Dynamics of Religious Group Growth and Survival

Tongzhou Chen*, Michael McBride**, Martin Short*

*Georgia Tech (Mathematics), **UC Irvine (Economics)

Background

Economic theory provides different accounts of group dynamics.

- Club theory suggests strict churches grow faster (Iannaccone 1992).
- Preference heterogeneity suggests diversity of groups can thrive (Stark and Finke 2000; McBride 2008).

We (Chen, McBride, Short 2019)

- combine both elements into single, dynamic framework,
- and add other elements neglected in existing religious market models such as population growth (Hout, et al. 2001; Scheitle, et al. 2011), cultural transmission (Bisin Verdier 2000; Carvalho 2012).

Outlines

- Single Group Model
- Multiple Group Model
- Simulations on Sequential Stackelberg Games

Individual Utility Function

Each person has a total time of 1, and individual i devotes t_i amount of time to in-group activities.

Each group has a parameter $\lambda \in [0, 1]$ called “group strictness”, which is the the amount of time members are expected to spend in the group.

In-group utility: $U_{in} = (\frac{\sum_j t_j}{N})^{1/2}$.

Out-group utility: $U_{out} = r_i(1 - t_i)$.

Punishment: $P = \beta_g(\lambda_g - t_i)_+$.

Total utility: $U_i = U_{in} + U_{out} - P$.

Single Group Nash Equilibrium

Everyone tries to choose her t_i so as to maximize her utility function U_i based on the choices of others.

Everybody has 5 options 0, λ, 1, and

$$t_a = \frac{1}{4Nr_i^2} - T_i \equiv a_i + (1-a_i)\lambda,\ 0 < a_i < 1, \tag{1}$$

$$t_b = \frac{1}{4N(r_i - \beta)^2} - T_i \equiv b_i\lambda,\ 0 < b_i < 1, \tag{2}$$

where $T_i = \sum_{j \neq i} t_j$.

Theorem

There exists a number $R_1 > 0$ that is a function of $\vec{r}$, N, λ, and β such that, the system is in a Nash Equilibrium if

- *all individuals with $r_i < R_1$ choose $t_i = 1$,*
- *all with $r_i = R_1$ choose t_a with a potentially specific value of a,*
- *all with $R_1 < r_i < R_1 + \beta$ choose $t_i = \lambda$,*
- *all with $r_i = R_1 + \beta$ choose t_b with a potentially specific value of b,*
- *all with $r_i > R_1 + \beta$ choose $t_i = 0$.*

Ideal Strictness and Punishment Levels

N unaffiliated people with the identical parameter r would like to form a group of strictness λ to get more payoff than being unaffiliated. Each individual will receive payoff $U = \sqrt{\lambda} + r(1-\lambda)$.

This payoff is maximized for ideal strictness $\lambda = 1/(4r^2)$ assuming that $r > 1/2$.

β needs to be at least $1/(2\sqrt{\lambda})$ to prevent the internal free-riding.

β needs to be at least $1/\sqrt{\lambda}$ to dissuade outsiders with differing $r_i > r$ from joining the group and playing $t_i = 0$.

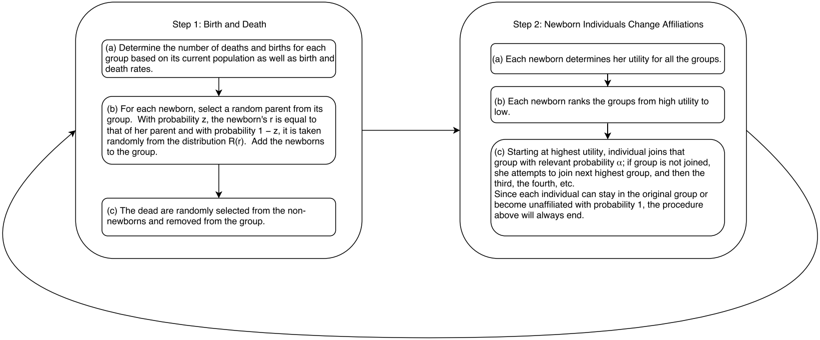
If an individual with $r_i < r$ joins the group, she will play λ if $r_i > r/(N+1)$ and a linear combination of 1 and λ otherwise.

Multiple Group Model

There are M groups with different strictnesses.

Unaffiliated Group of strictness 0. Utility of person i equals to r_i.

All groups will select a β that dissuades any possible free-riding, and therefore all of the members of any group will simply play $t_i = \lambda$.



Birth, Death and Inheritance

Each group has the same per capita death rate 1.

Each group has a per capita birth rate b_g which could potentially be group dependent.

Each new born child initially stays in the same group as their parents.

Whenever a new individual is born, with probability z her r_i is equal to that of her parent and with probability $1 - z$ her r_i is taken randomly from the distribution $R(r)$.

Dead people are simply removed from the population.

Changing Affiliation

Individual i could in principal associate with any of the groups g' and thereby obtain utility

$$U_{ig'} = \sqrt{\lambda_{g'}} + r_i(1 - \lambda_{g'}) \ . \tag{3}$$

Stationary Model: Each individual would simply determine which g' provides the maximum utility and choose that group.

If the birth rates are equal to 1 and everyone is currently in their ideal group, then in expectation the system will retain that configuration for all time.

Stationary model neglects the exposure to the group.

We define the exposure probability of an individual currently in group g to members of group g' as

$$\alpha_{gg'} = 1 - \left[1 - \frac{(1-\lambda_{g'})N_{g'}}{\sum_k (1-\lambda_k)N_k}\right]^{s(1-\lambda_g)}, \tag{4}$$

where $s > 0$ is a model parameter and N_g is the number of members in group g.

$\alpha_{gg} = 1$ and $\alpha_{g0} = 1$.

For each newborn i, we sort the groups such that g' is in front of g'' if

1. $U_{ig'} > U_{ig''}$.
2. $U_{ig'} = U_{ig''}$ and $\alpha_{gg'} > \alpha_{gg''}$.
3. $U_{ig'} = U_{ig''}$, $\alpha_{gg'} = \alpha_{gg''}$ and $\lambda_{g'} < \lambda_{g''}$.

This leaves us with a permutation of groups, denoted by $\sigma(j)$ where $j = 0, 1, \ldots, M-1$.

Then we simply march down the permutation starting with $j = 0$, at each point determining whether i chooses group $\sigma(j)$ via the exposure probability $\alpha_{g\sigma(j)}$ until she probabilistically joins a group.

Differential Equation Model for Group Size

Each potential r value from the distribution $R(r)$ can be classified by its permutation $\sigma_r(j)$ of the groups strictly in terms of the utility of the groups to a person with parameter $r_i = r$.

As such, we can divide the total population into a finite number S of subpopulations, each of which is labeled by the permutation of groups σ.

$n_{g\sigma}(t)$: number of individuals in group g that are members of subpopulation σ. $\sum_{\sigma}^{J} n_{g\sigma}(t) = N_g(t)$.

f_σ: fraction of the distribution $R(r)$ that encompasses subpopulation σ.

$$\frac{dn_{g\sigma}}{dt} = -n_{g\sigma} + \sum_{g'=0}^{M-1} b_{g'} \left[z n_{g'\sigma} + f_\sigma (1-z) N_{g'} \right] p_{g'g\sigma} \ . \qquad (5)$$

If group g takes position J in ordering σ, then

$$p_{g'g\sigma} = \alpha_{g'g} \prod_{j=0}^{J-1} \left(1 - \alpha_{g'\sigma(j)}\right) \ . \qquad (6)$$

We now recast (5) in terms of new variables $\tilde{n}_{g\sigma} = n_{g\sigma}/N$ and $\tilde{N}_g = N_g/N$, given that

$$\frac{dN}{dt} = -N + \sum_{g'=0}^{M-1} b_{g'} N_{g'} .$$

Two Group Case

Suppose we have only 2 groups, unaffiliated group 0 with strictness 0 and group 1 with strictness λ.

We denote $\sigma_0 = \{0, 1\}$ and $\sigma_1 = \{1, 0\}$ respectively and let $f_1 = f$, $n_{11} = n$, $K = z + f(1 - z)$. If birth rates are equal to 1, the system of ODE can be reduced to

$$\frac{dn}{dt} = (f - Kn)p + Kn - n \equiv g(n), \tag{7}$$

where

$$p(n) = 1 - \left[\frac{1-n}{1-\lambda n}\right]^s. \tag{8}$$

Theorem

If $g'(0) = fs(1-\lambda) + K - 1 \leq 0$, then the equation $dn/dt = g(n)$ has only the trivial equilibrium point $n = 0$ and it is stable. Otherwise the trivial equilibrium point becomes unstable and the equation has another stable equilibrium at a point n_0 in $(0, f)$.

Theorem

If $g'(0) = fs(1-\lambda) + K - 1 > 0$, then the non-trivial equilibrium n_0 satisfies $\partial n_0/\partial\lambda < 0$, $\partial n_0/\partial z > 0$, $\partial n_0/\partial s > 0$.

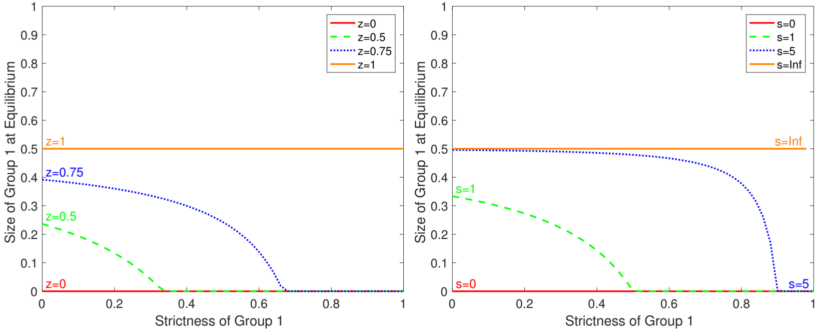


Figure: The size of group 1 at equilibrium is plotted as a function of its strictness λ as parameters s or z varies. When not varying, $s = 0.75$ and $z = 0.5$. The distribution $R(r)$ is chosen at each λ such that $f = 0.5$.

Theorem

For any $0 < \lambda \leq 1$, there exists a minimal birthrate b_{min} that allows for survival of the stricter group at equilibrium.

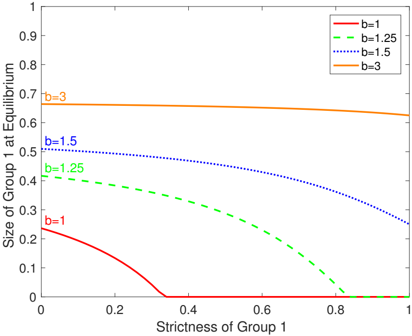


Figure: The size of group 1 at equilibrium is plotted as a function of its strictness λ for varying values of b, with $s = 0.75$ and $z = 0.5$ fixed and the distribution $R(r)$ chosen at each λ such that $f = 0.5$.

Going beyond Two Groups

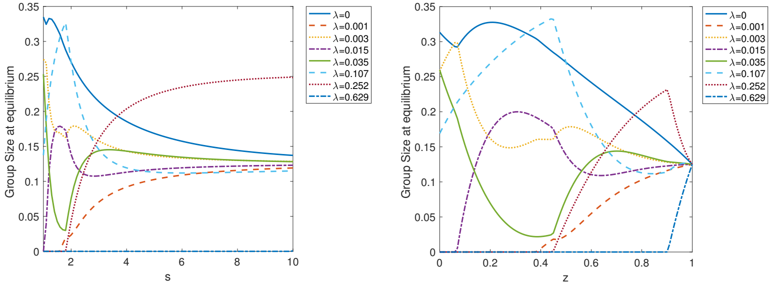


Figure: Equilibrium sizes for a system with eight groups, as parameters s or z varies. When not varying, $s = 2$ and $z = 0.5$. The distribution $R(r)$ is lognormal, and strictnesses are chosen such that every group is ranked most highly by an equal fraction of the population. Initial conditions set each subpopulation to an equal size.

Three Types of Groups

Type A Its utility function is the average time spent in group activities.

Type B Its utility function is the population of the group.

Type C Its utility function is the average individual payoff over the whole population.

Sequential Stackelberg Games

Initially there is only the unaffiliated group.

The groups then come into the marketplace sequentially.

Each group has to take a different strictness level.

When a group is joining the marketplace, it thinks one more step ahead that there will be another group following it.

The current group maximizes its payoff, anticipating the predicted response of the follower.

Groups need to meet the requirement of minimum population ϵ before maximizing their own utilities.

The group will not come to the marketplace if no λ values can satisfy the minimum population condition.

In addition, a Type C group will also not enter if existence of this group will lower the average payoff of individuals within the society.

After a group enters, the groups that die out will be removed from the market.

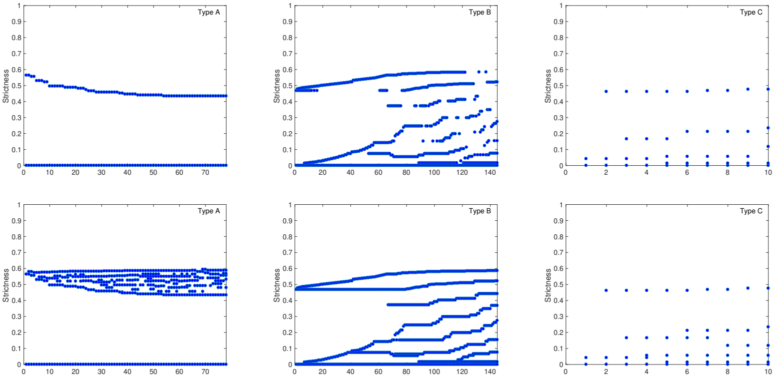


Figure: Simulation of the sequential Stackelberg game on the dynamic model. $r - 1/2 \sim \text{Lognormal}(\mu, \sigma^2)$ with $\mu = -0.5$ and $\sigma = 2$, $\epsilon = 0.01$, $s = 3$, $z = 0.5$, $b_g = 1$. The top and the bottom line show the strictness levels of groups with size larger than 0.1 and 0.01 at each step respectively. (Left) Type A groups only; (Middle) Type B groups only; (Right) Type C groups only.

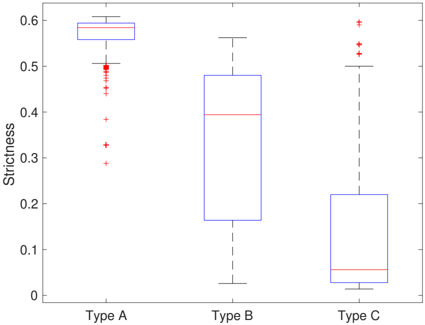


Figure: Boxplot of simulation of the sequential Stackelberg game on the dynamic model. $r - 1/2 \sim \mathrm{Lognormal}(\mu, \sigma^2)$ with $\mu = -0.5$ and $\sigma = 2$, $\epsilon = 0.01$, $s = 3$, $z = 0.5$, $b_g = 1$. Groups have an equal chance to be any of the three types. The procedure will end if 7 affiliations have been added.

T. Chen, M.McBride, and M.Short
Dynamics of Religious Group Growth and Survival
Journal for the Scientific Study of Religion, vol. 58, no.1, pp. 67-92, 2019

L. R. Iannaccona
Sacrifice and stigma: Reducing free-riding in cults, communes, and other collectives.
Journal of Political Economy, vol. 100, no. 2, pp. 271–291, 1992.

R. Stark, and R. Finke
Acts of Faith: Explaining the Human Side of Religion.
Univ of California Press, 2000.

M. McBride
Religious pluralism and religious participation: A game theoretic analysis.
American Journal of Sociology, vol. 114, no. 1, pp. 77–106, 2008.

A. Bisin and T. Verdier
Beyond the melting pot: Cultural transmission, marriage, and the evolution of ethnic and religious traits.
The Quarterly Journal of Economics, vol. 115, no. 3, pp. 955–988, 2000.

J.-P. Carvalho
Veiling
The Quarterly Journal of Economics, vol. 128, no. 1, pp. 337–370, 2012.

J. D. Montgomery
Adverse selection and employment cycles.
Journal of Labor Economics, vol. 17, no. 2, pp. 281–297, 1999.

M. D. Makowsky
A theory of liberal churches.
Mathematical Social Sciences 61.1 (2011): 41-51.

C. P. Scheitle, J. B. Kane, and J. V. Hook
Demographic imperatives and religious markets: Considering the individual and interactive roles of fertility and switching in group growth.
Journal for the scientific study of religion, vol. 50, no. 3, pp. 470–482, 2011.

Thank you for listening!

Tongzhou Chen
tchen308@gatech.edu