SIAM Annual Meeting 2017

Chaos and learning in spiking neural networks

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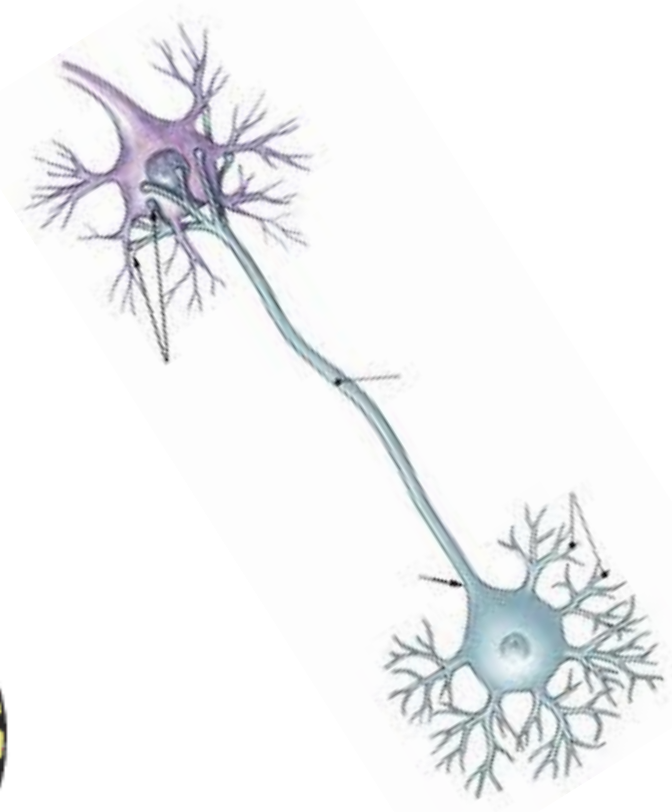
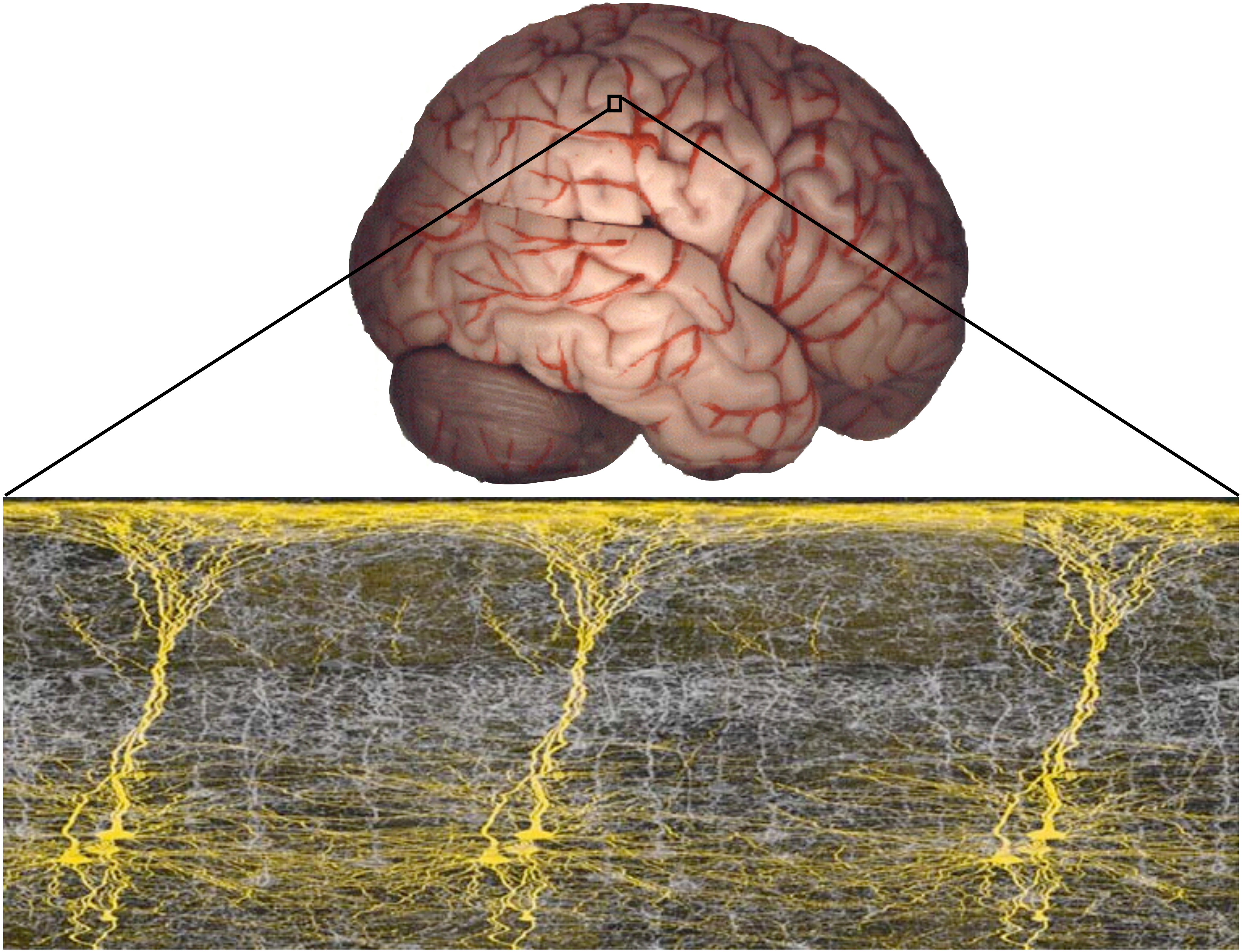
NIH
National Institute of Diabetes and Digestive and Kidney Diseases

National Geographic

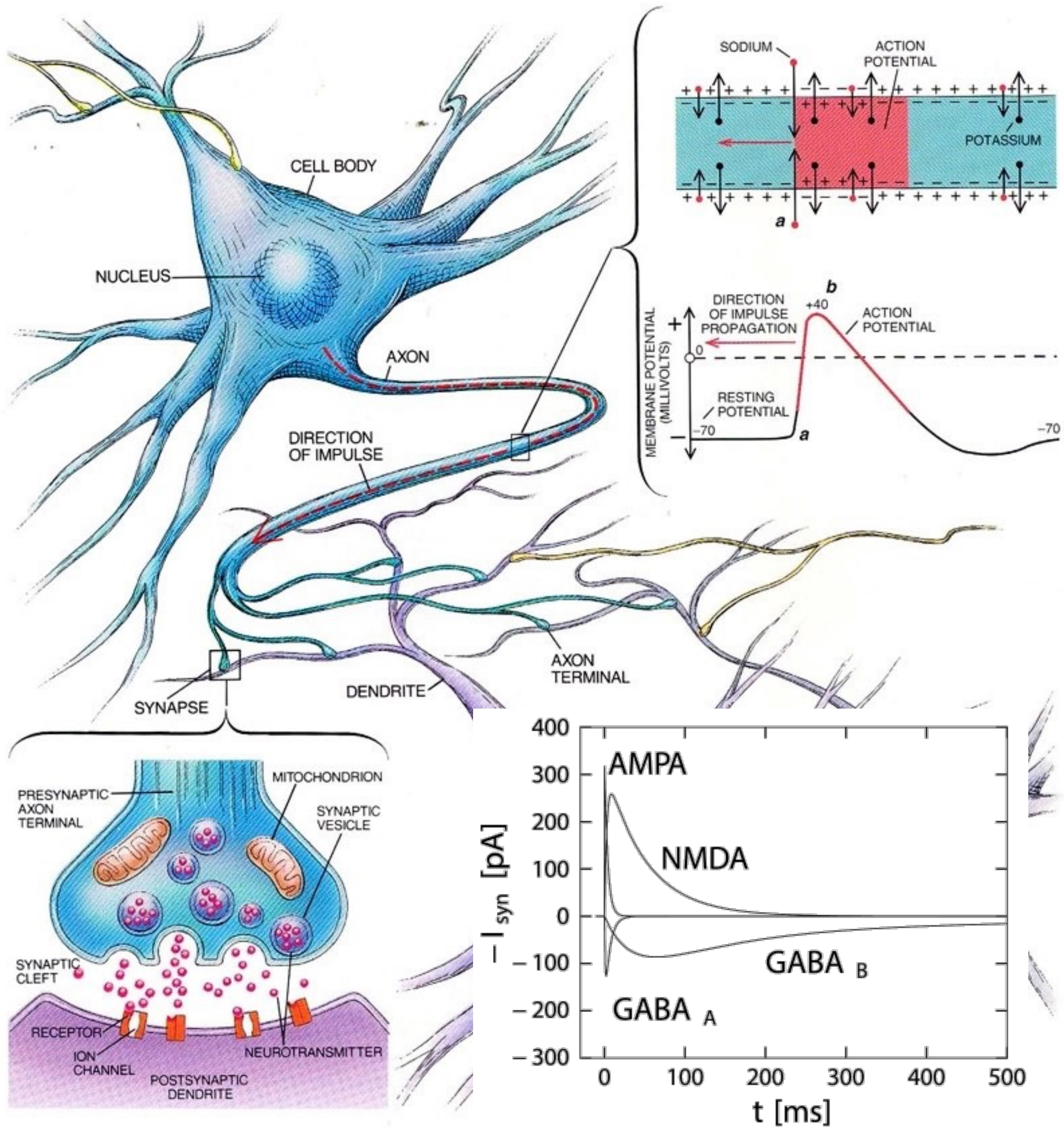
10^{11} neurons



10^{15} connections



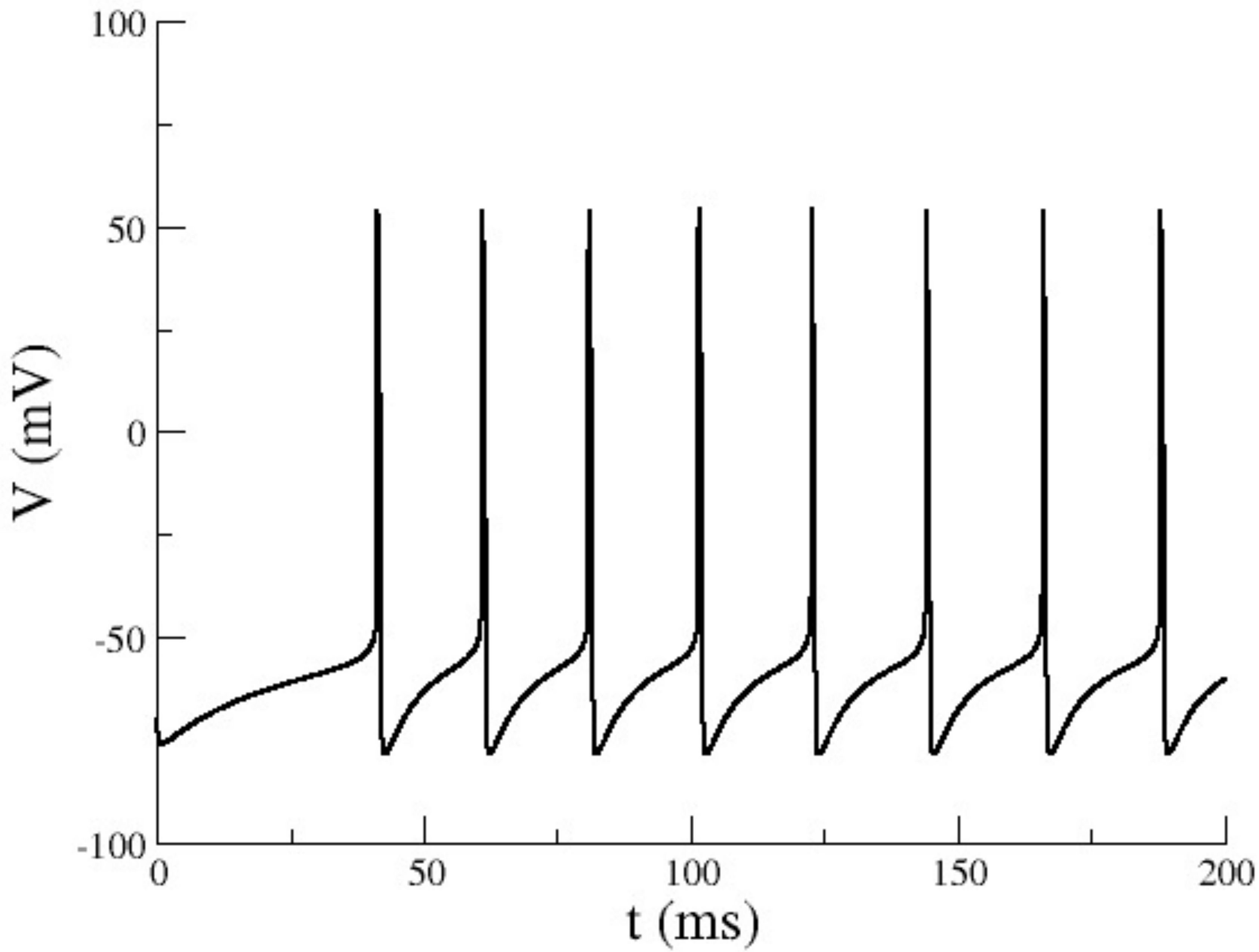
Cortex homogeneous at microscopic level



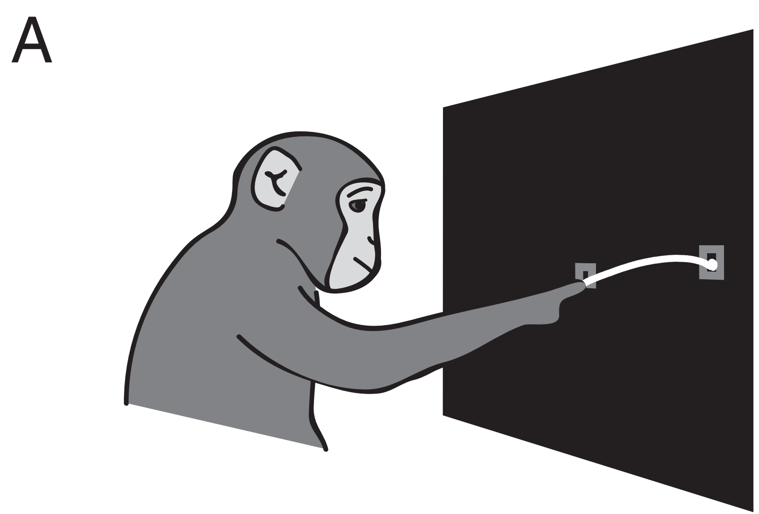
How Neurons Communicate

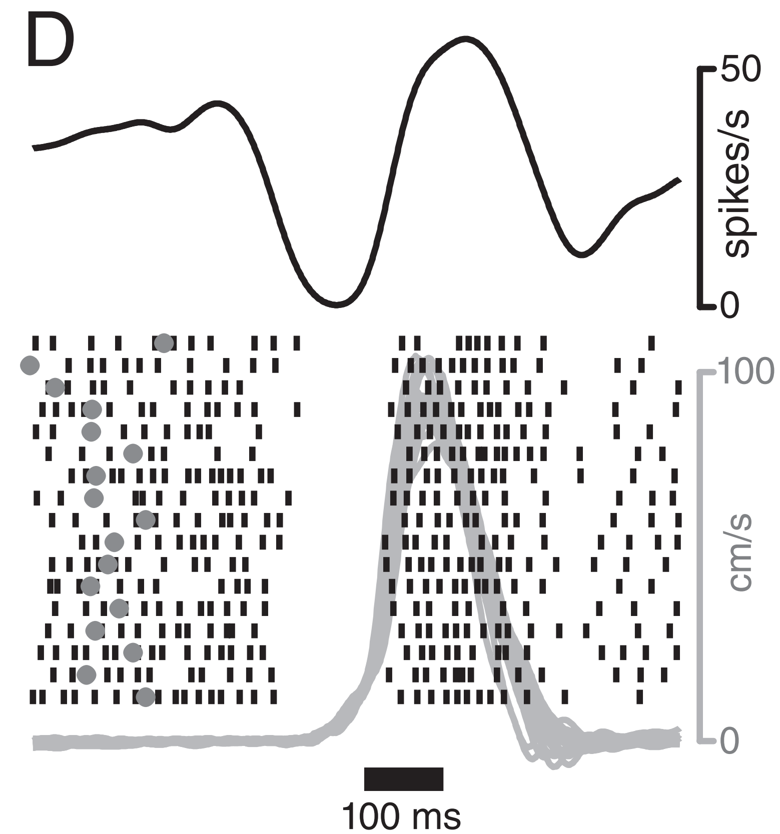
Scientific American

Neurons “spike”



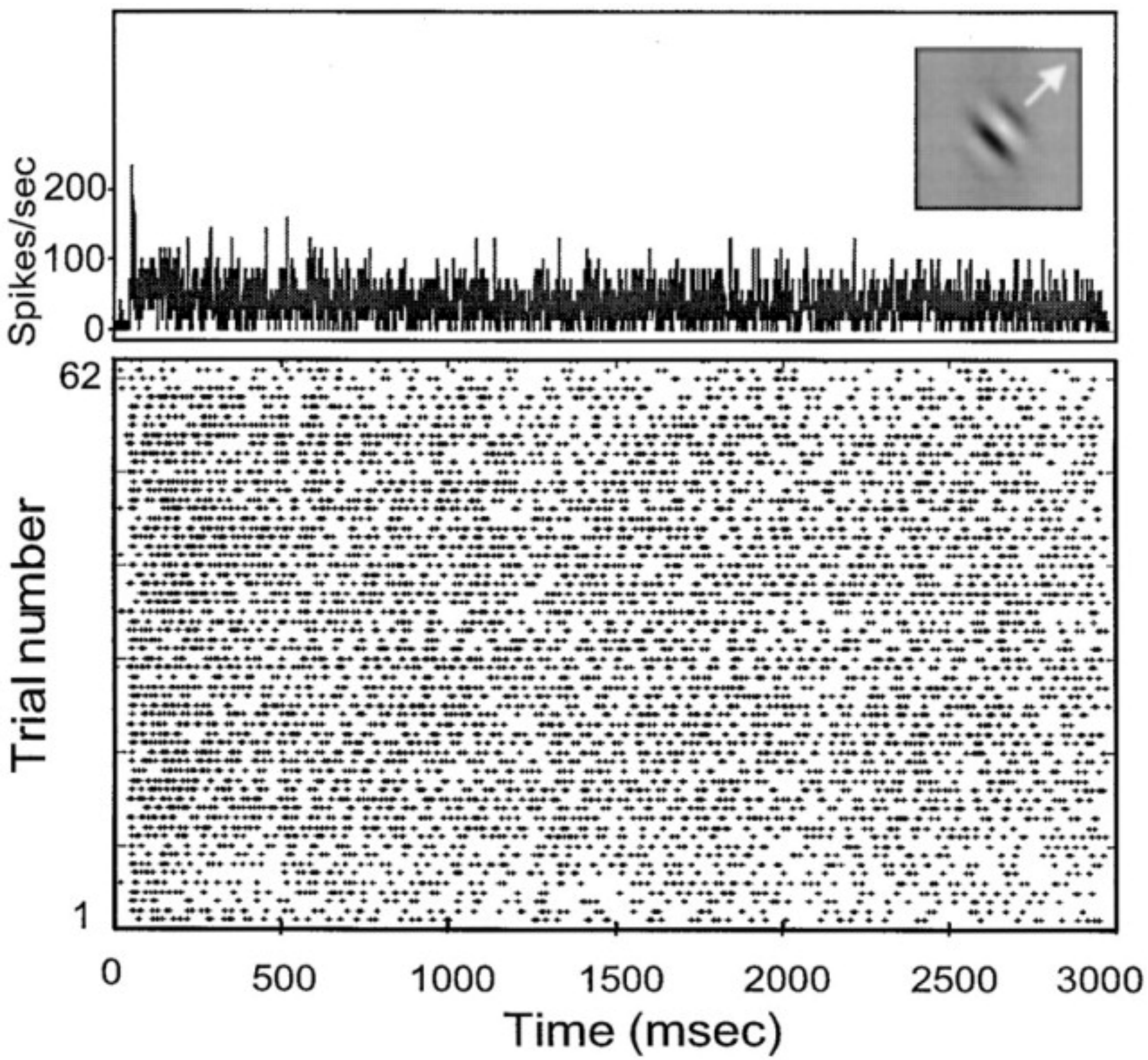
Spiking correlated to behavior





Churchland and Shenoy, 2007

Spiking is variable



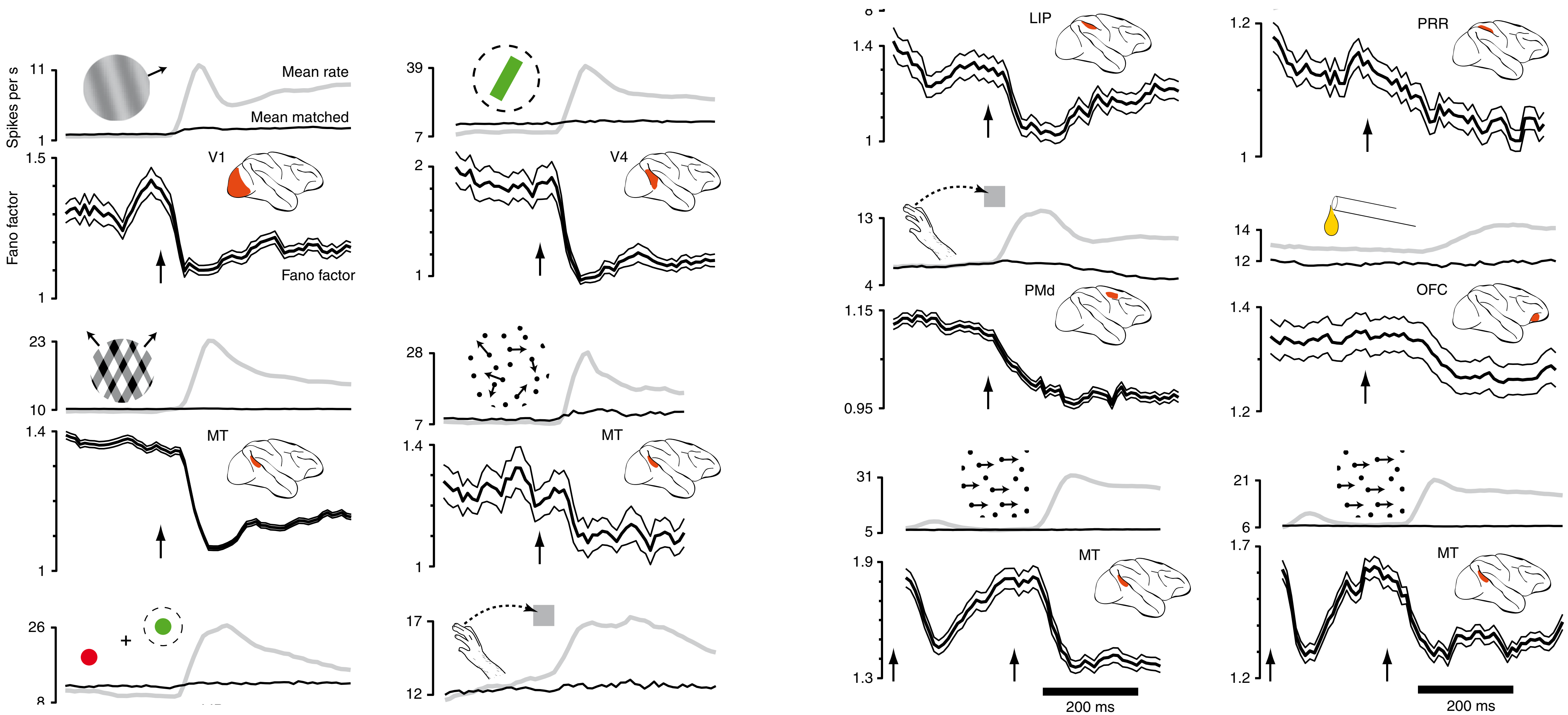
ISI CV ~ 1

Fano Factor ~ 1

Poisson process

Buracas et al. 1998

Spiking variability correlated to behavior



Churchland et al. 2010

Neurons are reliable

CV = .1

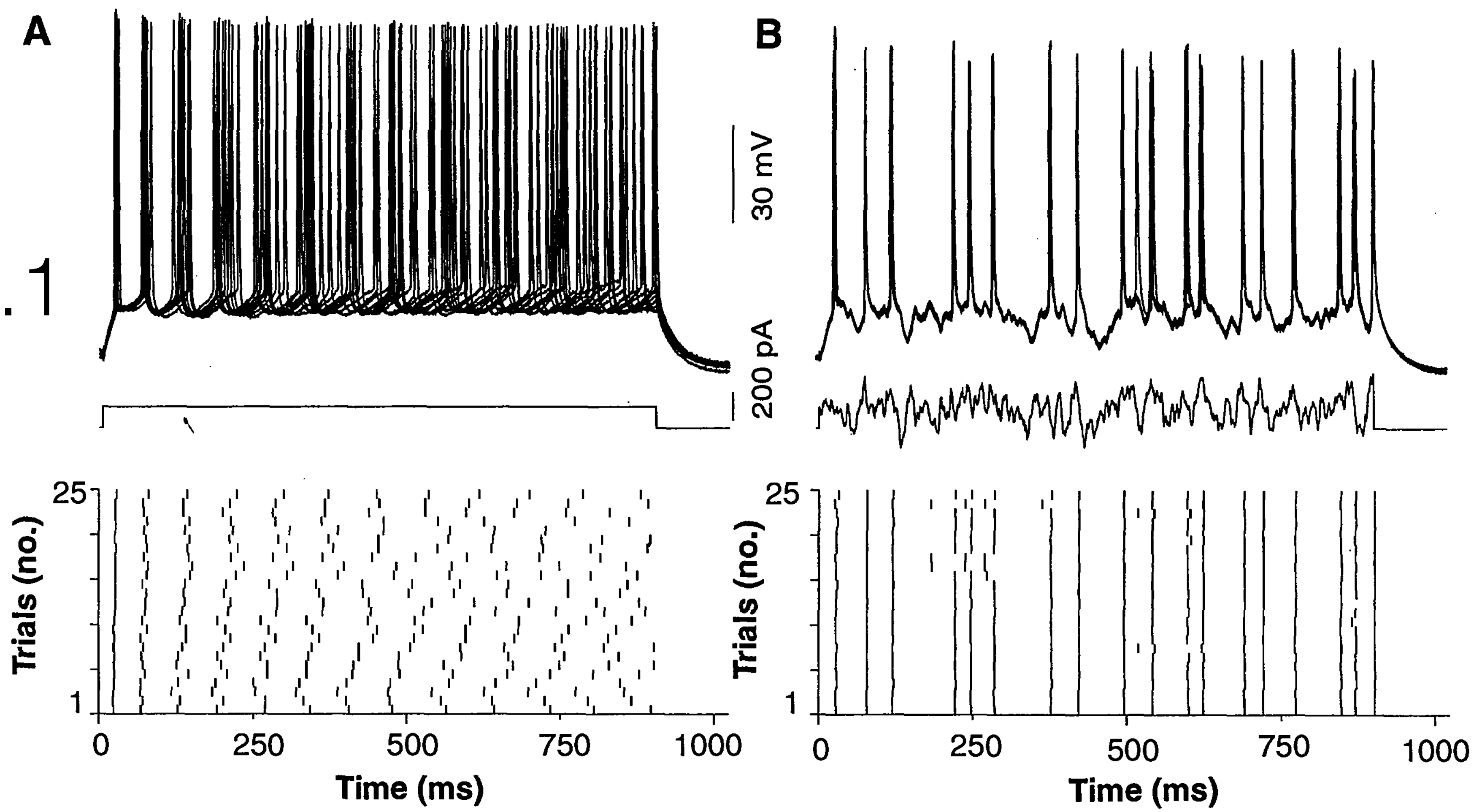


Fig. 1. Reliability of firing patterns of cortical neurons evoked by constant and fluctuating current. (**A**) In this example, a superthreshold dc current pulse (150 pA, 900 ms; middle) evoked trains of action potentials (approximately 14 Hz) in a regular-firing layer-5 neuron. Responses are shown superimposed (first 10 trials, top) and as a raster plot of spike times over spike times (25 consecutive trials, bottom). (**B**) The same cell as in (A) was again stimulated repeatedly, but this time with a fluctuating stimulus [Gaussian white noise, μ_s = 150 pA, σ_s = 100 pA, τ_s = 3 ms; see (*14*)].

Mainen and Sejnowski, 1995

Softy-Koch "Paradox", 1993

Spiking variable Neurons reliable

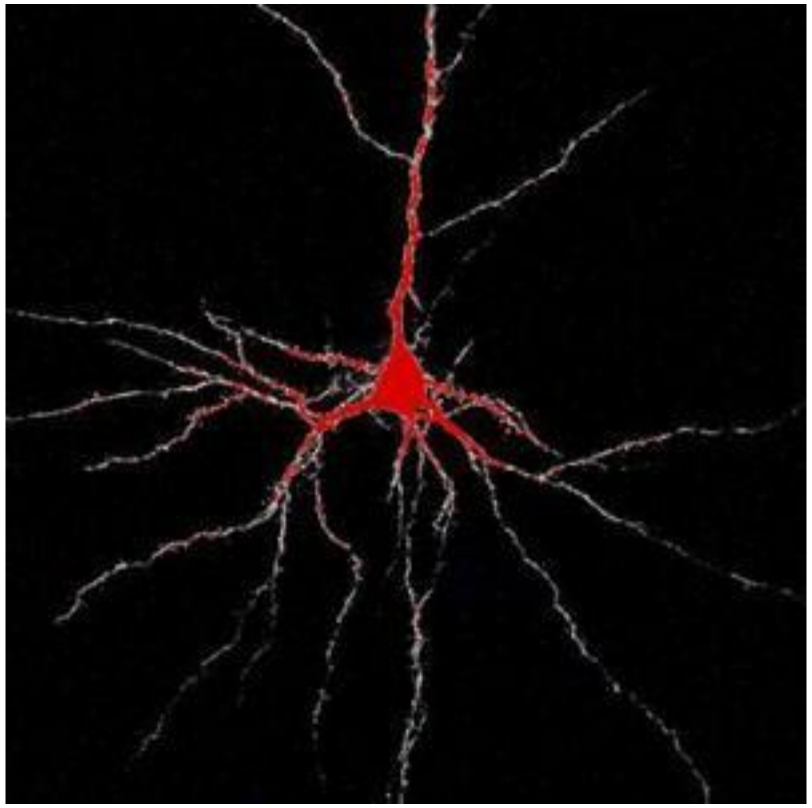
Open Questions Remain

Answer: Balanced State

Van Vreeswijk and Sompolinsky, 1996 & 1998

Connections strong and sparse,
Chaotic state is a fixed point

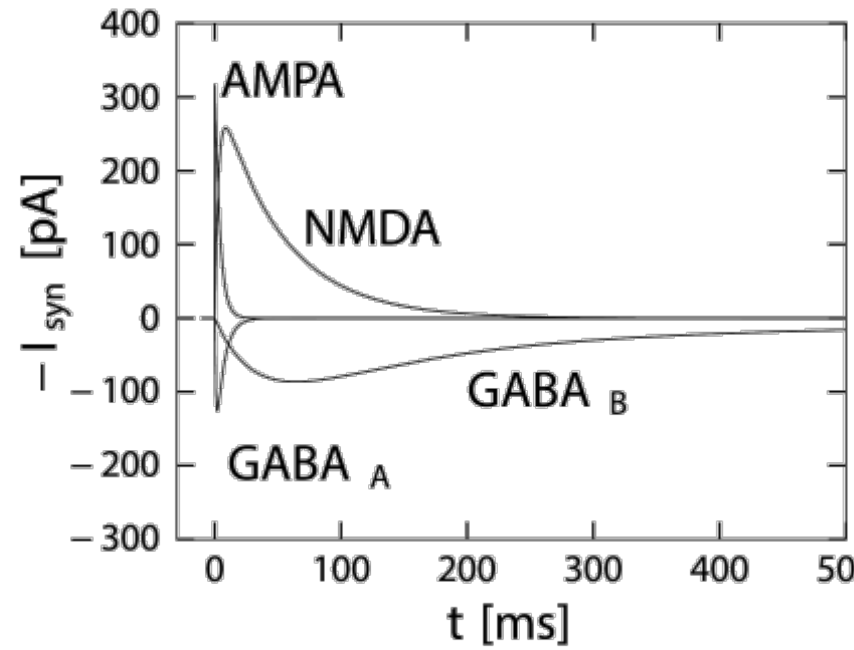
Network

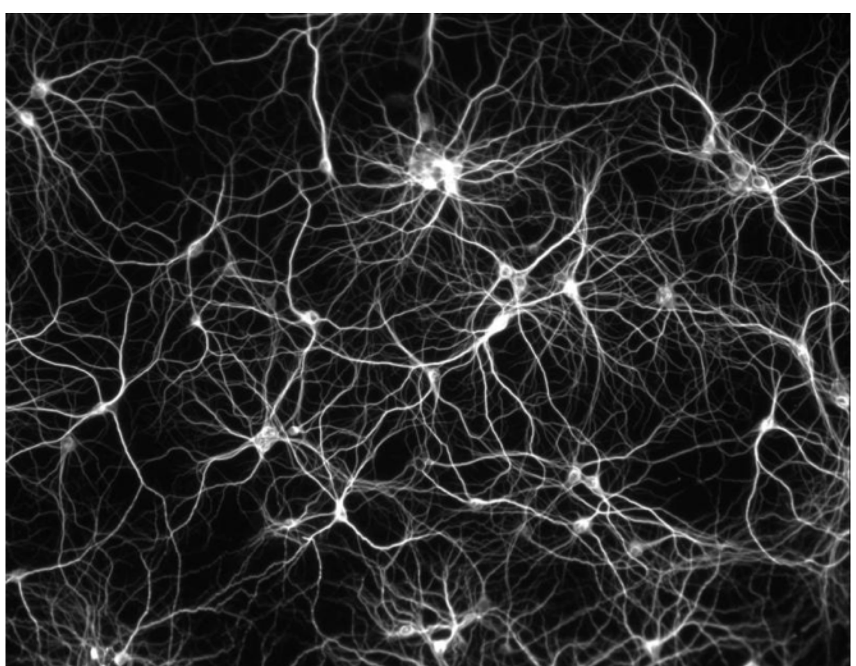


$$\frac{dV}{dt} = g(V, I)$$

Input

Synapse



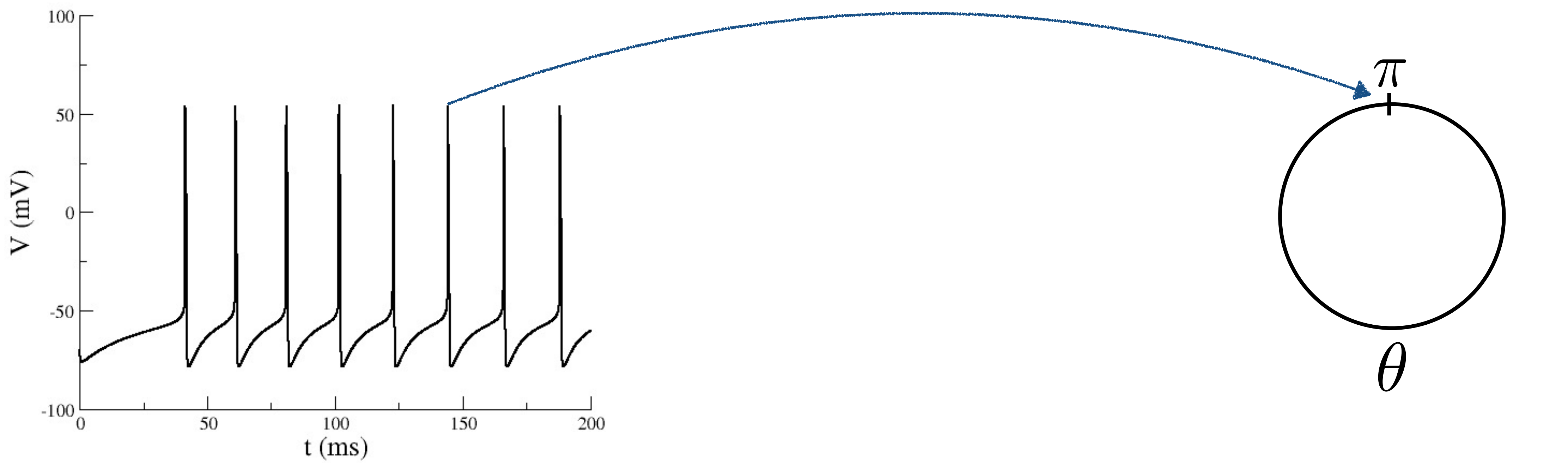


$$\frac{dV_i}{dt} = g\left(V_i, I + \sum_{j=1}^{N} w_{ij} s(V_j)\right)$$

Connection weights

What is the dynamical repertoire of a network of spiking neurons?

Neuron phase model



$$\frac{d\theta}{dt} = 1 - \cos\theta + I(1 + \cos\theta)$$

Theta neuron

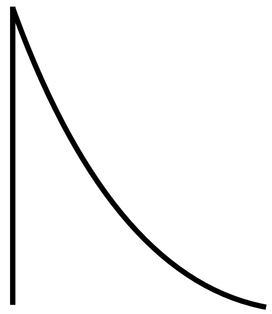
aka Ermentrout-Kopell canonical model

3 parameters

Network

$$\dot{\theta}_i = 1 - \cos\theta_i + (I + u_i)(1 + \cos\theta_i)$$

Synapse



$$\dot{u}_i = -\beta u_i + \beta \sum_{j,s} w_{ij}\delta(t - t_j^s(\theta_j))$$

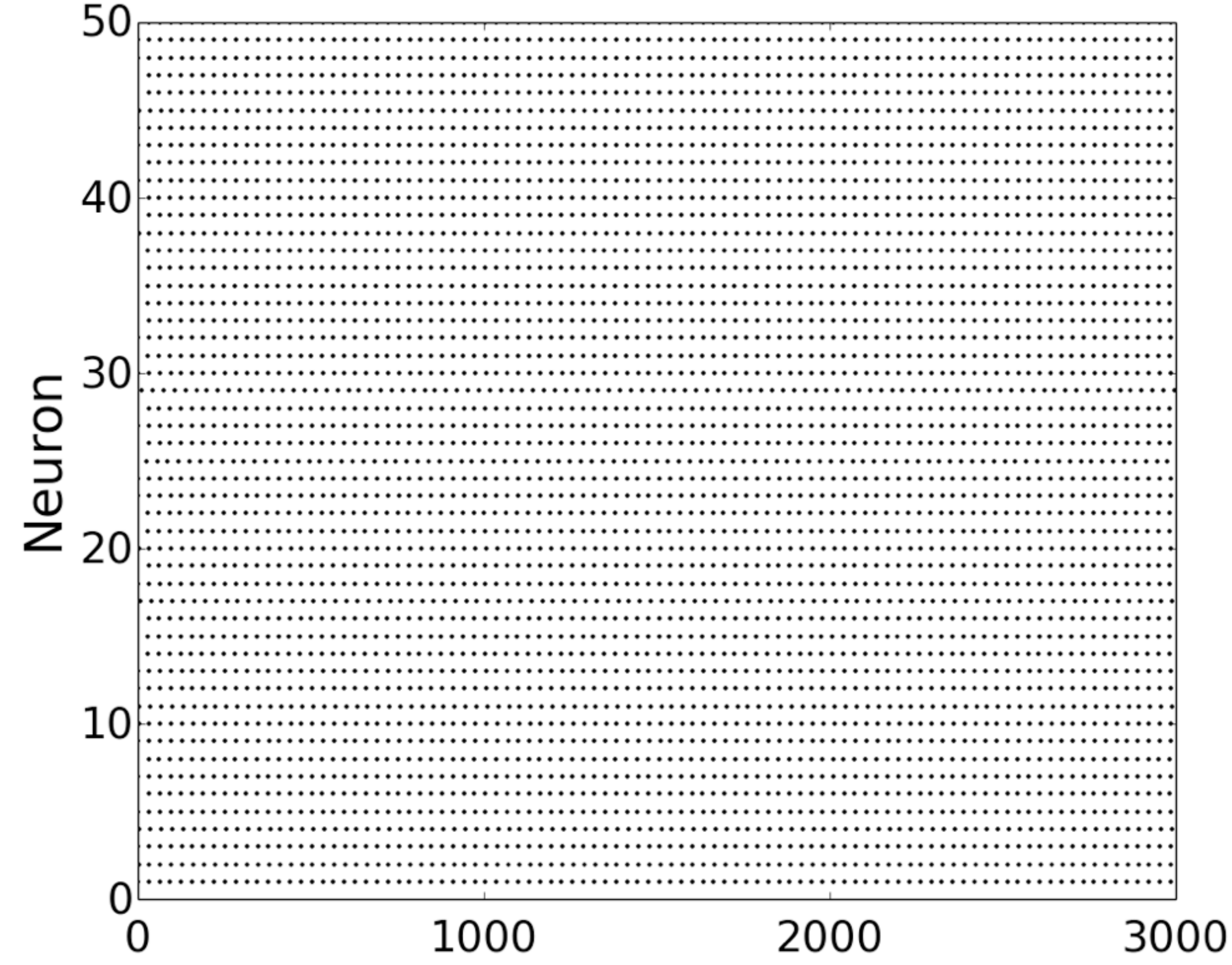
$$t_j^s = \{t \mid \theta_j(t) = \pi, \dot{\theta}_j > 0\}$$

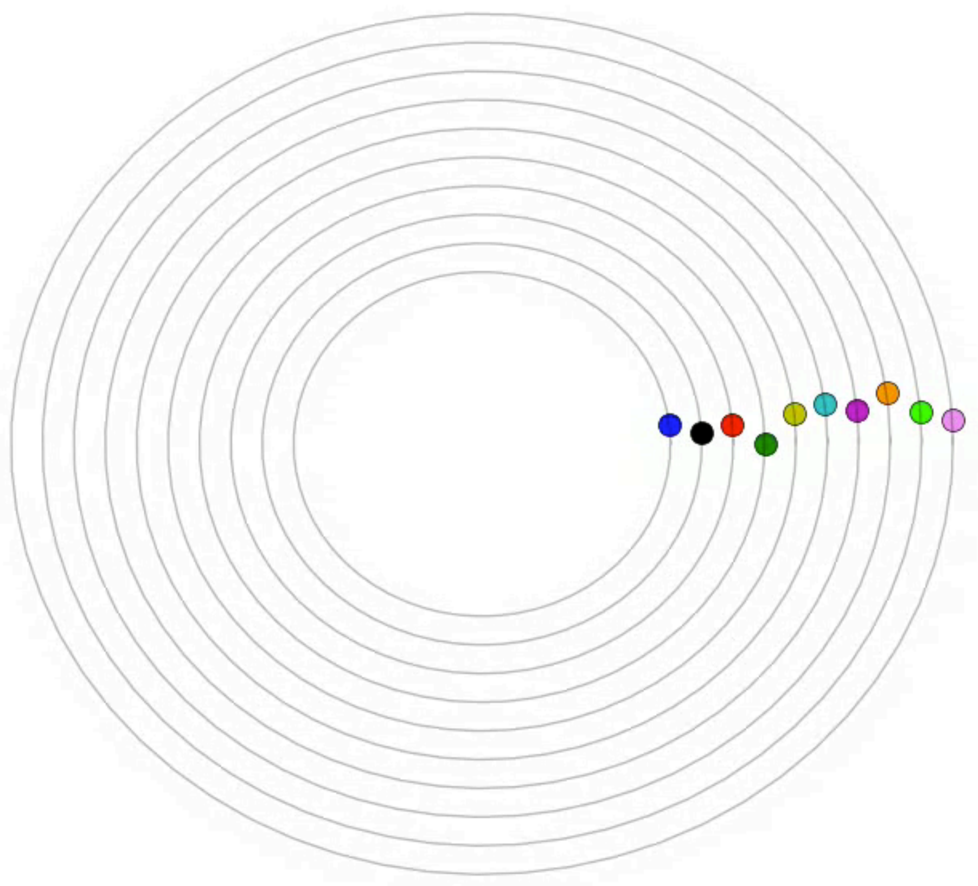
$$w_{ij} \sim N(0, \sigma^2/N)$$

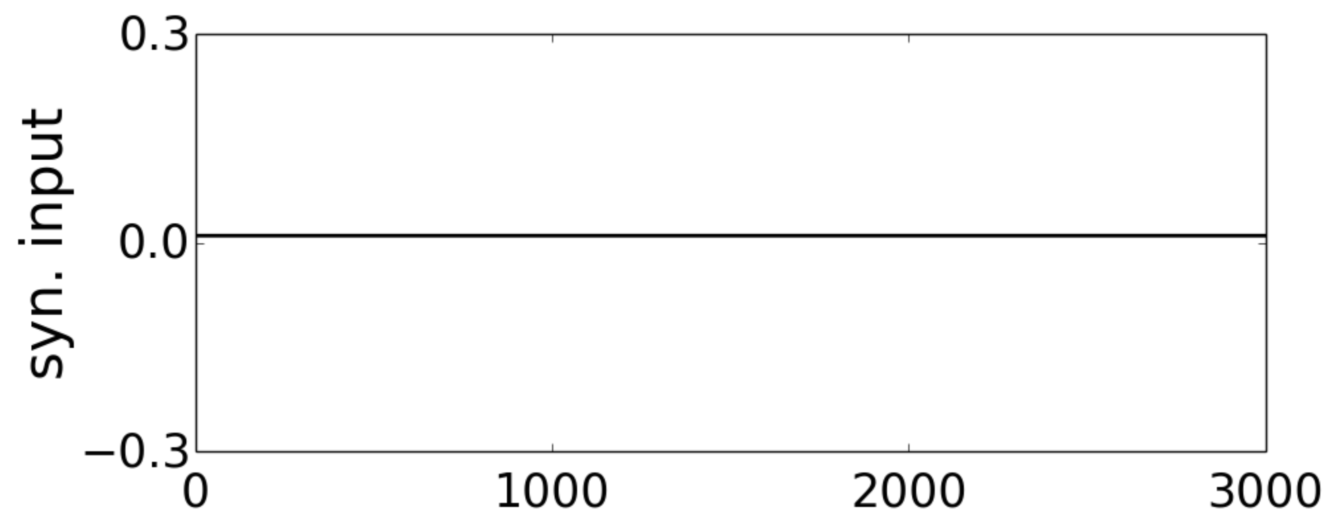
Random coupling

$\sigma = 0, I = 0.01, \beta = 0.1$

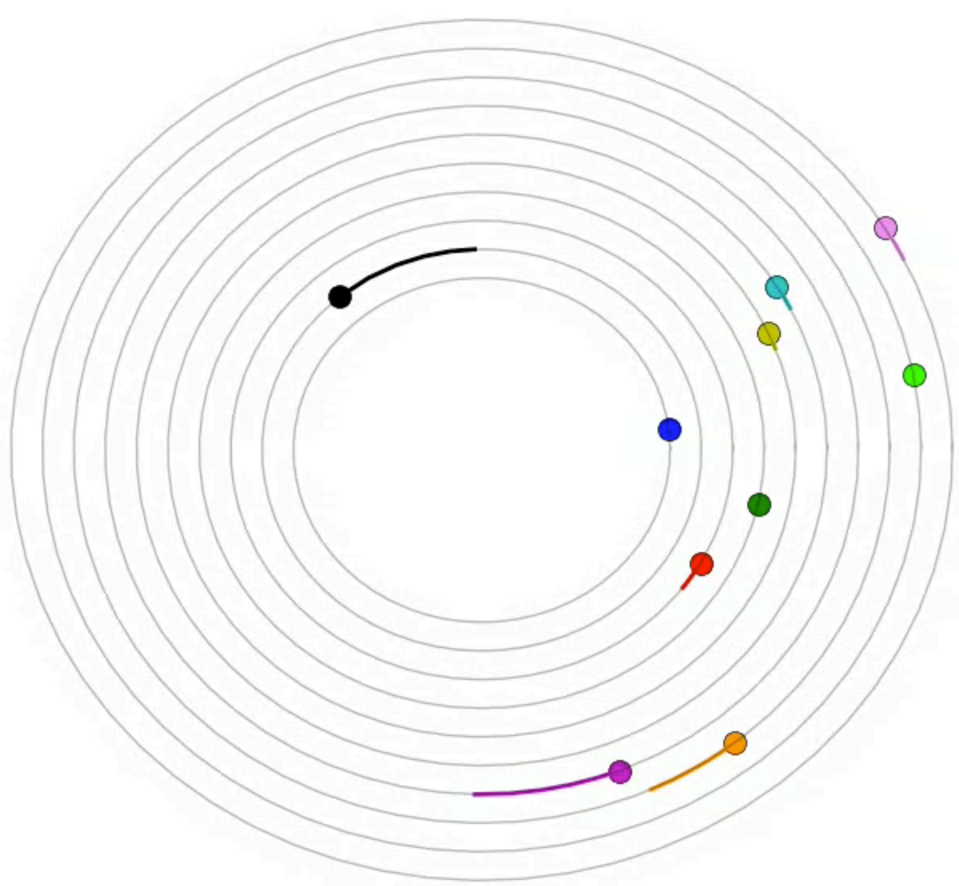
$N = 200$

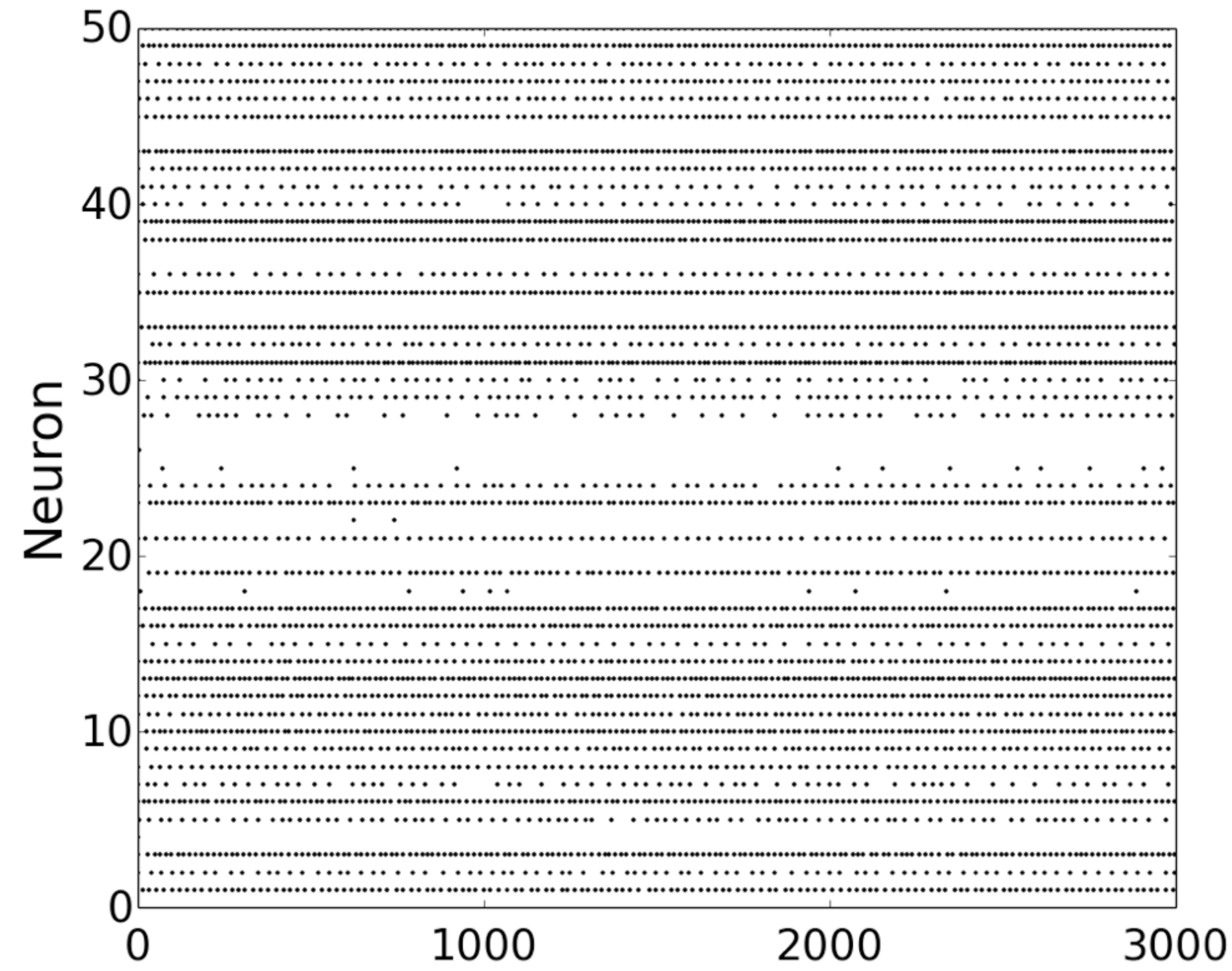




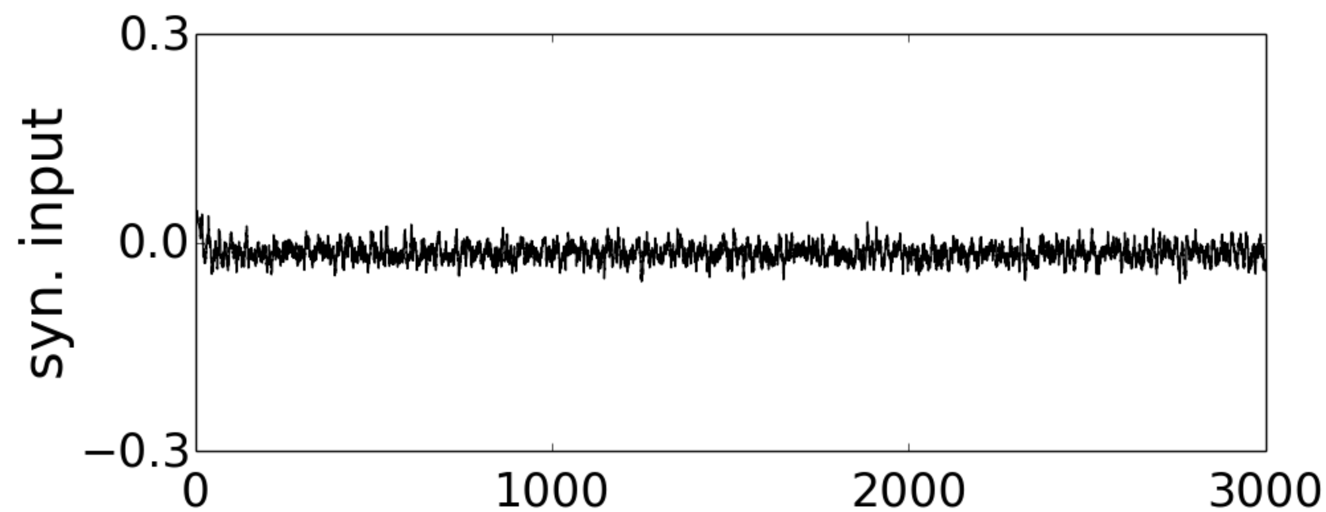


$\sigma = 0.5$

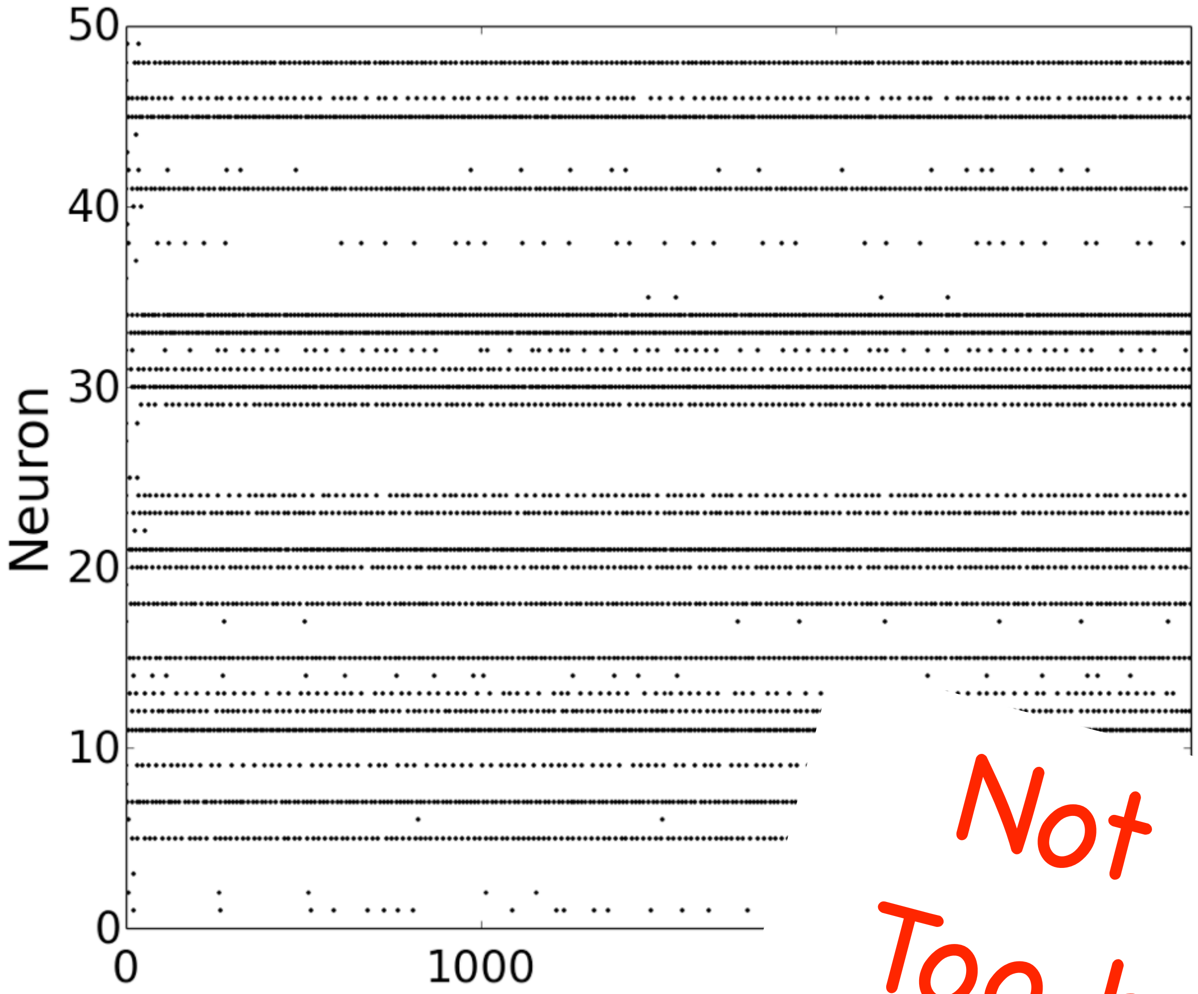




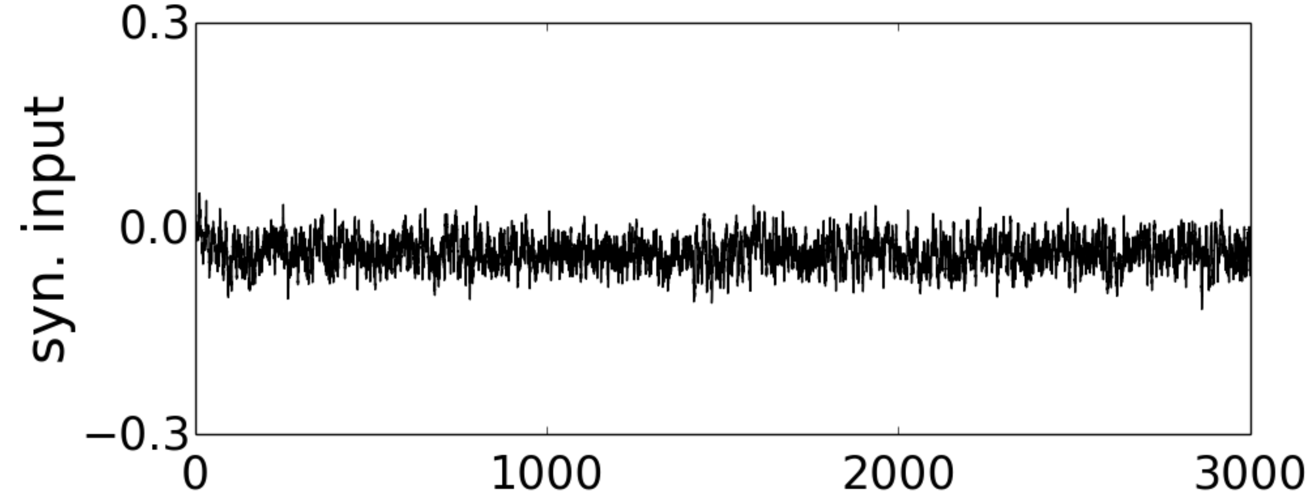
50
40
30
20
10
0
Neuron
0
1000
2000
3000



0.3
0.0
−0.3
syn. input
0
1000
2000
3000



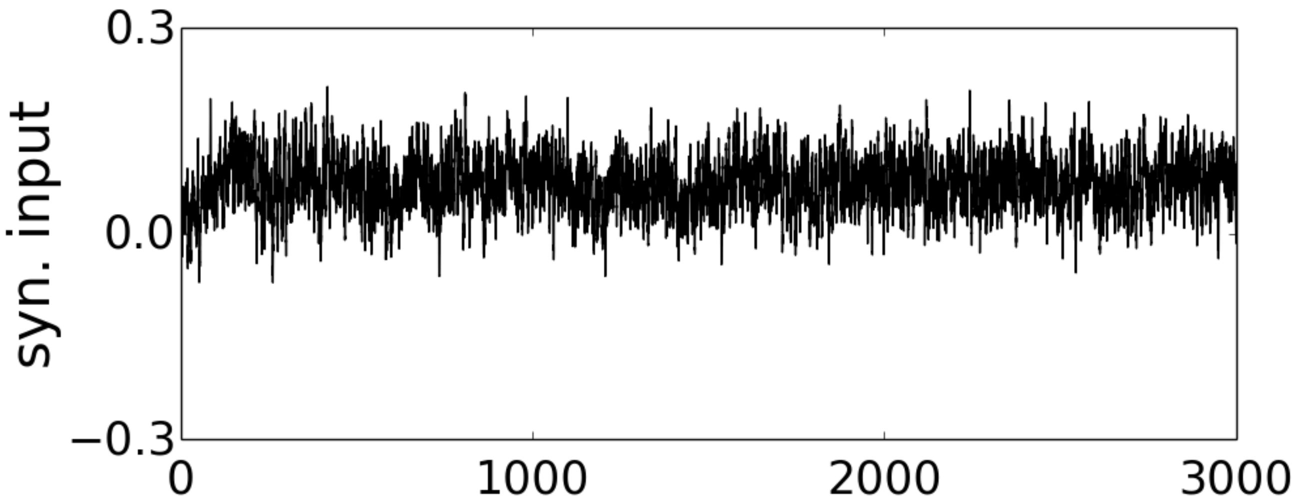
$\sigma = 1.0$

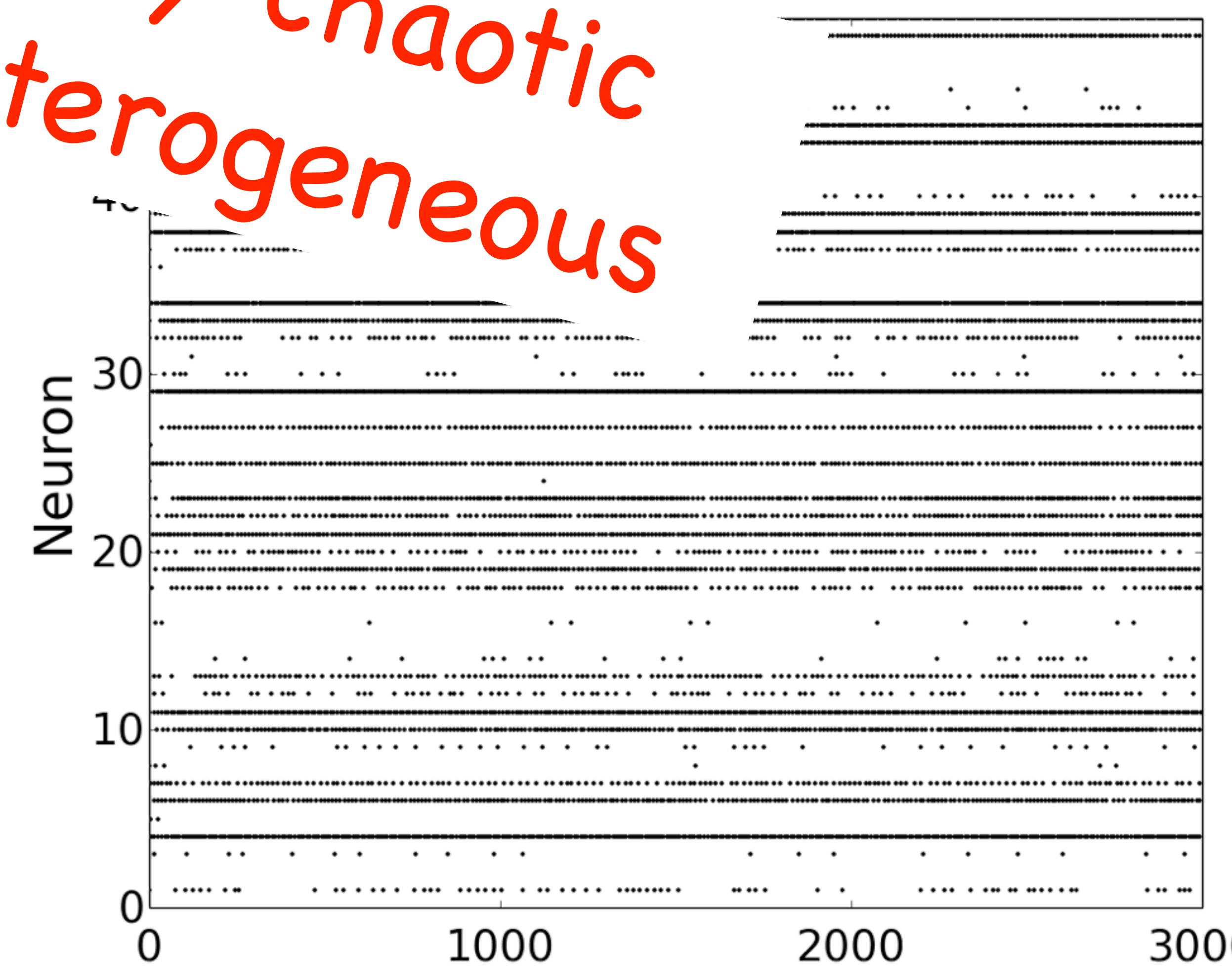


Not very chaotic

Too heterogeneous

$\sigma = 1.5$





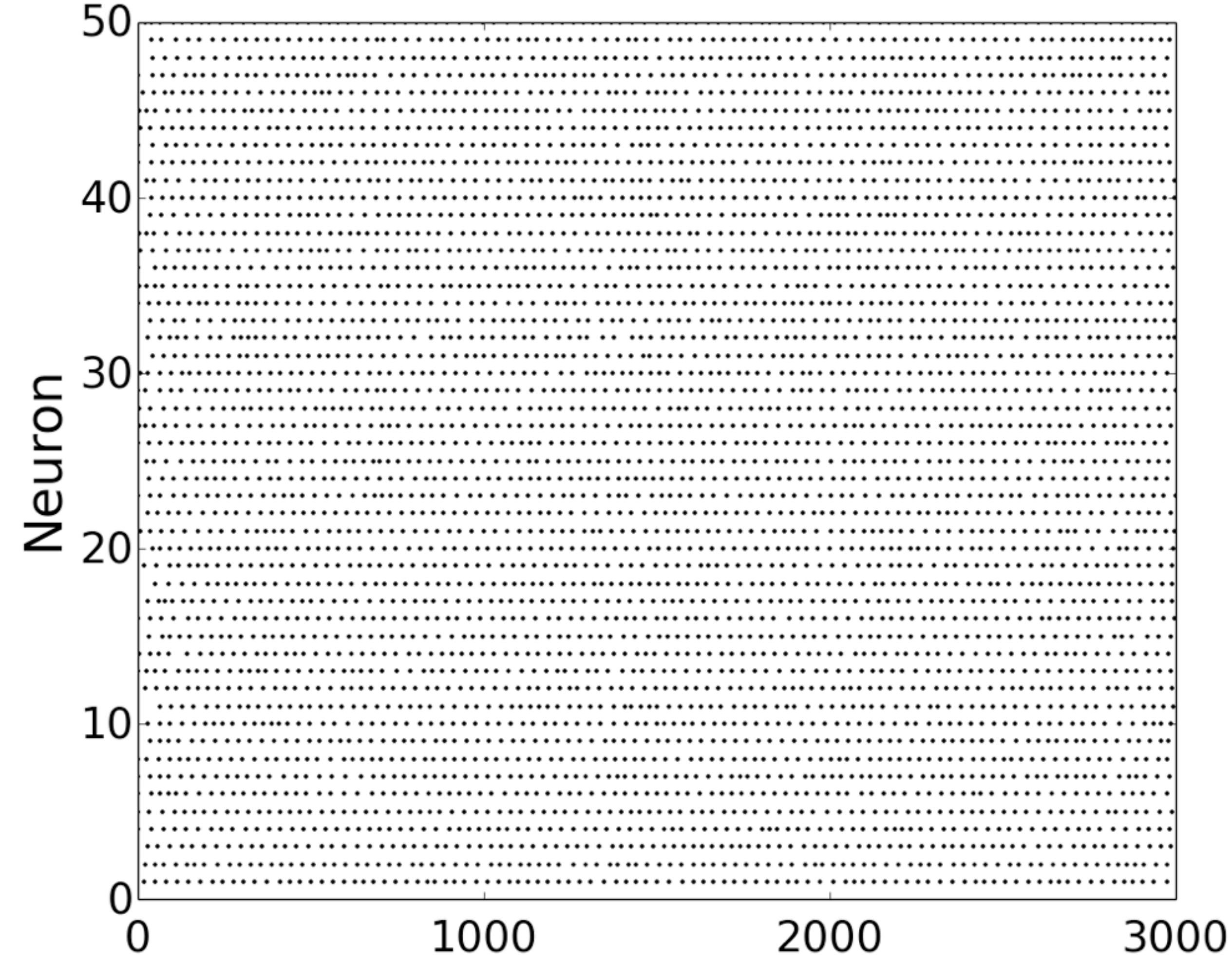
Column sum corrected

Network model

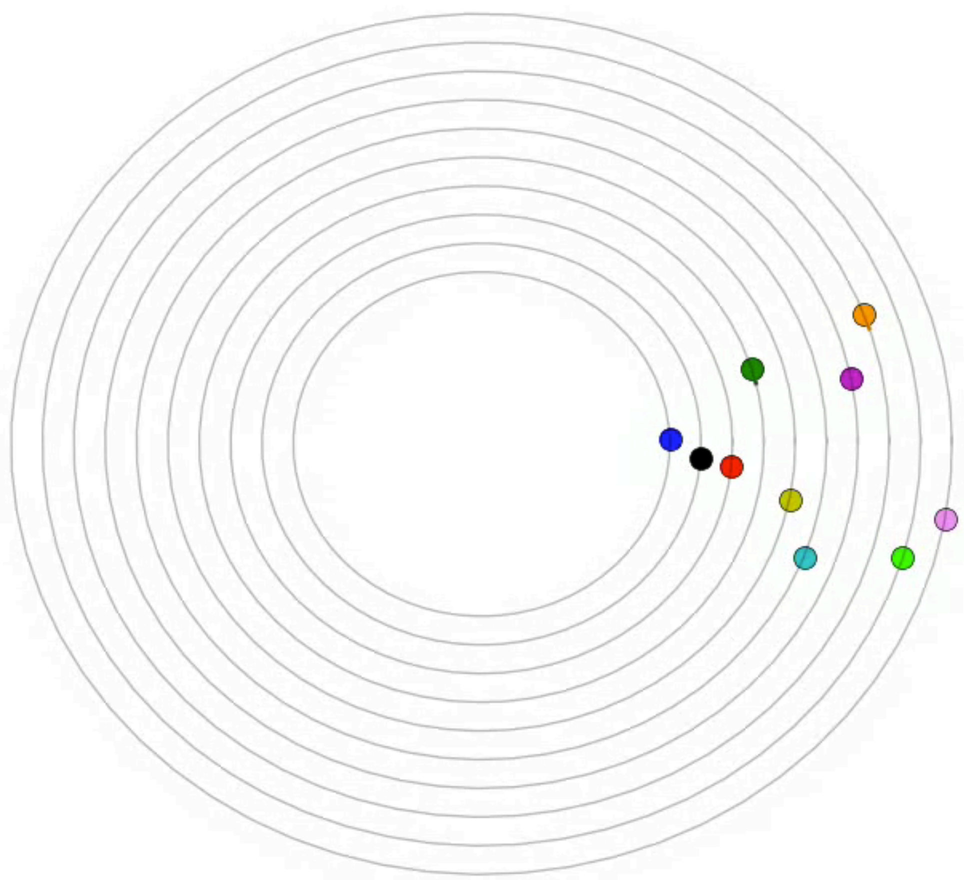
$$\dot{\theta}_i = 1 - \cos\theta_i + (I + u_i)(1 + \cos\theta_i)$$

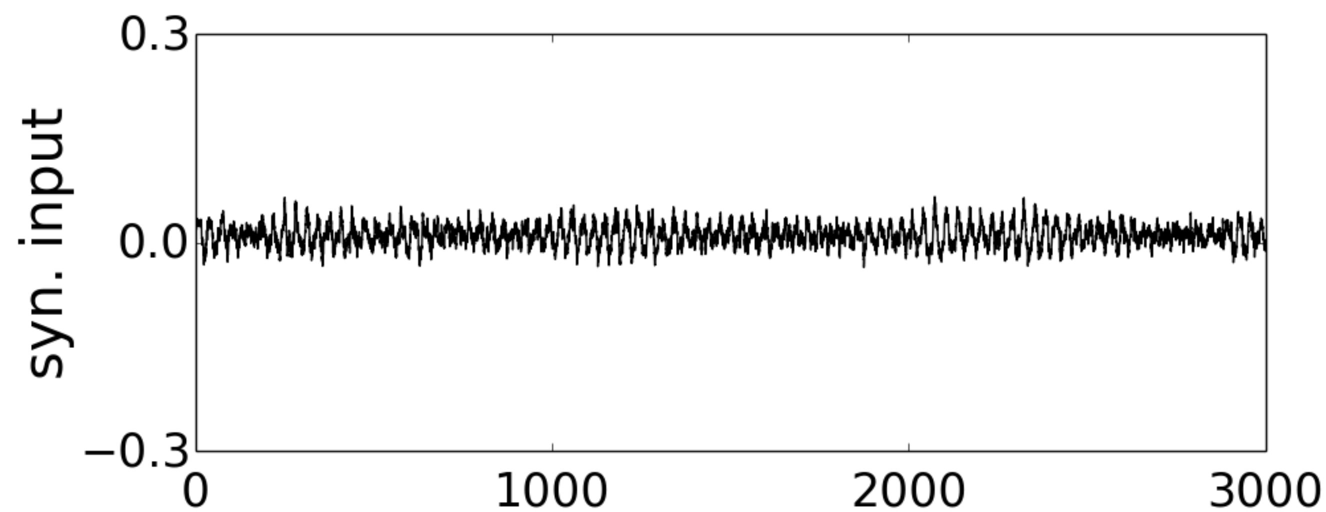
$$\dot{u}_i + \beta u_i = \beta \sum_{j,s} \left(w_{ij} - \frac{1}{N} \sum_k w_{ik} \right) \delta(t - t_j^s(\theta_j))$$

$\sigma = 0.5$



Neuron
0
10
20
30
40
50
0
1000
2000
3000

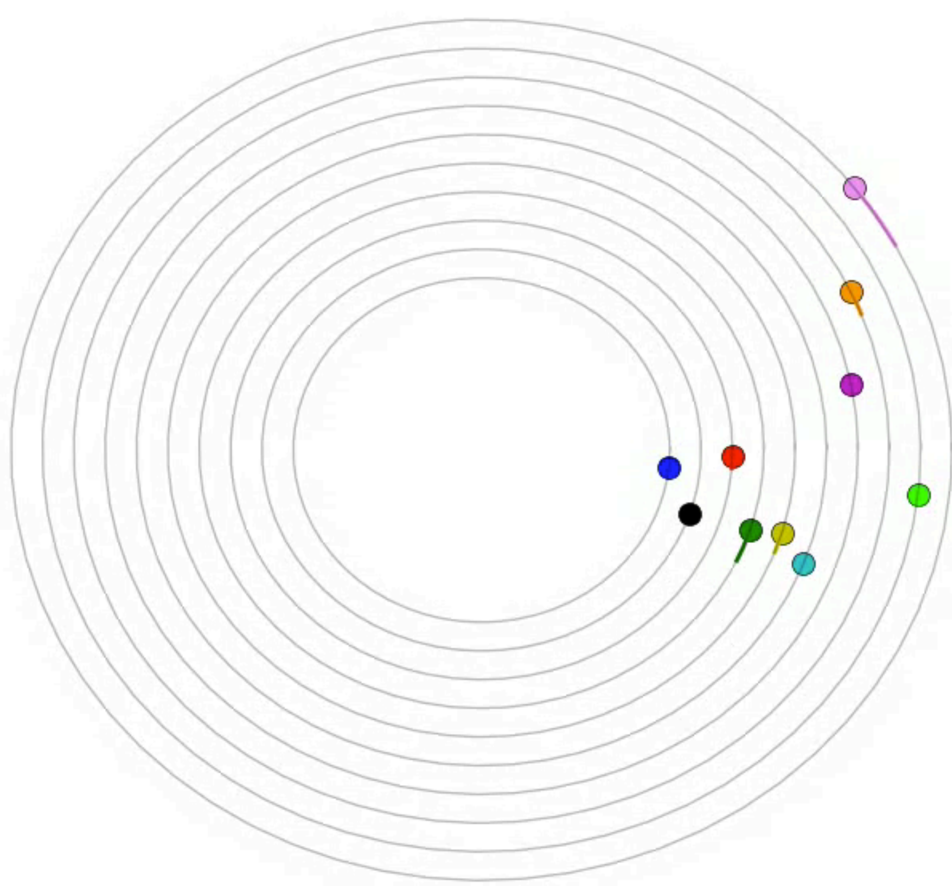


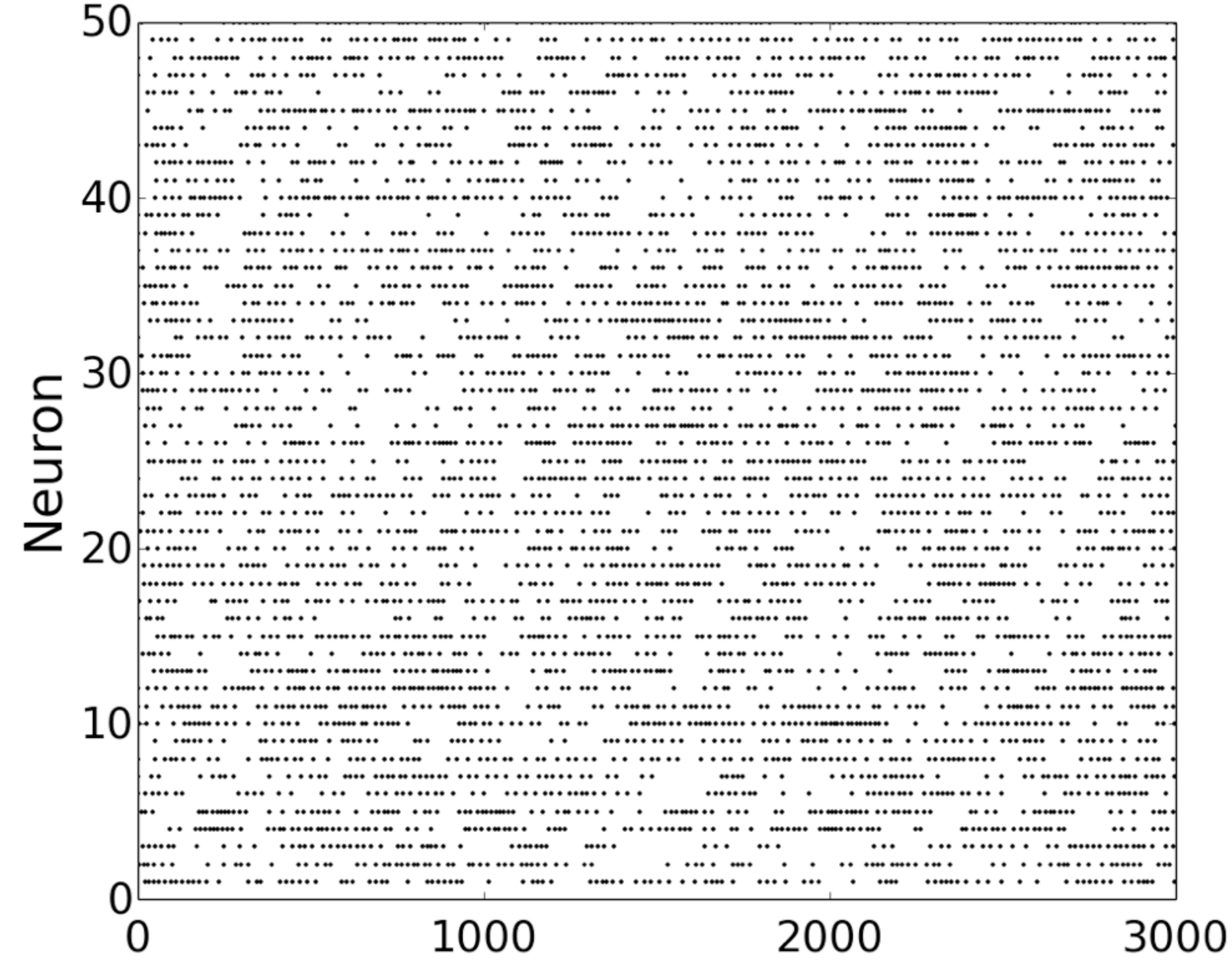


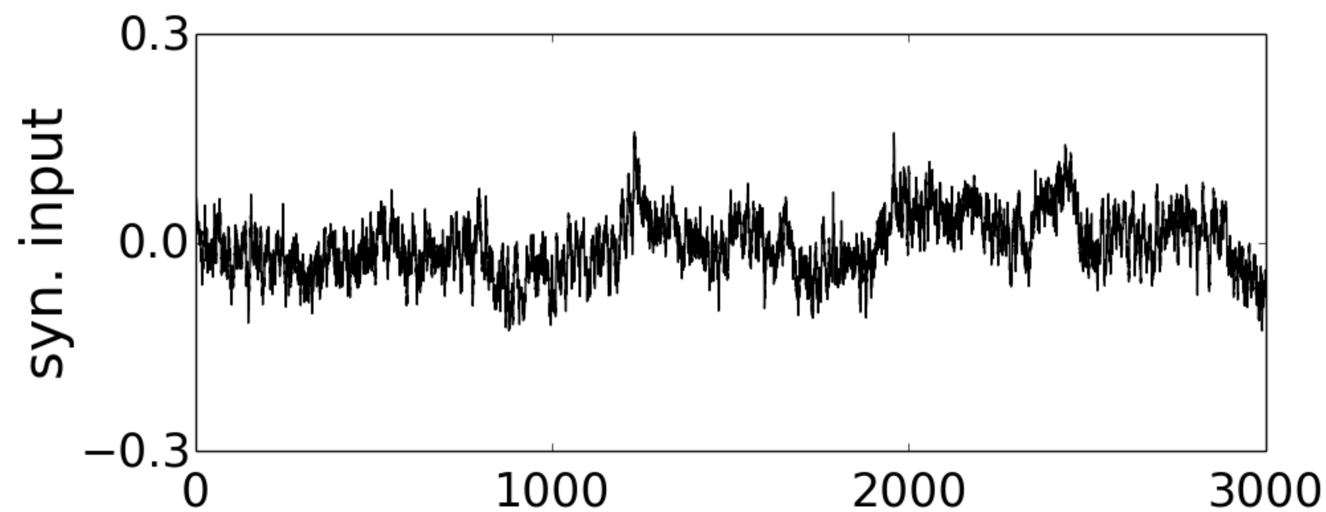
syn. input
0.3
0.0
−0.3
0
1000
2000
3000

FF < .1

$\sigma = 1.0$

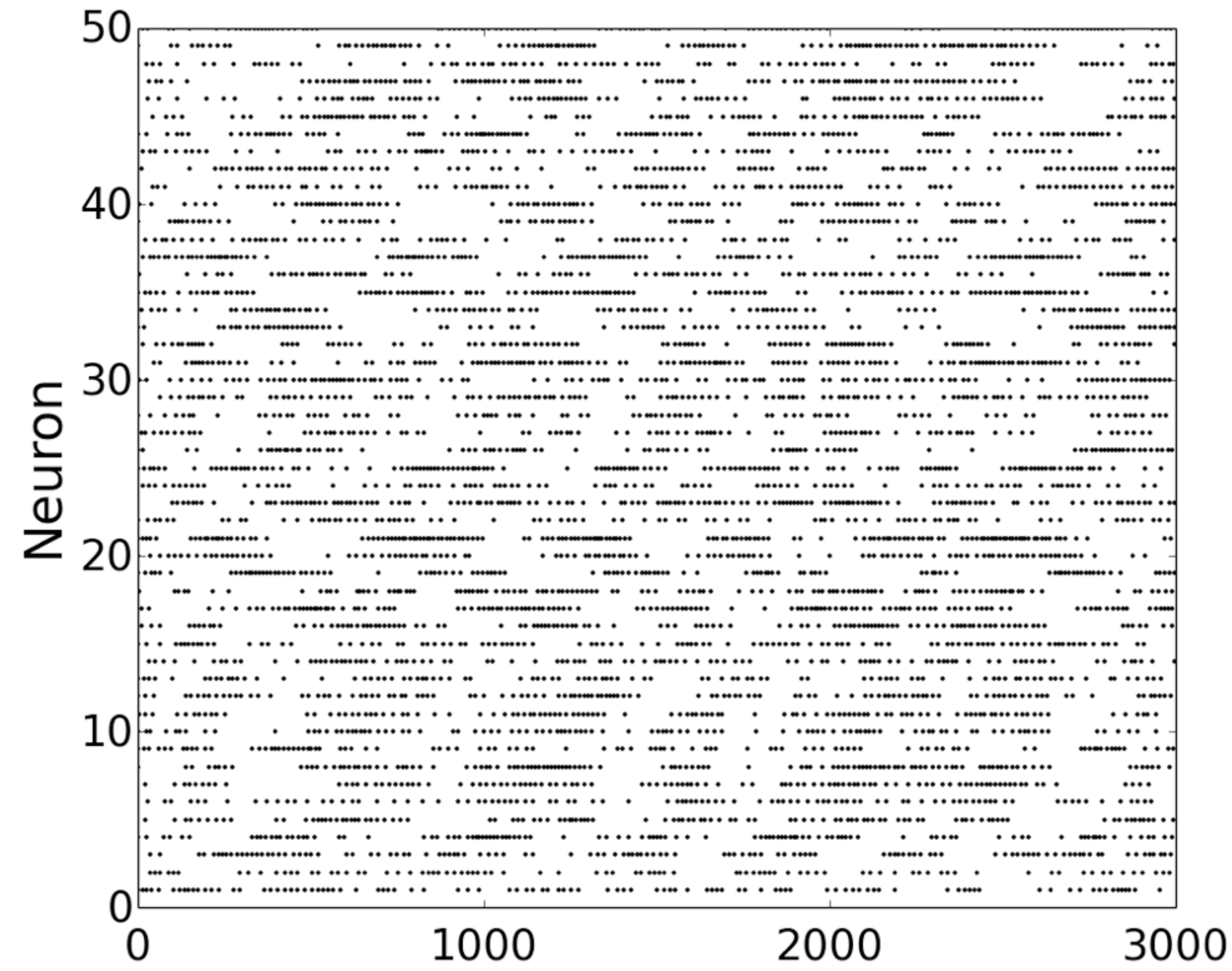


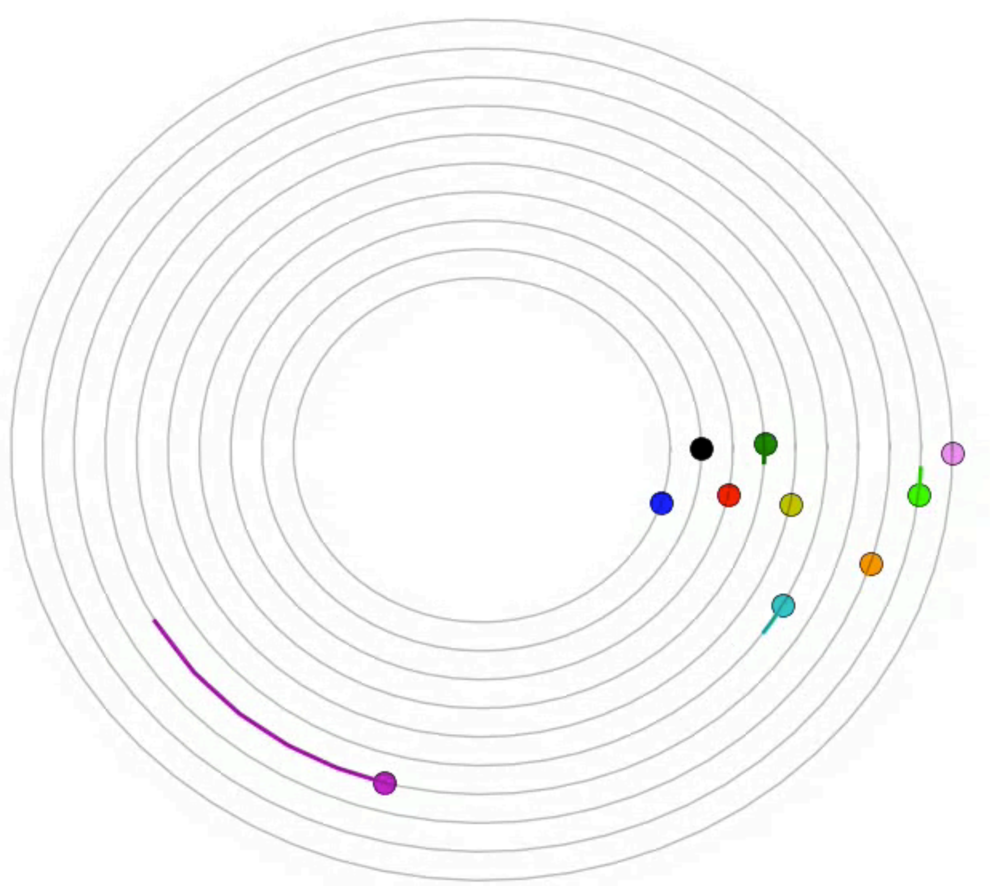


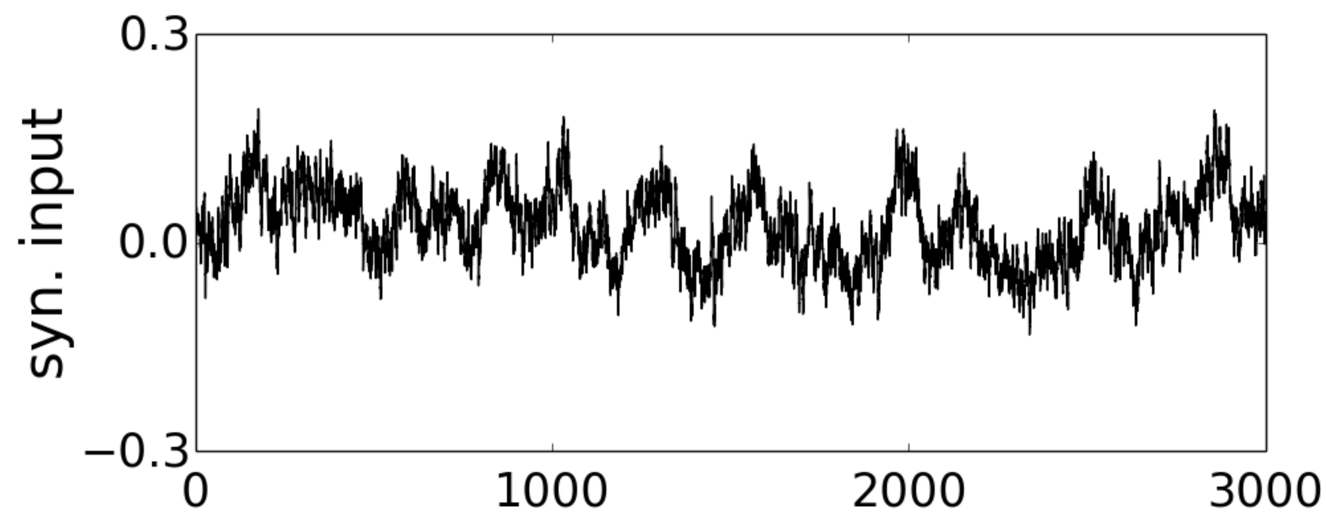


FF = .75

σ = 1.2

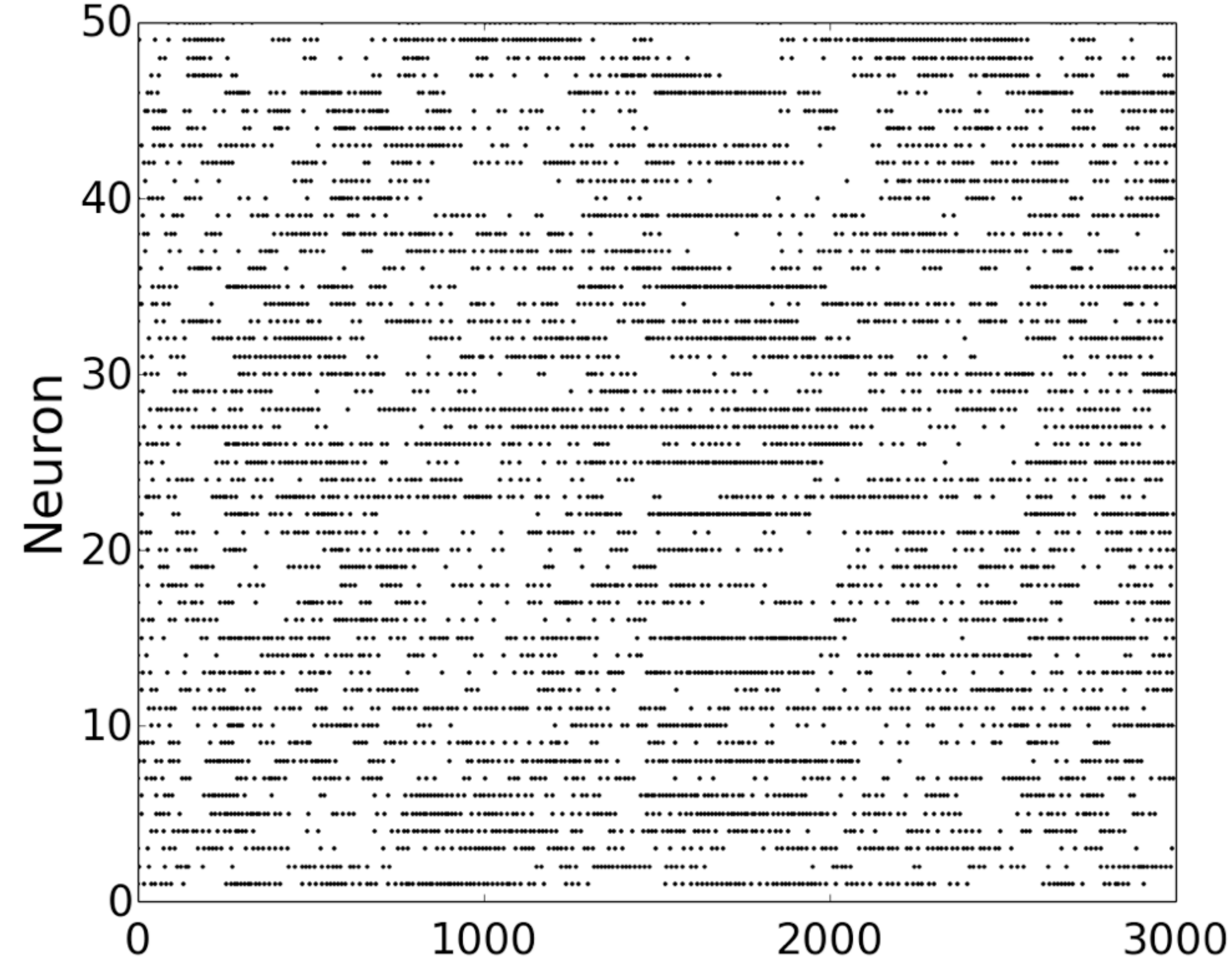

50
40
30
20
10
0
Neuron
0
1000
2000
3000

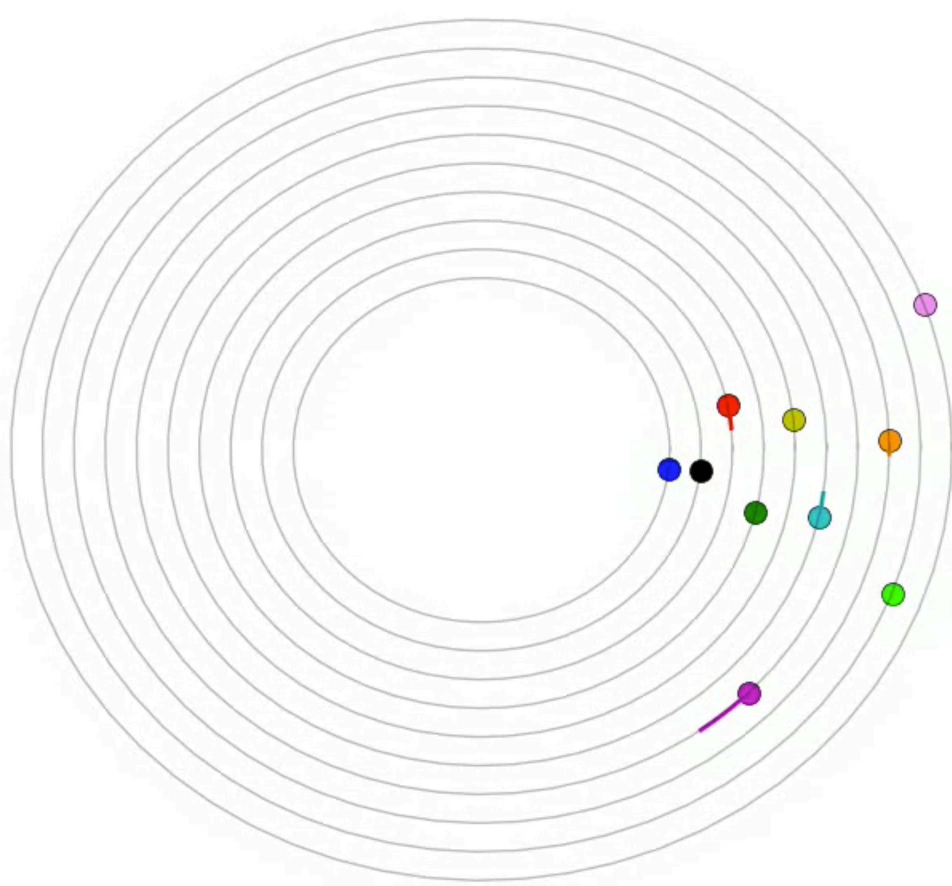


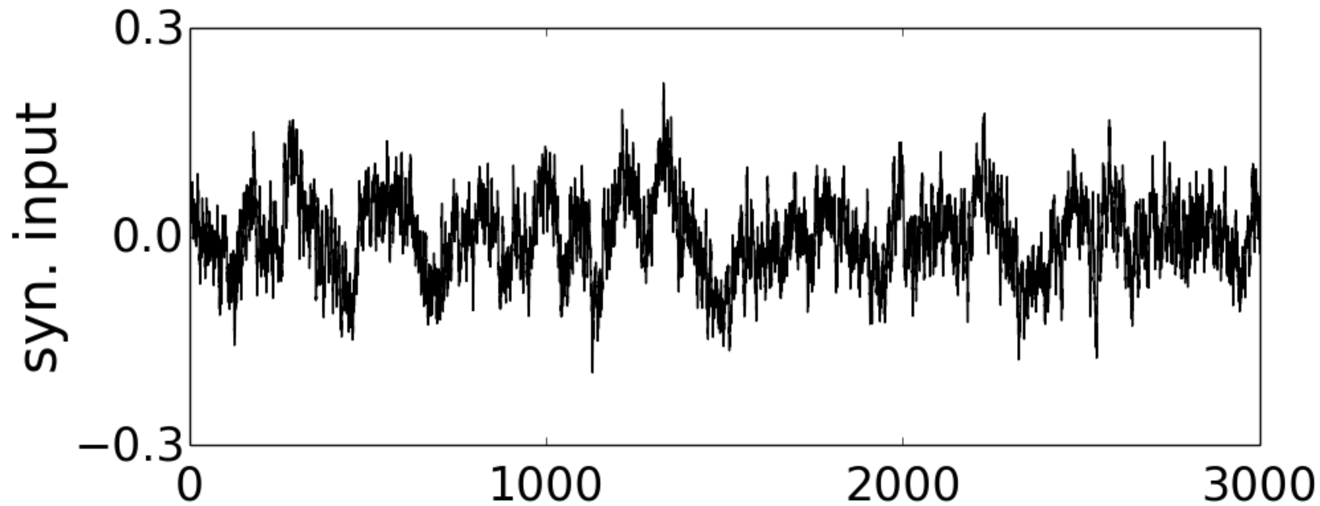

0.3
0.0
−0.3
syn. input
0
1000
2000
3000

FF = 1

$\sigma = 1.5$

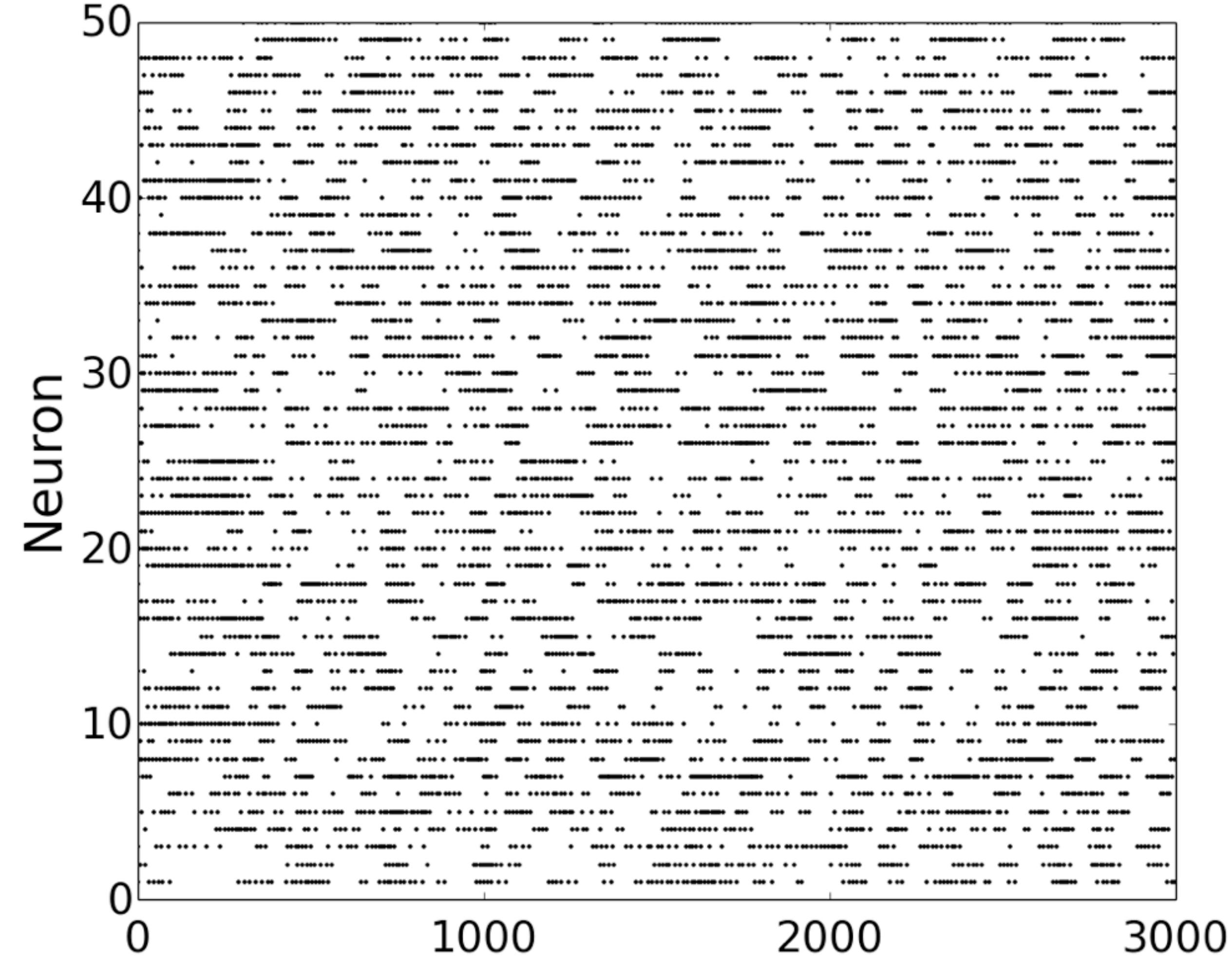


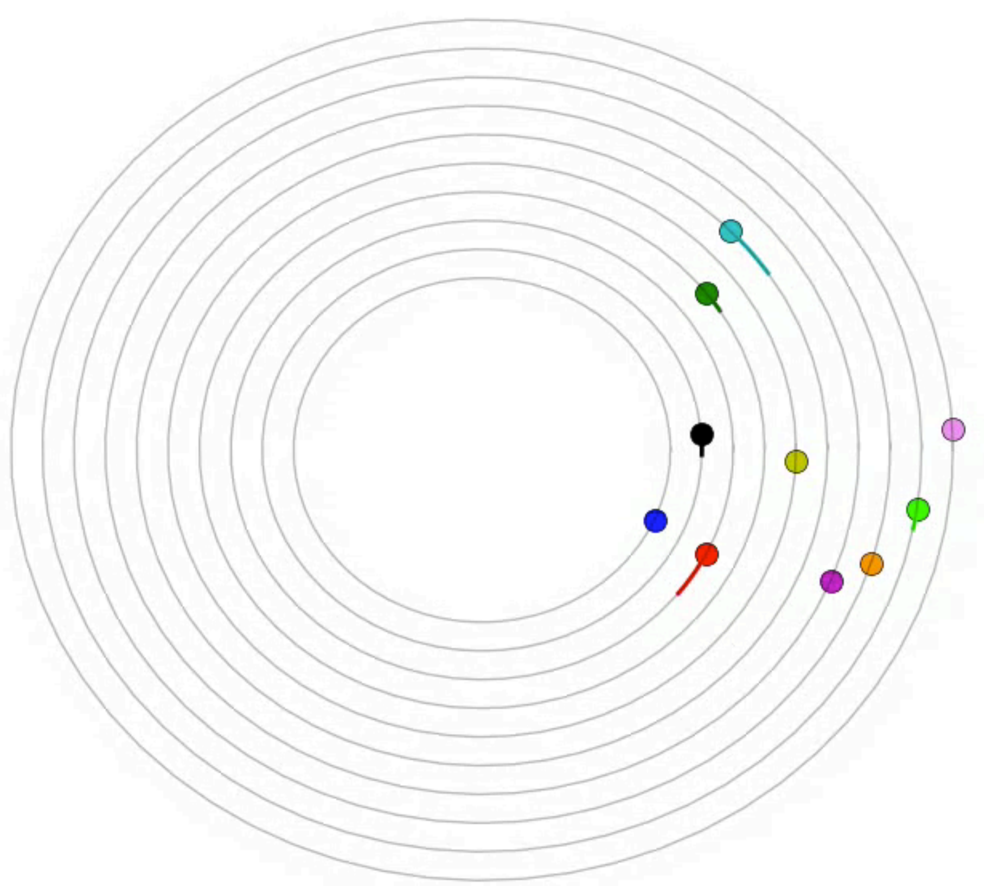


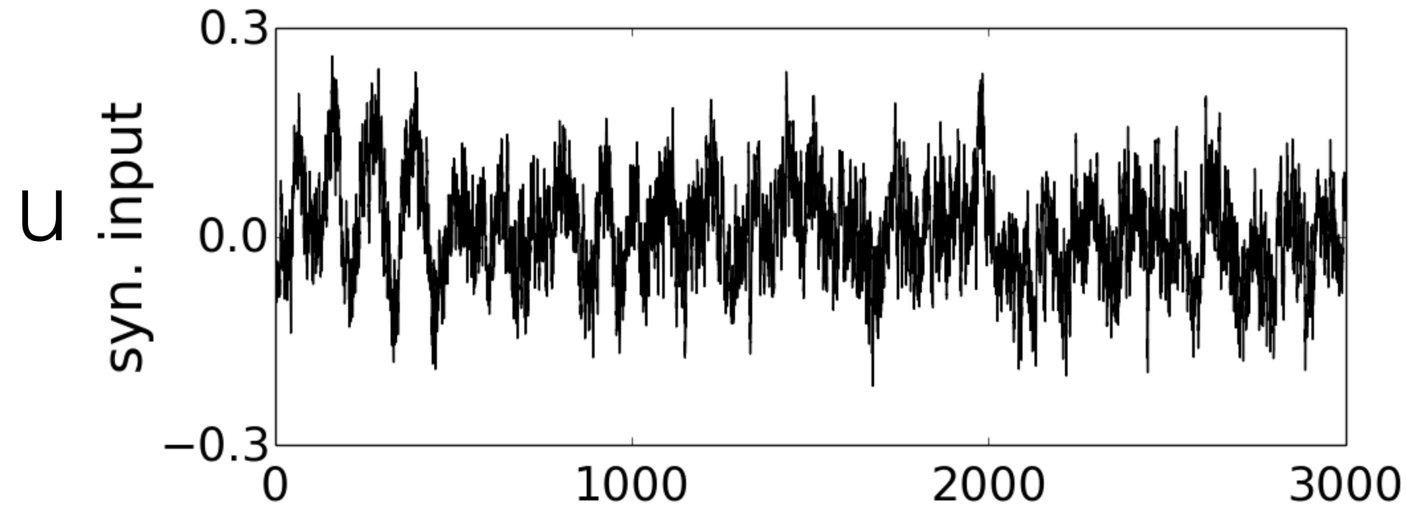


FF 1.4

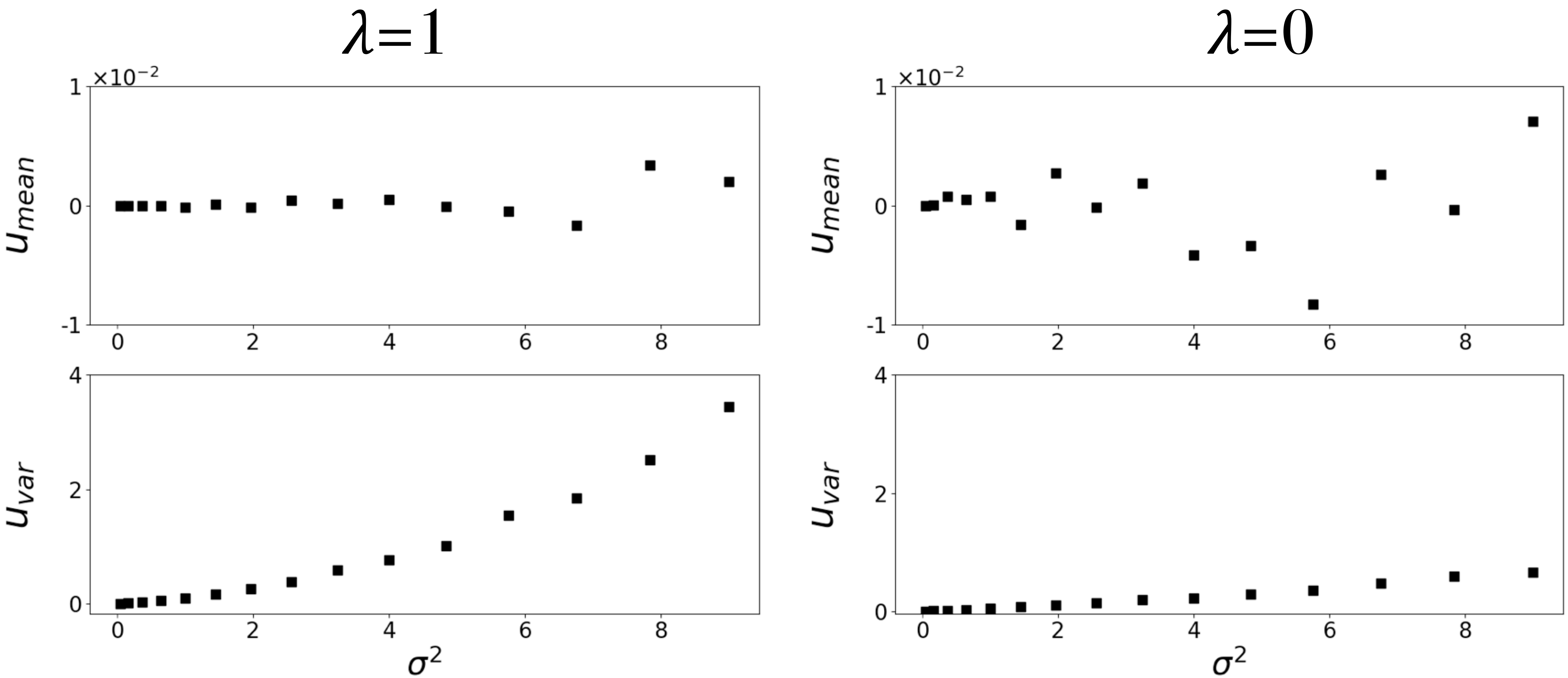
$\sigma = 2.0$



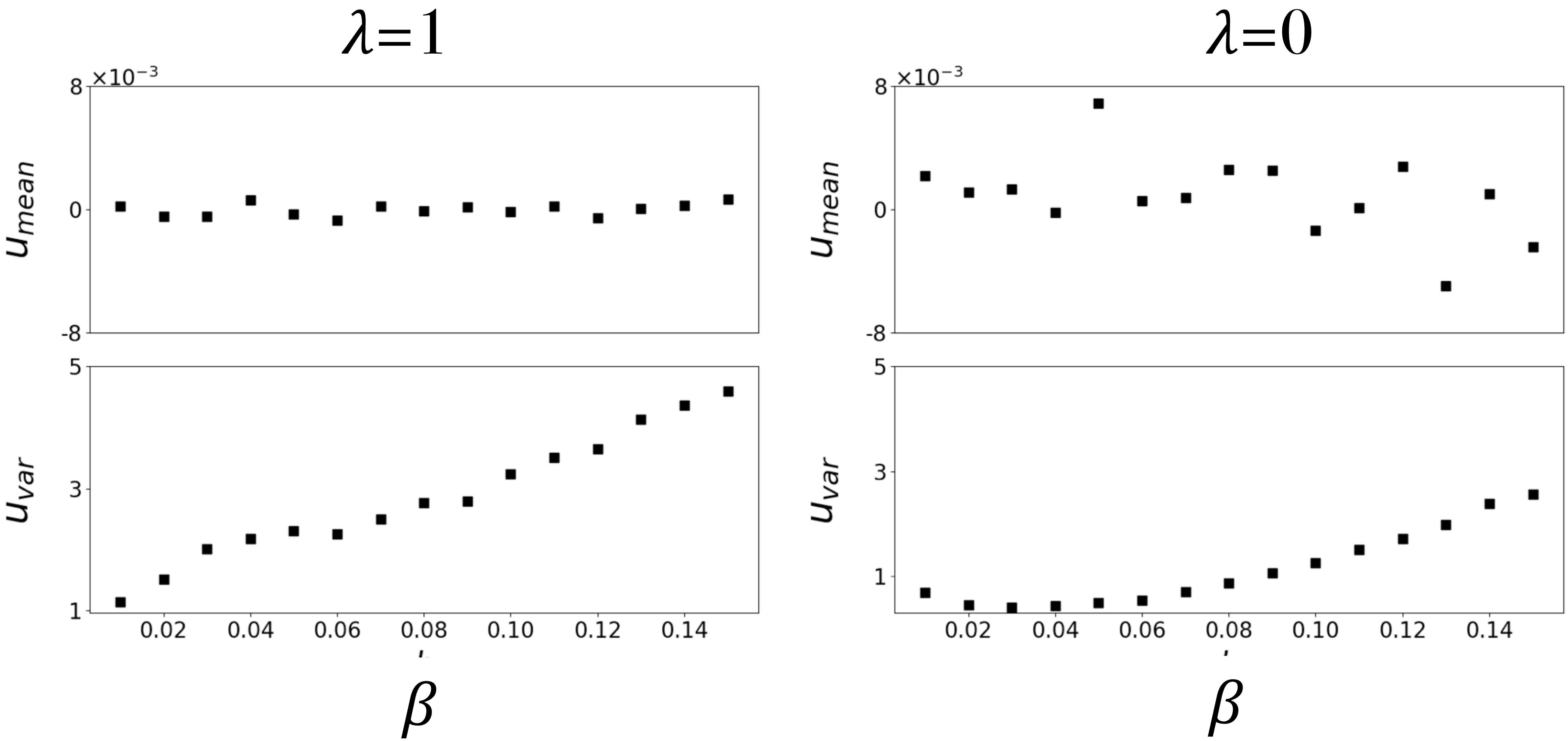




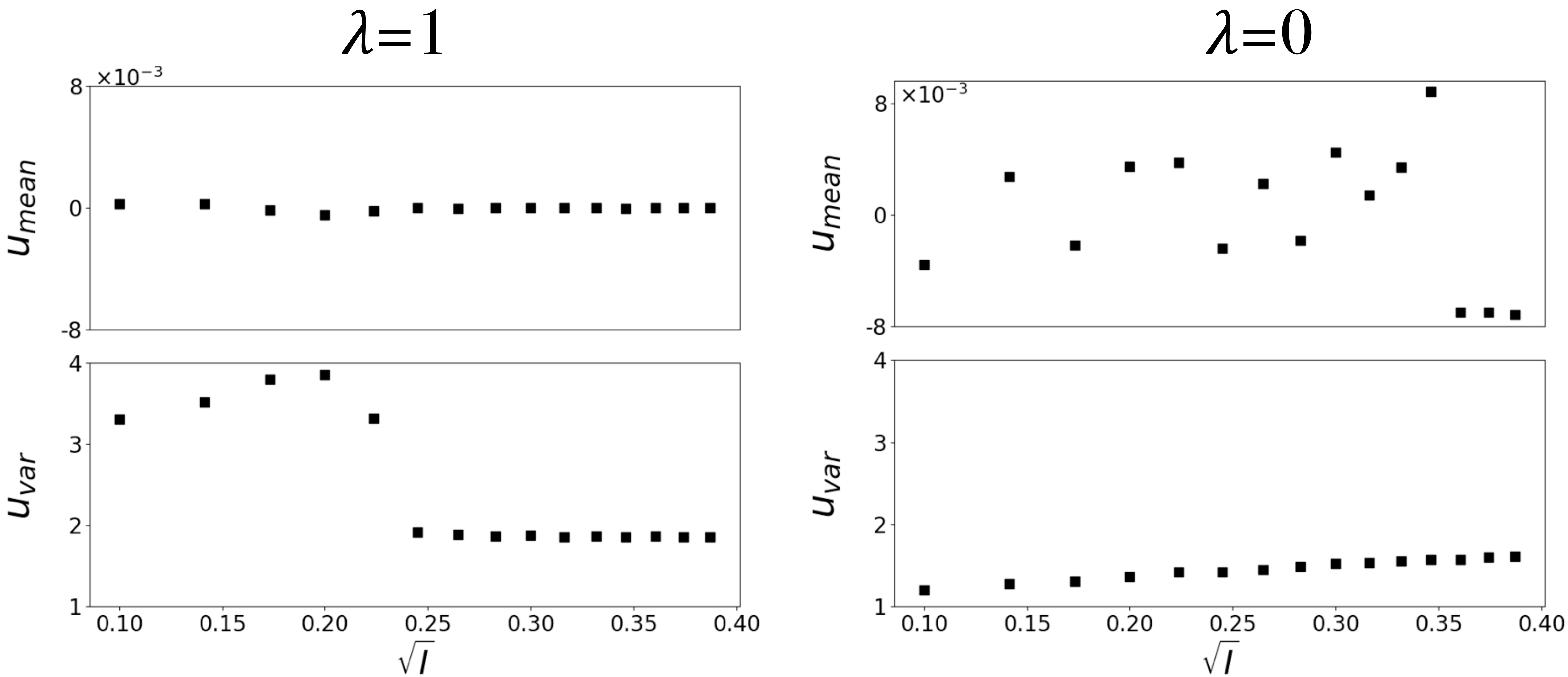
FF 1.7



λ=1
λ=0
×10⁻²
1
0
-1
u_{mean}
u_{var}
4
2
0
2
4
6
8
σ^2



λ=1
λ=0
×10⁻³
8
0
-8
u_{mean}
5
3
1
u_{var}
0.02
0.04
0.06
0.08
0.10
0.12
0.14
β



λ=1
λ=0
×10⁻³
8
0
-8
u_{mean}
4
3
2
1
u_{var}
0.10
0.15
0.20
0.25
0.30
0.35
0.40
$\sqrt{I}$

Can also use adaptation instead of sum correction

$$\dot{\theta}_i = 1 - \cos\theta_i + (I + u_i - a_i)(1 + \cos\theta_i)$$

$$\dot{u}_i + \beta u_i = \beta \sum_{j,s} w_{ij}\delta(t - t_j^s(\theta_j))$$

$$\boxed{\tau \dot{a}_i = u_i - a_i}$$

Only needs local information

Network revisited

$$\dot{\theta}_i = 1 - \cos\theta_i + (I + u_i)(1 + \cos\theta_i)$$

$$\dot{u}_i + \beta u_i = \beta \sum_{j,s} \left(w_{ij} - \frac{\lambda}{N} \sum_k w_{ik} \right) \boxed{\delta(t - t_j^s(\theta_j))}$$

Empirical density $\eta_j(\theta, t) = \delta(\theta - \theta_j(t))$

$$\sum_s \delta(t - t_j^s) = \eta_j(\pi, t)\dot{\theta}_j|_{\theta_j = \pi} = 2\eta_j(\pi, t)$$ Spiking rate

$$\dot{u}_i(t) + \beta u_i(t) - \sum_{j,s}\left(w_{ij} - \frac{\lambda}{N}\sum_k w_{ik}\right)\delta(t - t_j^s) = 0$$

$$\dot{u}_i(t) + \beta u_i(t) - 2\beta\left(\sum_j w_{ij} - \frac{\lambda}{N}\sum_k w_{ik}\right)\eta_j(\pi, t) = 0$$

Network mean $\eta(t) = \frac{1}{N}\sum_j \eta_j(t)$

Neurons are conserved

Exists in weak sense

$$\partial_t \eta_i(\theta, t) + \partial_\theta F_i(\theta, u_i)\eta_i(\theta, t) = 0$$

$$F_i = 1 - \cos\theta_i + (I + u_i)(1 + \cos\theta_i)$$

Regularize by integrating (averaging)

Reformulated network

$$\partial_t \eta_i(\theta, t) + \partial_\theta F_i(\theta, u_i)\eta_i(\theta, t) = 0$$

$$\dot{u}_i(t) + \beta u_i(t) - 2\beta \sum_j w_{ij}(\eta_j(\pi, t) - \lambda\eta(\pi, t)) = 0$$

Disorder to noise

$$\partial_t \eta_i(\theta, t) + \partial_\theta F_i(\theta, u_i)\eta_i(\theta, t) = 0$$

$$\dot{u}_i(t) + \beta u_i(t) = z_i(t)$$

$$z_i(t) = 2\beta \sum_j w_{ij}(\eta_j(\pi, t) - \lambda\eta(\pi, t))$$

$$P[w_{ij}] = \prod_{ij} \sqrt{\frac{N}{2\pi\sigma^2}} e^{-\frac{Nw_{ij}^2}{2\sigma^2}}$$

$$P[w_{ij}]dw_{ij} \to P[z(t)][dz(t)]$$

Reformulated network

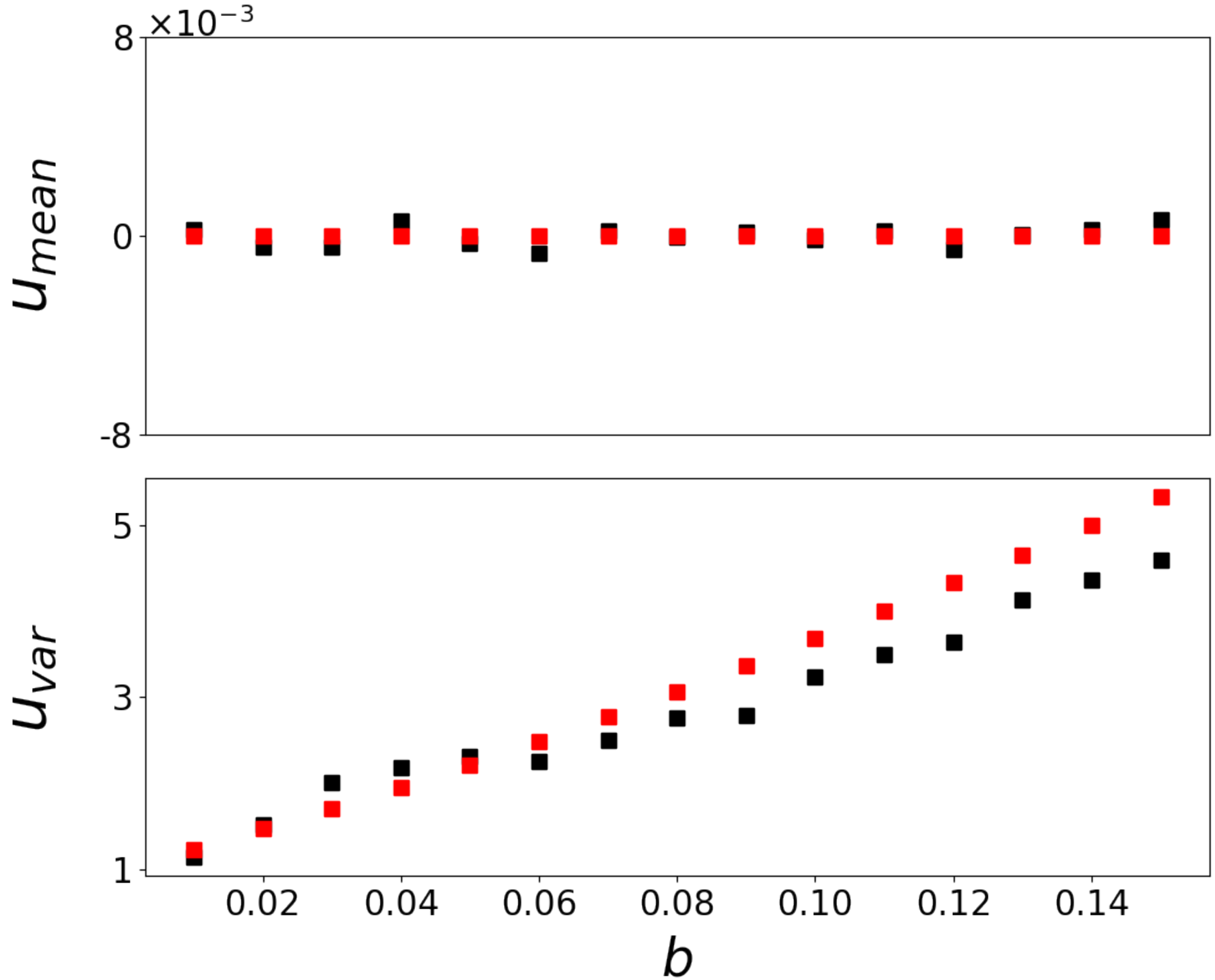
$$\partial_t \eta_i(\theta, t) + \partial_\theta F_i(\theta, u_i)\eta_i(\theta, t) = 0$$

$$\dot{u}_i + \beta u_i = z_i(t)$$

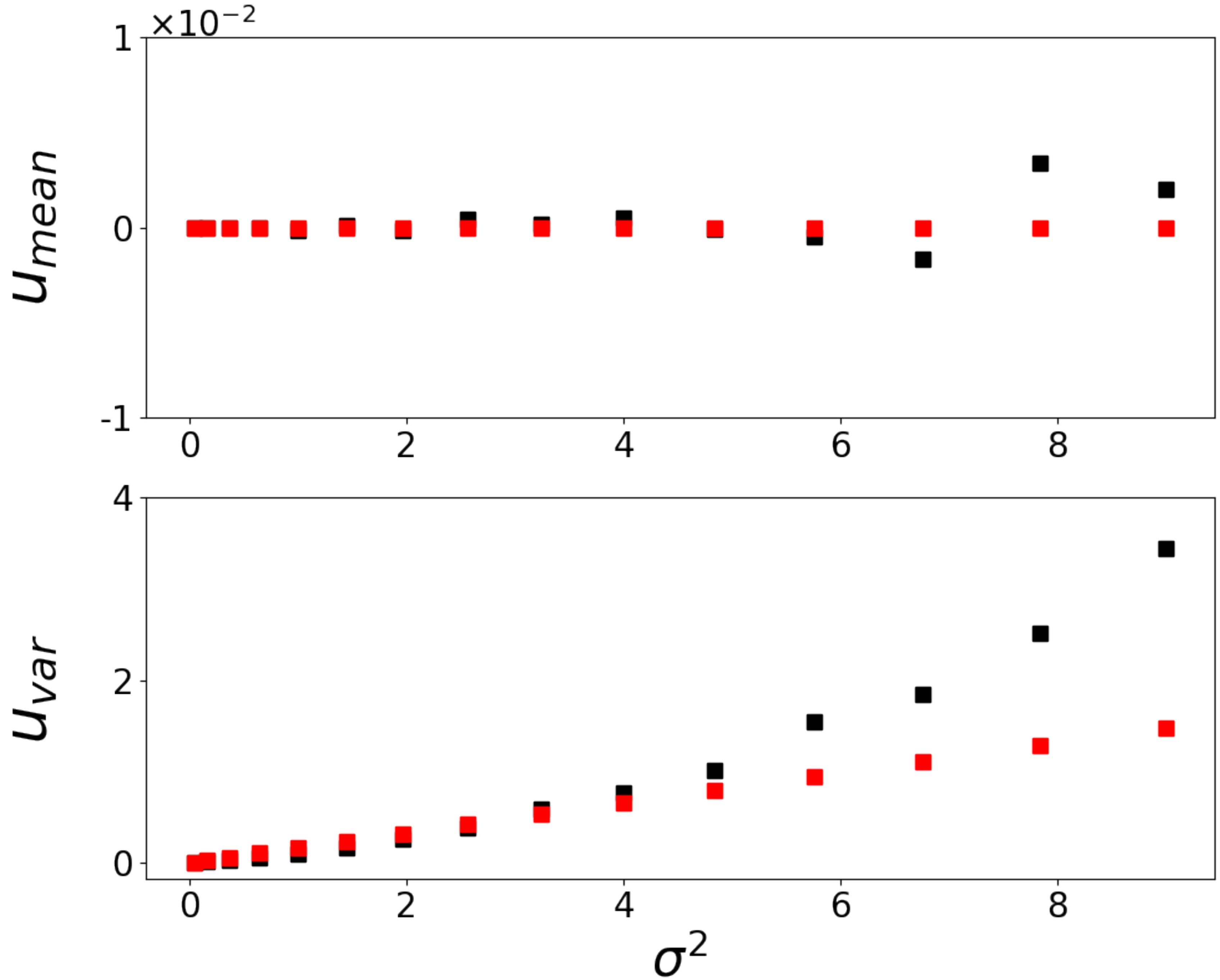
$$E[z_i(t)] = 0$$

$$\mathrm{Cov}[z_i(t), z_i(s)] = 4\beta^2\sigma^2 \int dtds \, \frac{1}{N} \sum_j \underbrace{[\eta_j(\pi, t) - \lambda\eta(\pi, t)] \, [\eta_j(\pi, s) - \lambda\eta(\pi, s)]}_{\text{Network covariance}}$$

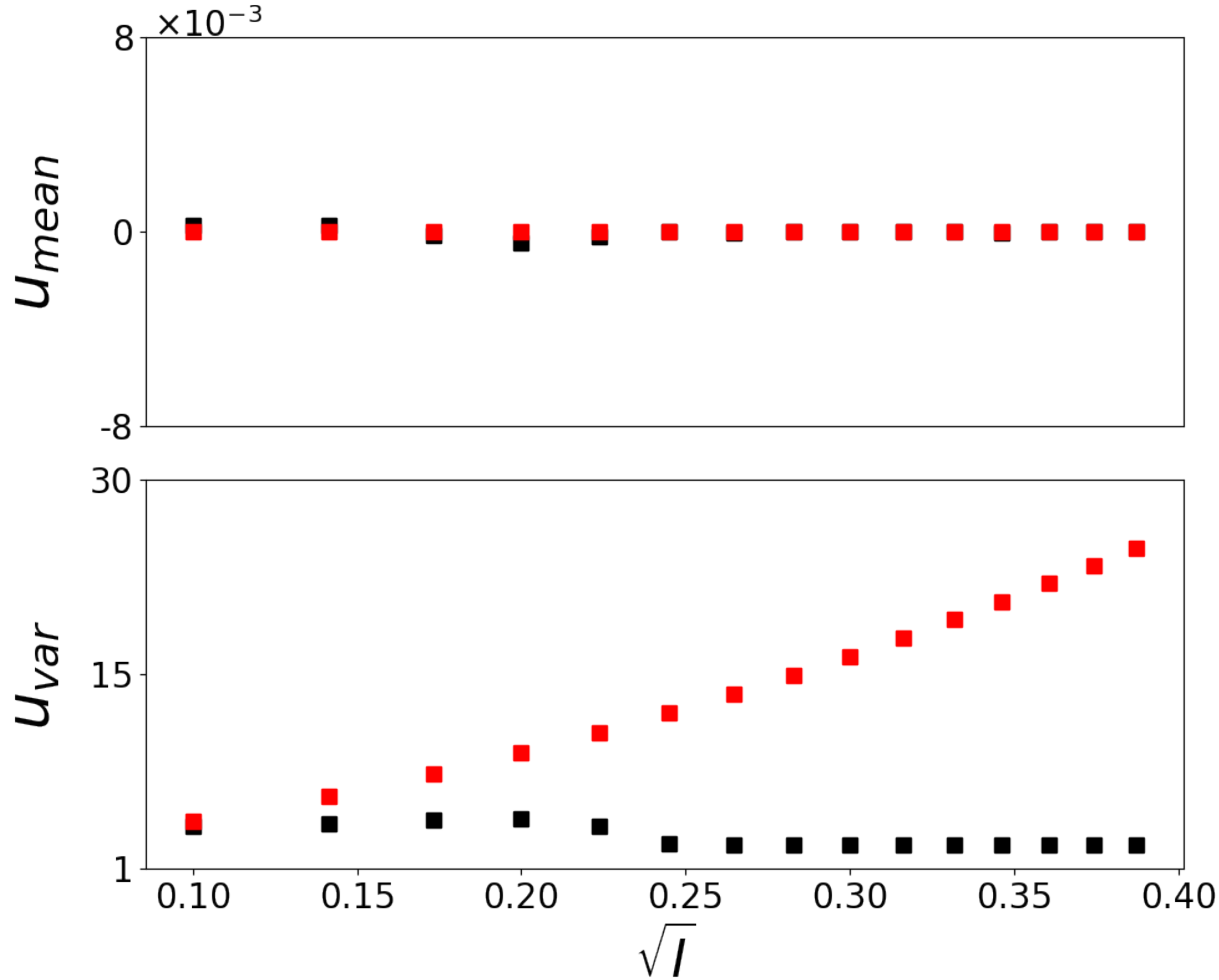
1st order expansion



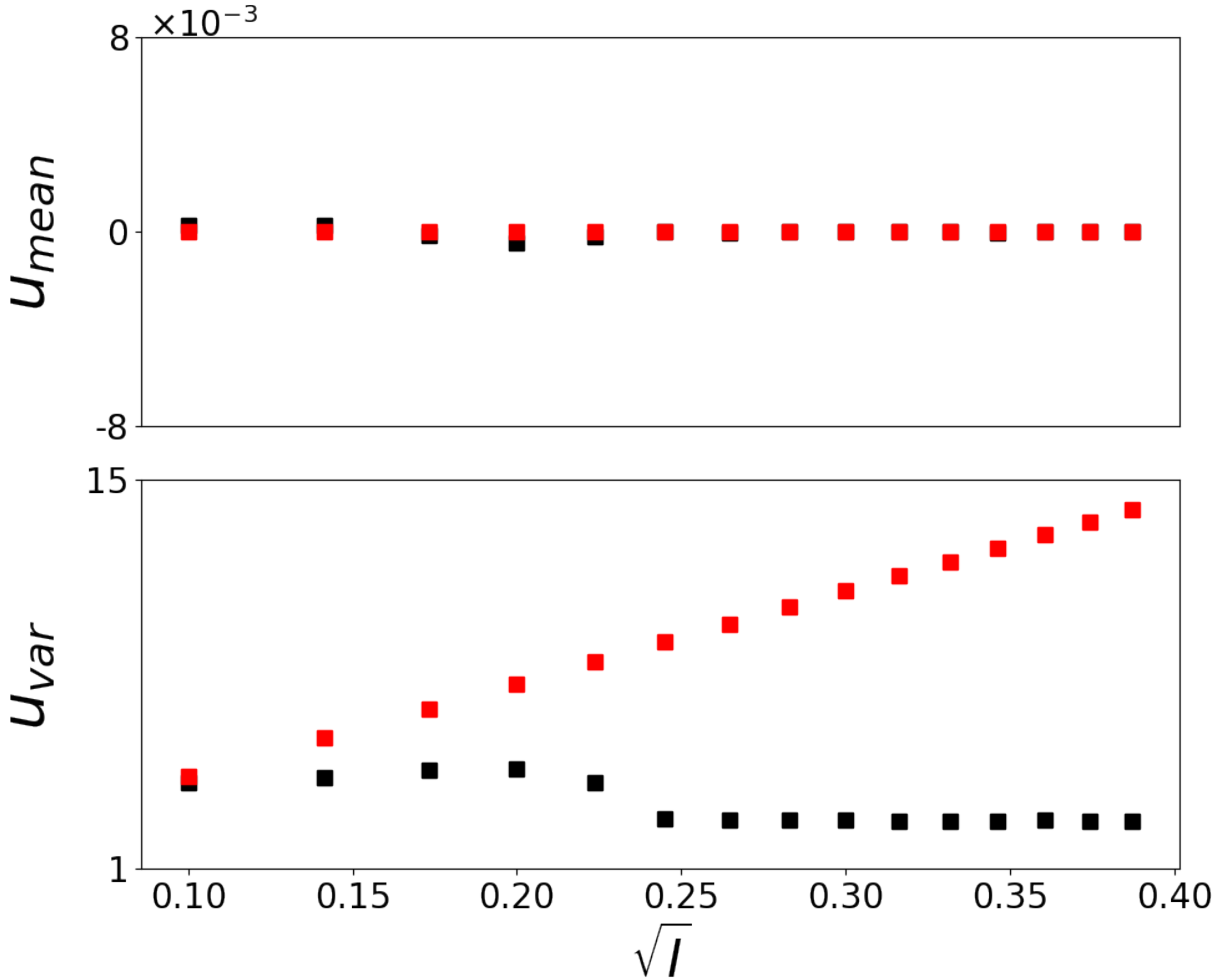
1st order expansion



1st order expansion



OU approximation



Can we train w_{ij} so network does what we want?

Network

$$\dot{\theta}_i = 1 - \cos\theta_i + (I + u_i)(1 + \cos\theta_i)$$

$$\dot{u}_i = \sum_j w_{ij} r_j$$

$$\dot{u}_i = -\beta u_i + \beta \sum_{j,s} w_{ij}\delta(t - t_j^s(\theta_j)$$

$$\dot{r}_j = -\beta r_j + \beta \sum_s \delta(t - t_j^s(\theta_j)$$

Goal: Train w_{ij} so u and r follow targets

Learning

Minimize over w

$$C_u(\mathbf{w}) = (\hat{\mathbf{u}} - \mathbf{u}(\mathbf{w}))^2$$

$$C_r(\mathbf{w}) = (\hat{\mathbf{r}} - \mathbf{r}(\mathbf{w}))^2$$

↑ Targets

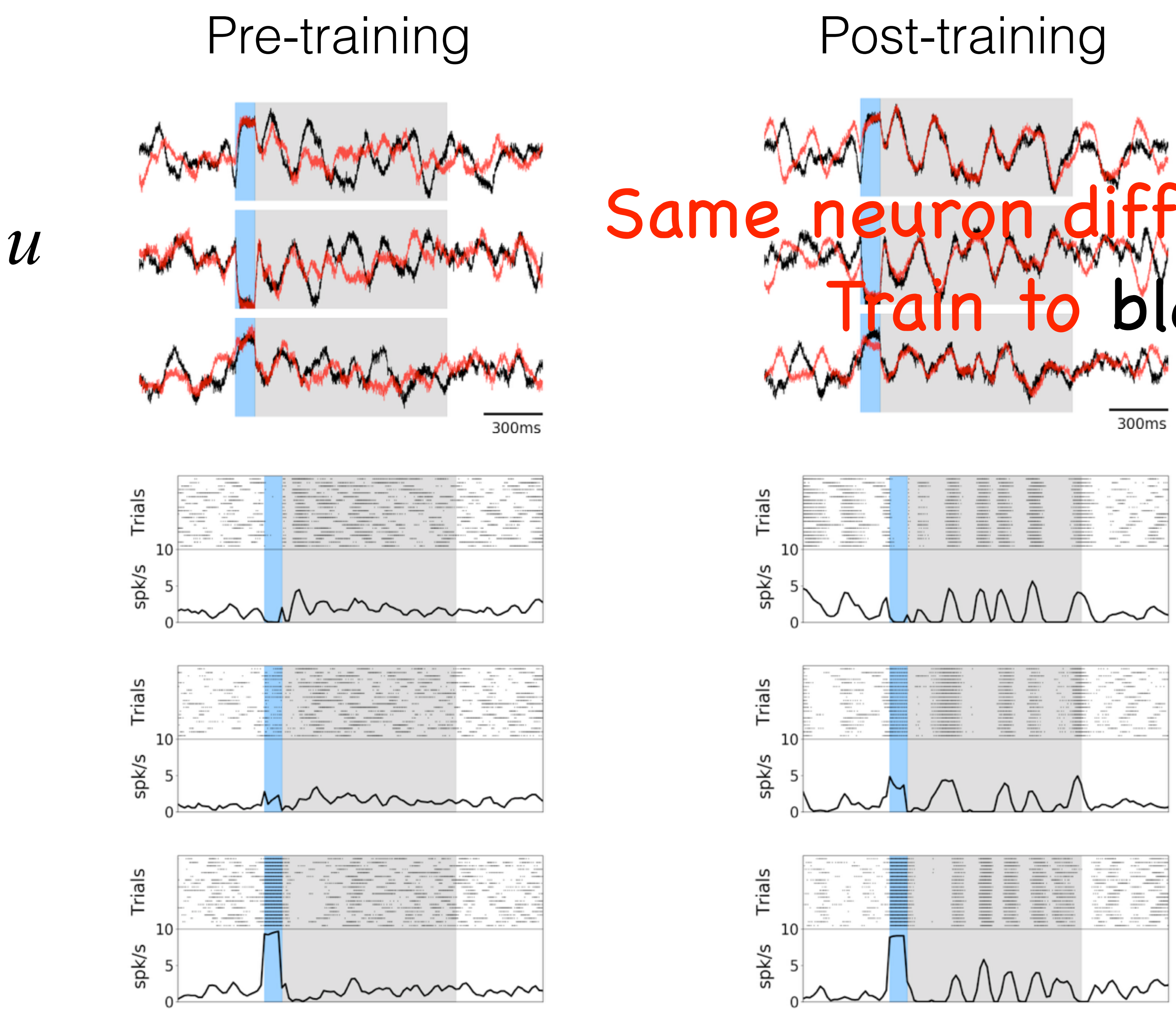
Super hard in general

since $\mathbf{u}(\mathbf{w}) = \mathbf{w}\mathbf{r}$ Linear in w

and $\mathbf{r}(\mathbf{w}) \approx \frac{1}{\pi}\sqrt{\mathbf{w}\mathbf{r}}$ Quasi-static approx

Recursive least squares or FORCE learning

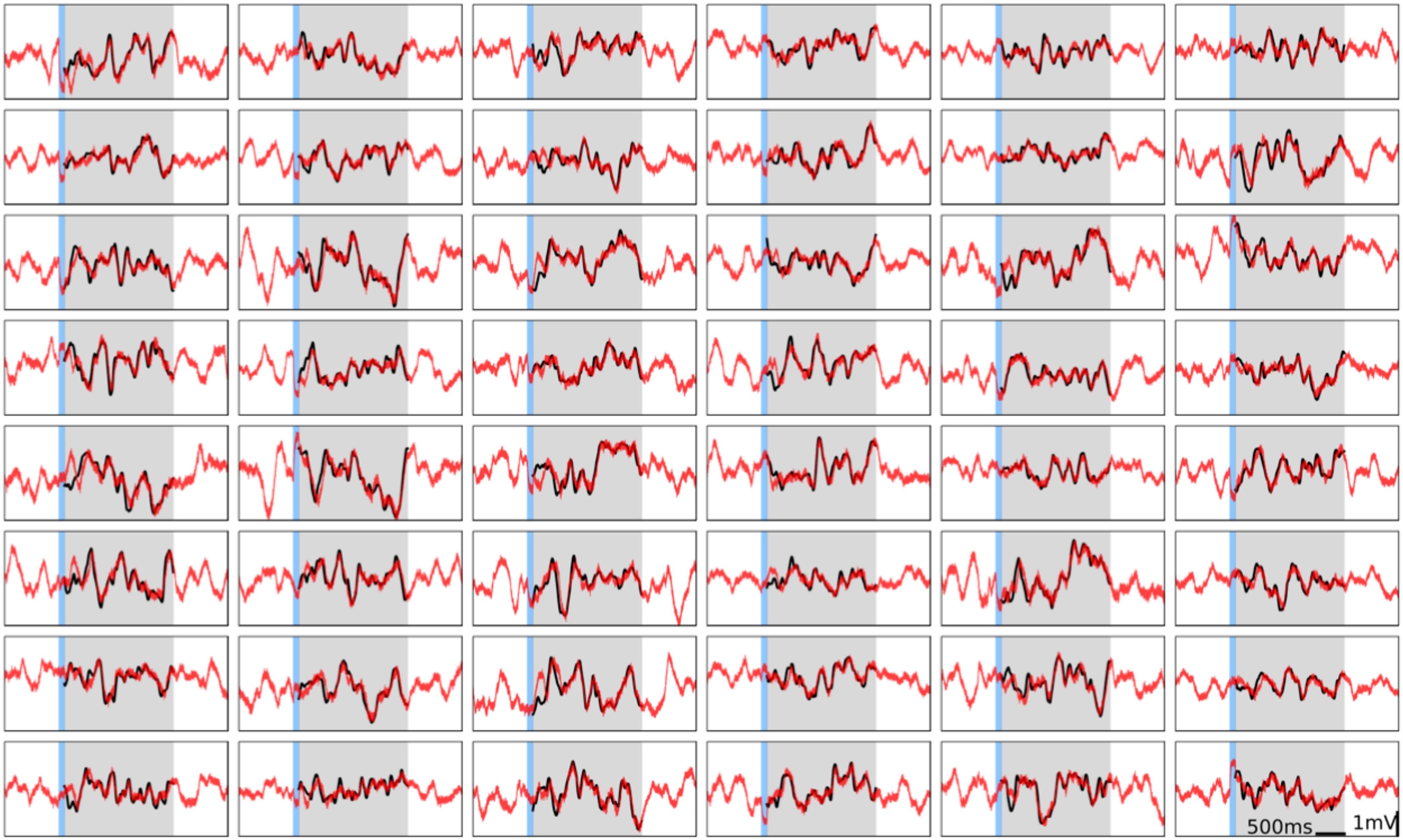
Learning innate trajectories



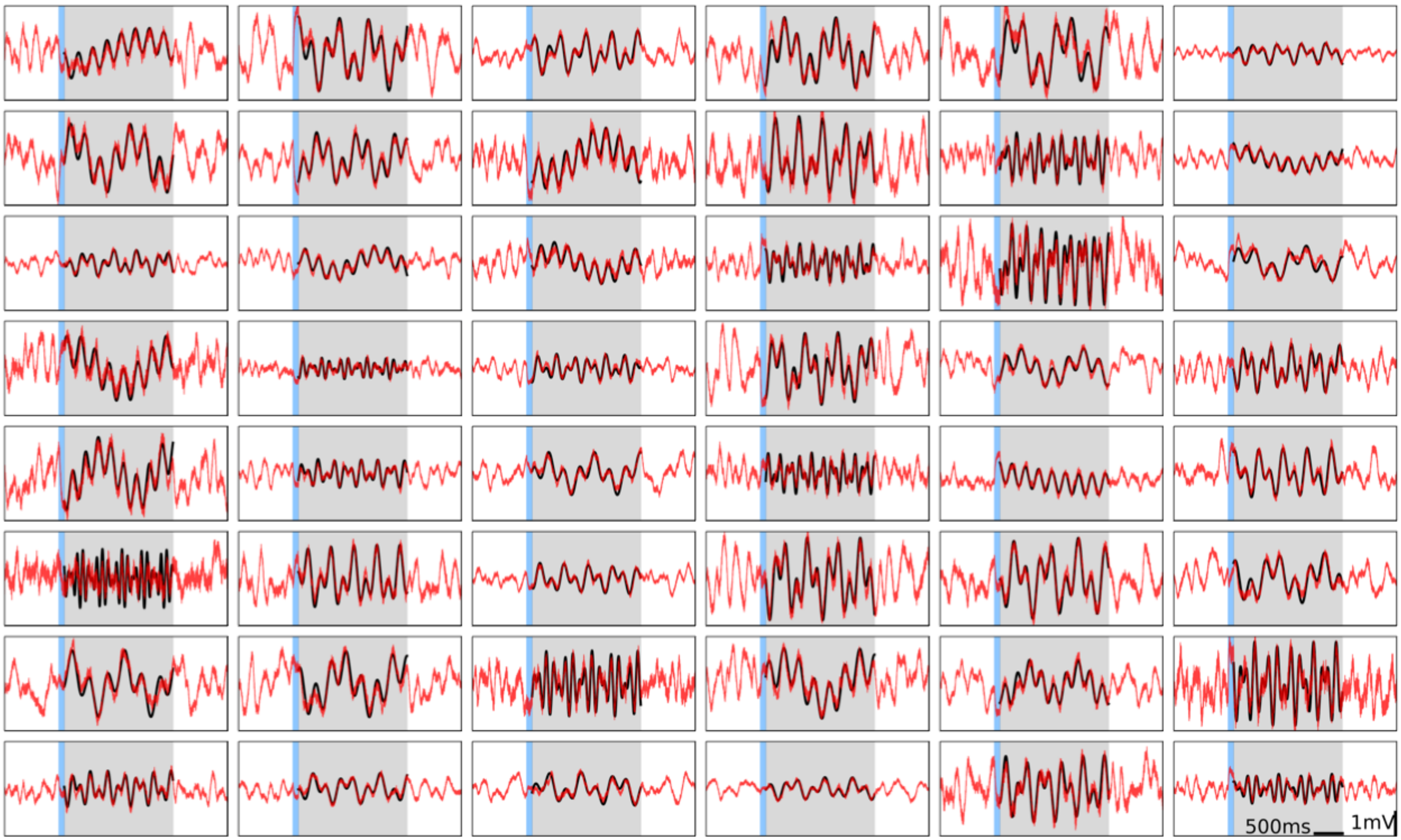
Same neuron different init conds,
Train to black curve

Extends Laje and Buonomano (2013) to spiking networks

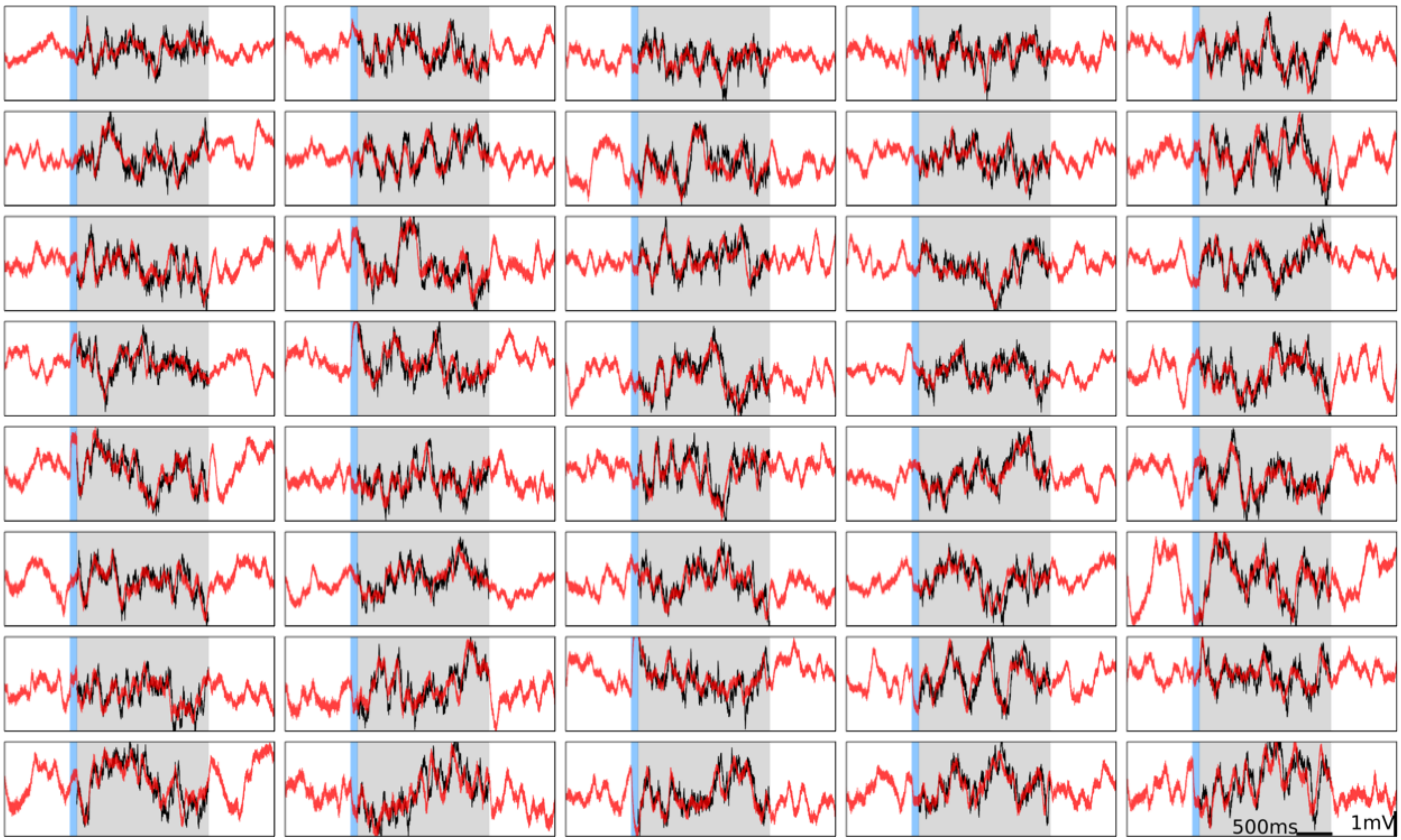
Chaotic trajectories from another system



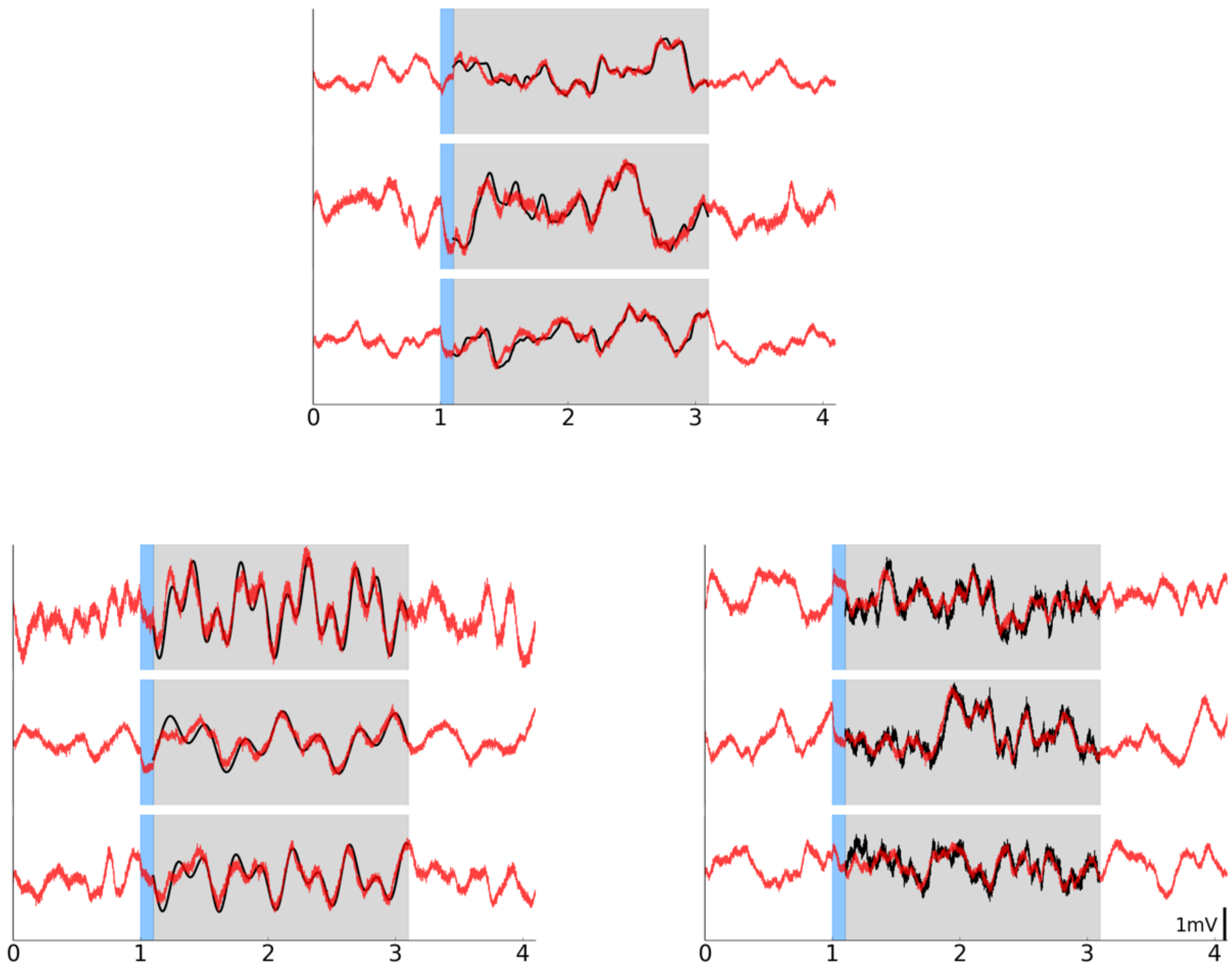
Periodic functions



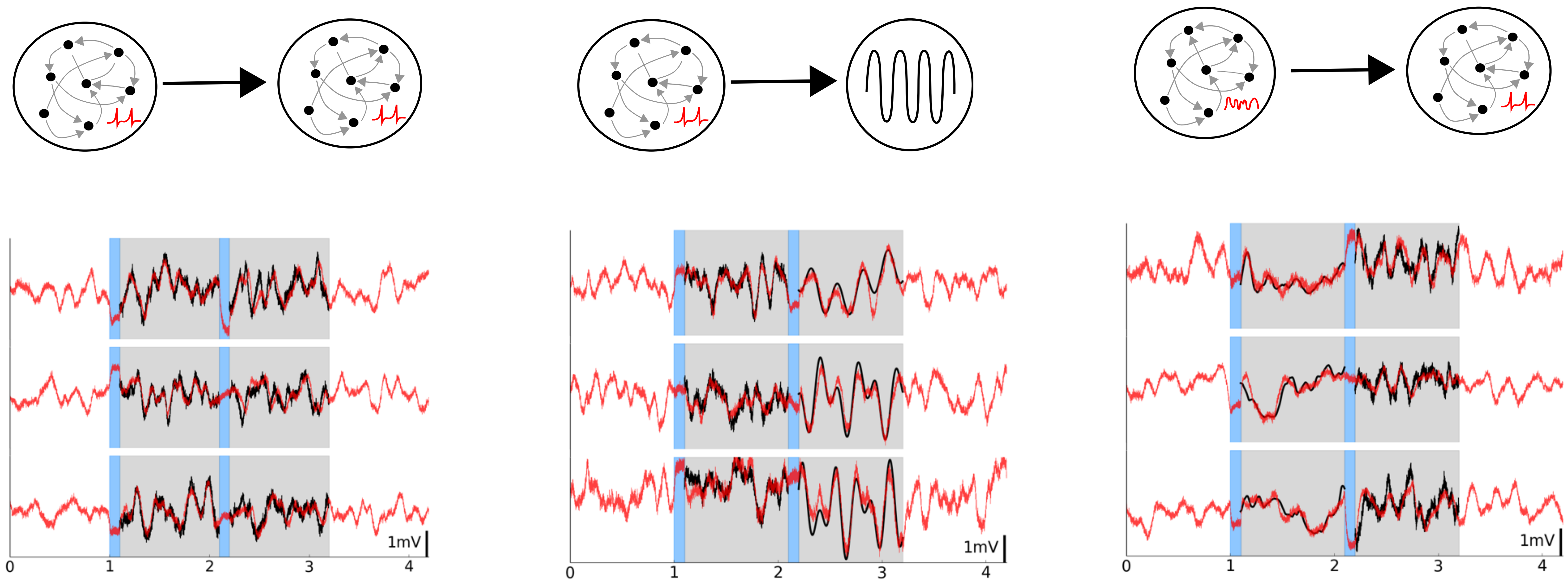
Stochastic OU process



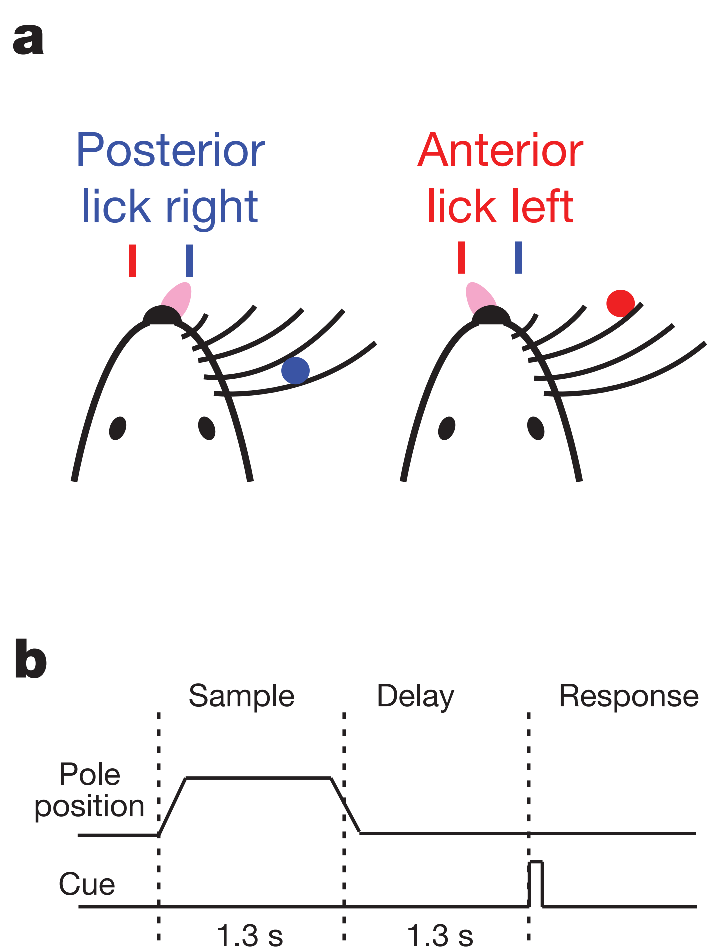
Arbitrary combinations



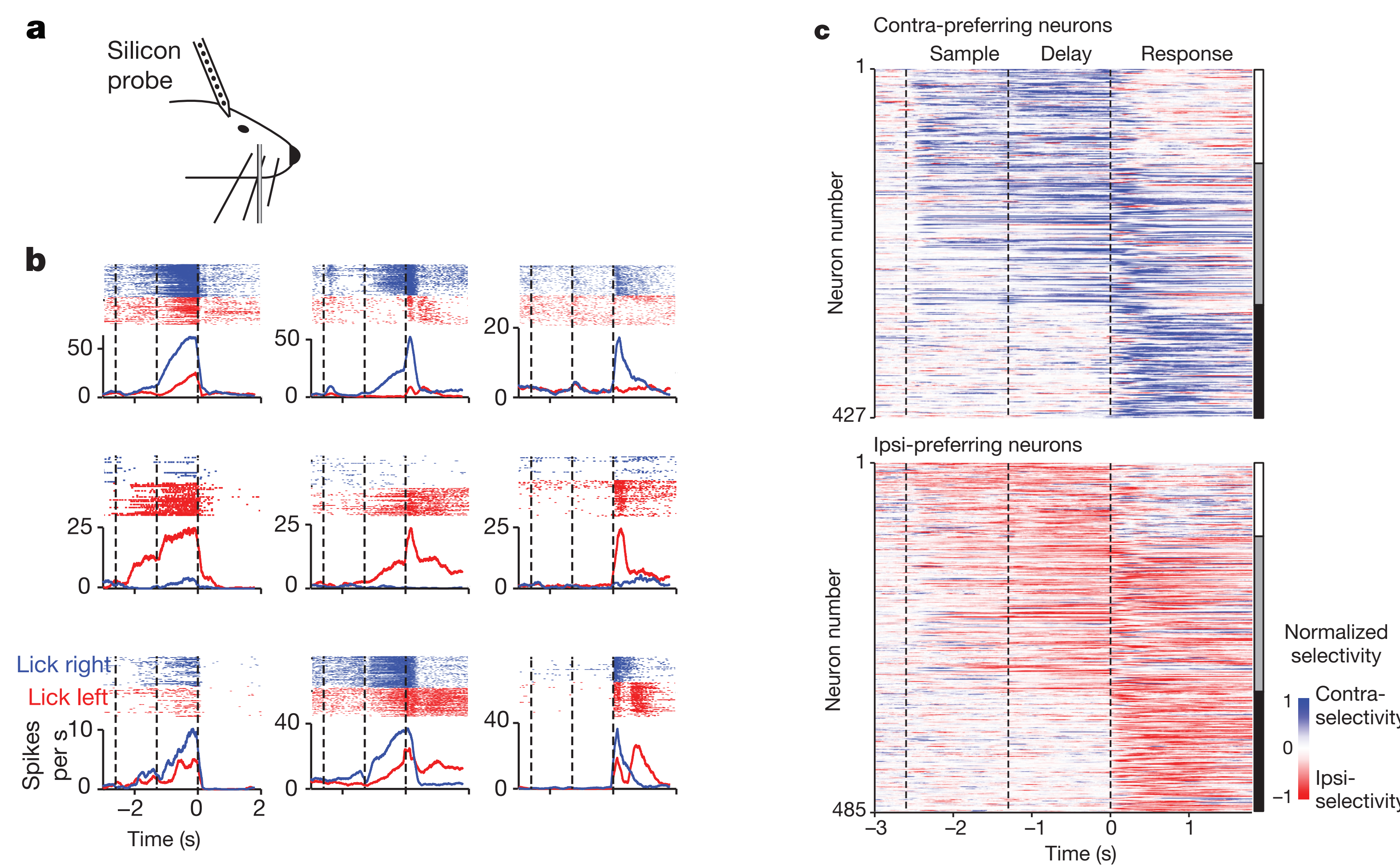
Multiple targets in one network

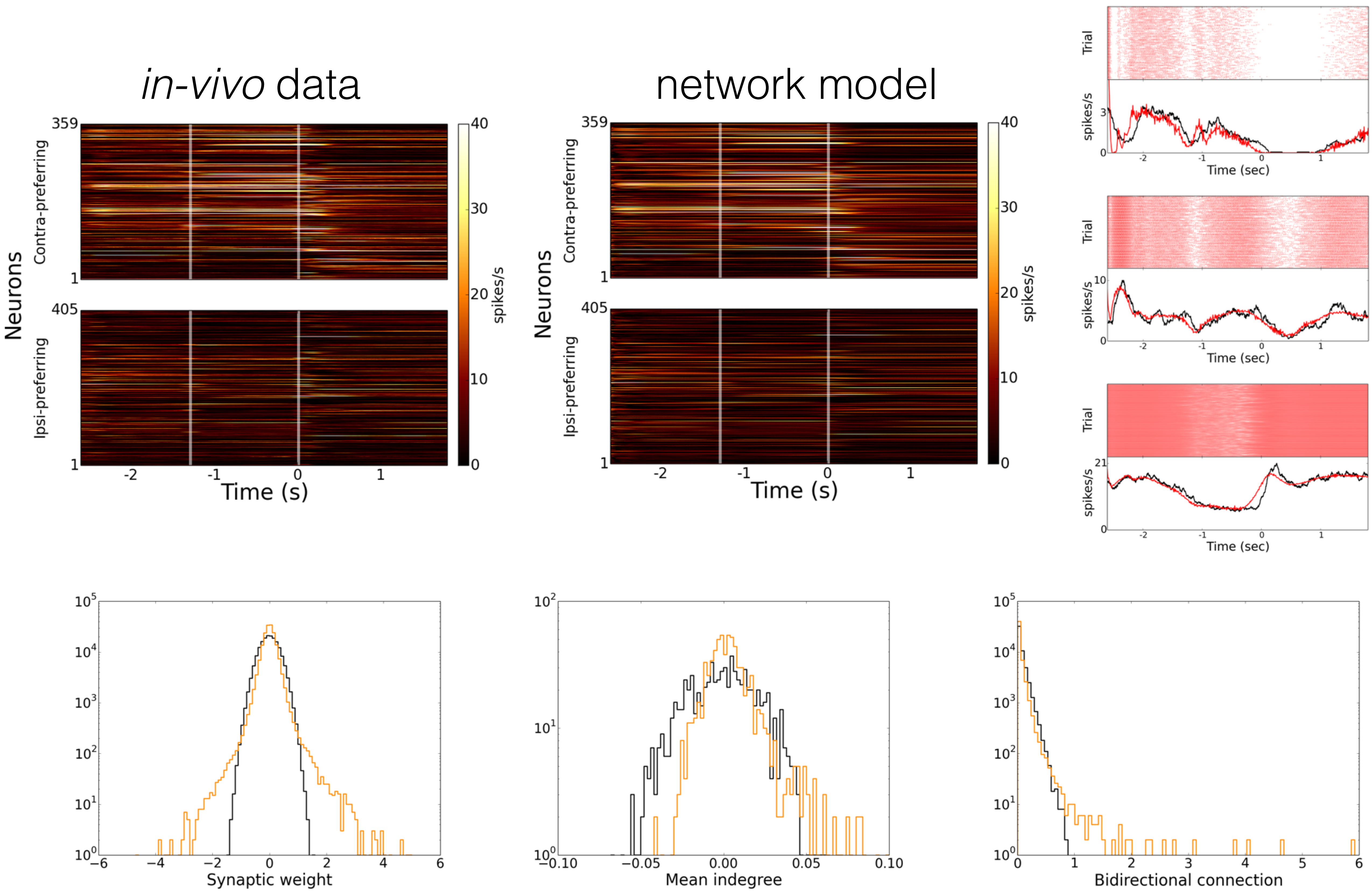


Real cortical neurons

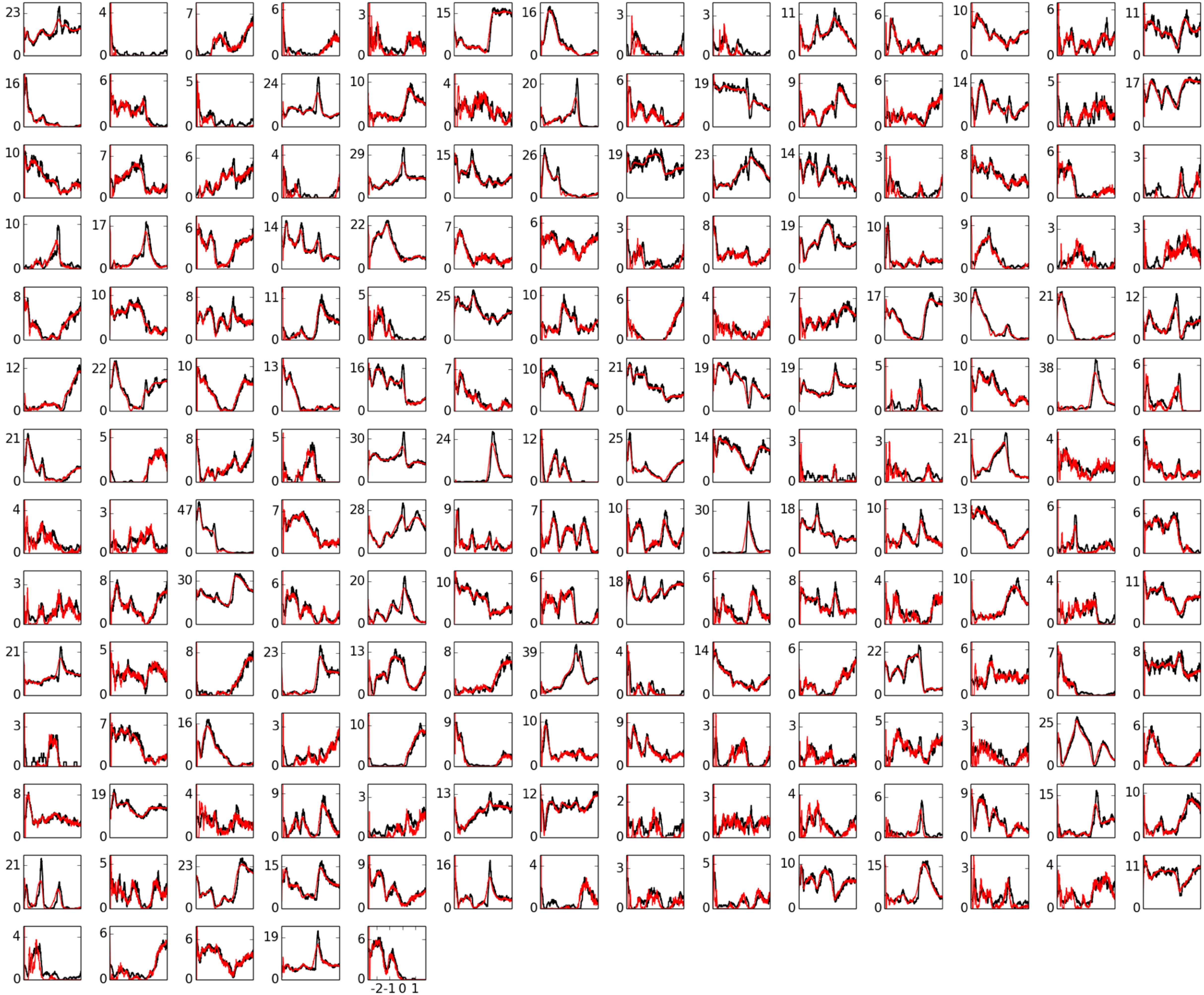


Li et al. 2015





in-vivo data
network model
Neurons
Contra-preferring
Ipsi-preferring
Time (s)
spikes/s
Trial
Time (sec)
Synaptic weight
Mean indegree
Bidirectional connection


-2-1 0 1

Universal Dynamical System?

$$\partial_t \eta_i(\theta, t) + \partial_\theta F_i(\theta, u_i)\eta_i(\theta, t) = 0$$

$$\dot{u}_i(t) + \beta u_i(t) - 2\beta \sum_j w_{ij}\eta_j(\pi, t) = 0$$

$$\mathbf{u}(t) = \mathbf{w}\varphi(u(t))$$

Conjecture: network can approximate an arbitrary set of continuous functions*

*under a mild set of conditions

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Slides to appear on sciencehouse.wordpress.com