Neurosensory Network Functionality, Adaptation and Robustness: Paradigms for Data-Driven Control and Learning

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Mathematical Foundations

Dimensionality Reductions
+ Machine Learning

Dynamical Systems + PDEs

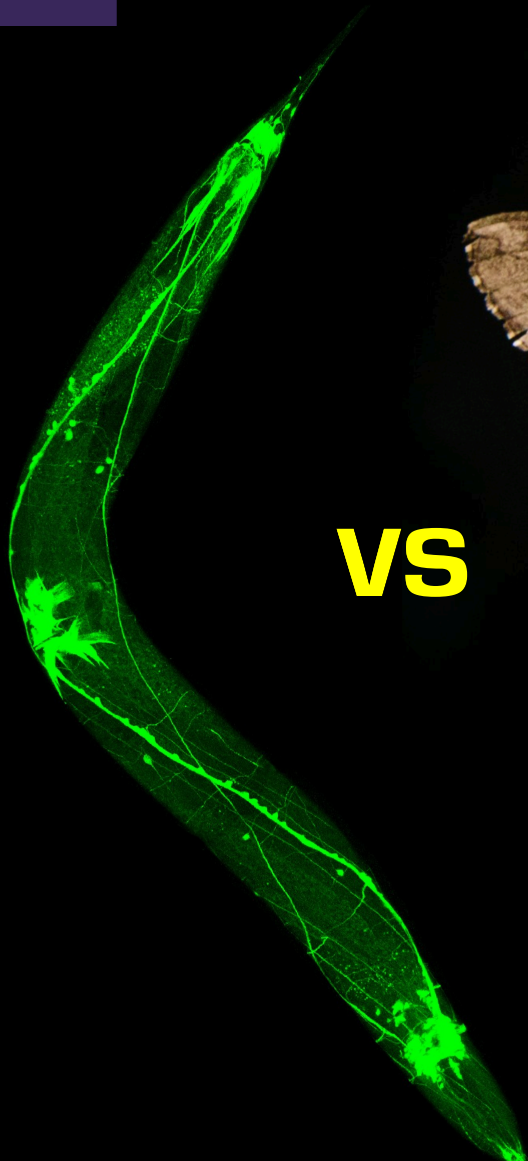
i. Generic nonlinear , time-dependent system

$$\frac{d\mathbf{x}}{dt} = N(\mathbf{x}, t; \mu) \qquad \mathbf{x}(0) = \mathbf{x}_0$$

ii. Measurements (assimilation) $G(\mathbf{x}, t_k) = 0$

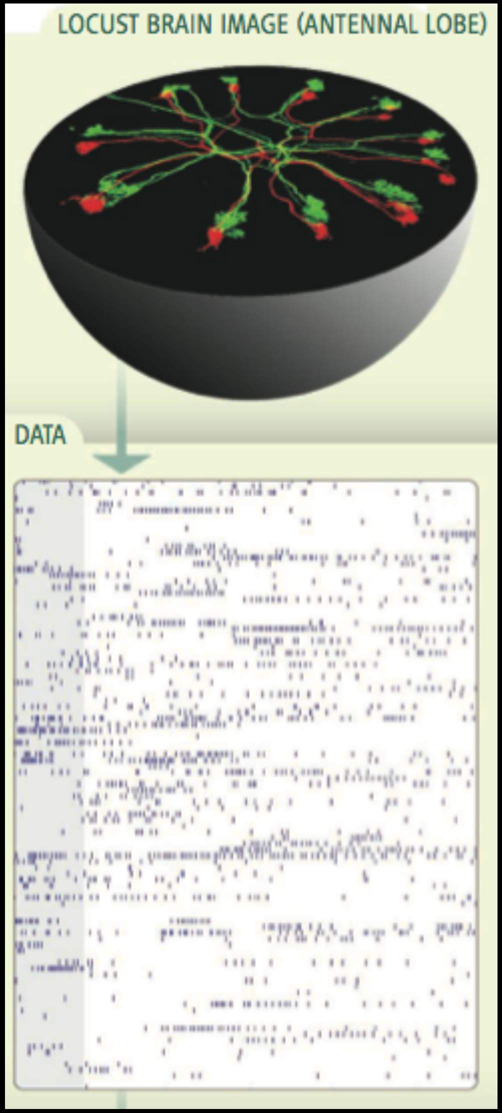
iii. My commitment – low-dimensional subspaces

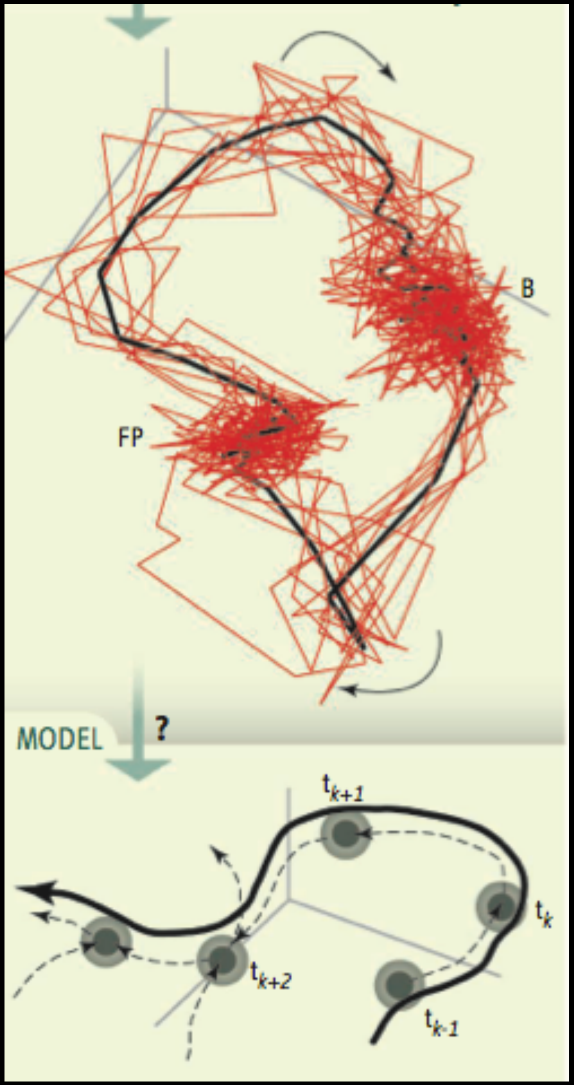
$$\mathrm{x}(t) \approx \mathbf{\Phi}_r \mathbf{a}(t)$$



VS

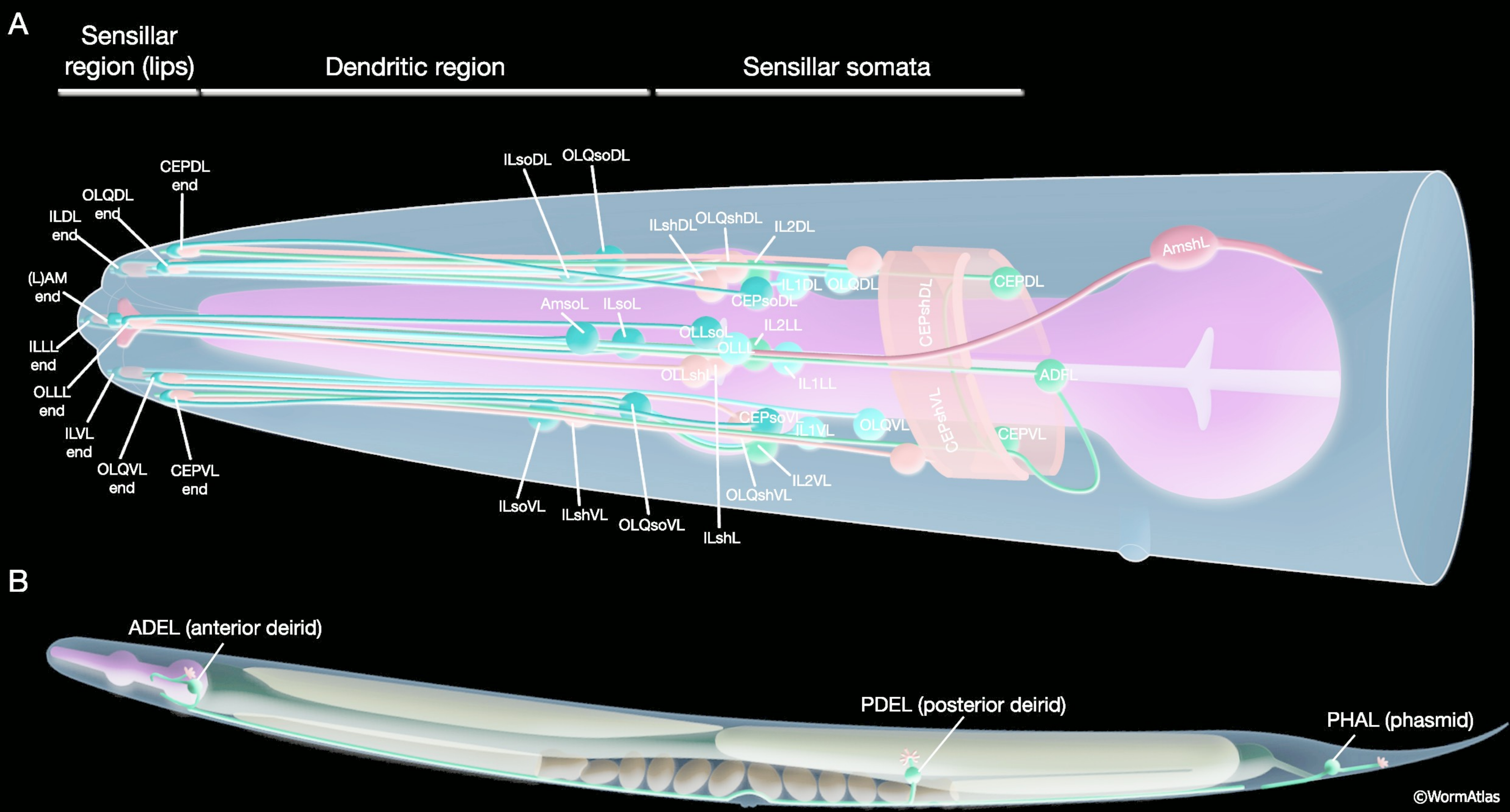
Encoding Dynamics



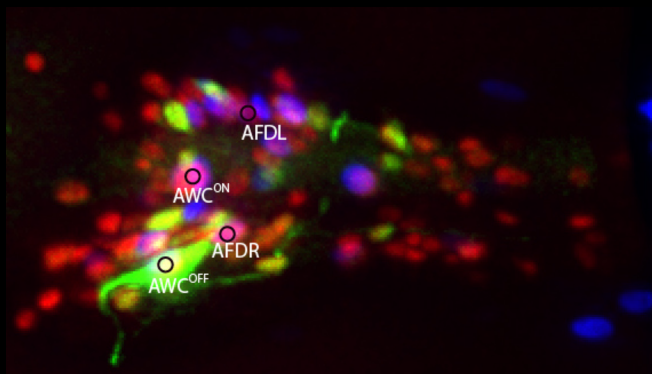


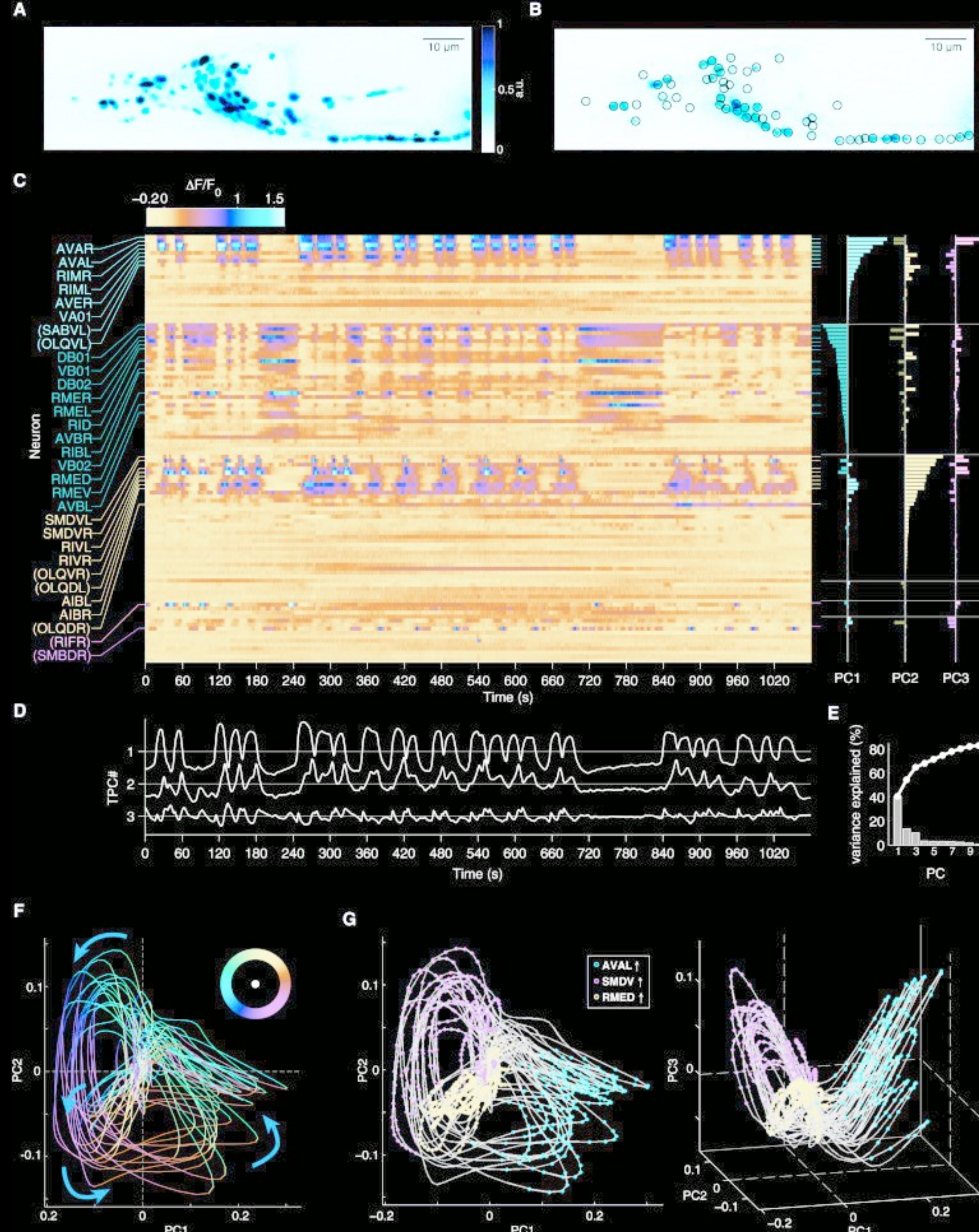
Rabinovich, Huerta, Laurent, Science (2008)

C. Elegans



Worm Atlas

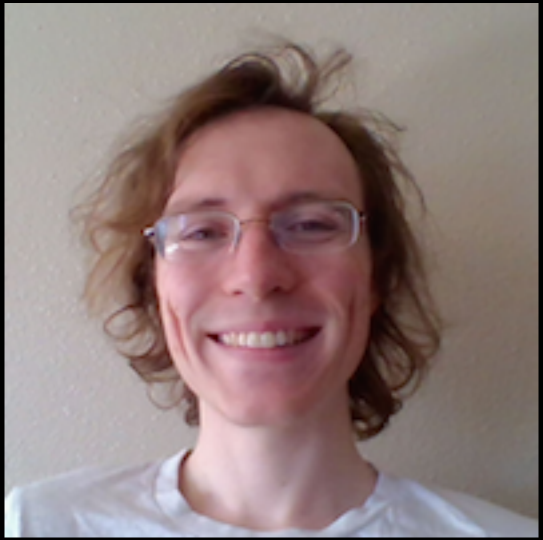




Manuel Zimmer
et al 2012-present

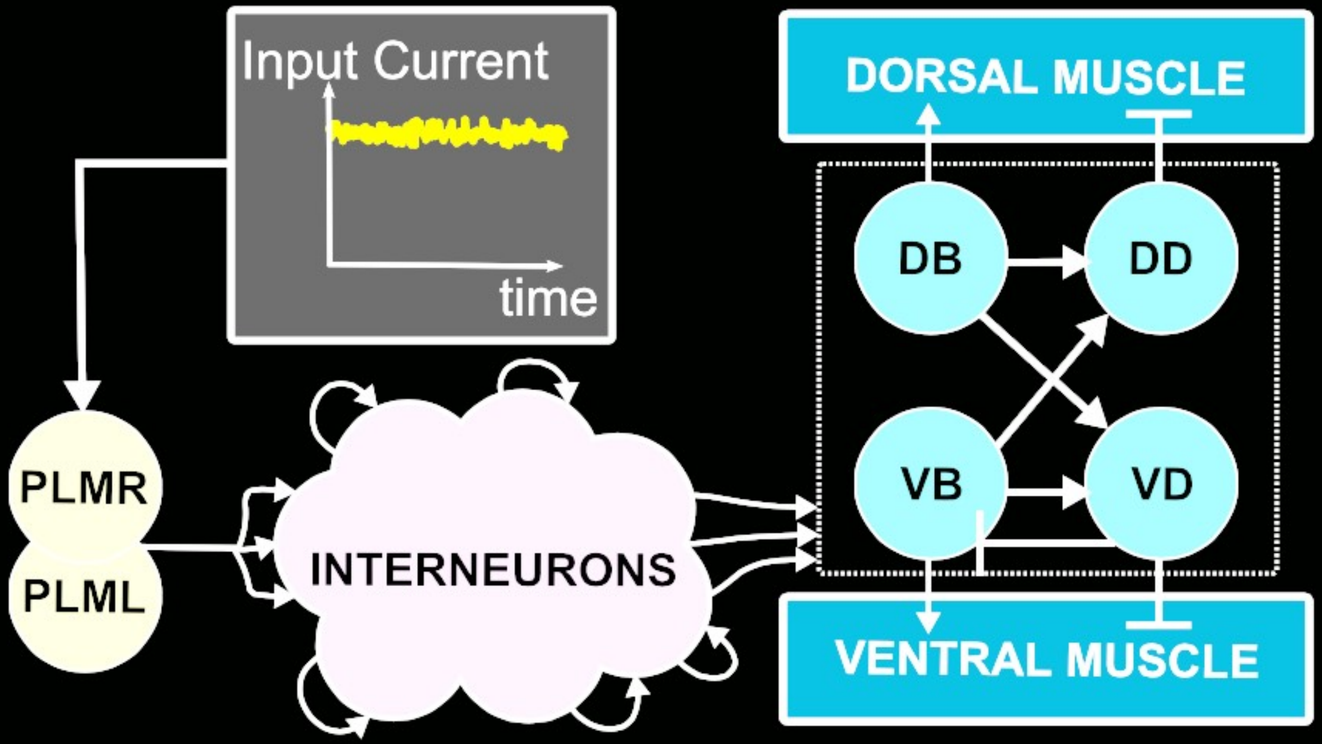
Models

C. Elegans
Stereotyped connectivity



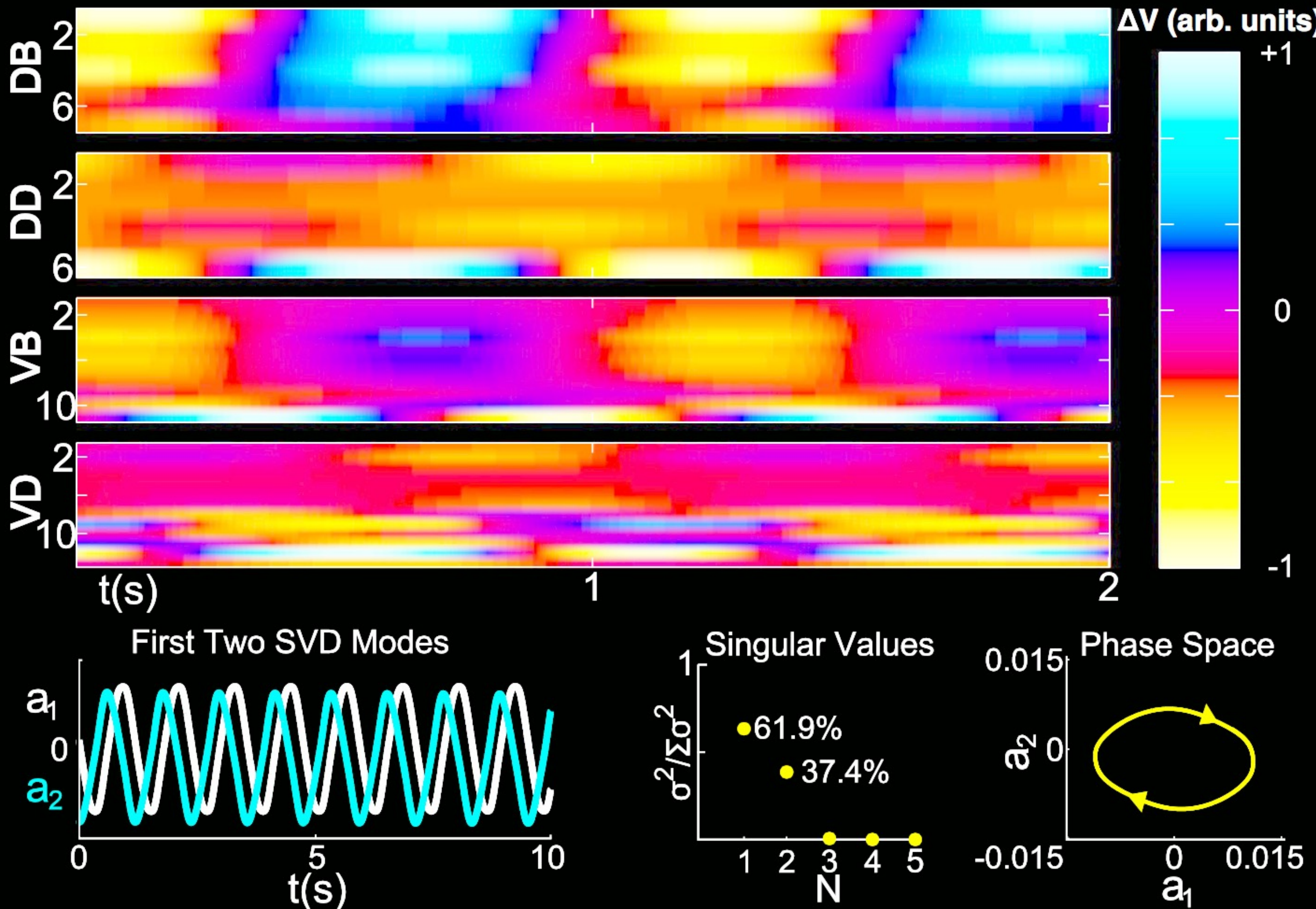
Charlie Fieseler

Connectomic Dynamics



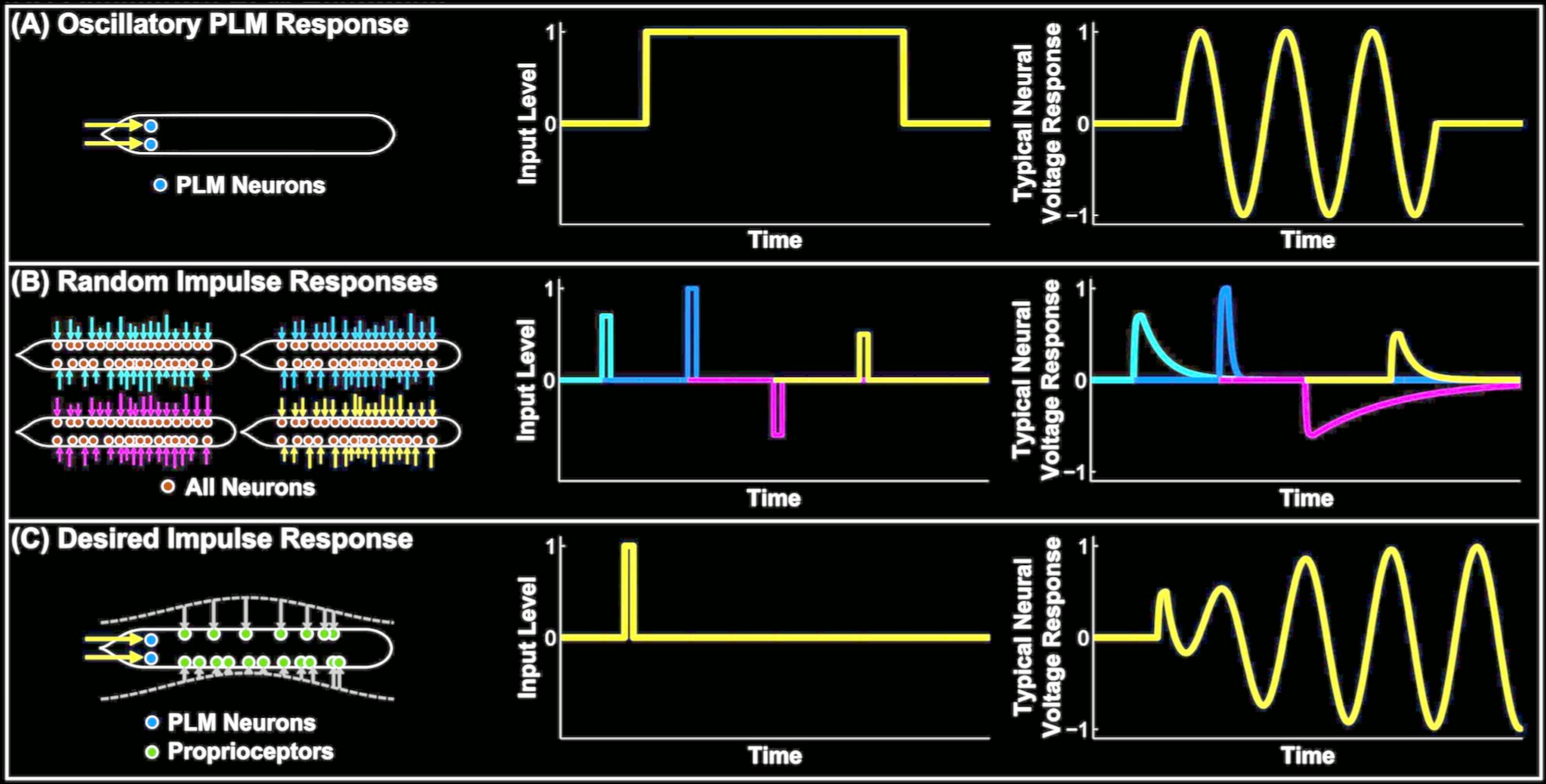
Kunert, Shlizerman & Kutz, PRE 2014

Connectomic Dynamics

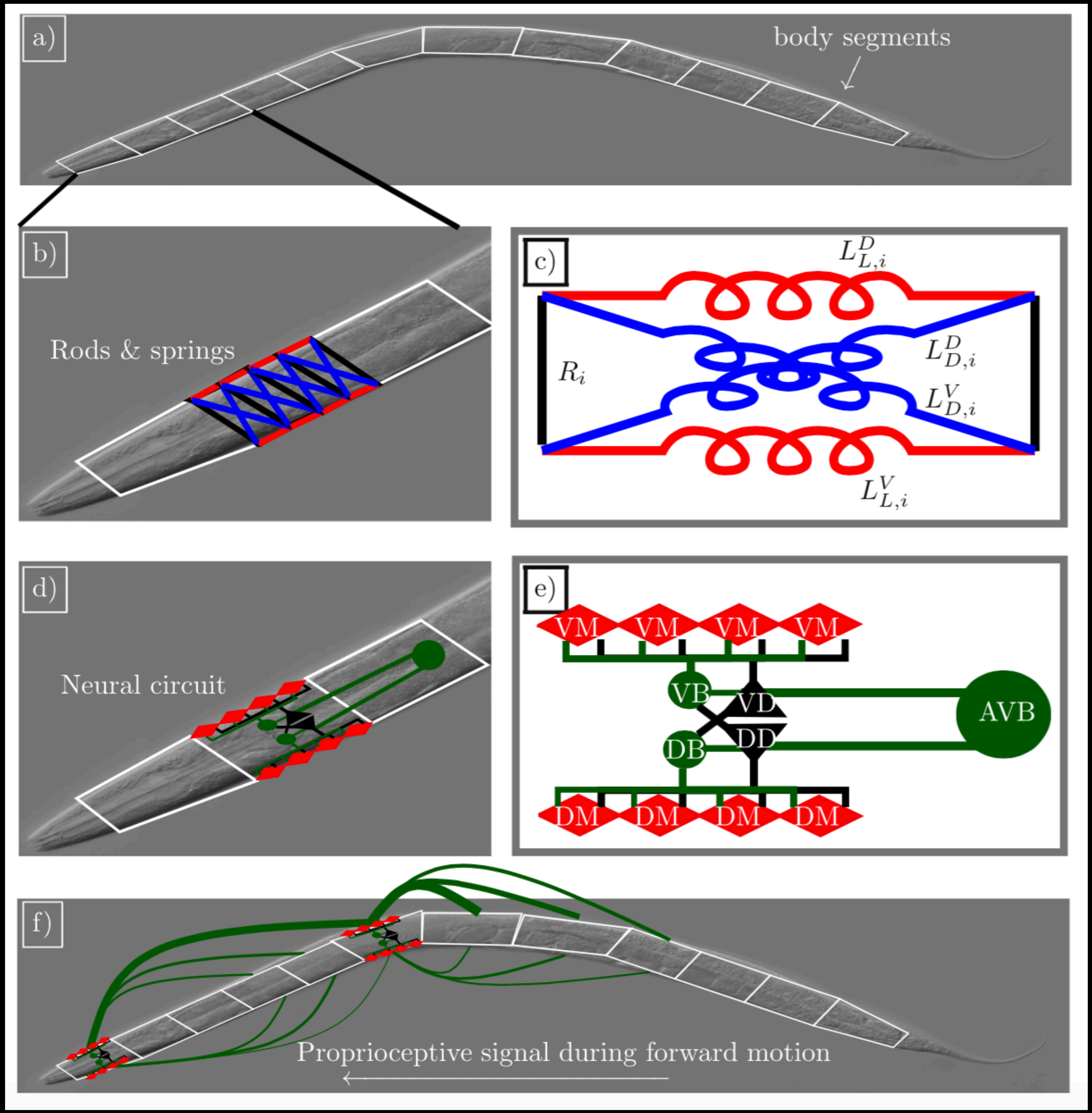


Kunert, Shlizerman & Kutz, PRE 2014

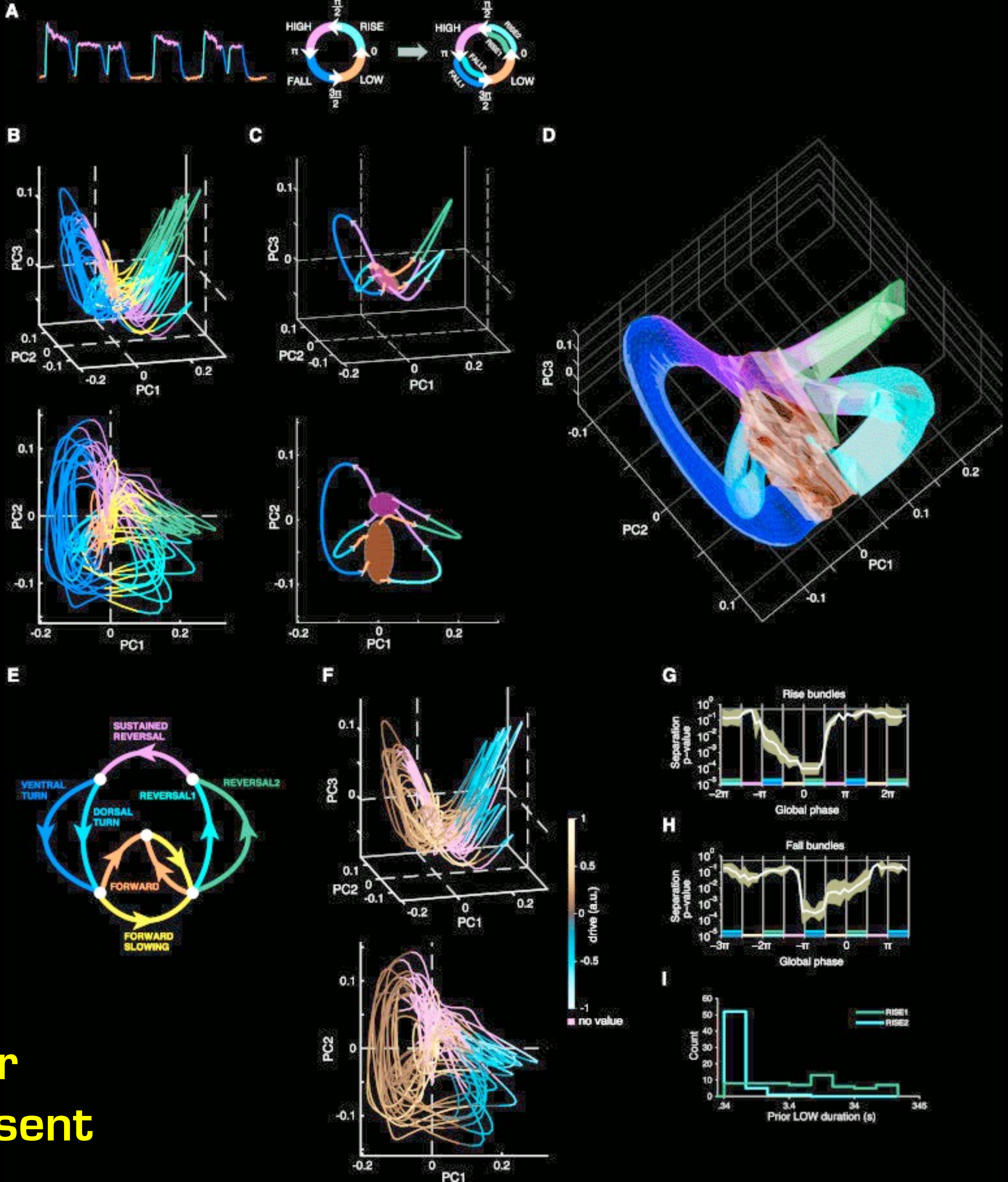
Proprioceptic Feedback



Kunert, Proctor, Brunton & Kutz, PLOS Comp Bio 2017



Fieseler, Kunert & Kutz, J. Biomechanics 2018



Manuel Zimmer
et al 2012-present

Dynamic Mode Decomposition

Definition: Dynamic Mode Decomposition (Tu et al. 2014 [1]): *Sup pose we have a dynamical system (1.17) and two sets of data*

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_M \\ | & | & & | \end{bmatrix}$$

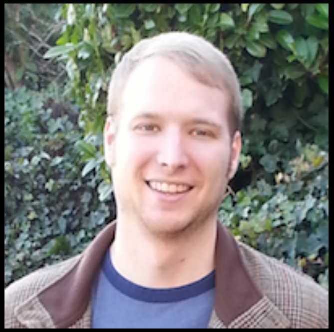
$$\mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}'_1 & \mathbf{x}'_2 & \cdots & \mathbf{x}'_M \\ | & | & & | \end{bmatrix}$$

with $\mathbf{x}_k$ *an initial condition to (1.17) and* $\mathbf{x}'_k$ *it corresponding output after some prescribed evolution time* τ *with there being* m *initial conditions considered. The DMD modes are eigenvectors of*

$$\mathbf{A}_{\mathbf{X}} = \mathbf{X}'\mathbf{X}^{\dagger}$$

where † *denotes the Moore-Penrose pseudoinverse.*

Travis Askham & Kutz (2017)



Approximate Dynamical Systems

Linear dynamics (equation-free)

$$\frac{d\tilde{\mathbf{x}}}{dt} = \mathbf{A}\tilde{\mathbf{x}}$$

Eigenfunction expansion

$$\tilde{\mathbf{x}}(t) = \sum_{k=1}^{K} b_k \psi_k \exp(\omega_k t)$$

Least-square fit

$$||\mathbf{x}(t) - \tilde{\mathbf{x}}(t)|| \ll 1$$

DMD with Control

Input

$$\mathbf{x}_{k+1} \approx \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k$$

Input Snapshots

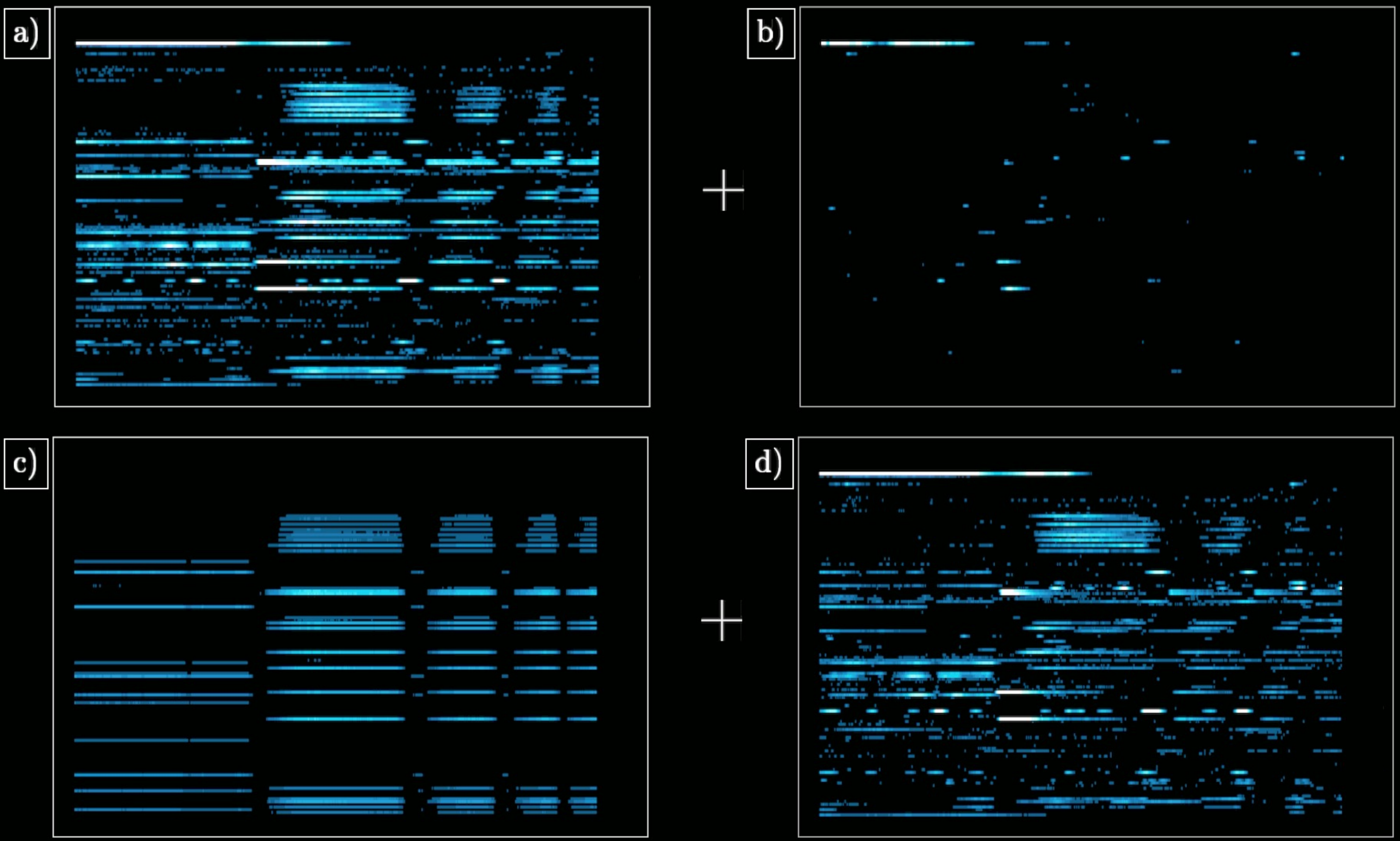
$$\Upsilon = \begin{bmatrix} | & | & & | \\ \mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_{m-1} \\ | & | & & | \end{bmatrix}$$

DMD generalization

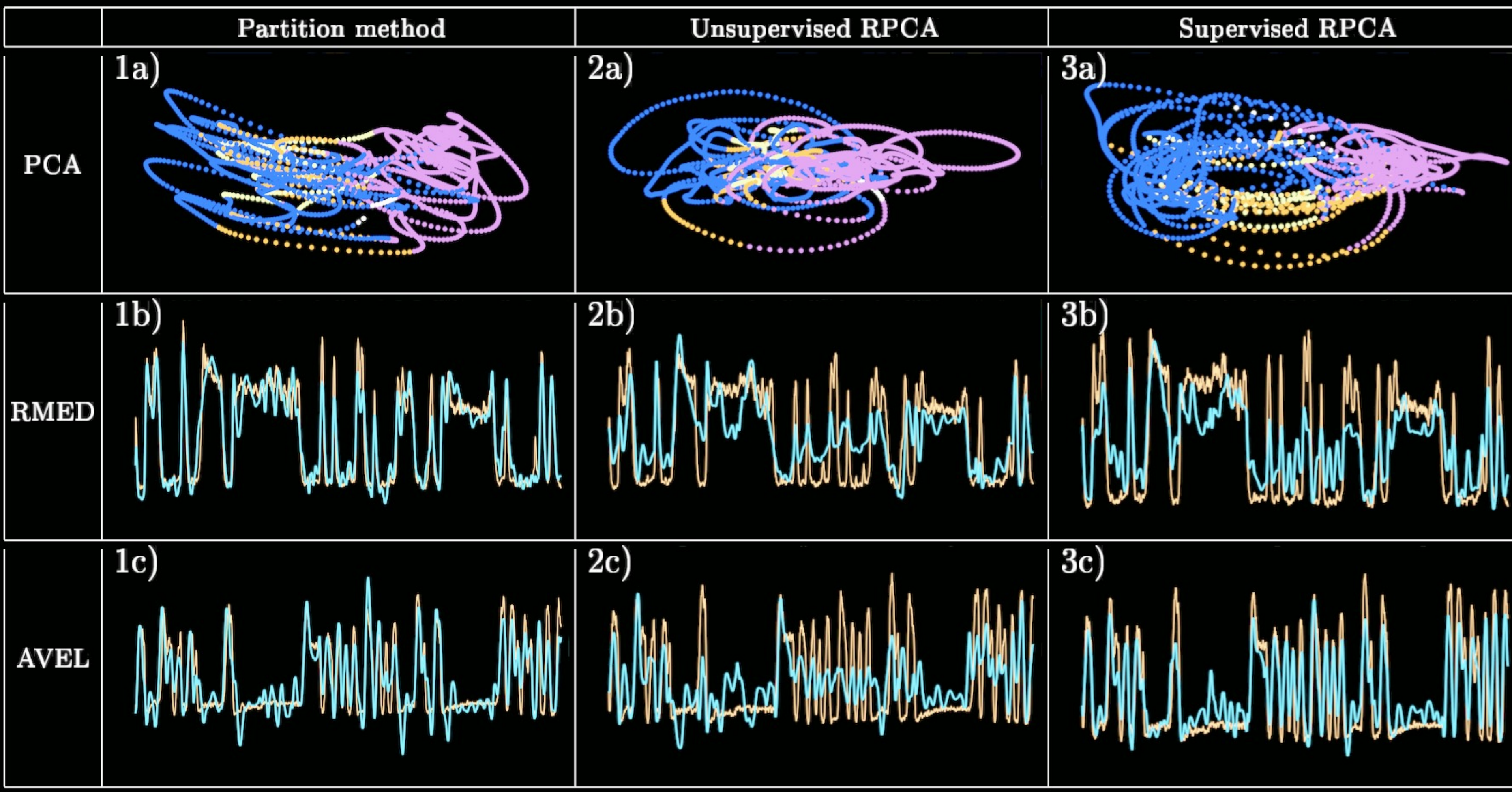
$$\mathbf{X}' \approx \mathbf{A}\mathbf{X} + \mathbf{B}\Upsilon$$

Proctor, Brunton & Kutz, SIADS (2016)

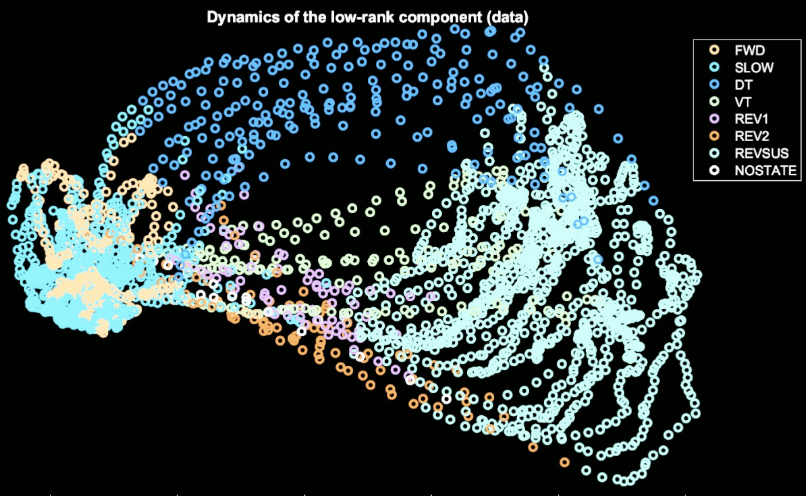
Sparse + Low-Rank Decomposition

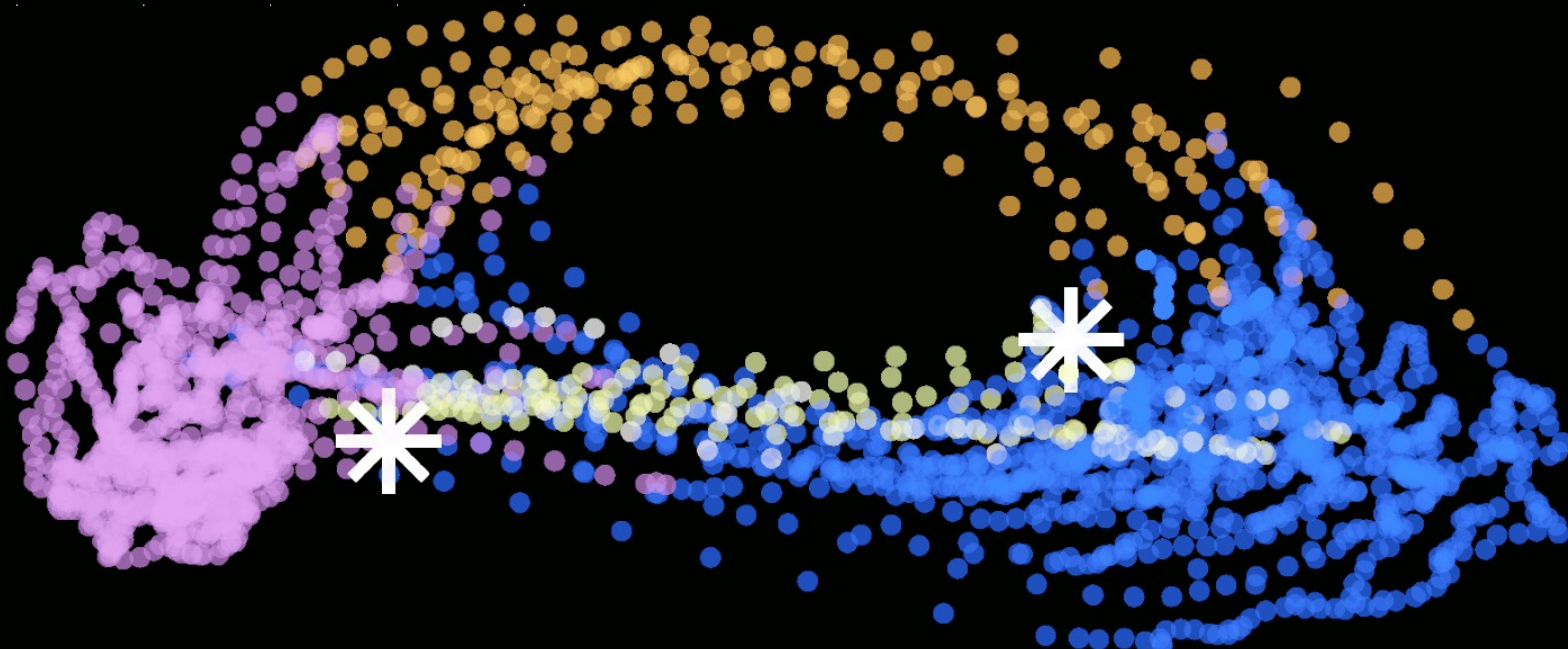


Supervised & Unsupervised Models



Intrinsic Bi-Stability



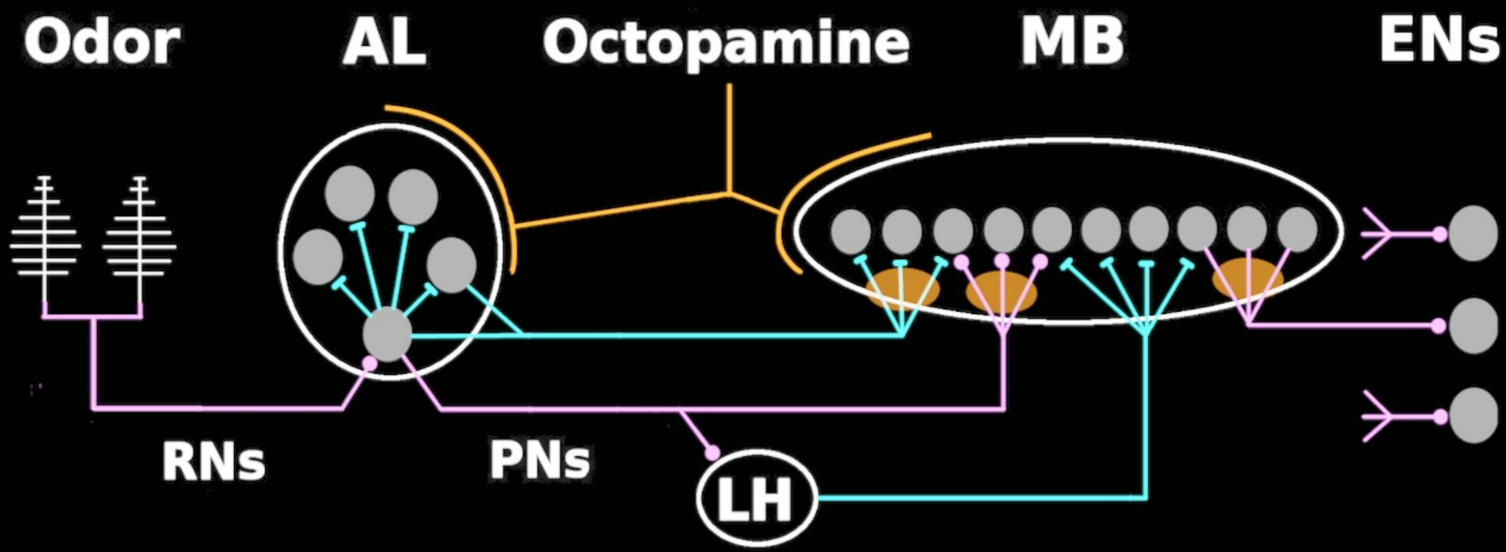


Fieseler, Zimmer & Kutz (soon on arxiv)

Manduca
Randomness and sparsity

Charles Delahunt

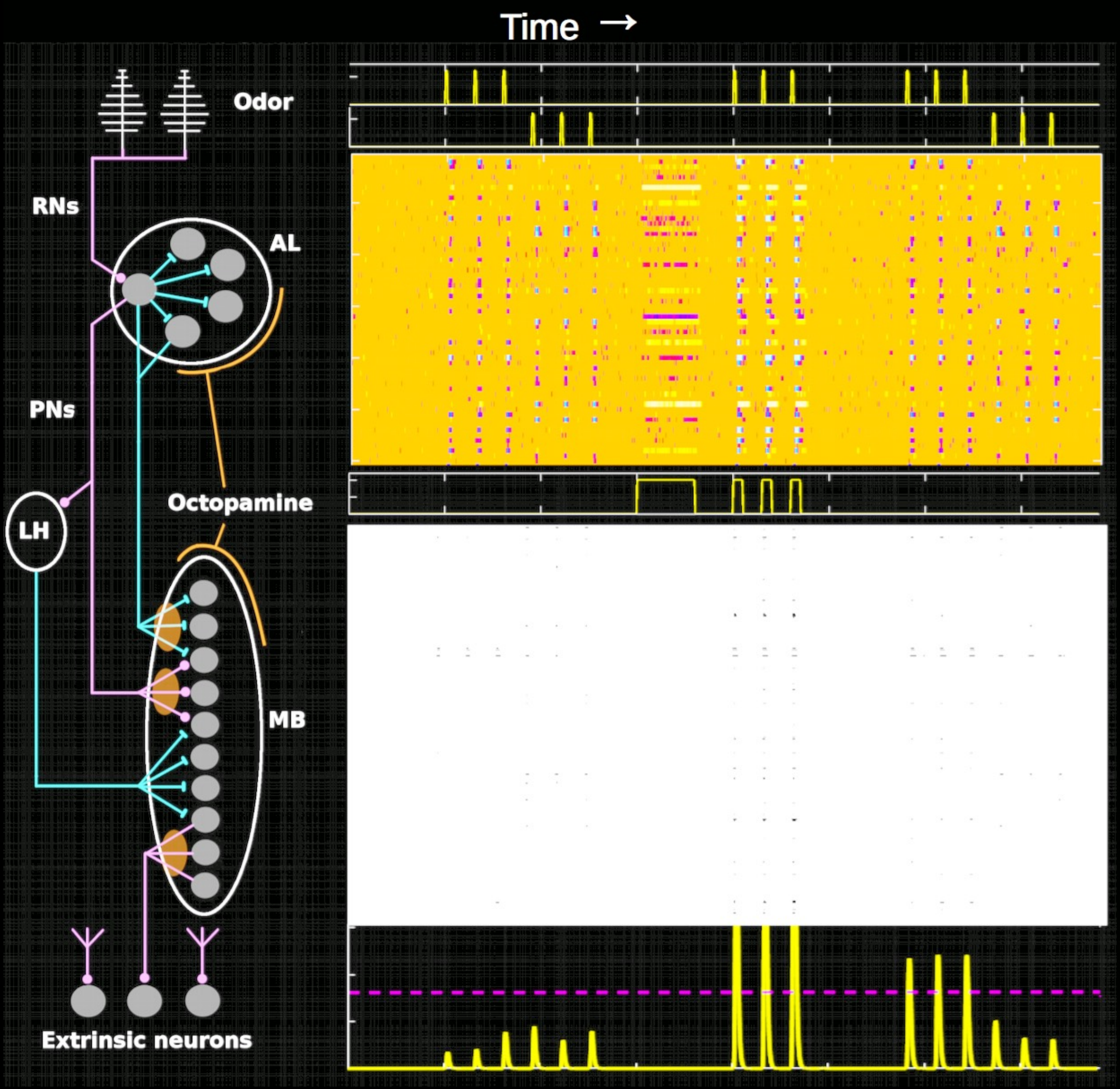
Moth Olfactory System



Riffell et al. Science 2013
Campbell et al. J Neuro 2013
Olson et al. Neuron 2010
Turner et al. J NeuroPhysiol 2008 Hong,
Wilson. Neuron 2015

Gupta, Stopfer. J NeuroSci 2012
Silbering et al. J NeuroSci 2003
Galizia. Eur J NeuroSci 2014
Caron et al. Nature 2013

Learning New Odors



Odor inputs

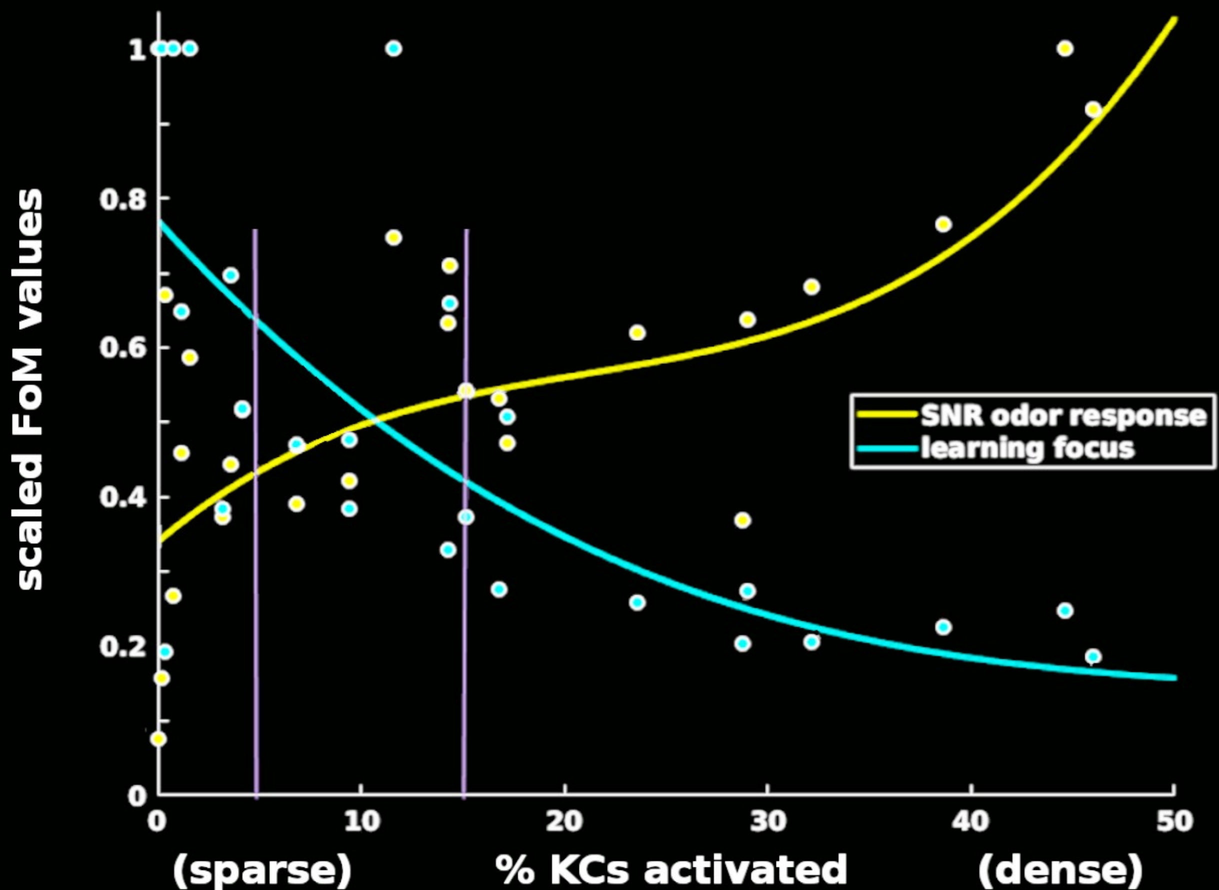
AL response

Octopamine

MB response

Readout neuron

Sparsity for Learning



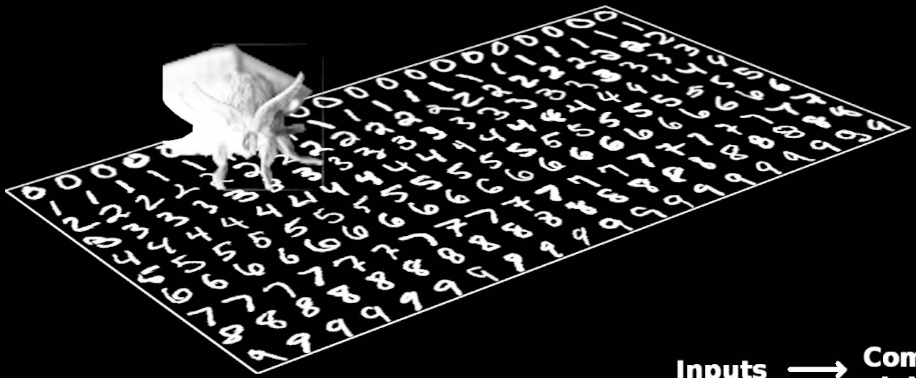
Signal to Noise = μ/σ of odor response.
high → reliable response.

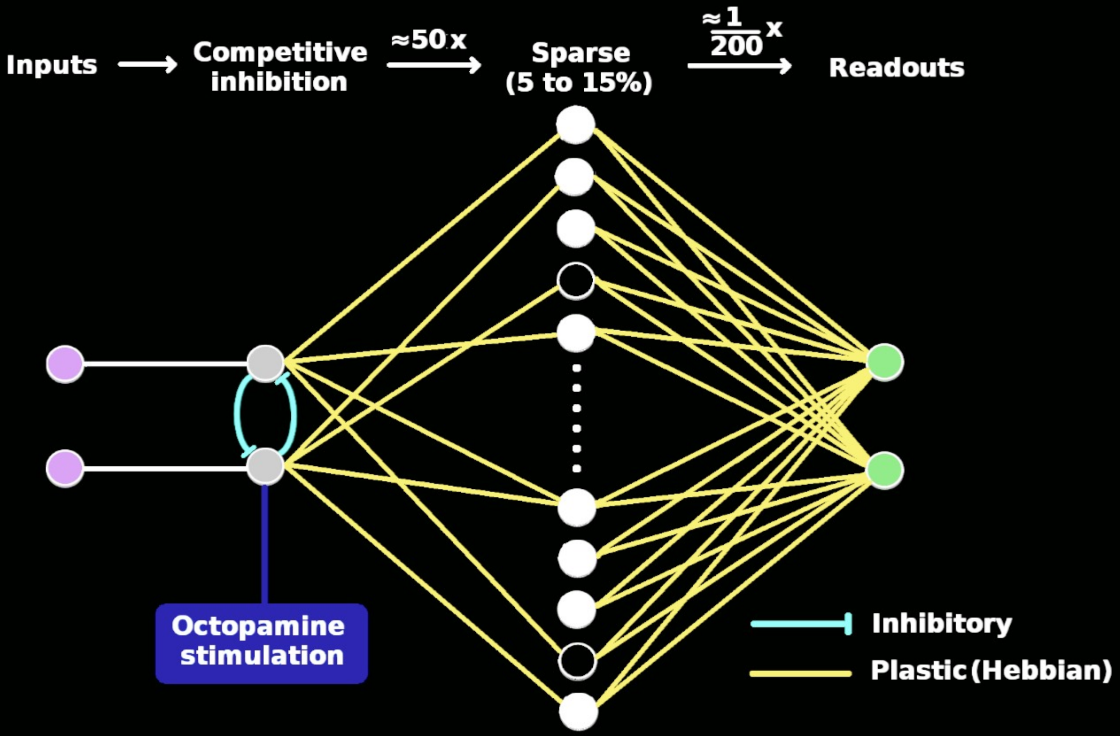
High → good.

Learning focus = ΔTrained / ΔControl.
high → focused learning.

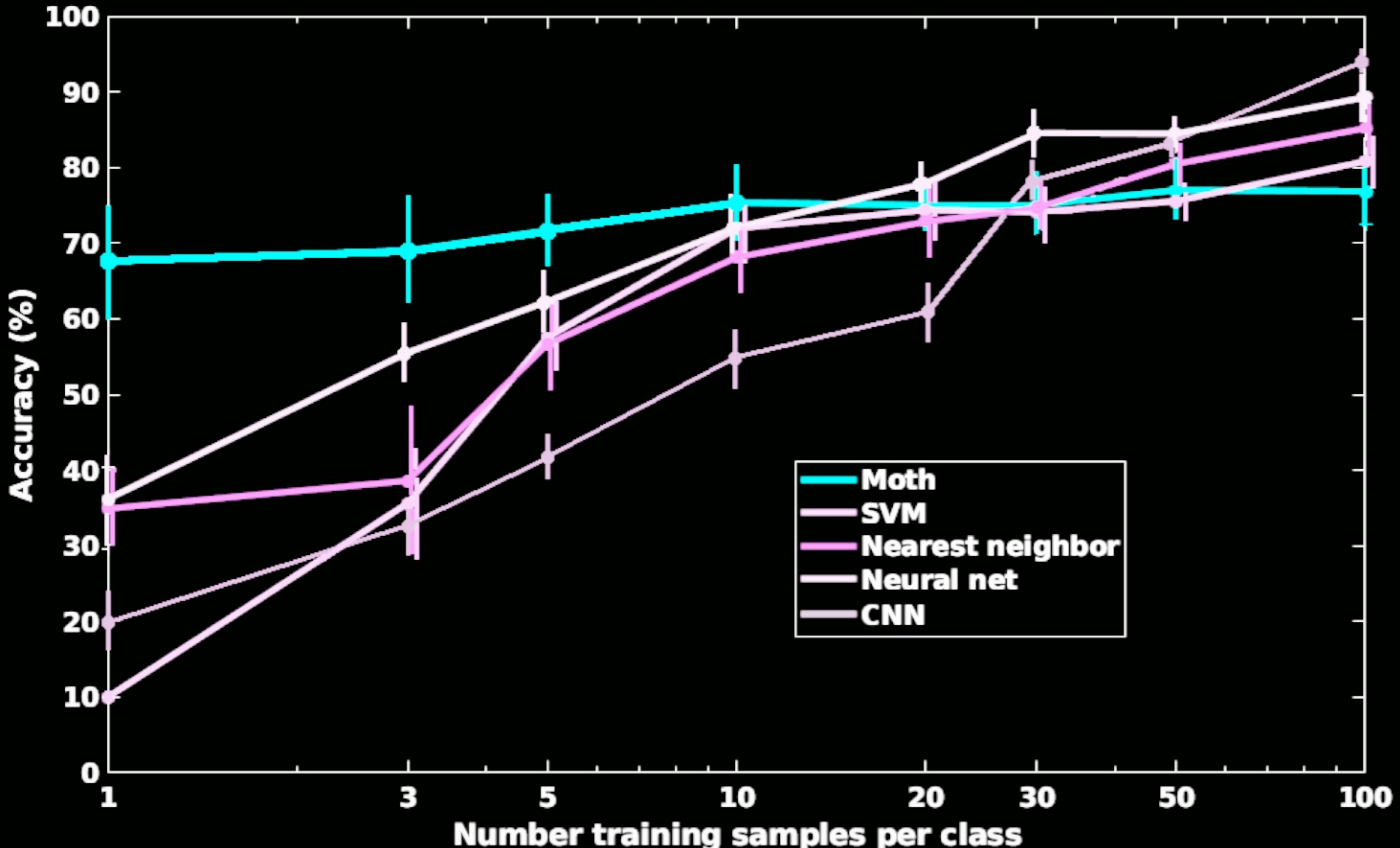
(Huerta, Nowotny; Peng, Chittka)

Rapid Learning in NNs



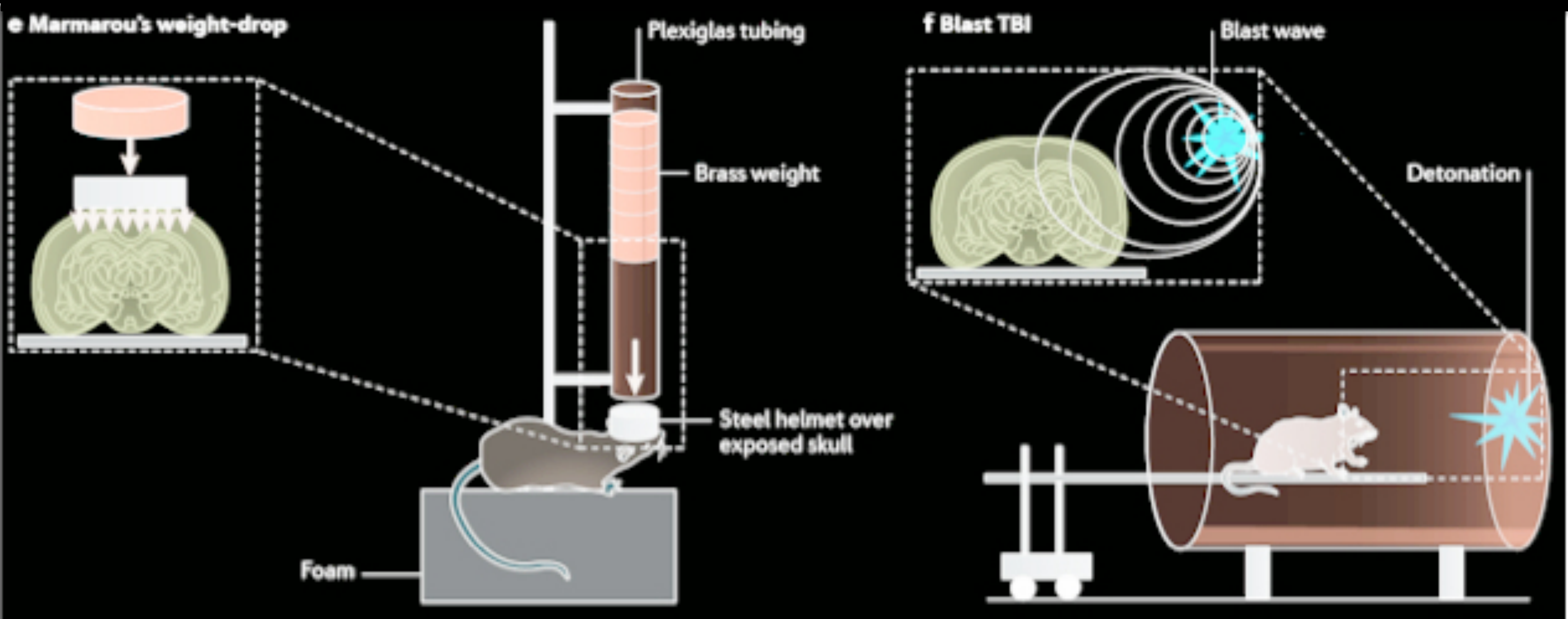


Comparisons



Neurodegenerative Diseases and Traumatic Brain Injuries

Pedro Maia, UC San Francisco

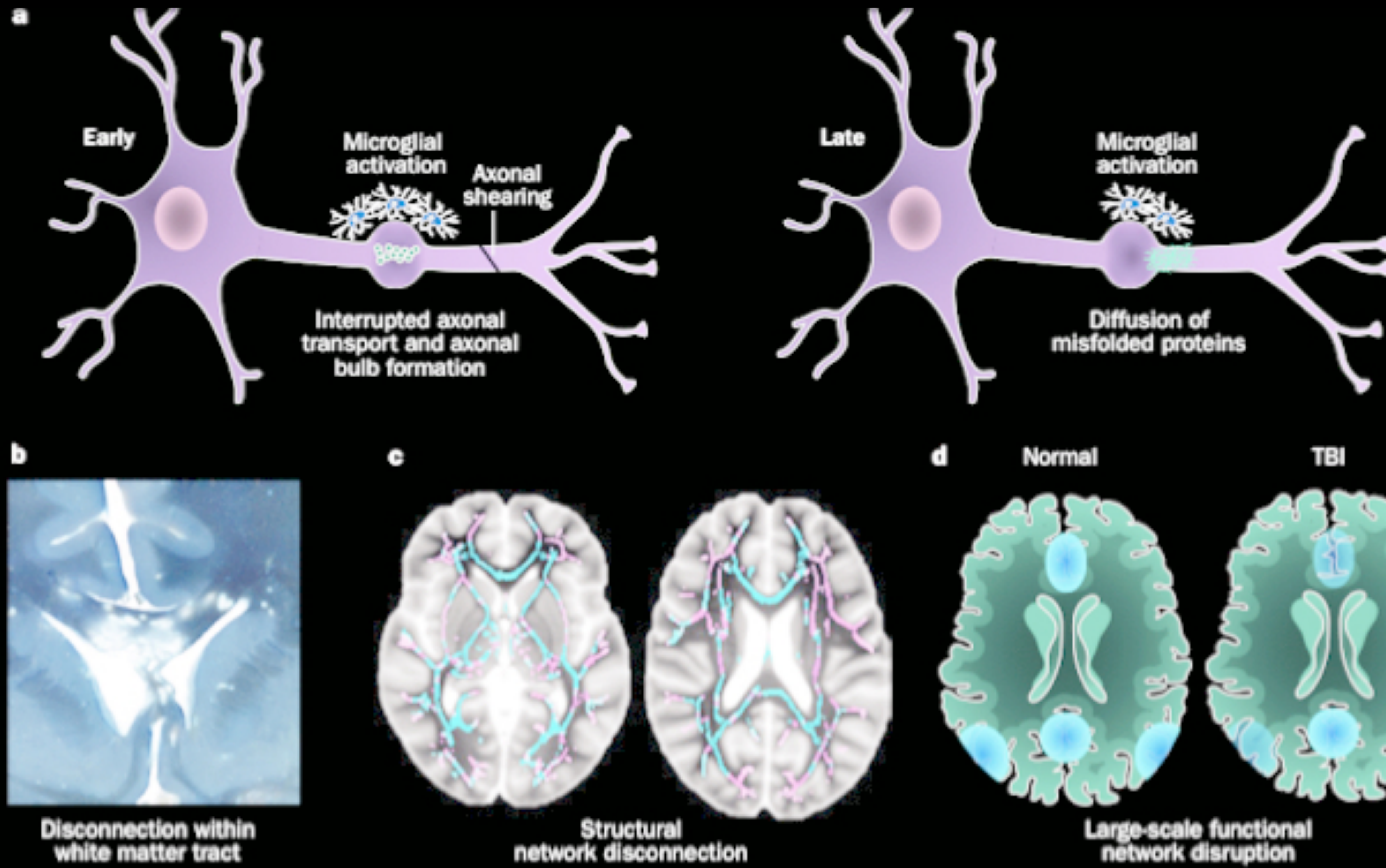


Weight drop with helmet

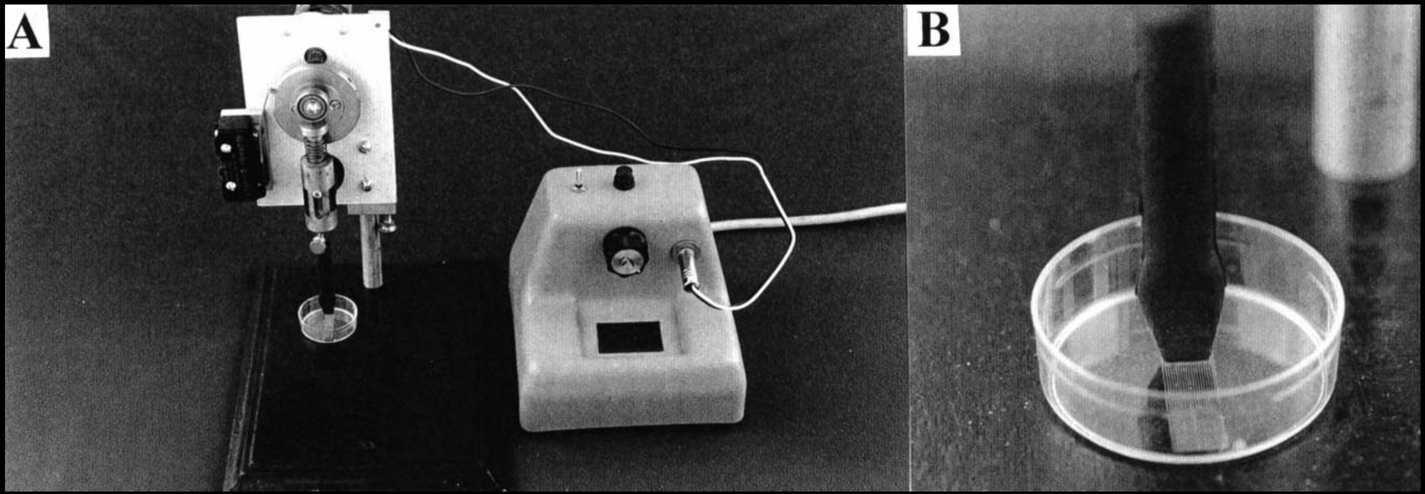
Blast injury

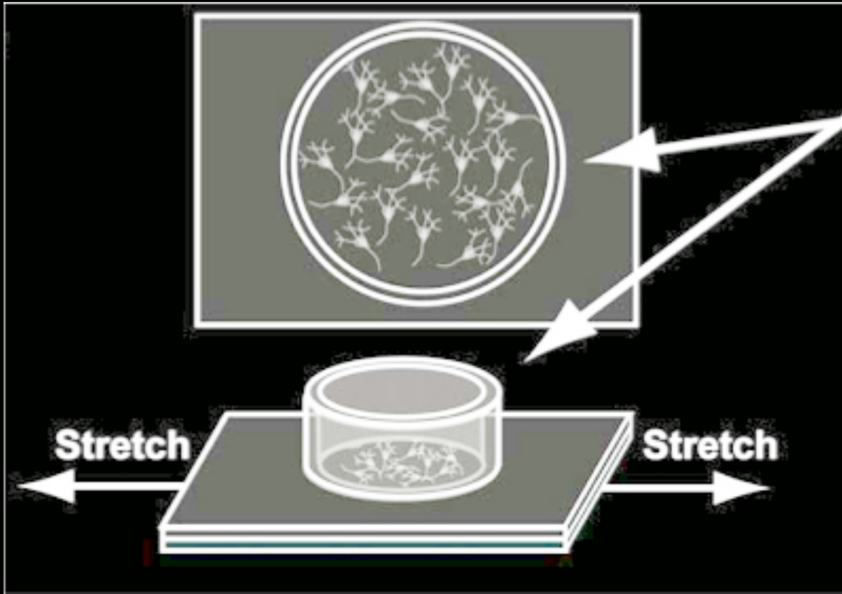
Xiong, Mahmood, Chopp
Nature Reviews Neuroscience (14) , 2013.

Sharp, Scott, Leech
Nature Reviews Neurology (10) , 2014.



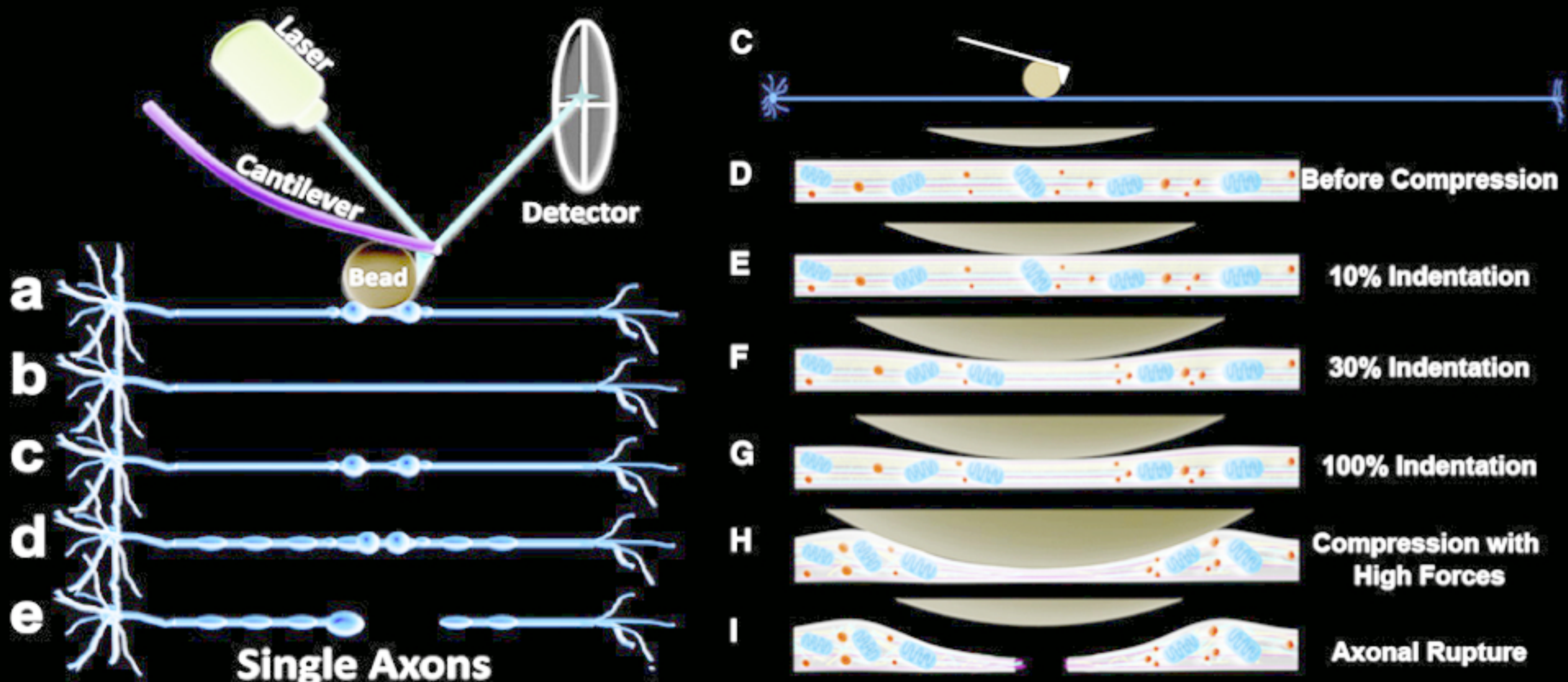
Fayaz, Tator
J. of Neuroscience Methods (102) , 2000





Hemphil, Dabiri, Parker et al
PLoS ONE (6) , 2011

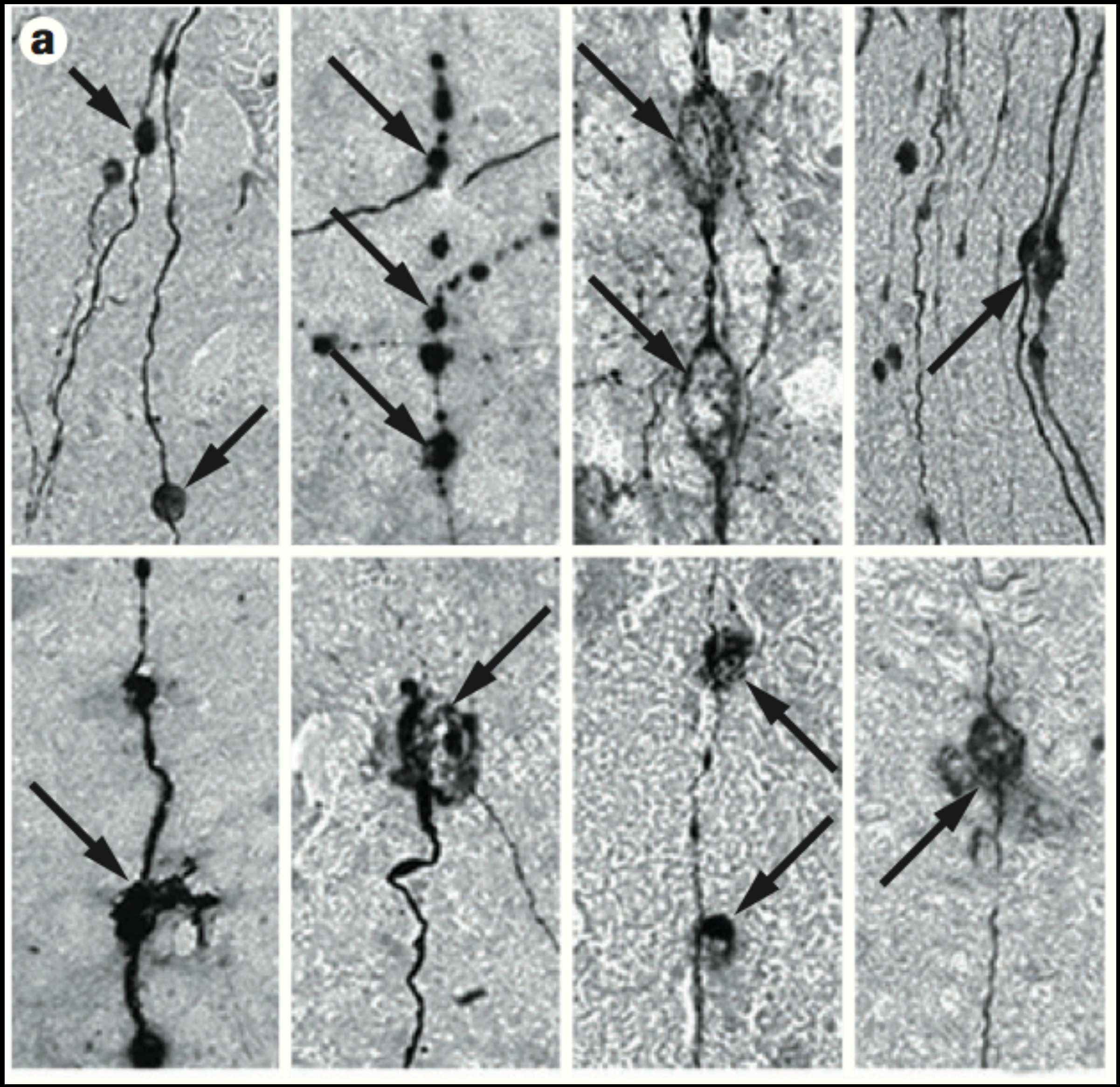
Atomic Force Microscopy



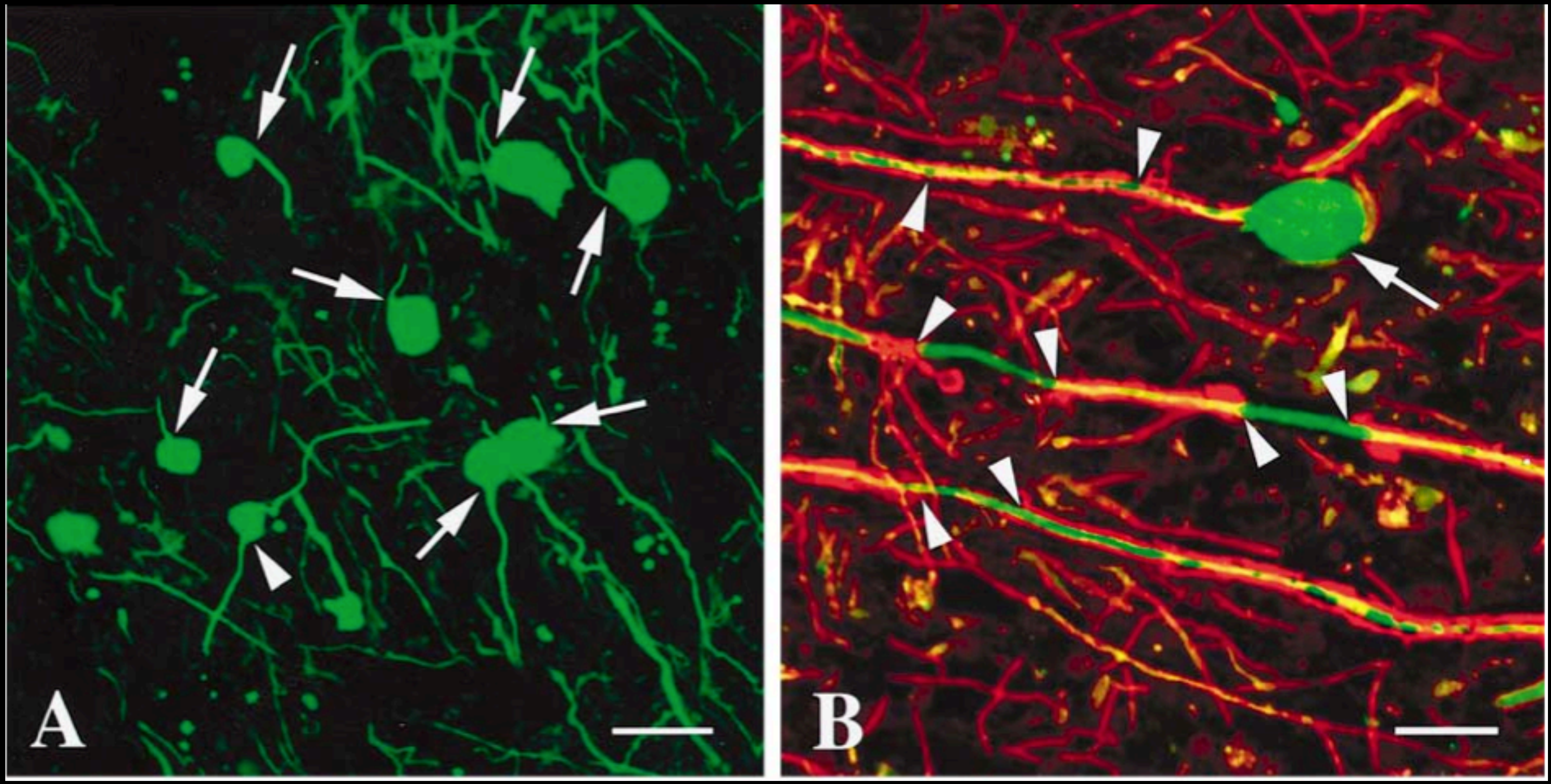
Magdesian et al. Biophys J.
2012 Aug 8; 103(3): 405–414.

Alzheimer's

Krstic, Knuesel
Nature Reviews Neurology
(9:25) , 2012.

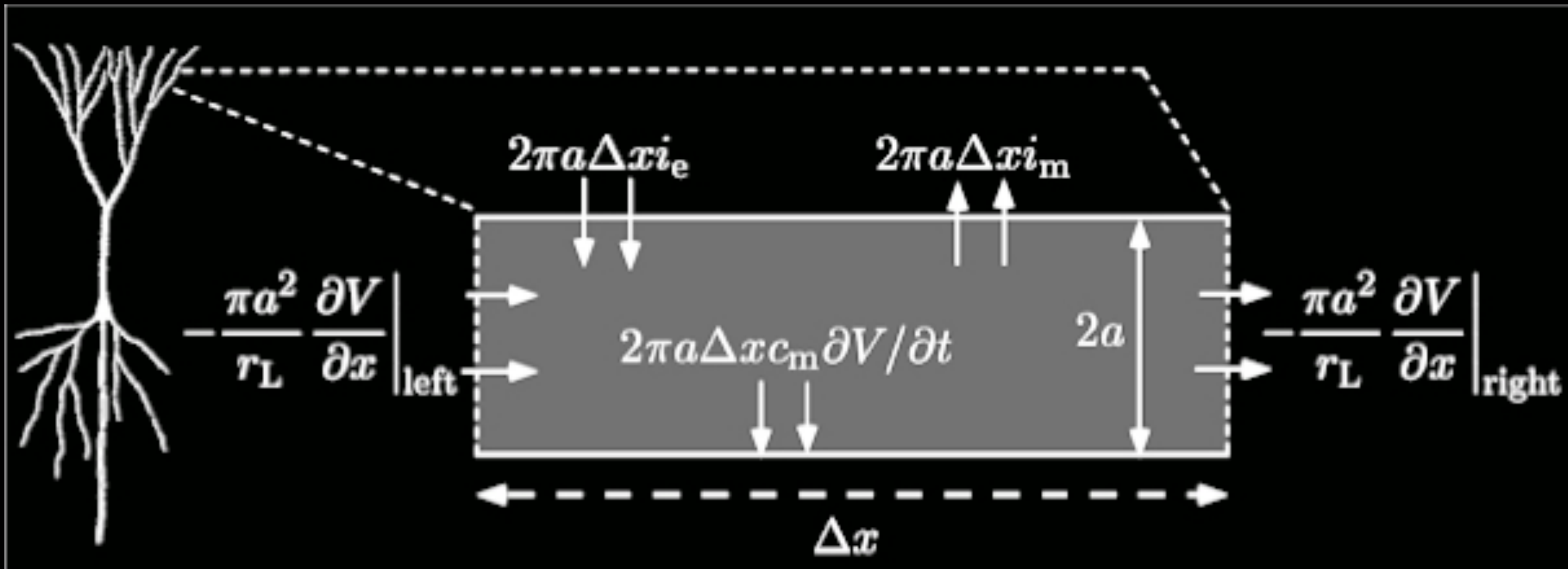


Multiple Sclerosis



Trapp, Peterson, et al
New England J. of Medicine (338) , 1998.

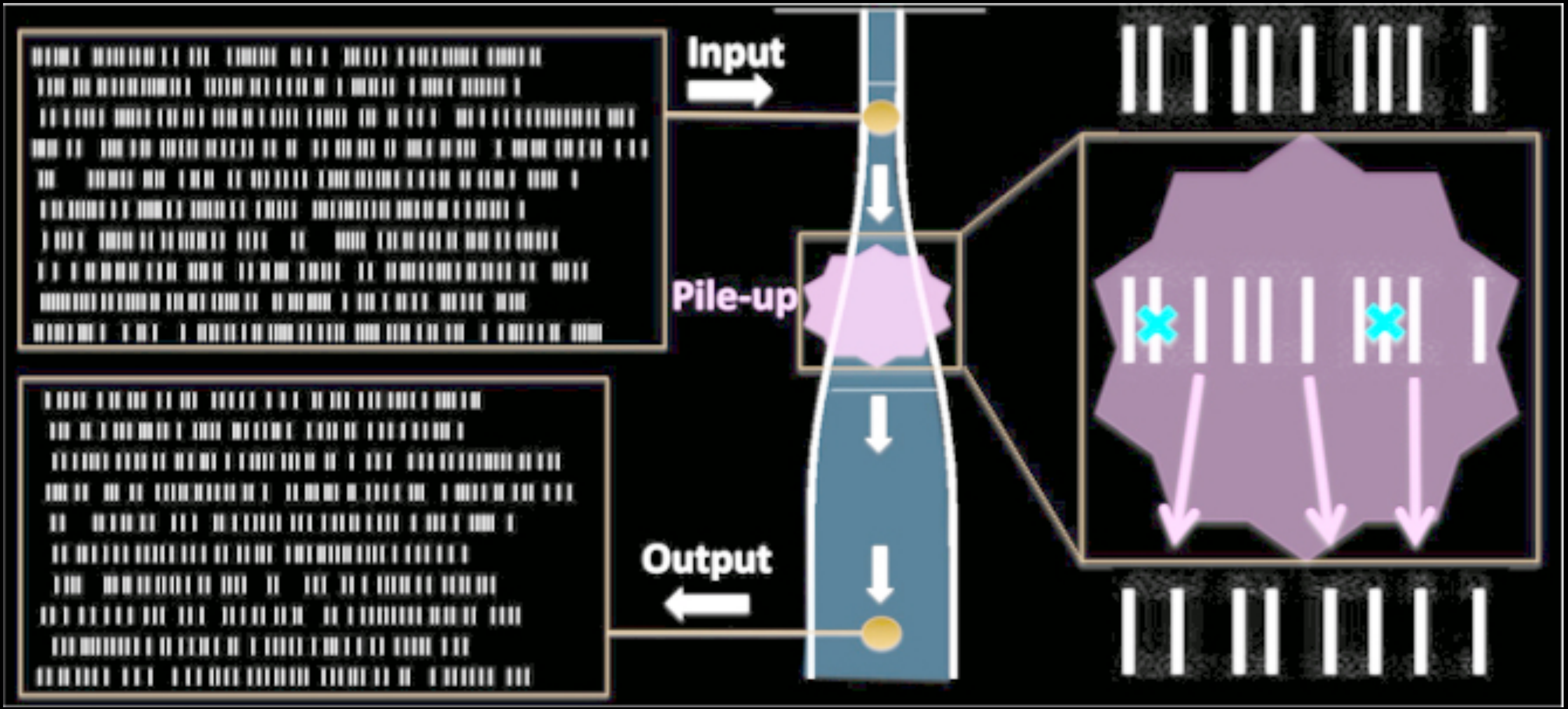
Cable Theory



$$c_m \frac{\partial V}{\partial t} = \frac{1}{2a(x)}\frac{\partial}{\partial x}\left(\frac{a^2(x)}{r_L(x)}\frac{\partial V}{\partial x}\right) - i_{ion} + i_{ext}.$$

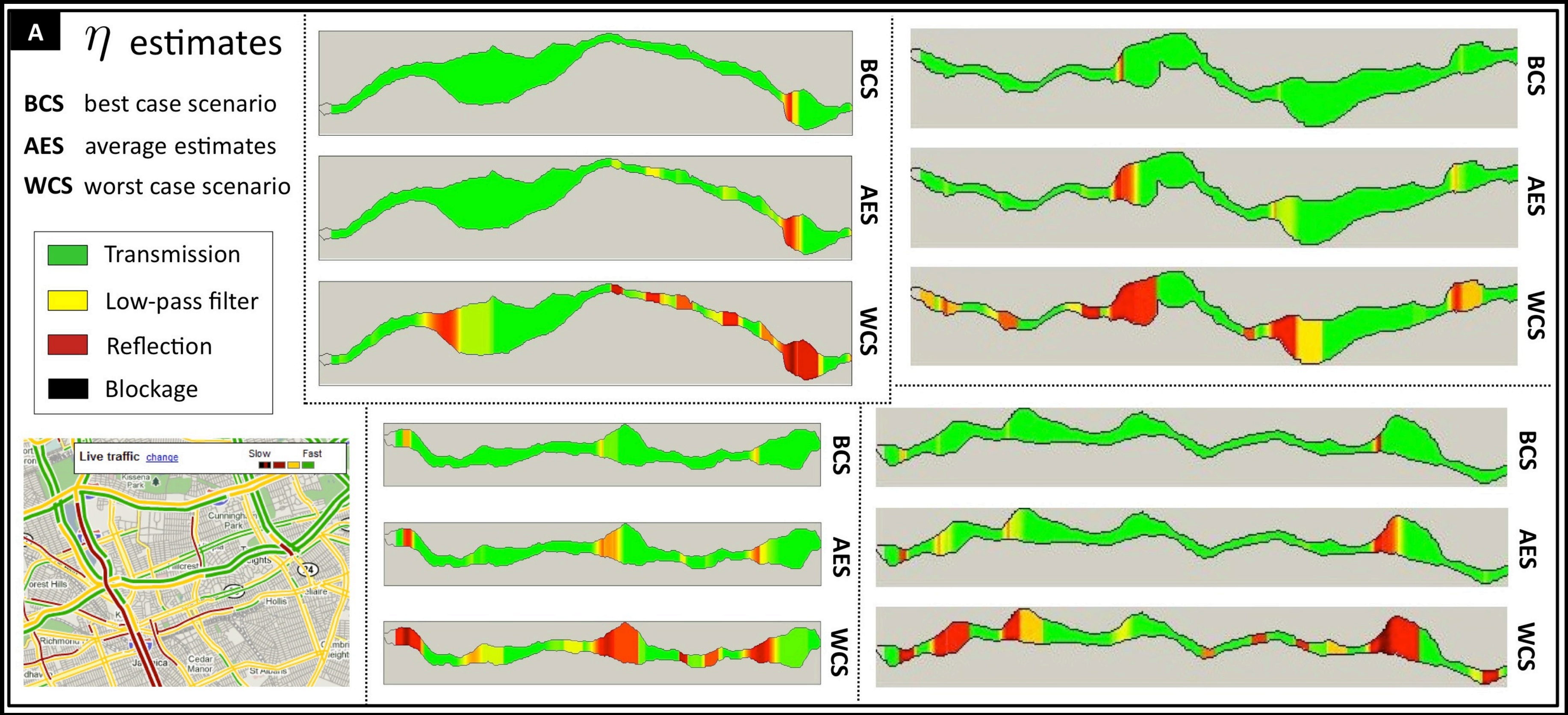
Dayan and Abbot
Theoretical Neuroscience, 2001.

Altering Spike Trains



Maia & Kutz, J. Comp. Neuro 2014

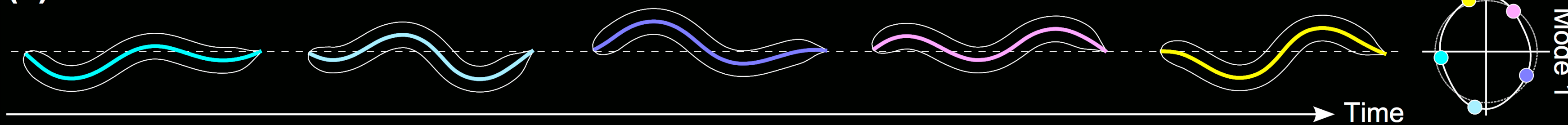
Evaluating Injuries



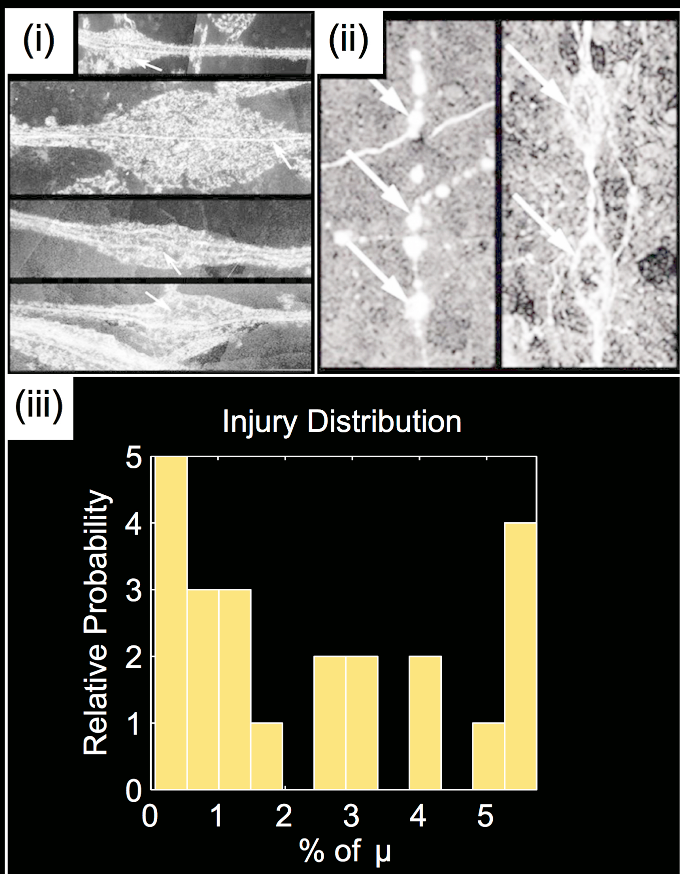
Maia…. Kutz, J. Neuroscience Methods 2015

Concussed C. Elegans

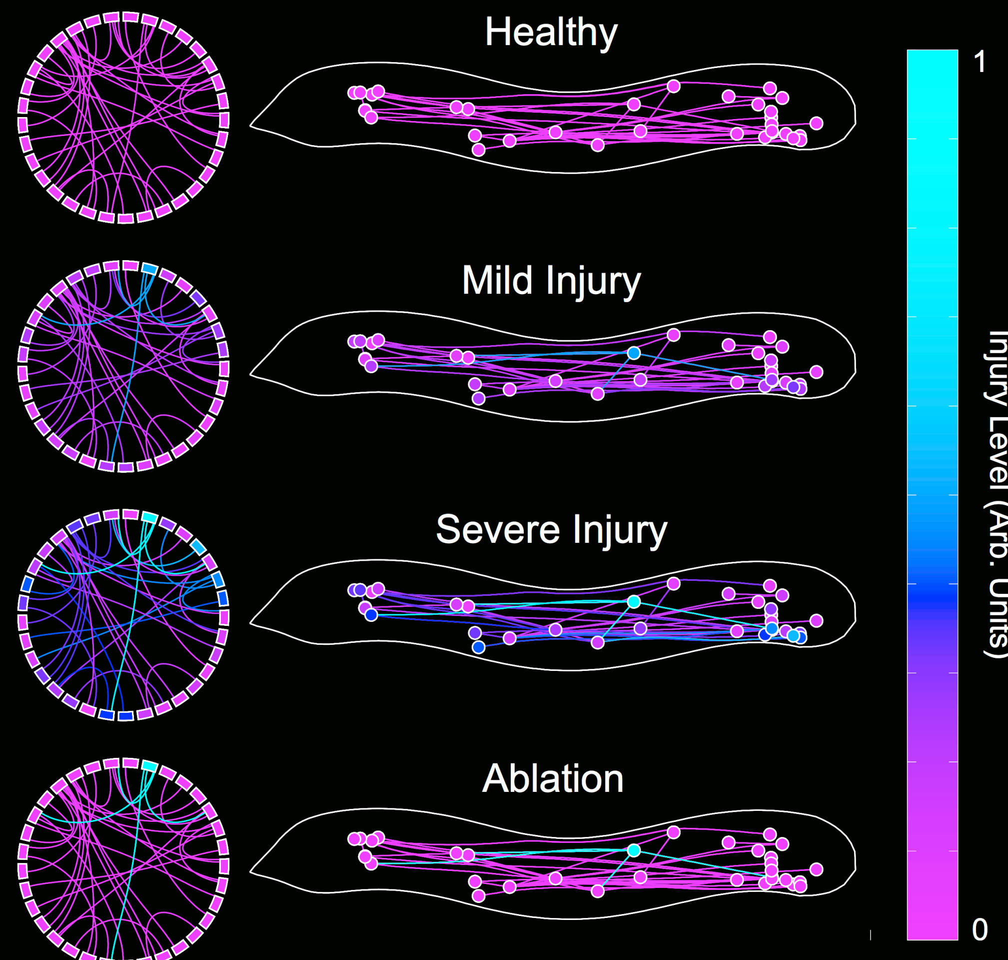
(a) Forward Motion



(b) Focal Axonal Swellings

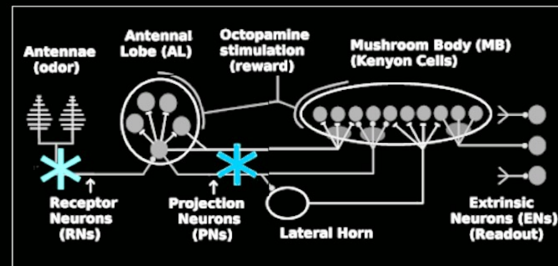


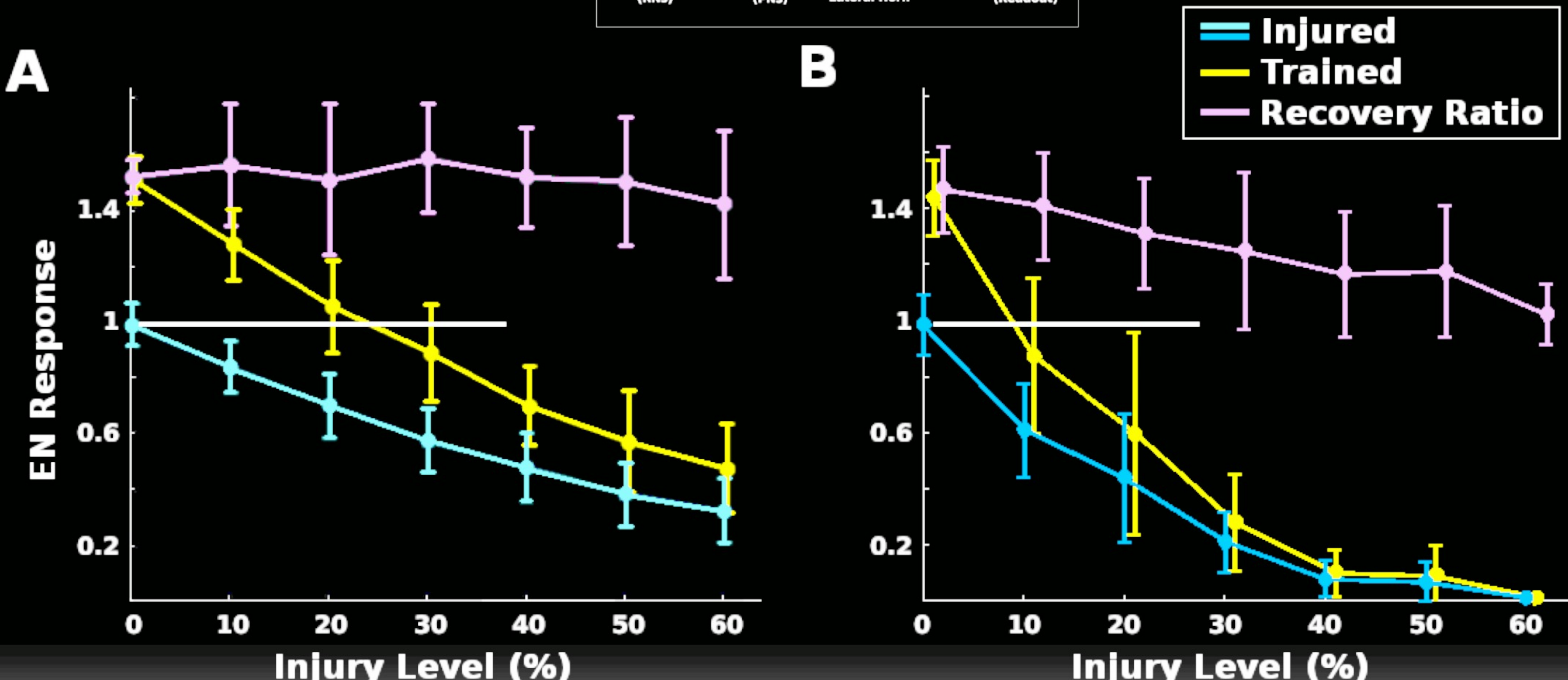
(c) Injured Connectomes



Behavioral Deficits

Learning (plasticity) for injury recovery





Dimensionality Reductions + Machine Learning

- Neural processing exploits low-dimensionality
- Both stereotyped and random connections are relevant
- Data-driven control strategies & architectures can be constructed

The math: Dynamical Systems integrated with Machine Learning