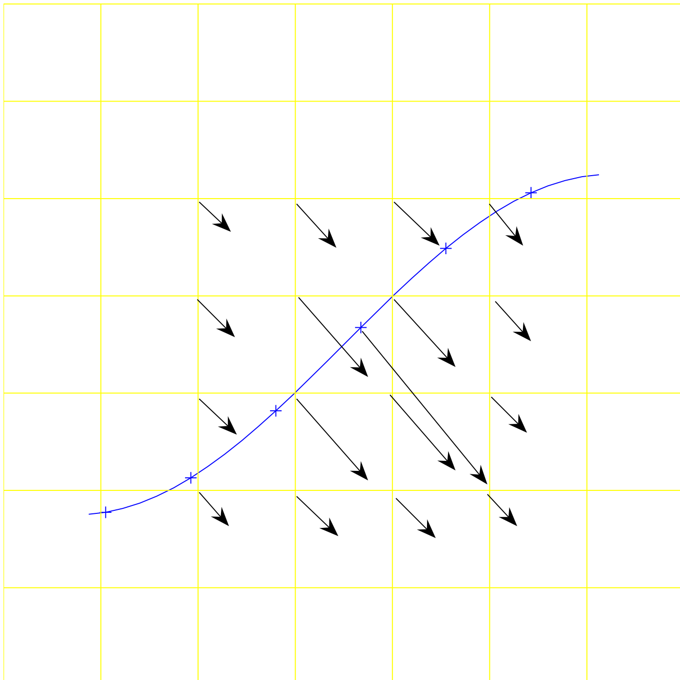

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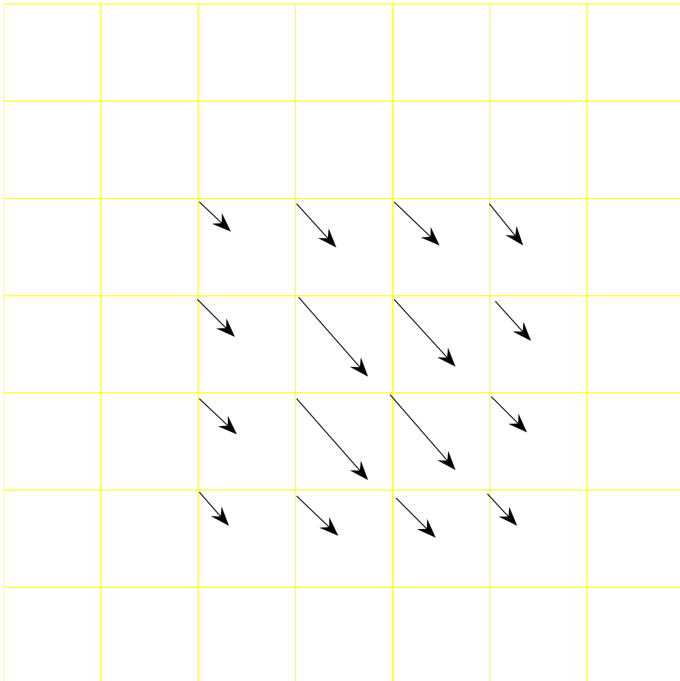

Equations of Motion

$$\mathbf{F} = -\frac{\partial E}{\partial \mathbf{X}} + \mathbf{F}_M,$$

$$\mathbf{F}_M = -M\frac{\partial^2 \mathbf{X}}{\partial t^2} - Mg\mathbf{e}_3,$$

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$$\rho(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f},$$

$$\nabla \cdot \mathbf{u} = 0,$$

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$$\begin{aligned} \frac{\partial \mathbf{X}}{\partial t}(r, s, t) &= \mathbf{u}(\mathbf{X}(r, s, t), t) \\ &= \int \mathbf{u}(\mathbf{x}, t)\delta(\mathbf{x} - \mathbf{X}(r, s, t))d\mathbf{x}. \end{aligned}$$

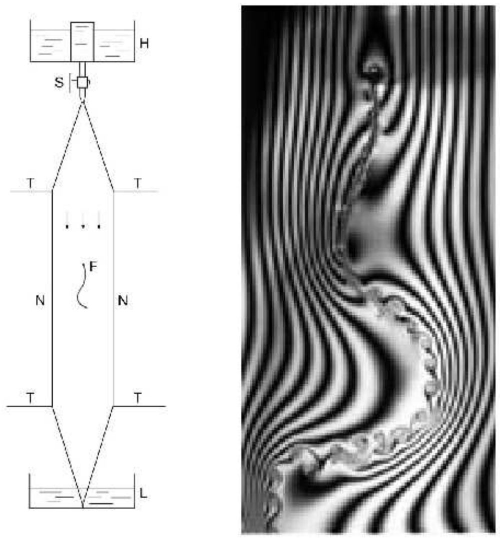
Mass of Immersed boundary

Mass of Immersed Boundary

3

Mass of Immersed boundary

Mass of the Elastic Boundary



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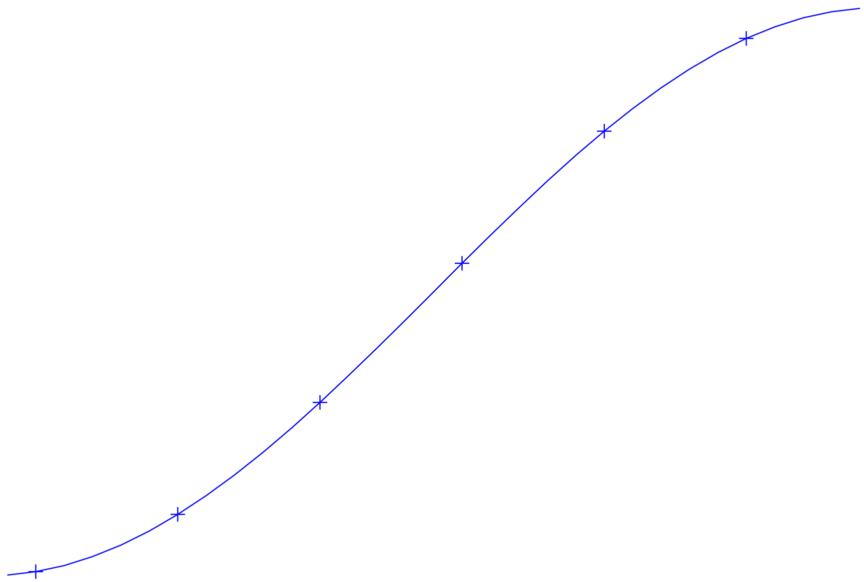
$$\rho(\mathbf{x},t) = \int \mathbf{M}(r,s)\delta(\mathbf{x} - \mathbf{X}(r,s,t))drds,$$

$$\rho(\mathbf{x},t)(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}) = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{f},$$

Penalty IB method I



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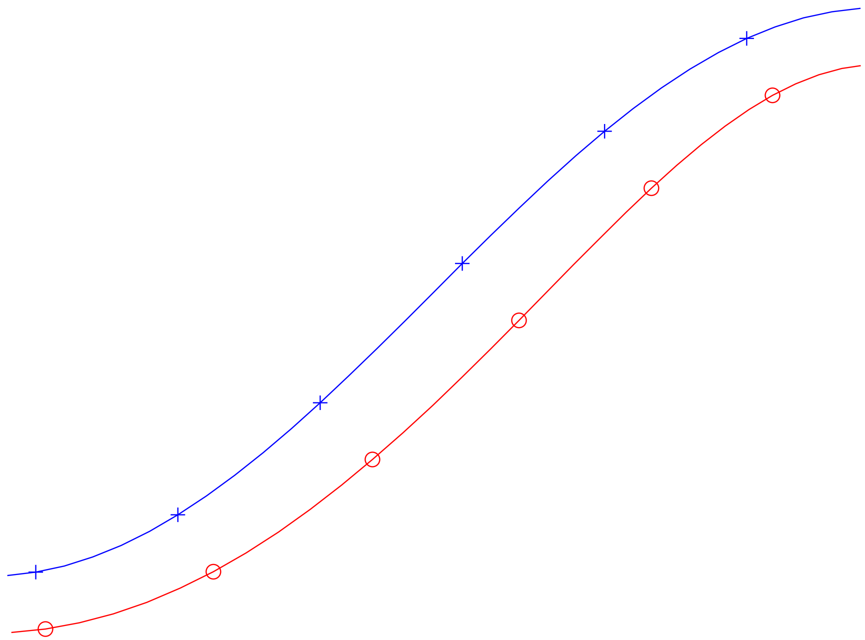

- Split the elastic boundary into two Lagrangian components: massive component $\mathbf{Y}(r, s, t)$ and massless component $\mathbf{X}(r, s, t)$.
- $\mathbf{Y}(r, s, t)$ does not interact with the fluid and moves by Newton's law.
- $\mathbf{X}(r, s, t)$ has no mass and plays the same role as in the IB method.

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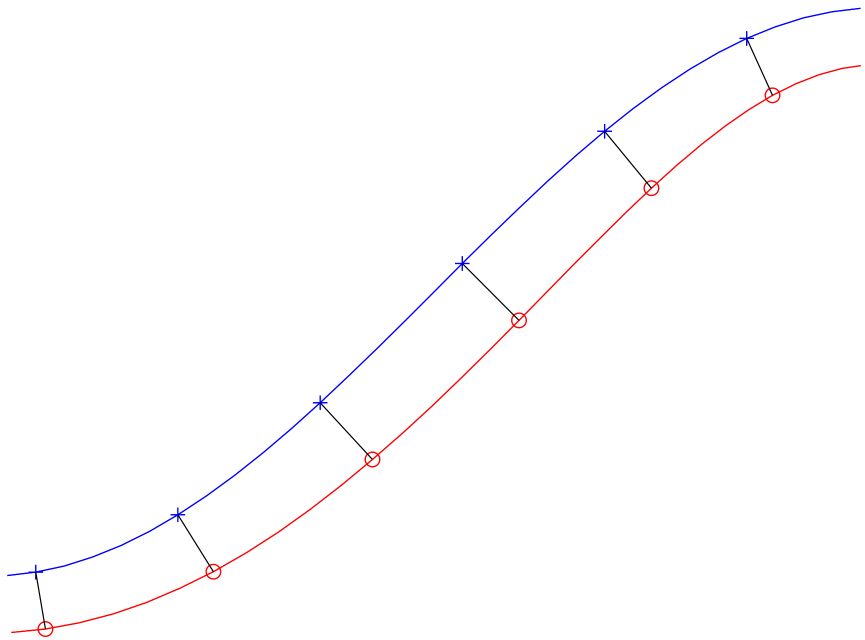


- Split the elastic boundary into two Lagrangian components: massive component $\mathbf{Y}(r,s,t)$ and massless component $\mathbf{X}(r,s,t)$.
- $\mathbf{Y}(r,s,t)$ does not interact with the fluid and moves by Newton's law.
- $\mathbf{X}(r,s,t)$ has no mass and plays the same role as in the IB method.
- The two components are connected by very stiff springs.
- The spring force acts on both components to keep them close.

Penalty IB method II

$$\mathbf{F} = -\frac{\partial E}{\partial \mathbf{X}} + \mathbf{F}_M. \tag{1}$$

$$\mathbf{F}_M = -M\frac{\partial^2 \mathbf{X}}{\partial t^2} - Mg\mathbf{e}_3. \tag{2}$$

are replaced by

Penalty IB method II

$$\mathbf{F} = -\frac{\partial E}{\partial \mathbf{X}} + \mathbf{F}_M. \tag{3}$$

$$\mathbf{F}_M = -M\frac{\partial^2 \mathbf{X}}{\partial t^2} - Mg\mathbf{e}_3. \tag{4}$$

are replaced by

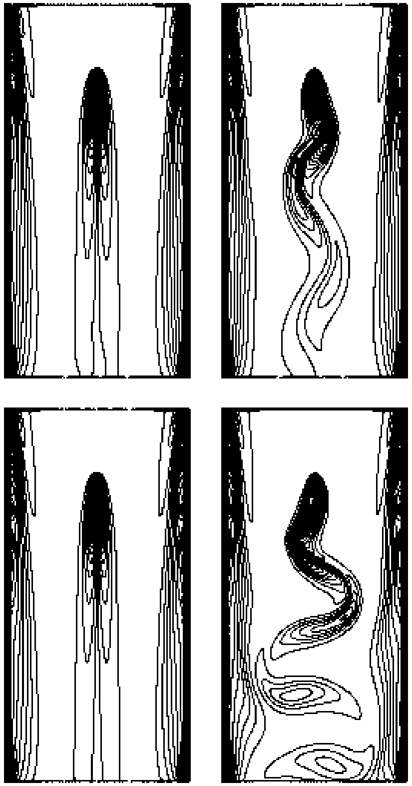
$$\mathbf{F} = -dE/d\mathbf{X} + \mathbf{F}_K, \tag{5}$$

$$\mathbf{F}_K(r,s,t) = K(\mathbf{Y}(r,s,t) - \mathbf{X}(r,s,t)) \tag{6}$$

$$M(r,s)\frac{\partial^2 \mathbf{Y}(r,s,t)}{\partial t^2} = -\mathbf{F}_K(r,s,t) - M(r,s)g\mathbf{e}_3. \tag{7}$$

Flapping Filament in a flowing soap film

Application I: Flapping Filament in a Flowing Soap Film



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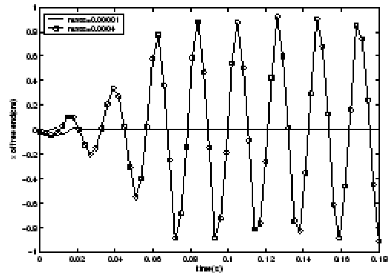
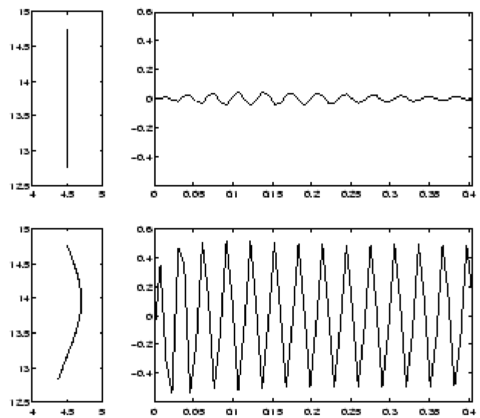
Flapping Filament in a flowing soap film

- Comparison between massless and massive boundaries.



- Bistability



Interaction with a rigid body

$$\mathbf{F}(q,r,s,t) = K(\mathbf{Y}(q,r,s,t) - \mathbf{X}(q,r,s,t)) \tag{8}$$

$$\mathbf{f}(\mathbf{x},t) = \int \mathbf{F}(q,r,s,t)\delta(\mathbf{x} - \mathbf{X}(q,r,s,t))dqdrds \tag{9}$$

Interaction with a rigid body

$$\mathbf{Y}(q,r,s,t) = \mathbf{Y}_{\mathrm{cm}}(t) + \mathcal{R}(t)\mathbf{C}(q,r,s) \tag{10}$$

$$M\frac{d\mathbf{V}_{\mathrm{cm}}}{dt} = -\int \mathbf{F}(q,r,s,t)dqdrds - M\mathbf{g} \tag{11}$$

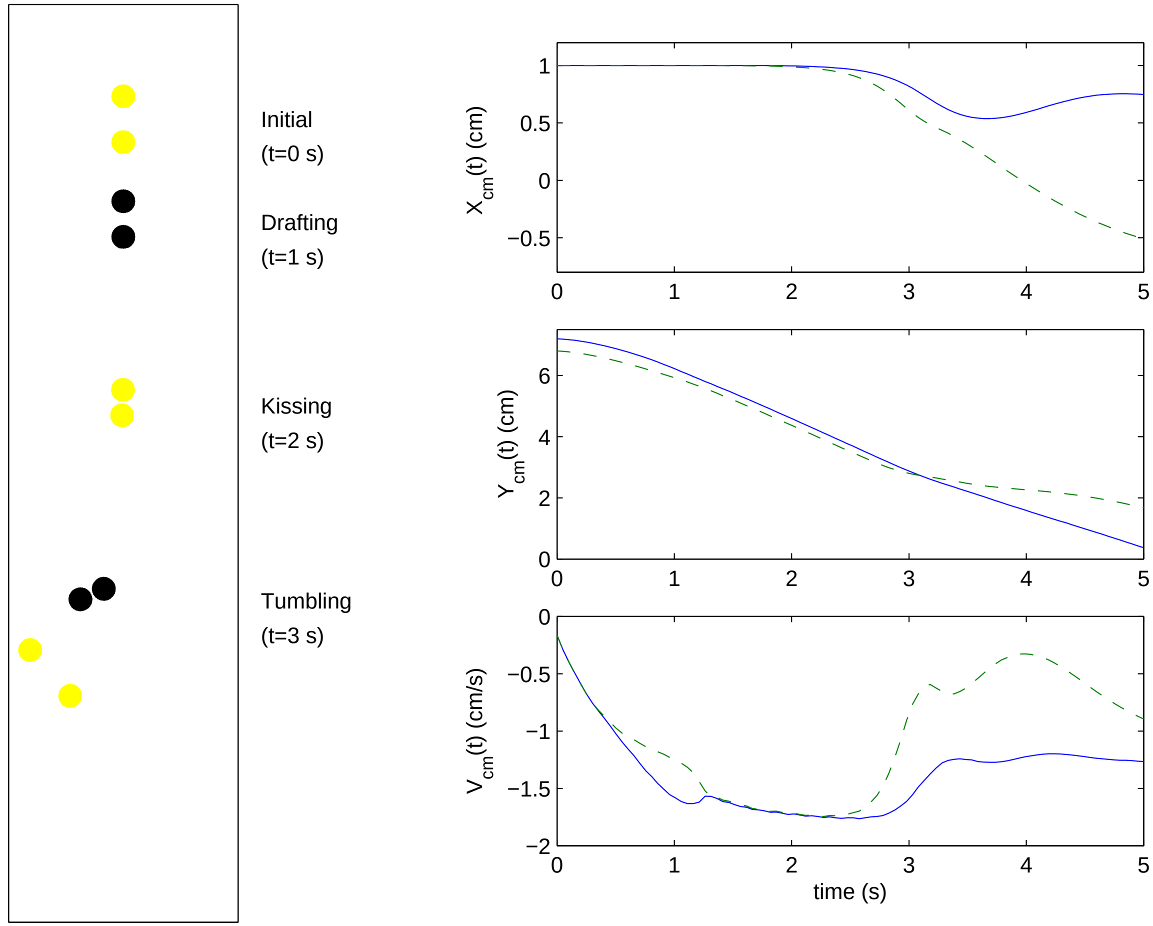
$$\frac{d\mathbf{Y}_{\mathrm{cm}}}{dt} = \mathbf{V}_{\mathrm{cm}}(t) \tag{12}$$

$$\frac{d\mathbf{L}}{dt} = \int (\mathbf{Y}(q,r,s,t) - \mathbf{Y}_{\mathrm{cm}}(t)) \times (-\mathbf{F}(q,r,s,t))dqdrds \tag{13}$$

$$\begin{aligned} \mathbf{L}(t) &= \int m(q,r,s)((\mathcal{R}(t)\mathbf{C})^T(\mathcal{R}(t)\mathbf{C})I_3 - (\mathcal{R}(t)\mathbf{C})(\mathcal{R}(t)\mathbf{C})^T)\,\Omega(t)\,dqdrds \\ &= \mathcal{R}(t)\int m(q,r,s)(\mathbf{C}^T\mathbf{C}I_3 - \mathbf{C}\mathbf{C}^T)\,dqdrds\,\mathcal{R}(t)^T\,\Omega(t) \end{aligned} \tag{14}$$

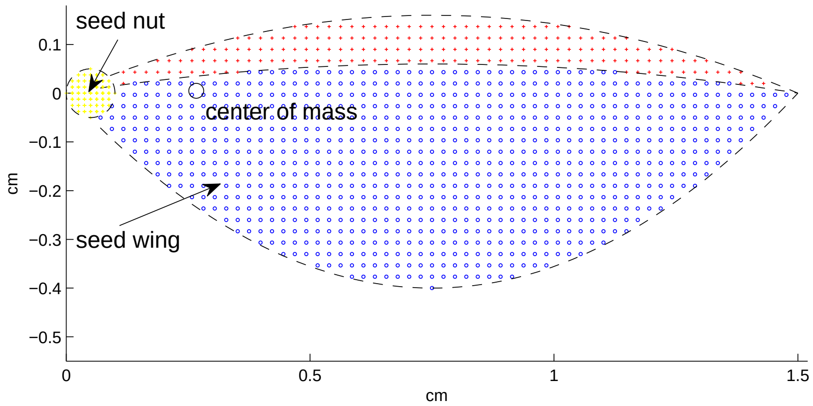
$$\frac{d\mathcal{R}}{dt} = \Omega(t) \times \mathcal{R}(t) \tag{15}$$

Two dropping discs



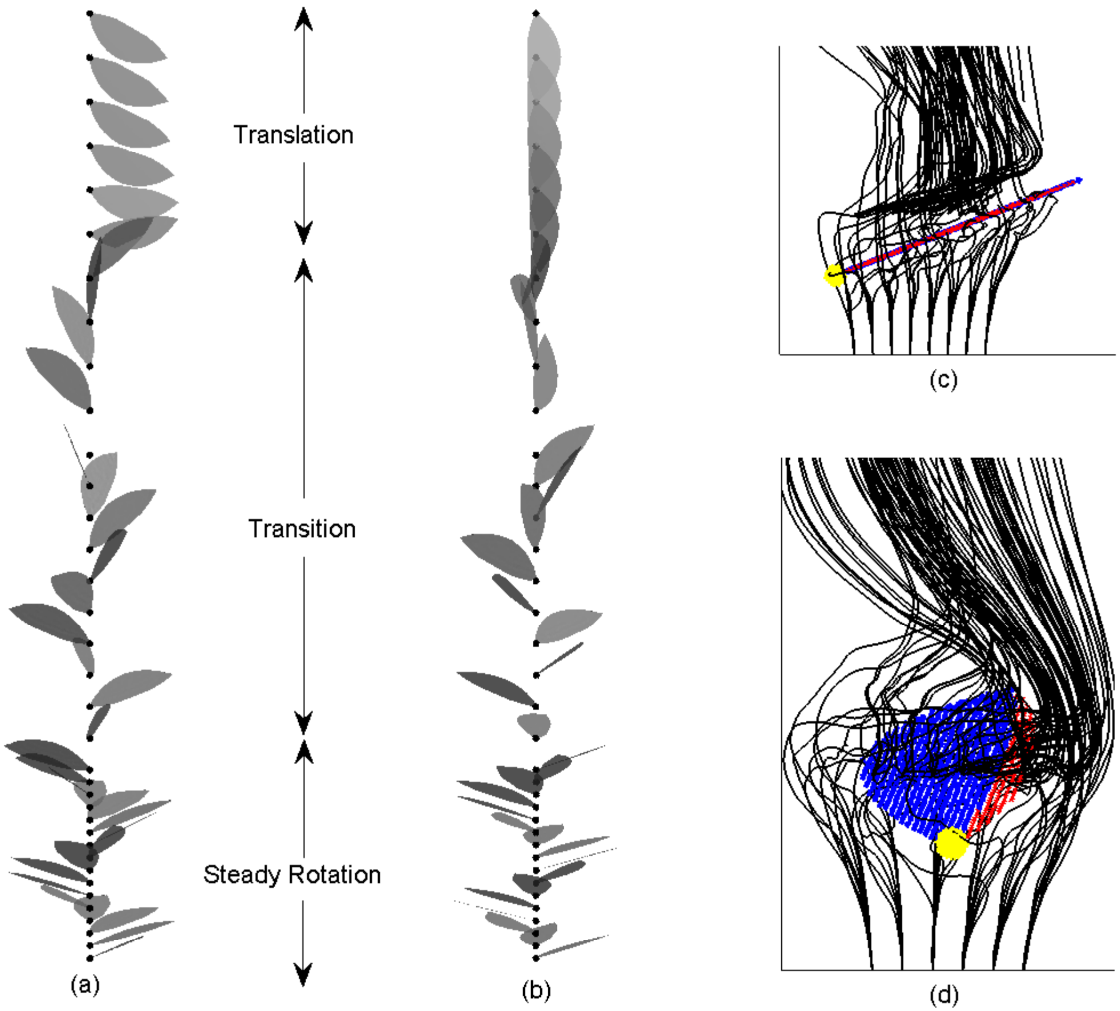
two falling discs

Maple seed



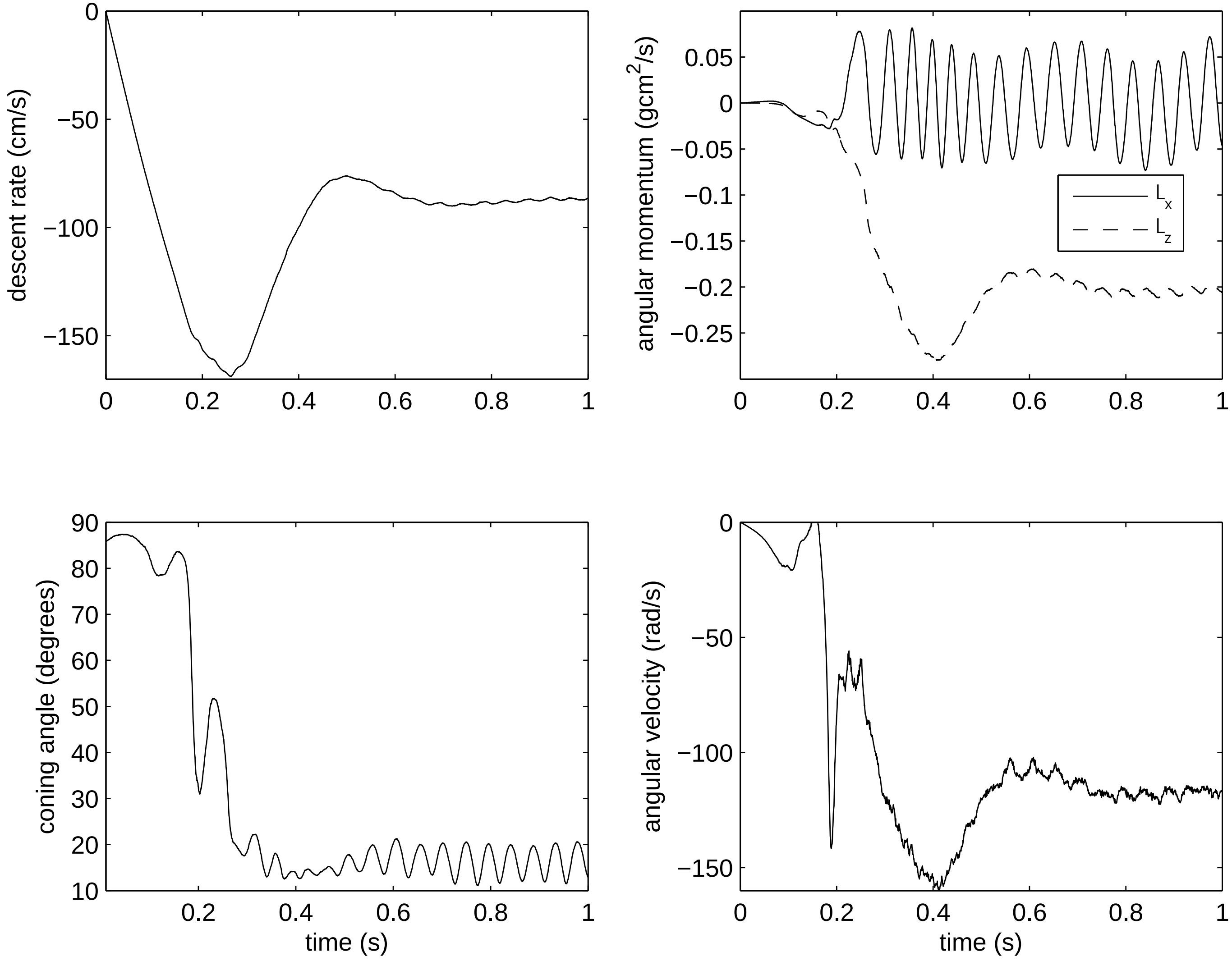
seed nut
center of mass
seed wing
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
cm
0
0.5
1
1.5
cm

Motion of Maple seed

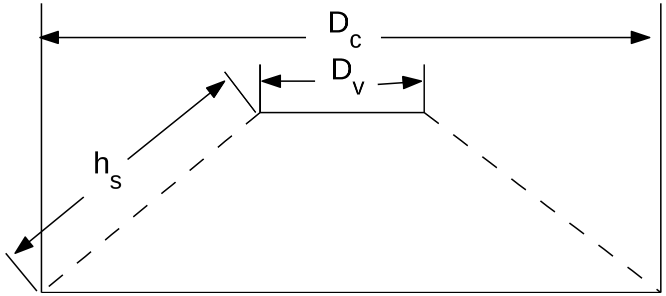


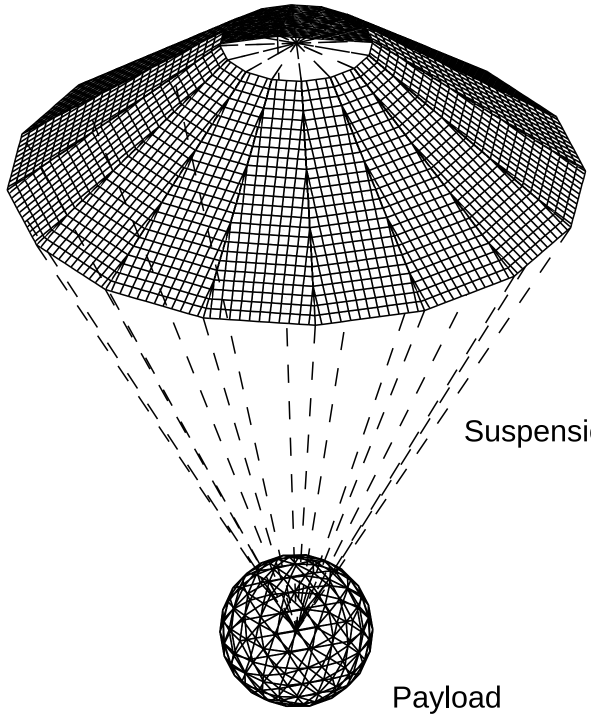
maple seed

Rotational speed of Maple seed

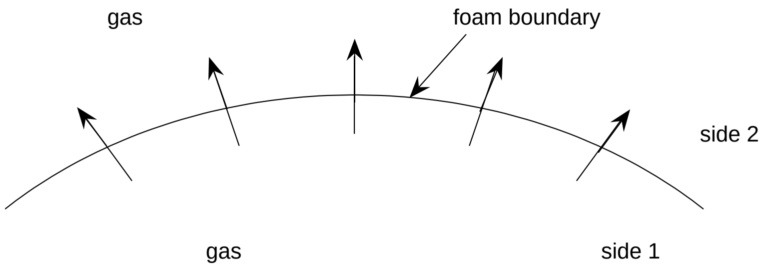


Parachute with Porous Canopy (2nd extension)





IB method with Porous Boundary



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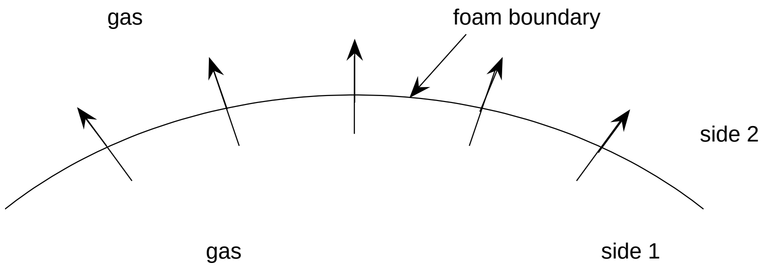

- Let $\mathbf{U}(s,t)$ be fluid velocity at $\mathbf{X}(s,t)$ and $\frac{\partial \mathbf{X}}{\partial t}(s,t)$ be the boundary velocity.

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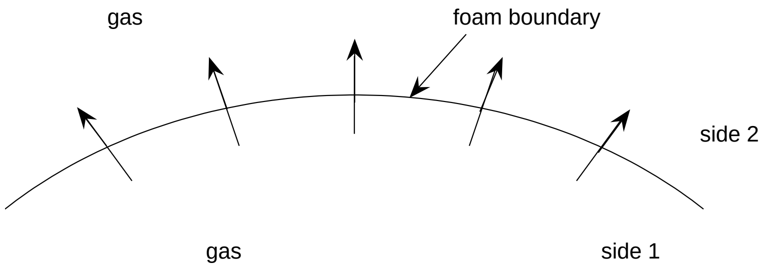

- Let $\mathbf{U}(s,t)$ be fluid velocity at $\mathbf{X}(s,t)$ and $\frac{\partial \mathbf{X}}{\partial t}(s,t)$ be the boundary velocity.
- Flux through a patch with the length $|\frac{\partial \mathbf{X}}{\partial s}|ds$:
  $(\mathbf{U}(s,t) - \frac{\partial \mathbf{X}}{\partial t}(s,t))|\frac{\partial \mathbf{X}}{\partial s}|ds = M(p_1 - p_2)|\frac{\partial \mathbf{X}}{\partial s}|ds$, where M is the permeability.

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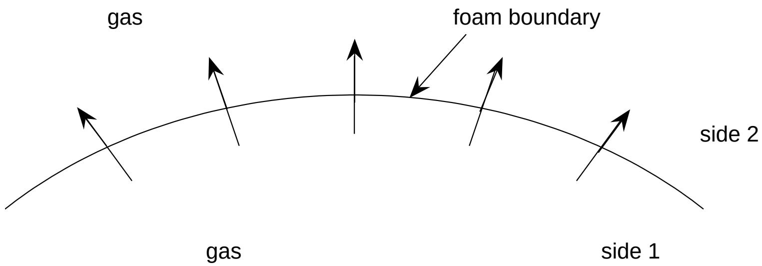

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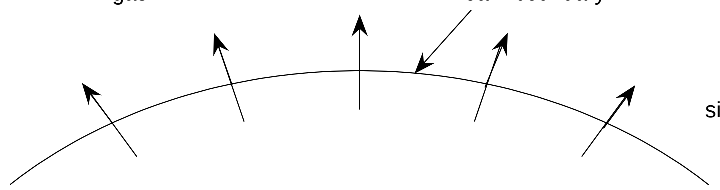
- Normal equilibrium of the boundary: $(p_1 - p_2)\left|\frac{\partial \mathbf{X}}{\partial s}(s,t)\right| + \mathbf{F}(s,t) \cdot \mathbf{n} = 0$.

IB method with Porous Boundary



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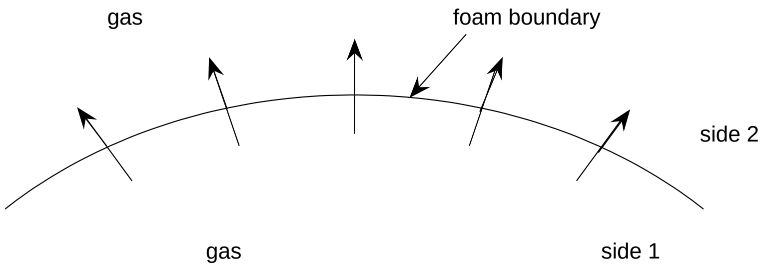
- $\frac{\partial \mathbf{X}}{\partial t}(s,t) - \mathbf{U}(s,t) = M\mathbf{F}(s,t)\cdot \mathbf{n}/|\frac{\partial \mathbf{X}}{\partial s}|$.

IB method with Porous Boundary



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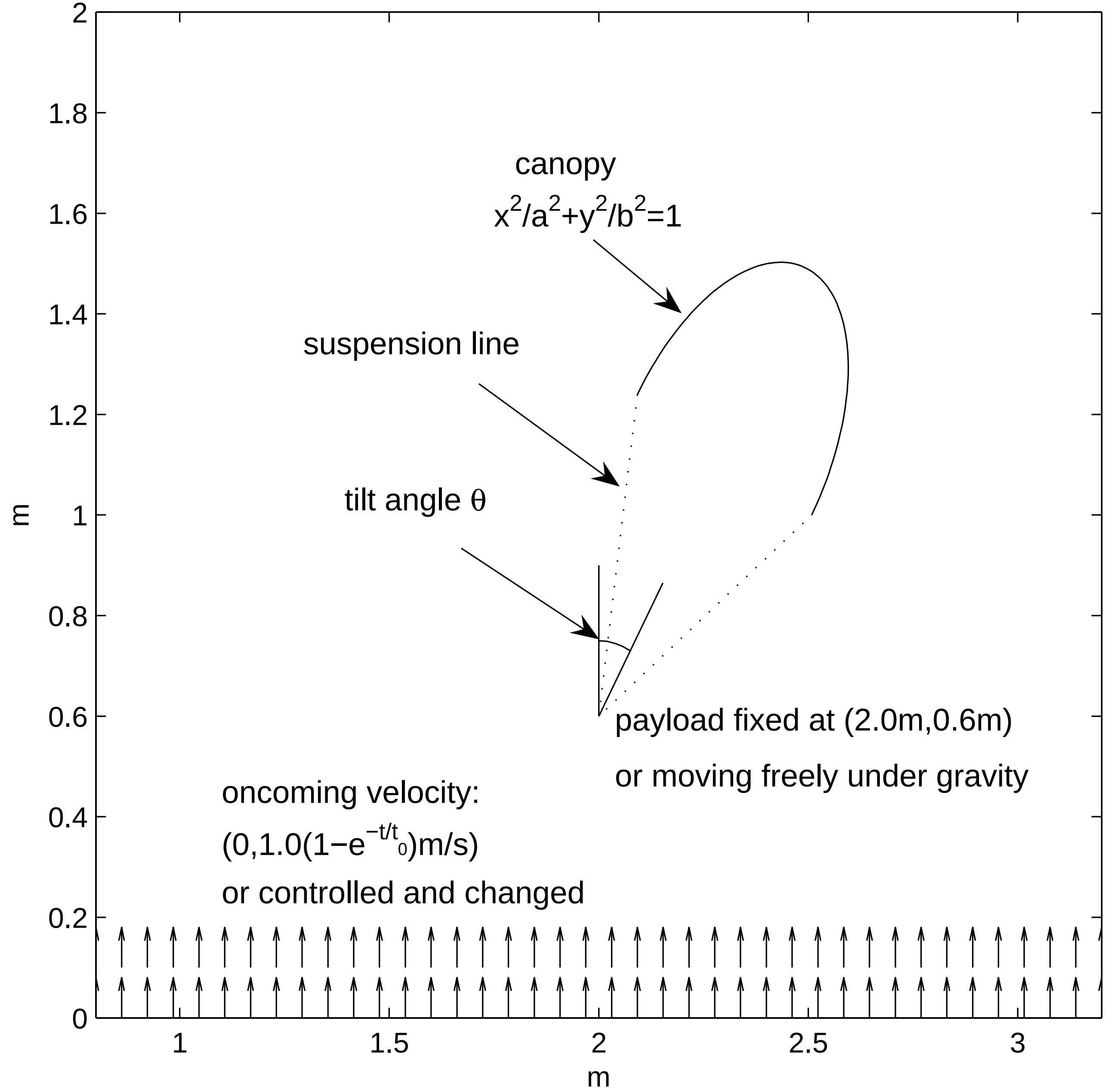
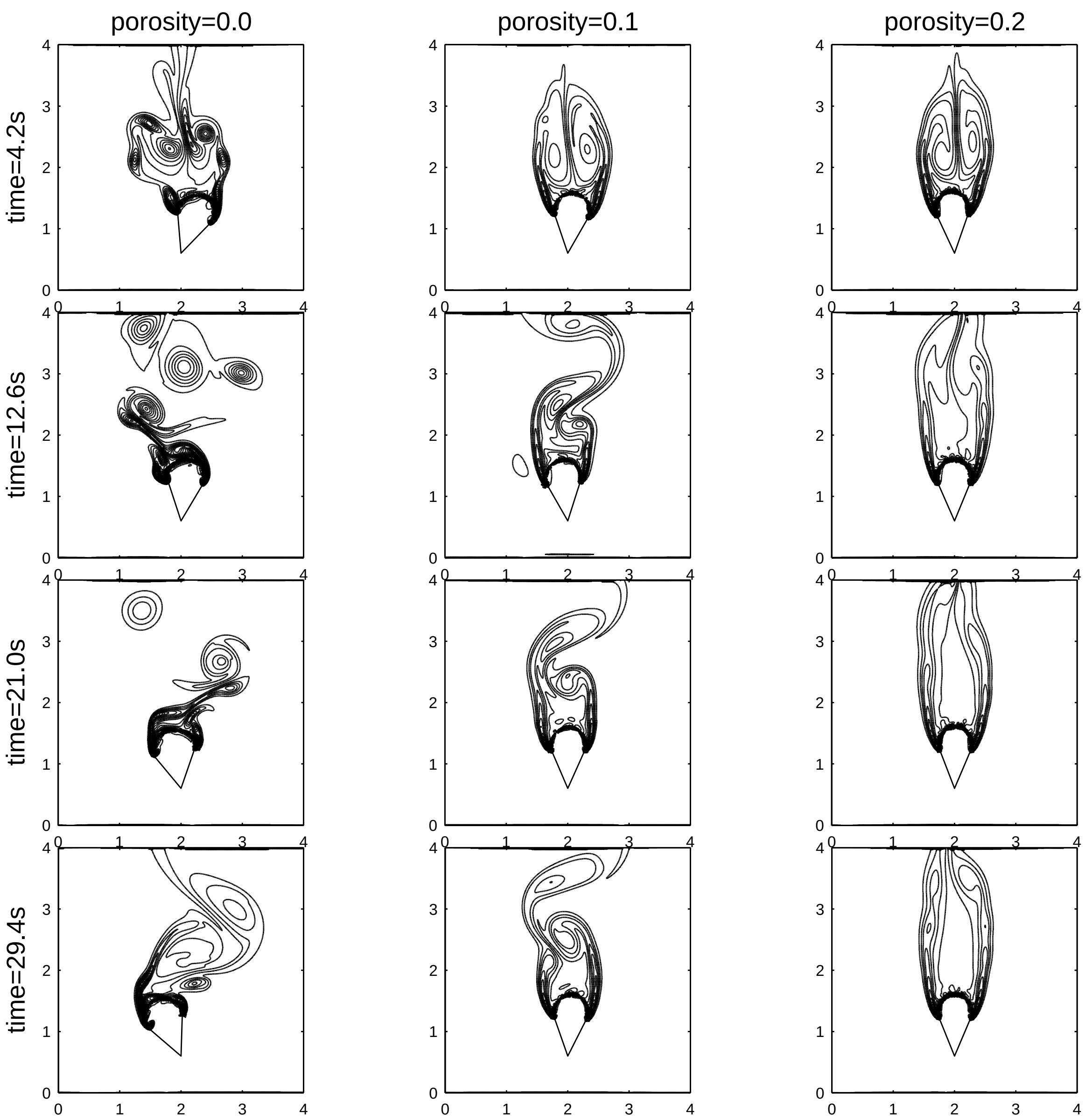
- Normal equilibrium of the boundary: $(p_1 - p_2)\left|\frac{\partial \mathbf{X}}{\partial s}(s,t)\right| + \mathbf{F}(s,t)\cdot\mathbf{n} = 0$.
- $\frac{\partial \mathbf{X}}{\partial t}(s,t) - \mathbf{U}(s,t) = M\mathbf{F}(s,t)\cdot\mathbf{n}/|\frac{\partial \mathbf{X}}{\partial s}|$.
- Since $\mathbf{F}(s,t)$ is normal to the boundary,

$$\begin{aligned} \frac{\partial \mathbf{X}}{\partial t}(s,t) &= \mathbf{u}(\mathbf{X}(s,t),t) + M\,\mathbf{F}(s,t)/|\frac{\partial \mathbf{X}}{\partial s}| \\ &= \int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(r,s,t))d\mathbf{x} + M\,\mathbf{F}(s,t)/|\frac{\partial \mathbf{X}}{\partial s}| \end{aligned} \tag{16}$$

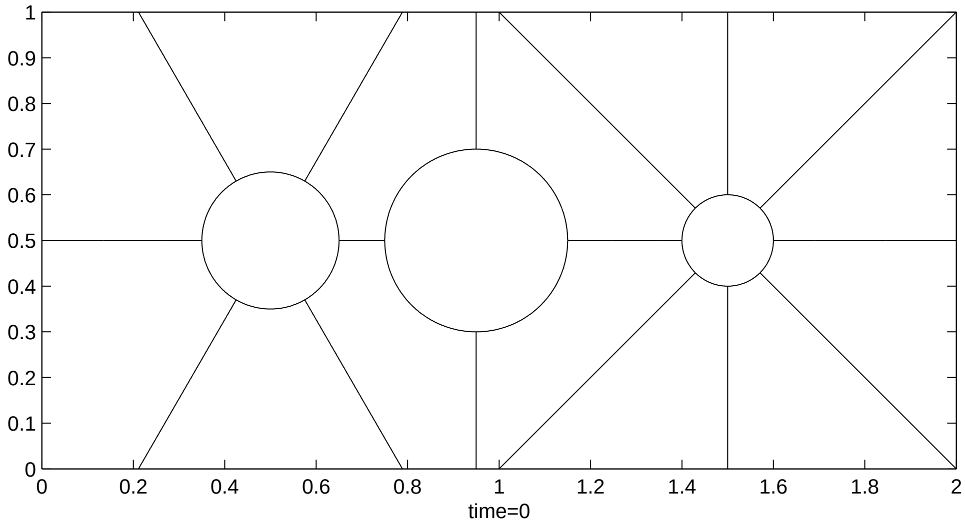
2-D Parachute with Porous Canopy



Motion of 2-D Parachute



2D Foam Dynamics: von Neumann relation



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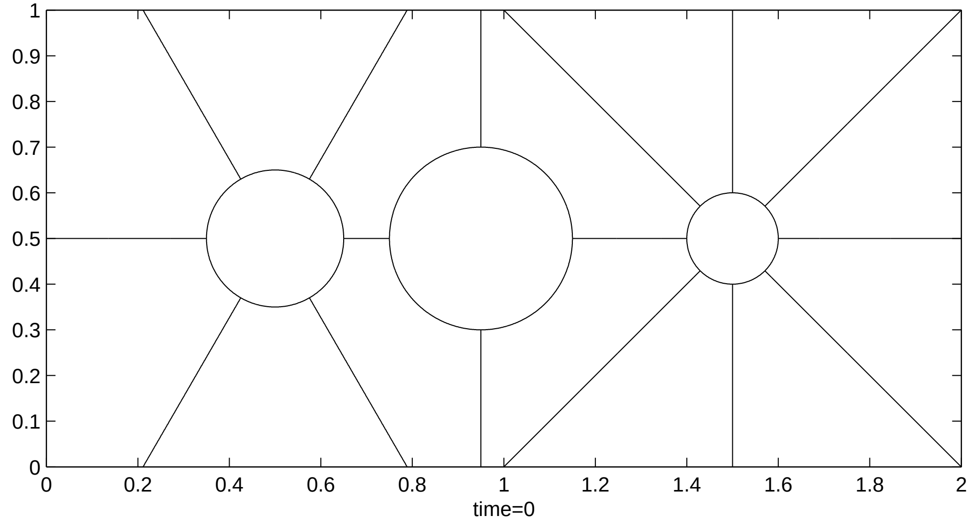

- M:permeability; γ:surface tension; κ:mean curvature.
- Assume that the gas diffuses through the wall at a rate $-M\,\gamma\,\kappa$ per unit length.
- $\frac{dA}{dt} = -M\,\gamma\,\int_\Gamma \kappa\,ds$.

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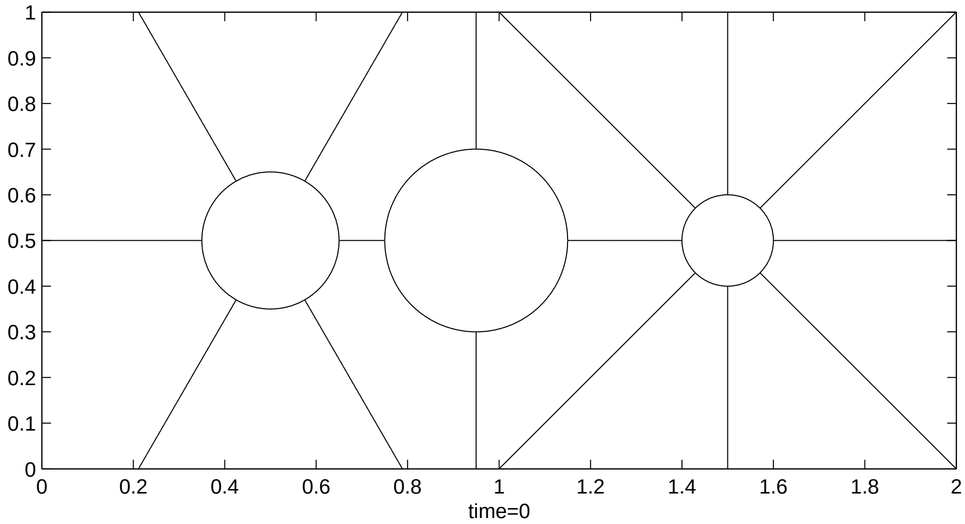

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- $\frac{dA}{dt} = -M\,\gamma\,\int_\Gamma \kappa\,ds$.
- $\frac{dA}{dt} = -M\gamma\big(2\pi - \sum_{i=1}^n \alpha_i\big) = -2\pi M\gamma\big(1 - n/6\big)$,
  where α_i: exterior angle; n: number of walls.

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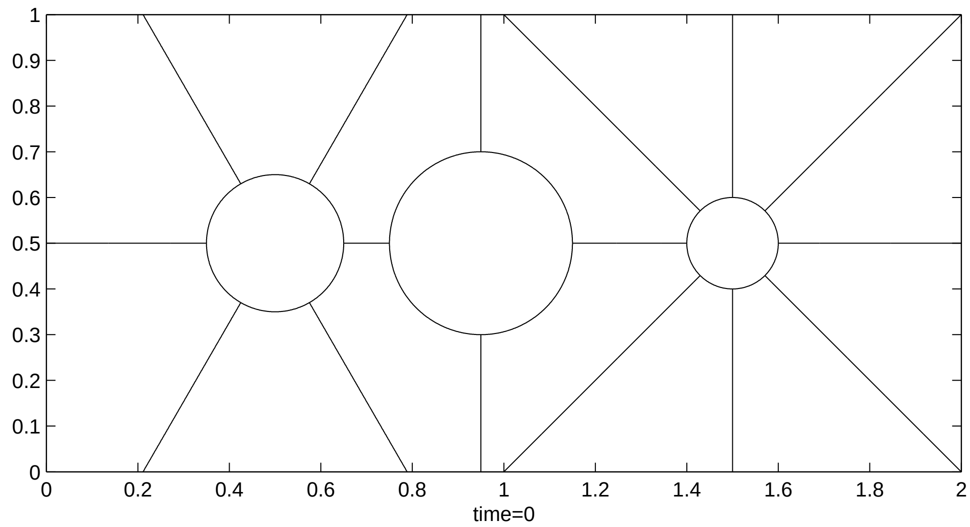

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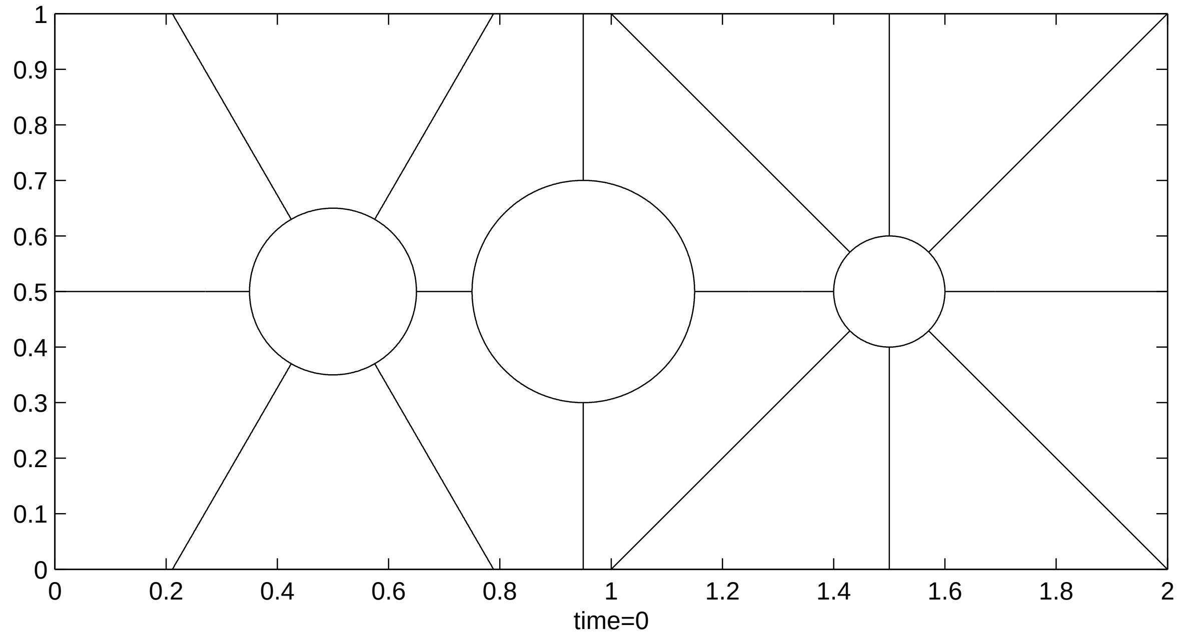
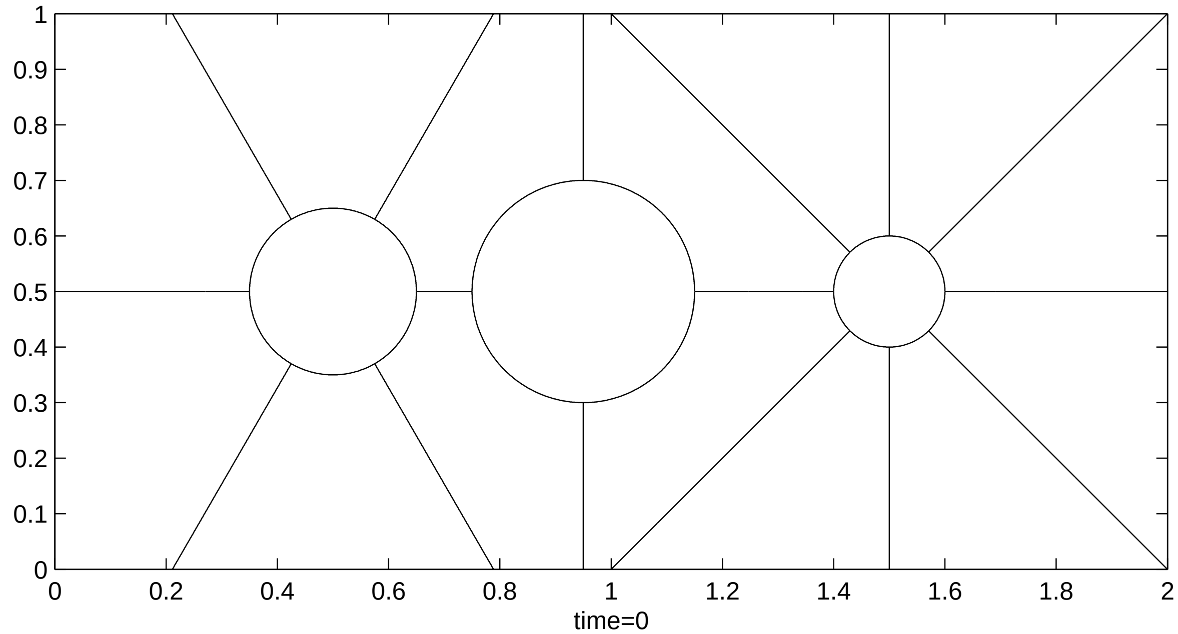
- $\frac{dA}{dt} = -M\,\gamma\,\int_\Gamma \kappa\,ds$.
- $\frac{dA}{dt} = -M\gamma\left(2\pi - \sum_{i=1}^{n} \alpha_i\right) = -2\pi M\gamma\left(1 - n/6\right)$,
  where α_i: exterior angle; n: number of walls.
- The rate of change of the area of a given cell is independent of cell size and solely dependent on the number of walls (or edges) of the cell.
- The area is constant for 6-sided bubbles, bubbles with fewer than 6 sides tend to shrink, and bubbles with more than 6 sides tend to grow.

2D Foam Dynamics: Force and Normal slip

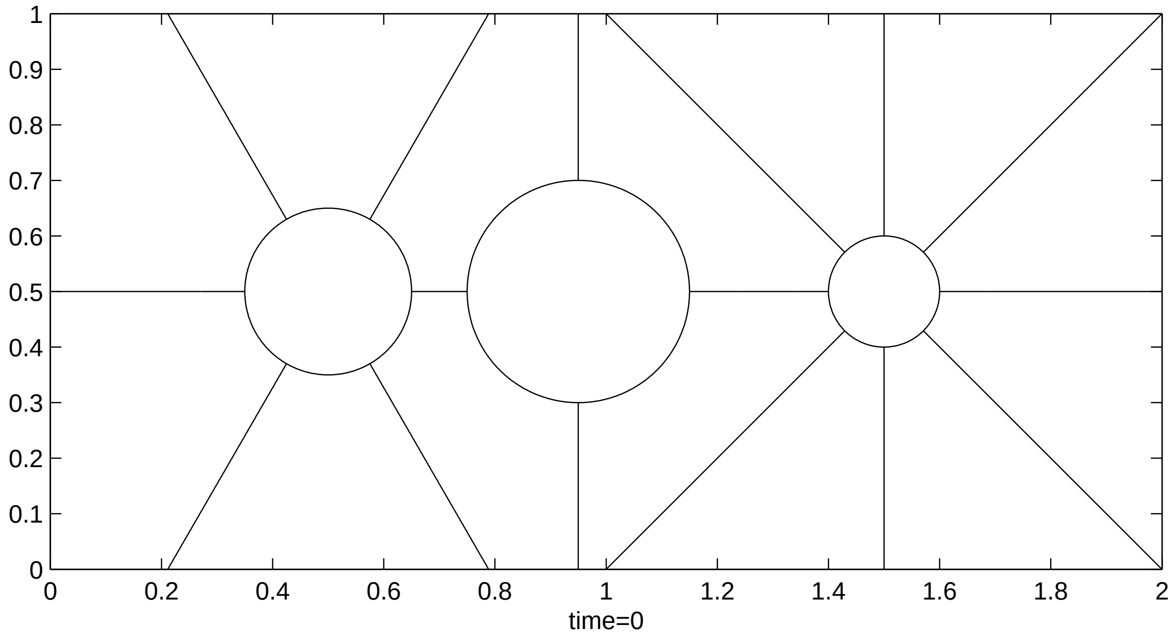


2D Foam Dynamics: Force and Normal slip



- $\mathbf{F}(s,t) = \frac{\partial}{\partial s}(\gamma\, \tau) = \gamma \frac{\partial \tau}{\partial s}$.
- $\tau(s,t) = \frac{\partial \mathbf{X}}{\partial s} / \left| \frac{\partial \mathbf{X}}{\partial s} \right|$.

2D Foam Dynamics: Force and Normal slip



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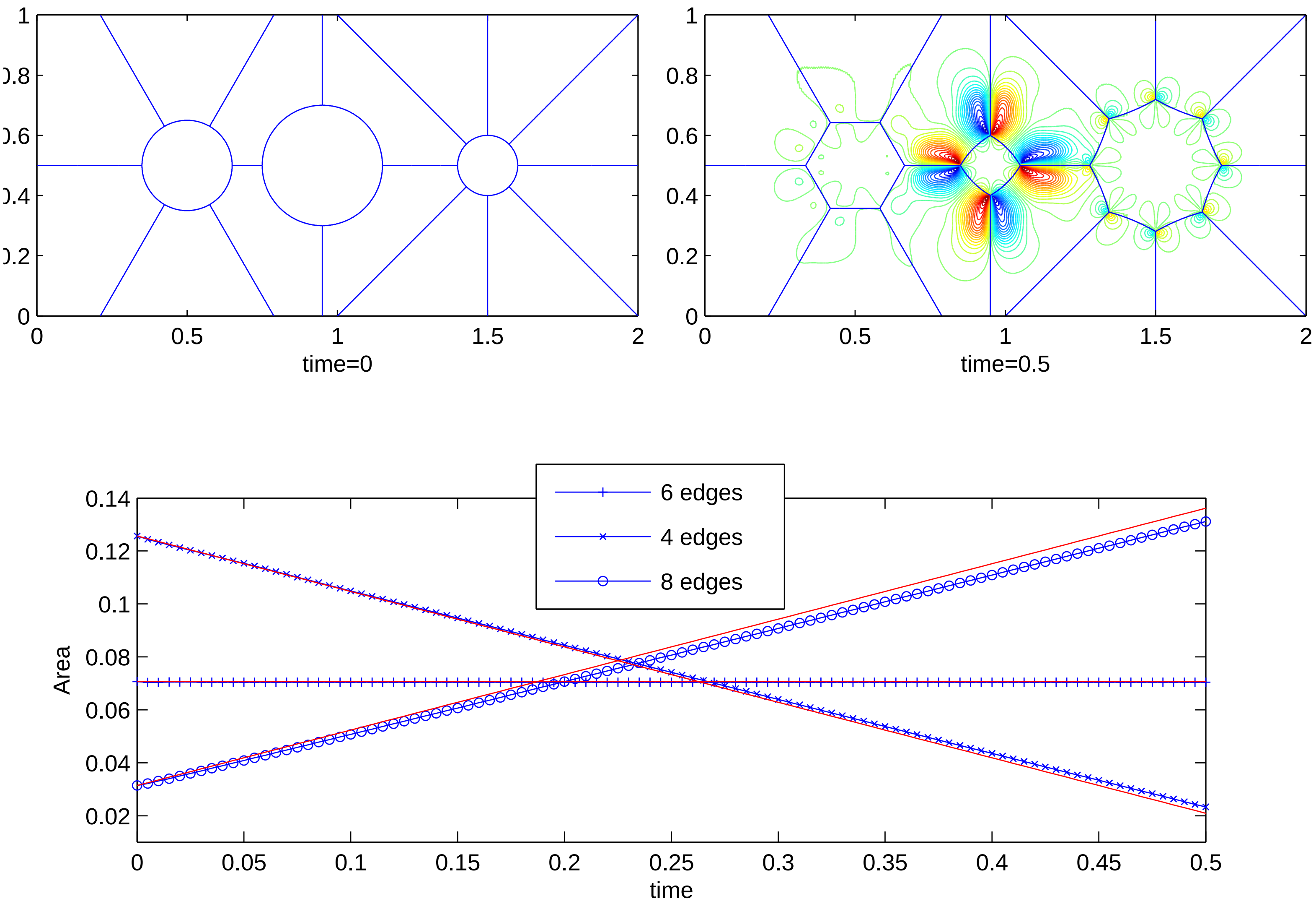
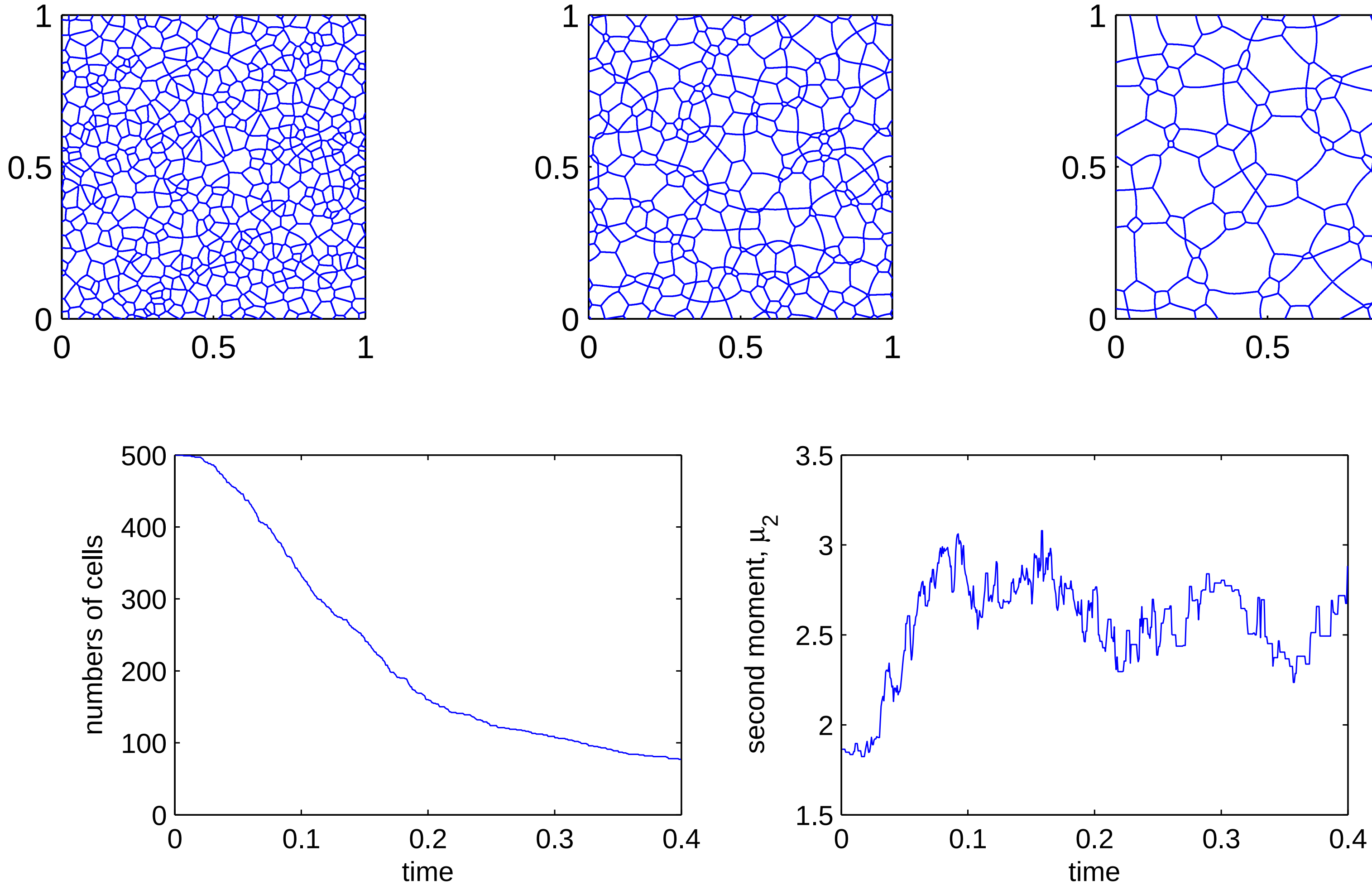
- $\tau(s,t) = \frac{\partial \mathbf{X}}{\partial s} / \left|\frac{\partial \mathbf{X}}{\partial s}\right|$.
- $\frac{\partial \mathbf{X}}{\partial t}(s,t) = \int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x} - \mathbf{X}(s,t))d\mathbf{x} + M\,\mathbf{F} / \left|\frac{\partial \mathbf{X}}{\partial s}\right|$.

Foam with 3 inner cells

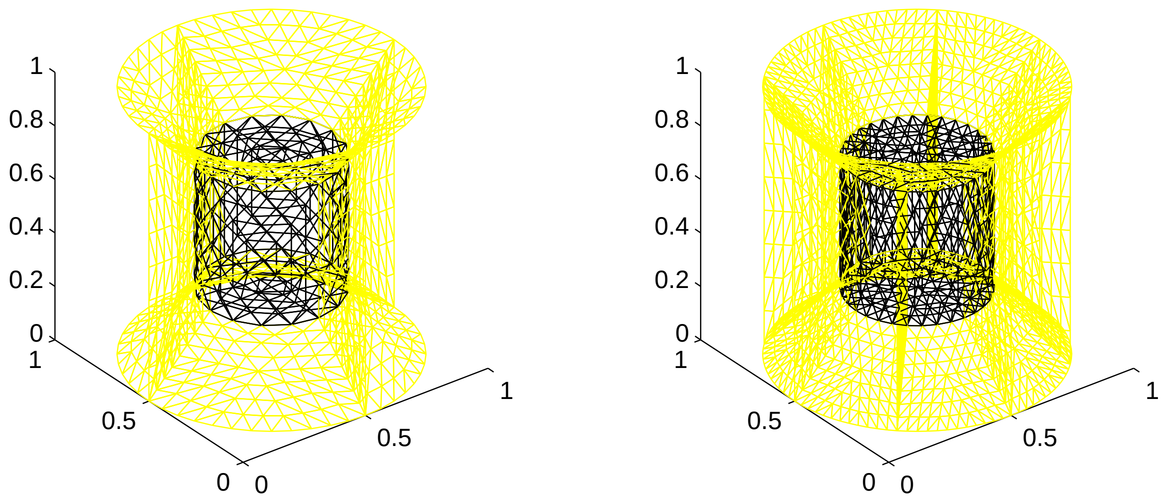


Foam with 500 cells



general foam animation with topological changes

3D Foam Dynamics



- R.D.MacPherson and D.J.Srolovitz, “The von Neumann relation generalized to coarsening of three-dimensional micro-structures”, Nature, 2007.
- $\frac{dV}{dt} = -2\pi M\gamma\big(L(D) - \frac{1}{6}\sum_{i=1}^{6} e_i(D)\big)$, where $L(D)$ is a natural measure of the linear size of domain D and e_i is the length of triple line (edge) i.
- Descretzied version of the 3D von Neumann relation:

$$\frac{dV}{dt} = -M\gamma \sum_{e\in E} L_e\,\theta_e,$$

where L_e is the length of edge e of the triangular facet, and θ_e is the angle between the two facets with the same edge e.

3D foam: Continuous force and normal slip

- Let $\mathbf{X}(r, s, t)$ be a foam boundary,

$$\mathbf{F}(r, s, t) = -\frac{\partial E}{\partial \mathbf{X}},$$

- $E[\mathbf{X}(\cdot, \cdot, t)] = \gamma \int \left| \frac{\partial \mathbf{X}}{\partial r} \times \frac{\partial \mathbf{X}}{\partial s} \right| dr ds$, where γ is the surface tension.

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- Normal slip is

$$\begin{aligned}
\frac{\partial \mathbf{X}}{\partial t}(r,s,t) &= \mathbf{u}(\mathbf{X}(r,s,t),t) + M\,\mathbf{F} / \left| \frac{\partial \mathbf{X}}{\partial r} \times \frac{\partial \mathbf{X}}{\partial s} \right|, \\
&= \int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x} - \mathbf{X}(r,s,t))d\mathbf{x} + M\,\mathbf{F} / \left| \frac{\partial \mathbf{X}}{\partial r} \times \frac{\partial \mathbf{X}}{\partial s} \right|.
\end{aligned}$$

3D foam: Discrete force and slip using Triangulation

- After triangulation of the foam boundary,

$$E[\mathbf{X}^n] = \gamma \sum_k |T^k| = \gamma \sum_k \frac{1}{2} |(\mathbf{X}_2^k - \mathbf{X}_1^k) \times (\mathbf{X}_3^k - \mathbf{X}_1^k)|,$$

  where T^k is a triangle with vertices $\{\mathbf{X}_1^k, \mathbf{X}_2^k, \mathbf{X}_3^k\}$ and $|T^k|$ is the area of the triangle T^k.

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- $\mathbf{F}_1^k = -\frac{\gamma}{\Delta r \, \Delta s} \sum_{k=1} \frac{1}{2} \frac{\partial}{\partial \mathbf{X}_1^k} |(\mathbf{X}_2^k - \mathbf{X}_1^k) \times (\mathbf{X}_3^k - \mathbf{X}_1^k)|,$

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- $\mathbf{F}_1^k = \frac{\gamma}{2 \Delta r \, \Delta s} \sum_{k=1} (\mathbf{X}_3^k - \mathbf{X}_2^k) \times \mathbf{n}_1^k,$
  where $\mathbf{n}_1^k = (\mathbf{X}_2^k - \mathbf{X}_1^k) \times (\mathbf{X}_3^k - \mathbf{X}_1^k) / |(\mathbf{X}_2^k - \mathbf{X}_1^k) \times (\mathbf{X}_3^k - \mathbf{X}_1^k)|$.

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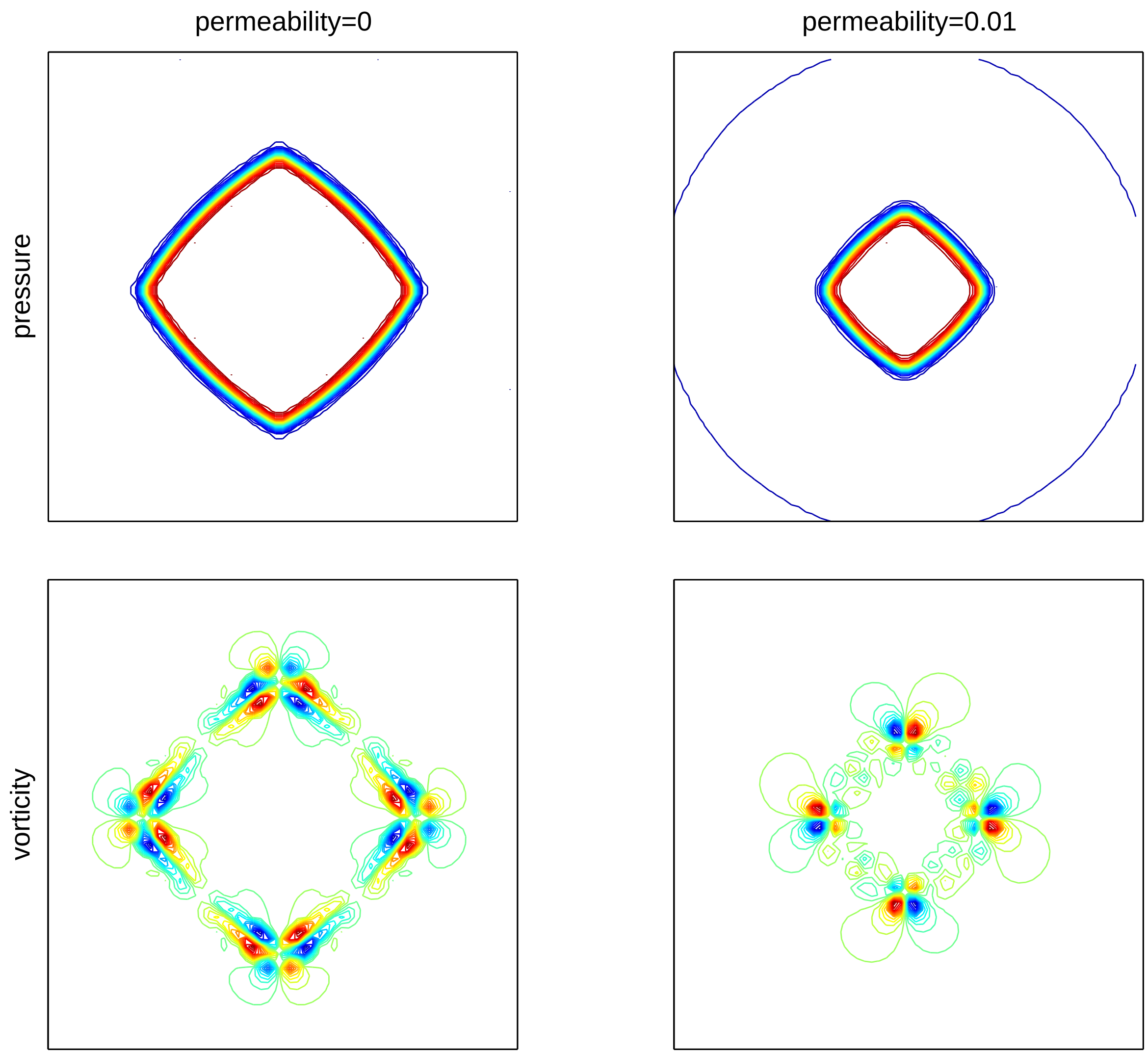
- $\frac{\mathbf{X}_1^{n+1} - \mathbf{X}_1^n}{\Delta t} = \sum_{\mathbf{x}} \mathbf{u}^{n+1}(\mathbf{x}) \delta_h (\mathbf{x} - \mathbf{X}_1^n) h^3 + \frac{M \mathbf{F}_1^k \Delta r \Delta s}{\sum_{j=1}^m |T^{k_j}|/3}$.

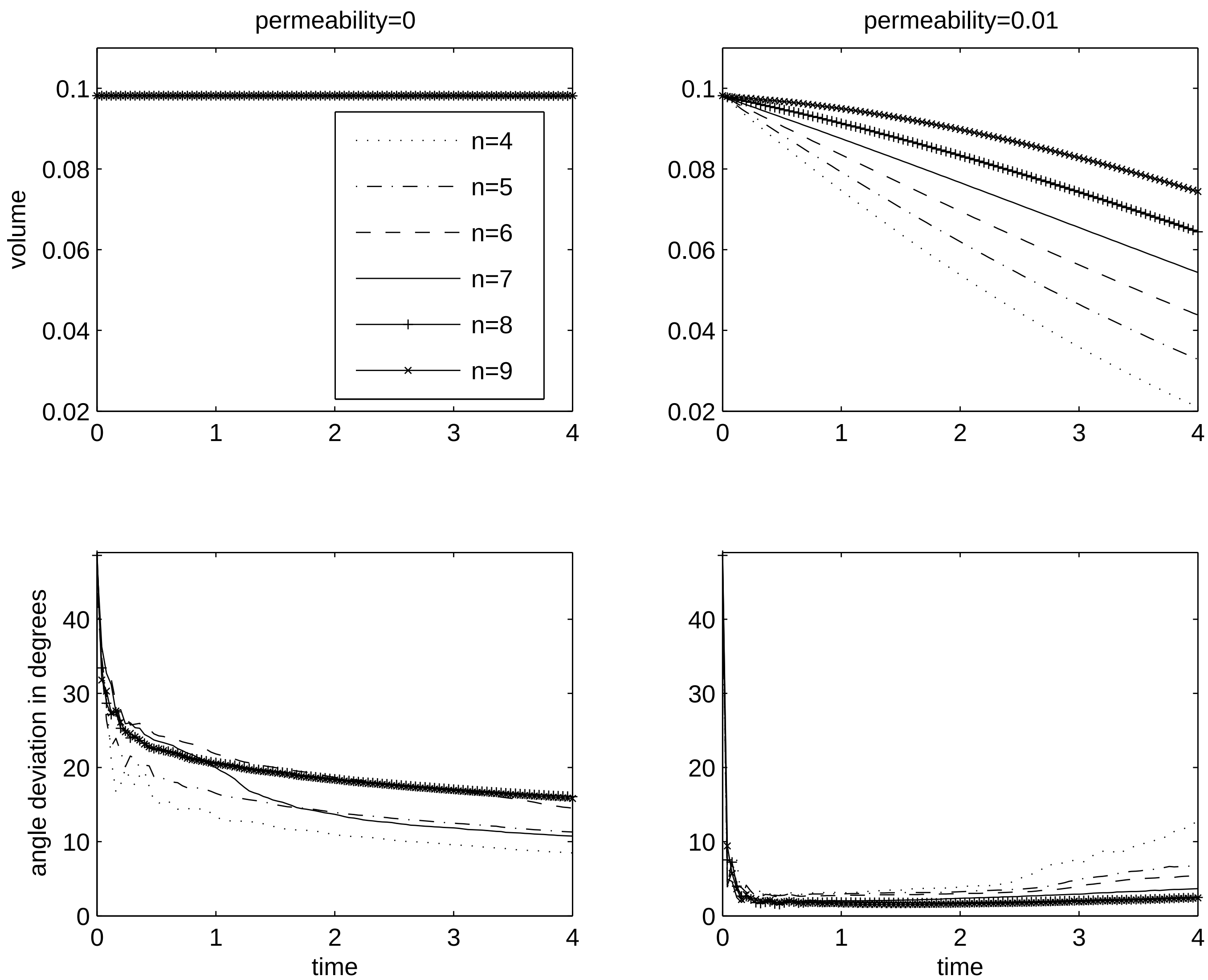
3D Foam Dynamics with a single inner cell



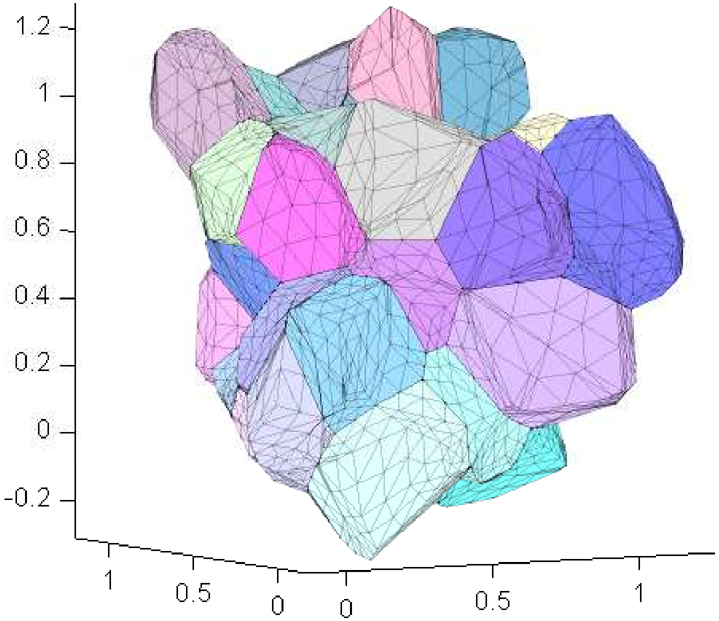
permeability=0

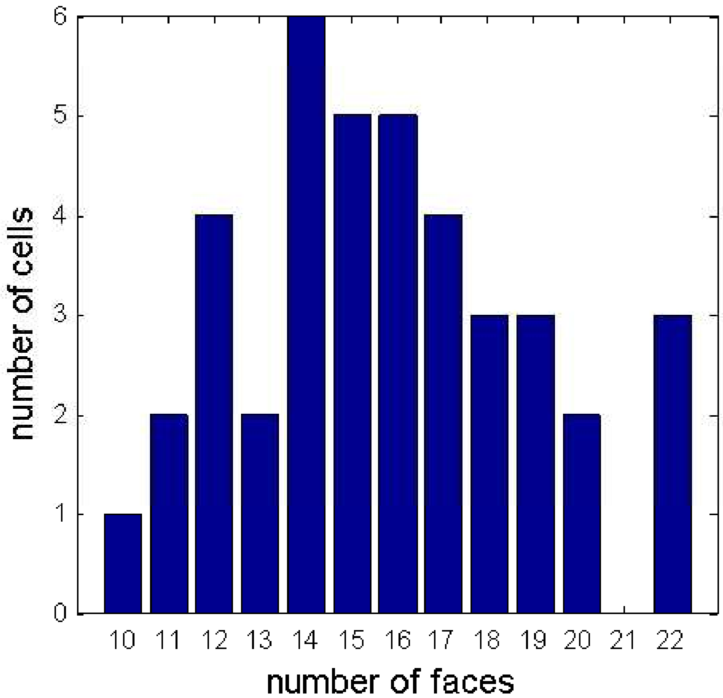
permeability=0.05

3D Foam Dynamics: 3D von Neumann relation



3D General Foam with 40 Cells





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Conclusions and Future Work:

- The pIB method is useful for the interaction between massive boundary and fluid.
- The pIB method can be applied to problems by decoupling the structural dynamics from the fluid dynamics.
- The results verify 2D and 3D von Neumann relations.
- The IB method can handle the interaction between porous elastic material and the surrounding fluid.
- Improve the stability condition generated from elastic force.
- Increases in the Reynolds number are needed in various ways: improved fluid solvers, global mesh refinement, adaptive mesh refinement, and direct numerical simulation of turbulence models.