CRITIR: *Model-Based Reconstruction for Diffraction Tomography*

Charles A. Bouman
School of ECE, Purdue University
SIAM Imaging Science – 2016-05-25

Co-authors:
Aditya Mohen, Purdue University,
Xianghui Xiao, Argonne National Laboratory

Special Thanks:
Dula Parkinson, Lawrence Berkeley National Laboratory

Supported by:
Air Force Office of Scientific Research, MURI contract # FA9550-12-1-0458

INTEGRATED IMAGING

Outline

- TIMBIR: Time interlaced 4D reconstruction
  - Time-space reconstruction from sparse views
- CRITIR: Complex diffraction tomographic reconstruction
  - Diffraction tomography

Time Interlaced Model Based Iterative Reconstruction (TIMBIR)

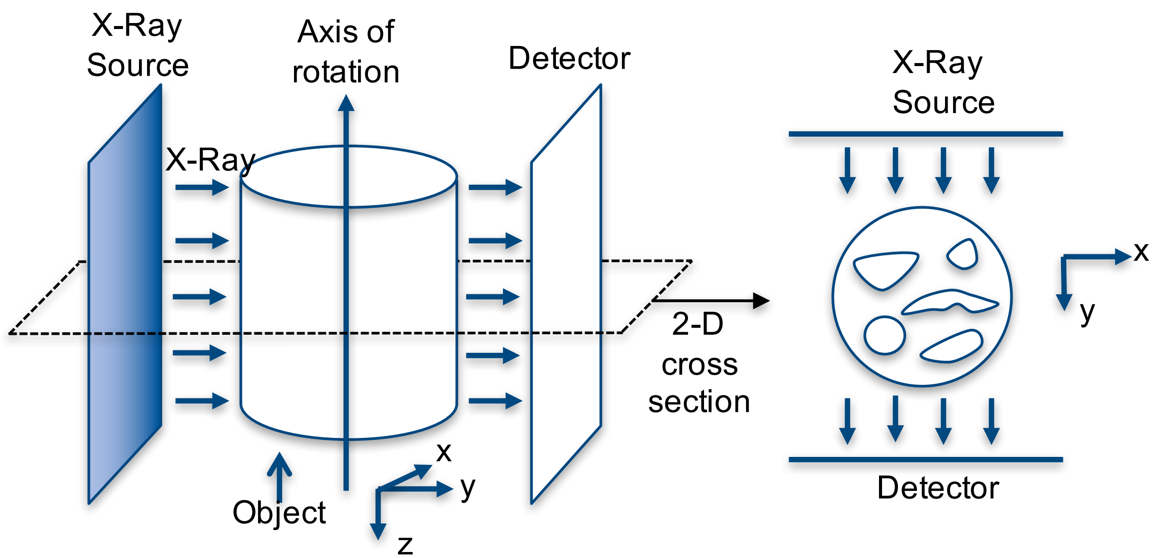
K. Aditya Mohan, Purdue
John Gibbs, NW
Prof. Peter Voorhees, NW
Prof. Marc De Graef, CMU
Dr. Xianghui Xiao, APS
Prof. Charles Bouman, Purdue

Synchrotron Imaging

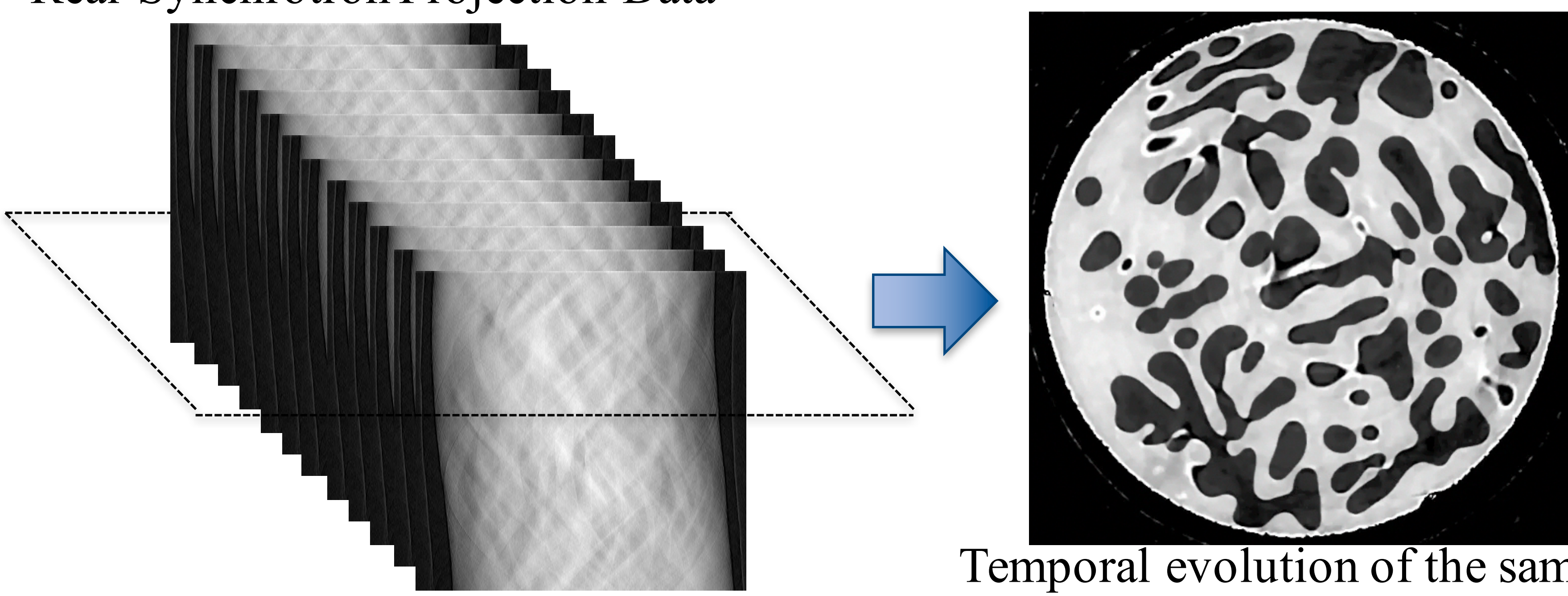
An aerial view of the APS synchrotron at Argonne National Laboratory (Argonne, IL).

- Why are they important?
  - Intense, columnated, monochromatic source of X-rays
  - Have become more widely available
- Facilities
  - Advanced Photon Source (APS), Argonne National Labs; Advance Light Source (ALS), Lawrence Berkeley Labs; Cornell High Energy Synchrotron Source (CHESS); Stanford Synchrotron Radiation lightsource (SLAC); National Synchrotron Light Source, Brookhaven; Swiss Light Source.

Synchrotron Imaging of Time-Varying Sample



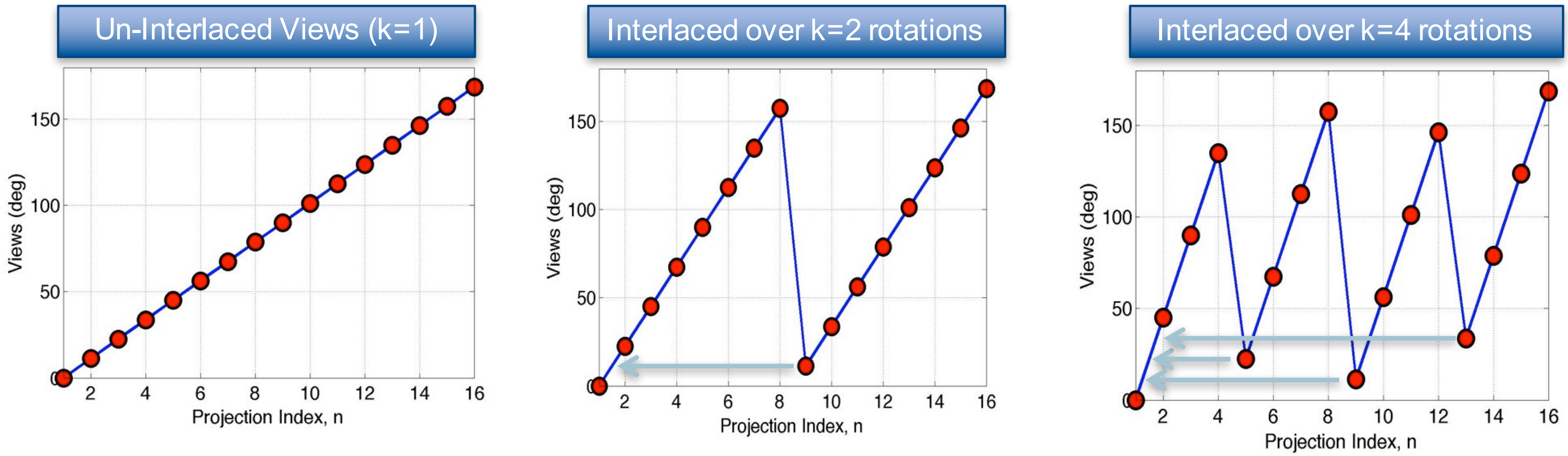
Real Synchrotron Projection Data



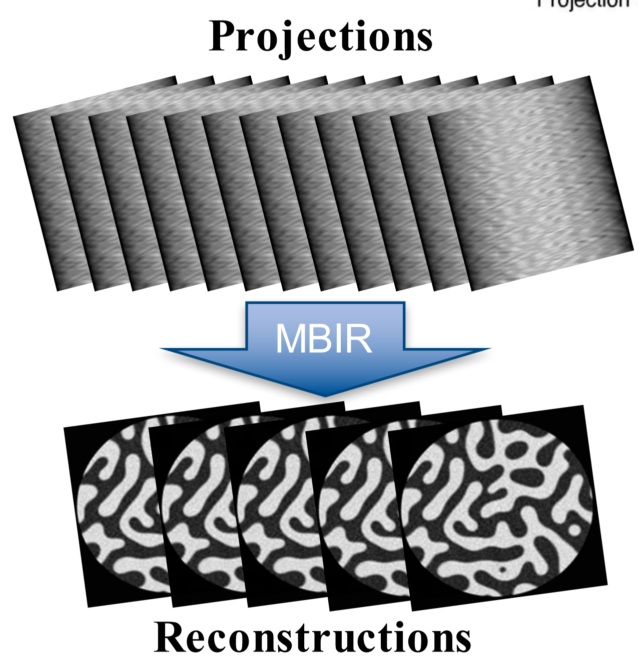
Temporal evolution of the sample

TIMBIR: Time Interlaced Model Based Iterative Reconstruction

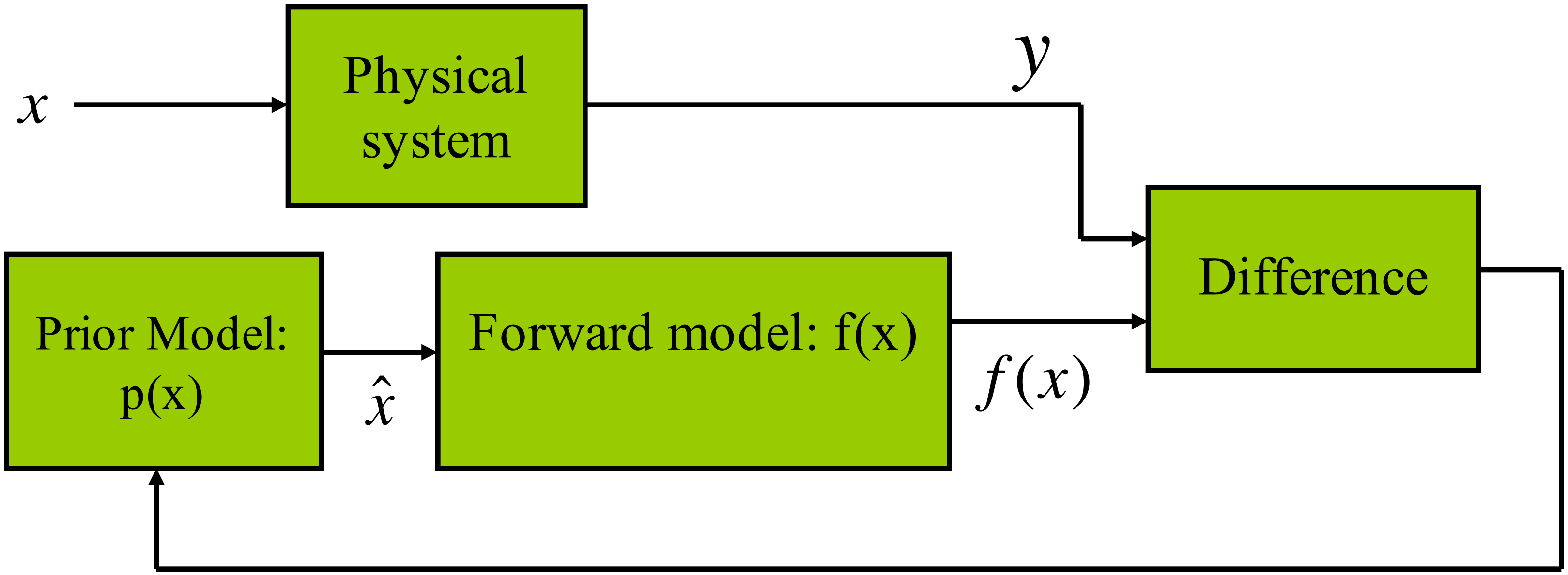
- Interlace the views over K rotations of the object.



- Perform 4D MBIR reconstruction at any desired temporal resolution.



Model Based Iterative Reconstruction (MBIR): A General Framework for Solving Inverse Problems



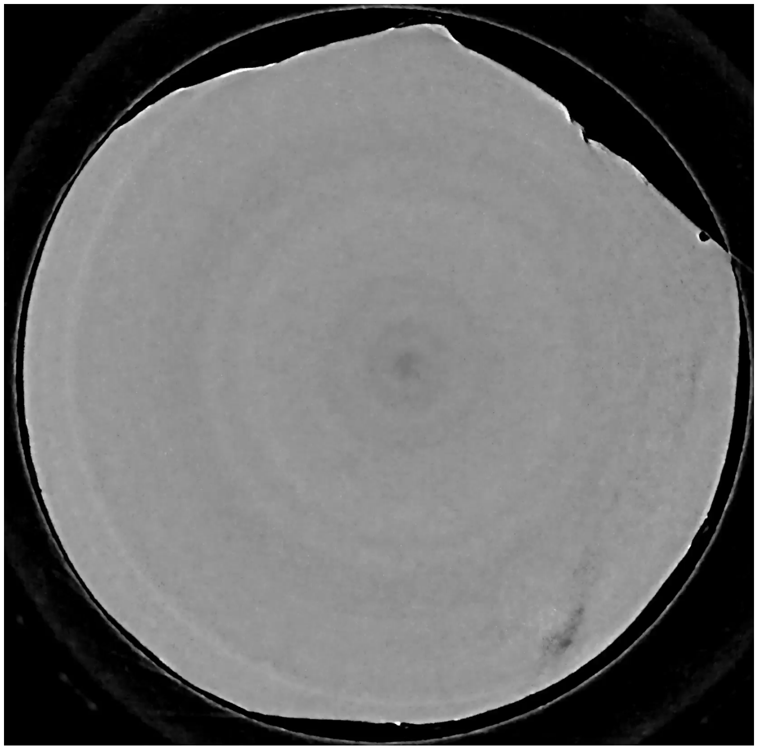
$$\hat{x} \leftarrow \arg\max_{x} \{ \log p(y \mid x) + \log p(x) \}$$

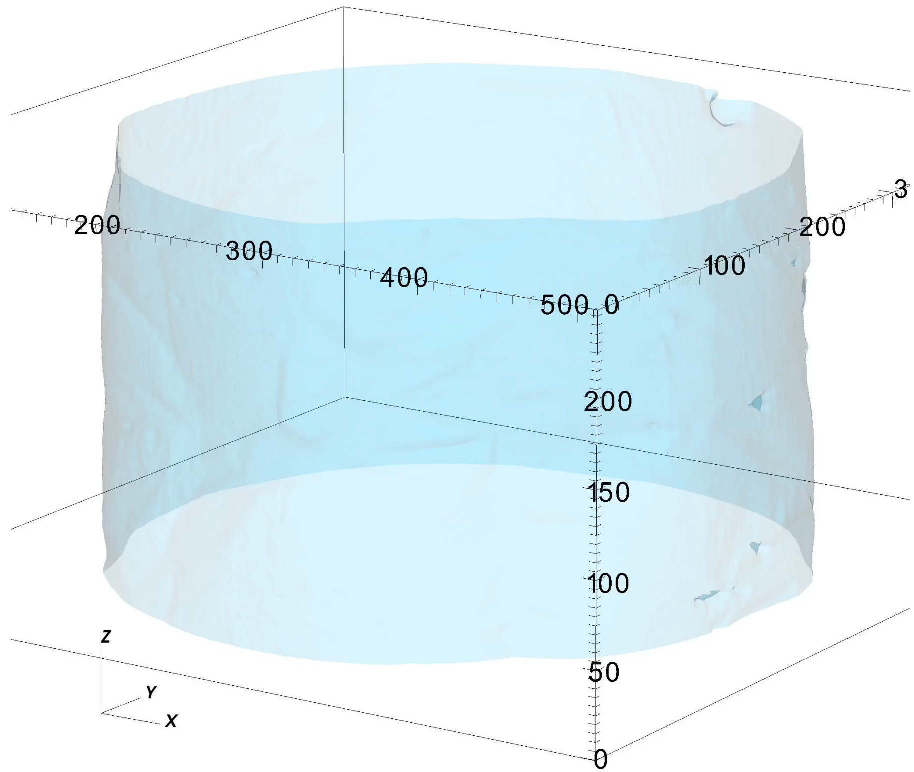
forward model prior model

$\hat{x}$ – Reconstructed object

y – Measurements from physical system

TIMBIR: 16x Speed-Up (k=16)





Experiment - APS

- Solidification of aluminum and copper mixture
- Temperature decreased at 2^0 Celsius per minute
- 2000 views in a frame, interlaced over 16 sub-frames
- 16x speed up

Reconstruction

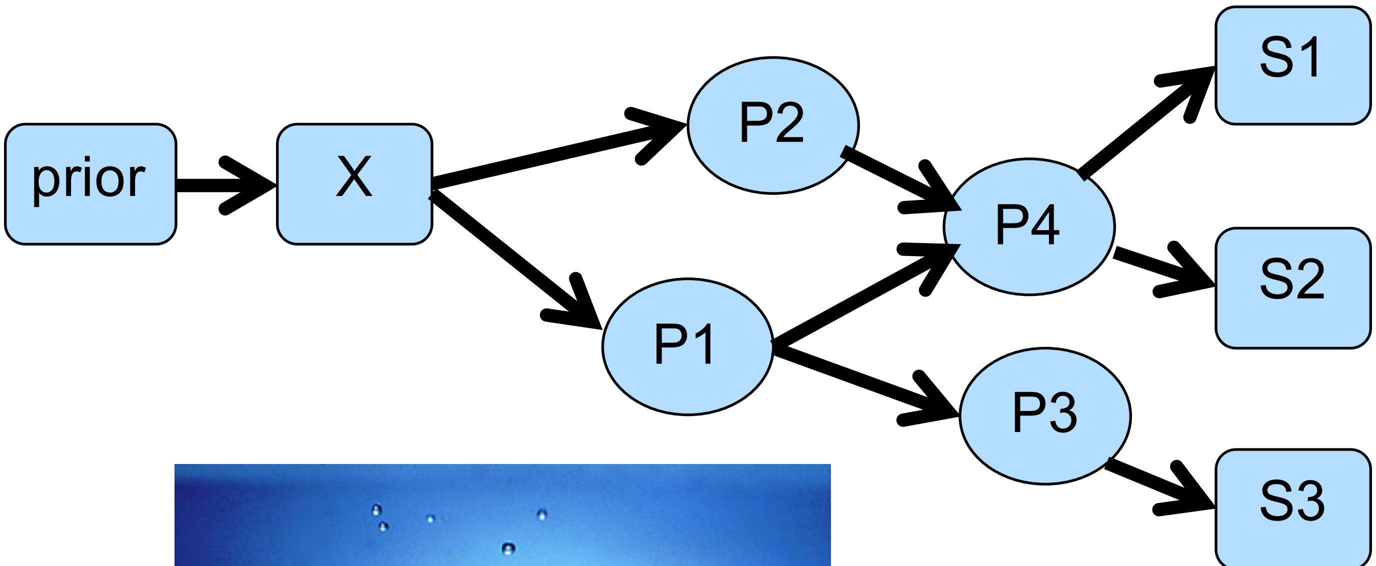
- (2048 x 2048 x 1000) space x 16 time
- $(0.65\ \mu m)^3$ voxel size
- 1.8 sec time step
- Image scaling: 10000 HU to 60000 HU

Complex Refractive Index Tomography

- CRITIR:
  - Complex refractive index tomographic iterative reconstruction
- Combine:
  - Phase retrieval
  - Complex refractive index
  - Tomographic reconstruction

Forward Models for Complex Systems

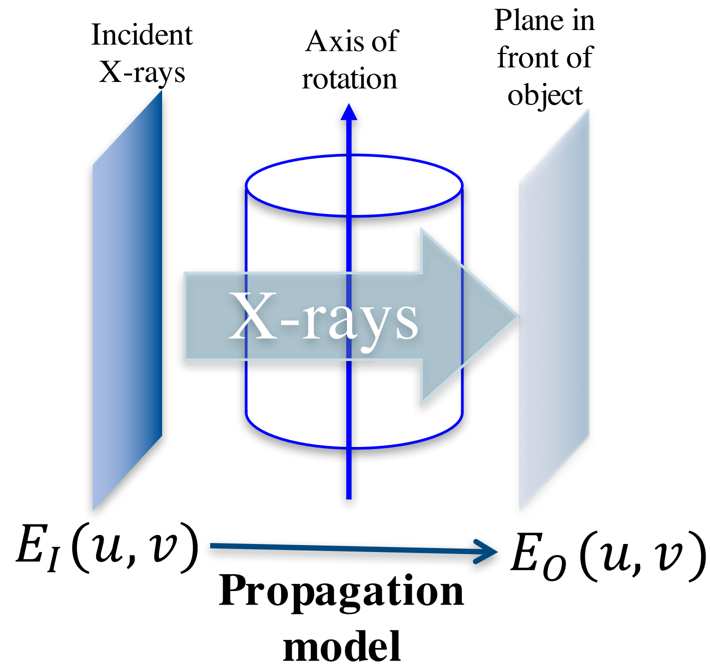
- Real data results from a series of nested, nonlinear, networked interactions. How do we deal with this?



Traditional Tomography Model

- Attenuation (Beer's Law)

$$\mu(u, v, w) \rightarrow \text{Attenuation}$$



- Transfer function

$$E_O(u, v) = E_I(u, v) \exp\left(-\int \mu(u, v, w) dw\right)$$

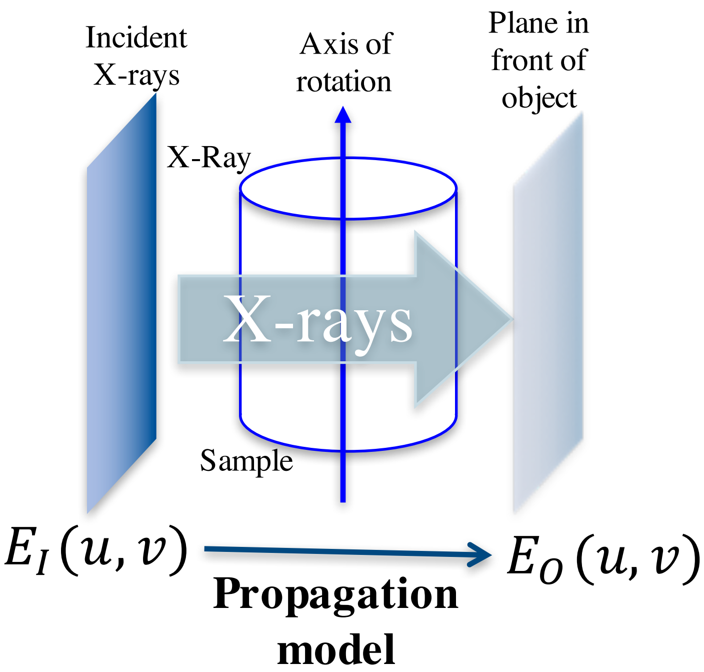
Complex Tomography Model

- Complex refractive index

$$n(u, v, w) = 1 - \delta(u, v, w) + i\beta(u, v, w)$$

$\delta \rightarrow$ Refractive index decrement

$\beta \rightarrow$ Absorption index



- Input/Output

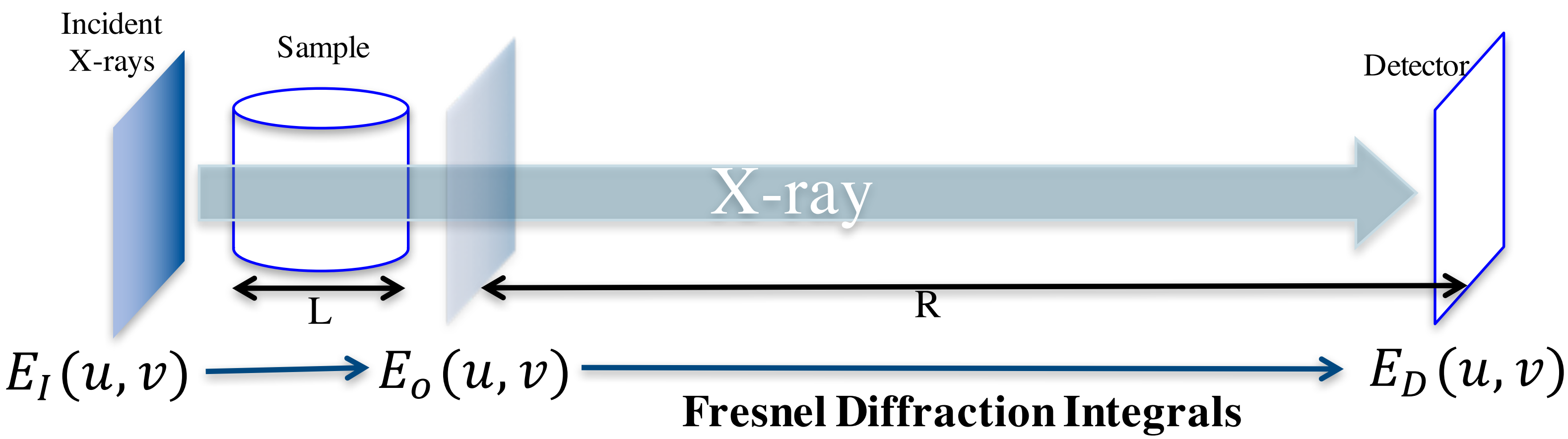
$$E_O(u, v) = E_I(u, v)F(u, v)$$

- Transfer function

$$F(u', v') = \exp\left(-\frac{2\pi i}{\lambda}\int n(u, v, w)dw\right)$$

$$= \exp\left(-\frac{2\pi i}{\lambda}\int (1 - \delta(u, v, w))dw\right)\exp\left(-\frac{2\pi i}{\lambda}\int \beta(u, v, w)dw\right)$$

Fresnell Diffration



- Fresnel integral

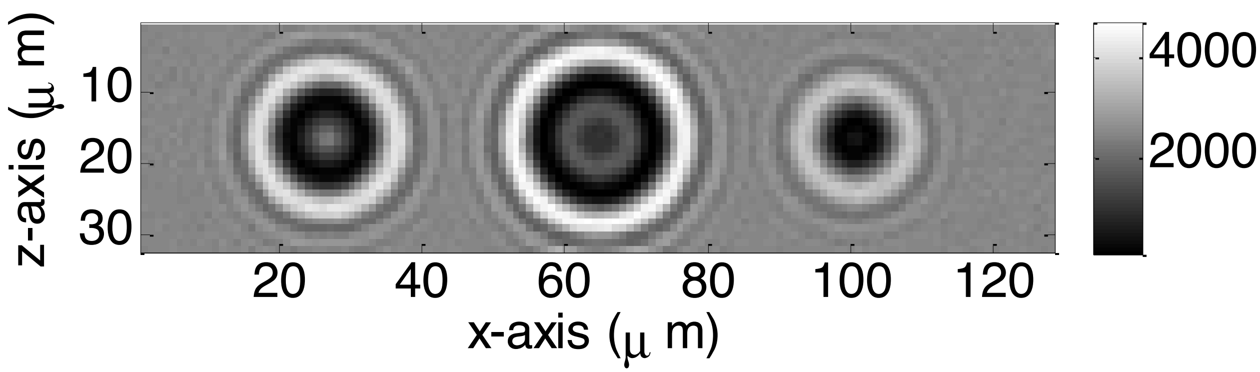
$$E_D(u,v) = \iint_{u'v'} E_O(u',v') \exp\left(-\frac{ik}{2R}\left[(u-u')^2 + (v-v')^2\right]\right) du'dv'$$

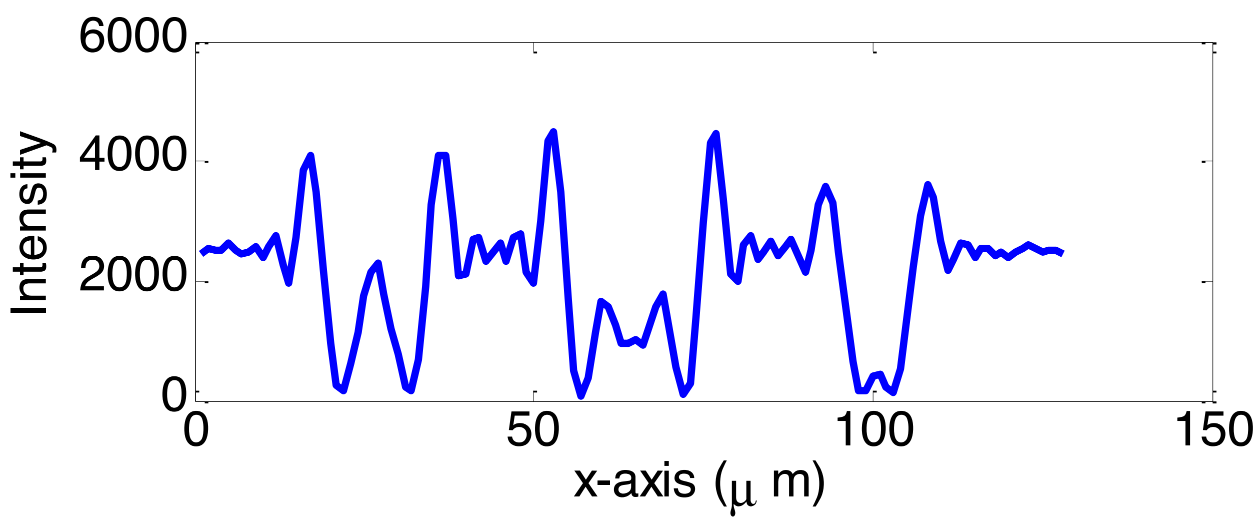
- Energy detector

$$Y(u,v) = |E_D(u,v)|$$

Simulated Measured Image

Object to detector distance, R = 400mm

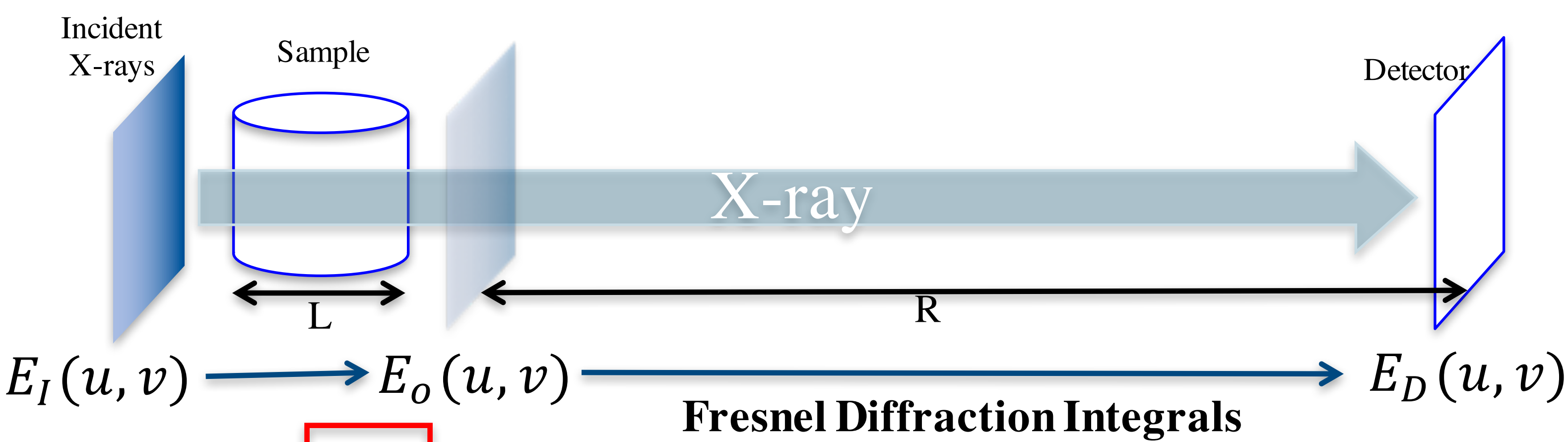




Forward Projection

- Pixel width – 0.7 μm
- Size – 32 x 128

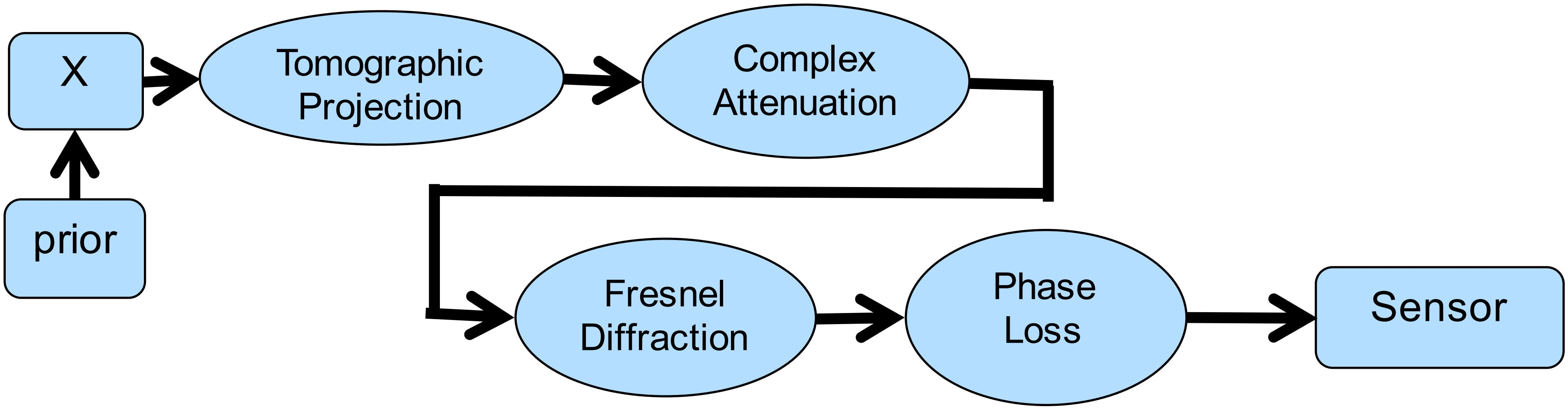
CRITIR Forward Model



$$F(u',v') = \exp\left(-\frac{2\pi i}{\lambda}\int n(u',v',w\,)dw\right)$$

$$Y(u,v) = \left|\iint_{u'v'} E_O(u',v')\exp\left(-\frac{ik}{2R}[(u-u')^2+(v-v')^2]\right)du'dv'\right|$$

CRITIR Measurement Model



Fresnel Diffraction

Tomographic Projection

Noise

Measurement

$$y_i = \left| H \exp\left(-A_i x\right) \right| + w_i$$

Phase Loss

Complex Attenuation

CRITIR Optimization problem

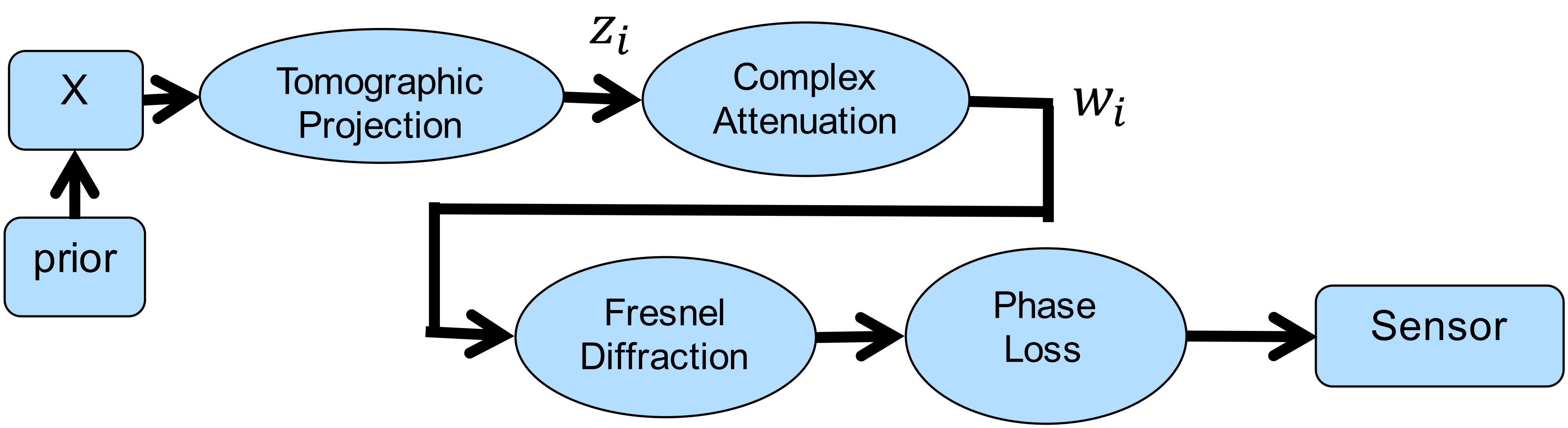
- MAP reconstruction given by

$$\hat{x} = \arg\min_x \left\{ \sum_{i=0}^{M-1} \| y_i - |H \exp(A_i x)| \|_{\Lambda_i}^2 + R(x) \right\}$$

$R(x)$ − Prior model for the object
Λ_i − Noise matrix
M − Number of views

- Problem:
  - Non-convex optimization
  - Non-linear attenuation
  - Phase recovery

Decomposing Optimization Problem



- Tomographic forward projection constraint

$$z_i = A_i x$$

- Complex attenuation constraint

$$w_i = \exp(-z_i)$$

- Regularized phase recovery

$$\hat{x} = \arg\min_{w} \left\{ \sum_{i=0}^{M-1} \| y_i - |H\, w_i| \|_{\Lambda_i}^2 + R(x) \right\}$$

Reconstruction using modified ADMM

Phase Retrieval

For all l in 1 to L_{max}

For all k in 1 to K_{max}

$$(\widehat{w}, \widehat{\Omega}) \leftarrow argmin_{w,\Omega} \sum_{i=1}^{M} \left(\|y_i - \Omega_i H D w_i\|_{\Lambda_i}^2 + \frac{\gamma}{2} \|\exp(-\hat{z}_i) - w_i + v_i\|^2 \right)$$

$$s.t \left|\Omega_{i,k,k}\right| = 1$$

$$\hat{z} \leftarrow argmin_z \sum_{i=1}^{M} \left(\frac{\mu}{2} \|A_i \hat{x} - z_i + u_i\|^2 + \frac{\gamma}{2} \|\exp(-z_i) - \widehat{w}_i + v_i\|^2 \right)$$

Estimate complex exponential

$$v_i \leftarrow v_i + (\exp(-\hat{z}_i) - \widehat{w}_i)$$

end

$$\hat{x} \leftarrow argmin_x \left\{ \frac{\mu}{2} \sum_{i=1}^{M} \|\hat{z}_i - A_i x - u_i\|^2 + R(x) \right\}$$

Tomographic inversion

$$u_i \leftarrow u_i + (A_i \hat{x} - \hat{z}_i)$$

end

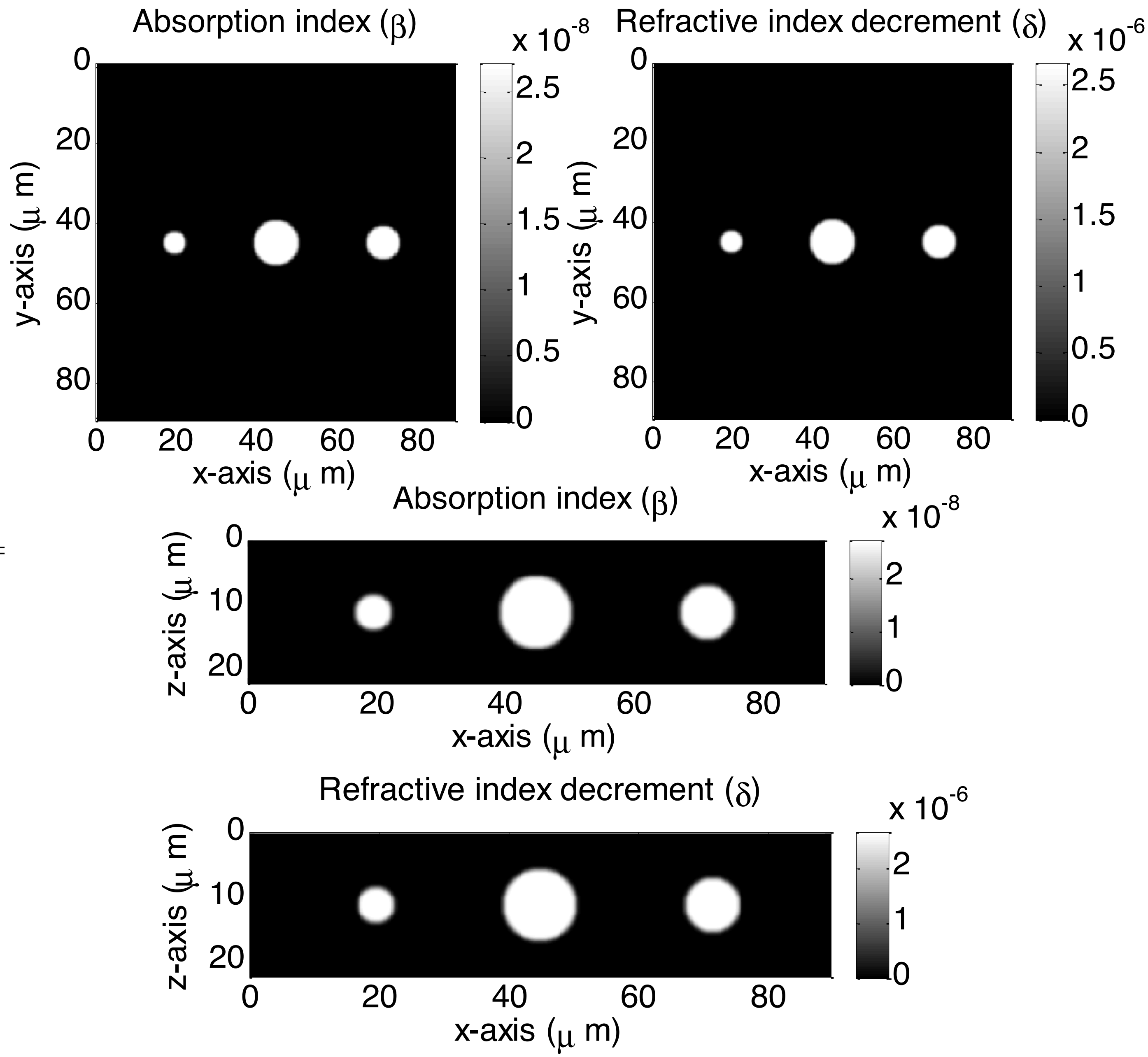
Simulated Phantom

X-ray parameters

- Energy – 3keV
- Wavelength – 41.328 pm

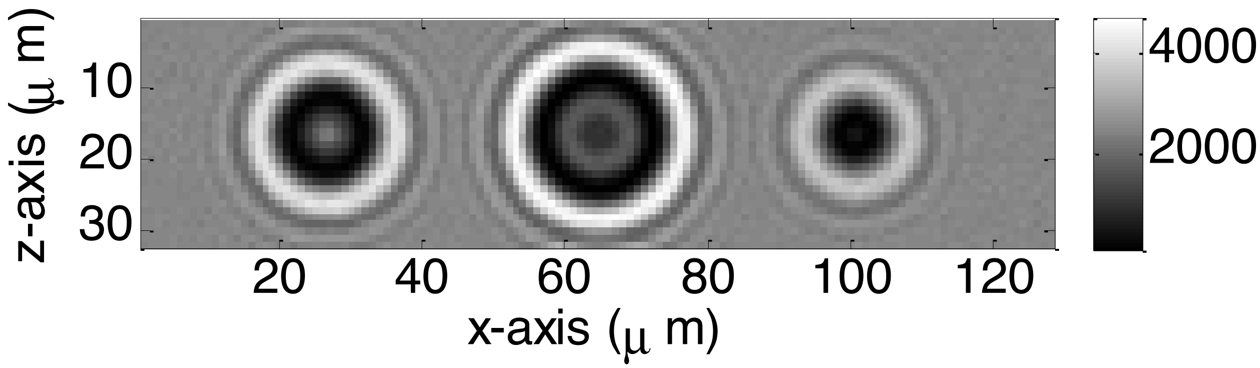
Latex Spheres Phantom

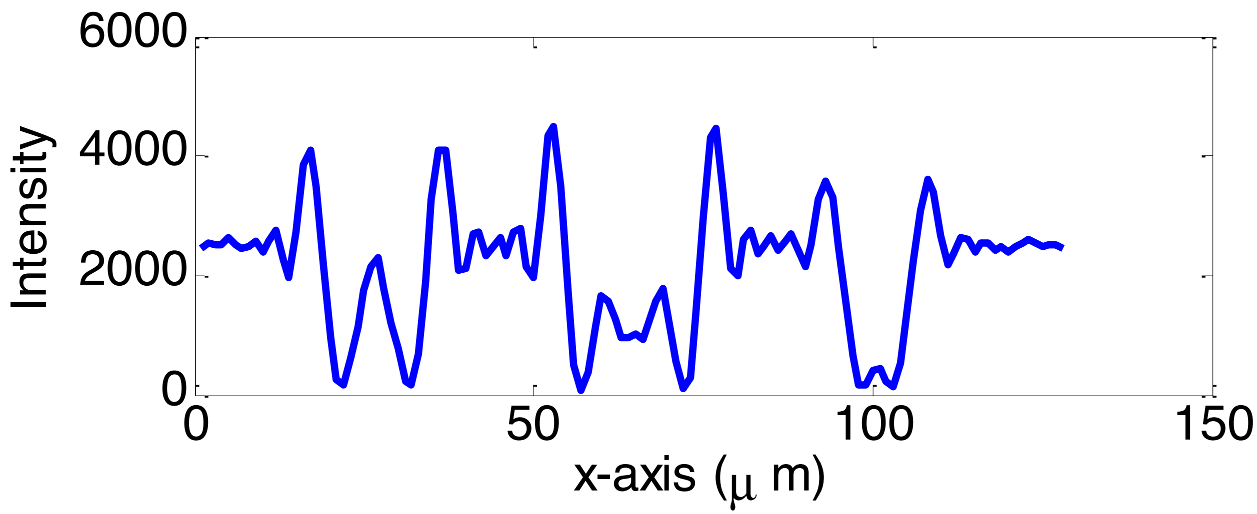
- Voxel width – 0.175 µm
- Size – 128 x 512 x 512
- Absorption index of spheres, $\beta = 2.7218 \times 10^{-8}$
- Refractive index decrement of spheres, $\delta = 2.6639 \times 10^{-6}$
- Surrounding material is weakly absorbing.



Noisy Measured Image

Object to detector distance, R = 400mm

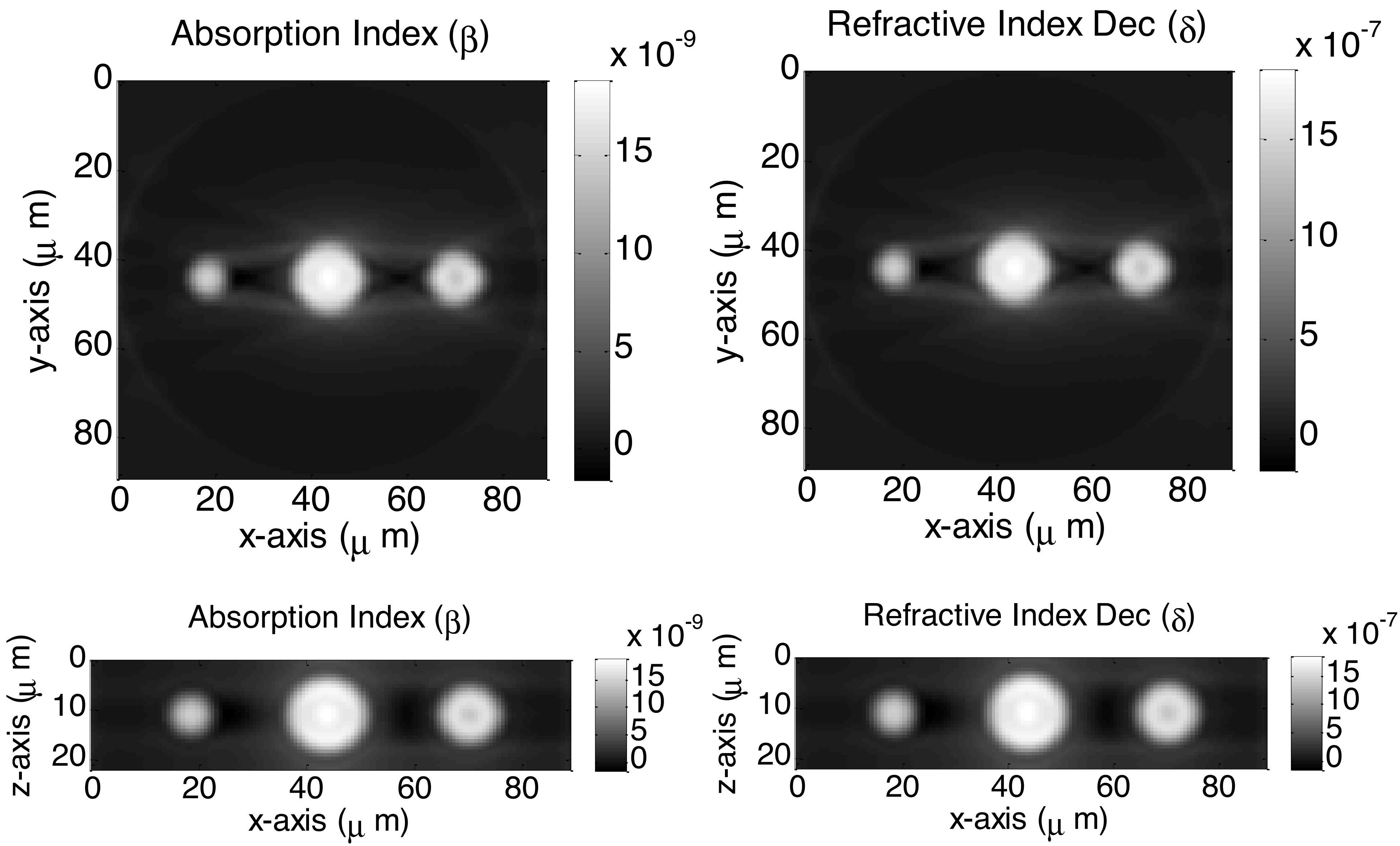




Forward Projection

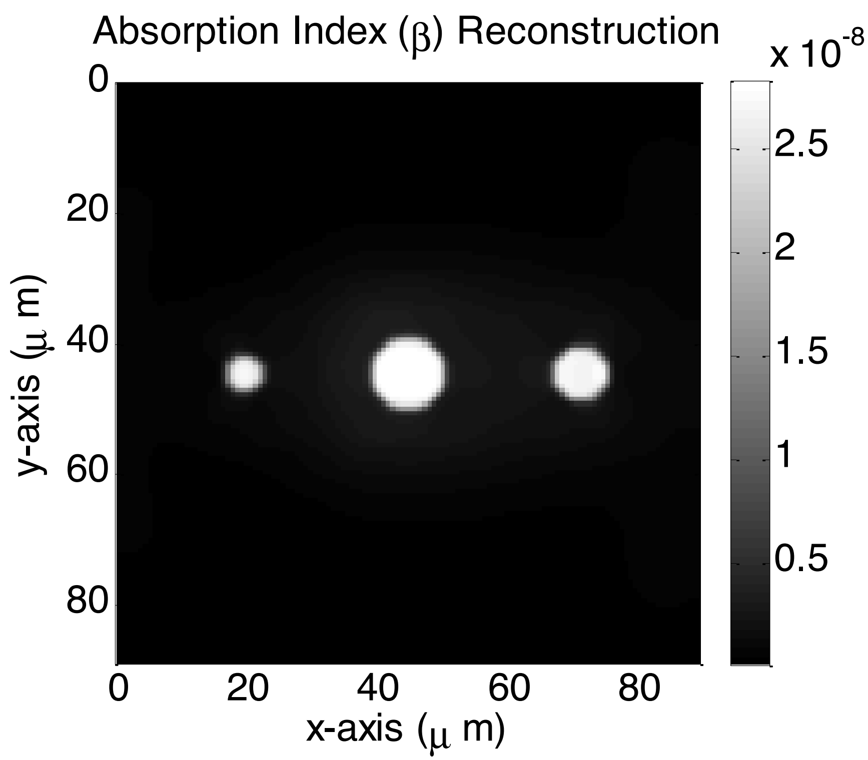
- Pixel width – 0.7 µm
- Size – 32 x 128

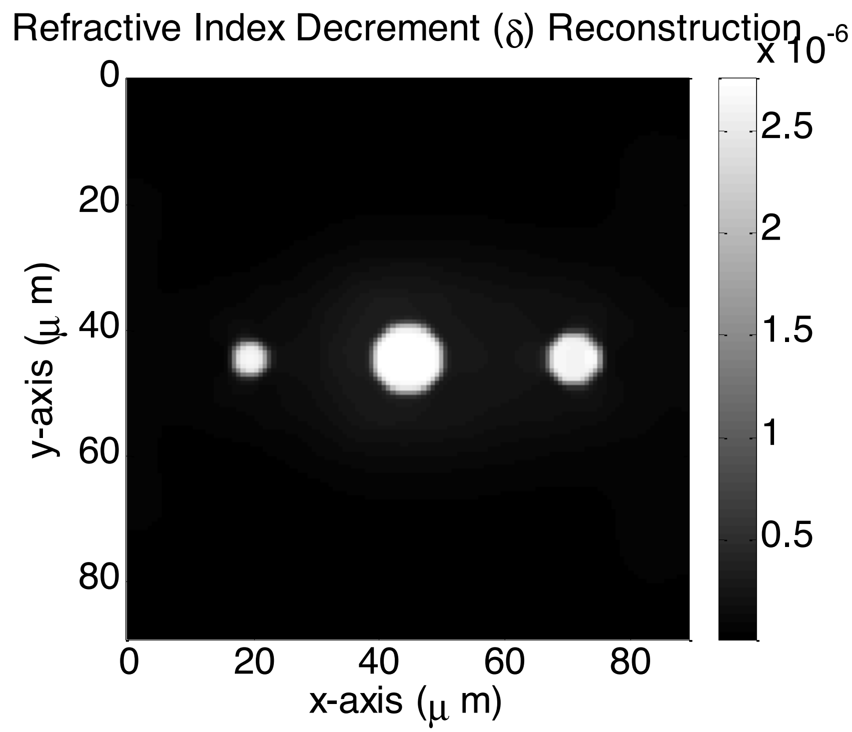
Paganin & FBP

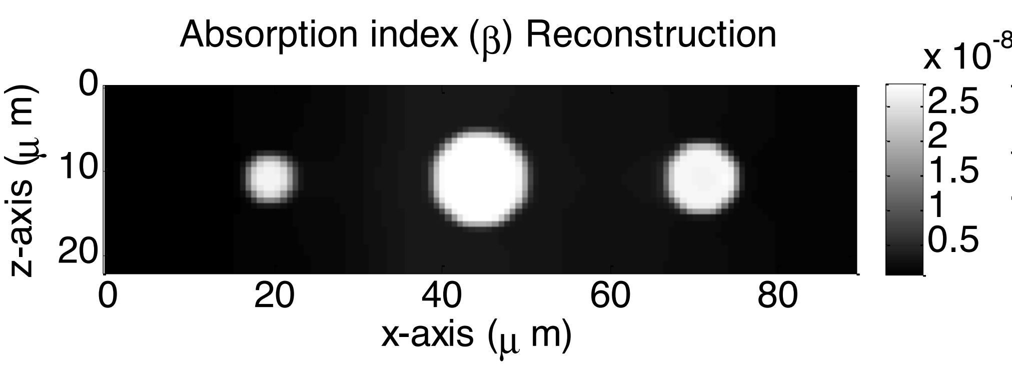


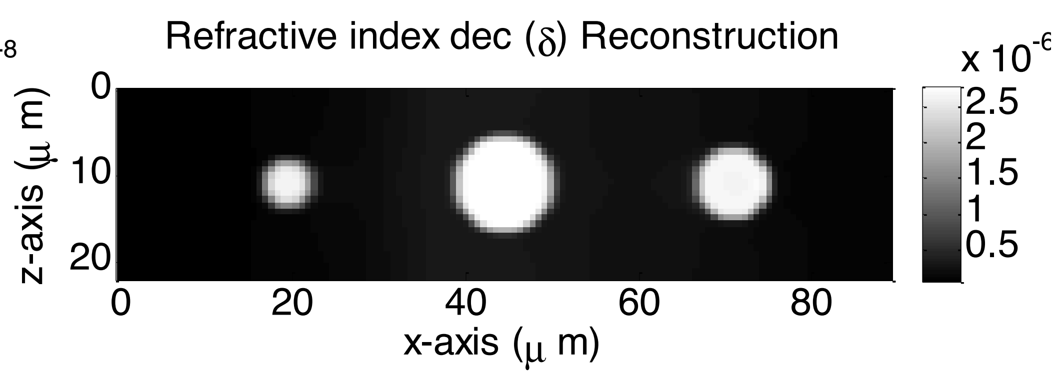
Absorption Index (β)
x 10^-9
15
10
5
0
0
20
40
60
80
y-axis (μ m)
x-axis (μ m)
Refractive Index Dec (δ)
x 10^-7
Absorption Index (β)
z-axis (μ m)
Refractive Index Dec (δ)

CRITIR Reconstruction

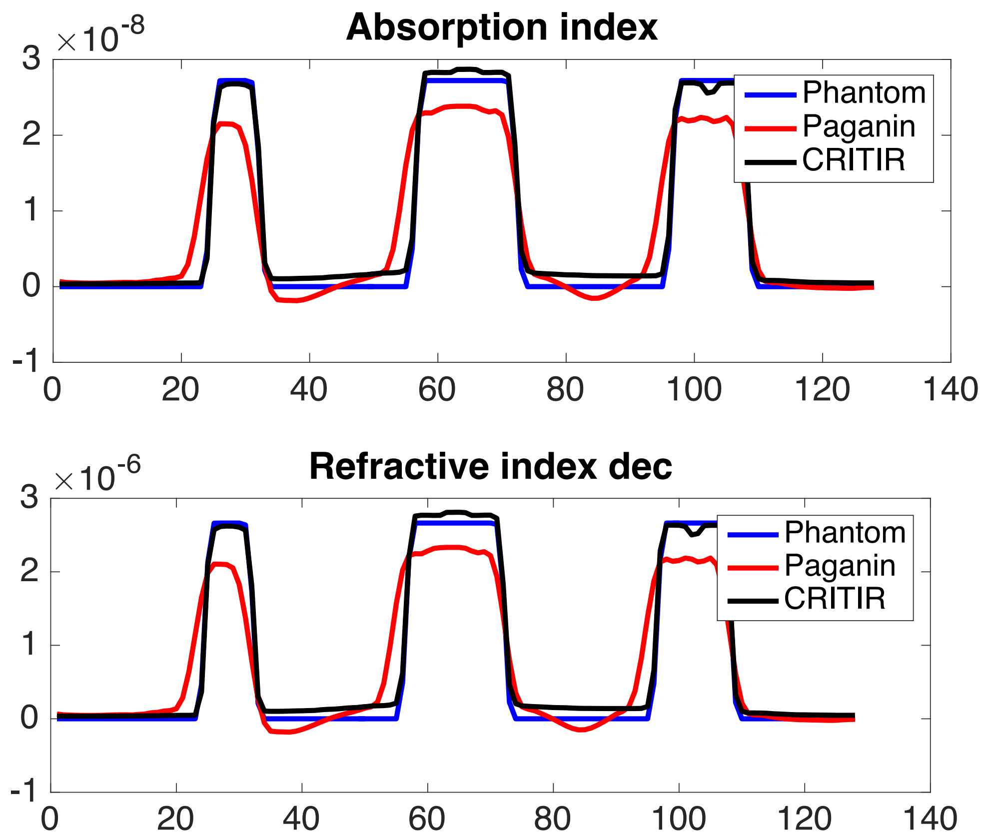








Line Plot and RMSE Comparisons



RMSE Comparisons

	Absorption Index	Refractive Index Decrement
Paganin & FBP	1.1770×10^{-9}	1.1520×10^{-7}
CRITIR	5.1365×10^{-10}	5.0274×10^{-8}

Take-Aways

- ADMM/variable splitting can be used to break down large problems into smaller ones.
- Complex diffraction tomography is non-convex optimization problem, but it can be done.
- Questions?
  - Will this work for larger amounts of diffraction?
  - Are the better ways to solve optimization problem?