Assessing Critical Mass at UC-Berkeley

Creating Predictive Models for Racial Affirmative Action Policies in U.S. Undergraduate Admissions

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Outline

- Background, Contributions, and Motivation
- Quantifying Critical Mass
- Markov Chain Modeling
- Results, Conclusions, and Next Steps

Background, Contributions, and Motivaion

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- A term called *critical mass* is used to meet this criterion
- Critical mass has not been defined in its legal use

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- Provide a quantitative framework for critical mass in the context of affirmative action assessment and litigation
- Create predictive models for university demographics over time using Markov Chains
- Assess a current affirmative action policy according to critical mass projections and the Markov Chain model's results

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- We can understand a current policy's future impact rather than waiting 5-10 years to see how it plays out
- This new approach can save a lot of time, money, and energy for universities and litigators

Quantifying Critical Mass

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- State of California Department of Finance and U.S. Census Bureau both provide racial demographic projections for their respective domains from 2016-2060
- University of California Undergraduate Admissions Summaries also provide data on state residency by race/ethnicity and year
- We will use the CA and U.S. projections provided, but will use a stochastic process to make our own projections for the UC-Berkeley in- vs. out-of-state breakdown

Ideal Critical Mass Projections (Null Model)

Then, our critical mass projection, CM_{ij} for year i and group j is:

$$CM_{ij} = CA_{ij} \times UCB_{i,\text{in-state}} + US_{ij} \times UCB_{i,\text{out-of-state}}$$

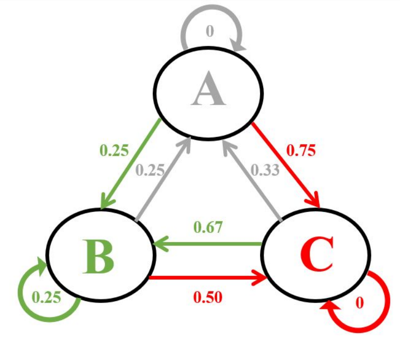
Critical Mass Projections from CA and US Data*

	Year				
Race/Ethnicity	2018	2019	2020	2021	2022
African-American	0.07152	0.07137	0.07130	0.07114	0.07105
Asian-American	0.12982	0.13055	0.13122	0.13196	0.13262
Hispanic/Latino	0.35844	0.36110	0.36336	0.36584	0.36807
White/Caucasian	0.44022	0.43699	0.43412	0.43105	0.42826

*These are means from 10,000 iterations of the stochastic process

Markov Chain Modeling

Markov Chain Example



$$
\begin{array}{c} \\ A \\ B \\ C \end{array}
\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & 0.25 & 0.75 \\ 0.25 & 0.25 & 0.50 \\ 0.33 & 0.67 & 0 \end{pmatrix} \end{array}
$$

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- A Markov Chain is *absorbing* if it contains at least one absorbing state and we can reach any absorbing state in finite steps

Model States

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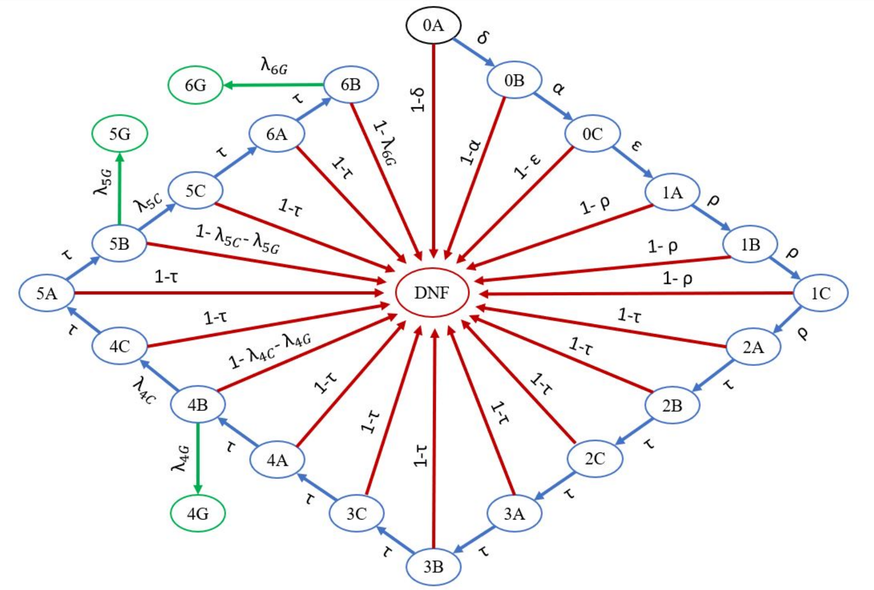
- Transient States:
  - High School Terms (0A, 0B, 0C)
  - College Terms (1A, 1B, 1C, 2A, 2B, 2C, ..., 5A, 5B, 5C, 6A, 6B)

Model Schematic



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- National Center for Education Statistics (NCES)
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  - Number of applicants, acceptances, and enrollment per year at UC-Berkeley by race/ethnicity
- UC-Berkeley's Office of Planning and Analysis
  - Overall freshman retention rate by incoming class
  - Overall 4-, 5-, and 6-year graduation rates by incoming class

Data Prediction for 2018 - 2027

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Markov Chain Model Predictions

We run our model for 2018 - 2027 to see our predicted results

These are percentages of the predicted enrollment for a given year

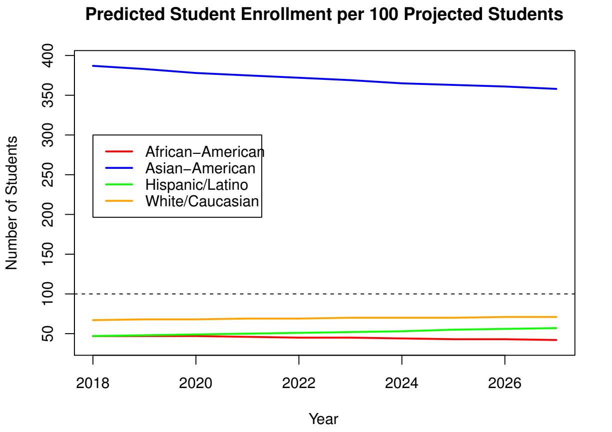
Critical Mass Predictions from Markov Chain Model

	Year				
Race/Ethnicity	2018	2019	2020	2021	2022
African-American	0.03328	0.03274	0.03241	0.03187	0.03135
Asian-American	0.49584	0.49437	0.49307	0.49163	0.49017
Hispanic/Latino	0.17559	0.18057	0.18549	0.19064	0.19591
White/Caucasian	0.29528	0.29233	0.28903	0.28586	0.28257

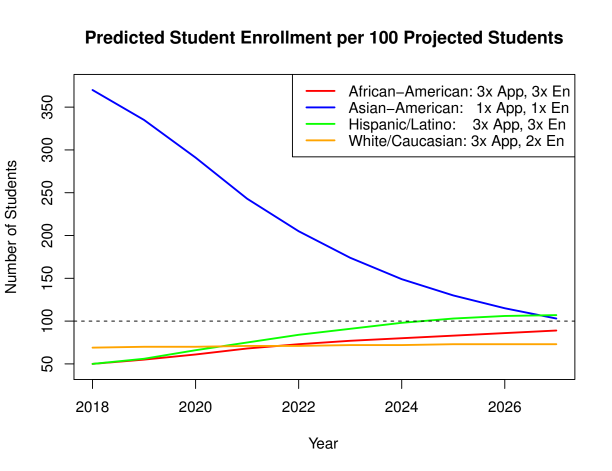
Results

Assessing the Predicted Performance

Assessment Metric: $\left(\frac{\text{Markov Chain Predictions}}{\text{Critical Mass Projections}}\right) \times 100$



Optimizing the 10-Year Overall Demographics



Conclusions and Future Work

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- Examples of strategies that could help:
  - More admissions recruiting events for groups that are underrepresented
  - Provide better on campus resources to increase enrollment yield for underrepresented groups

Future Work

- Apply this predictive modeling technique to:
  - Another university with available data
  - Another context (i.e. gendered affirmative action or employment)

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- Find a faster algorithm for optimizing results over the next 10 years
- Develop another (better) assessment criterion

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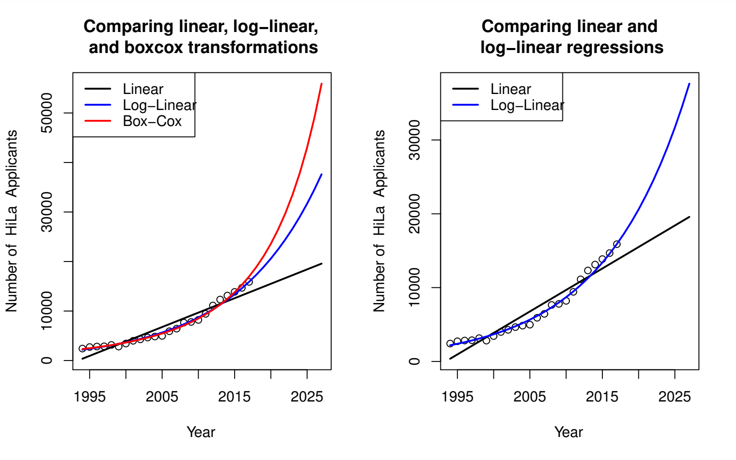
- Chad Topaz (Advising/Professional Support)
- Williams College Department of Mathematics & Statistics (Undergraduate Support)
- Mellon Mays Undergraduate Fellowship (Research Funding)
- SIAM (Student Travel Award)
- University of Michigan, Rackham Graduate School (Travel Funding and Graduate Support)

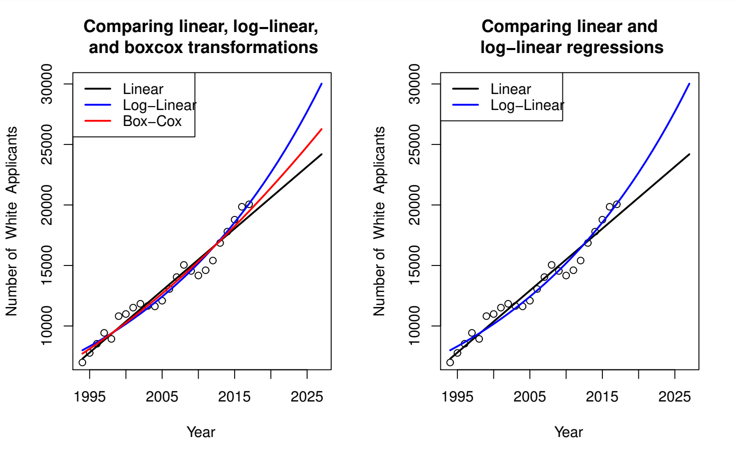
Questions?

Supplementary Material

Log-Linear Example 1: Lower Est. than Box-Cox



Log-Linear Example 2: Higher Est. than Box-Cox



Crtical Mass Quantification Process

- Find the kernel density for our observed data
- Draw a sample from that density
- Add that sample as an observed data point
- Repeat the process for as many years as we want to predict (10)

Initial Conditions for Individual Markov Chains

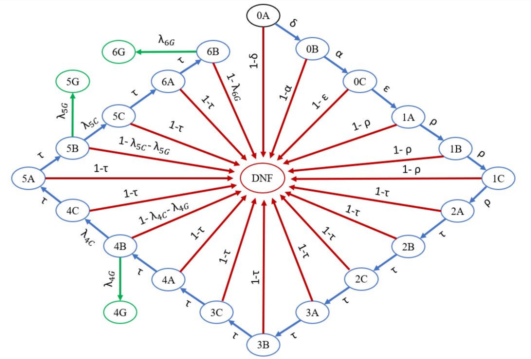
- We will have a starting vector $\mathbf{u}$ of initial conditions for our model
- Since we only want to introduce students in our model to start from the beginning of their senior year in high school ($0A$), our vector will have the form

$$\mathbf{u}_{ij} = \begin{bmatrix} n_{ij} & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix}$$

- To find this, for a given year we take the overall number of graduates and multiply it by the graduation demography
  - Let N_i be the overall number of high school graduates for year i
  - Let p_{ij} be the percentage makeup of graduates for group j in year i
  - Then $n_{ij} = N_i \times p_{ij}$ is the n_{ij} we want for $\mathbf{u}_{ij}$

Deriving Transition Rates: τ

We have 4-, 5-, and 6-year graduation rates and will call them $\gamma_4, \gamma_5,$ and , γ_6 respectively. Derive the ***polynomial of transition***: $\mathrm{T}(x) = \nu x^{13} - \gamma_4(x^6 - x^3) - \gamma_5(x^3 - 1) - \gamma_6$ to find our basic transition rate, τ, which is the root of $\mathrm{T}(x) \in (0, 1]$.



Deriving the Polynomial of Transition

We know that going from $1A \rightarrow 2A = \nu$, the first-year retention rate.

We would like to find the basic transition rate, that is, the rate that takes an individual starting in $1A$ all the way to $6B$.

We would like to know this since $1A \rightarrow 6B$ is the longest time one can stay in the model without being forced to go to an absorbing state, and we don't have explicit data on when students drop out.

We know that there are seven time steps between $2A$ and $4B$, so we have that $1A \rightarrow 4B = \nu x^7$, where x is some number.

We then take into account the loss of those graduating in 4 years, γ_4, to get $1A \rightarrow 4C = (\nu x^7 - \gamma_4)x$.

Deriving the Polynomial of Transition, Continued

Then after progressing through two more steps to $5B$, we get that $1A \to 5B = (\nu x^7 - \gamma_4)x^3$.

We find $1A \to 5C$ in a similar process to finding $1A \to 4C$ to get that $1A \to 5C = [(\nu x^7 - \gamma_4)x^3 - (\gamma_5 - \gamma_4)]x$.

Then, progressing two more steps to $6B$, we get that $1A \to 6B = [(\nu x^7 - \gamma_4)x^3 - (\gamma_5 - \gamma_4)]x^3$.

Lastly, we can multiply this out and simplify it to get the polynomial of transition: $\mathrm{T}(x) = \nu x^{13} - \gamma_4(x^6 - x^3) - \gamma_5(x^3 - 1) - \gamma_6$.

Proof of Root in $(0, 1]$ for Polynomial of Transition

We know that $\mathrm{T}(x) = \nu x^{13} - \gamma_4(x^6 - x^3) - \gamma_5(x^3 - 1) - \gamma_6$

We know that $\nu, \gamma_4, \gamma_5, \gamma_6 \in [0, 1]$ and $\nu \geq \gamma_6 \geq \gamma_5 \geq \gamma_4$

$\mathrm{T}(0) = -\gamma_6$, since we have that $\gamma_6 \geq 0 \implies \mathrm{T}(0) \leq 0$

$\mathrm{T}(1) = \nu - \gamma_6$, since we have that $\nu \geq \gamma_6 \implies \mathrm{T}(1) \geq 0$

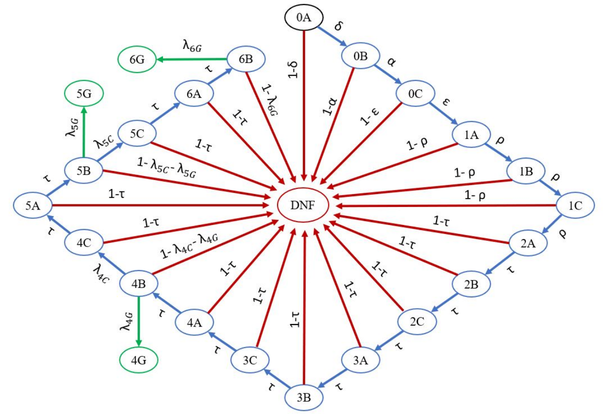
Since $\mathrm{T}(x)$ is just a polynomial, we know $\mathrm{T}(x)$ is continuous

Then, by the Intermediate Value Theorem, there is at least one root of $\mathrm{T}(x)$ which exists in $[0, 1]$ ■

Deriving Transition Rates: λ_{4G}

Conditional probability gives us that $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$4B \rightarrow 4G : \lambda_{4G} = P(4G|4B) = \frac{P(4G \cap 4B)}{P(4B)} = \frac{P(4G)}{P(4B)} = \frac{\gamma_4}{\nu\tau^7}$$



Deriving Transition Rates: λ_{4C}

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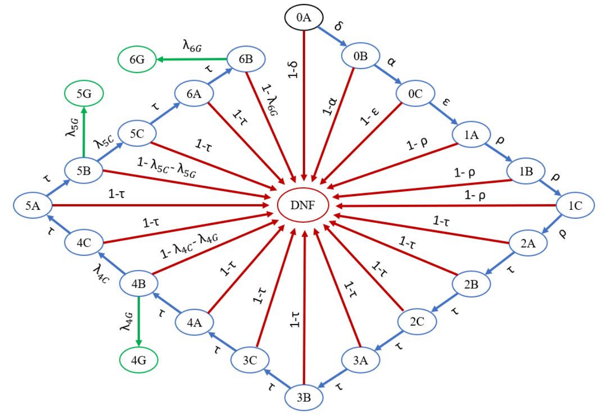
$$4B \to 4C : \lambda_{4C} = \frac{P(4C)}{P(4B)} = \frac{[P(4B) - P(4G)]\tau}{P(4B)} = \frac{(\nu\tau^7 - \gamma_4)\tau}{\nu\tau^7} = \frac{\nu\tau^7 - \gamma_4}{\nu\tau^6}$$



Deriving Transition Rates: $\lambda_{5G}, \lambda_{5C}, \lambda_{6G}$

$$5B \to 5G : \lambda_{5G} = \frac{P(5G)}{P(5B)} = \frac{\gamma_5 - \gamma_4}{(\nu\tau^7 - \gamma_4)\tau^3} = \frac{\gamma_5 - \gamma_4}{\nu\tau^{10} - \gamma_4\tau^3}$$

$$5B \to 5C : \lambda_{5C} = \frac{P(5C)}{P(5B)} = \frac{[P(5B) - P(5G)]\tau}{P(5B)} = \cdots = \frac{\nu\tau^{10} - \gamma_4(\tau^3 - 1) - \gamma_5}{\nu\tau^9 - \gamma_4\tau^2}$$

$$6B \to 6G \text{ is } \lambda_{6G} = \frac{P(6G)}{P(6B)} = \cdots = \frac{\gamma_6 - \gamma_5}{\nu\tau^{13} - \gamma_4(\tau^6 - \tau^3) - \gamma_5\tau^3}$$