Nontrivial collective dynamics in networks of pulse-coupled oscillators

Antonio Politi, Ekkehard Ullner

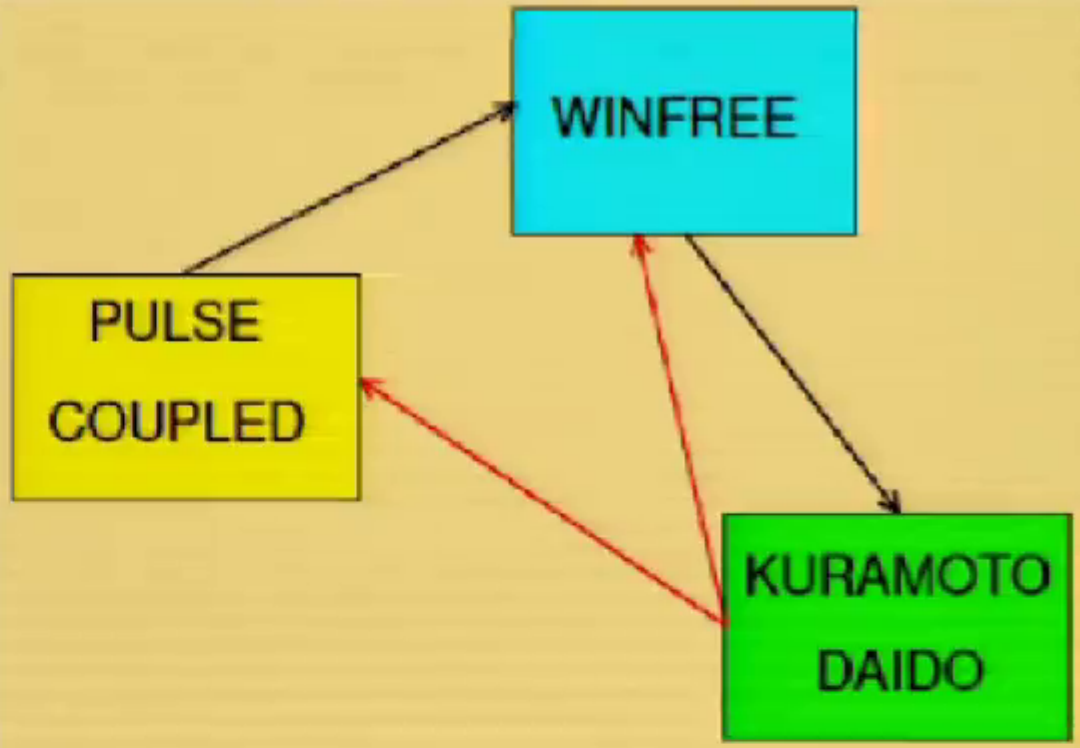
Department of Physics, University of Aberdeen, UK

Snowbird, 17-21 May 2015

Outline

- Collective processes in networks of 1d oscillators
- Pulse-coupled version of the Kuramoto model
- Characterization of the microscopic and macroscopic dynamics

Different classes of 1d dynamical systems



Winfree setup

$$\dot{\phi}_i = \nu + g\Gamma(\phi_i)\frac{1}{N}\sum_j S(\phi_j)$$

Forcing function $\quad S(\phi)$

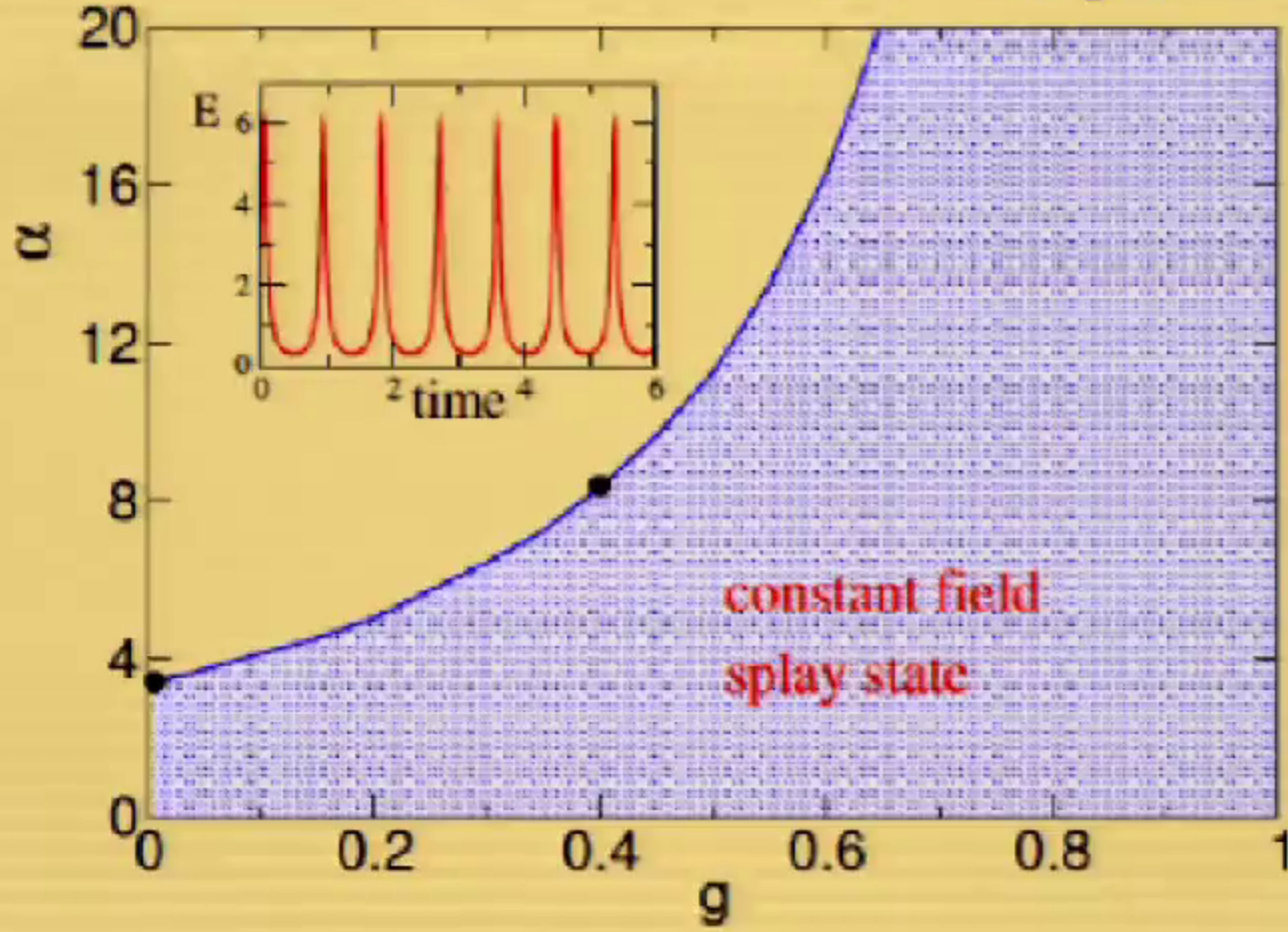
Kuramoto-Daido setup

$$\dot{\phi}_i = \nu + g\frac{1}{N}\sum_j G(\phi_i - \phi_j)$$

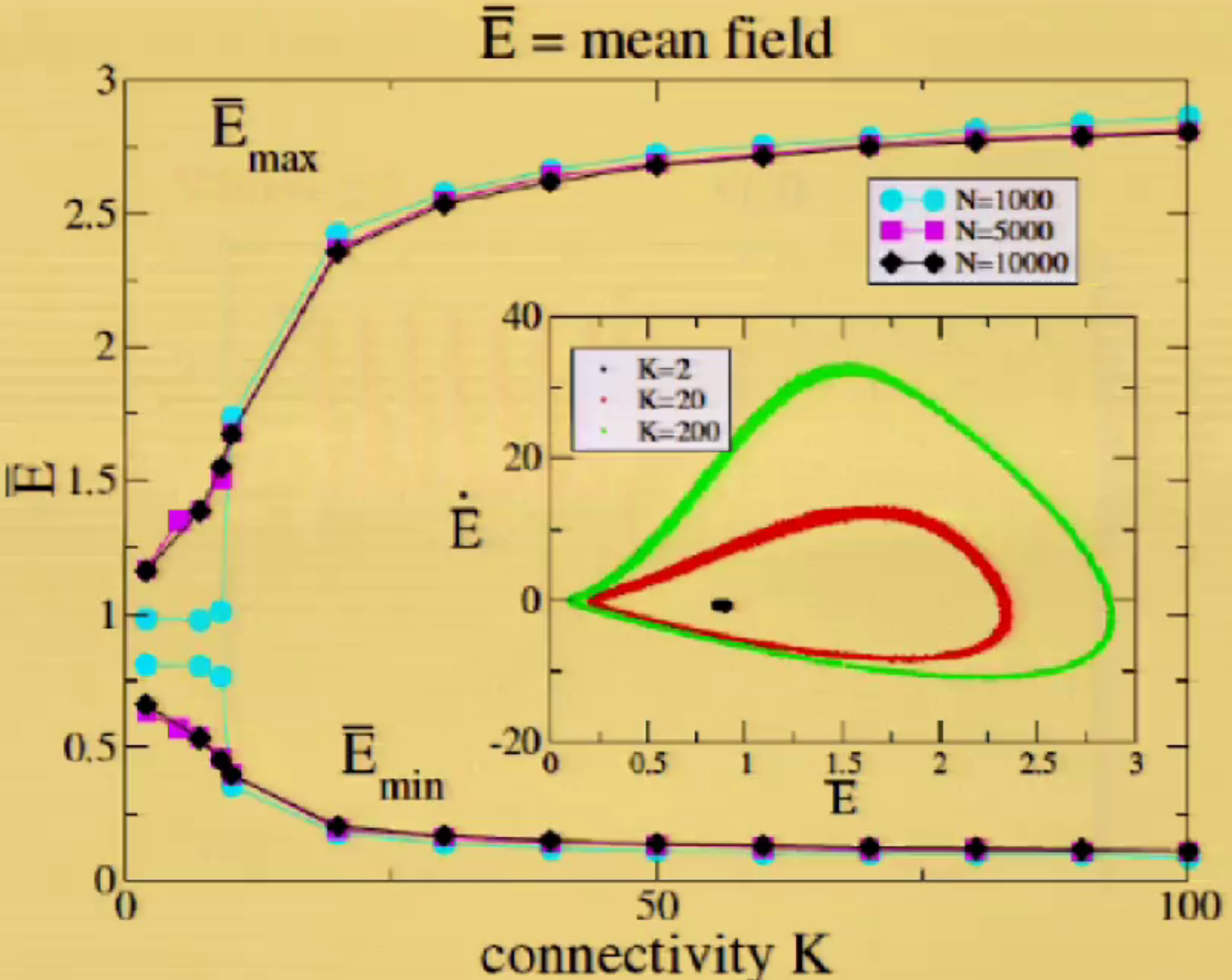
Coupling function $\quad G(\phi)$

Identical leaky-integrate and fire (LIF) $F = a - v$
Self-consistent partial synchronization

With α-pulses $\Gamma(\phi) = \frac{\nu \exp(\phi/\nu)}{a + gE_0}$

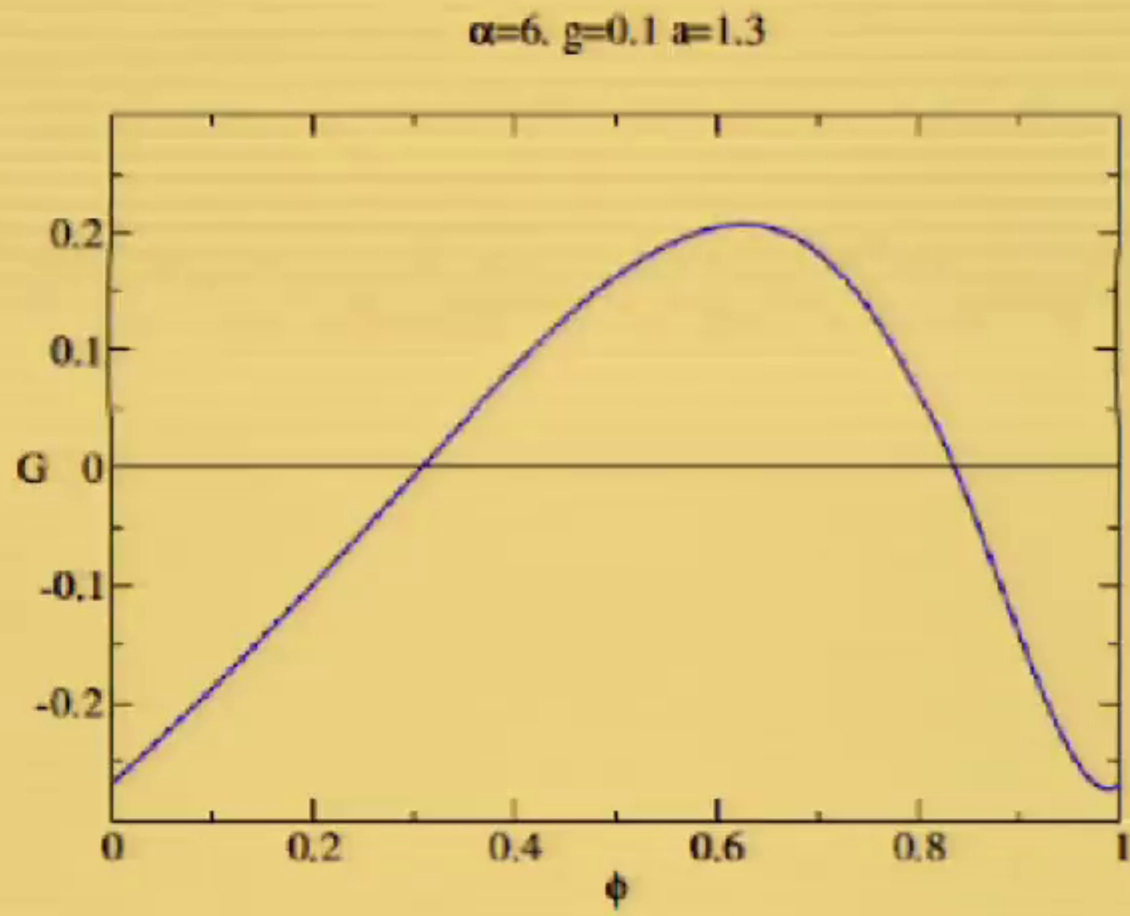


Introducing sparseness



Kuramoto-Daido with a specific coupling function

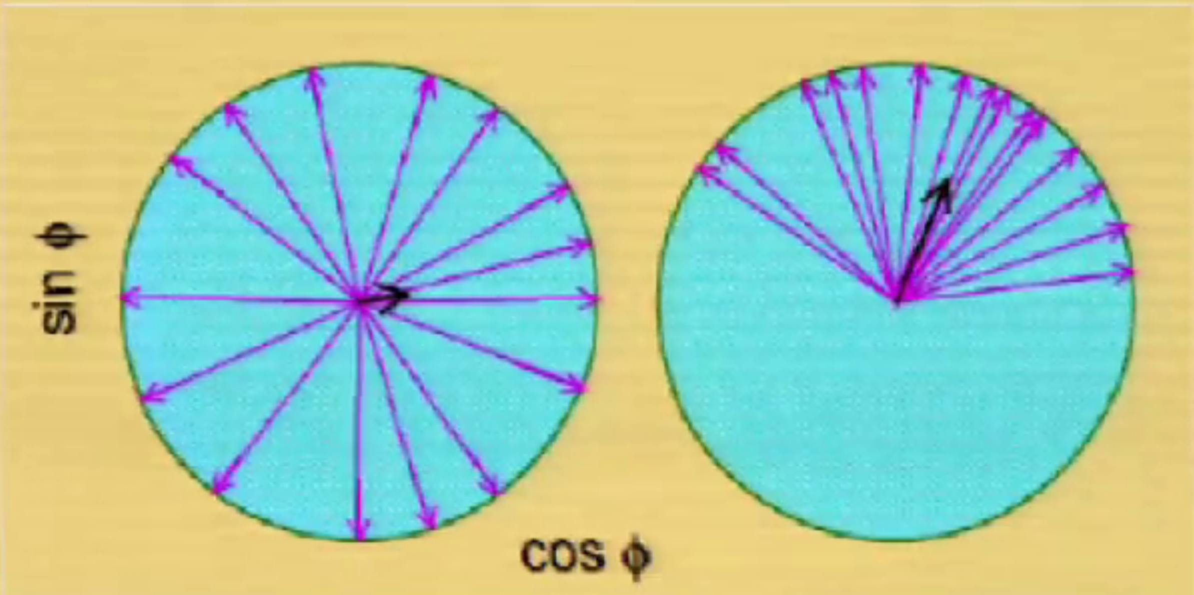
Again self-consistent partial synchronization



Adding disorder: the Kuramoto synchronization transition

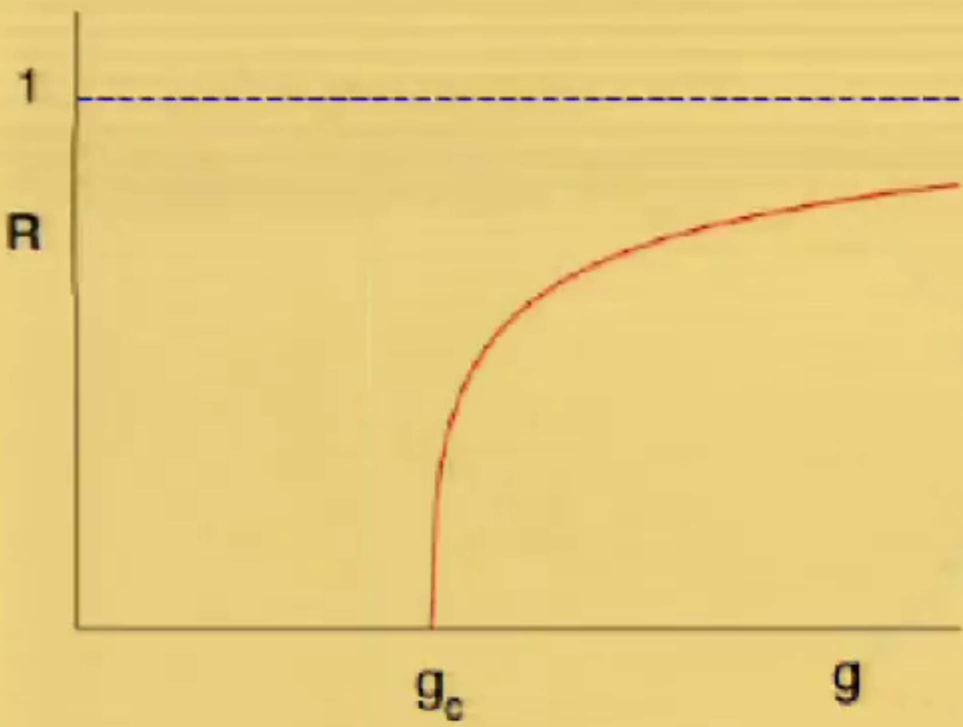
$$\dot{\phi}_i = \omega_i + \frac{g}{N} \sum_{j=1}^{N} \sin(\phi_j - \phi_i) + \xi_i(t)$$

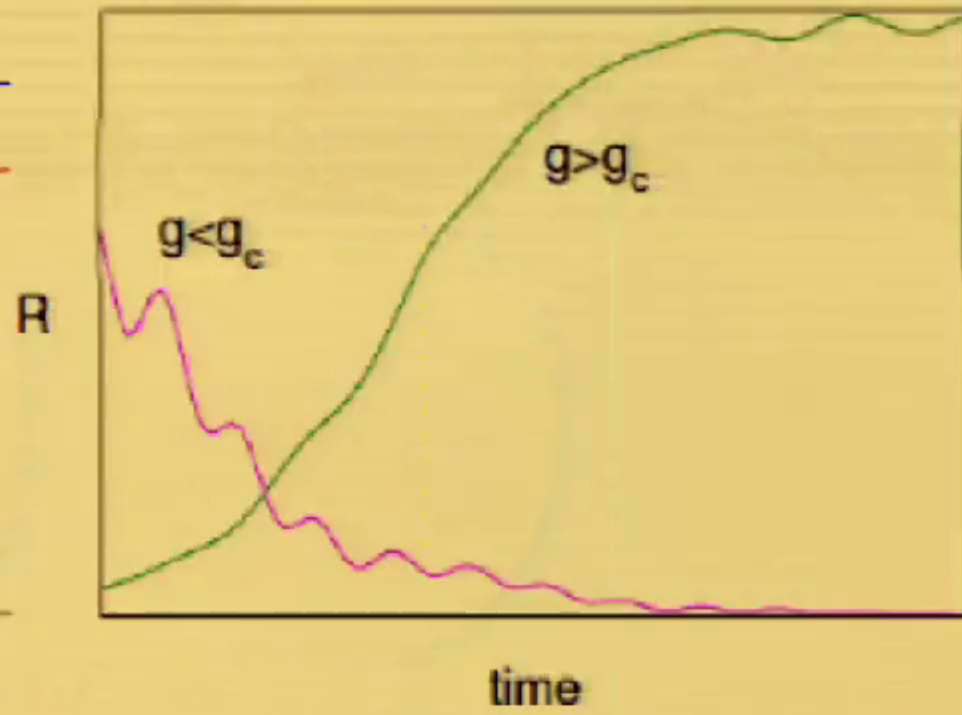
order parameter $\quad Re^{i\psi} = \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j}$



$$\dot{\phi}_i = \omega_i + gR \sin(\psi - \phi_i) + \xi_i(t)$$

Kuramoto model





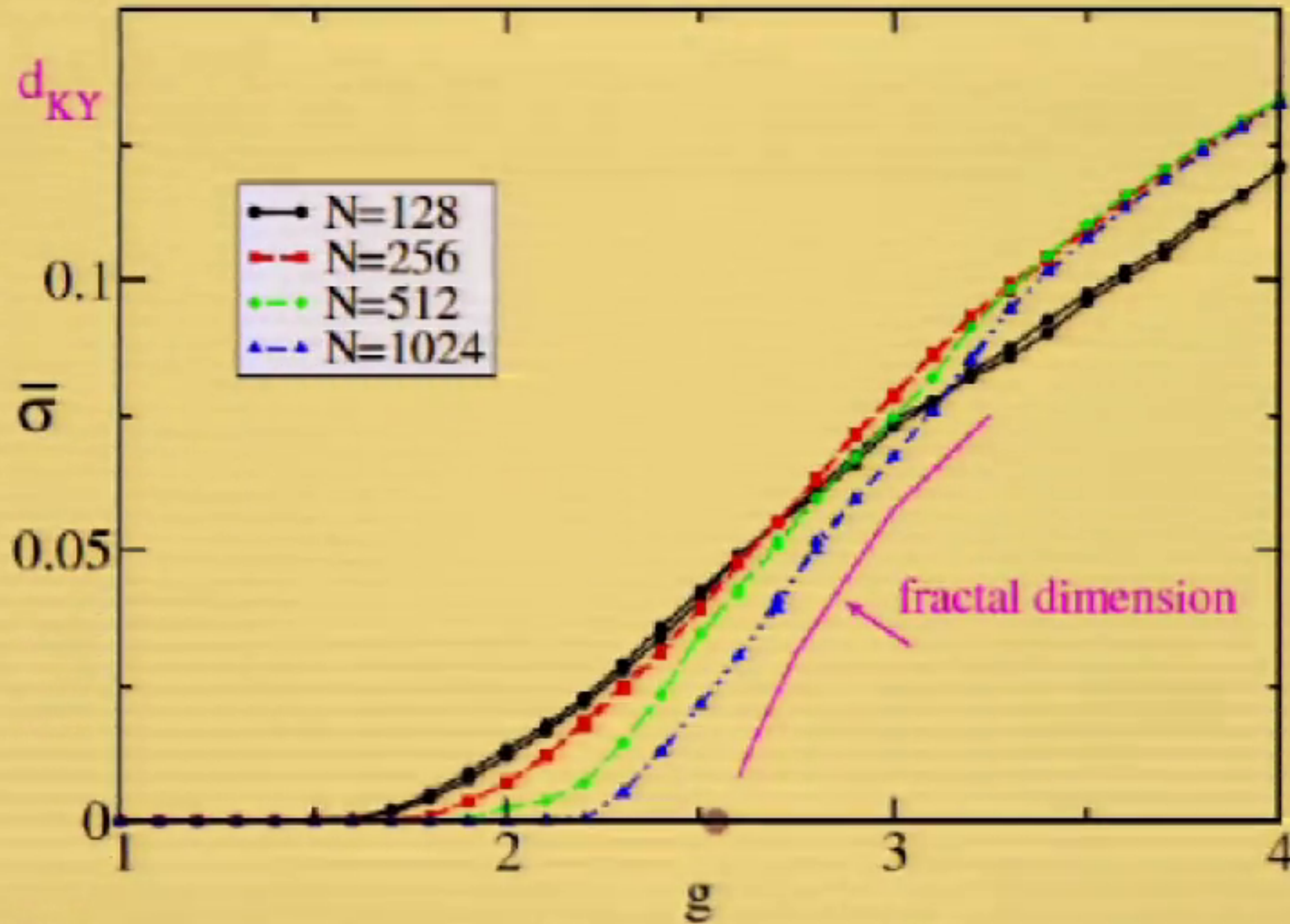
Balanced networks (discrete-time model)

$$h_i(t+1) = \frac{g}{\sqrt{N}} \sum_j J_{ij} \left[1 + \tanh h_j(t)\right] - \theta_i$$

$$h_i(t+1) = \left[\frac{g}{\sqrt{N}} \sum_j J_{ij} - \theta_i\right] + \frac{g}{\sqrt{N}} \sum_j J_{ij} \tanh h_j(t)$$

Fixed point solution for small g

Dynamic phase-transition

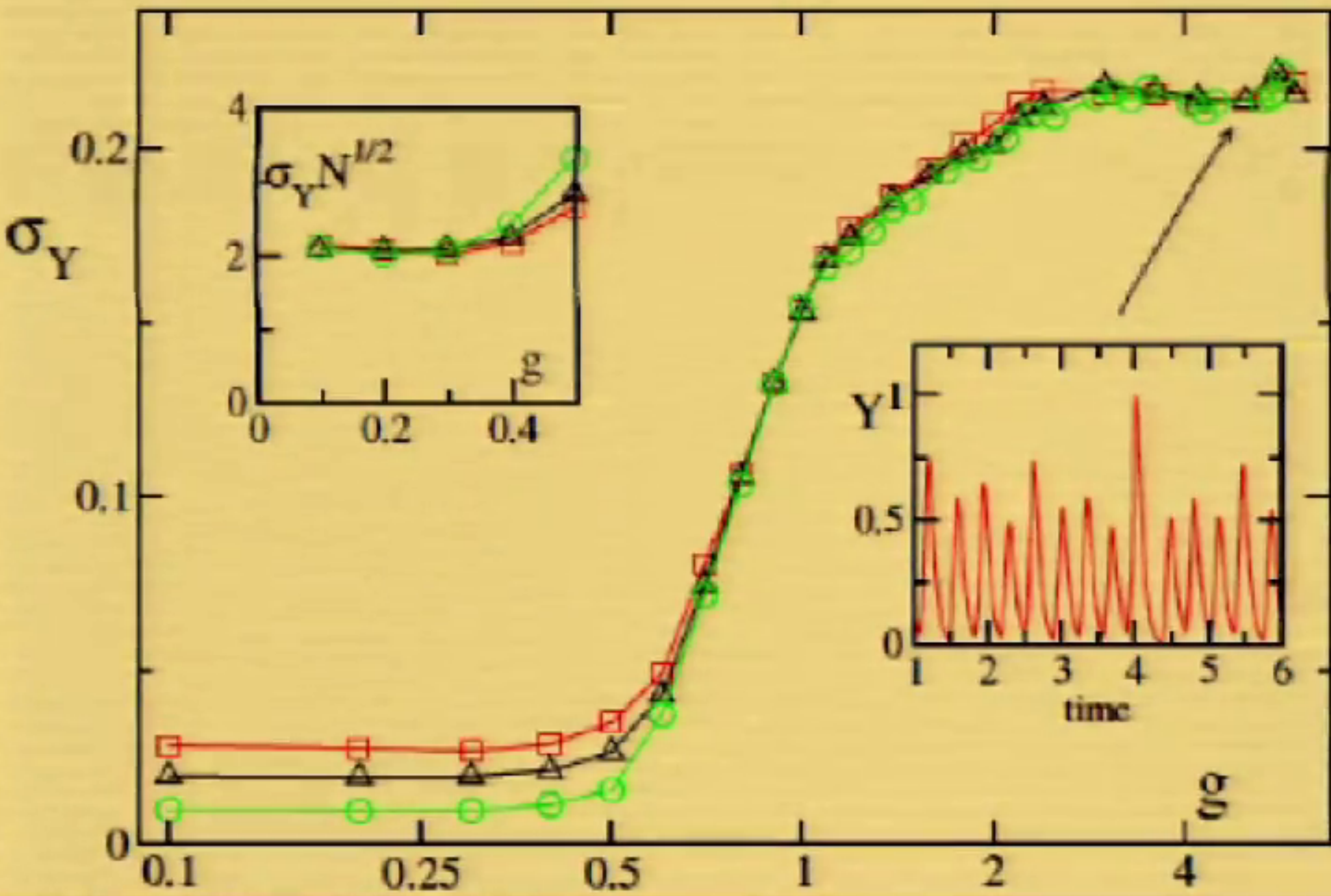


$$\overline{\sigma} = \sum_i \left\langle (h_i(t) - \langle h_i \rangle)^2 \right\rangle$$

High-dimensional dynamics in a Kuramoto-like setup (S. Luccioli, AP, 2011)

$$\dot{v}_i = a_i - v_i - \frac{g}{N} \sum_{n|t_n<t} \delta(t - t_n - t_d)$$

ORDER PARAMETER σ_Y = standard deviation of Y

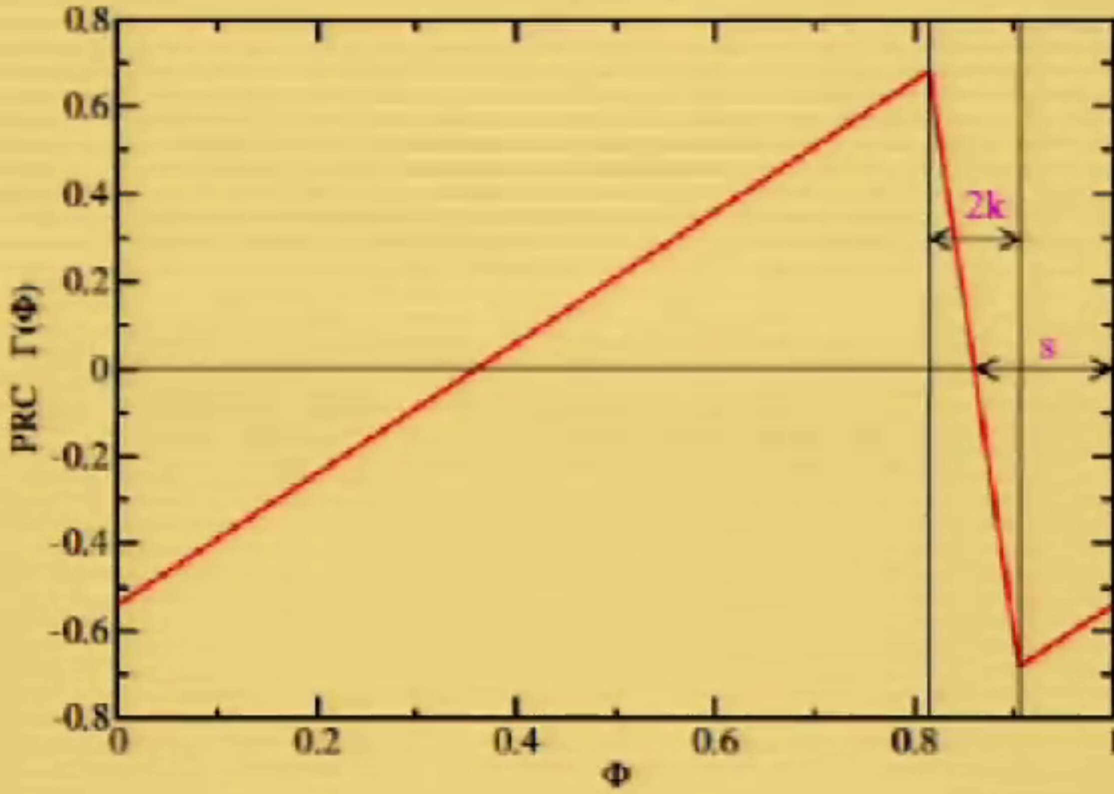


Robustness of the scenario

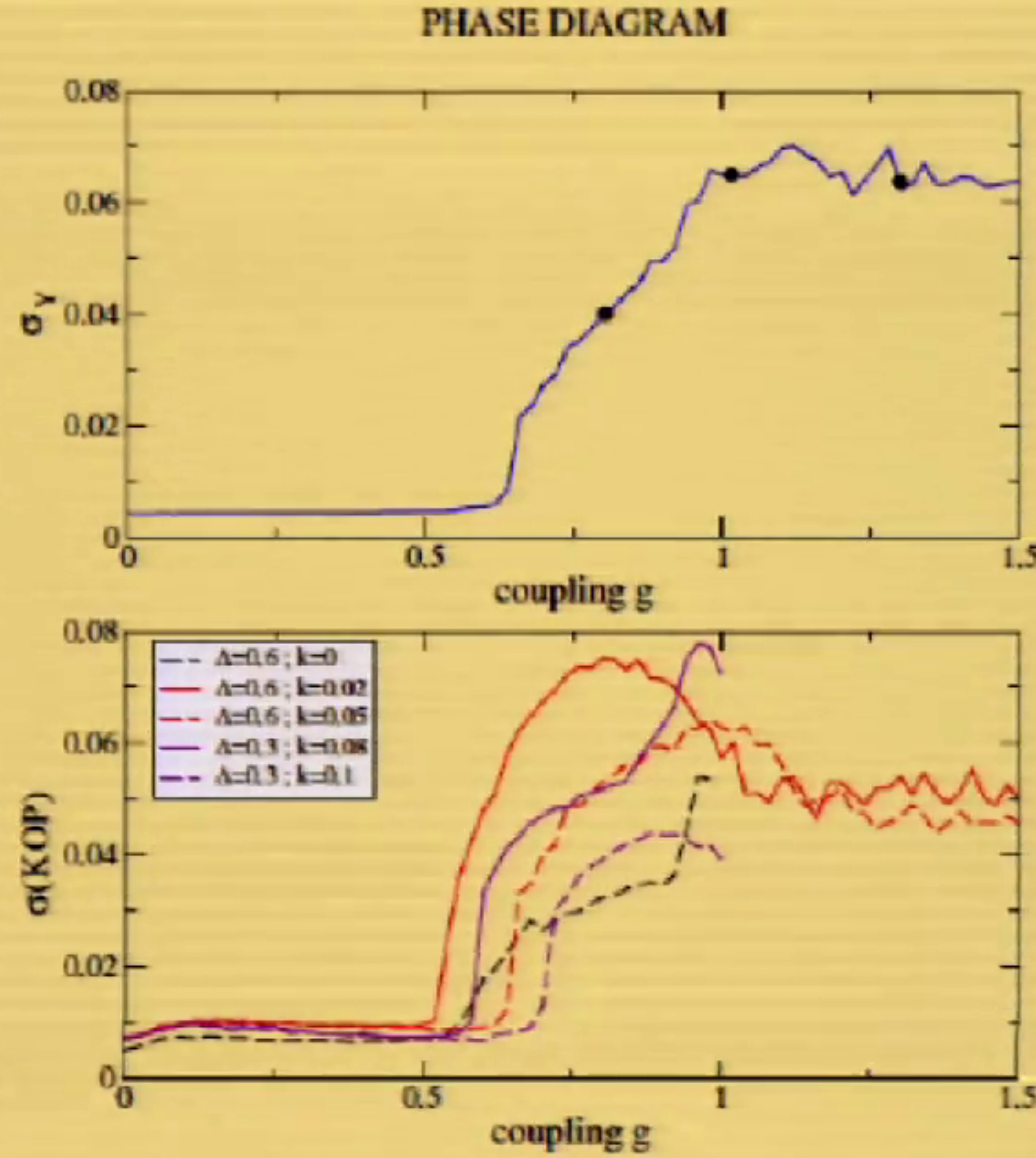
- Against adjustments of the phase response curve
- Against a vertical shift of the phase response curve (to minimize the net effect of the coupling on the effective frequency)
- Against removal of delay (replaced by a rotation of the PRC)
- Against removal of the discontinuity in the PRC

A case study

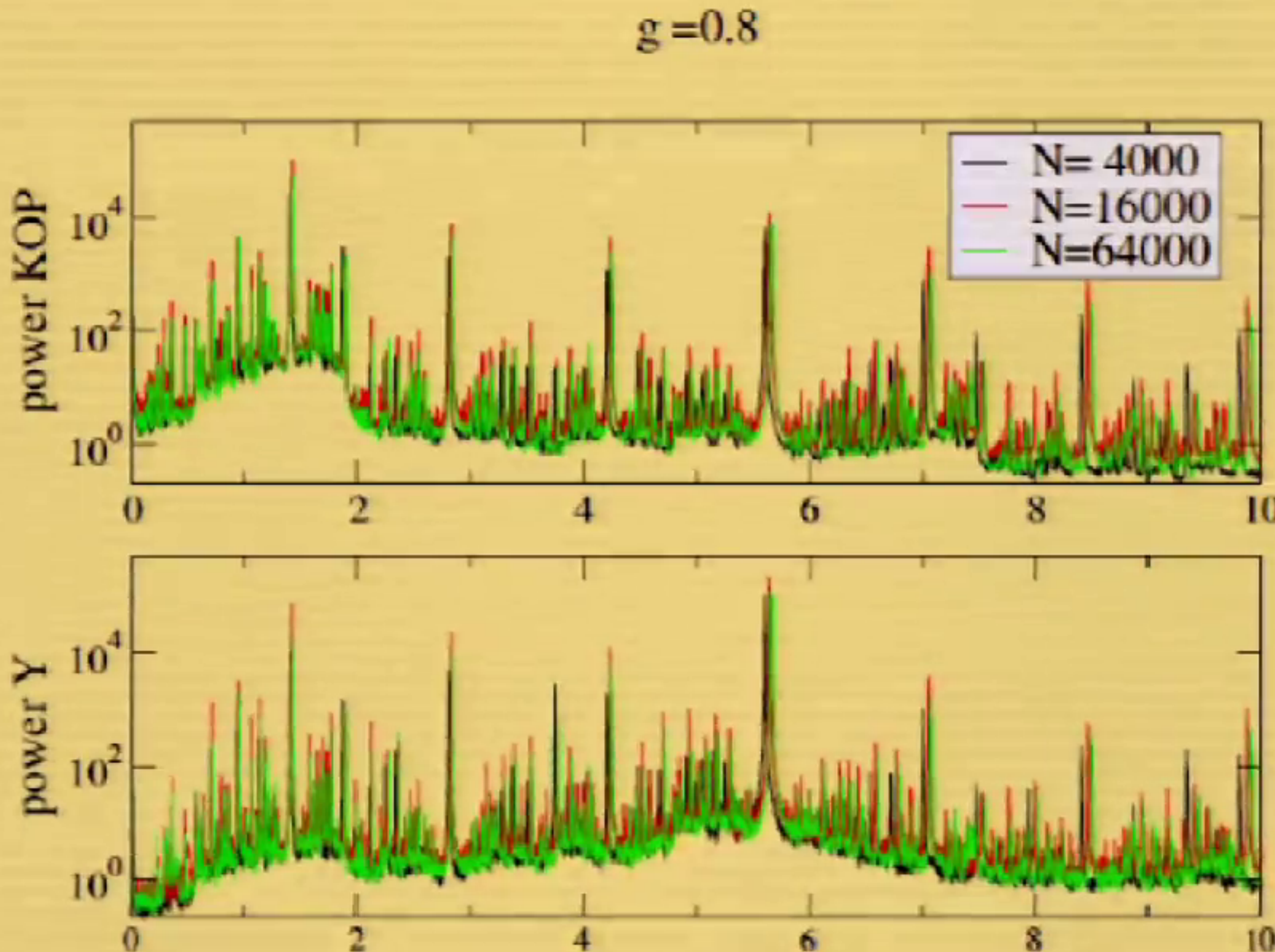
$$\dot{\phi}_i = \omega_i - \frac{g}{N}\Gamma(\phi_i)\sum_{n|t_n<t}\delta(t-t_n)$$



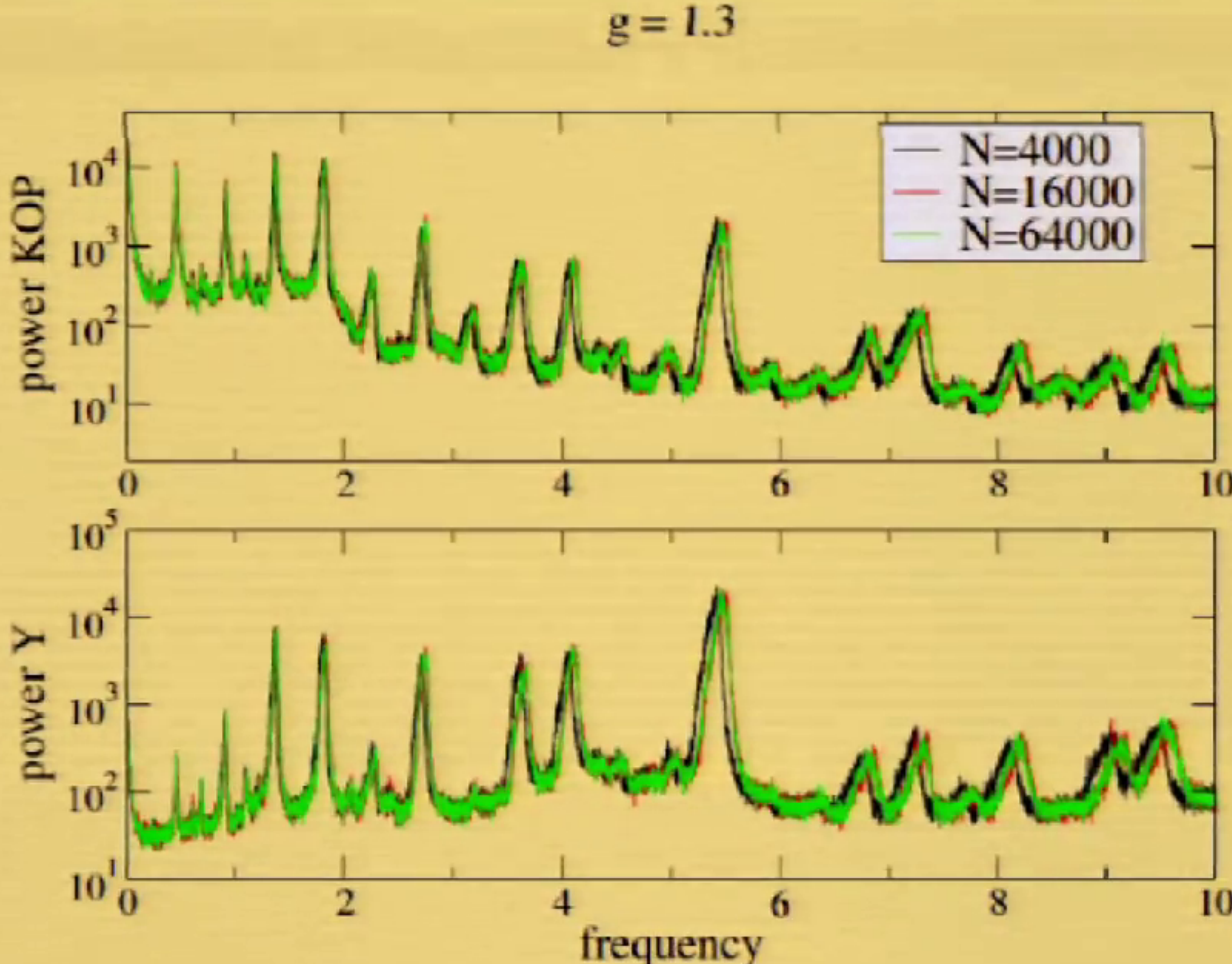
Phase diagrams



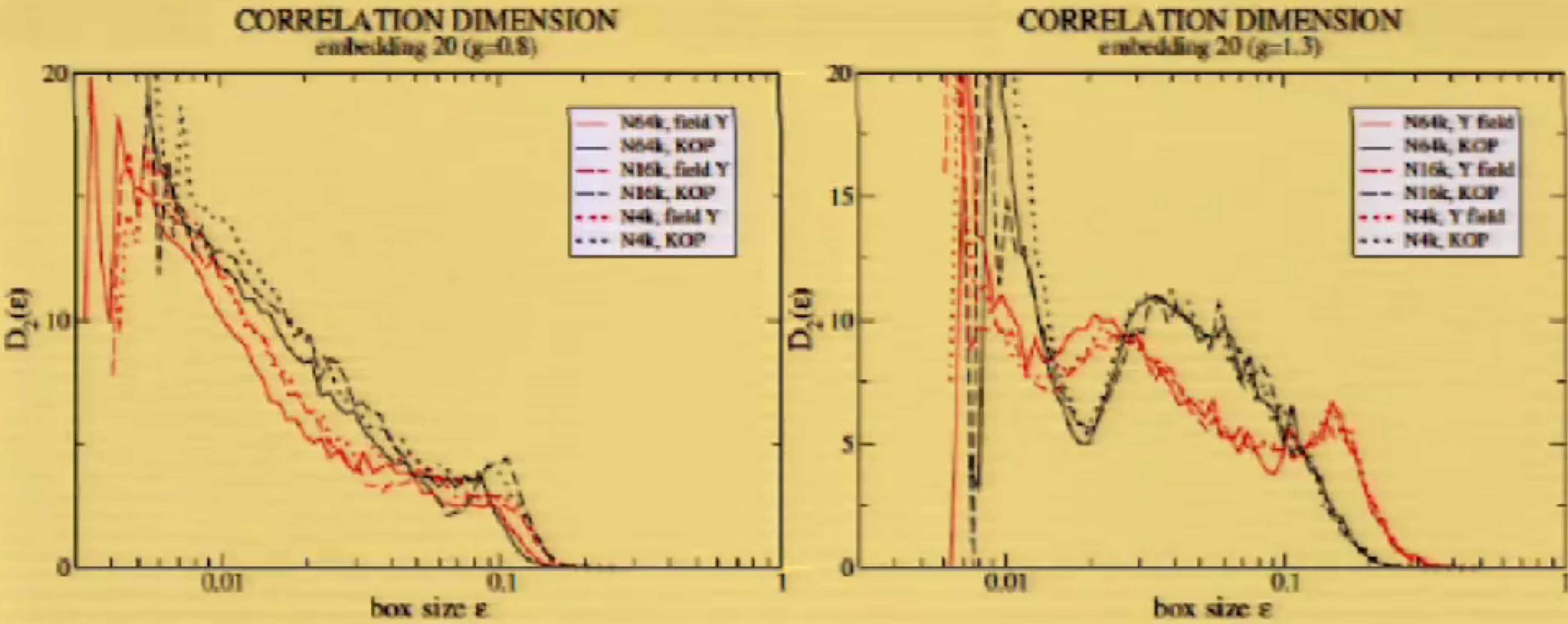
Power spectrum of the order parameters



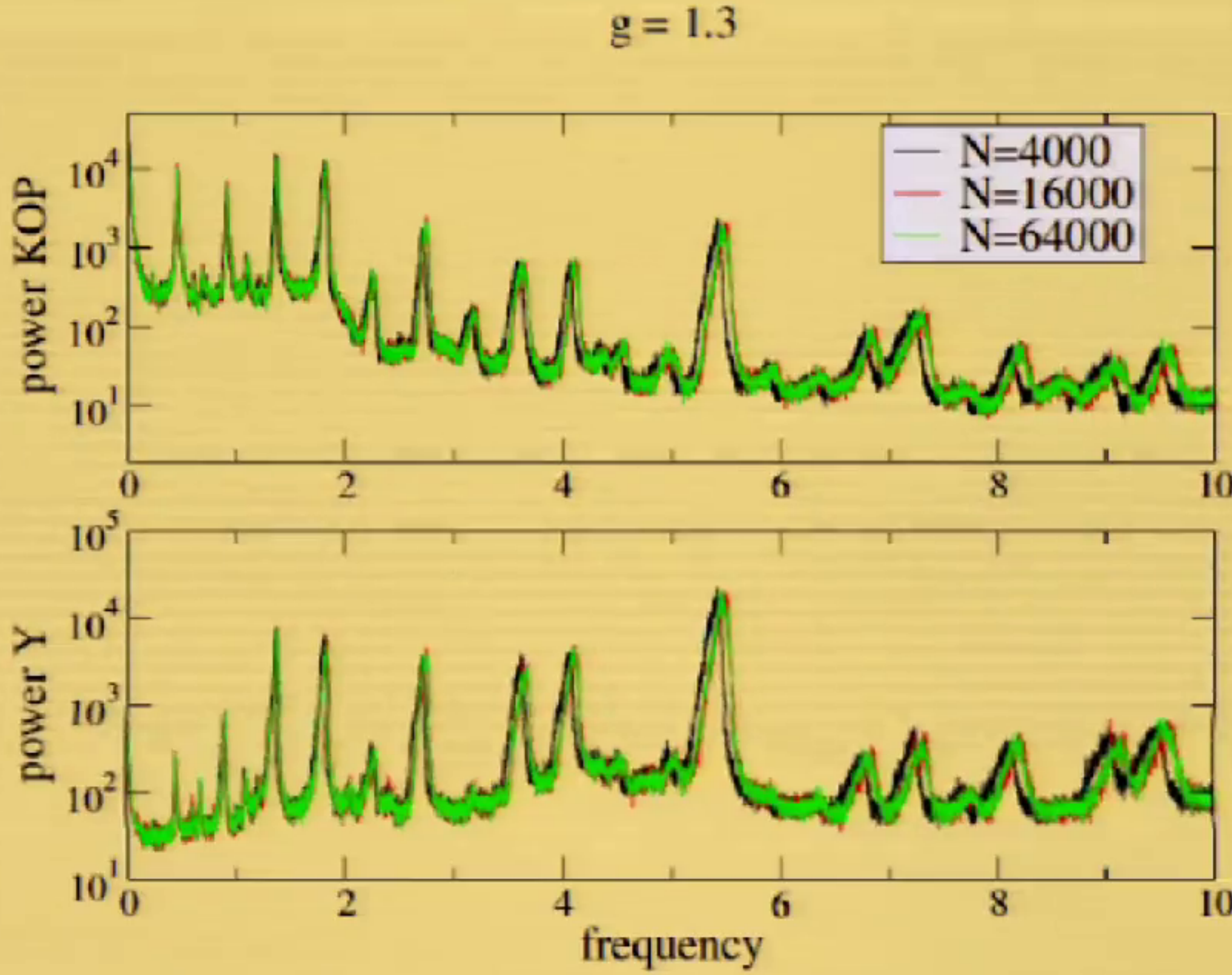
Power spectrum of the order parameters



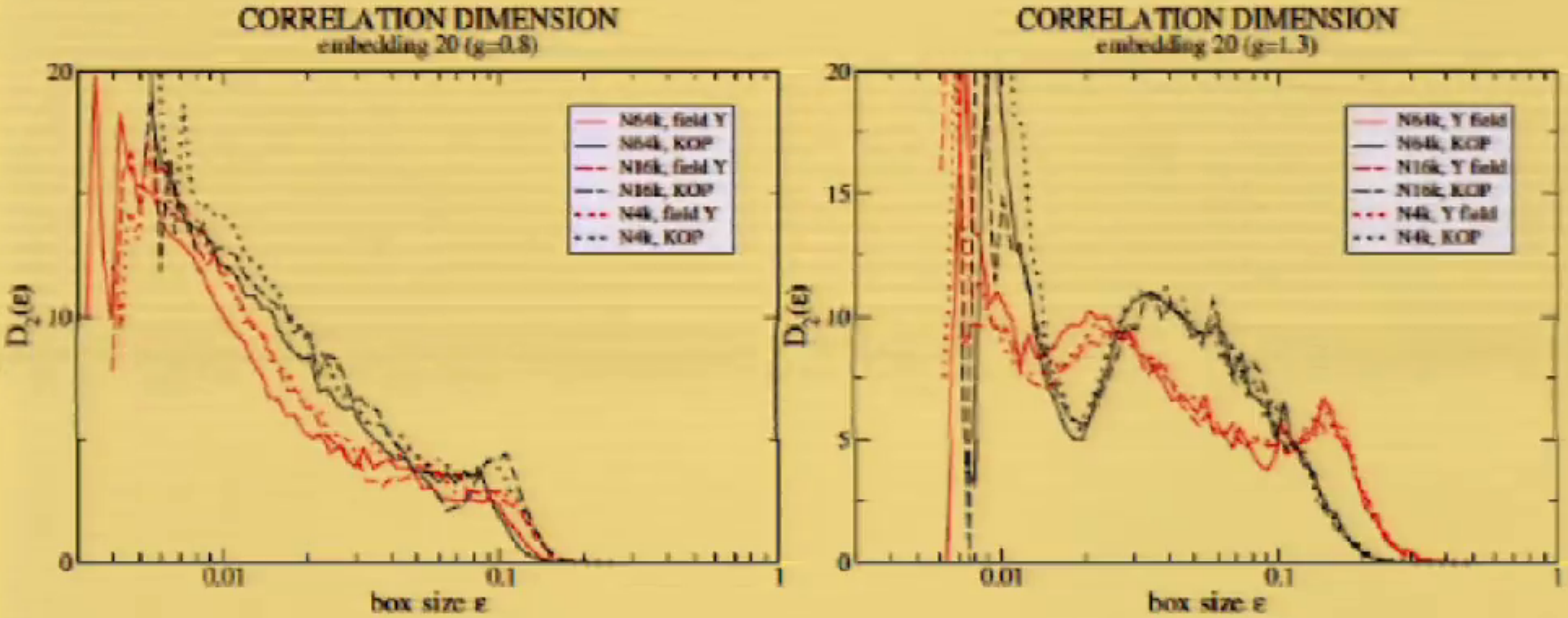
Fractal dimension of the order parameter



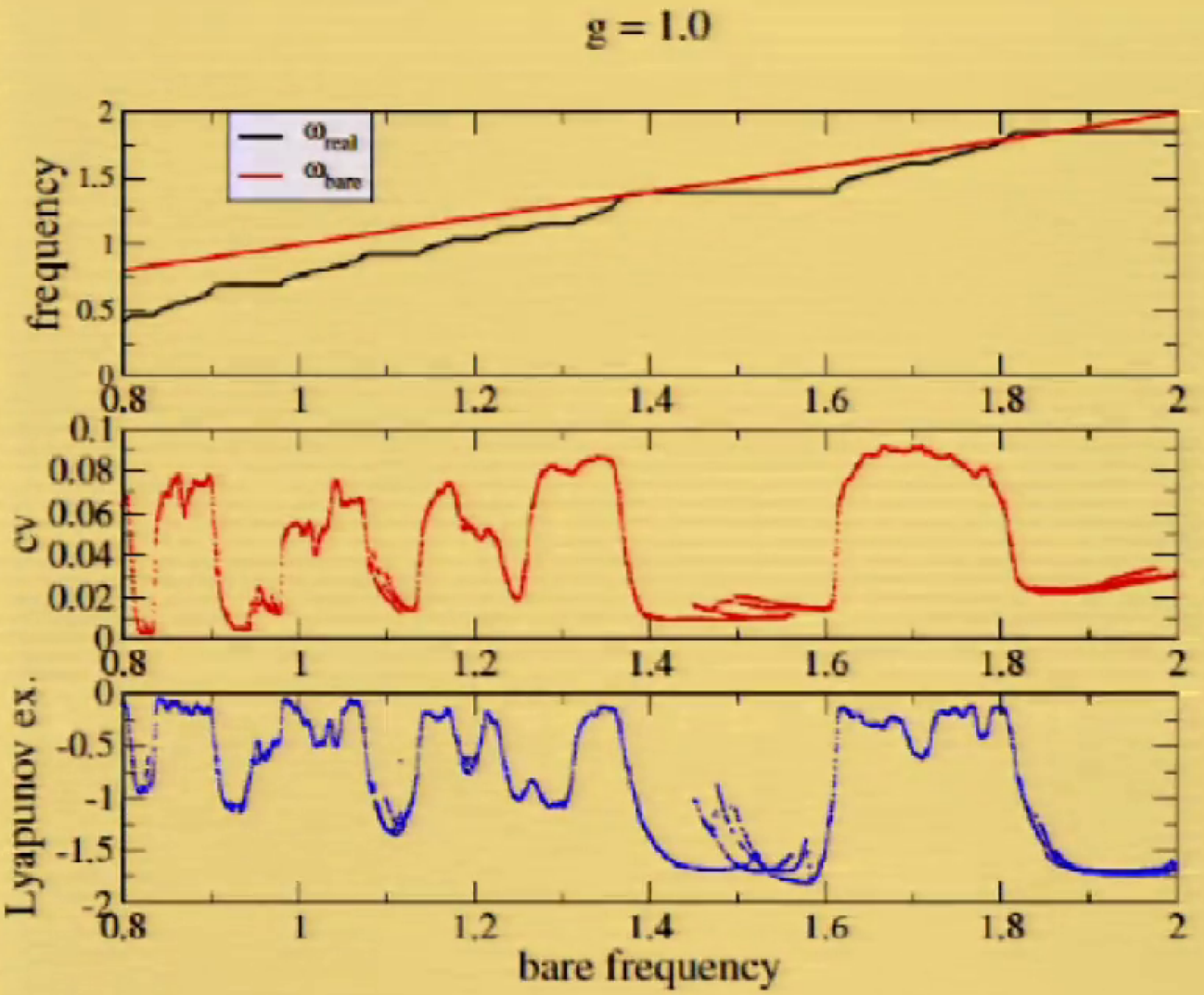
Power spectrum of the order parameters



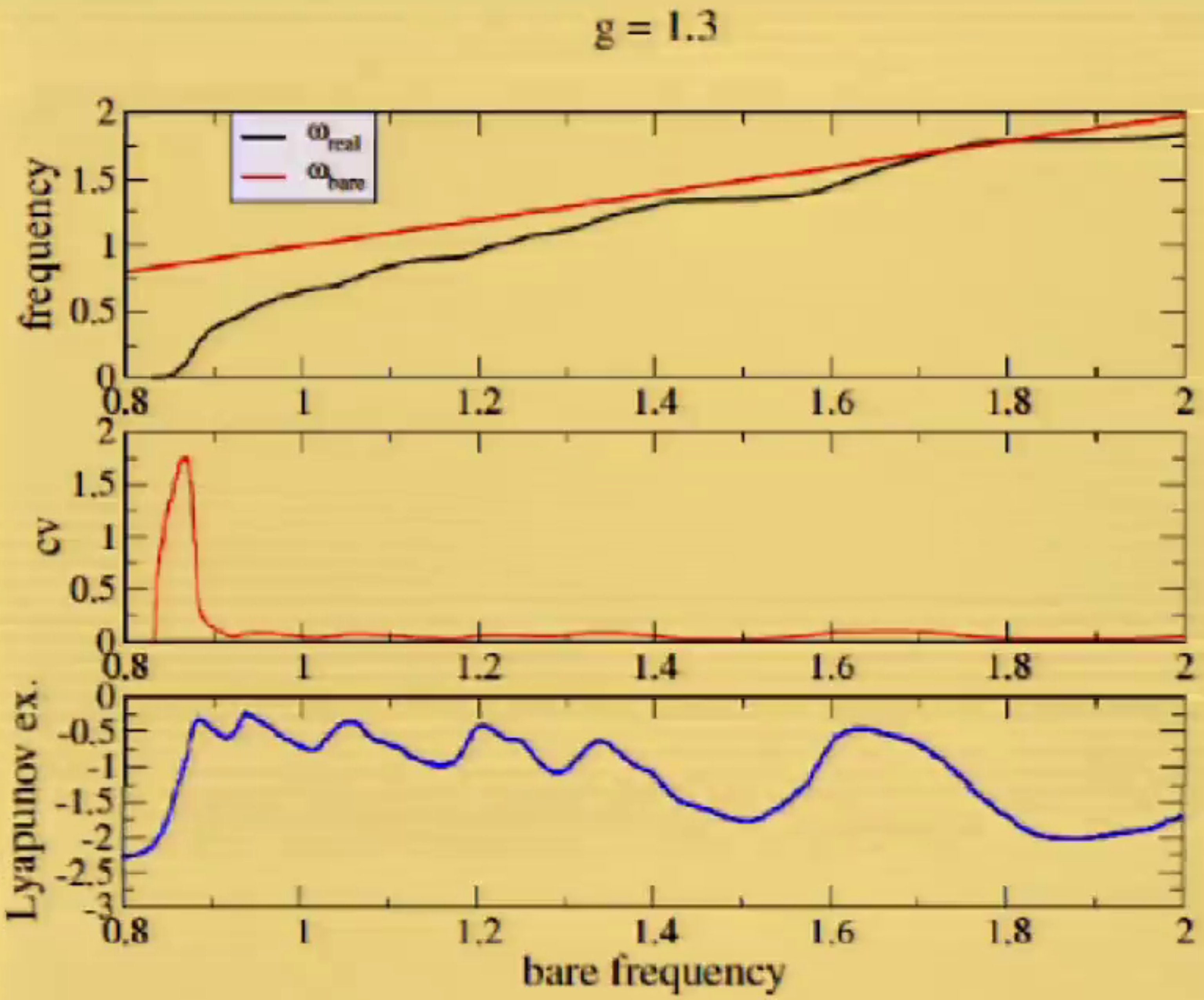
Fractal dimension of the order parameter



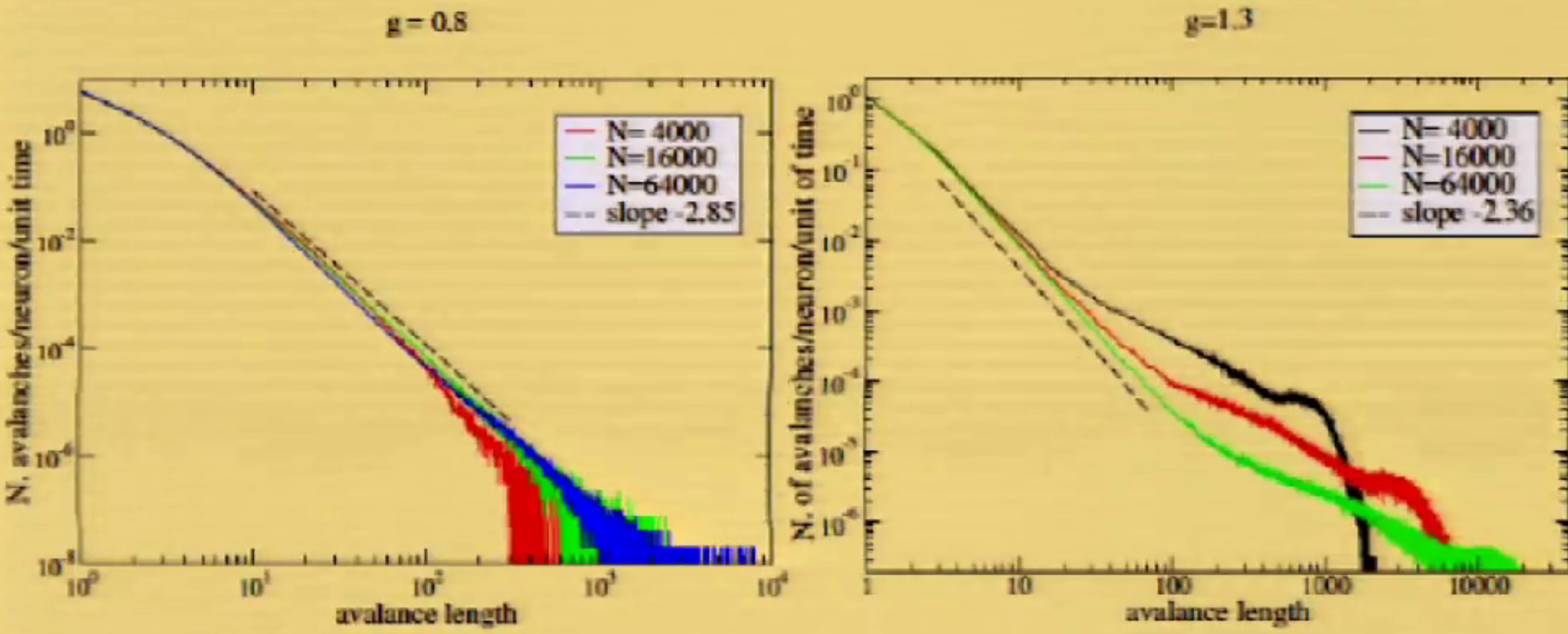
Characterization of the microscopic dynamics



Characterization of the microscopic dynamics

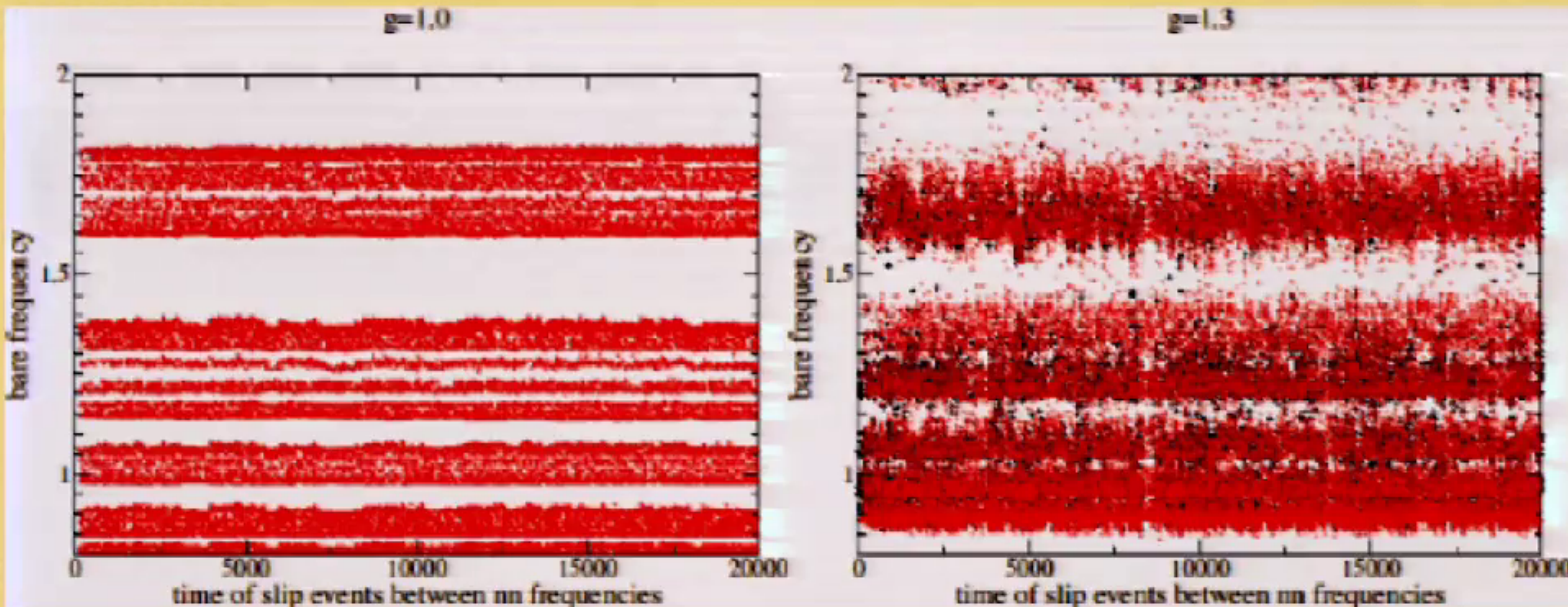


Avalanches



g = 0.8
N= 4000
N=16000
N=64000
slope -2.85
N. avalanches/neuron/unit time
avalance length
g=1.3
N= 4000
N=16000
N=64000
slope -2.36
N. of avalanches/neuron/unit of time
avalance length

Phase slips



g=1.0
g=1.3
bare frequency
time of slip events between nn frequencies
2
1.5
1
0
5000
10000
15000
20000

Snapshots

$g = 1.3$