Learning Invariants and Representation Spaces of Shapes and Forms

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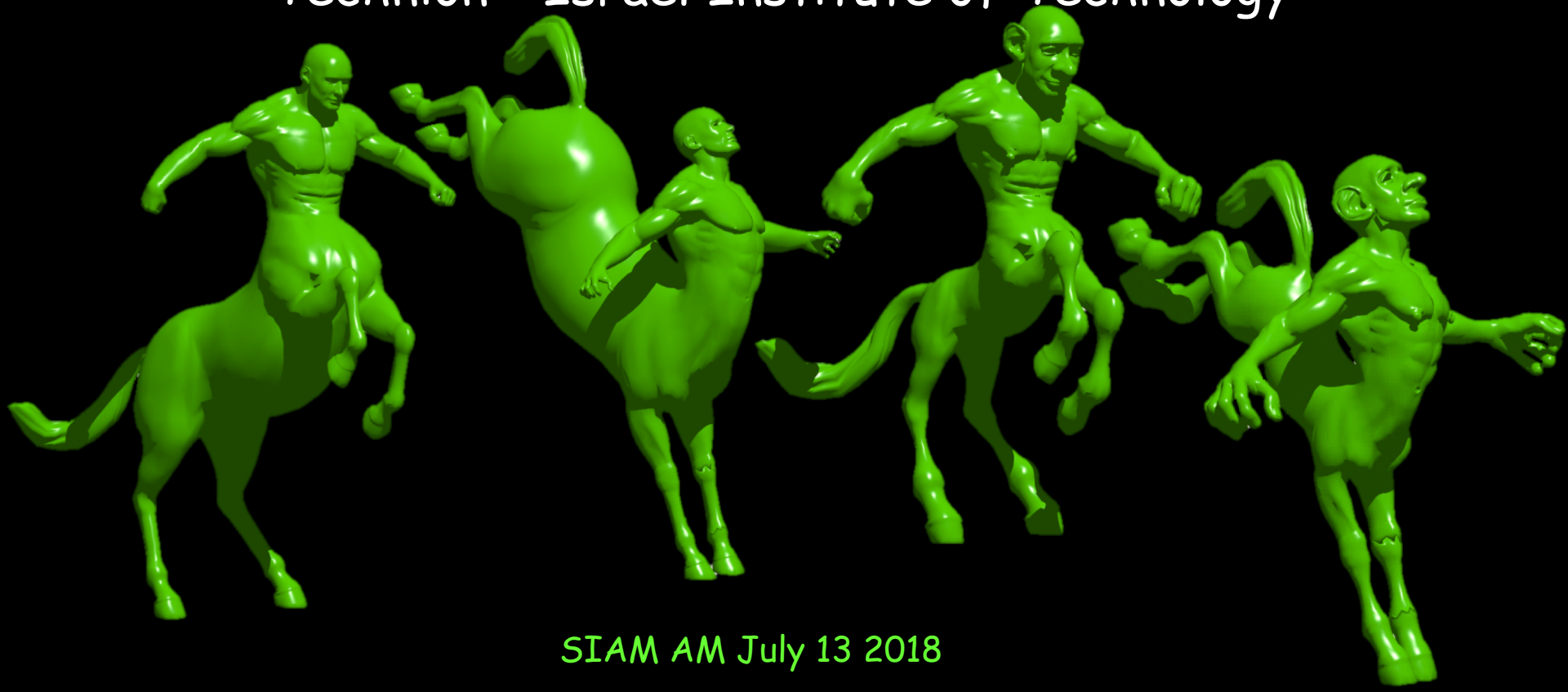
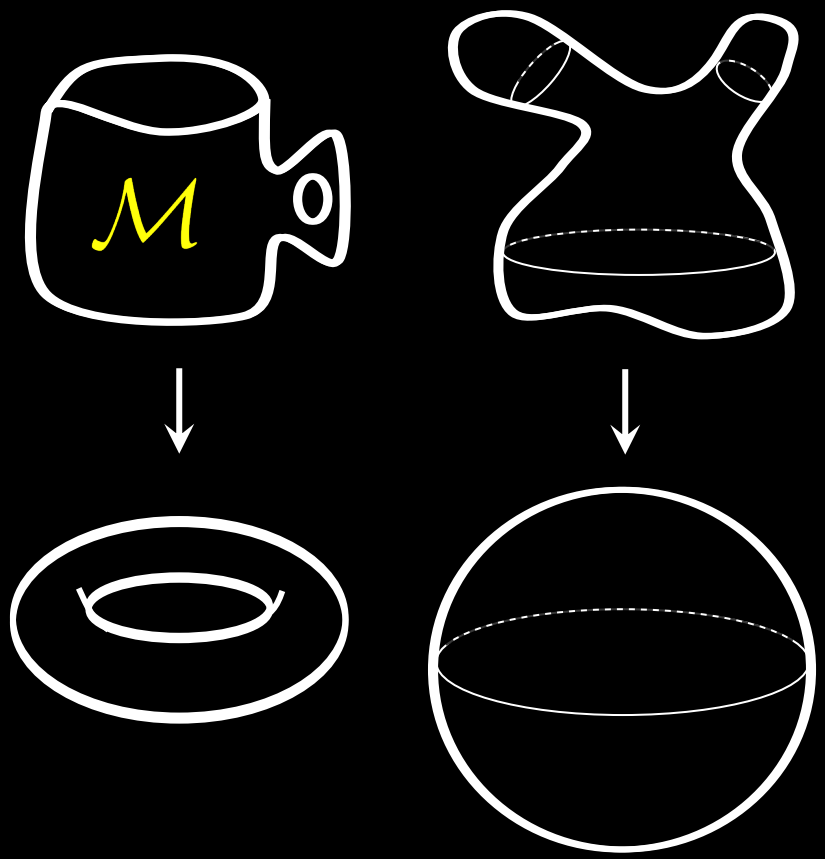
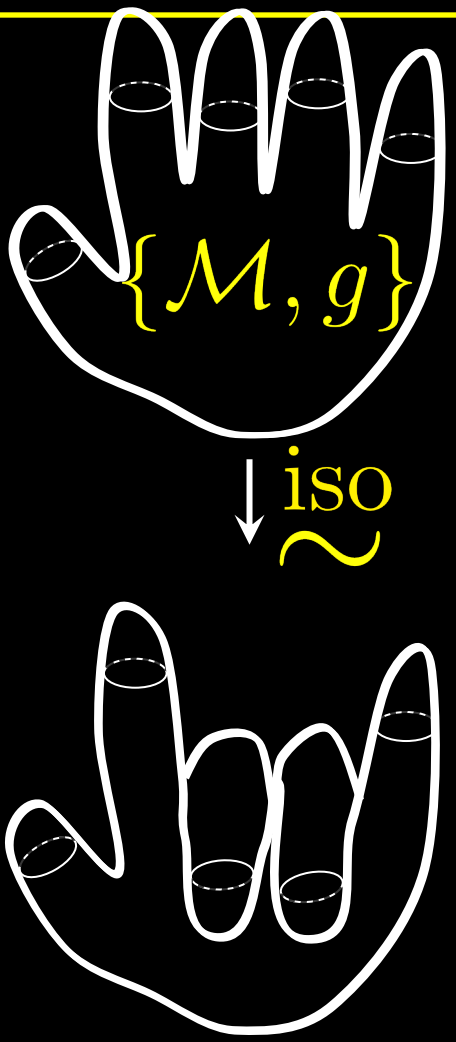


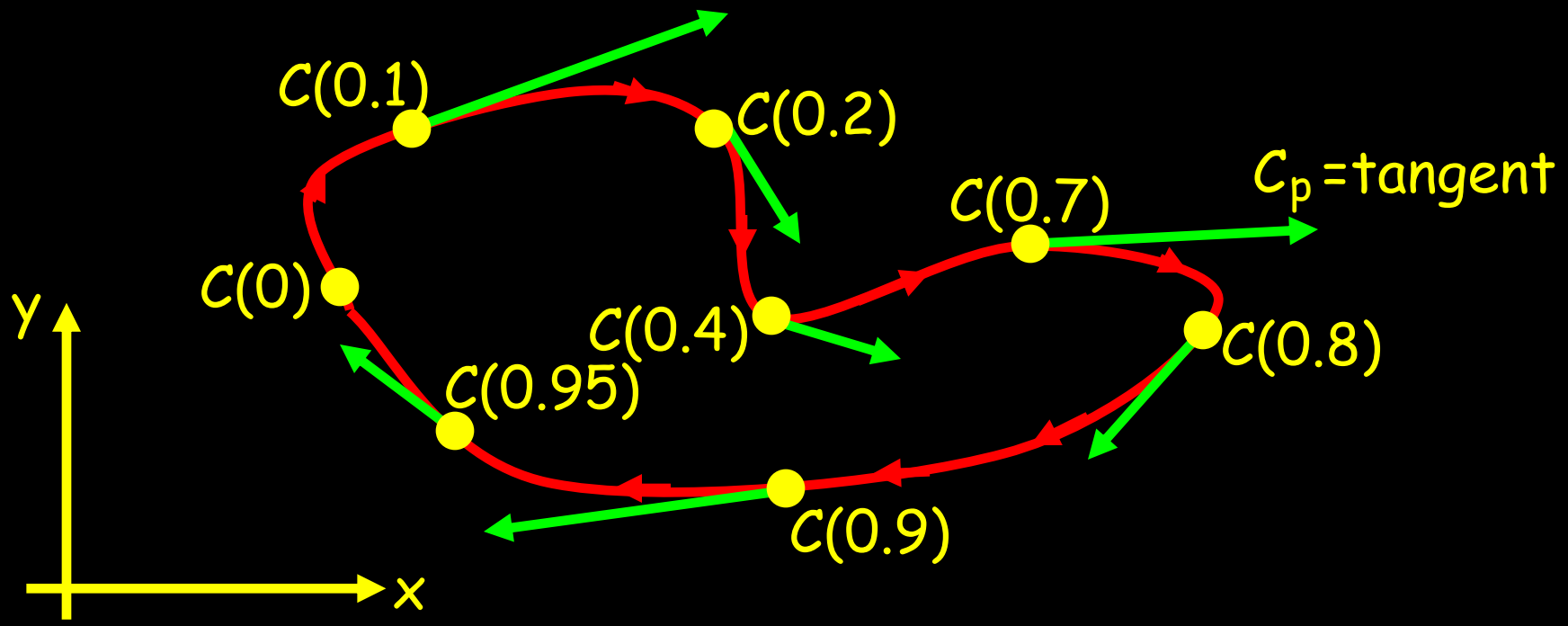
SIAM AM July 13 2018

Manifold vs. Riemannian Manifold





Planar Curves

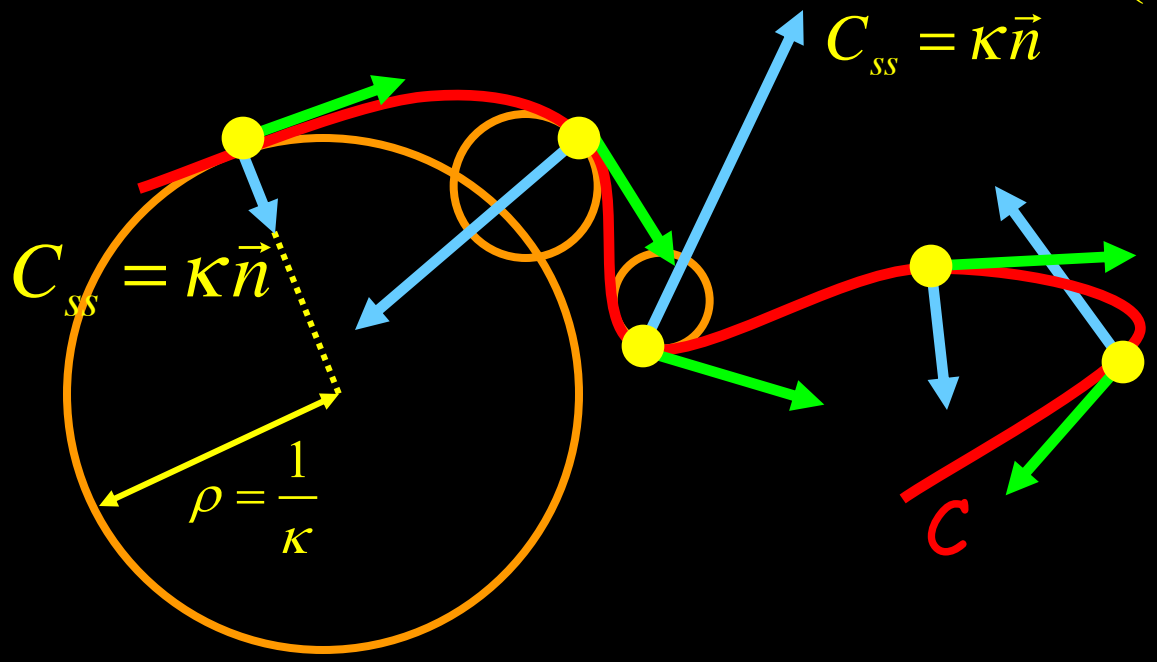


$$C(p) : [0,1] \in \mathrm{R}^1 \rightarrow \mathrm{R}^2$$

$$C(p) = \{x(p), y(p)\} \quad p \in [0,1]$$

Arc-length and Curvature

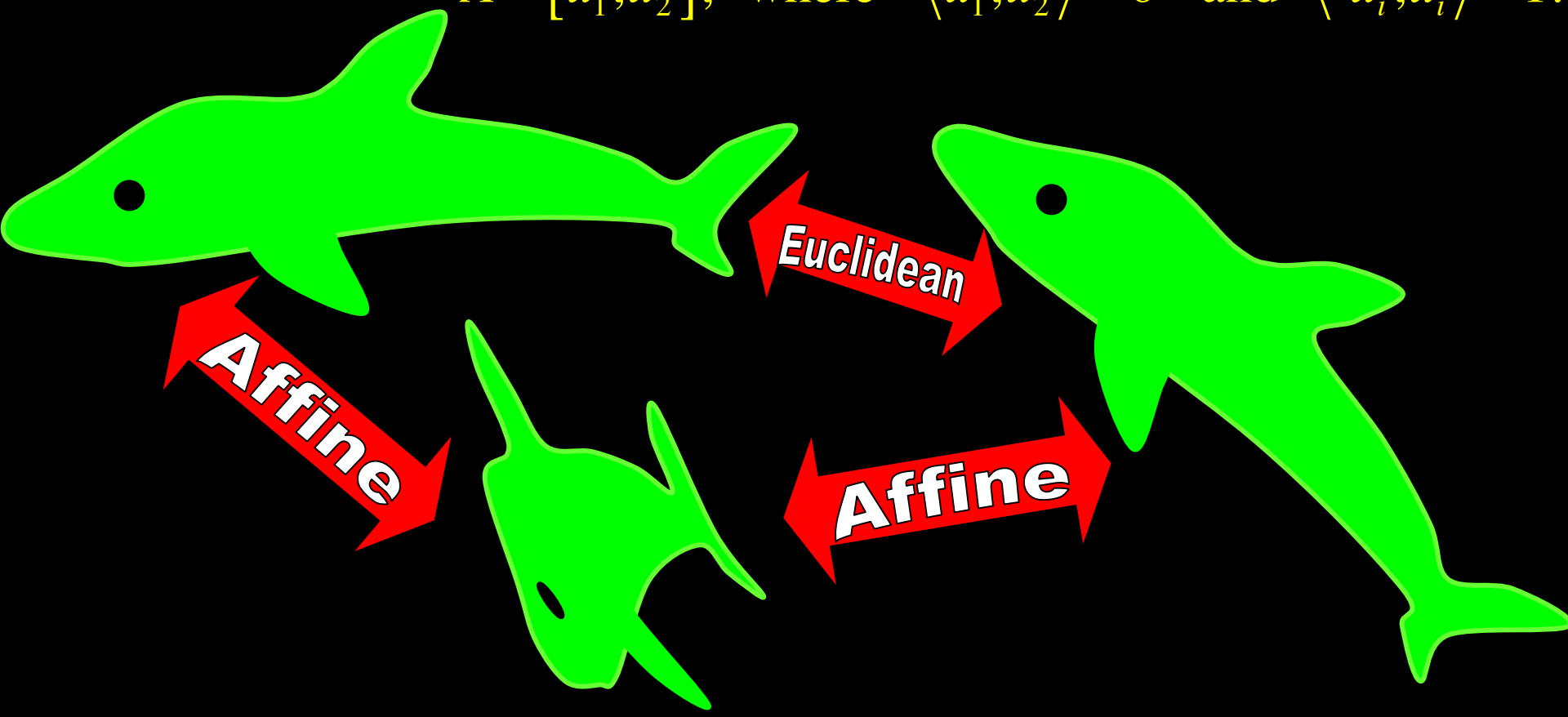
$$s(p) = \int_0^p |C_{\hat{p}}| d\hat{p} \quad \Rightarrow \quad |C_s| = 1, \qquad \left(\vec{t} = C_s = \frac{C_p}{|C_p|} \right)$$



Linear Transformations

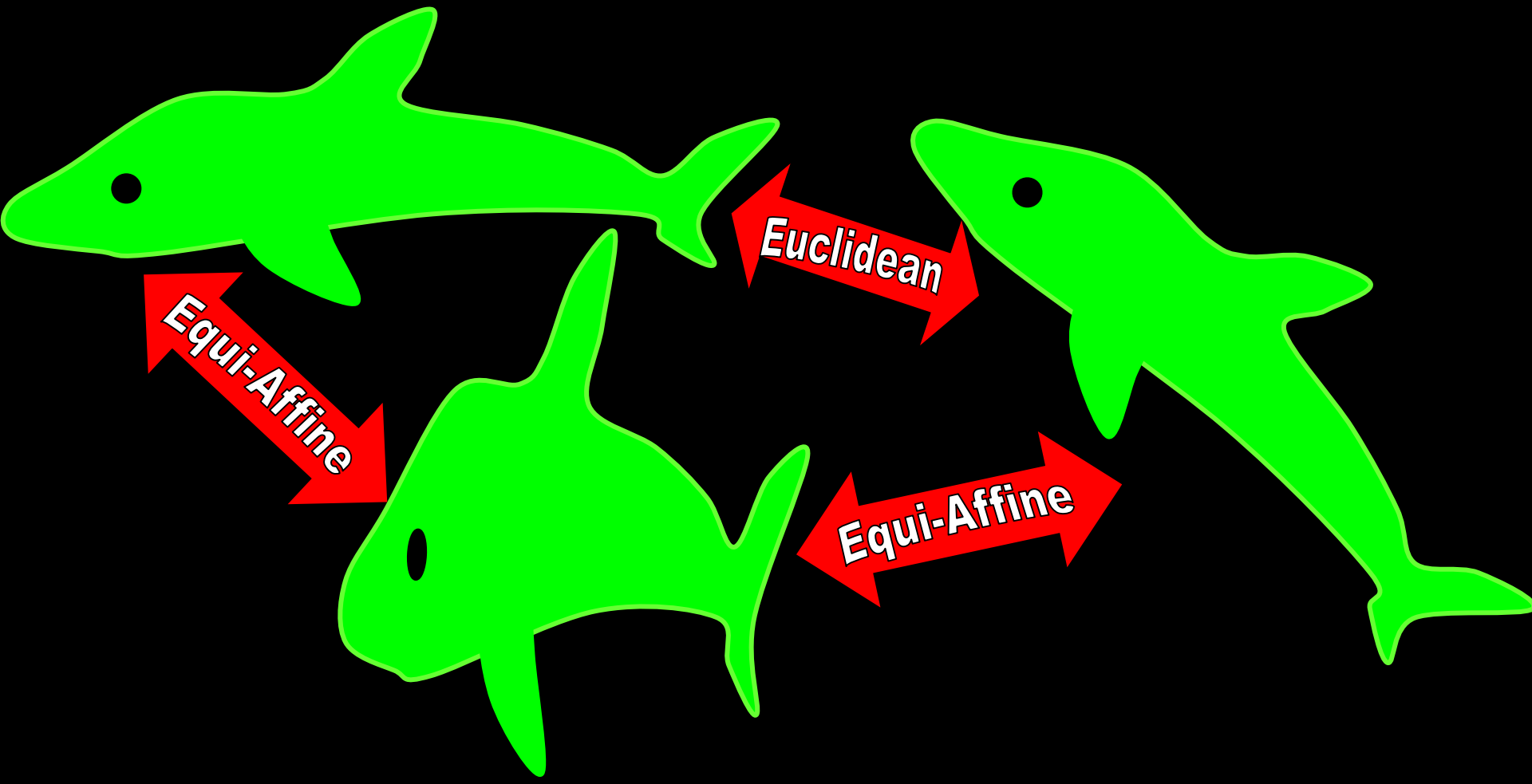
Affine: $\{\tilde{x},\tilde{y}\}^T = A\{x,y\}^T + \bar{b}$,

Euclidean: $A = [\bar{u}_1, \bar{u}_2]$, where $\langle \bar{u}_1, \bar{u}_2 \rangle = 0$ and $\langle \bar{u}_i, \bar{u}_i \rangle = 1$.



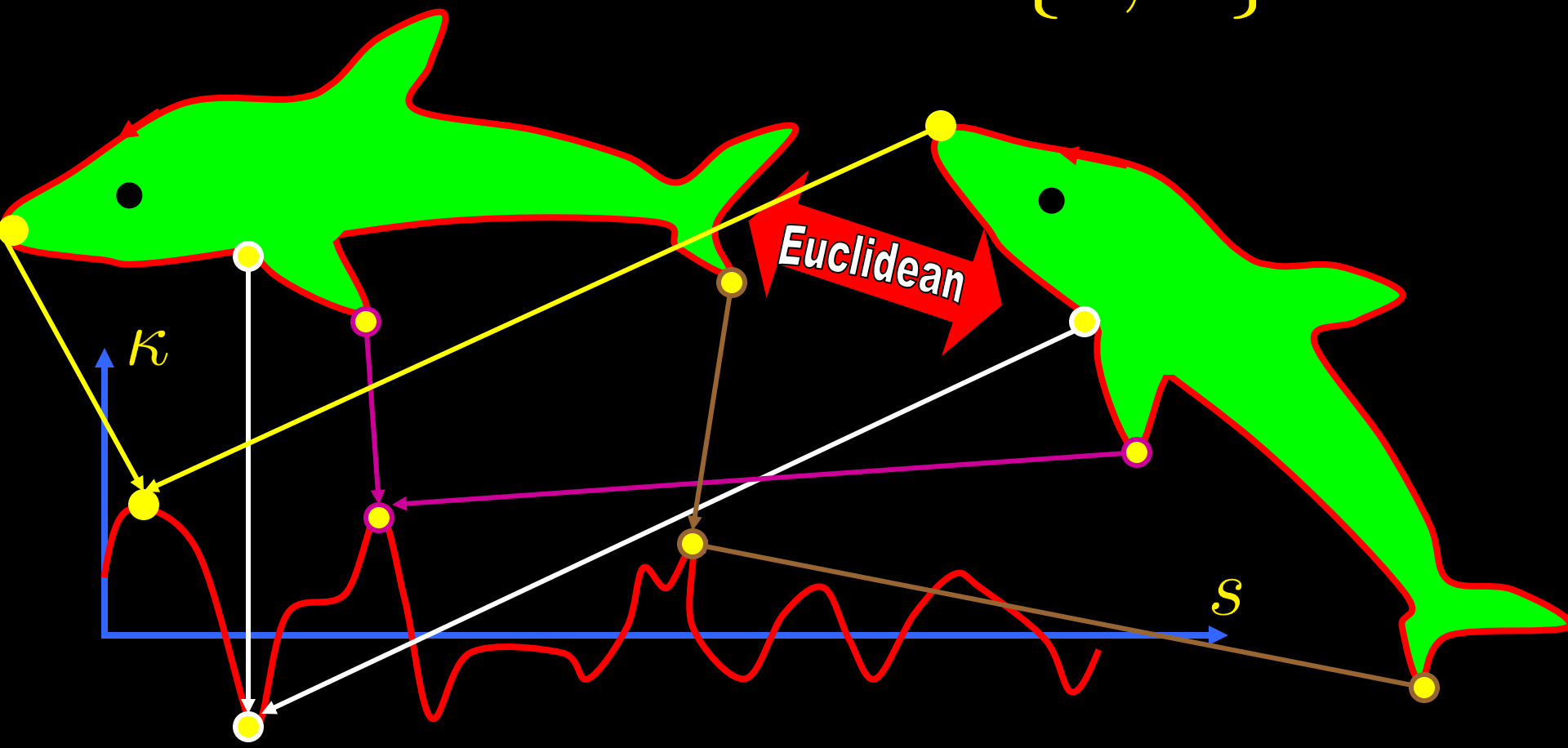
Linear Transformations

Equi-Affine: $\{\tilde{x},\tilde{y}\}^T = A\{x,y\}^T + \overline{b}, \quad \det(A) = 1.$



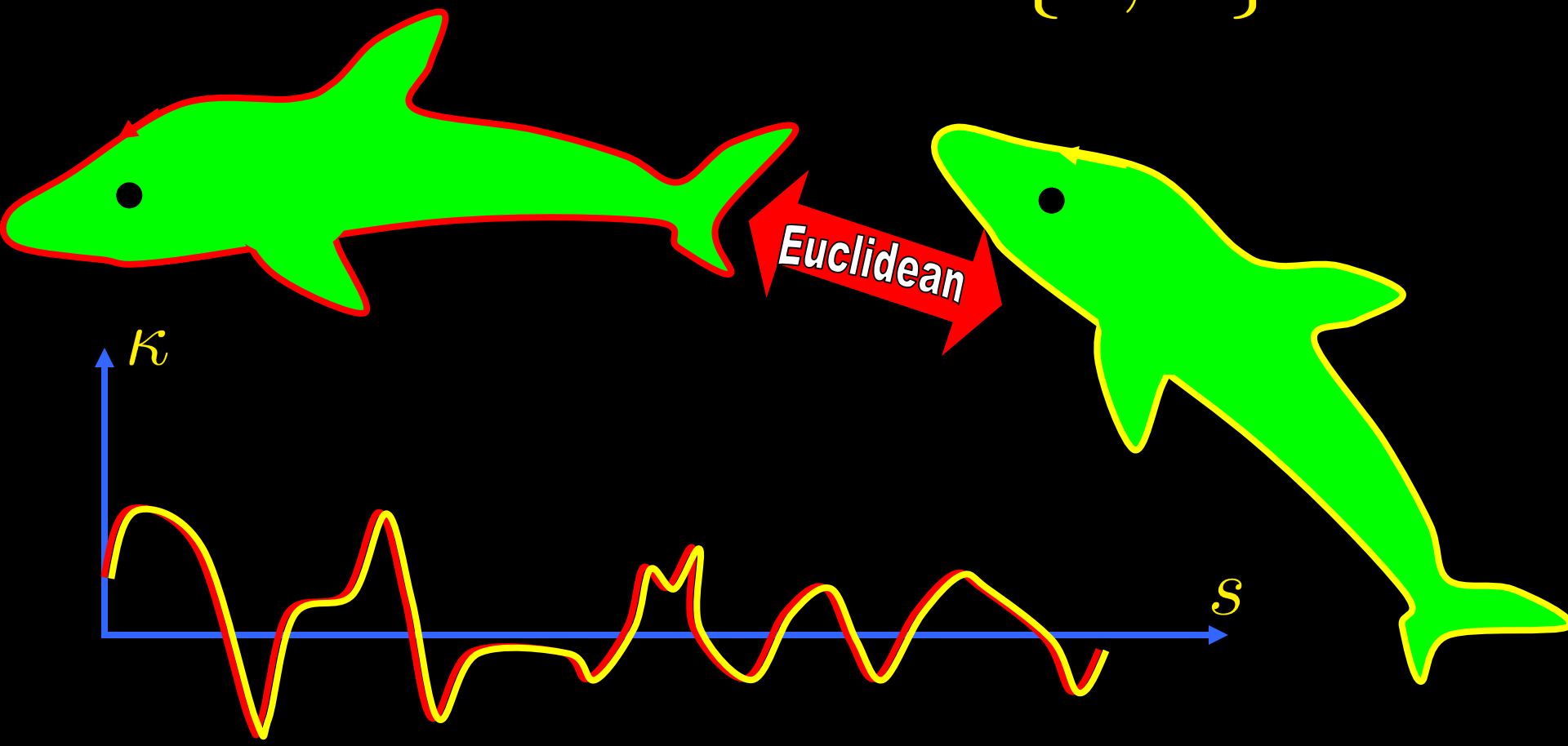
Differential Signatures

- Euclidean invariant signature $\{s, \kappa\}$

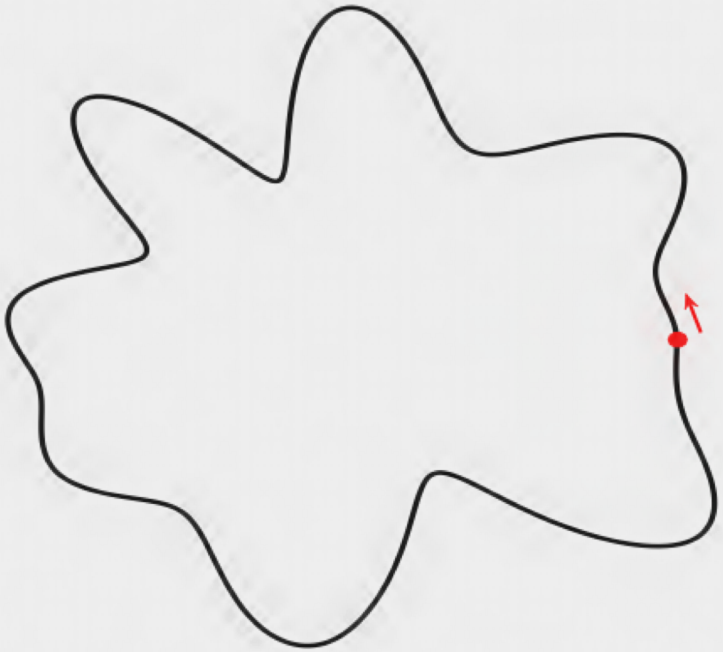


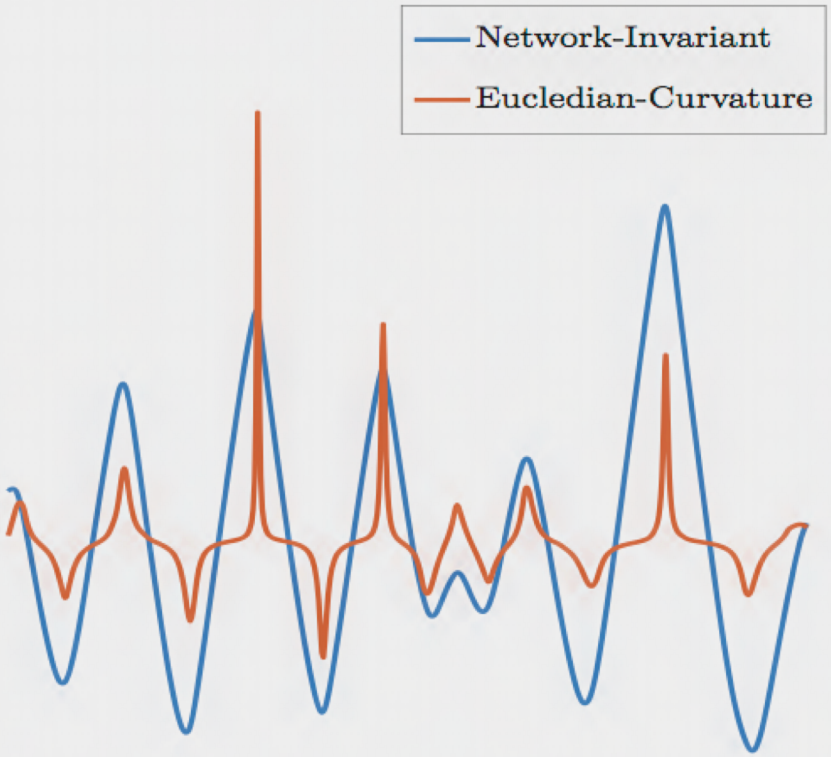
Differential Signatures

- Euclidean invariant signature $\{s, \kappa\}$



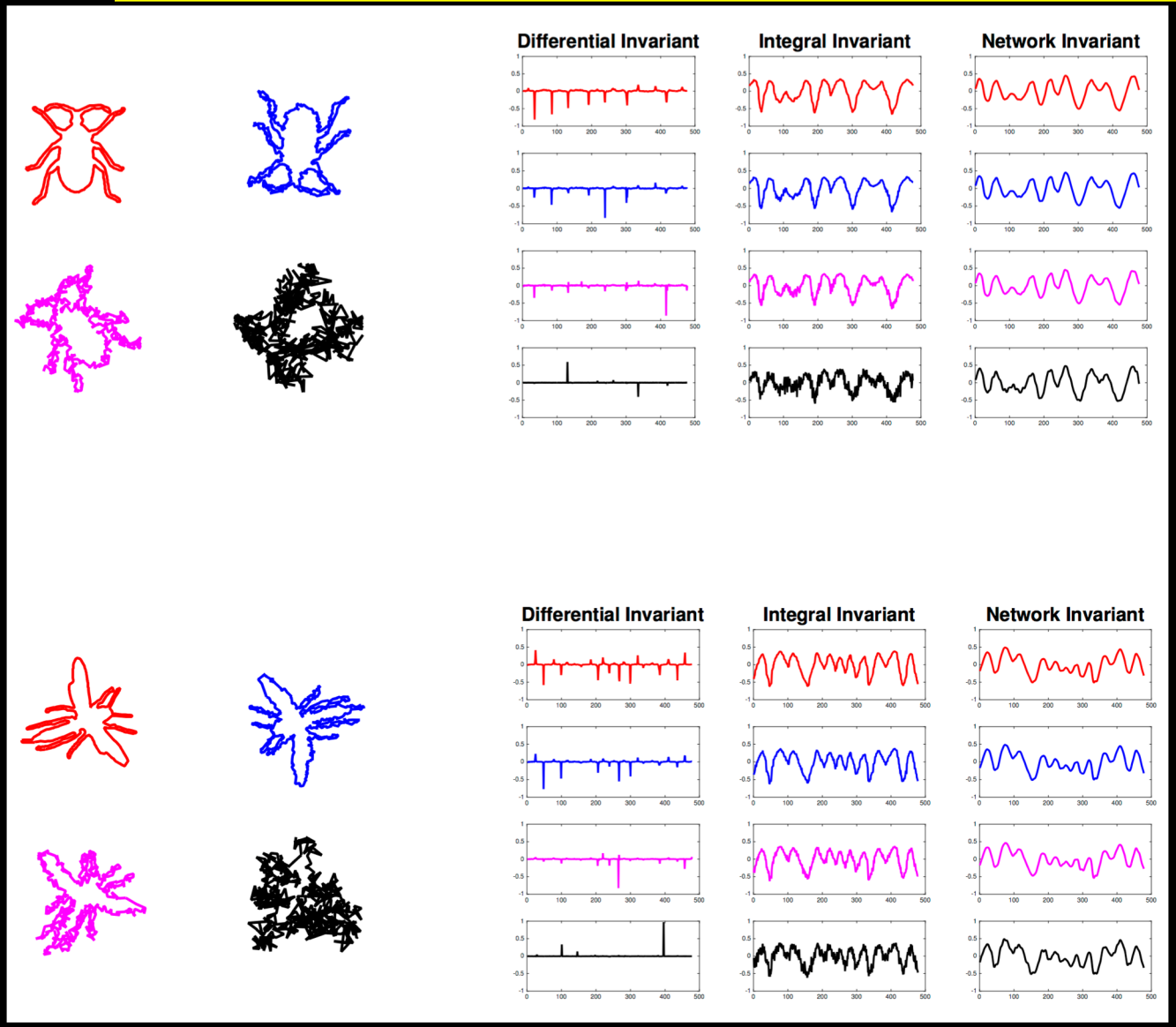
Learning invariants





Pai Wetzler K. ICLR 2017

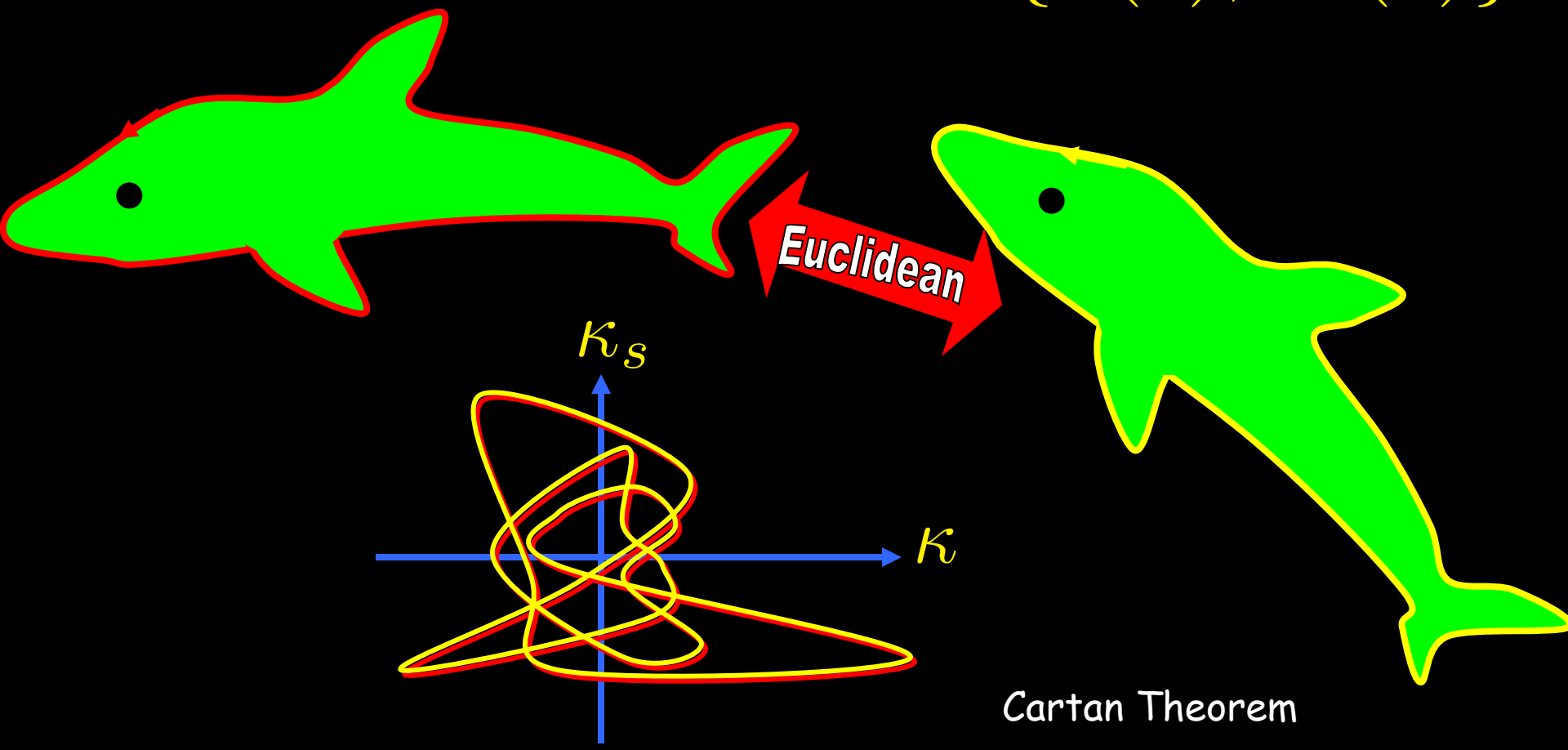
Robustness to noise



Pai Wetzler K. ICLR 2017

Differential Signatures

- Euclidean invariant signature $\{\kappa(s), \kappa_s(s)\}$



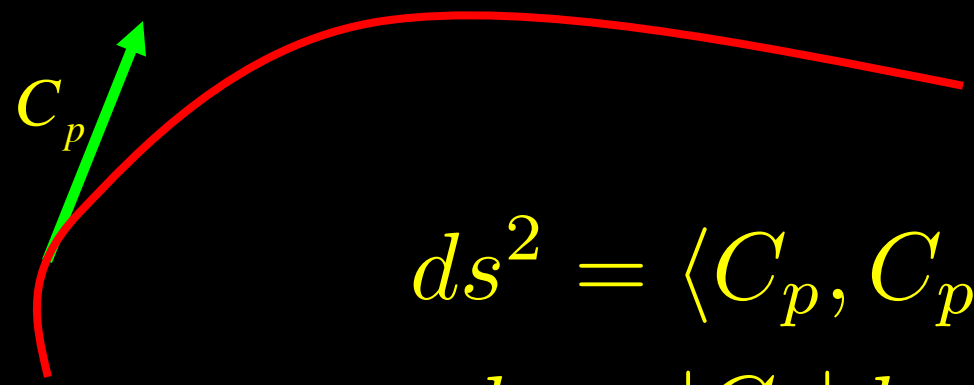
Cartan Theorem

Euclidean arclength

- Length is preserved, thus $1 = \langle C_s, C_s \rangle$

$$= \langle C_p p_s, C_p p_s \rangle$$

$$= \langle C_p, C_p \rangle p_s^2$$



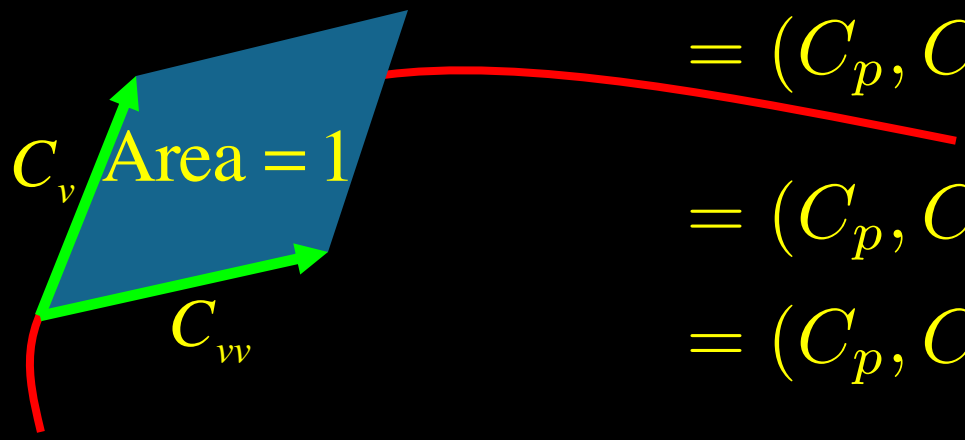
$$ds^2 = \langle C_p, C_p \rangle dp^2$$

$$ds = |C_p| dp$$

Equi-affine arclength

- Area is preserved,

$$\begin{aligned} 1 &= (C_v, C_{vv}) \\ &= (C_p p_v, \frac{d}{dp}(C_p p_v) p_v) \\ &= (C_p, C_{pp} p_v + C_p \frac{d}{dp} p_v) p_v^2 \\ &= (C_p, C_{pp} p_v) p_v^2 \\ &= (C_p, C_{pp}) p_v^3 \end{aligned}$$

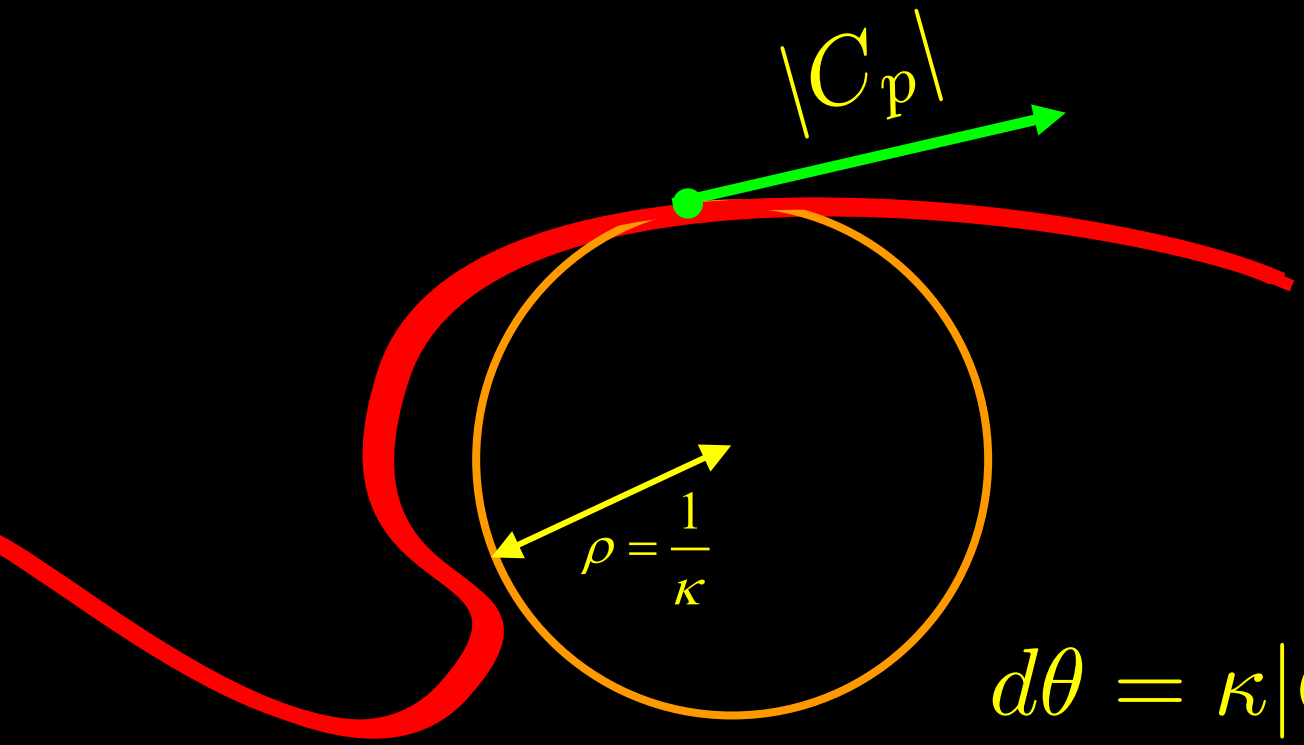


$$dv = (C_p, C_{pp})^{1/3} dp = (C_s, C_{ss})^{1/3} ds = |\kappa|^{1/3} ds$$

Scale inv. arclength

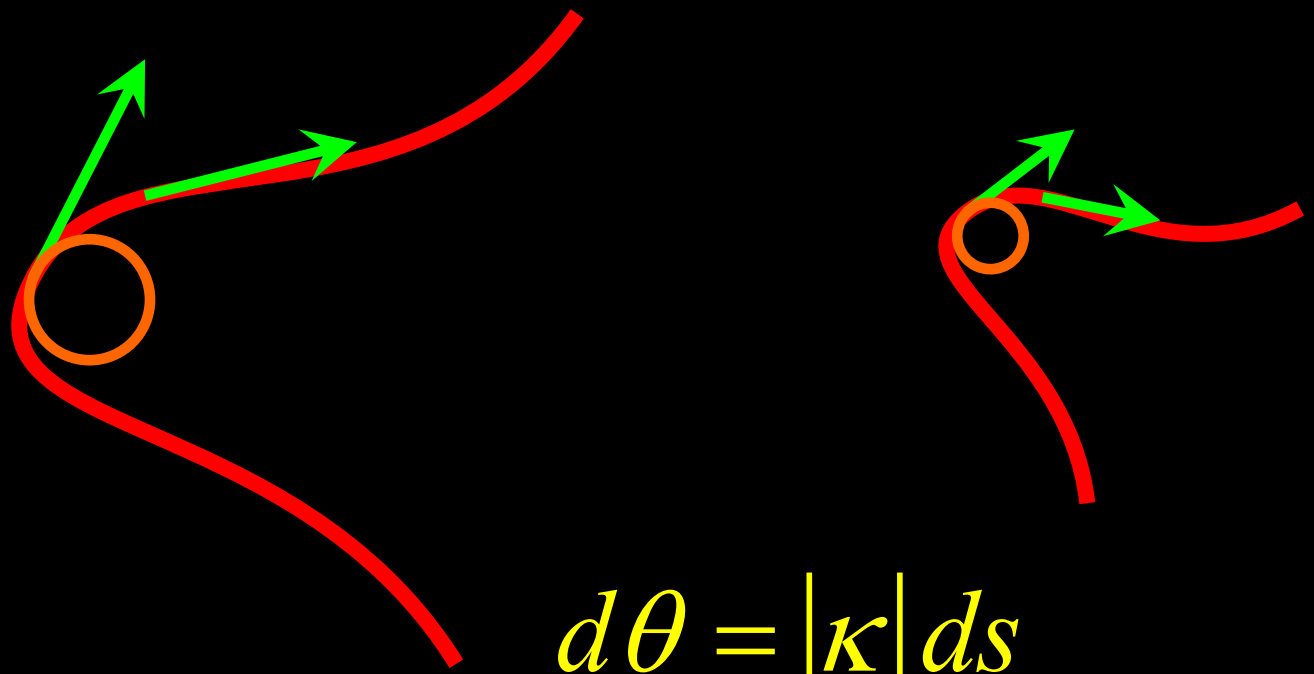
- Ratio is preserved,

$$1 = \frac{|C_\theta|}{\rho} = \kappa |C_\theta| = \kappa |C_p p_\theta| = \kappa |C_p| p_\theta$$



$$d\theta = \kappa |C_p| dp = \kappa ds$$

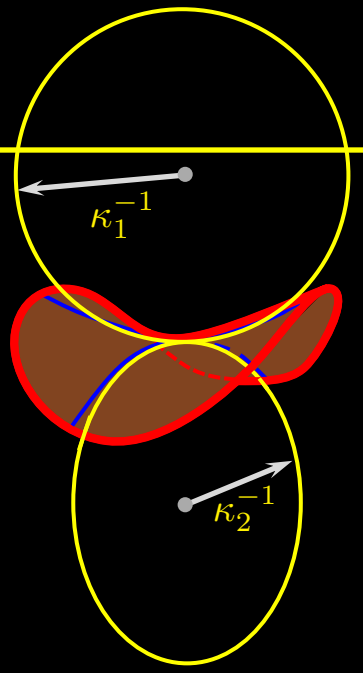
Scale invariance?



$$d\theta = |\kappa| ds$$

From curves to surfaces

$$K = \kappa_1 \kappa_2$$



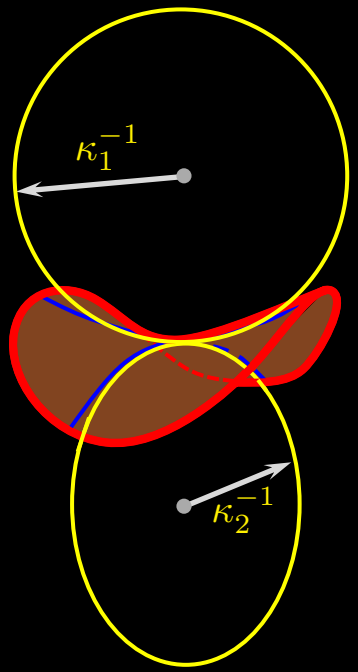
$$S(u,v) = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix}$$

$$dx^2 = (x_u du + x_v dv) = x_u^2 du^2 + 2x_u x_v du dv + x_v^2 dv^2$$

$$\begin{aligned} ds^2 &= dx^2 + dy^2 + dz^2 \\ &= (du\, dv) \begin{pmatrix} S_u^2 & \langle S_u, S_v \rangle \\ \langle S_u, S_v \rangle & S_v^2 \end{pmatrix} \begin{pmatrix} du \\ dv \end{pmatrix} \\ &= (du\, dv) \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} du \\ dv \end{pmatrix} \end{aligned}$$

From curves to surfaces

$$
\begin{aligned}
d\tilde{s}^2 &= \kappa_1\kappa_2(dx^2+dy^2+dz^2) \\
&= (du\,dv)\begin{pmatrix} KS_u^2 & K\langle S_u, S_v\rangle \\ K\langle S_u, S_v\rangle & KS_v^2 \end{pmatrix}\begin{pmatrix} du \\ dv \end{pmatrix} \\
&= (du\,dv)\,(K g_{ij})\begin{pmatrix} du \\ dv \end{pmatrix}
\end{aligned}
$$

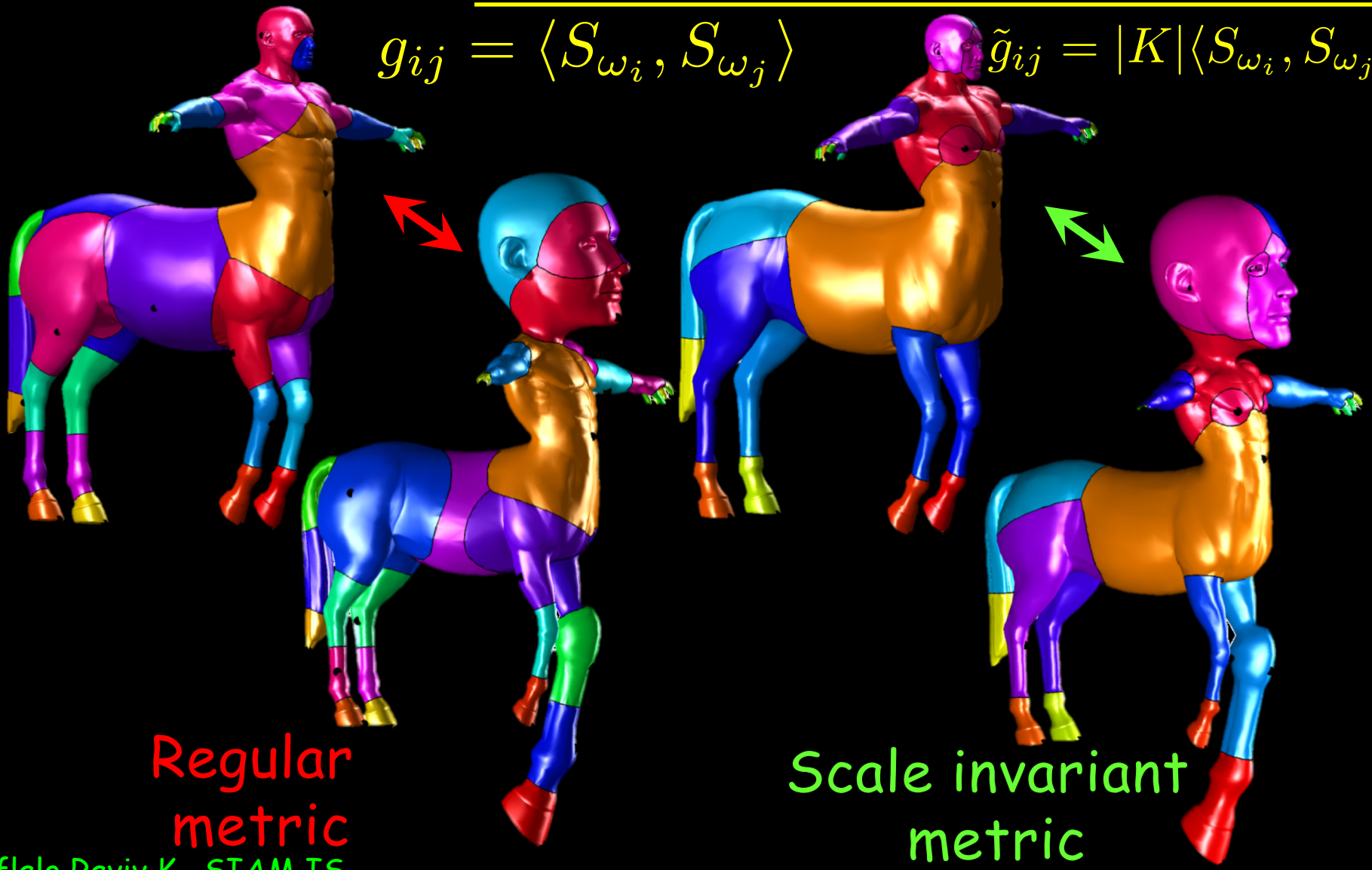


$$\tilde{g}_{ij} = |K| g_{ij} = |\kappa_1\kappa_2|\langle S_i, S_j\rangle$$

Farthest Point Sampling - Voronoi

$$g_{ij} = \langle S_{\omega_i}, S_{\omega_j} \rangle$$

$$\tilde{g}_{ij} = |K| \langle S_{\omega_i}, S_{\omega_j} \rangle$$

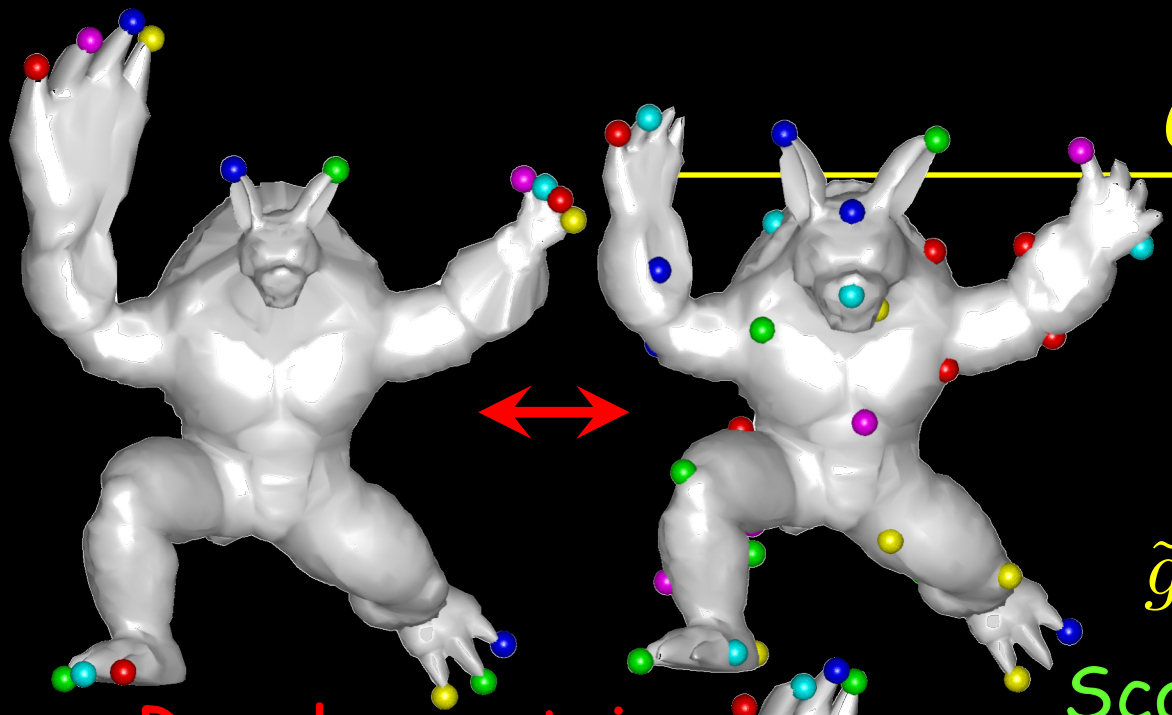


Regular metric

Scale invariant metric

Aflalo Raviv K. SIAM IS

Generalized MDS

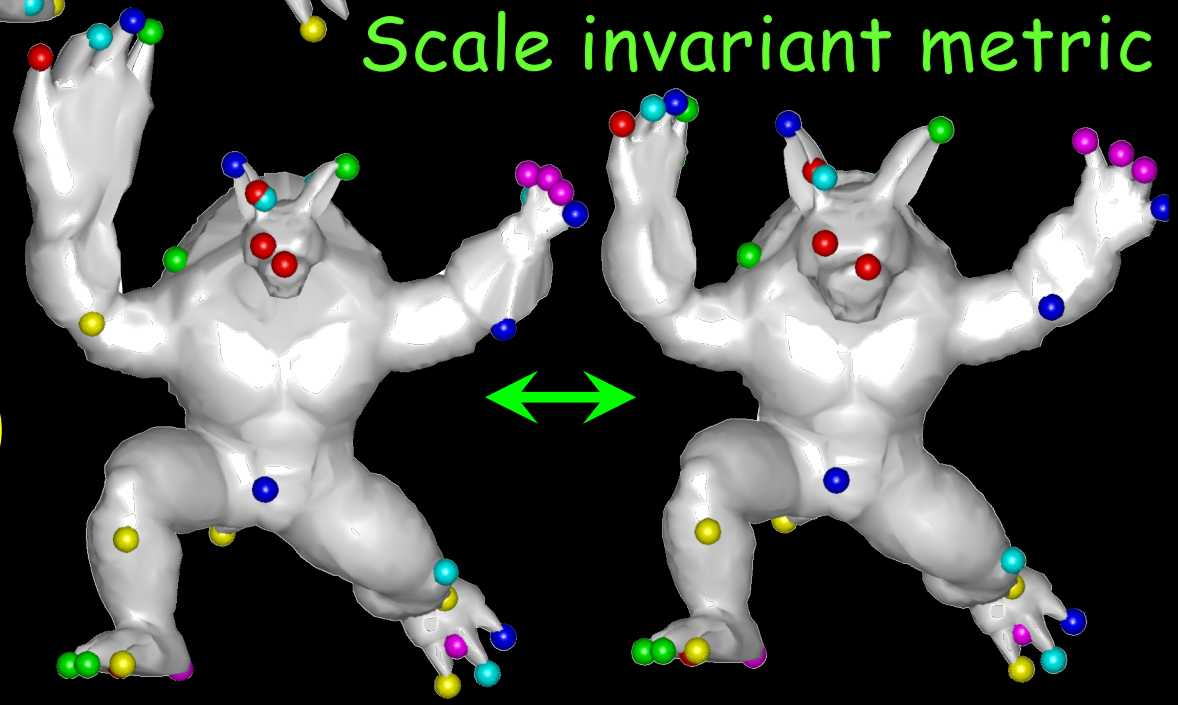


Regular metric

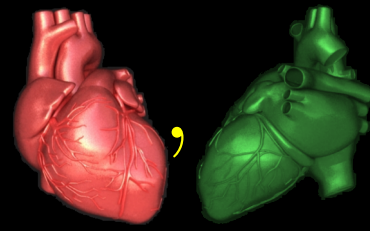
$$g_{ij} = \langle S_{\omega_i}, S_{\omega_j} \rangle$$

$$\tilde{g}_{ij} = |K| \langle S_{\omega_i}, S_{\omega_j} \rangle$$

Scale invariant metric



$$d\left(\quad , \quad \right)$$



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Surface Laplacian

$$\Delta_{\tilde{g}} \equiv -\frac{1}{\sqrt{\tilde{g}}}\partial_i \sqrt{\tilde{g}}\tilde{g}^{ij}\partial_j$$

g

$$\tilde{g}_{ij} = \quad = |\kappa_1\kappa_2|\langle S_i, S_j\rangle$$

Regular metric $g_{ij} = \langle S_{\omega_i}, S_{\omega_j} \rangle$

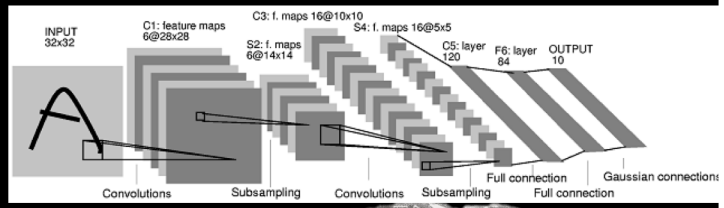
$\Delta_g \phi_i = \lambda_i \phi_i$ Eigenfunctions

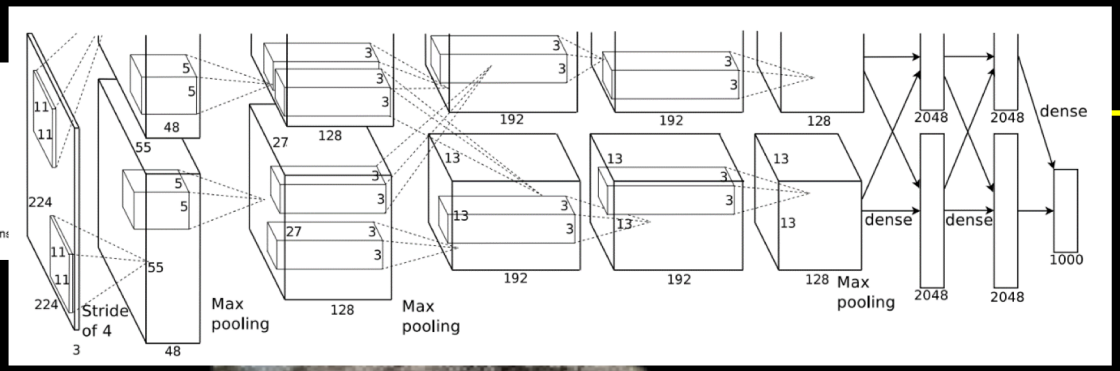
Scale invariant metric $\tilde{g}_{ij} = |K|\langle S_{\omega_i}, S_{\omega_j} \rangle$

$\Delta_g \phi_i = \lambda_i \phi_i$ Eigenfunctions

How could we use algebra to describe geometry?

René Descartes

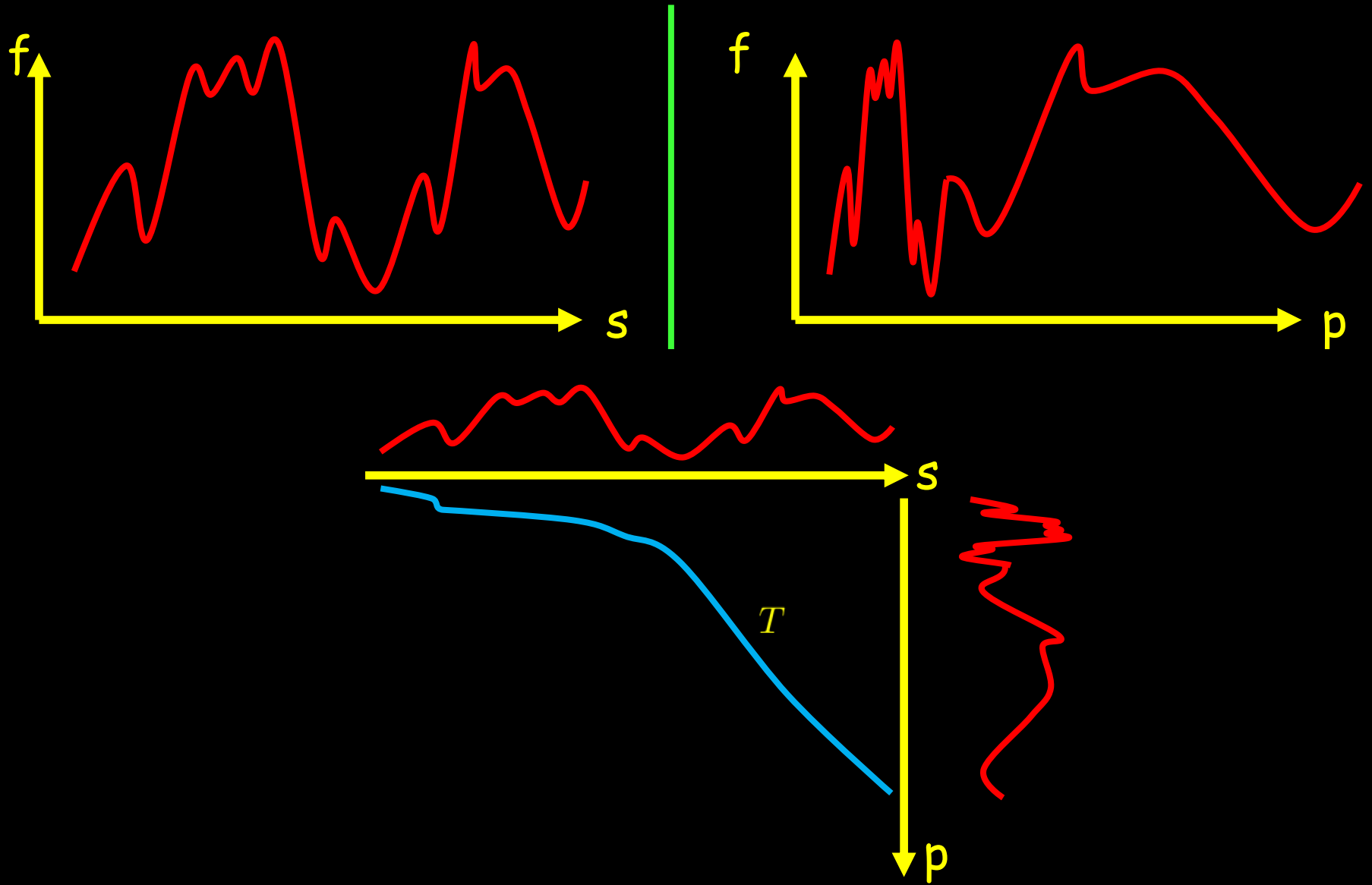




Yann LeCunn

Geoff Hinton

Functional Maps Ovsjanikov et al. 2012



Functional Maps Ovsjanikov et al. 2012

$\{\psi_i\}$

$$f = \sum_i \langle f, \psi_i \rangle \psi_i = \sum_i \alpha_i \psi_i$$

$\{\phi_i\}$

$$f = \sum_i \langle f, \phi_i \rangle \phi_i = \sum_i \beta_i \phi_i$$

$$\beta = C\alpha$$

$$C_{ij} = \langle T(\phi_i), \psi_j \rangle$$

Functional Maps

$\{\psi_i\}$

$$f = \sum_i \langle f, \psi_i \rangle \psi_i = \sum_i \alpha_i \psi_i$$

$\{\phi_i\}$

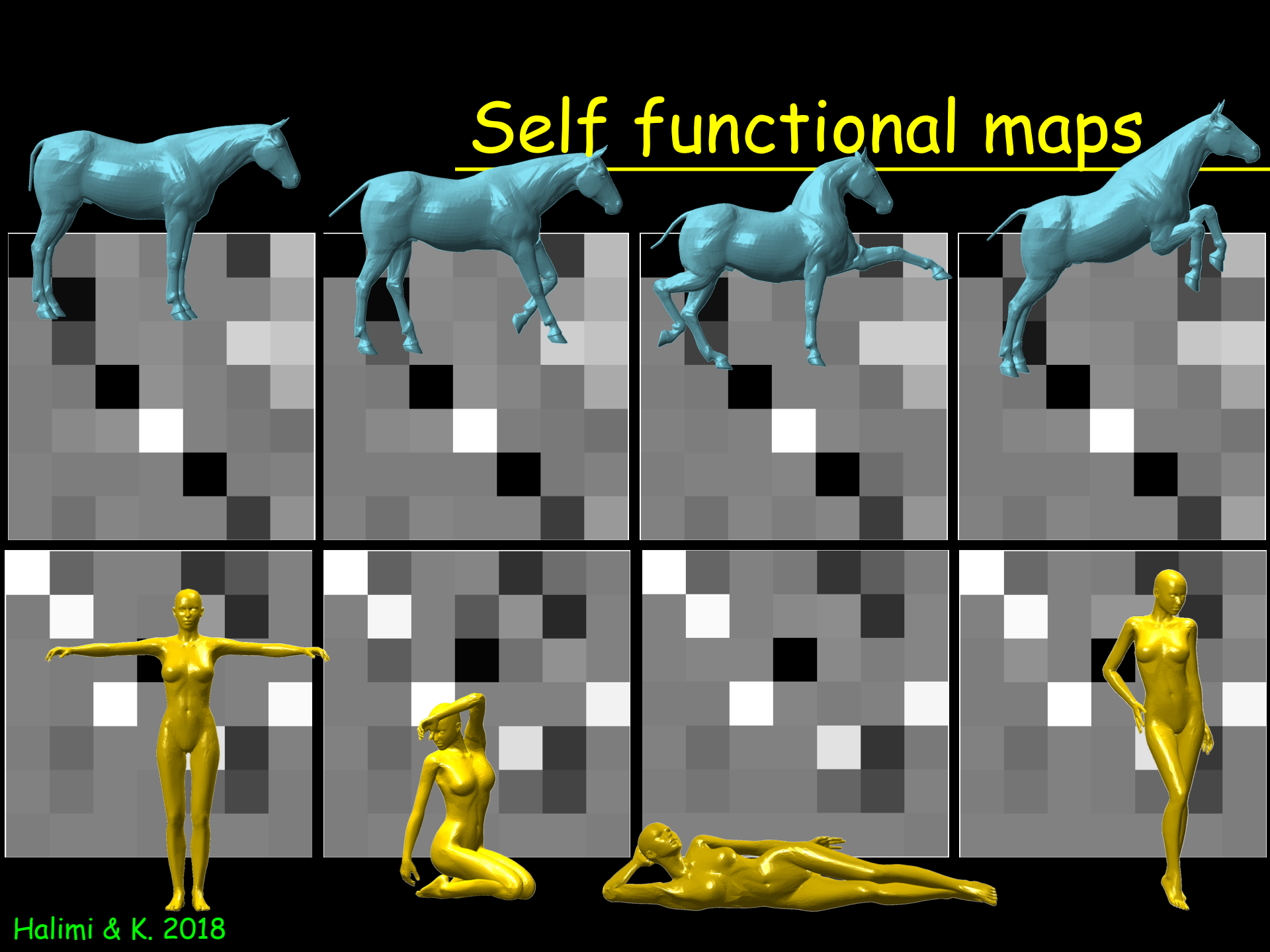
$$f = \sum_i \langle f, \phi_i \rangle \phi_i = \sum_i \beta_i \phi_i$$

$$\beta = C\alpha$$

$$C_{ij} = \langle T(\phi_i), \psi_j \rangle$$

Halimi & K. 2018

Self functional maps



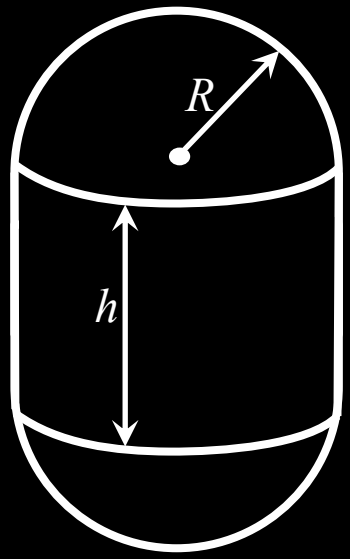
Halimi & K. 2018

Eigenfunctions

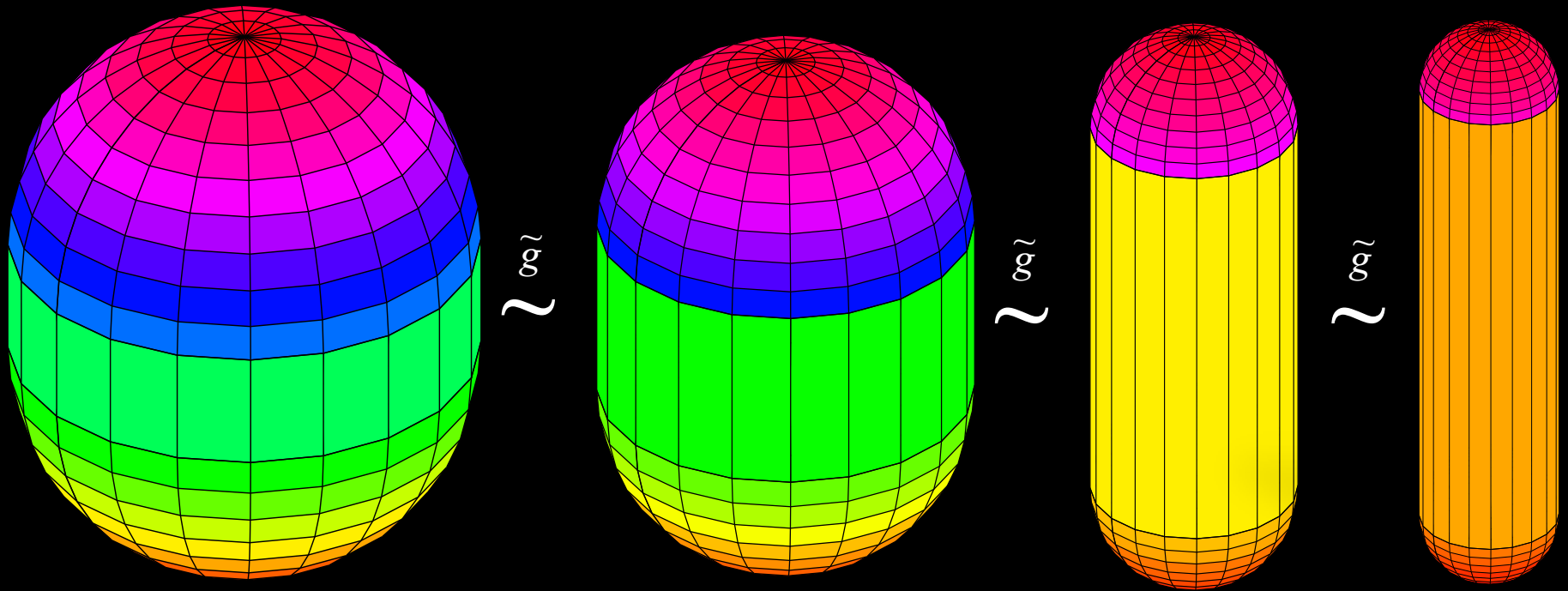
$$g_{ij} = \langle S_{\omega_i}, S_{\omega_j} \rangle \qquad -\Delta_g \psi_i = \lambda_i \psi_i$$

$$\tilde{g}_{ij} = |K| g_{ij} \qquad -\Delta_{\tilde{g}} \tilde{\psi}_i = \tilde{\lambda}_i \tilde{\psi}_i$$

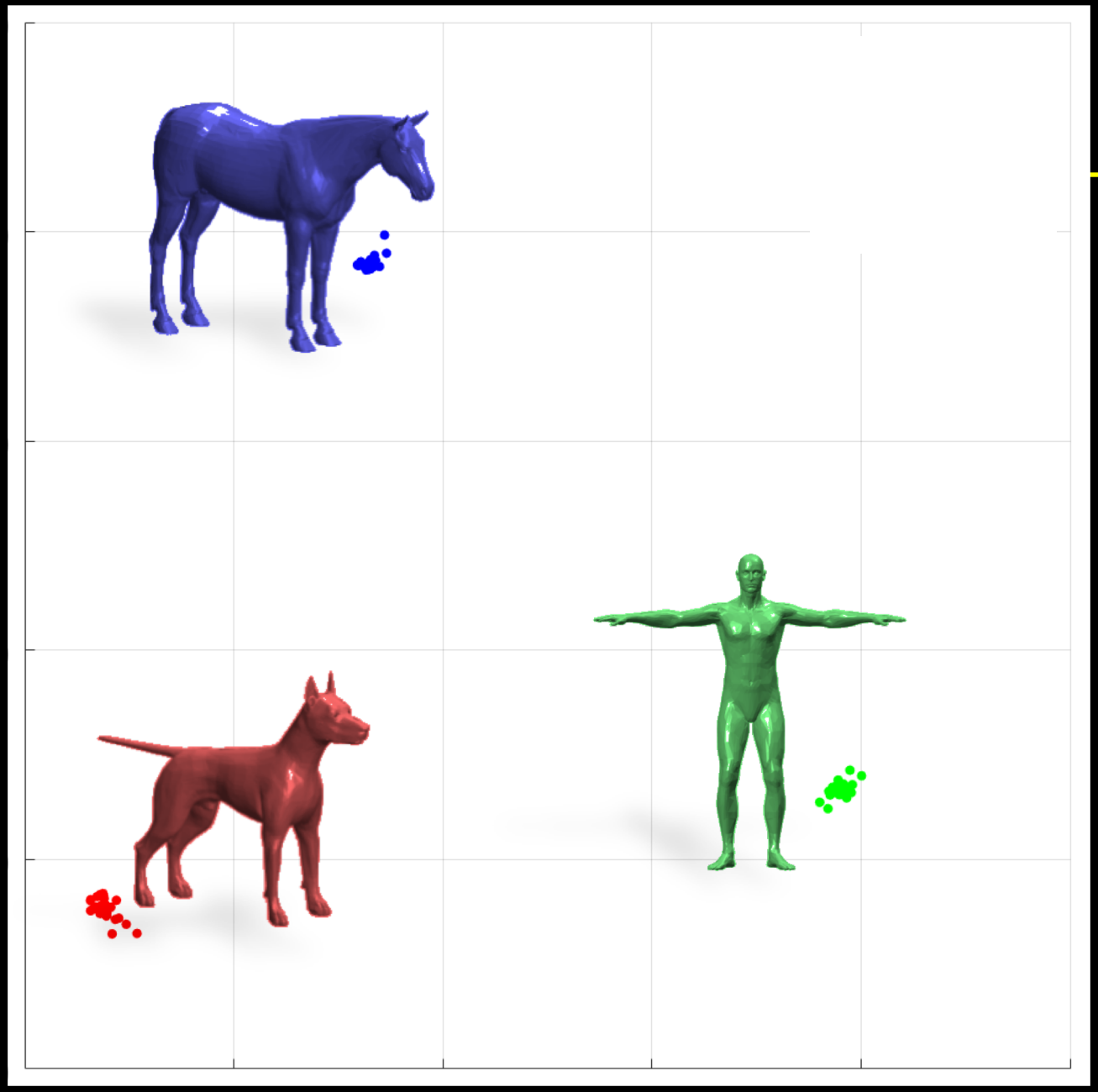
Halimi & K. 2018



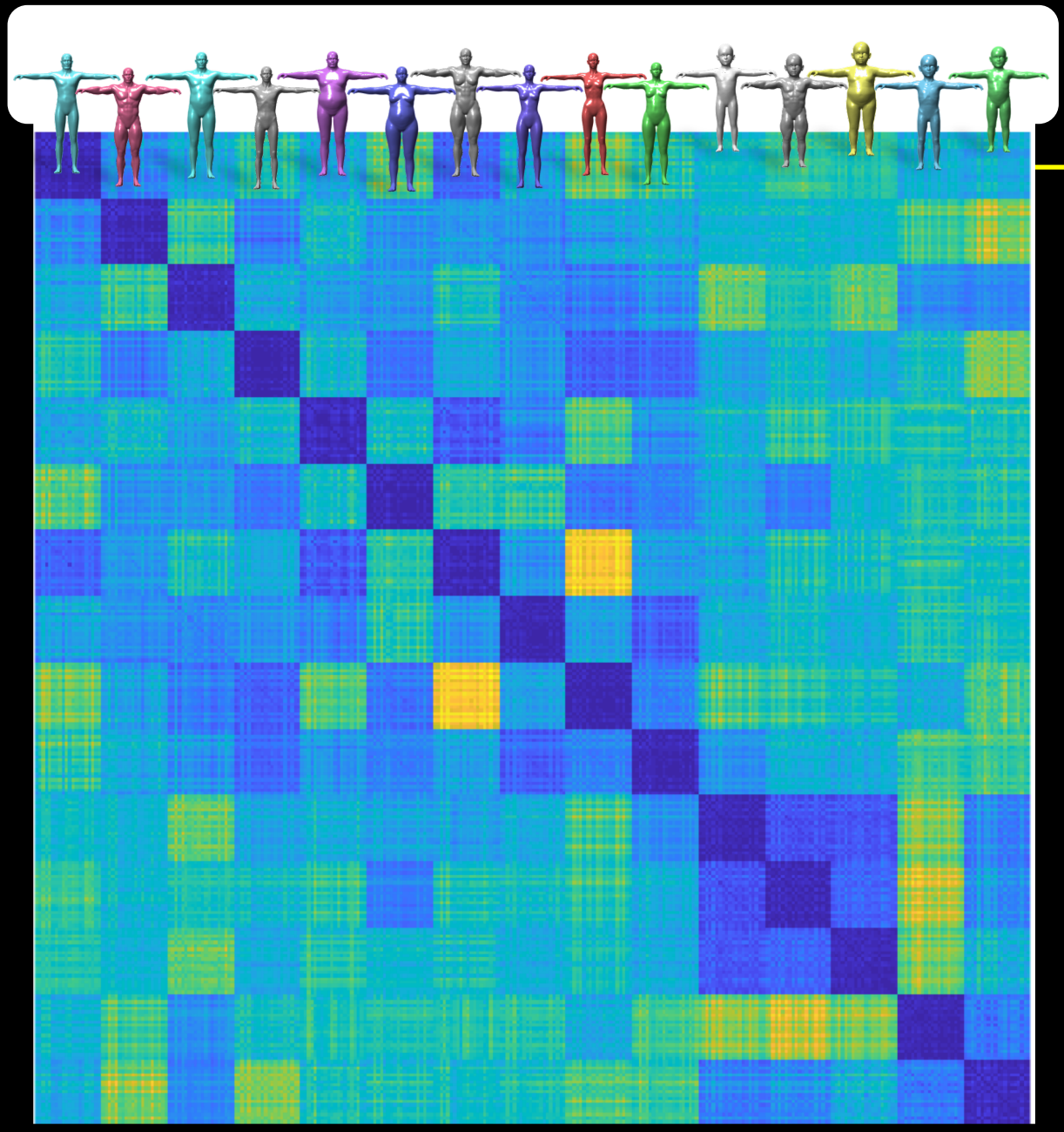
Isometry classes



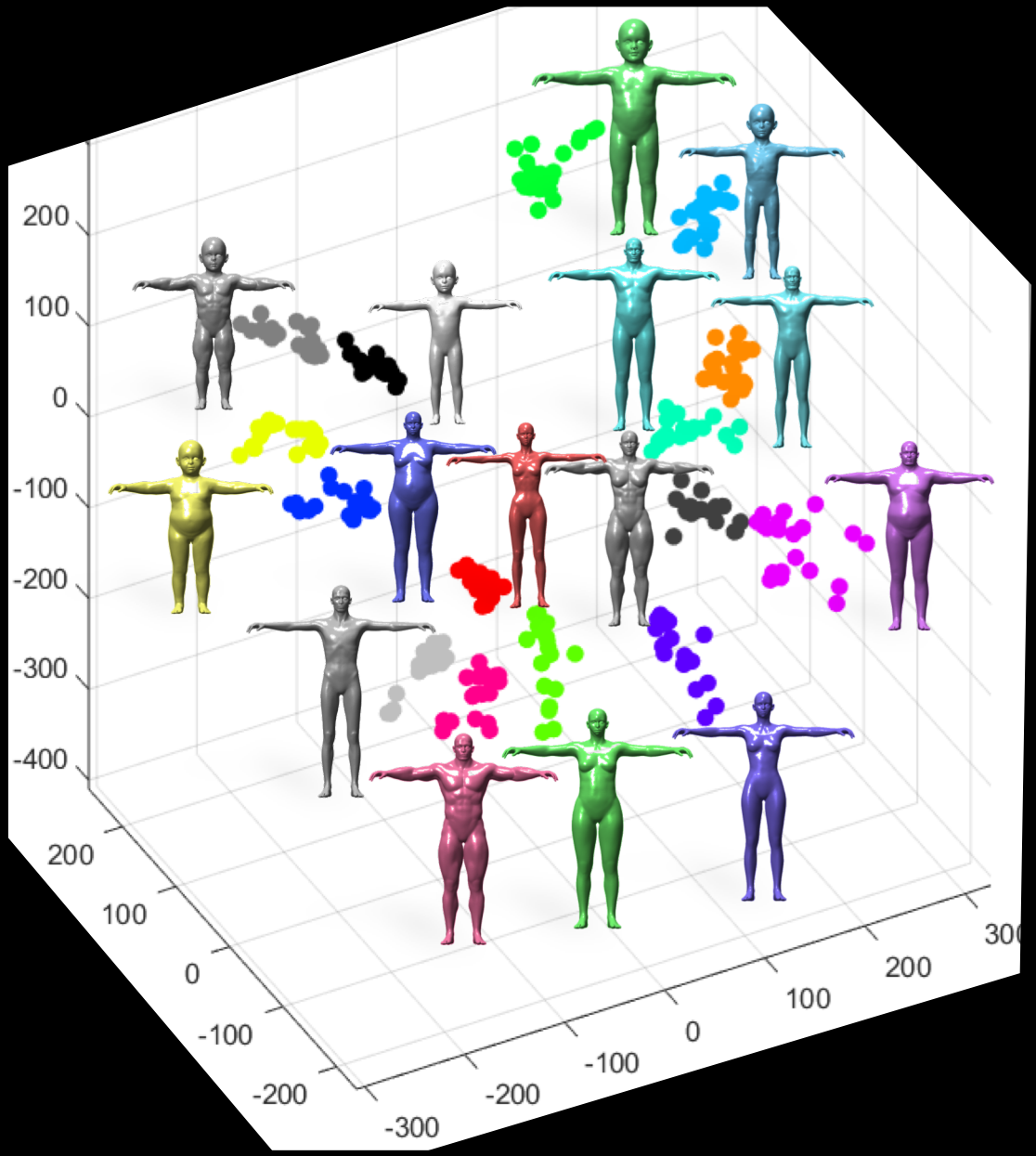
Halimi & K. 2018



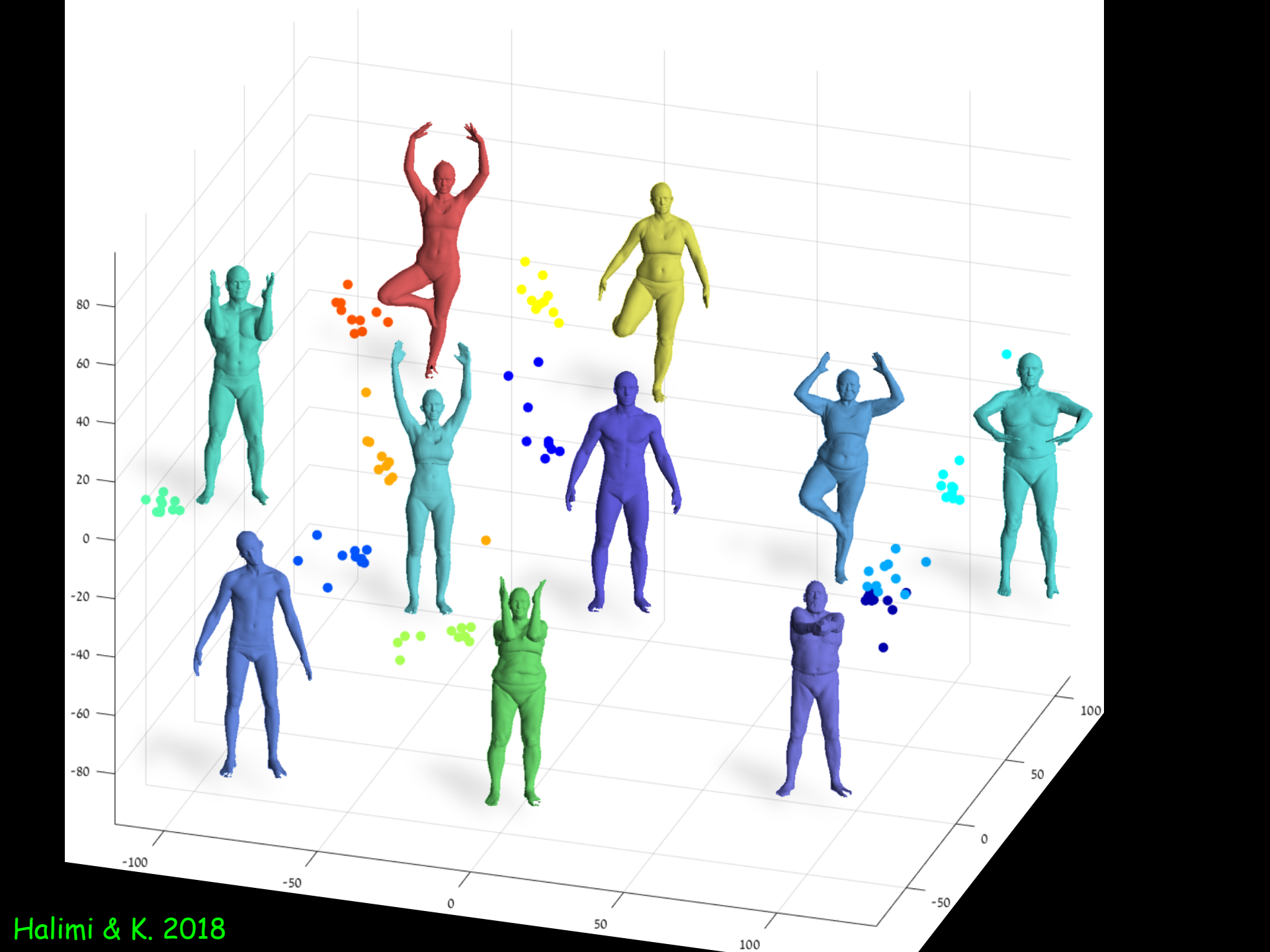
Halimi & K. 2018



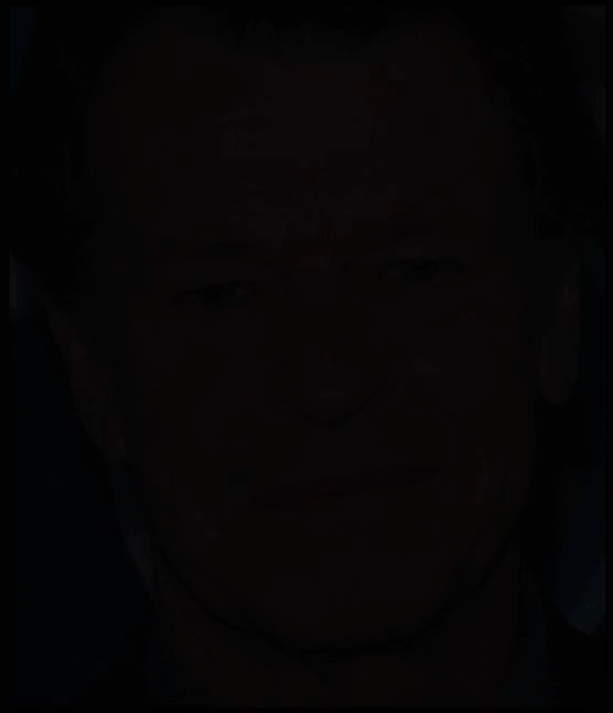
Halimi & K. 2018



Halimi & K. 2018



Halimi & K. 2018



input 2D image

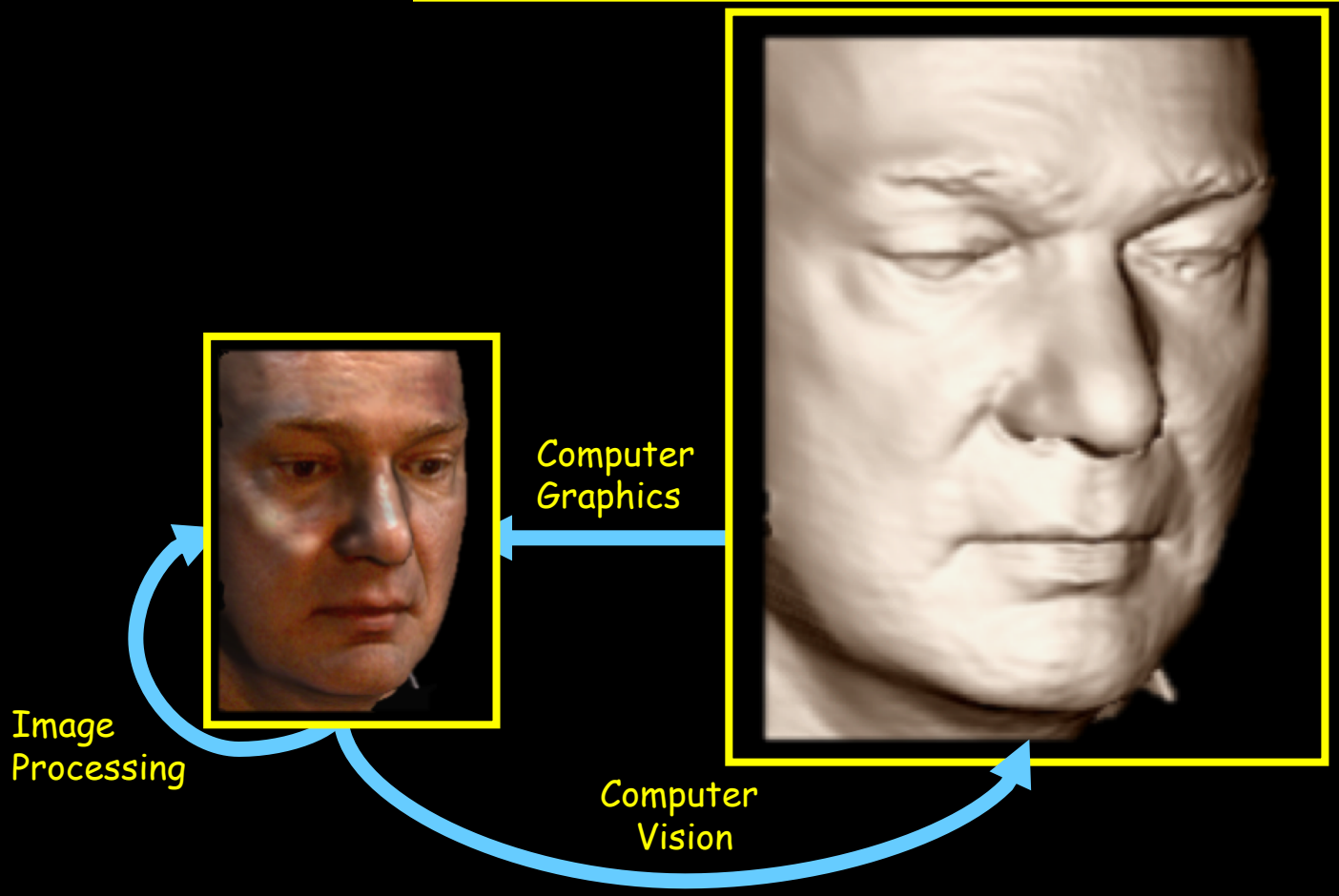
output 3D face

output 3D face
with texture

Richardson Sela K. ICCV 2017

Thank you for your attention

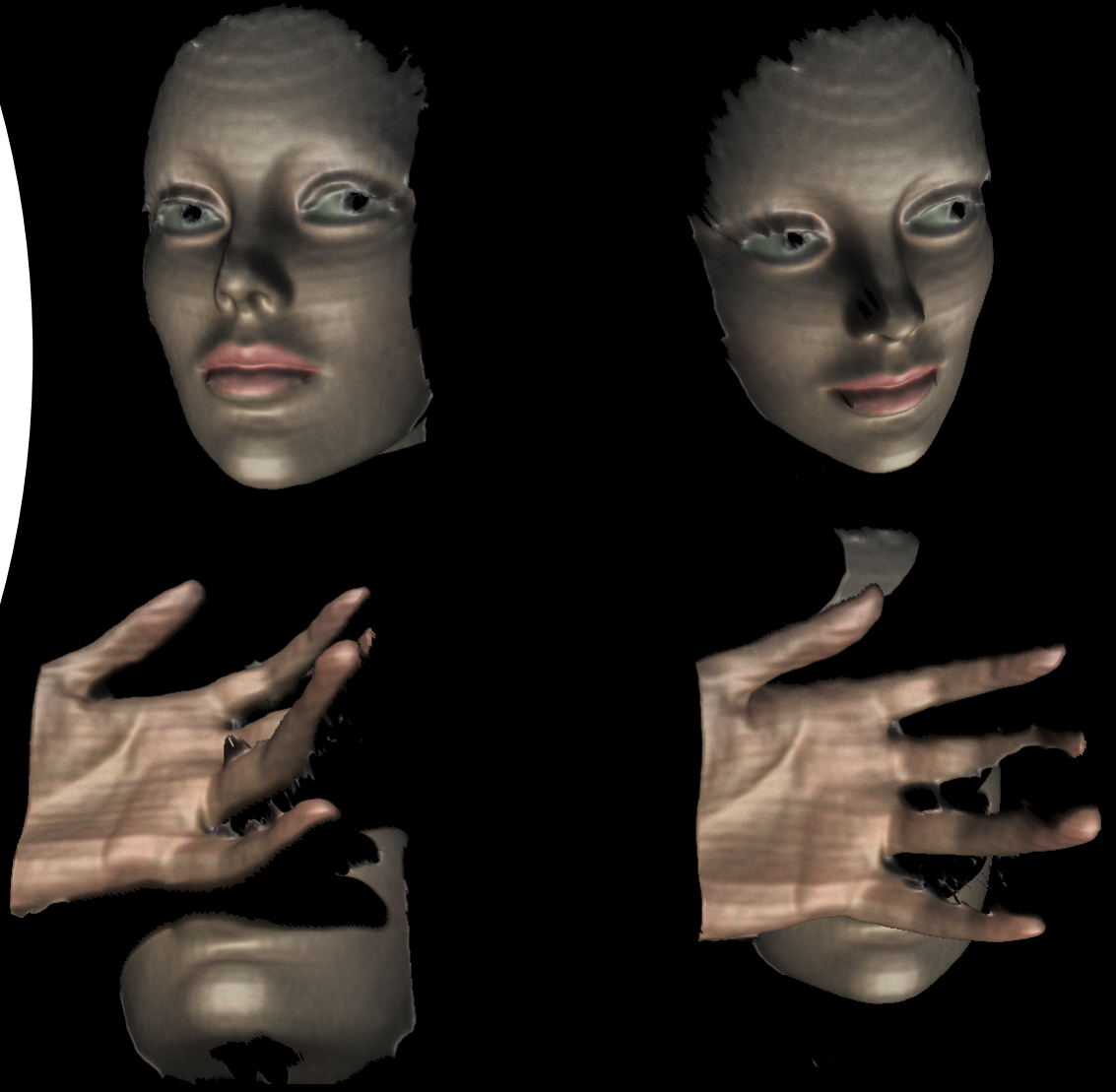
Our domain



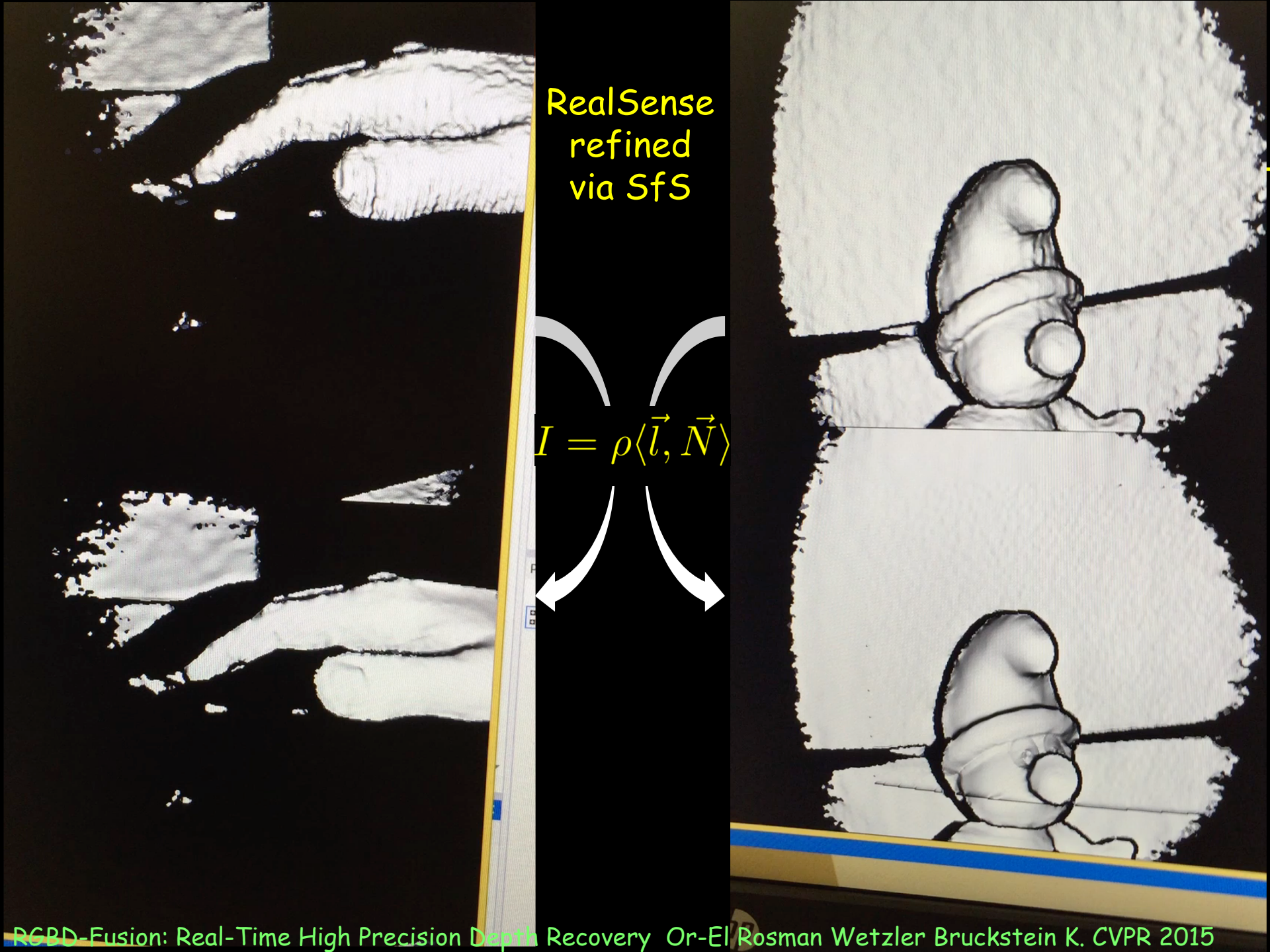
Coded light 3D scanner

INVISION
Ultimate 3D Experience

intel




intel REALSENSE
TECHNOLOGY
$\iiint dxdtd\$$

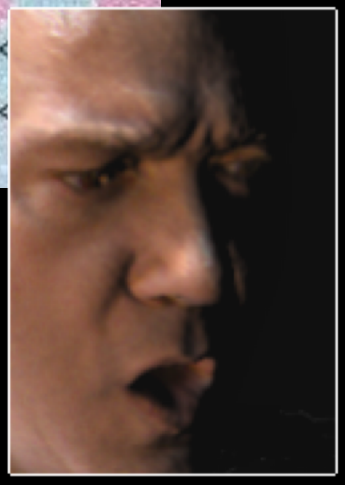


RGBD-Fusion: Real-Time High Precision Depth Recovery Or-El Rosman Wetzler Bruckstein K. CVPR 2015

B^2 = Bronstein^2

Face recognition challenges

Is this the same person?



Face decomposition

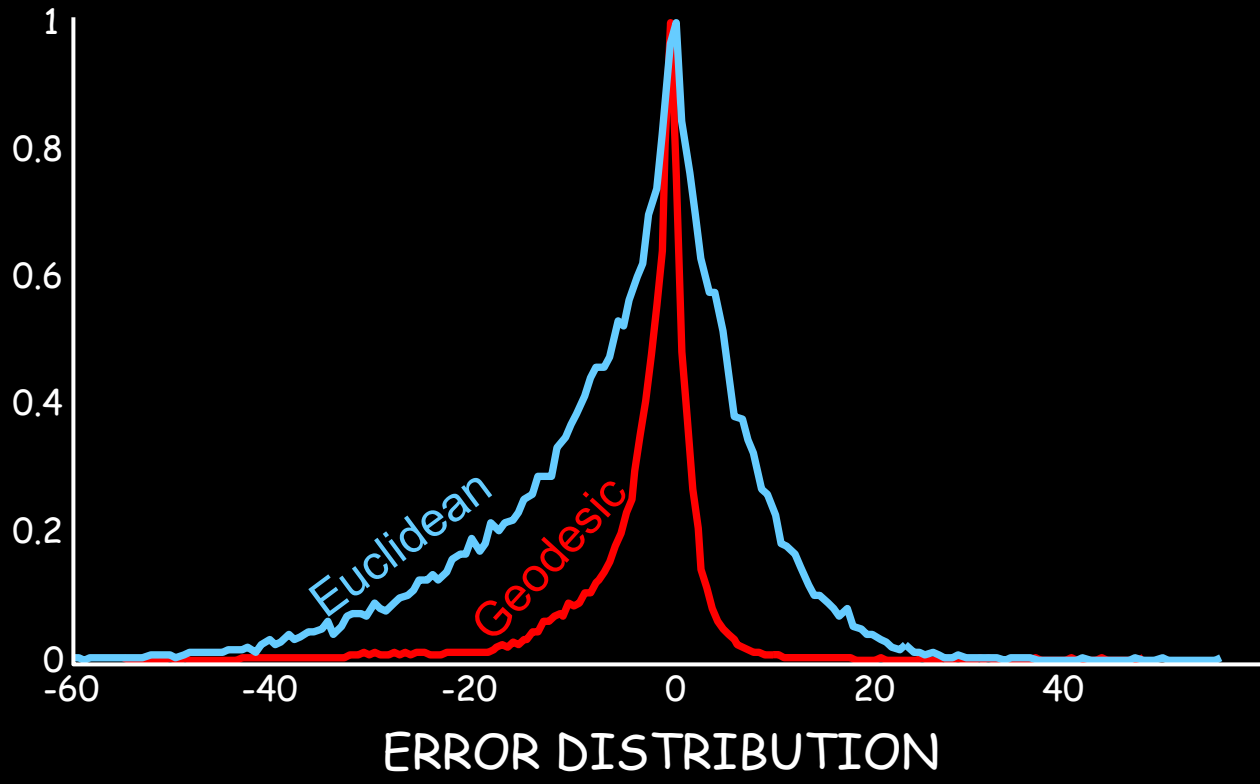
=

PHOTOMETRY
(2D)

+

GEOMETRY
("3D")

Expressions ≈ Isometries



- Extrinsic (Euclidean) geometry = expressions
- Intrinsic (Riemannian) geometry = identity

Bronstein[2] K. 2007

Most models have exceptions

from youtube

$$\inf_{\varphi:S\to Q}\left|d_Q(\varphi(s),\varphi(s')) - d_S(s,s')\right|$$

Morphing

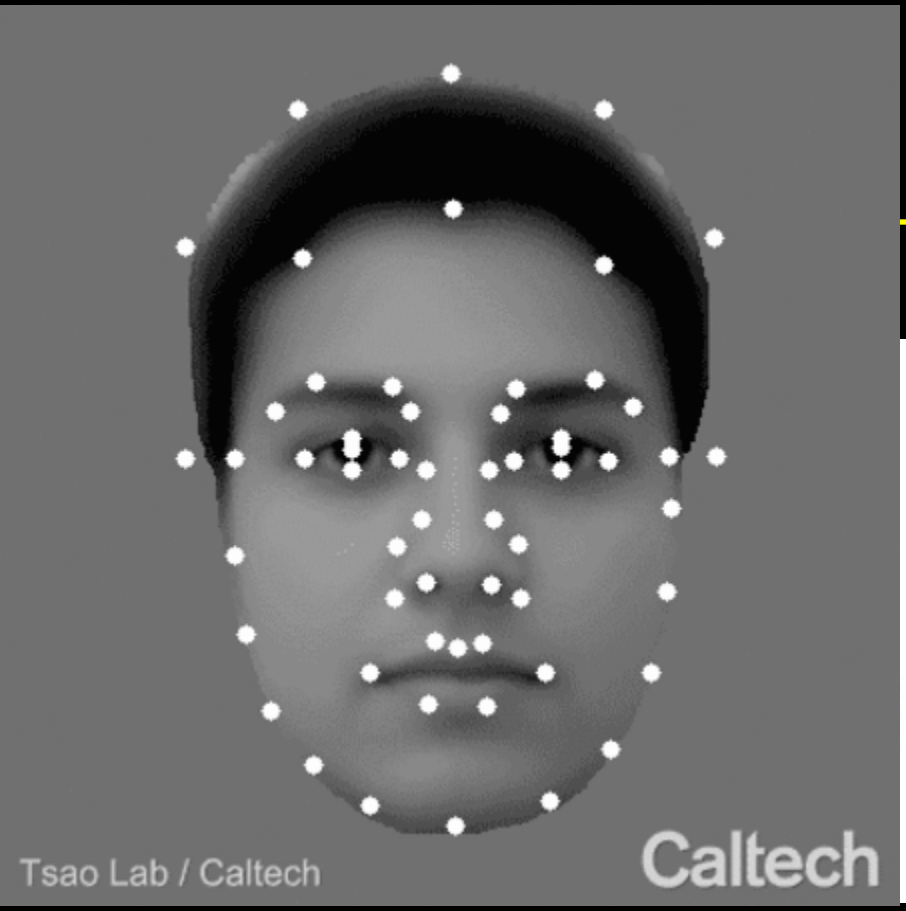
Texture

S

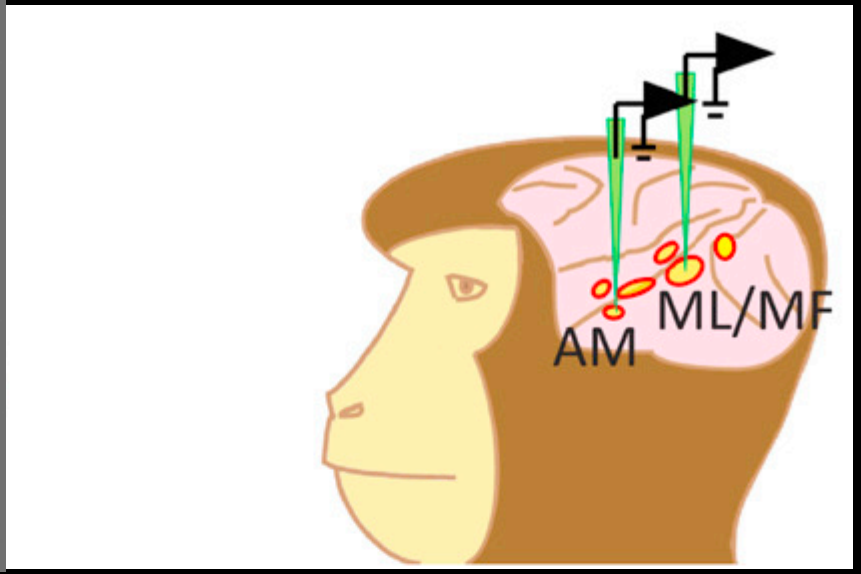
Q

$(1-\lambda)S + \lambda Q$

Bronstein[2] & K. 06



Remarkable result



Facial images can be linearly reconstructed using responses of ~200 face cells

Chang & Tsao The code for facial identity in the primate brain. Cell, 2017

3DMM

Vetter & Blanz, A morphable model for the synthesis of 3D faces, Siggraph 1999

Kemelmacher & Basri, 3D face reconstruction from a single image … PAMI 2011

Learning using Axiomatic Knowledge

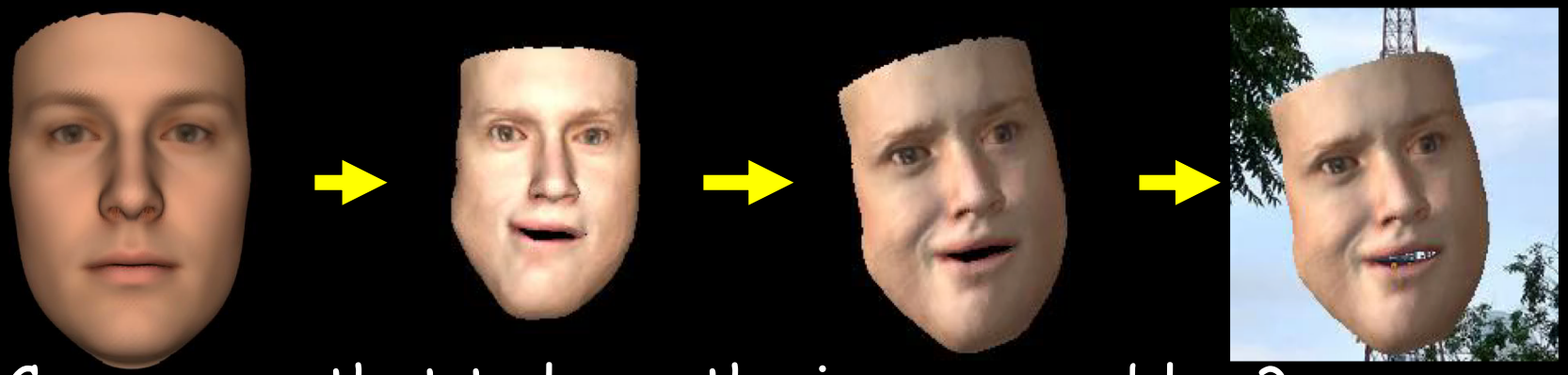
Richardson Sela Or-El K. CVPR'17

Learning using Axiomatic Knowledge

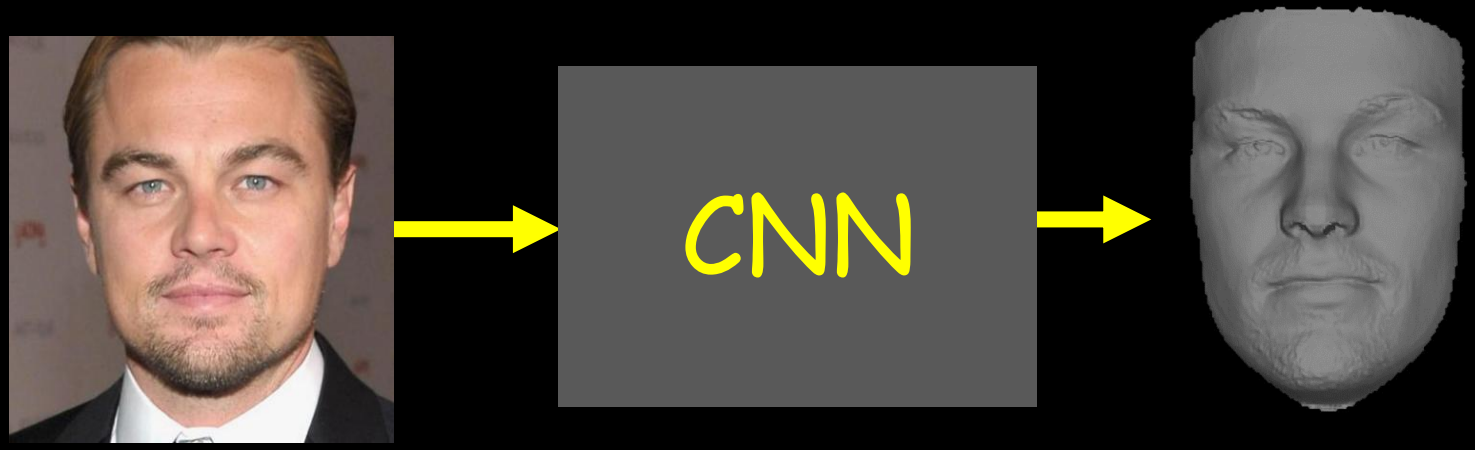
Richardson Sela Or-El K. CVPR'17

Learning using Axiomatic Knowledge

We know how to model faces



Can we use that to learn the inverse problem?



Richardson Sela Or-El K. CVPR'17

Face reconst n evolution

3DMM CNN I→3DMM

3DMM to Z

SfS Z2z

3DMM CNN I→3DMM

3DMM to Z

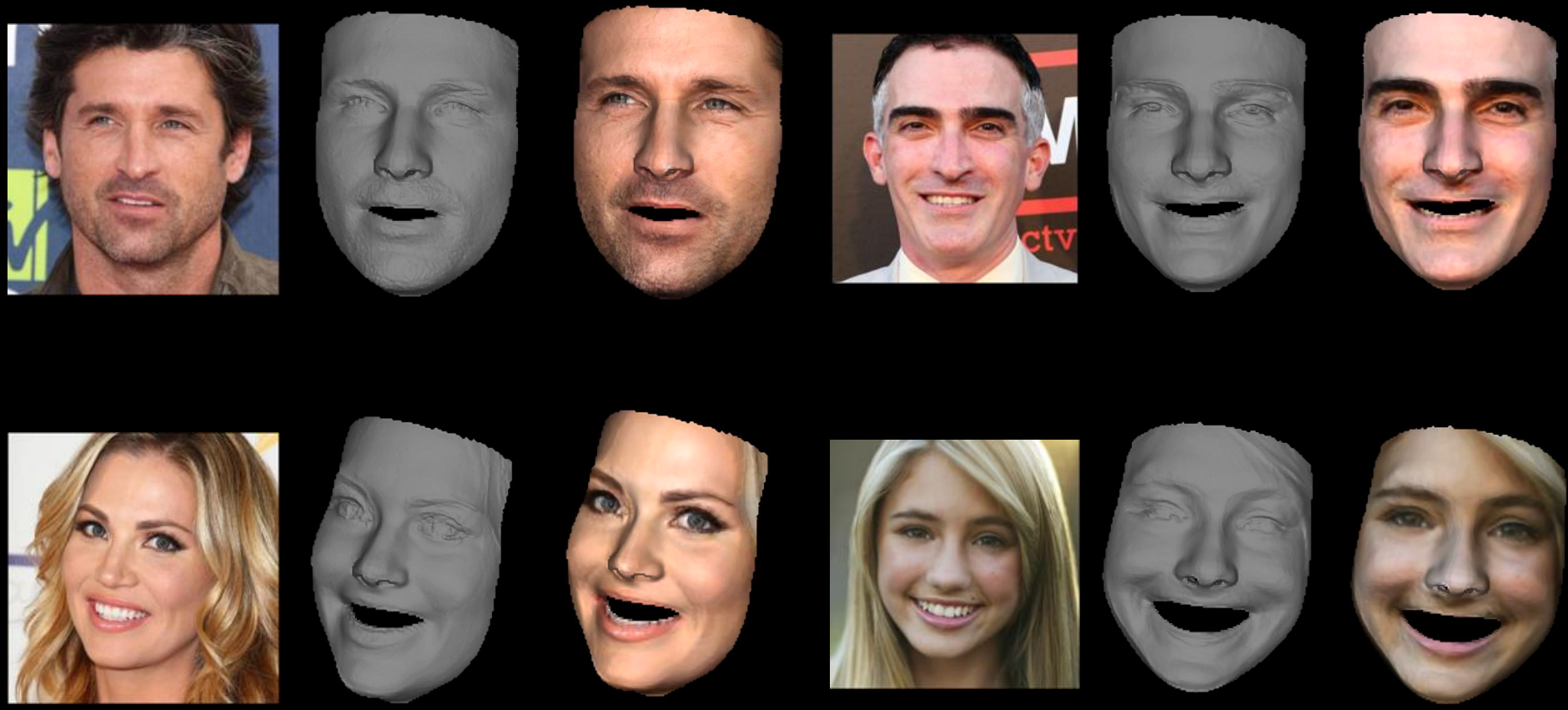
SfS CNN Z2z

3DMM CNN I→Z

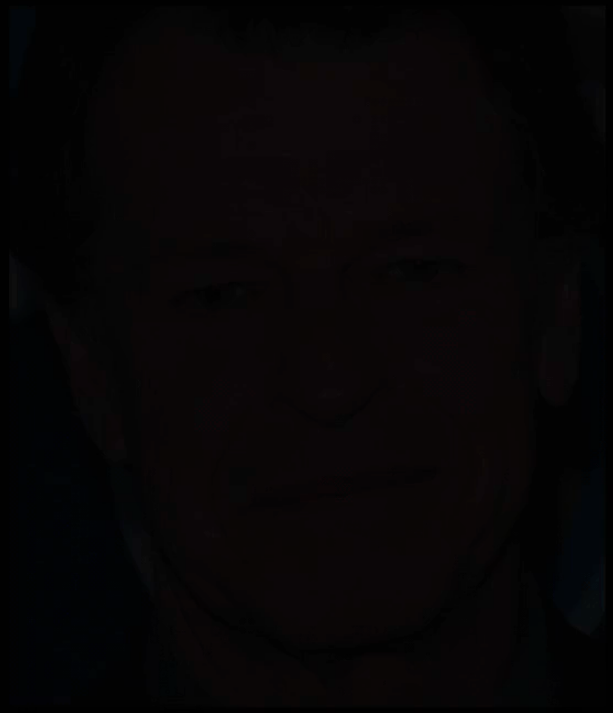
SfS CNN Z2z

Richardson Sela K. 3DV'16/ Richardson Sela Or-El K. CVPR'17/ Richardson Sela K. ICCV'17

More results



Richardson Sela Or-El K. CVPR'17



input 2D image

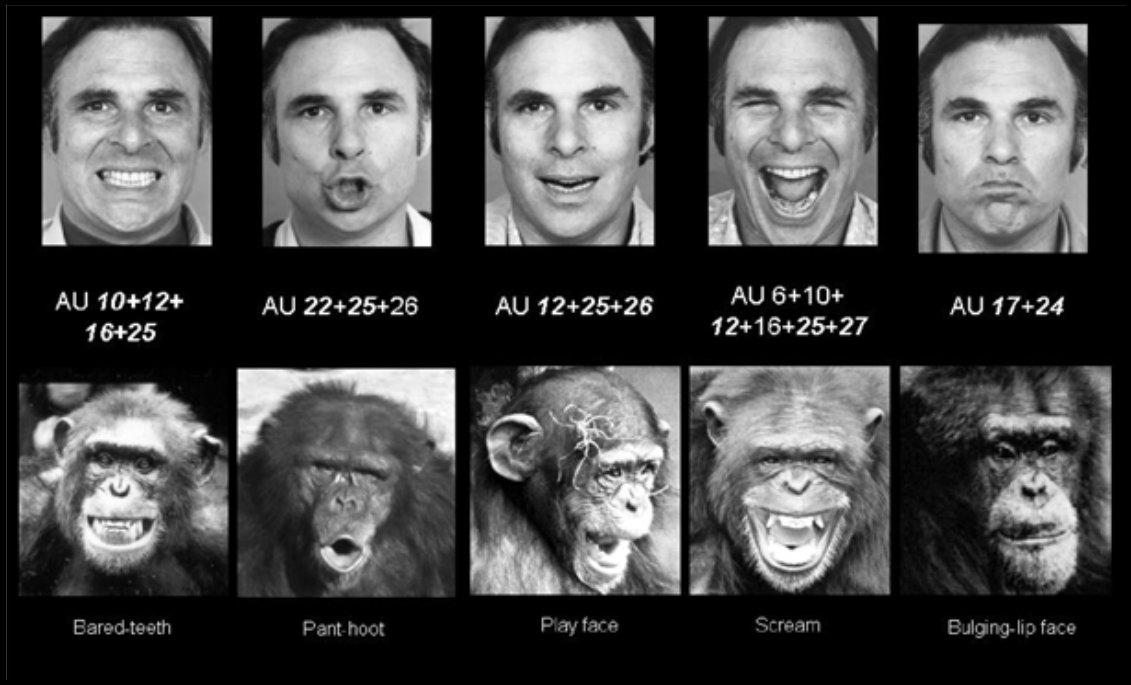
output 3D face

output 3D face
with texture

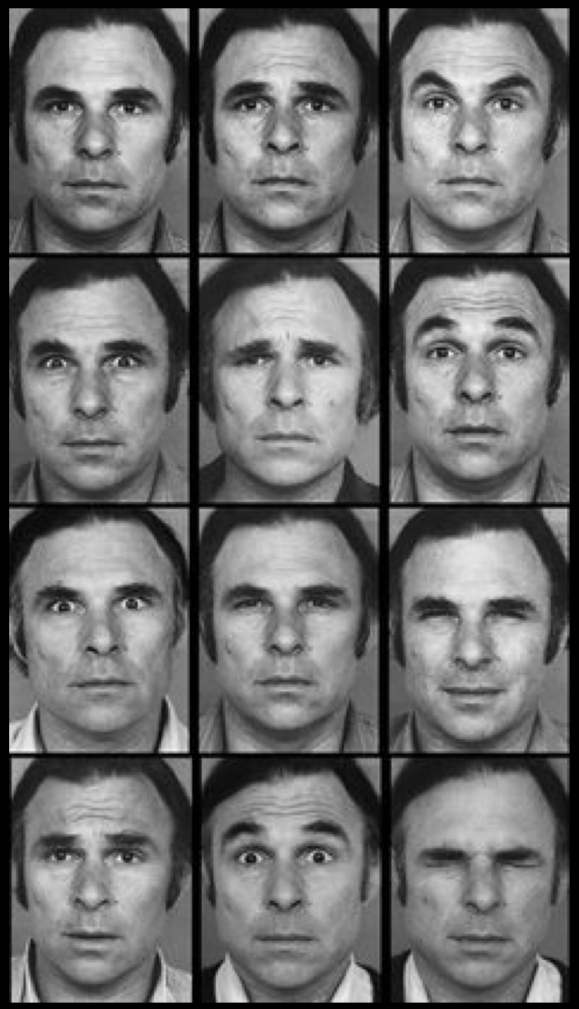
Richardson Sela K. ICCV 2017

Analyzing Facial Expressions

- 1800s - Charles Bell, Duchenne de Boulogne
- Darwin – Expressions as an evolutionary mechanism



Buwel ~1680 - Paul Ekman 1970,



Universal Expressions & Emotions

Paul Ekman 1970

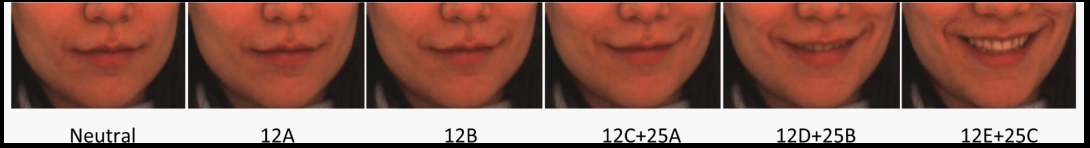
Universal Expressions & Emotions

- Joy (Happiness)
- Sadness
- Fear
- Disgust
- Anger
- Contempt
- Surprise

Sounds familiar?

Paul Ekman 1970

Insideout - Disney Studios



Facial Action Coding System

- Ekman & Friesen 1978
- Taxonomized facial movements by appearance based on muscle clusters & contractions

Ekman, Friesen 1978

Upper Face Action Units

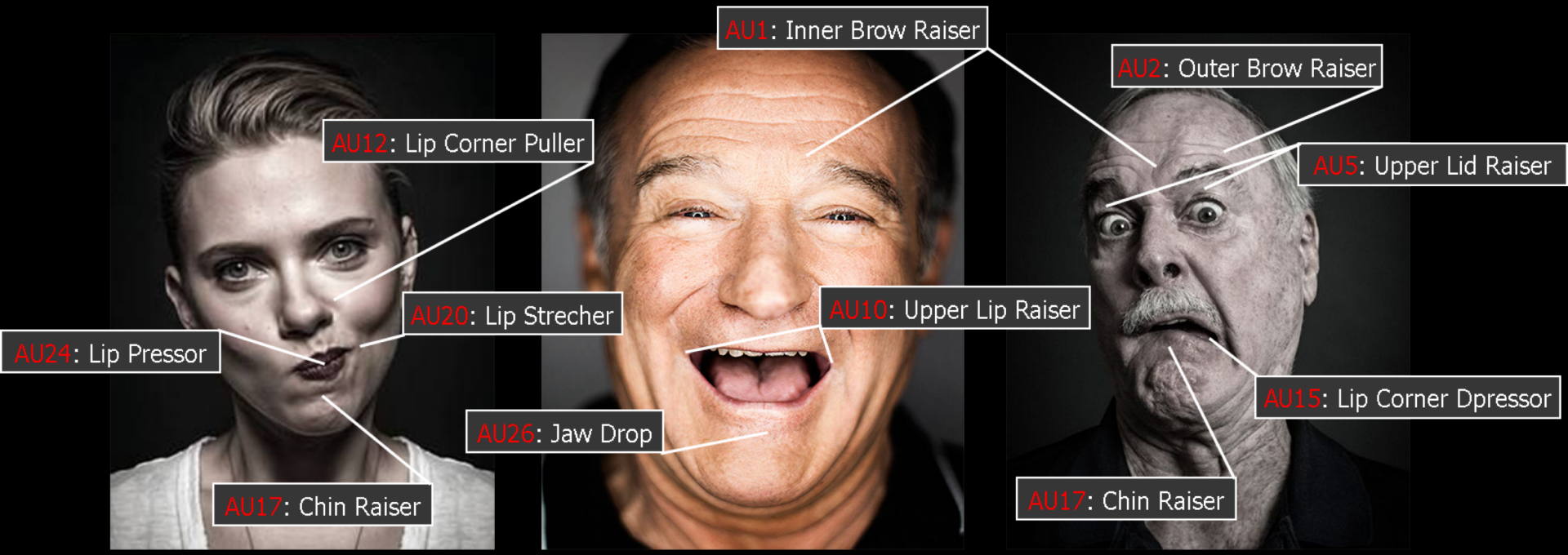
AU 1	AU 2	AU 4	AU 5	AU 6	AU 7
					
Inner Brow Raiser	Outer Brow Raiser	Brow Lowerer	Upper Lid Raiser	Cheek Raiser	Lid Tightener
*AU 41	*AU 42	*AU 43	AU 44	AU 45	AU 46
					
Lid Droop	Slit	Eyes Closed	Squint	Blink	Wink

Lower Face Action Units

AU 9	AU 10	AU 11	AU 12	AU 13	AU 14
					
Nose Wrinkler	Upper Lip Raiser	Nasolabial Deepener	Lip Corner Puller	Cheek Puffer	Dimpler
AU 15	AU 16	AU 17	AU 18	AU 20	AU 22
					
Lip Corner Depressor	Lower Lip Depressor	Chin Raiser	Lip Puckerer	Lip Stretcher	Lip Funneler
AU 23	AU 24	*AU 25	*AU 26	*AU 27	AU 28
					
Lip Tightener	Lip Pressor	Lips Part	Jaw Drop	Mouth Stretch	Lip Suck

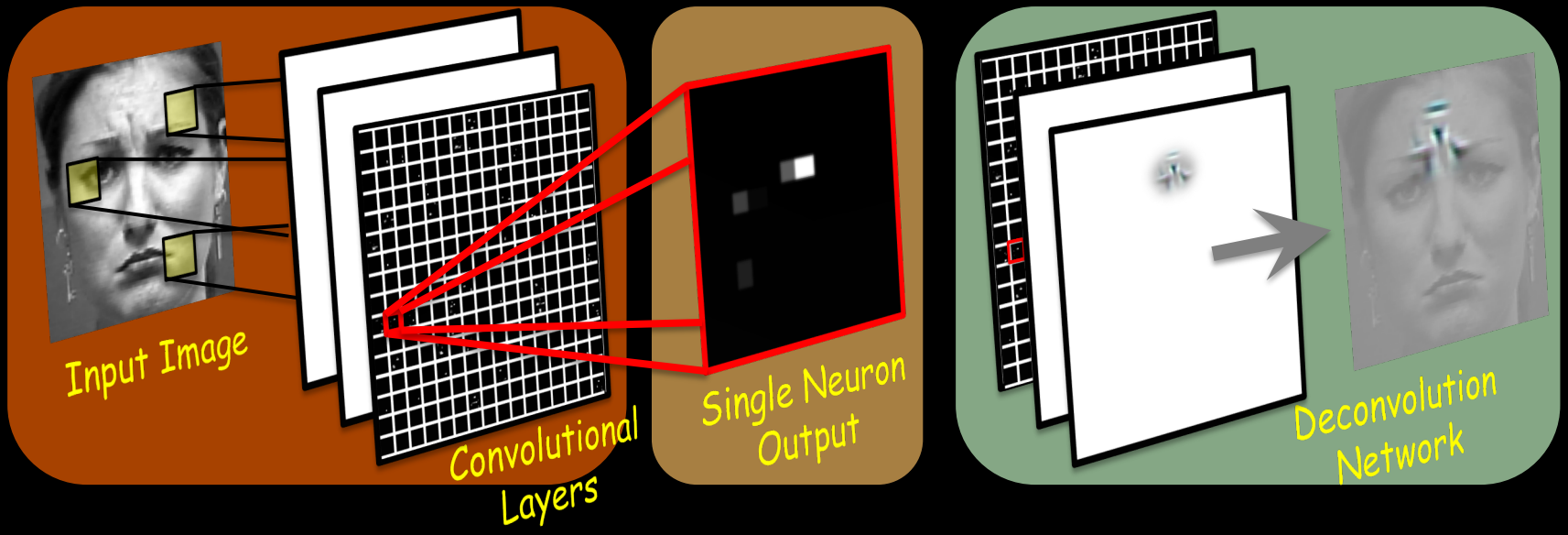
W/O Macaque Monkeys

What can be said about expressions?



Breuer & K. A deep learning perspective on the origin of facial expressions, BMVC/workshops 2017

W/O macaque monkeys



Breuer & K. A deep learning perspective on the origin of facial expressions, BMVC/workshops 2017

Visualizing CNN Filters

Breuer & K. A deep learning perspective on the origin of facial expressions, BMVC/workshops 2017


Thank you for your attention
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