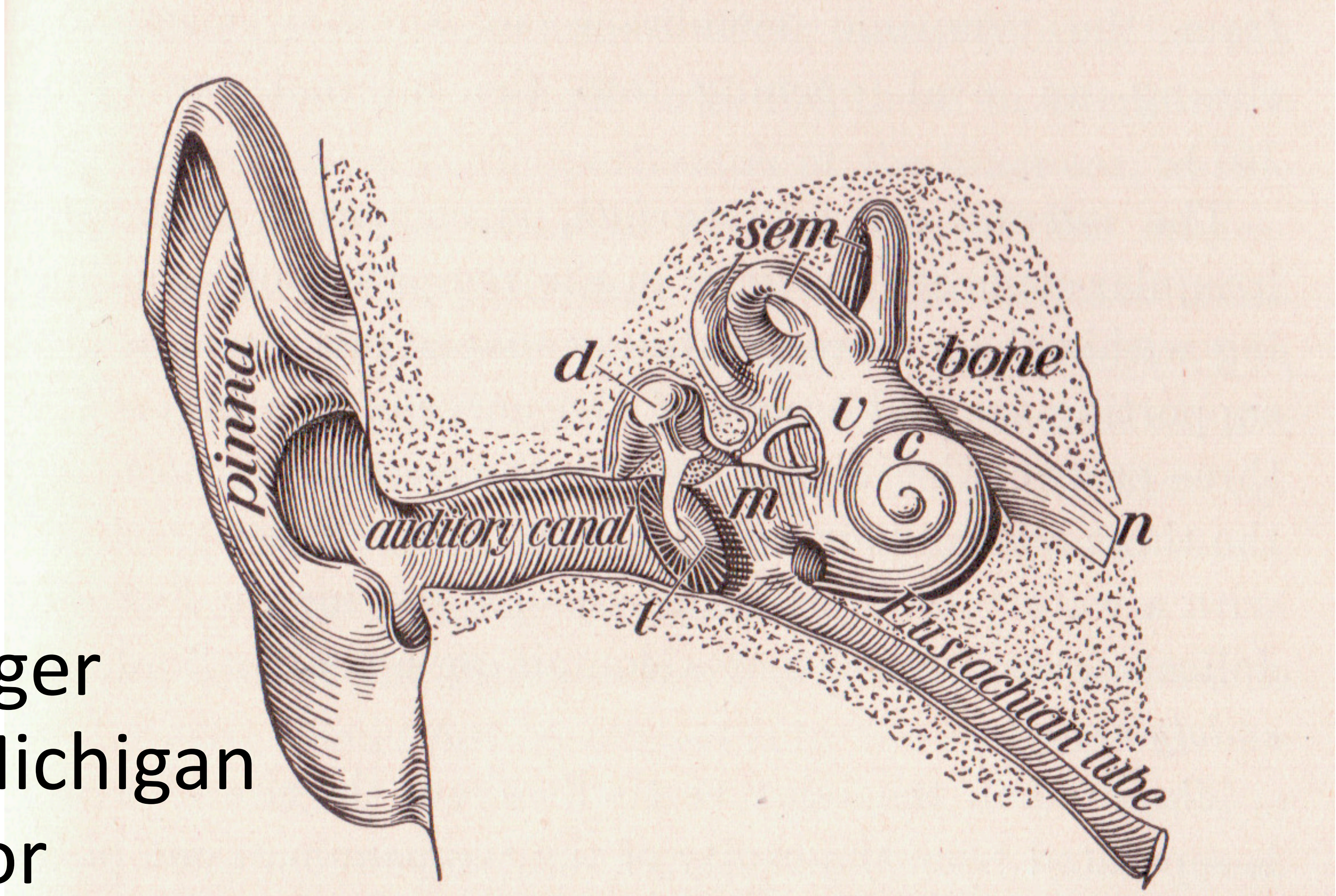
Mathematical Models of the Inner Ear



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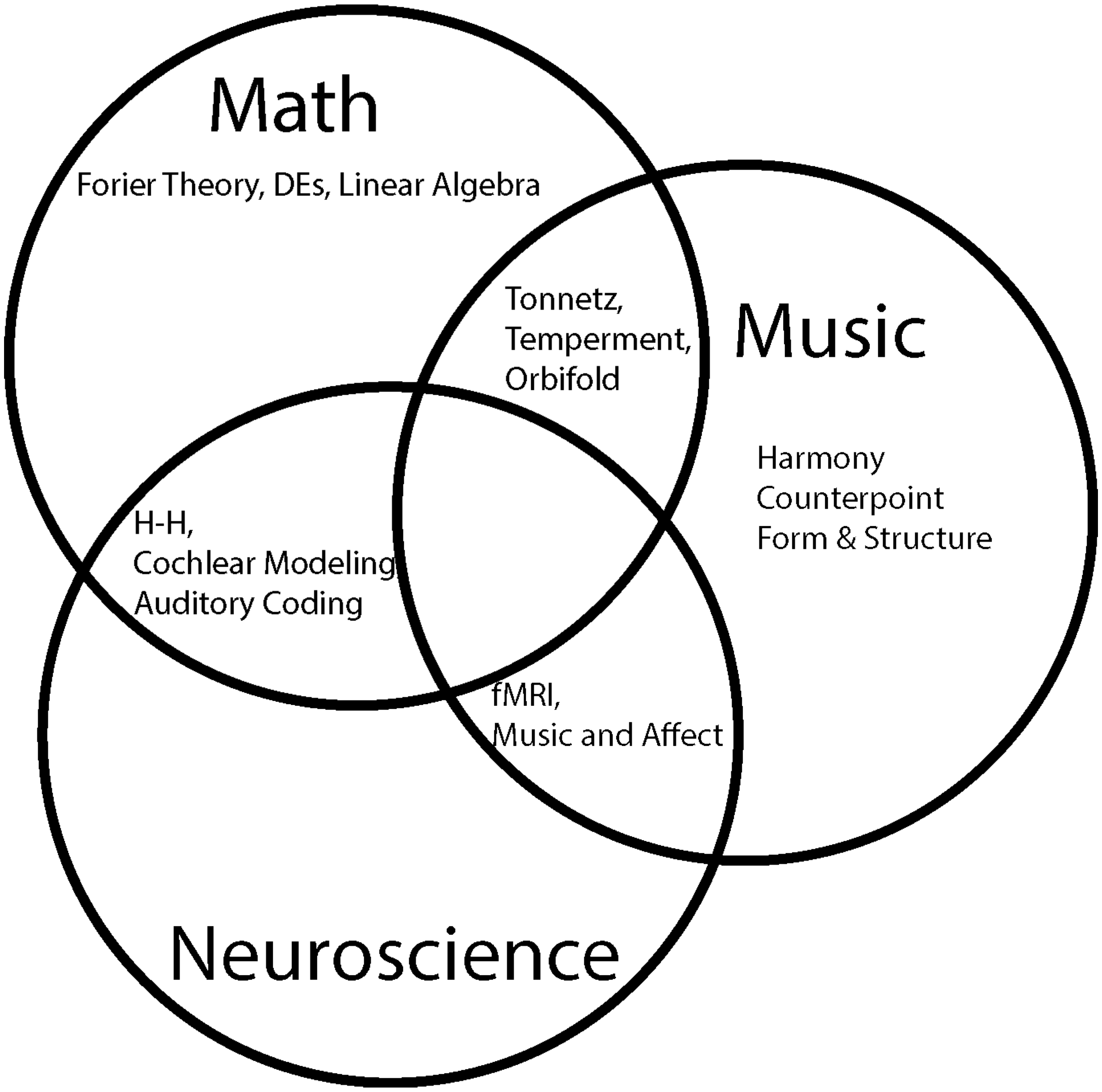
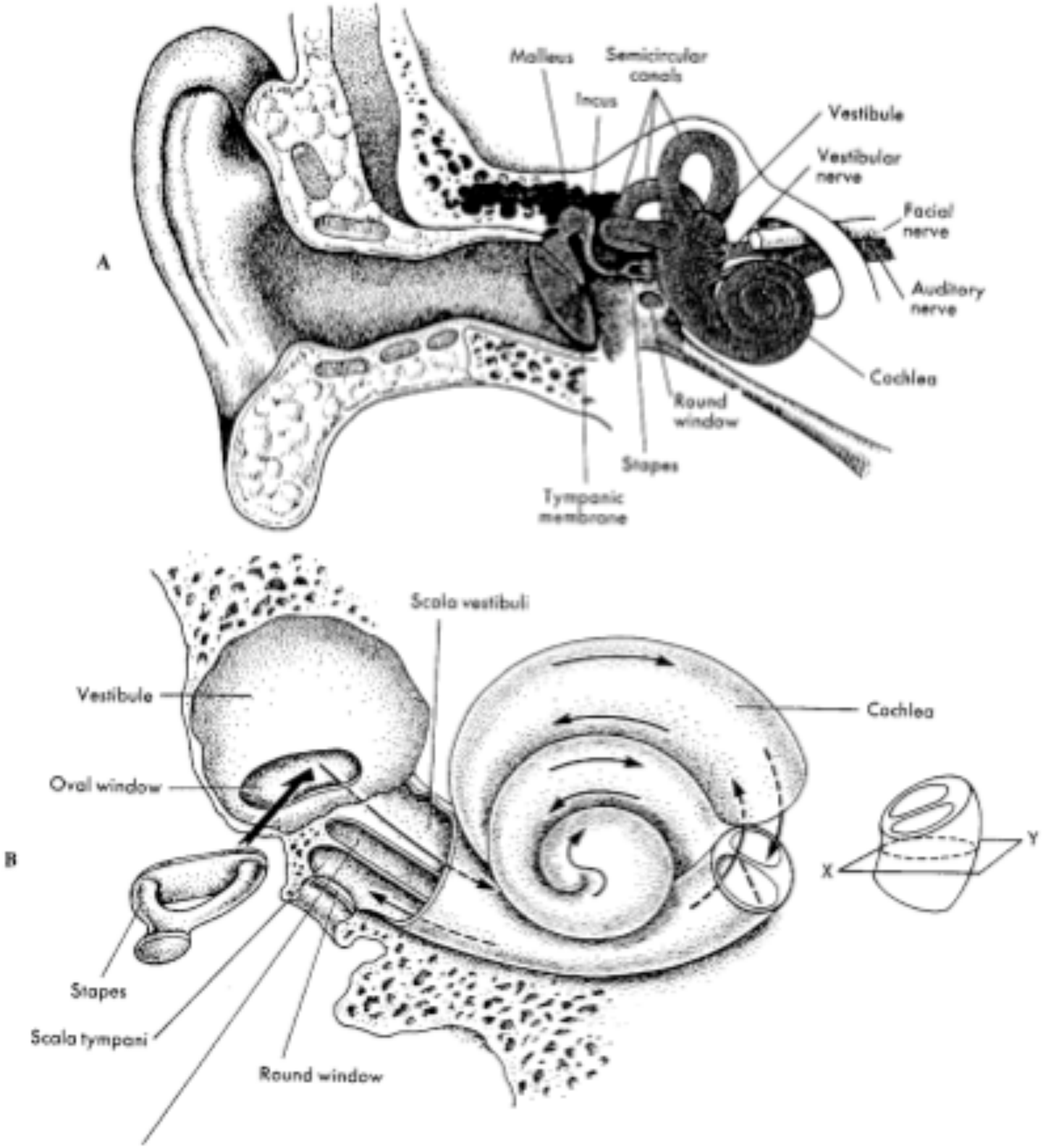

Math
Forier Theory, DEs, Linear Algebra
Tonnetz,
Temperment,
Orbifold
Music
Harmony
Counterpoint
Form & Structure
H-H,
Cochlear Modeling
Auditory Coding
fMRI,
Music and Affect
Neuroscience

Physiology of inner ear

Keener and Sneyd Chapter 12 (BernevandvLevy, 1993, Fig.10-6.)



General References and setup

- Peskin (P) Partial Differential Equations in Biology (web)
  - Neu/Keller (1985)
- Keener and Sneyd (KS)
  - Lesser/Berkley (1972)
- "Physiology and mathematical modeling of the auditory system" Alla Borisyuk, Springer Tutorial in Mathematical Biosciences (2005)

Navier Stokes equations, possibly viscous, incompressible, constant density, ignore $\rho(\nabla \bullet u)u$ (small velocity) p = pressure, ρ = density, u = (u_1, u_2) fluid velocity

(KS)

$$\rho \partial u/\partial t + \nabla p = 0$$

$\nabla \cdot u = 0$

Potential flow

$u = \nabla \phi$

$\rho \partial \phi/\partial t + p = 0$

$\nabla^2 \phi = 0$

(P)

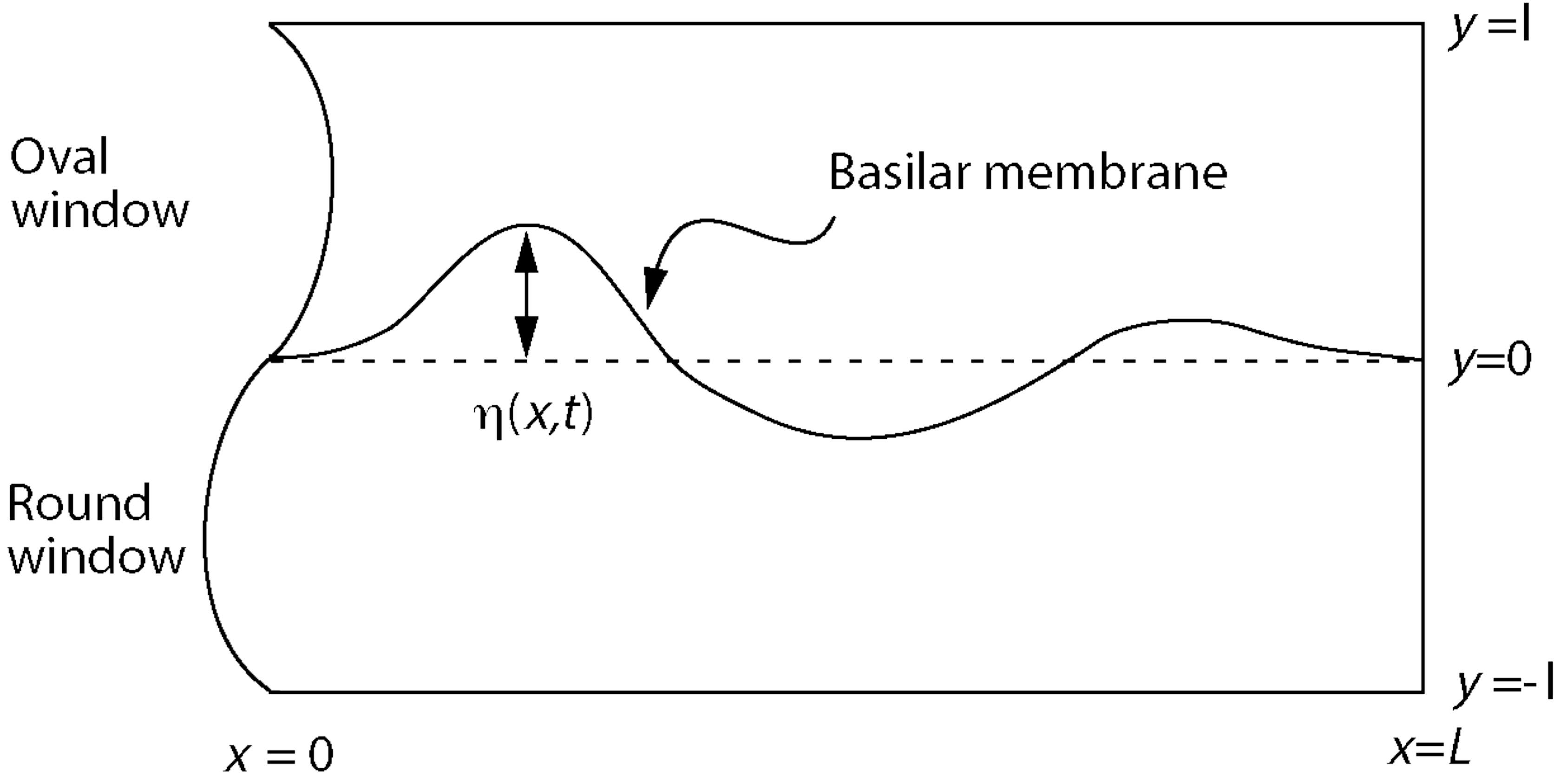
$\rho \partial u/\partial t + \partial p/\partial x = \mu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2)$

$\rho \partial u/\partial t + \partial p/\partial x = \mu(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2)$

$\partial u/\partial x + \partial v/\partial y = 0$

So the main difference thus far is that (P) considers viscosity.

Basilar membrane physiology



Keener Sneyd, Figure 23.6

(KS) from Lesser and Berkley

Each point on the basilar membrane has mass m, damping r and stiffness k.

These parameters vary along the membrane

$$m(x)\frac{\partial^2 n}{\partial t^2} + r(x)\frac{\partial n}{\partial t} + k(x)n = P_2(x, n(x,t), t) - P_1(x, n(x,t), t)$$

n in (P) = h in (KS), $P_2(x, n(x,t), t) = p(x, 0^-, t)$, $P_1(x, n(x,t), t) = p(x, 0^+, t)$

(P) From Neu Keller

Basilar membrane can be treated as massless but active with

At y =0,

u(x, 0, t) = 0, v(x, 0, t) = $\frac{\partial h}{\partial t}(x, t)$

$$p(x, 0^-, t) - p(x, 0^+, t) = s_0 e^{-\lambda x}\left(h + B\frac{\partial h}{\partial t}\right)(x, t)$$

What gives frequency tuning?

- I can find at least three mechanisms which give this
- Details are sometimes hidden in the methods
- Figuring out which (or which combination) is physiologically relevant remains an important challenge!

Resonance (KS)

(KS) oscillations can be inherent to basilar membrane. Frequency tuning can arise from resonance from input signal to basilar membrane oscillations.

$$m(x)\frac{\partial n^2}{\partial t^2}+r(x)\frac{\partial n}{\partial t}+k(x)n=P_2(x,n(x,t),t)-P_1(x,n(x,t),t)$$

Assume m(x) not zero, and drop r(x) for moment

$$m(x)\frac{\partial n^2}{\partial t^2}+k(x)n=F\cos(wt)$$

$$n(t)=C_1\cos(w_0 t)+C_2\sin(w_0 t)+\frac{F}{w^2-w_0^2}\cos(wt)$$

Solutions grow as $w_0 = k(x)^{1/2}$ -> w as one typically finds in resonance.

If *r(x)* > 0 we have damped oscillations. If r(x) <0 we have sustained oscillations, which would then be complex and likely coupled.

Recent work shows how damped oscillators are easier to entrain.

Boundary and other effects

- If we had viscosity, and a bounded domain, there would need to be a no slip condition at $x = l$. This would keep the basilar membrane fixed at $y = 0$ when $x = l$, and dampen oscillations as we approach the boundary.
- But also, even if we have no viscosity, we can have frequency tuning and energy dissipation due to the stiffness. This is seen in the (KS) analysis.

Lesser and Berkley

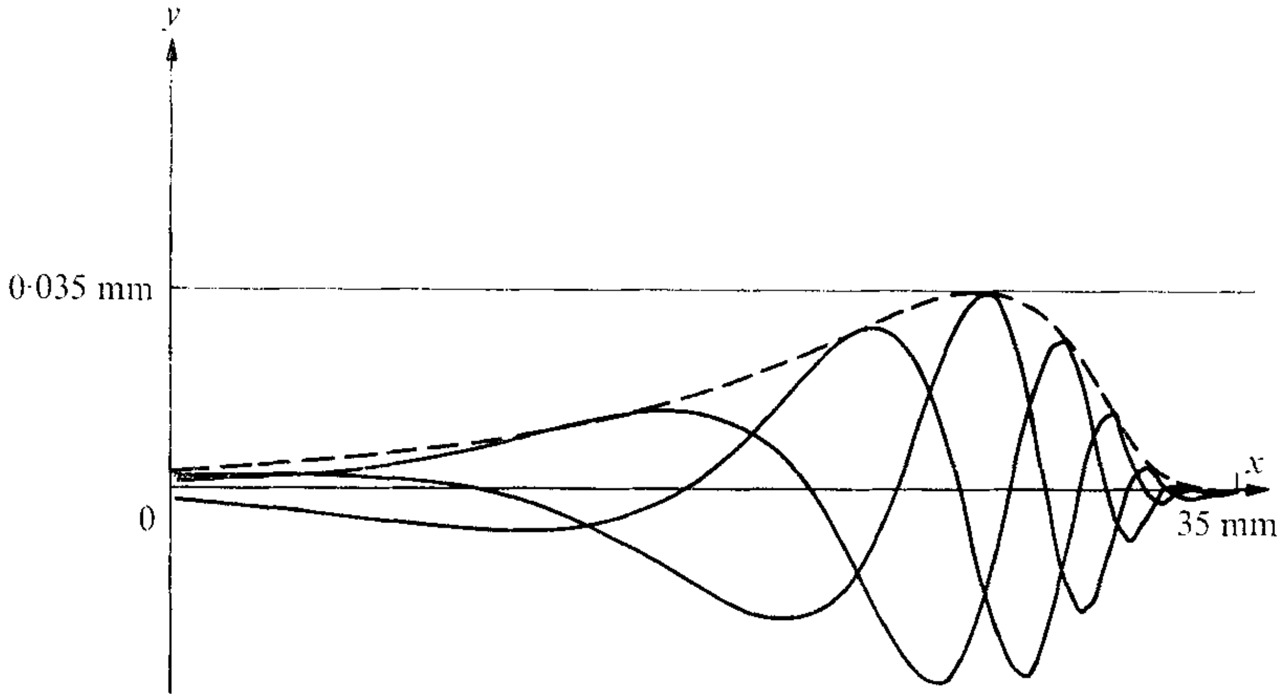


FIGURE 6. Wave envelope and motion of basilar membrane.
$m = 0{\cdot}05$, $k = 10^7 e^{-1{\cdot}5x}$, $r = 3000e^{-1{\cdot}5x}$, $\omega = 1000$.

Balance of viscosity and changing stiffness (P)

Peskin notes that from von Békésy (NP 1961), BM acts as if it were massless. We can calculate a viscous boundary layer in several ways (dimensional analysis, moving plate, acoustics) to be:

$\sqrt{\mu/\rho w}$

The appropriate length scale is λ giving us

$\epsilon=\lambda\sqrt{\mu/\rho w} \;=x^{-\lambda} x_{\epsilon}$

Note this constant is very small (e.g., *w* = 600, μ = 0.02). Now appeal to WKB theory:

$\begin{pmatrix}U\\V\\P\end{pmatrix}(x, y, t, \epsilon)= \begin{pmatrix}U\\V\\P\end{pmatrix}(x,-x_{\epsilon} ,y/\epsilon , \epsilon)e^{i(wt+\Phi(x-x_{\epsilon})/\epsilon)}$

$h(x, t, \epsilon)=H(x_{0} -x_{\epsilon})e^{i(wt+\Phi(x-x_{\epsilon})/\epsilon)}$

$X=x-x_{\epsilon} , Y=y/\epsilon , \xi(x)= \Phi'(x)$

$p(x, y, t)= -p(x, -y, t), u(x, y, t)= -u(x, -y, t) v(x, y, t=x(x, -y, t)$

Solutions of form: $U=U_0+\epsilon U_1 \ldots$, $V=V_0+\epsilon V_1$, $P=\epsilon(P_0+\epsilon P_1 \ldots)$, $H=H_0+\epsilon V_1 \ldots$

(Change in pressure proportional motion of basilar membrane which is size of viscus boundary)

Substitute into model equations. To zero order:

$$\rho w\left(i-\frac{1}{\lambda^2}\left(-\xi^2+\frac{\partial^2}{\partial y^2}\right)\right)U_0+i\xi P_0=0$$

Which has solution: $P_0(X,Y)=P_0(X,0)e^{\sqrt{\xi^2}\,Y}$, $U_0(X,Y)=-P_0(X,0)\frac{i\xi}{iw\rho}\left(e^{\sqrt{\xi^2}\,Y}-e^{\sqrt{\xi^2-i\lambda^2}\,Y}\right)$

At basilar membrane:

$U(X,0,\epsilon)=0$, $V(X,0,\epsilon)=iwH(x,0,\epsilon)$

$2P(X,0,\epsilon)=s_0 e^{-\lambda x}(1+iwB)e^{-\lambda x_\epsilon}H(X,0,\epsilon)$

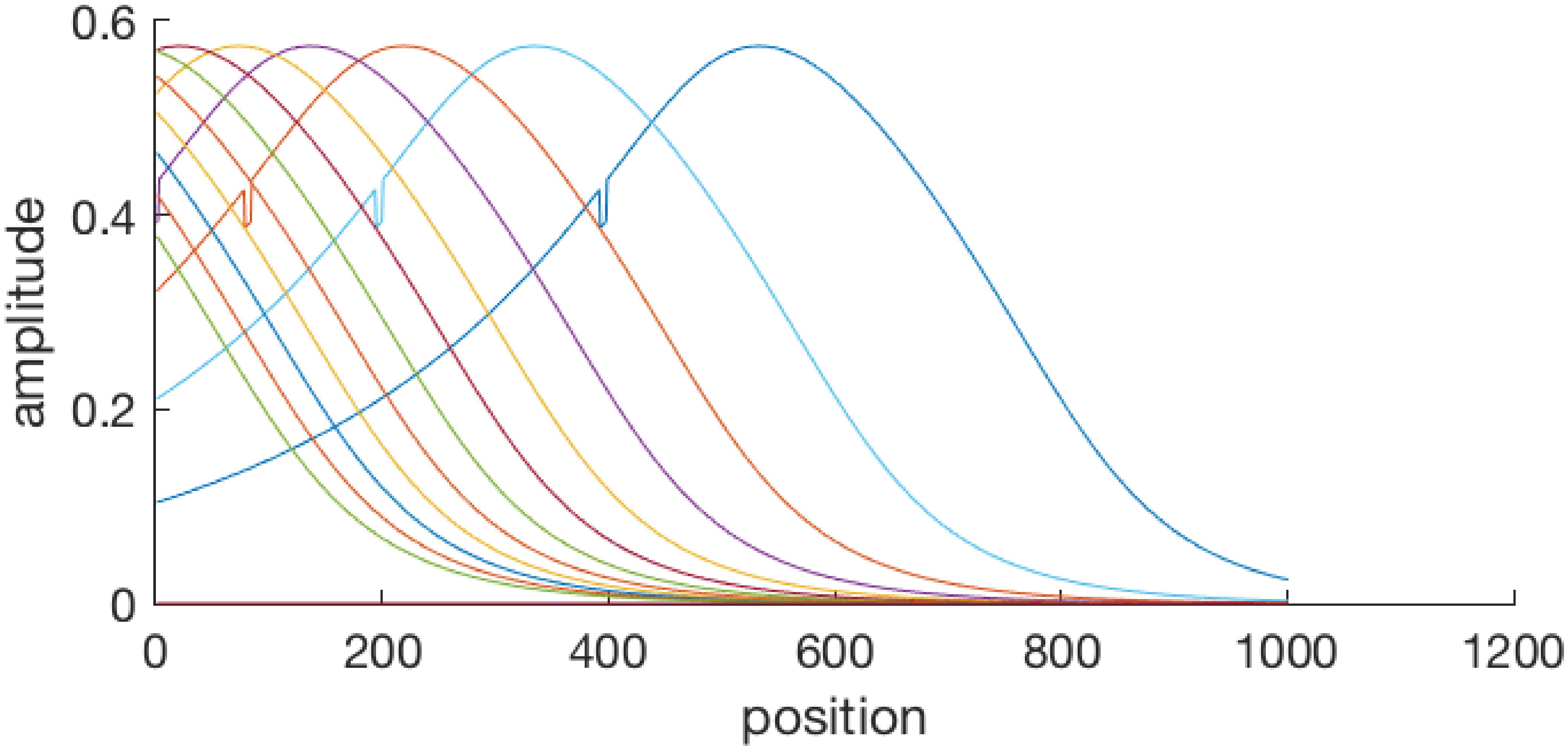
This gives an equation for ξ in terms of *w* and X.

Finally, we integrate the zero order solution with the first order solution, to make sure that none o

creeps into the 1st order solution. After much algebra, this gives:

$$P_0(X,0)^2=\frac{2w\rho C_0\left(\sqrt{\xi^2+i\lambda^2}\right)^3\sqrt{\xi^2}}{\xi\left(\sqrt{\xi^2+i\lambda^2}-\sqrt{\xi^2}\right)\left(\sqrt{\xi^2+i\lambda^2}\sqrt{\xi^2}-i\lambda^2\right)}$$

Numerical Solution



What is really happening

- As stiffness of basilar membrane decreases as we move along the membrane, the basilar membrane moves more freely vertically due to a pressure difference
- As (P) assumes that the Cochlea has infinite length, this would continue to happen indefinitely,
- However, as we move along the basilar membrane, the viscous forces dissipate the energy from the motion of the stapes.
- Dissipation could also occurs through B, but (P) chooses negative B which gives better frequency tuning. (Outer hair cells)
- This combined effect gives us the frequency tuning since the dissipation of energy depends on the stimulus frequency.

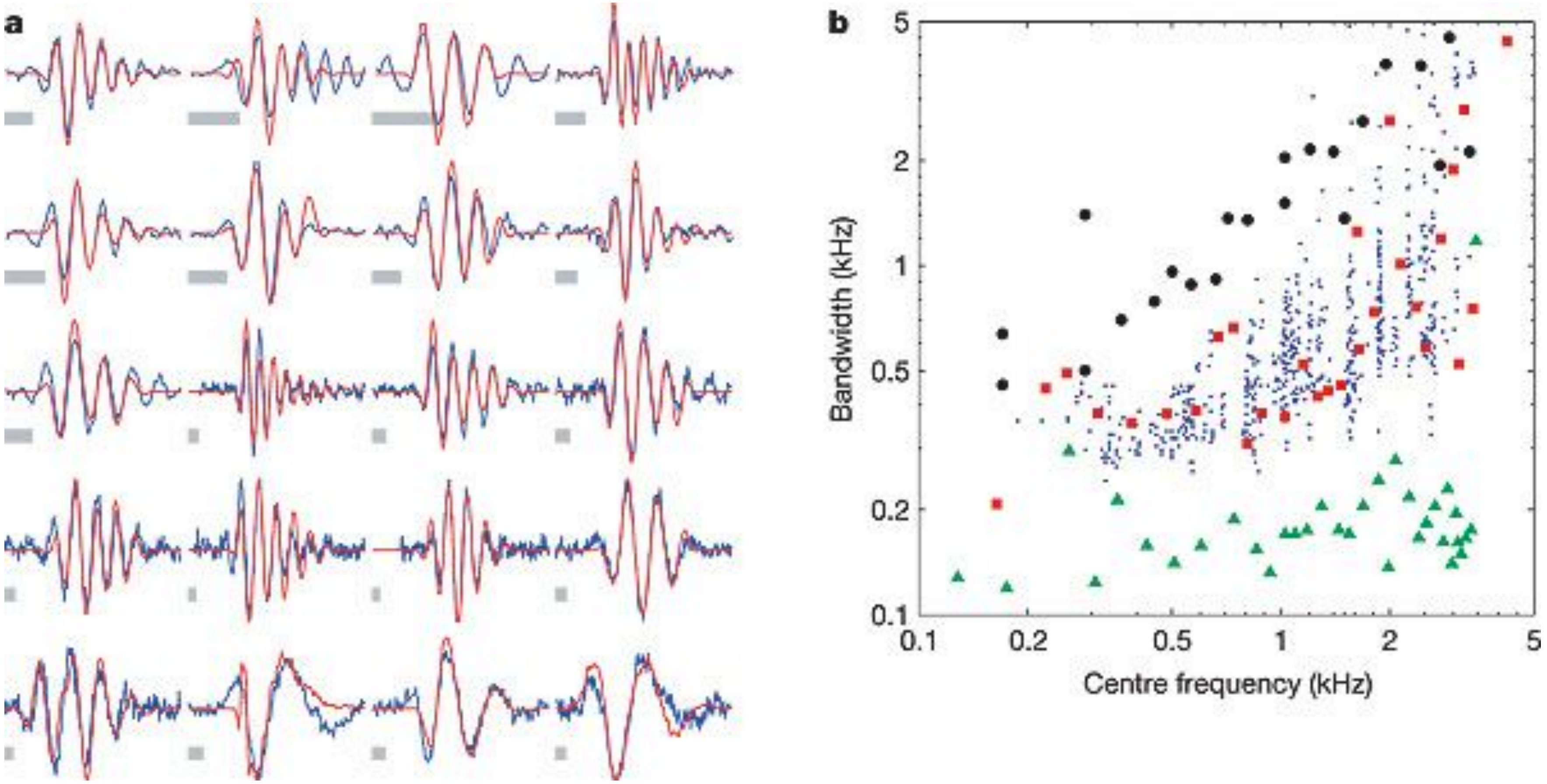
Efficient auditory coding

Smith and Lewicki, Nature 2006

- White noise analysis can be used to determine the response of neurons in the auditory nerve. White noise is given, and spike triggered averaging is conducted (Irino and Patterson, 1997)
- We can also represent sounds by kernel functions
- $x(t) = \sum \sum s_i^m \phi_m (t - \tau_i^m) + \epsilon(t)$
- What properties of ϕ_m would give the best representations of sounds?
- Trained on a library of natural sounds

Found direct match with data

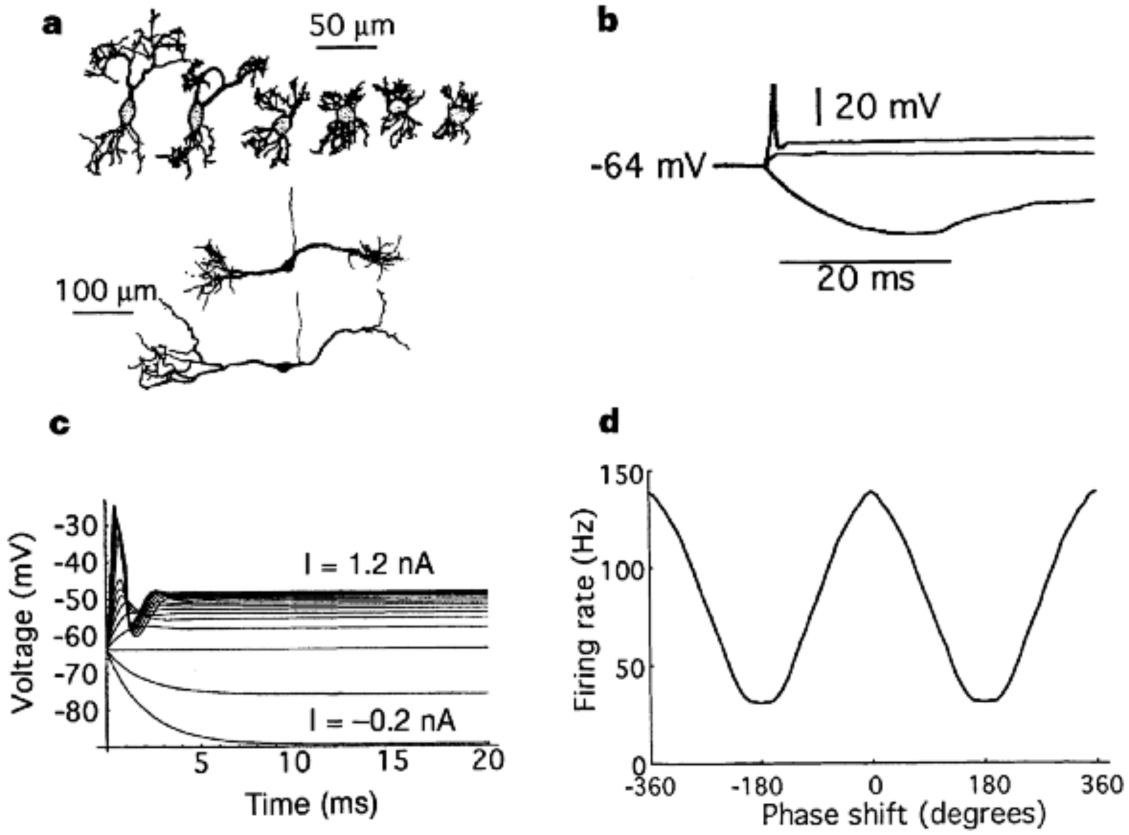
black environmental sounds along, green animal voicalizations



What’s next?

- Coincidence detectors in brainstem
  - Work of Rinzel and Colleagues

  Agmon=Snir et al. Nature 1998

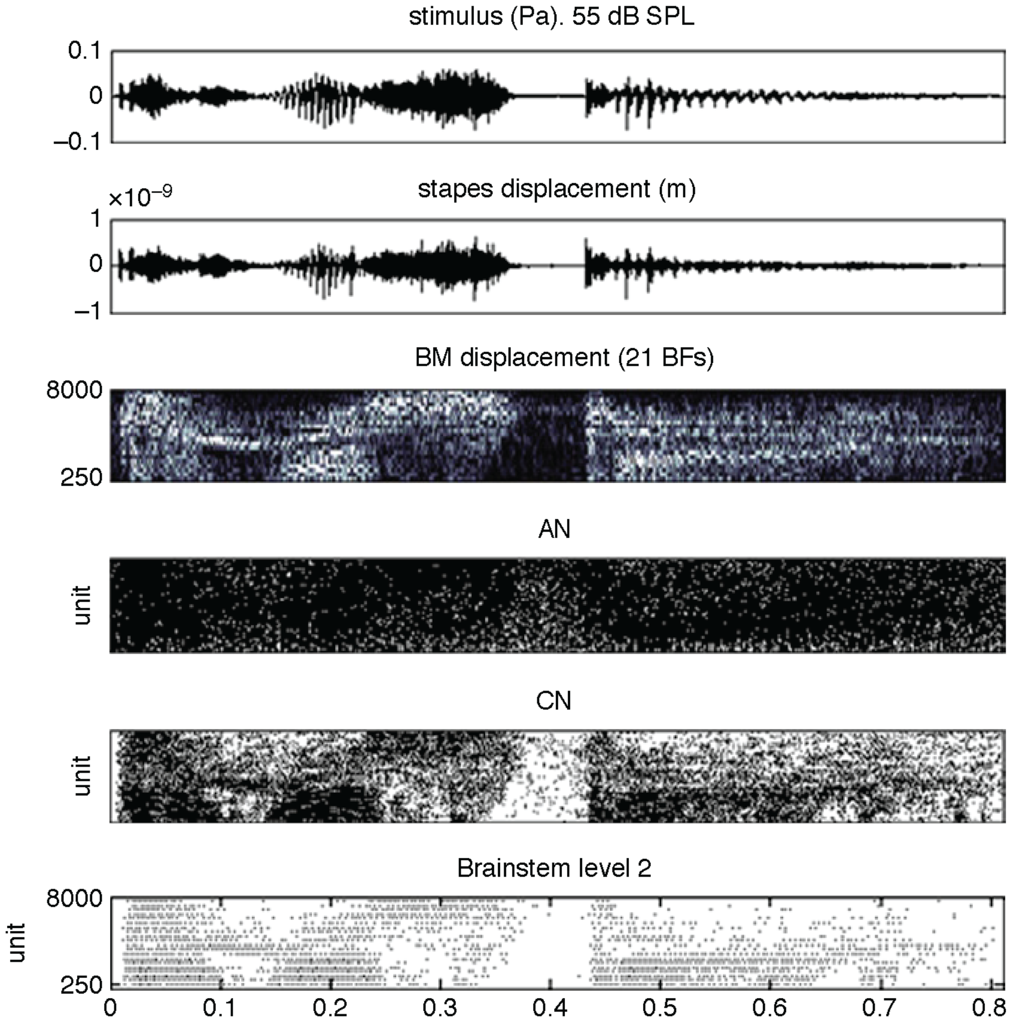


Integrated models (Meddis)

- Models including
  - Basilar membrane
  - Inner hair cell response
  - Auditory nerve firing rate
  - Cochlear Nucleus response

  (Meddis ISH 2013)

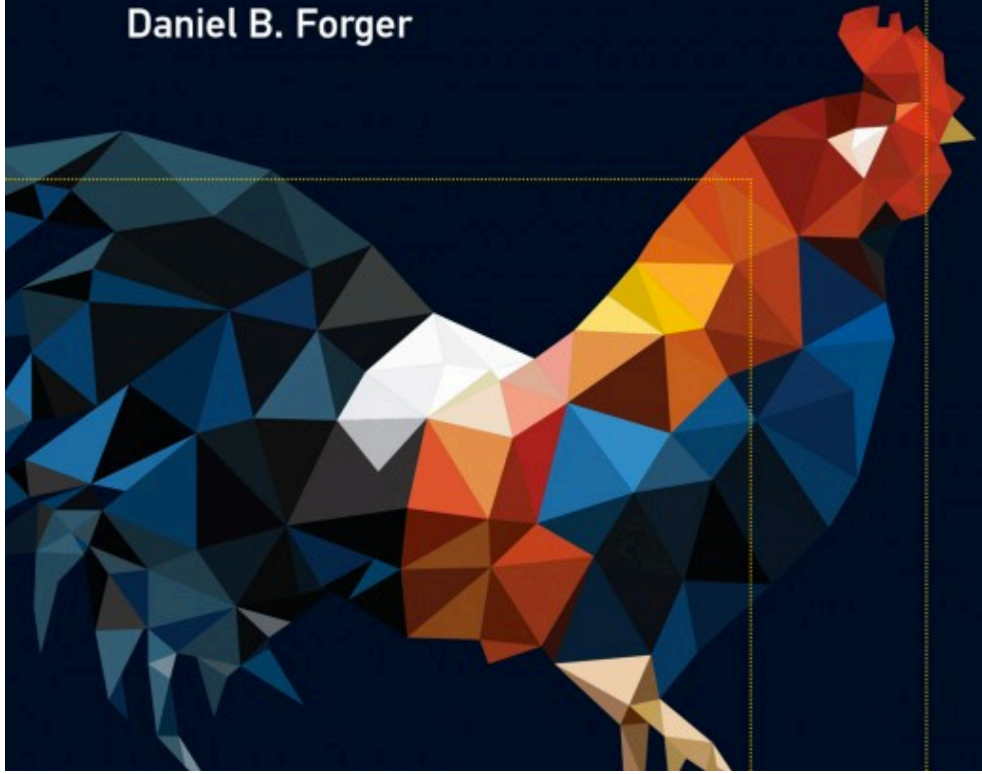
- Used in a new hearing aid
  - Bioaid



Biological Clocks, Rhythms, and Oscillations

The Theory of Biological Timekeeping

Daniel B. Forger



Mathematical Representations of Music

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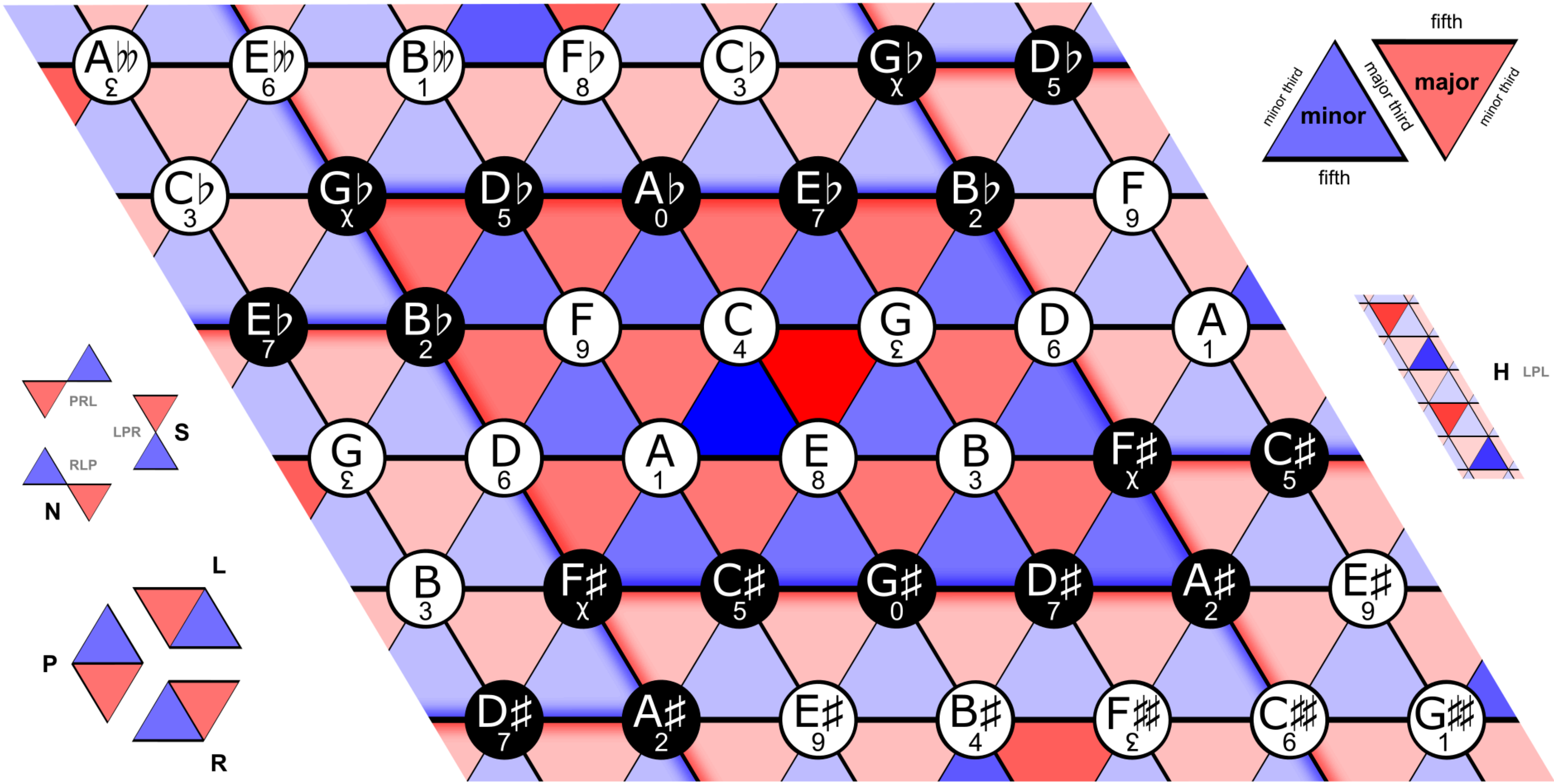
Chords

- 12 notes on keyboard, repeated
- Groups of notes form chords
- Heinichen’s conMusicalischer Circul (1711), (circle of keys)
- “emergence of twenty-four major and minor keys in Bach’s time and the well—tempered tuning ‘that made their use possible’” Gardiner
- Rameau (1722) chords preserve 1) octave shifts, 2) permutations and 3) doublings
- Typically consider chords of 2 or three notes
- 1 = C, 2 = C#, 3 = D, 4 = D#, …
- 3 note chords can be major (+4, +3) or minor (+3, +4)
- (C, E, G), (C, E ♭ , G),

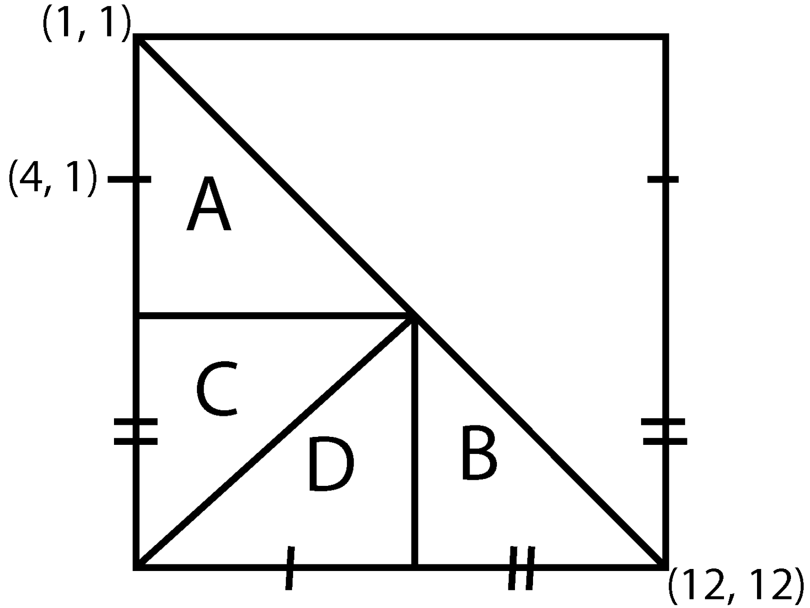
Euler Tonnetz (1739)

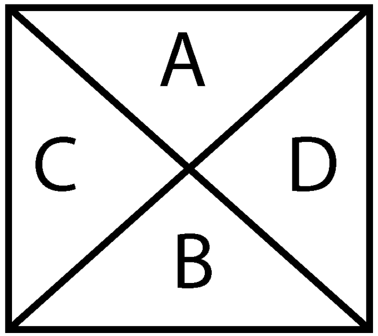
Wikipedia Tonnetz



Orbifold Analysis

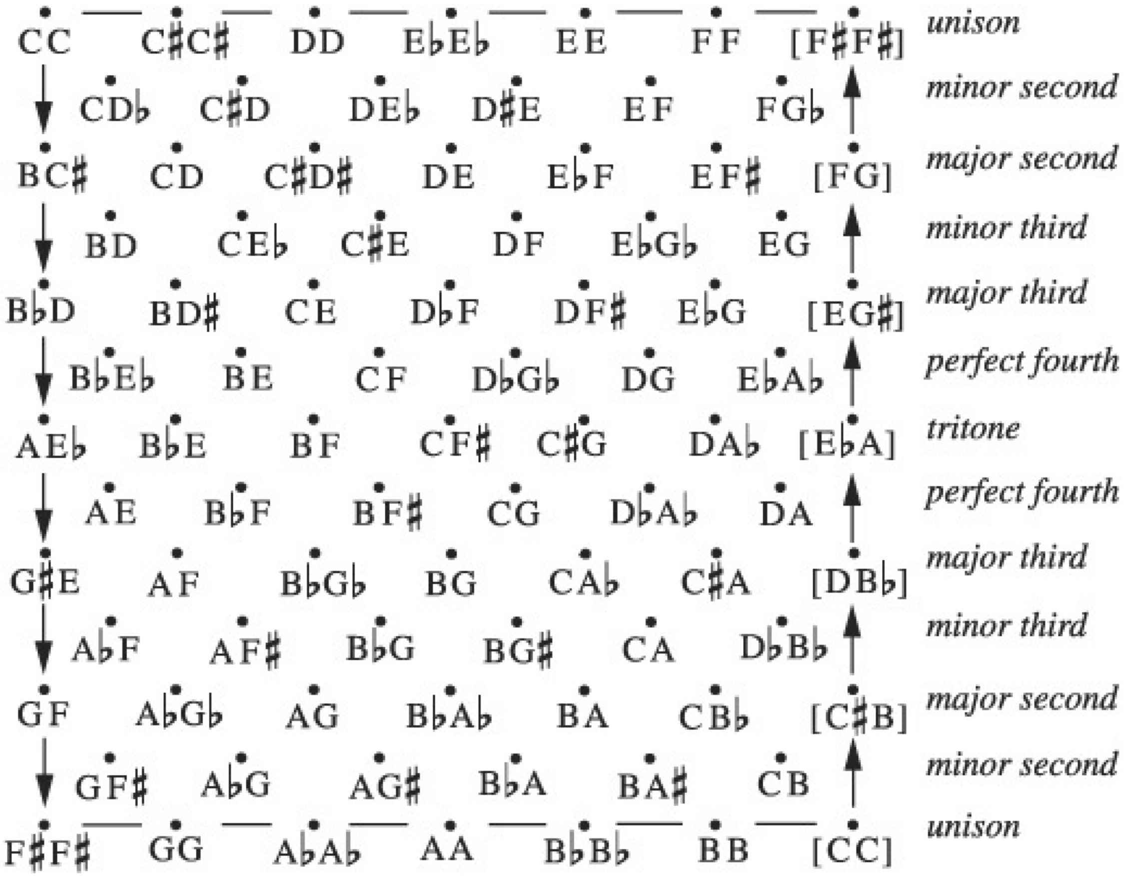
Tymoczko, “A Geometry of Music” Oxford, 2011





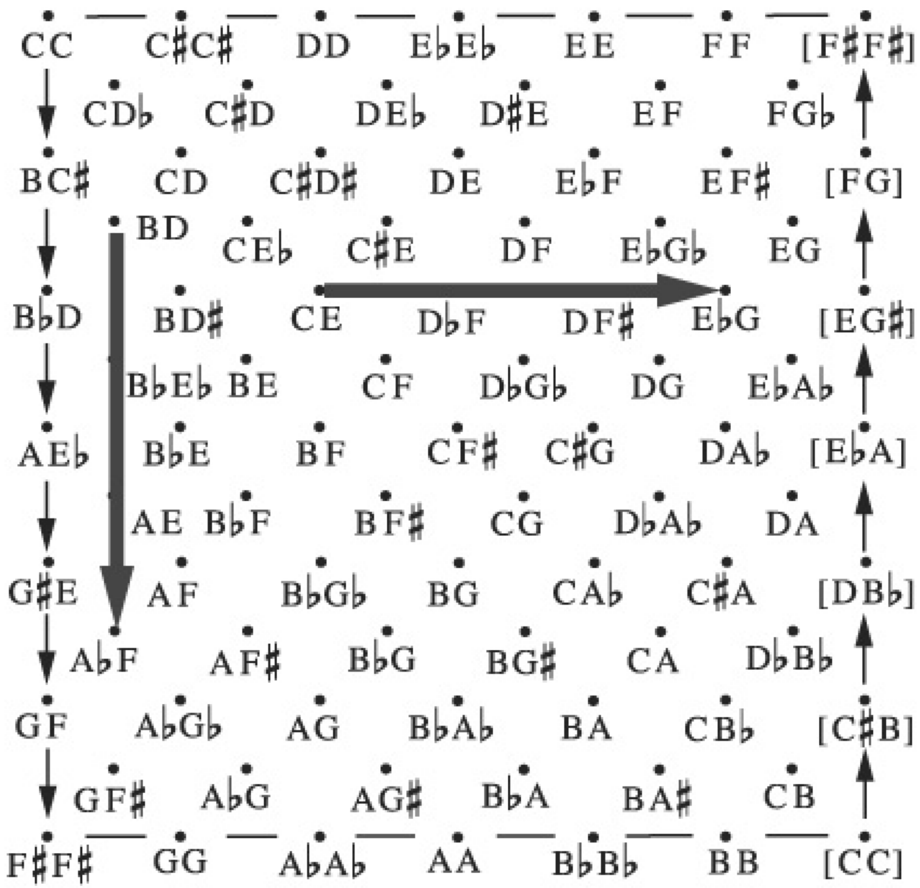
and so on. The result will be a new geometrical space, analogous to our single tile of wallpaper, in which points represent two-note *chords*—or unordered pairs of pitch classes—rather than ordered pairs of pitches.

Figure 3.3.1 Two-note chord space. The left edge is “glued” to the right, with a twist.

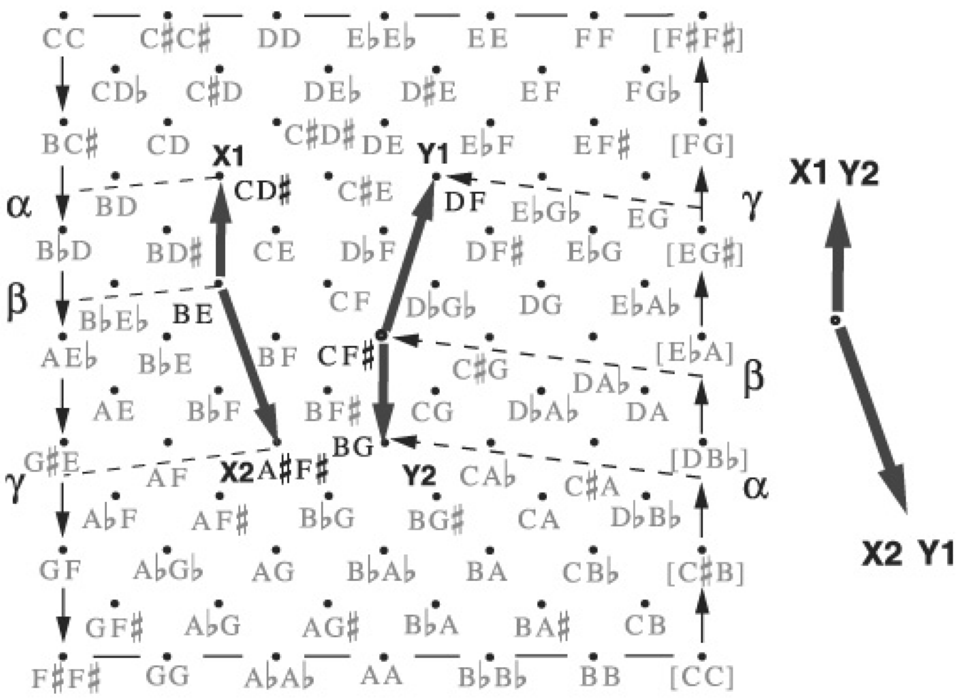


What about the lower left and upper left quadrants? Here, the relationship is somewhat harder to grasp. Imagine that there were a hinge connecting them,

Figure 3.4.1 Voice leadings are represented by line segments. Parallel motion is horizontal, while perfect contrary motion is vertical. The voice leadings (C, E)→(E♭, G) and (B, D) → (A♭, F) are shown.



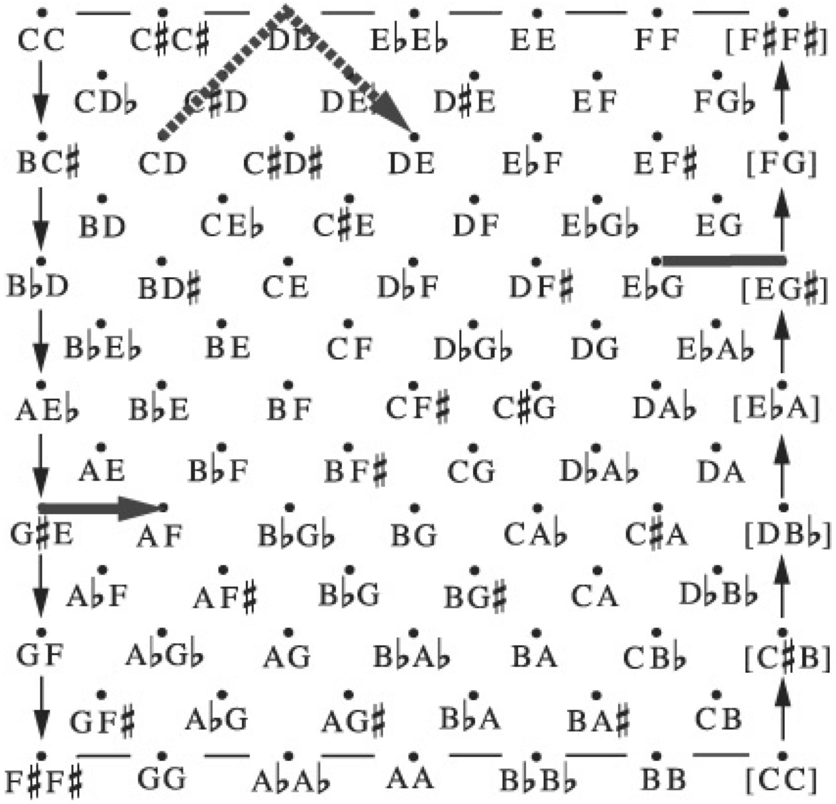
(*c*)


CC C♯C♯ DD E♭E♭ EE FF [F♯F♯]
CD♭ C♯D DE♭ D♯E EF FG♭
BC♯ CD C♯D♯ DE E♭F EF♯ [FG]
X1 Y1
X1 Y2
α CD♯ C♯E DF E♭G♭ EG γ
BD
B♭D BD♯ CE D♭F DF♯ E♭G [EG♯]
β CF D♭G♭ DG E♭A♭
B♭E♭ BE
AE♭ B♭E BF CF♯ C♯G DA♭ [E♭A] β
AE B♭F BF♯ CG D♭A♭ DA
BG
γ G♯E AF X2 A♯F♯ Y2 CA♭ C♯A [DB♭] α
X2 Y1
A♭F AF♯ B♭G BG♯ CA D♭B♭
GF A♭G♭ AG B♭A♭ BA CB♭ [C♯B]
GF♯ A♭G AG♯ B♭A BA♯ CB
F♯F♯ GG A♭A♭ AA B♭B♭ BB [CC]

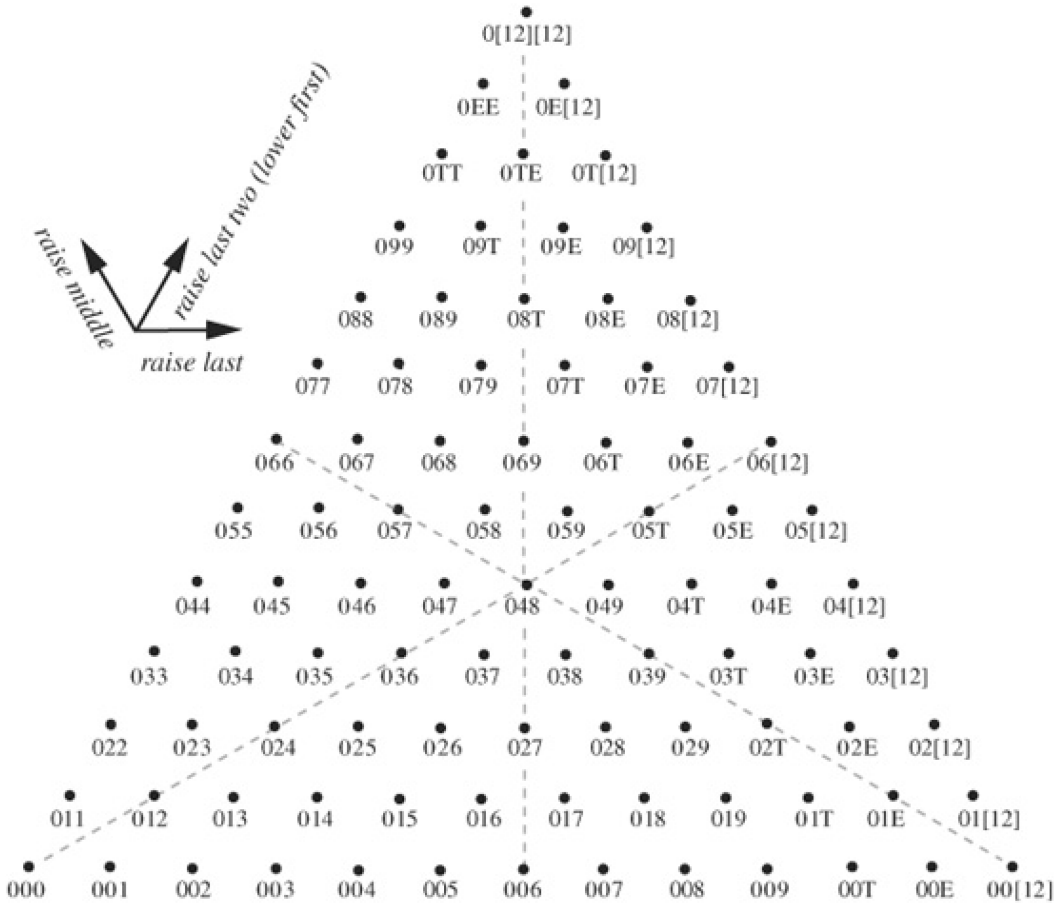
(*d*)


X1 X2 Y2 Y1
P N P N

Figure 3.4.2 The horizontal boundaries act like mirrors, whereas the vertical boundaries are glued together with a "twist." Voice leadings thus disappear off the left edge to reappear on the right, and vice versa. Here, the voice leadings (E♭, G) → (F, A) and (C, D) → (E, D) are shown.

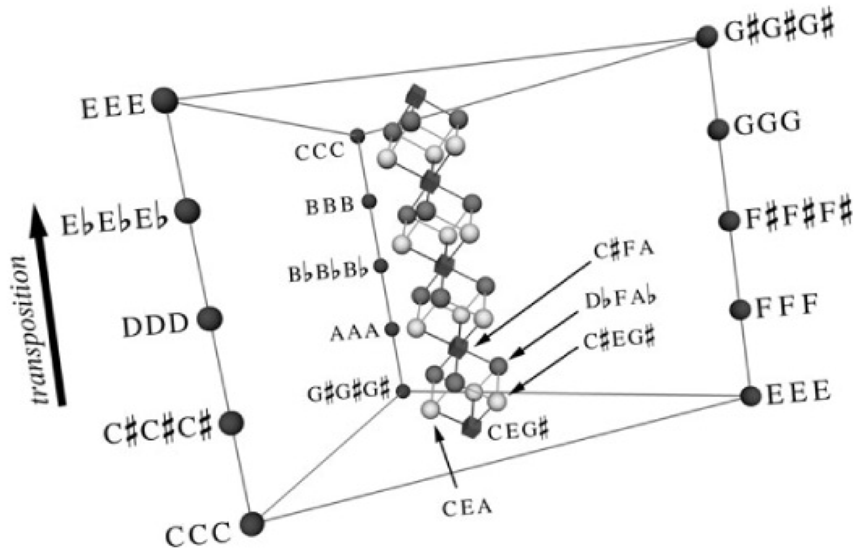


Voice leadings are therefore represented by what might be called "generalized line segments" that can bounce off a mirror boundary, or

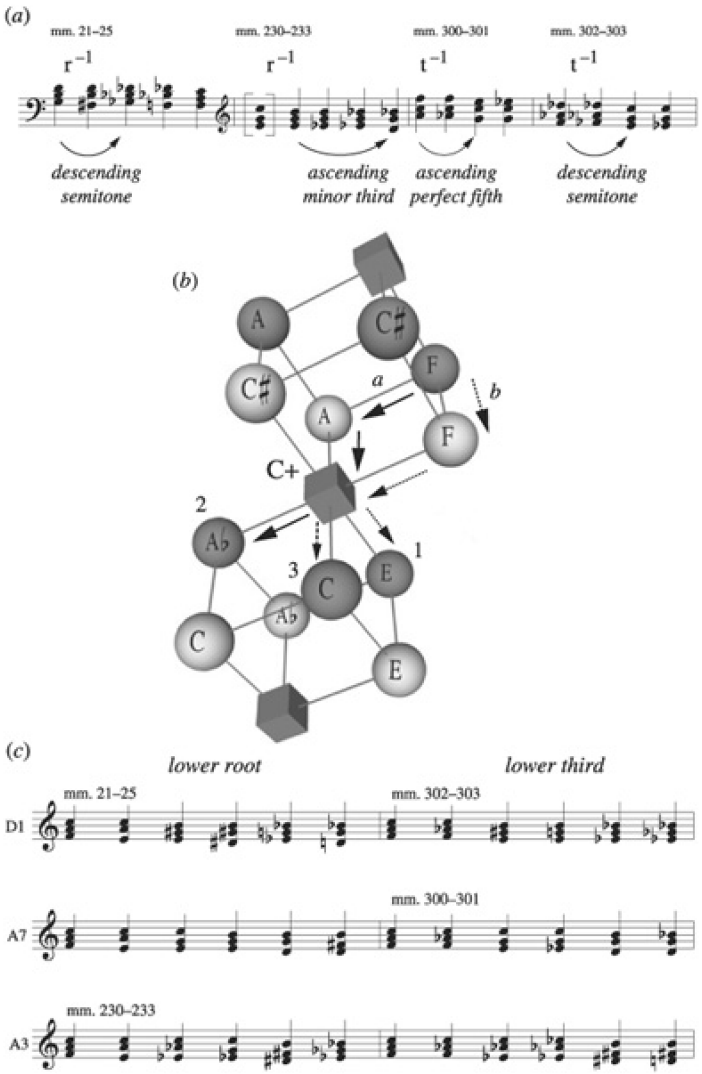


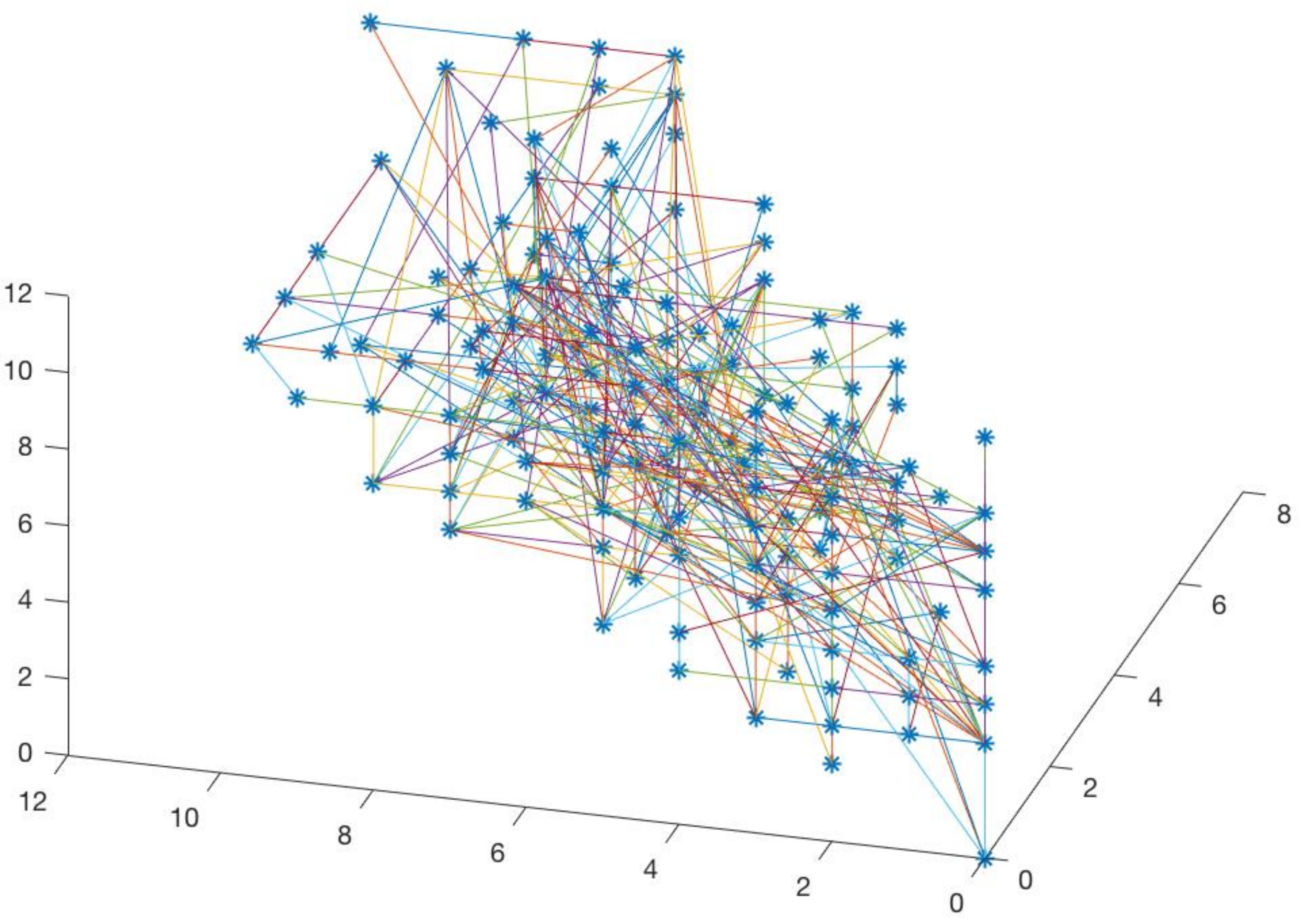
Finally, observe that the structure of the space again demonstrates the relation between evenness and efficient voice leading. Suppose you want to find an efficient (three-voice) voice leading between transpositionally related three-note chords. As in §3.7, we decompose our voice leading into pure

The lines in the center of the space connect chords that can be linked by voice leading in which only a single voice moves, and it moves by only a single semitone.



As before, voice leadings are “generalized line segments” that stretch from one point to another. Ascending parallel motion in all three voices corresponds to ascending vertical motion on the prism.[19] Here, however, line segments disappear off the top face and reappear on the bottom, rotated by one third of a turn. Thus as the three voices ascend in parallel from {C, C, C} to {E, E, E}, the associated line segment in Figure 3.8.2 ascends along the left edge of the


(a)
mm. 21–25
mm. 230–233
mm. 300–301
mm. 302–303
r⁻¹
r⁻¹
t⁻¹
t⁻¹
descending semitone
ascending minor third
ascending perfect fifth
descending semitone
(b)
A
C♯
C♯
F
A
a
b
F
C+
2
A♭
3
C
E
1
A♭
C
E
(c)
lower root
lower third
mm. 21–25
mm. 302–303
D1
mm. 300–301
A7
mm. 230–233
A3

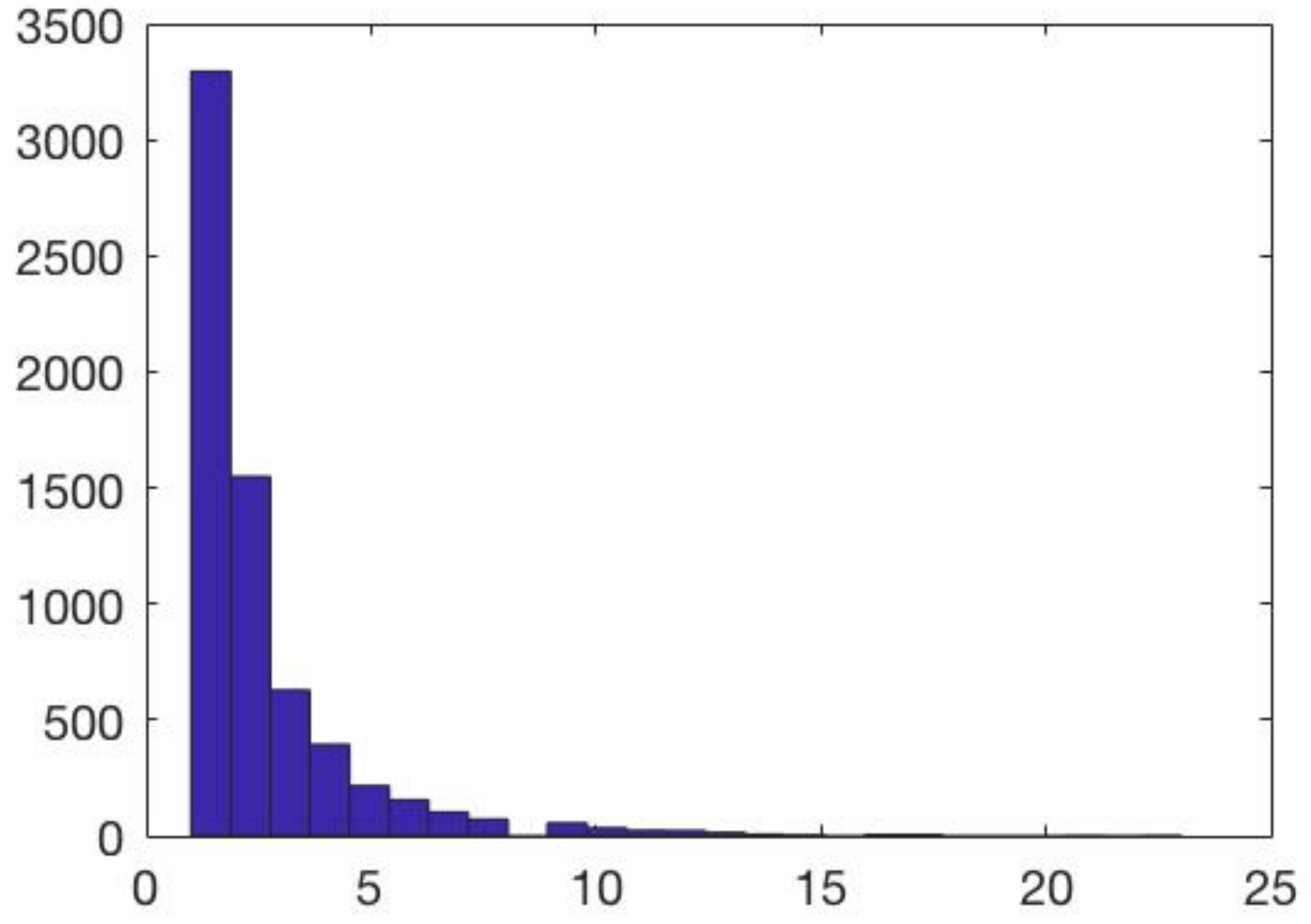


12
10
8
6
4
2
0
12
10
8
6
4
2
0
0
2
4
6
8

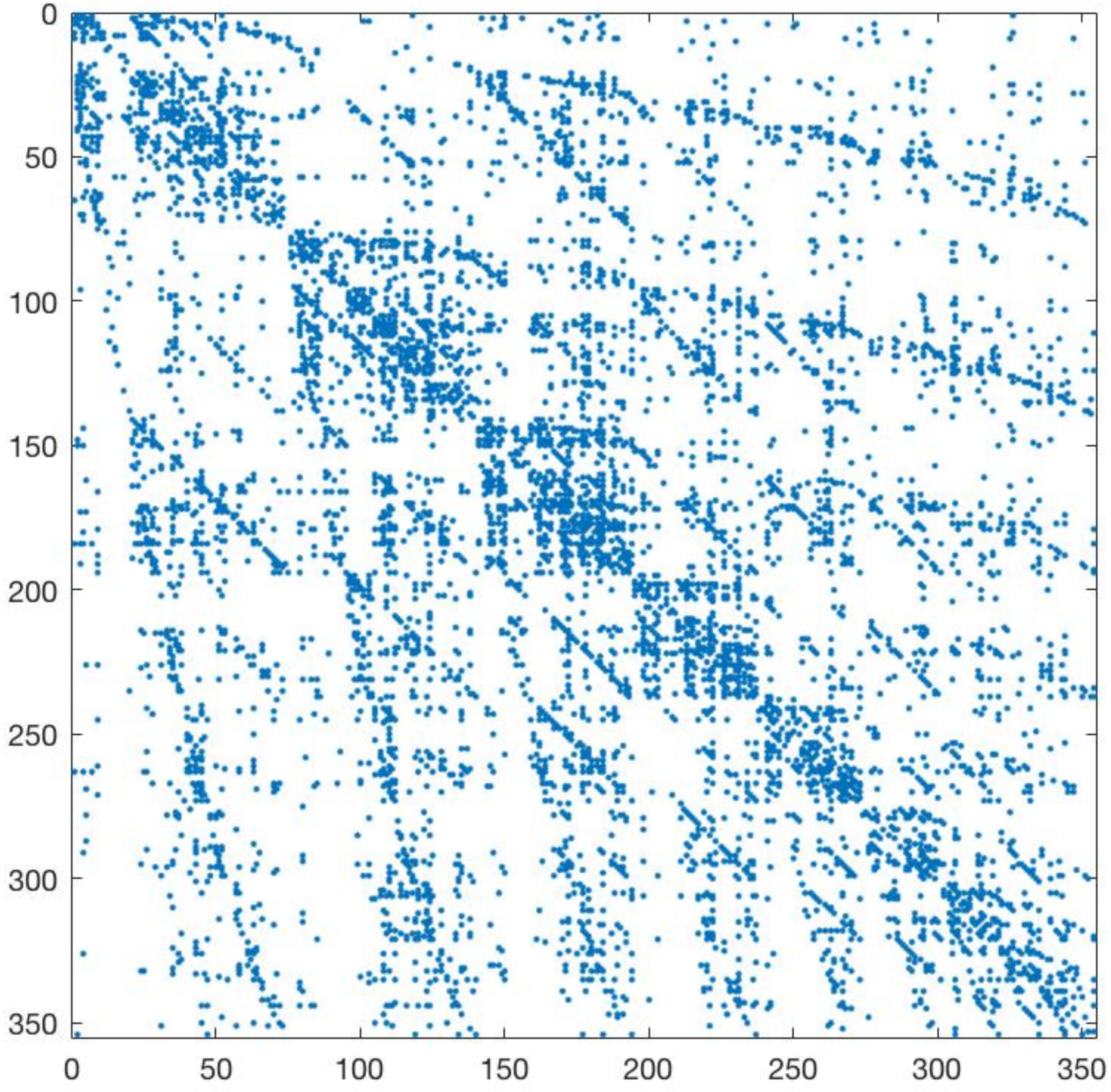
Bach uses many chords, and many progressions for a composer, but a fraction of all progressions

- Bach uses 354 of 364 chords (almost all possible chords)
- 15073 chord progressions recorded
- 6582 unique chord progressions (so we likely discover most)
- This is 11% of all possible chord progressions

Most progressions are used once

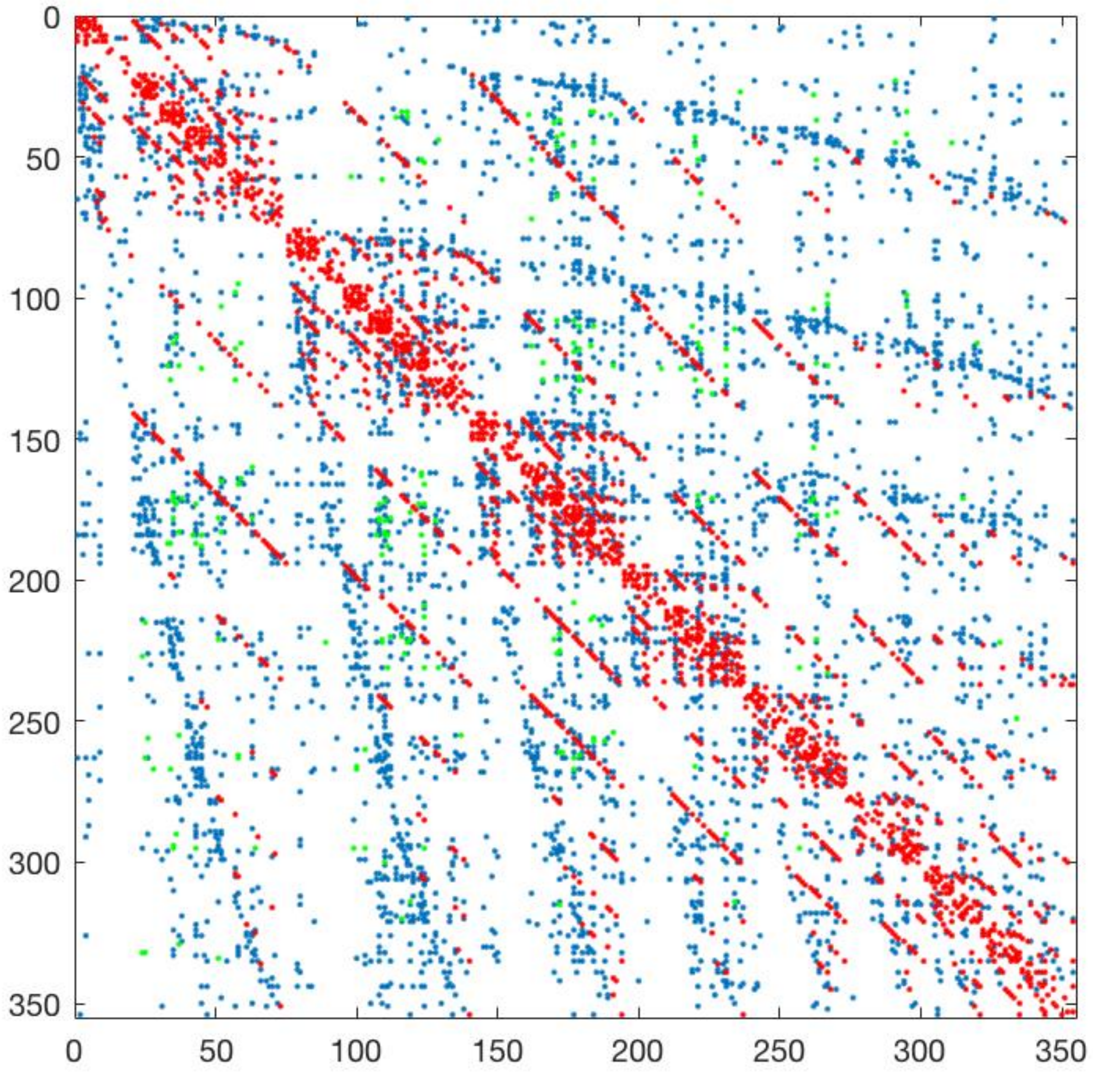


Chord Progressions



How many voices are moving

Red = 1, Blue = 2, Green = 3 (start on three notes, end on three different notes)
7525 one voice (50%), 7244 two voice motions (48%), 304 three voice motions (2%),



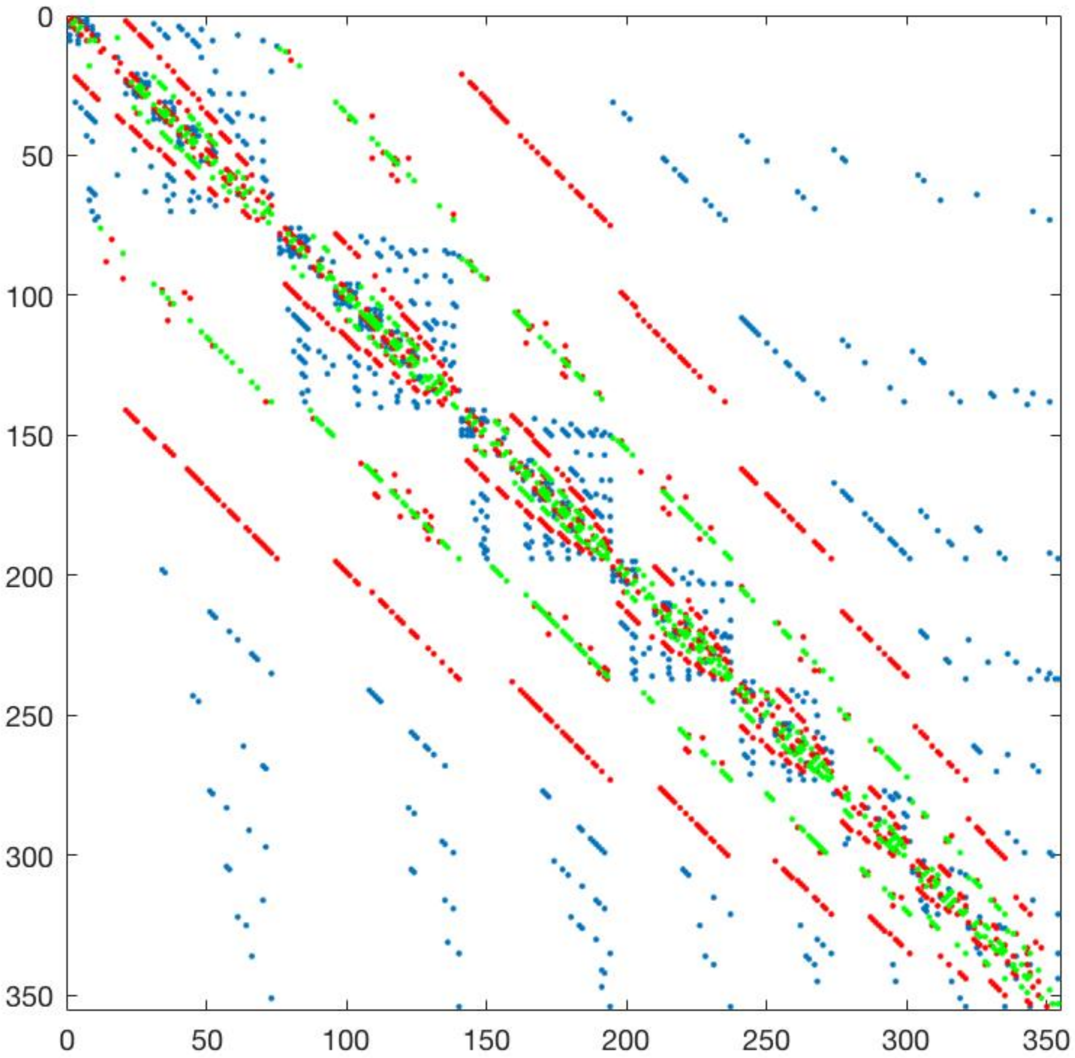
Three voice motions should be ~1/3 of chord progressions

- Three voice progress appear 304 times using 208 progressions, the most common only appears 6 times
- Most common, F G# B -> F# A# C, D F# A# -> C G A

One voice motion

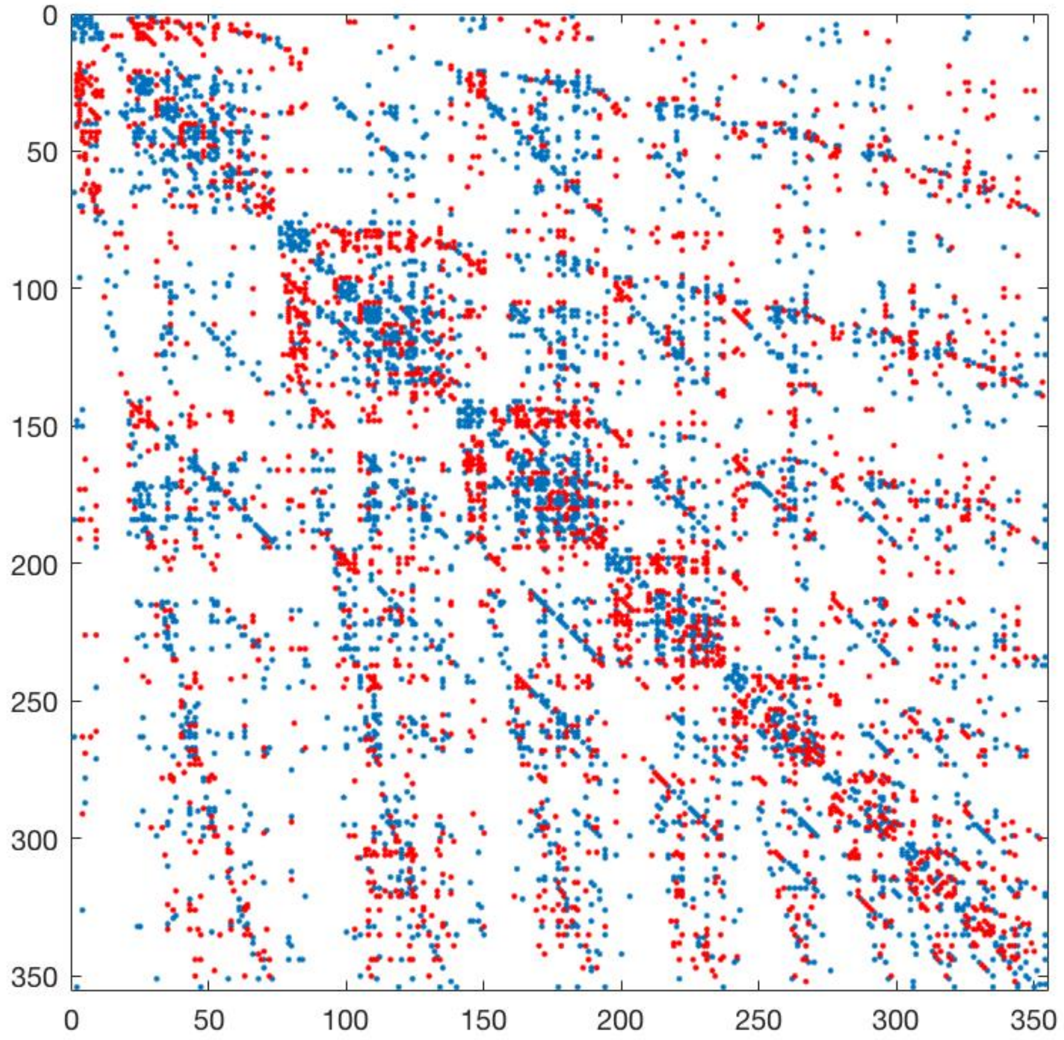
Green – motion by half step, Blue motion by whole step

2485 half step, 3449 whole step, 39.4% of all chord motions, 78.9% of all one voice motions



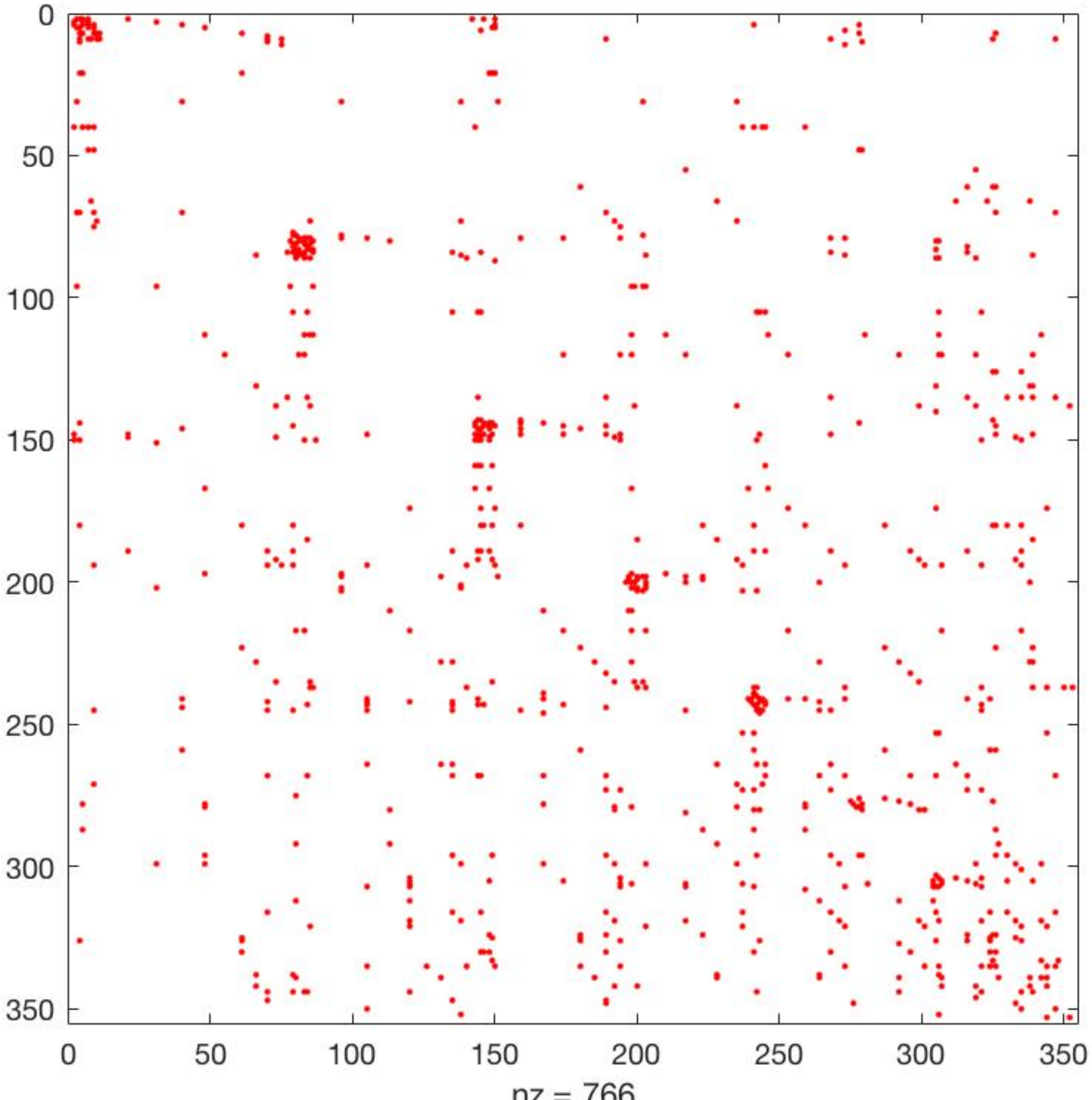
Voices coming together or coming apart

5967 times (40%), 2774 of the possible chords (42%)



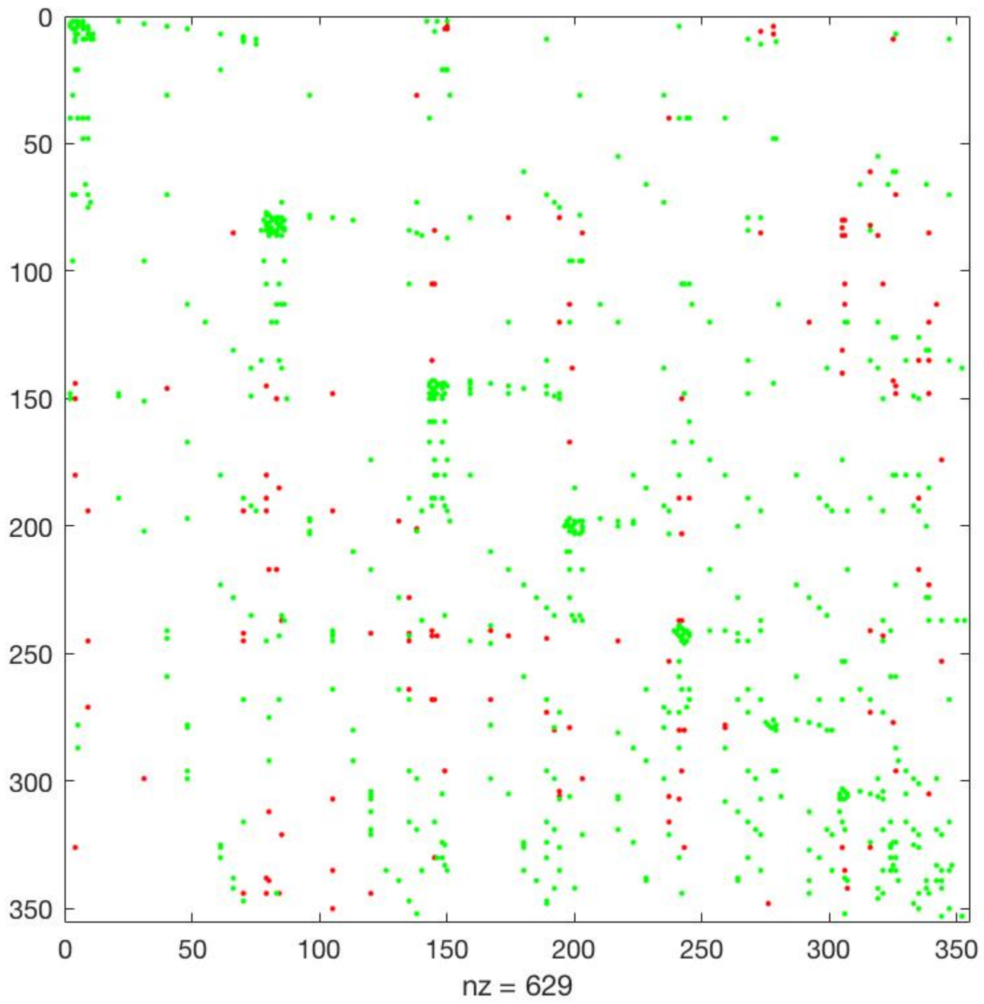
But voiceparts typically don’t stay together

1606, 766



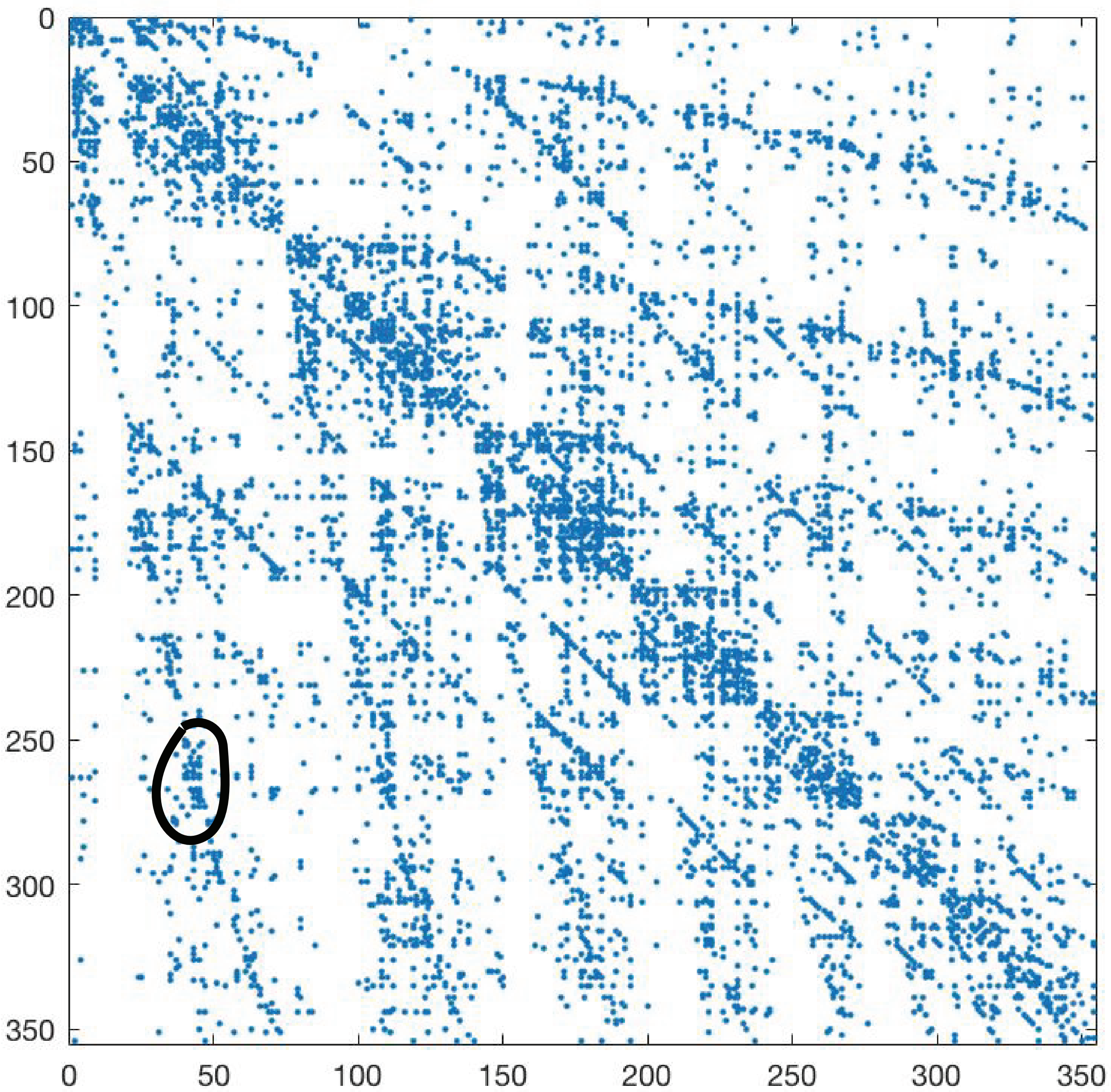
And very rarely travel together

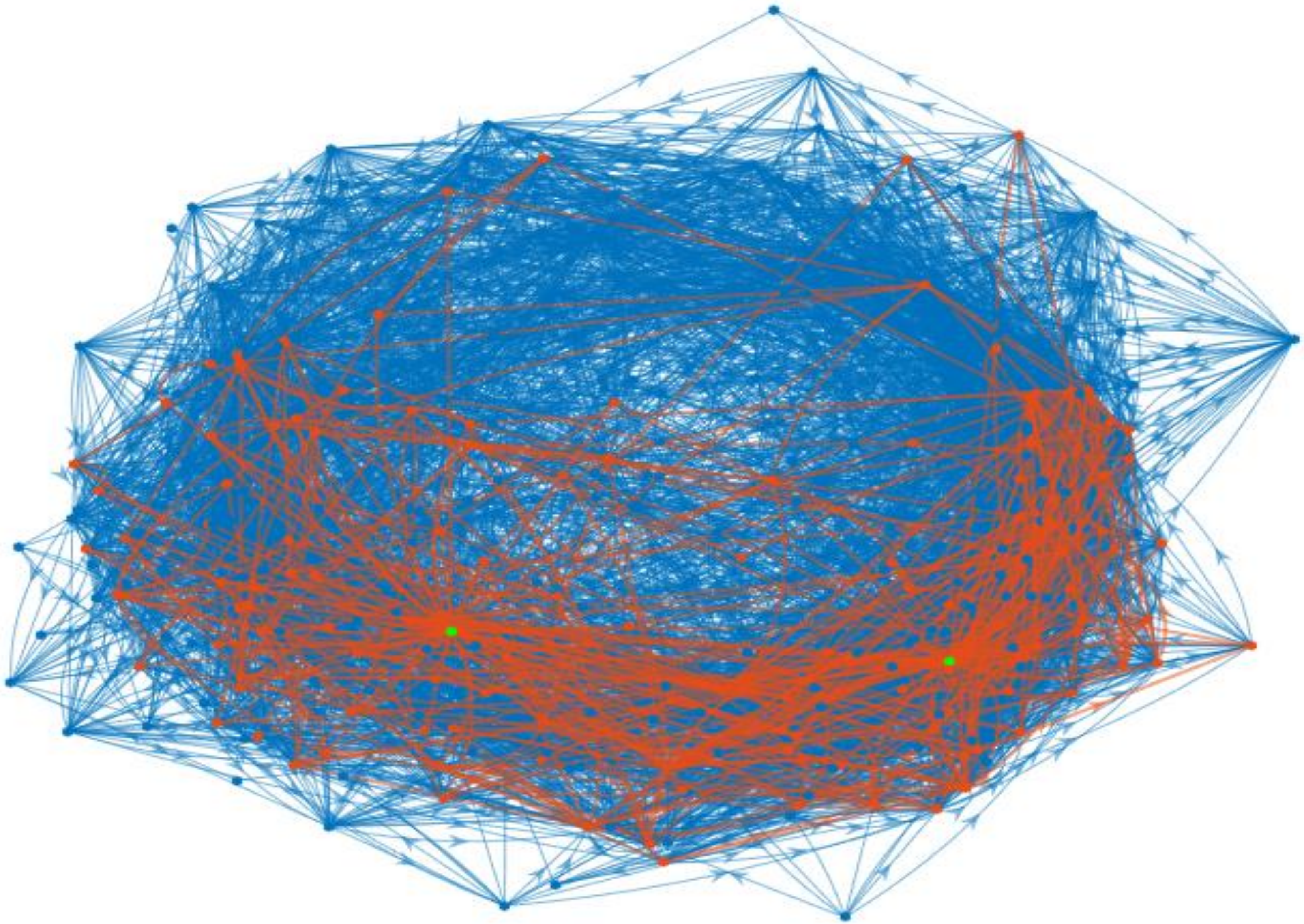
191, 67 green stay put, red move



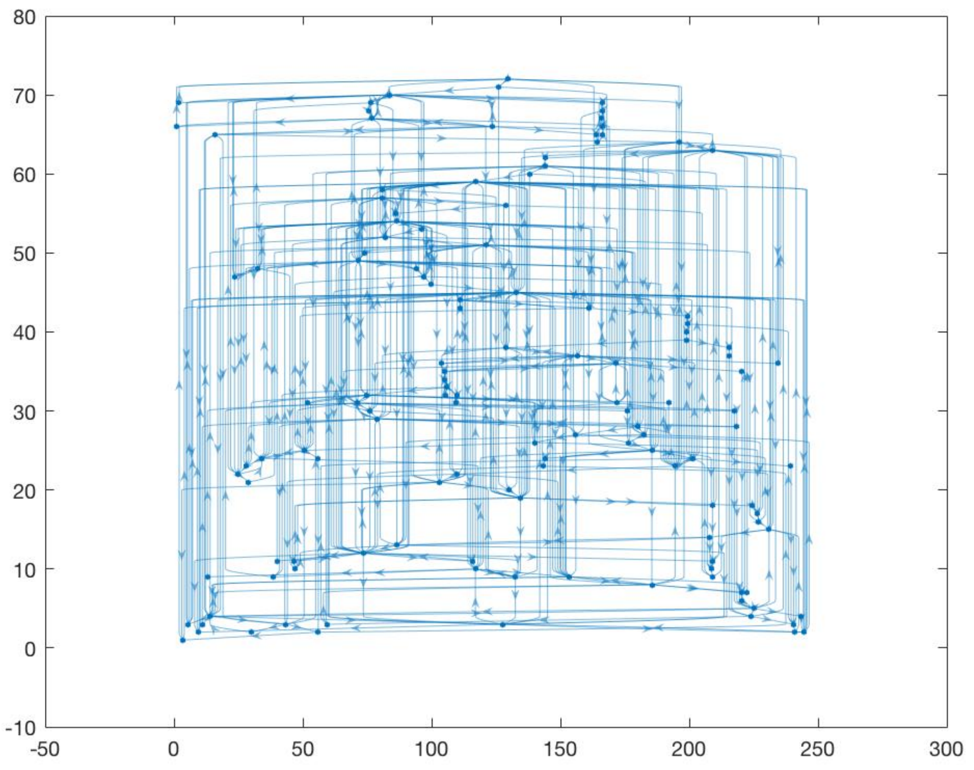
What are the near vertical or horizontal lines

- E A A# to C E A
- E G# A# to C E A
- E A A to C E A
- E A B to C E A
- E A# A# to C E A
- …





Trio 1, 1 layered



Analyzing the chord structure

- Consider all chords that are used in the trio sonatas, after all have been transposed into the key of C major or A minor and treating all inversions of a chord as the same chord
- Chord progressions form a graph
- Most central (pagerank) chords are:

(C, E, G), (E, G# B), (F, A, C), (A, C#, E), (F#, A, C#), (G, B,D), (C, E ♭ , G), (A ♭ , C, E ♭).

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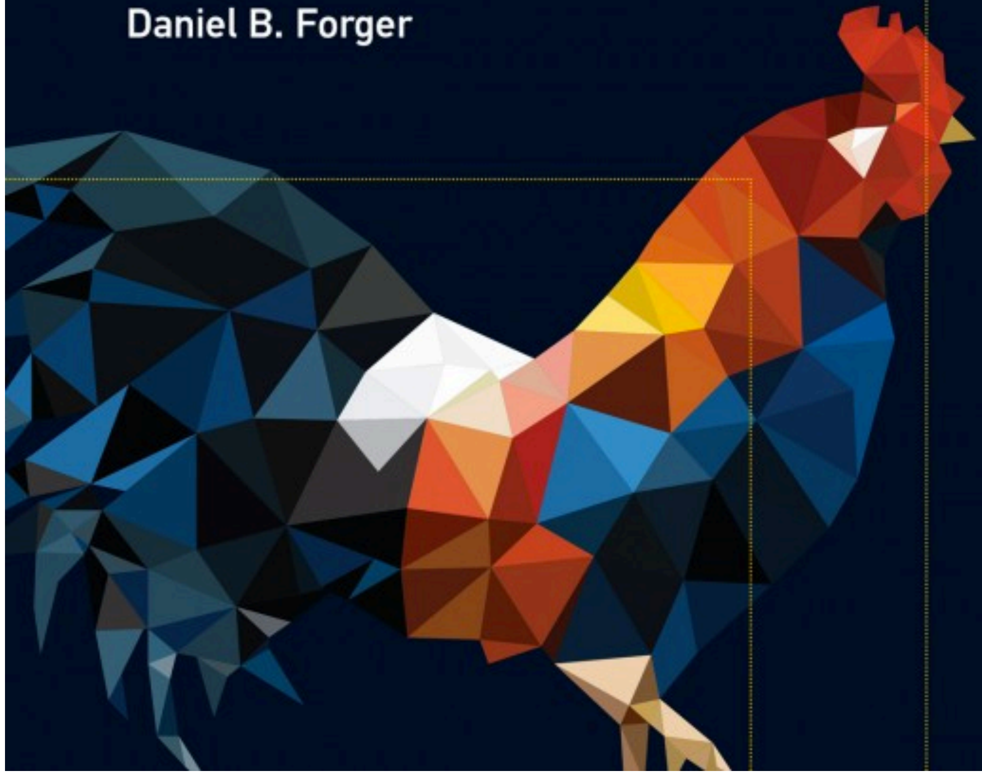

Big Data Approaches to the Neuroscience of Performance

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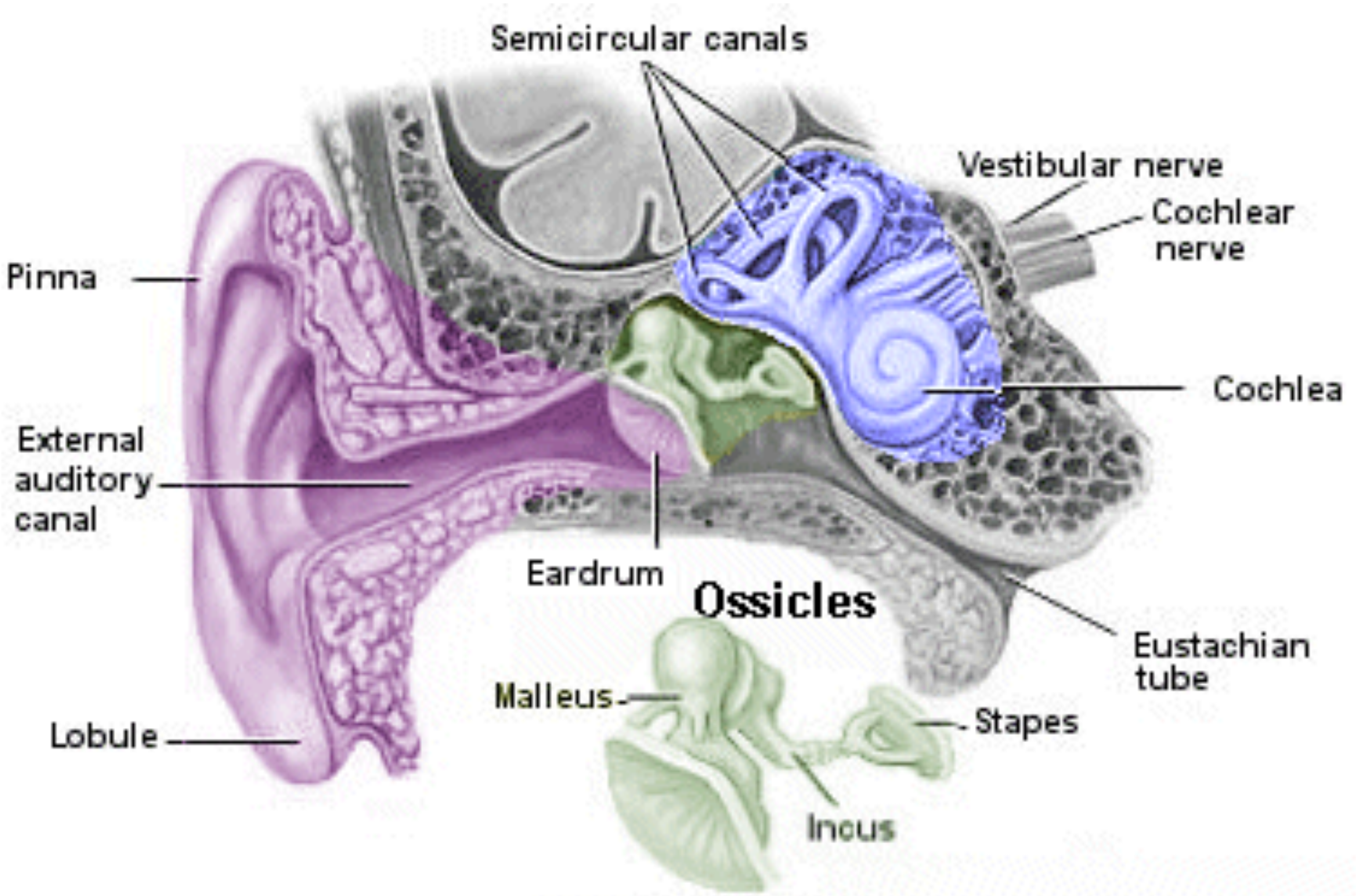
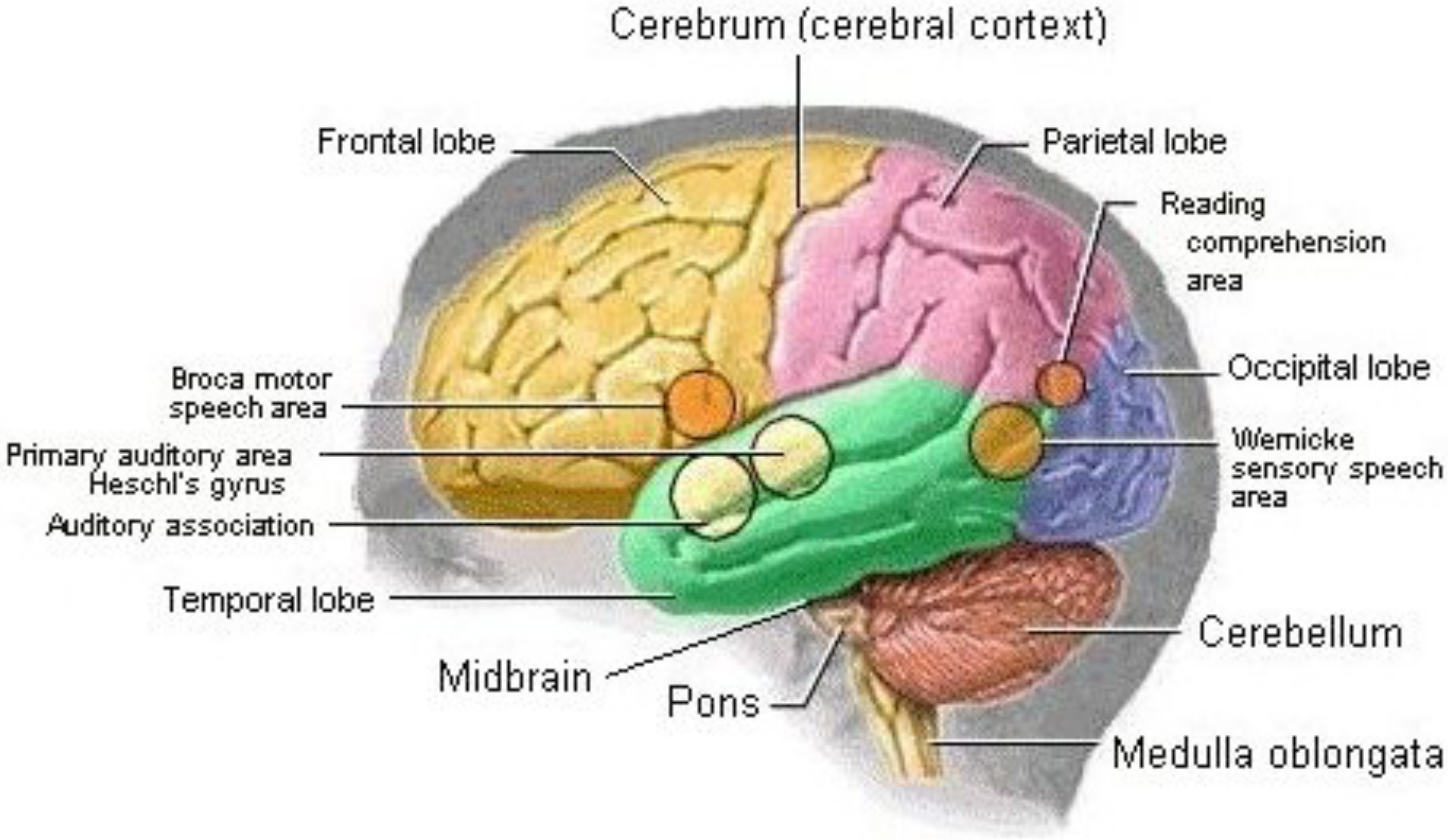


http://firstyears.org/anatomy/ear.htm



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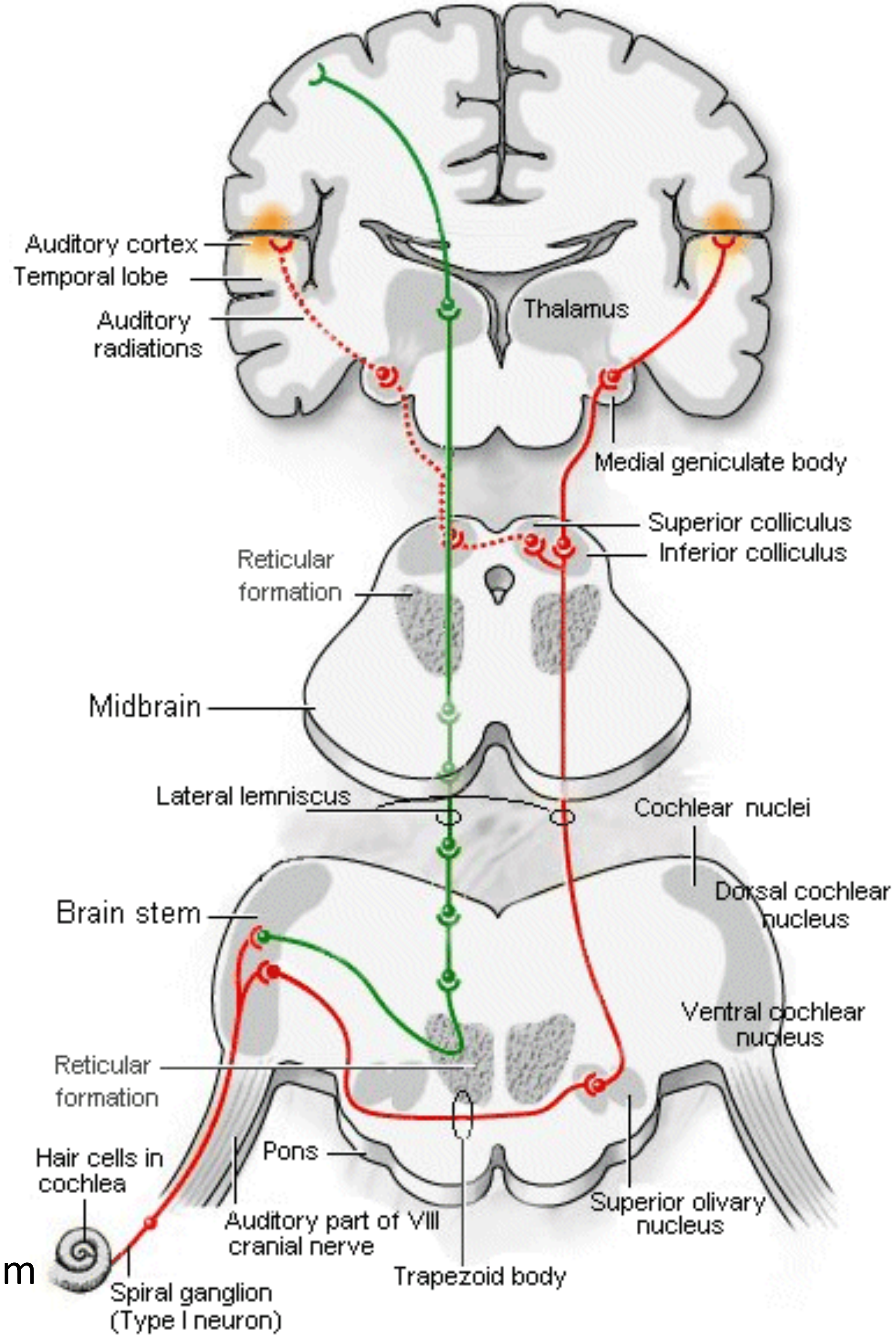
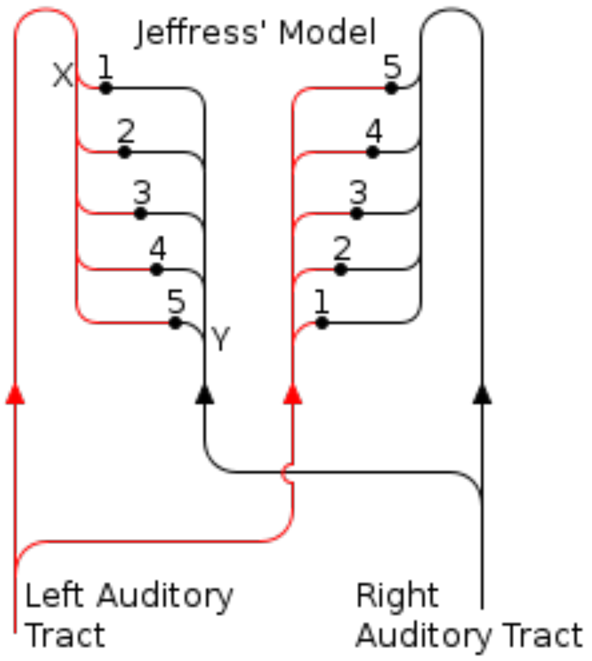
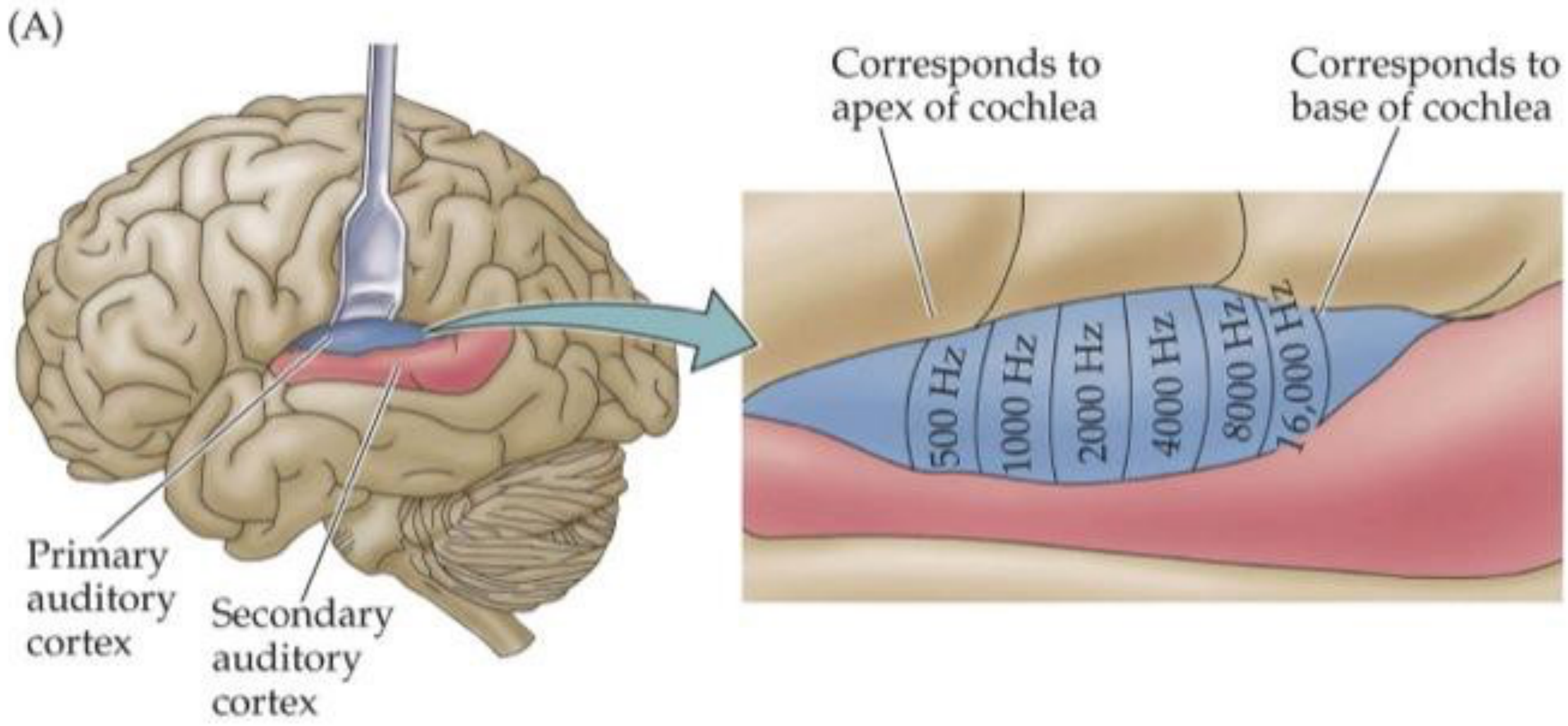


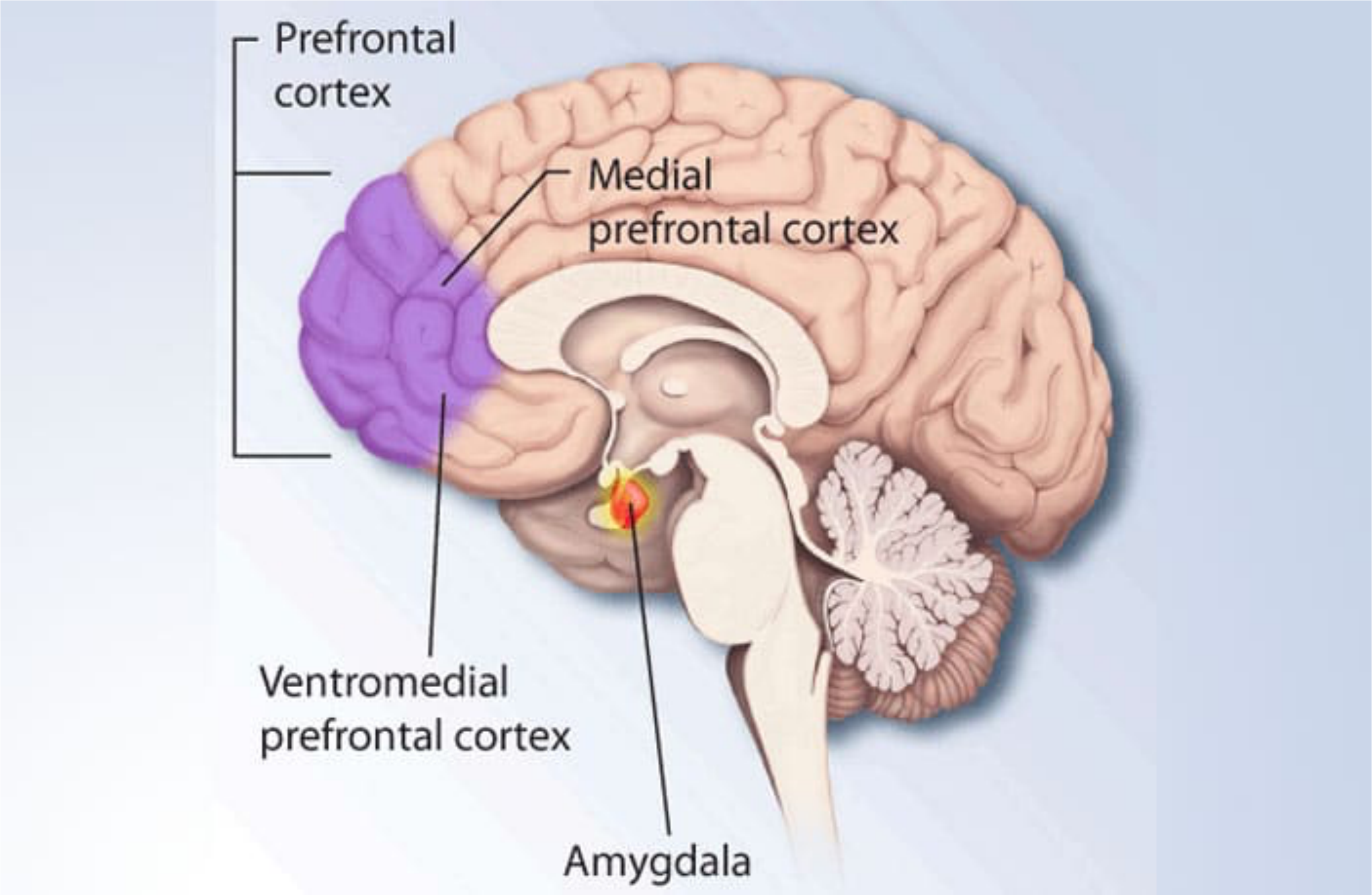
http://firstyears.org/anatomy/ear.htm

Wikipedia coincidence detection in Neurobiology



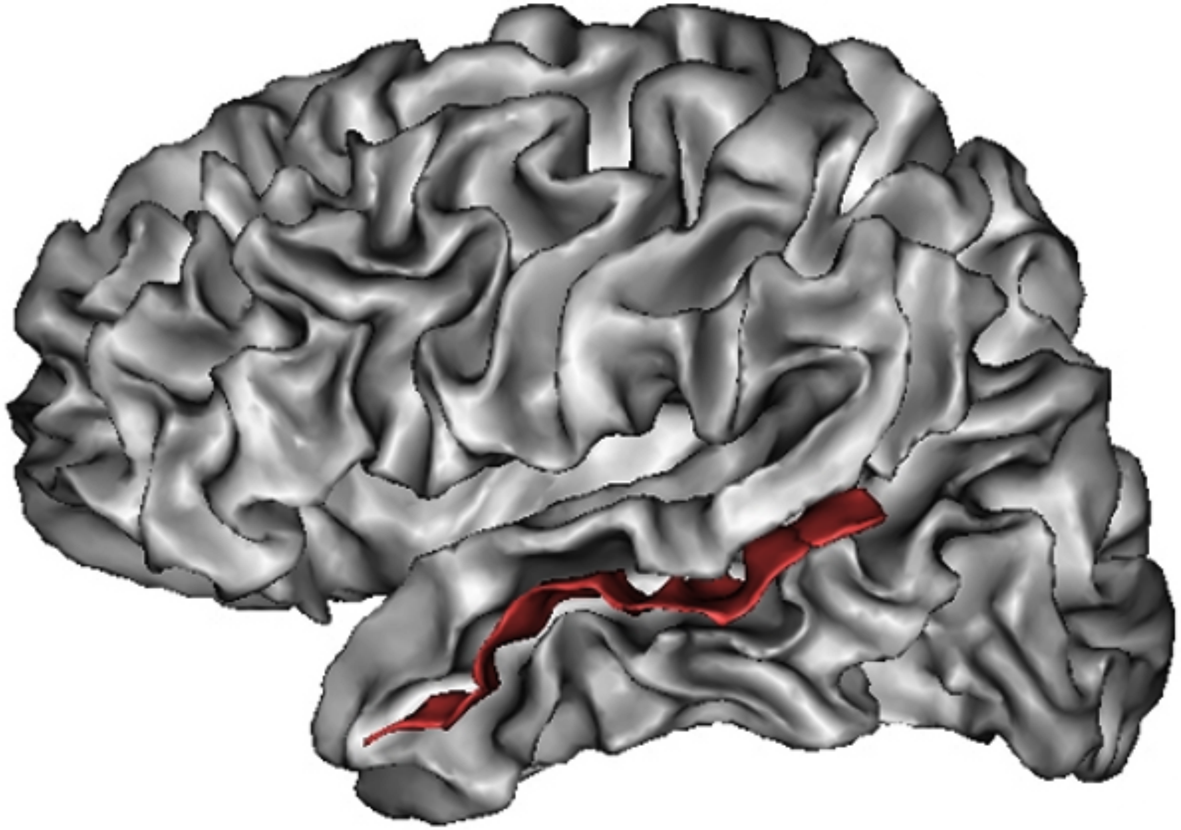

(A)
Corresponds to apex of cochlea
Corresponds to base of cochlea
500 Hz
1000 Hz
2000 Hz
4000 Hz
8000 Hz
16,000 Hz
Primary auditory cortex
Secondary auditory cortex

Expectations

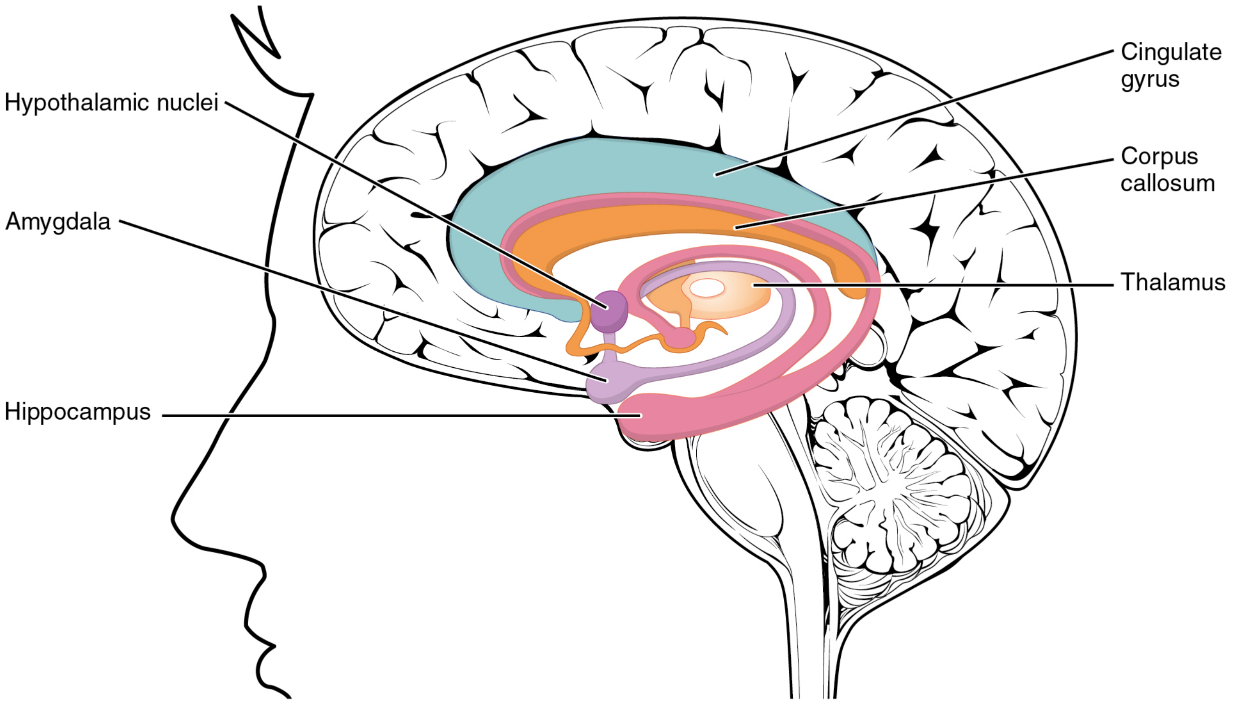


Timbre

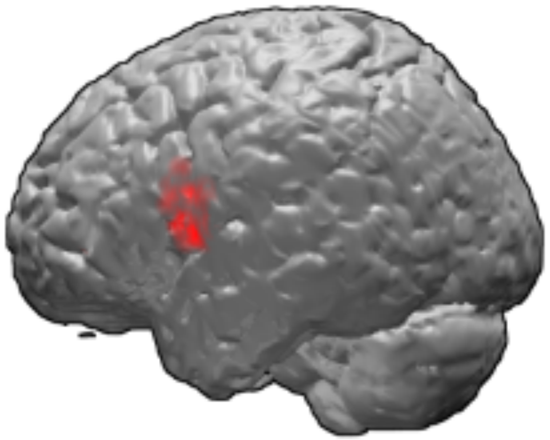
Wikipedia: superior temporal gyrus

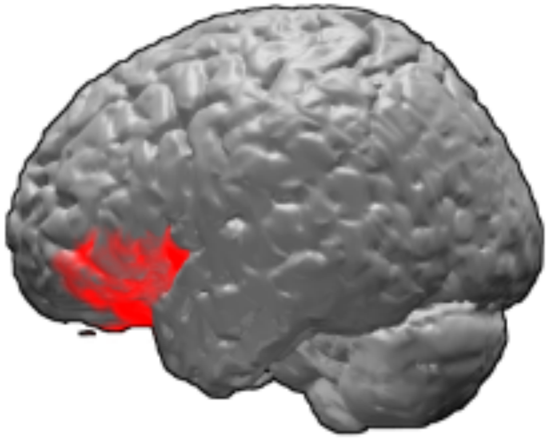


Memory


Hypothalamic nuclei
Amygdala
Hippocampus
Cingulate gyrus
Corpus callosum
Thalamus

BA 44 and 47 (contours)





A bit about organ history

- Tracker organs before 1850's, where a performer physically opens the pipes
- This is not always practical as the organ console (keyboards) may need to be in a different location than the pipes. Some instruments have too many pipes for tracker action
- Most instruments post 1850's involve some sort of coding system, where each note is converted to an electrical signal, which is sent to the pipes.
- Many instruments encode this signal via the MIDI standard
  - Which note was played, on which keyboard, when it started, when it ended

Thus, unlike other instruments we have an exact representation of what was played

- Some recent instrumental allow for a playback feature
  - actually an old idea
- Some instruments have midi ports
  - This can allow a connection to a computer to record these signals.
  - I have done this for a practice instrument at my home
- Others have thumb drives to record
  - Hill Auditorium with recent updates
  - Other instruments in the area

‘**trio sonata.** A sonata for two melody instruments and continuo. It was the central instrumental form of the Baroque period.” Grove dictionary of music

1st ten notes of my performance of Trio 4 M 1 in Midimusic format

0.4000	4.6380	1.0000	52.0000	64.0000	0.2000	2.3190
2.4300	2.3880	2.0000	64.0000	64.0000	1.2150	1.1940
4.9780	2.2740	2.0000	67.0000	64.0000	2.4890	1.1370
7.2640	1.6020	1.0000	51.0000	64.0000	3.6320	0.8010
7.2840	2.1640	2.0000	71.0000	64.0000	3.6420	1.0820
9.5220	1.7760	2.0000	60.0000	64.0000	4.7610	0.8880
9.5840	3.3500	1.0000	52.0000	64.0000	4.7920	1.6750
11.5940	0.8180	2.0000	60.0000	64.0000	5.7970	0.4090
12.4000	0.3380	2.0000	62.0000	64.0000	6.2000	0.1690
12.8200	0.3640	2.0000	64.0000	64.0000	6.4100	0.1820

Ongoing Project

1) Develop a library of digitized performances of Bach’s trio sonatas by many performers and in multiple locations.

2) Analyze this library using existing and novel tools from data science to learn about the trio sonatas, Bach’s compositional methods and the performance practice of these works.

3) Review the findings of our analysis with performers and members of the organ department to compare our results with their performance practice.

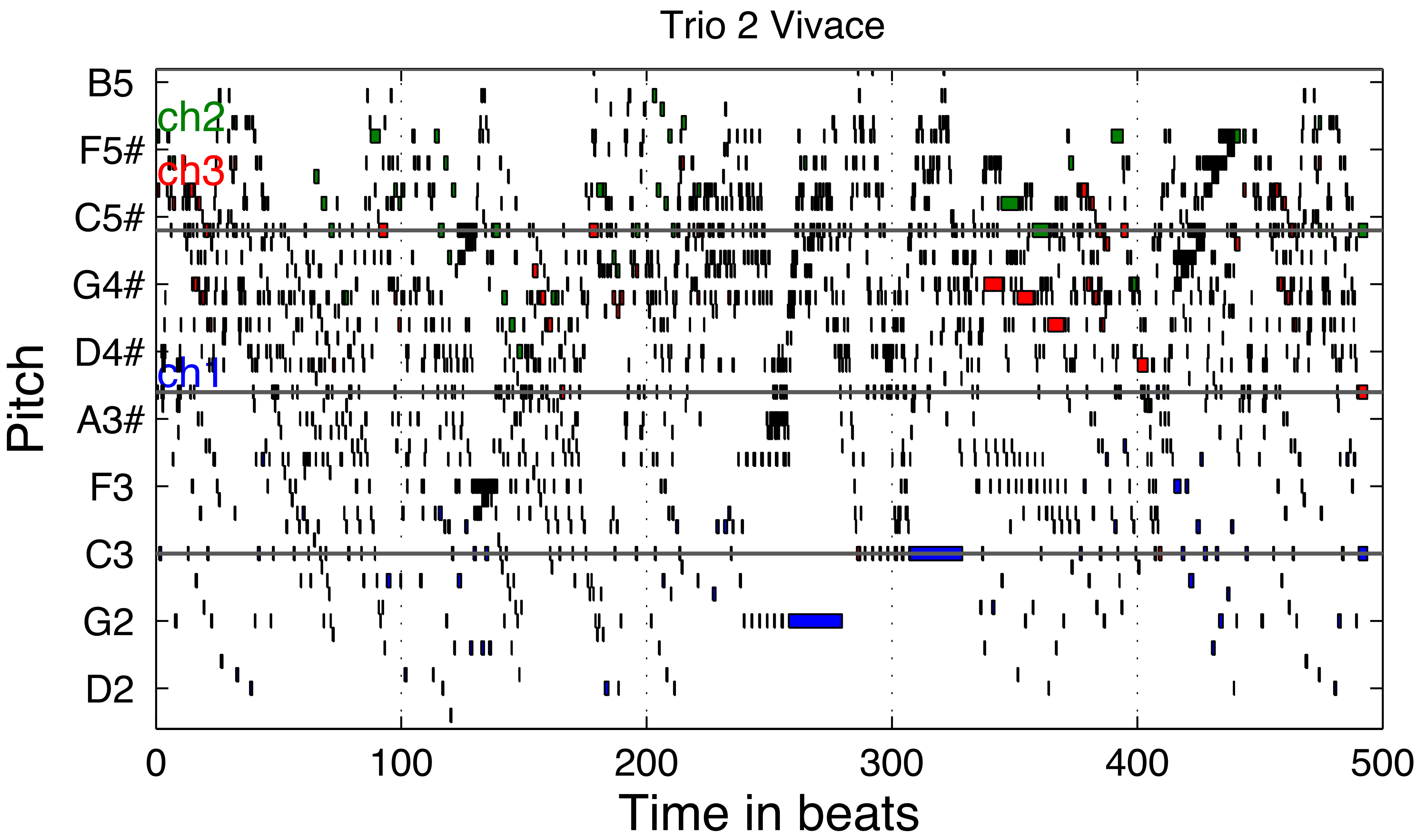
4) Disseminate results through coursework, conferences, open access code and publications.

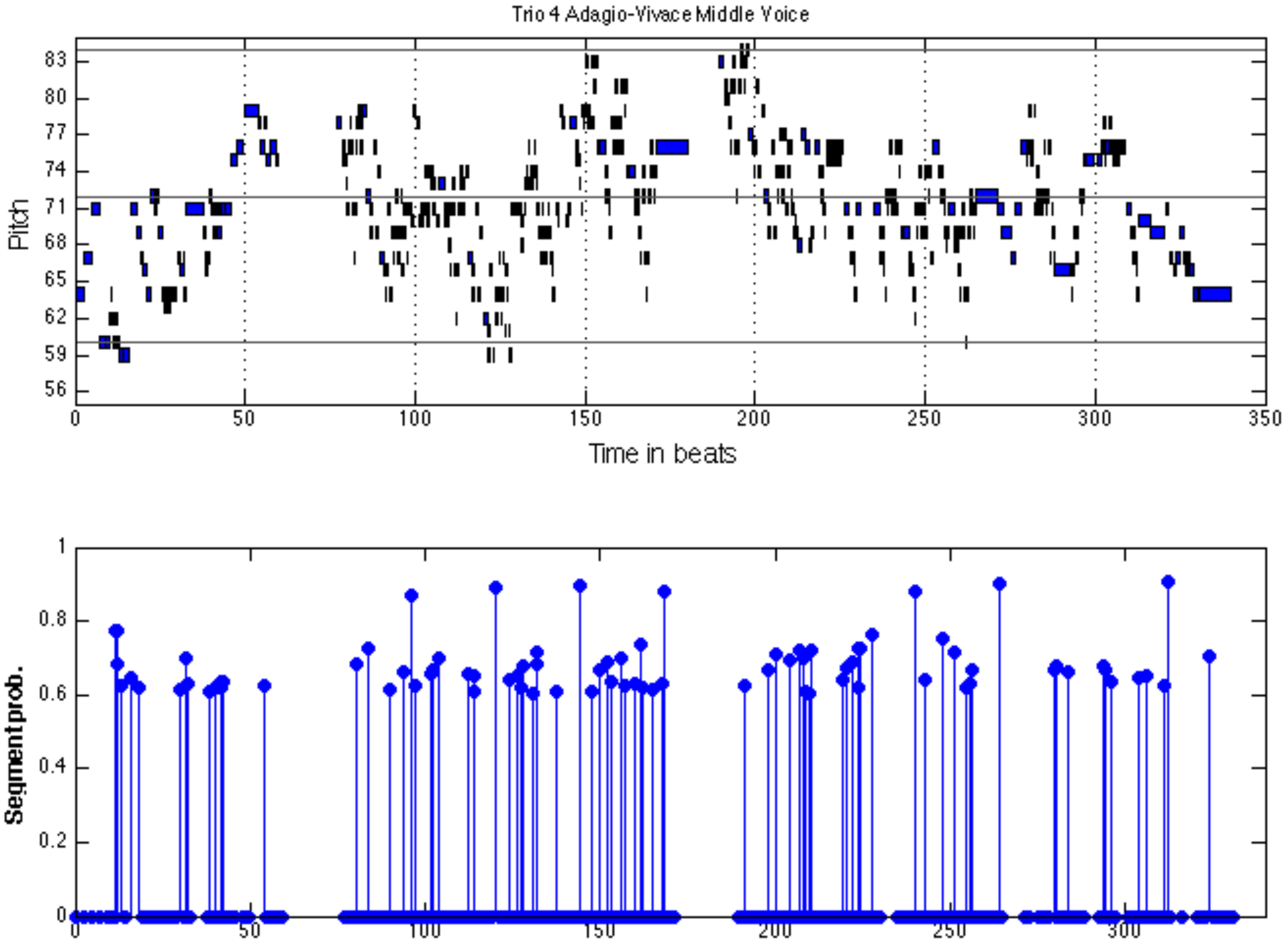
Many other questions

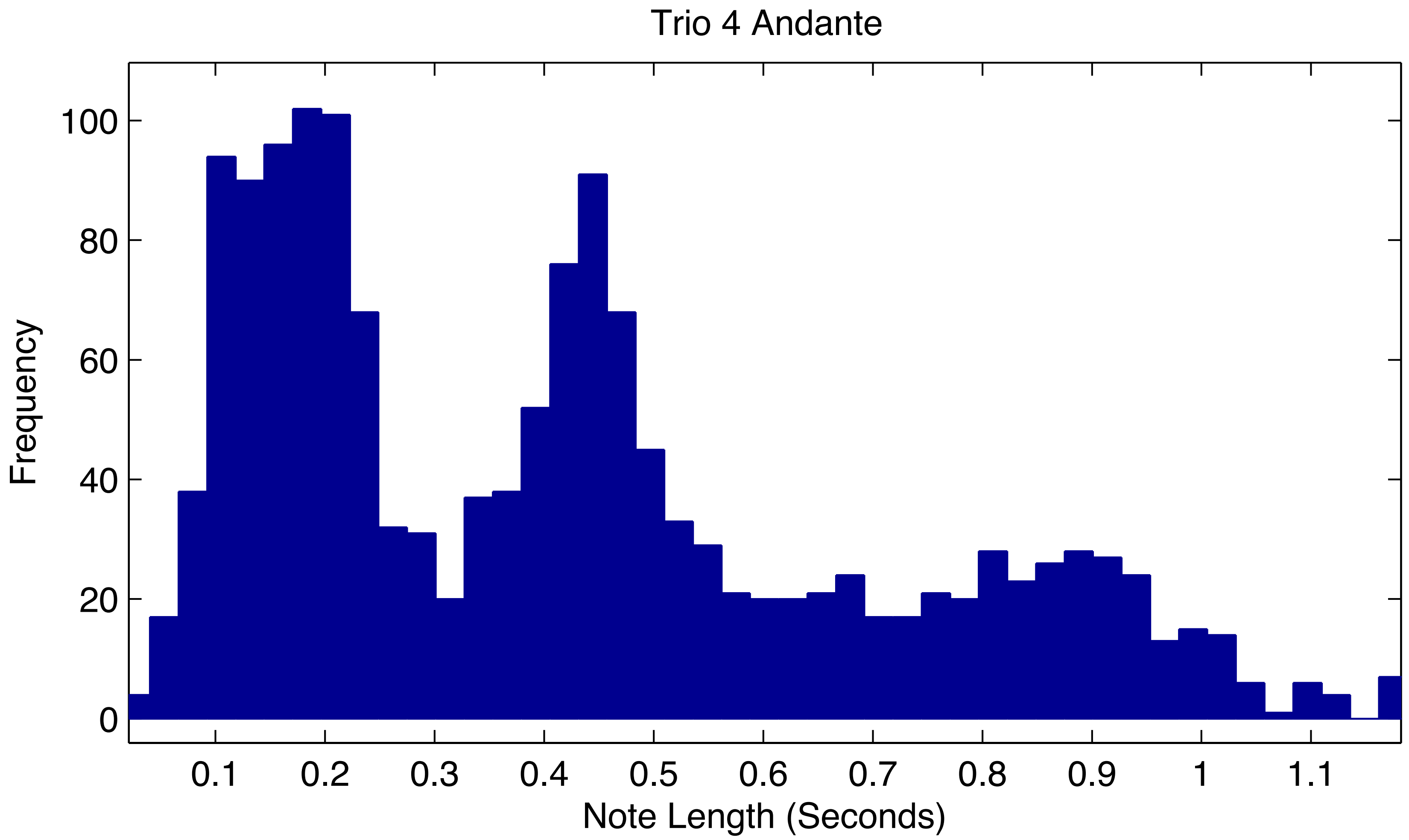
- Physiology: What aspects of the physiology of hearing, learning and memory impact our appreciation of these works?
- Mathematics: What mathematical representations of these works reveal interesting patterns and features that could affect our appreciation of these works?
- Composition: Can we identify patterns in these works that can be used to help guide future compositions.
- Networks: How can we use network theory to represent the chord progressions in these works.
- Performance: How can we quantify differences between performances? Which parts of the works are sped up or slowed down? Are there beats that receive particular emphasis? Are there common mistakes that can be quantified? How are performances different on different instruments?

Preliminary Results

Some used MIDI matlab tools by Eeroal and Toiviainen




Trio 4 Adagio-Vivace Middle Voice
Pitch
83
80
77
74
71
68
65
62
59
56
0
50
100
150
200
250
300
350
Time in beats
Segment prob.
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300


Trio 4 Andante
Frequency
Note Length (Seconds)
0
20
40
60
80
100
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1

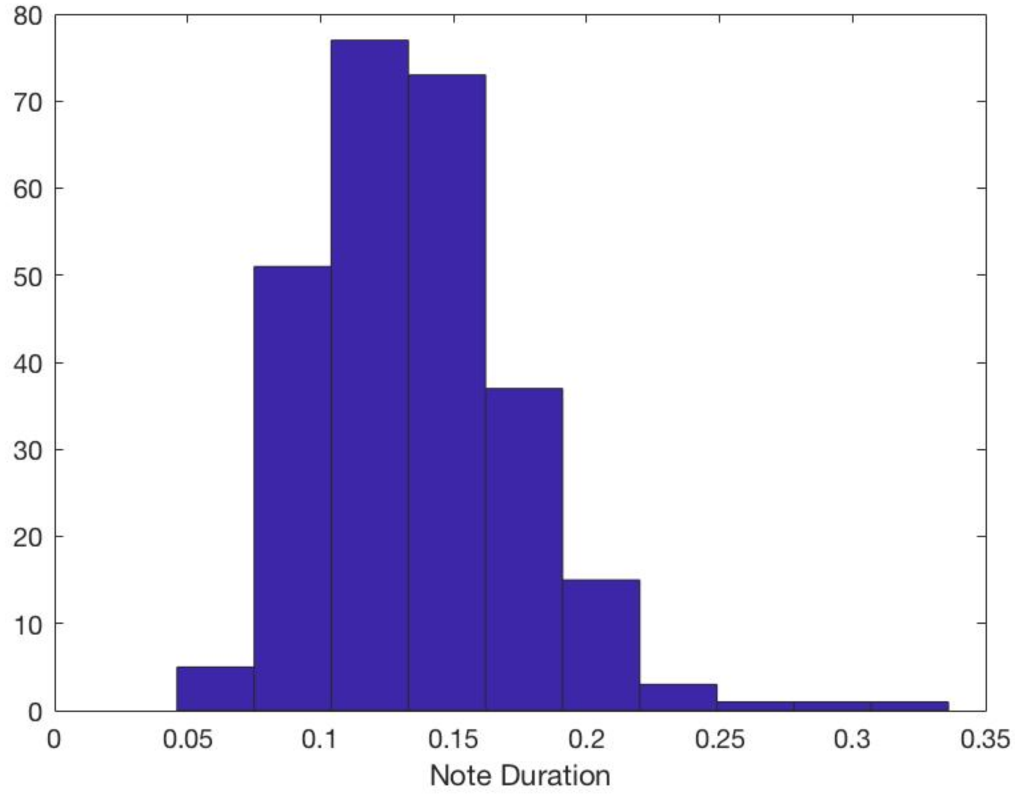
Artistry of Organ Playing is attack and release

- BWV 540 begins with a cannon. Right and left hands play same notes, on top of each. Then the pedal plays a very similar melody with the same motifs. This then repeats with the left hand starting
- Good test case to determine differences in touch between right hand, left hand and pedal.
- Look at 16^{th} notes (most common). What is their average duration? What is the std of their duration? Coefficient of variation?

Left Hand

Notes 0-300 on Chanel 2 and notes 279-573 on chanel 3

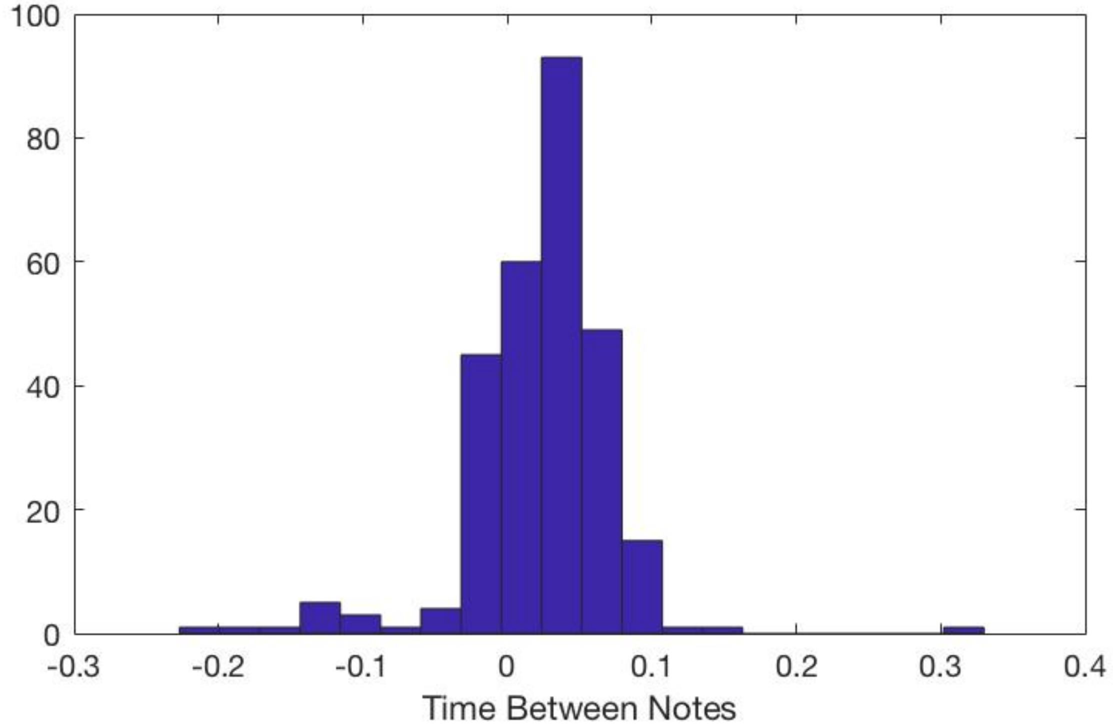
- Mean 16^{th} note duration .16 seconds
- Coefficient of variation .27
- Mean time between 16^{th} notes .03 seconds
- Variation of time between notes .045



Right Hand

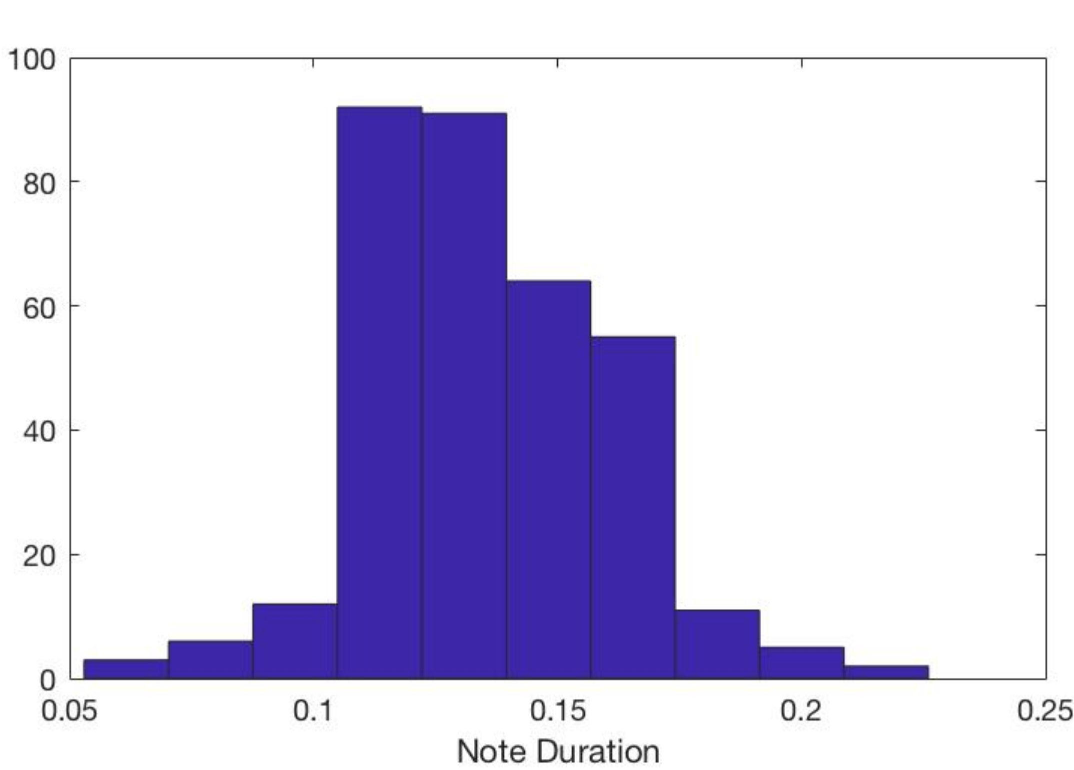
notes 1-273 on channel 3 and notes 320:580 on chanel 2

- Mean 16th note duration .14 seconds
- Coefficient of variation .27
- Mean time between 16th notes .05 seconds
- Variation of time between notes .045



Pedal
Chanel 1

- Mean 16^{th} note duration .14 seconds
- Coefficient of variation .18
- Mean time between 16^{th} notes .07 seconds
- Variation of time between notes .039

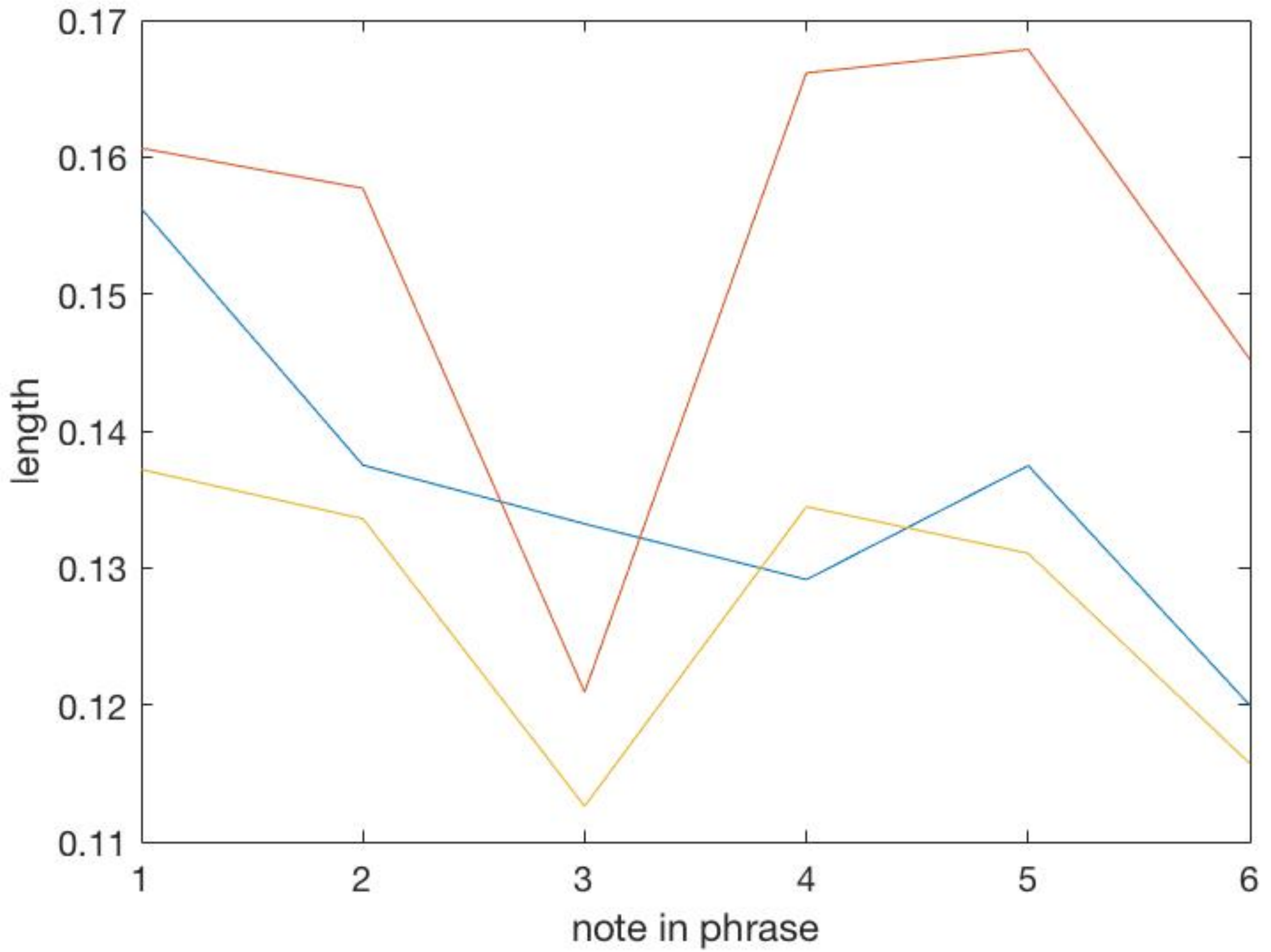


The first section of the work is based on a motif

- F – E – F –C – A – F
- But this is transposed and key is accommodated
- Also performers sometimes make mistakes
- Look for leaps of [- + - + -] and which you return after 2 notes and 5 notes
- The general problem of discovering phrases in music remains open
- Smith Waterman Algorithm variants are probably what you should use.
- Here I’ll use:
    - grs = (AA((ij+1):(ij+5), 4) - AA(ij:(ij+4), 4));
    - if max(abs((grs <0)'+ [-1 0 -1 0 -1]))+abs(sum(grs(1:2)) + abs(sum(grs))) == 0

Note lengths in repeated phrase

blue pedal, yellow right hand, red left hand



What is the coefficient of variation of length of each note

- 0.2991
- 0.2905
- 0.2935
- 0.2978
- 0.2969
- 0.3034

- This likely represents neuronal limits of information processing

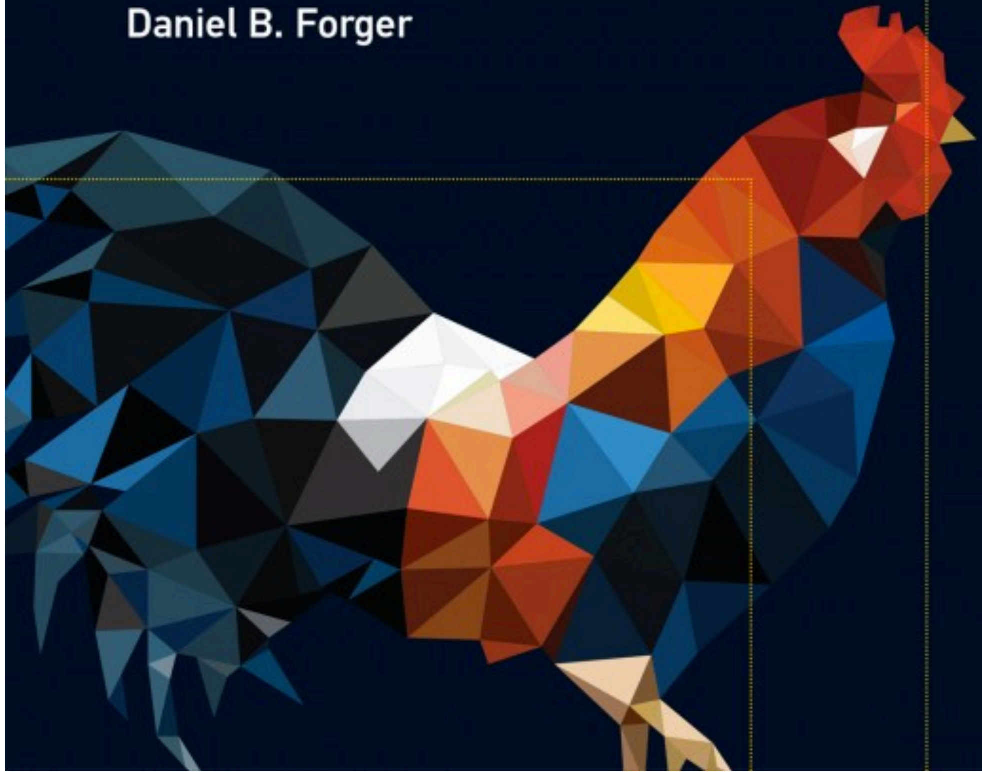
Future Questions

- Are there physiological limits to performance
- Can these be improved by practice?
- How are they different for different individuals?
- How are these limits changes when sight reading (learning a piece)?
- Do they depend on time of day?

Biological Clocks, Rhythms, and Oscillations

The Theory of Biological Timekeeping

Daniel B. Forger



Using Math and Neuroscience to Generate New Music

Daniel Forger
University of Michigan
Ann Arbor

“The more art is controlled, limited worked over, the more it is free.”
Stravinsky Poetics of Music

Bach Trio Sonatas

1. Bach is go to composer for theory
   Never wrote a bad piece
2. Trio Sonatas were composed for pedagogical reasons
3. Fairly strict three voiceparts
4. Organ perfect for digitization

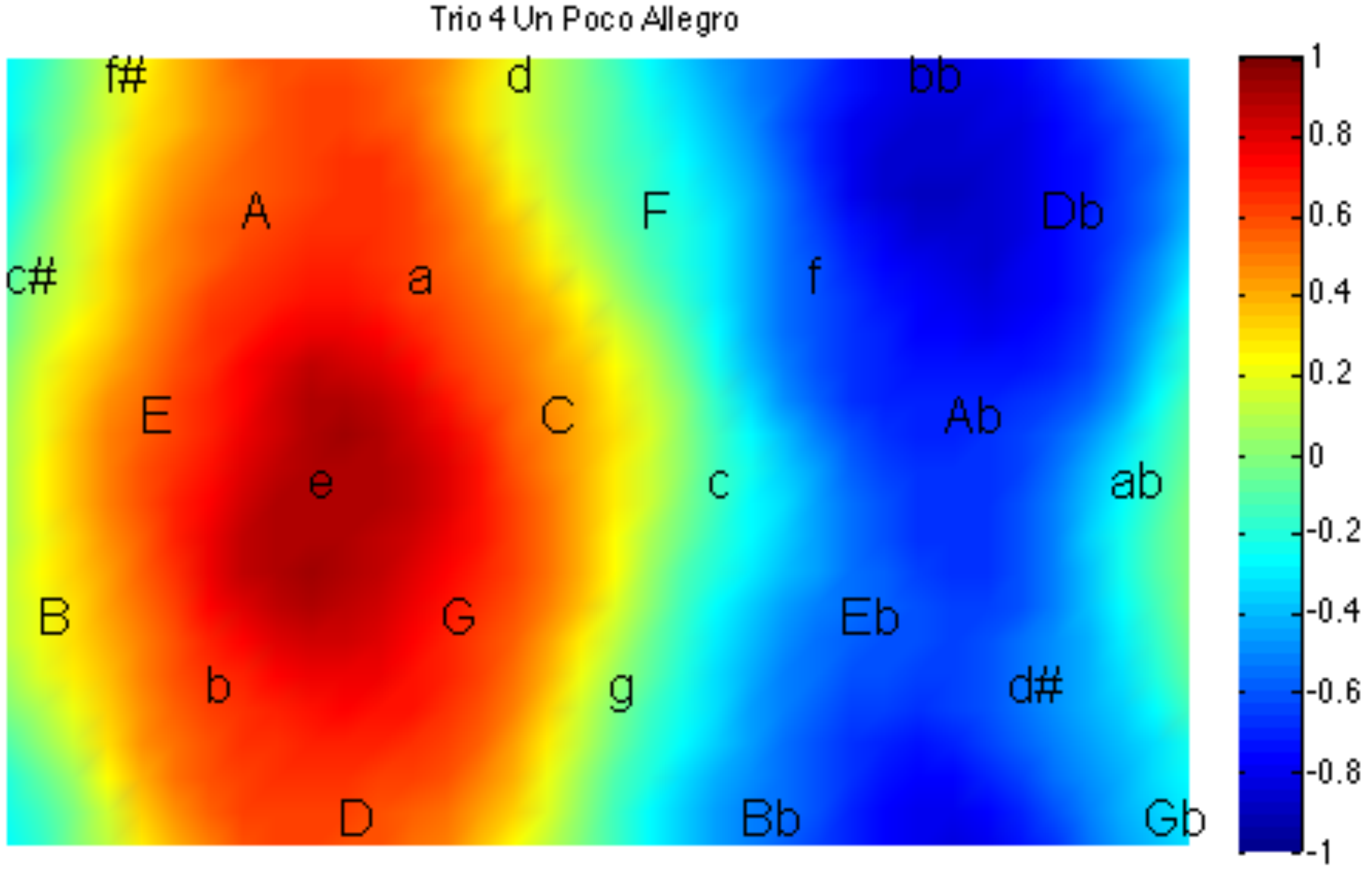
Great Resource: MATLAB Tools for Music Research
Eerola and Toiviainen

key finding algorithms

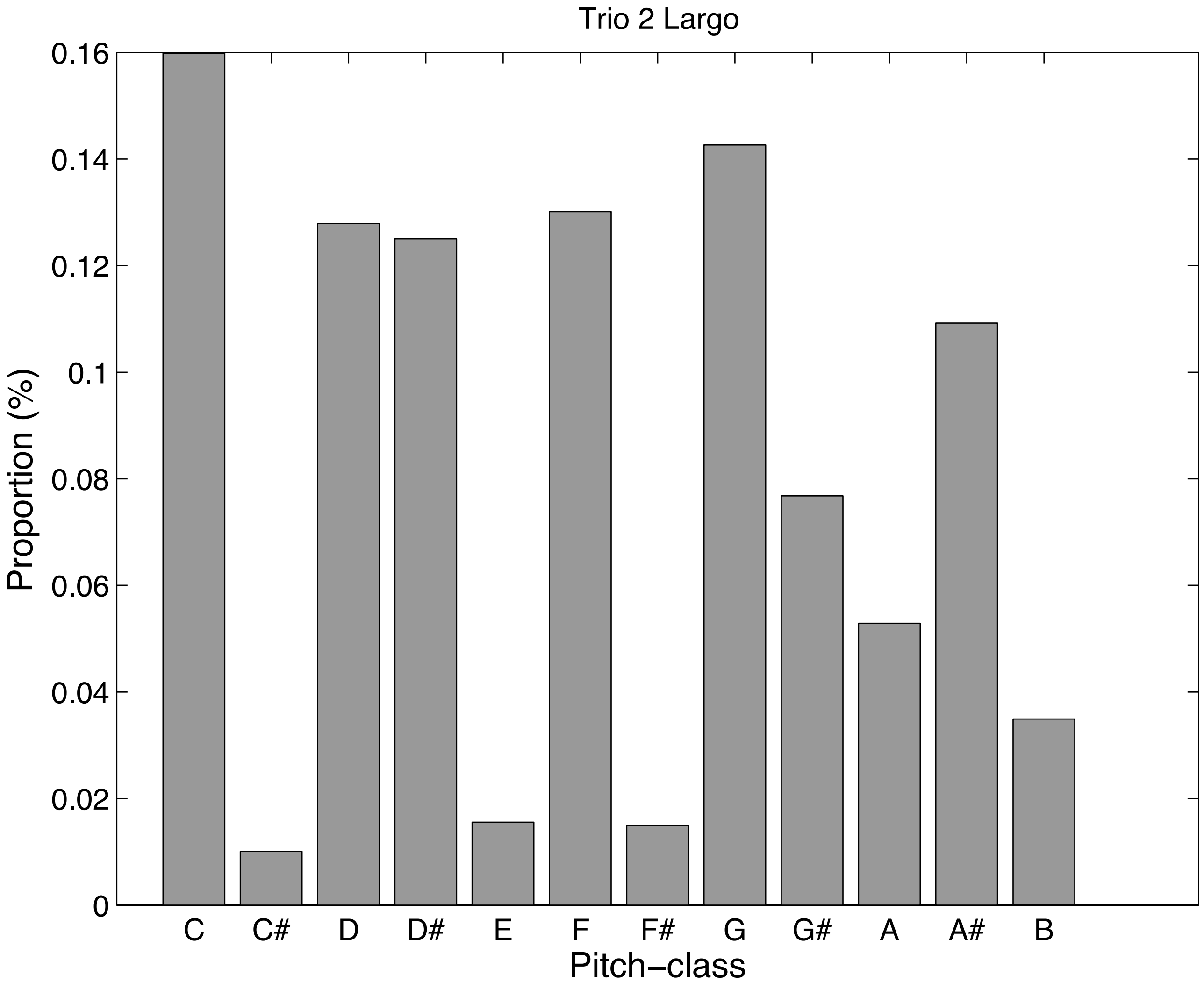
- Based on the work by Krumhansl (1990) we know the chance an individual thinks a note is in a particular key after being trained to that key
- From that, we can get a correlation coefficient for any particular piece of music for any key.
- Choose most likely key.

```
load datforsiam

keysomanim(gfantwo,2,1);
```


Trio 4 Un Poco Allegro
f#
d
bb
A
F
Db
c#
a
f
E
C
Ab
e
c
ab
B
G
Eb
b
g
d#
D
Bb
Gb
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

Probability of finding notes

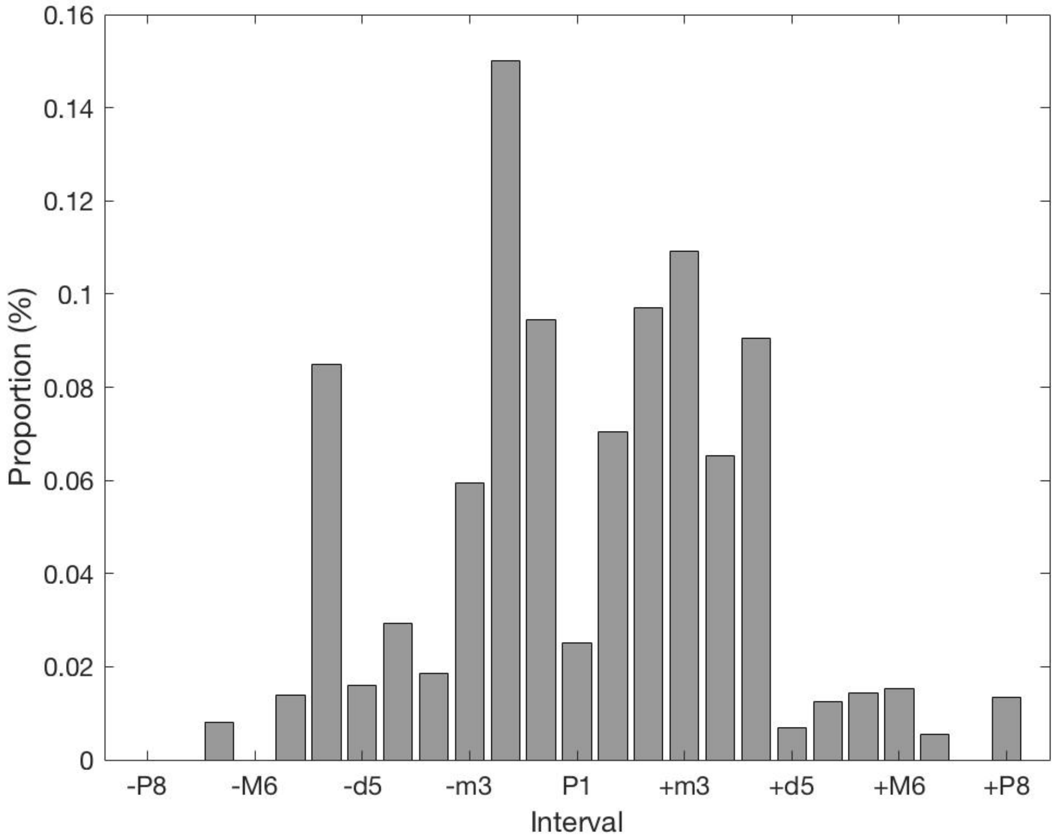


Generate melody based on interval distribution

- Look at the possible intervals
- Choose among them weighted by their probabilities (e.g. as in Gillespie)

Interval distribution from trio sonata

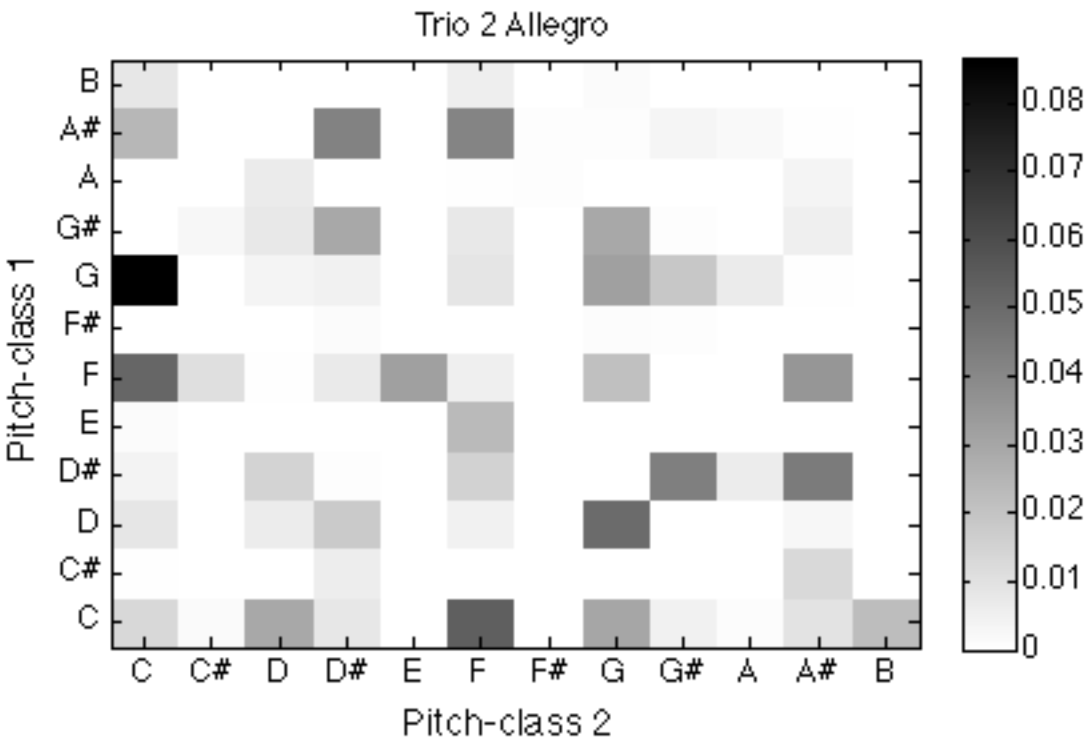
genmusic.m



Generate a Trio Sonata

genmusicc.m

- Start on three notes
- For each note, look to that voicepart to determine the probability of going to any other note.
- For each voicepart, chose a new note
- Combine all voiceparts


Trio 2 Allegro
Pitch-class 1
Pitch-class 2
B
A#
A
G#
G
F#
F
E
D#
D
C#
C
C C# D D# E F F# G G# A A# B
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0

We can then draw many possible chords Chose randomly based on the probability that Bach uses the chord

- genmusicd

What would "Hail to the Victors" sound like if Bach had scored it as part of one of his trio sonatas.

- Start on one of the chords which Bach could use and which contains one of the notes of the melody.
- Find in this database where the chord was used by Bach and chose randomly among the possible subsequent chords one which contains the next note of the melody.
- Repeat until we have chords for notes in the melody.
- Chose one voicepart to have the melody, and the two remaining voiceparts from the available notes.

Hail to the Victors

Need more constraints

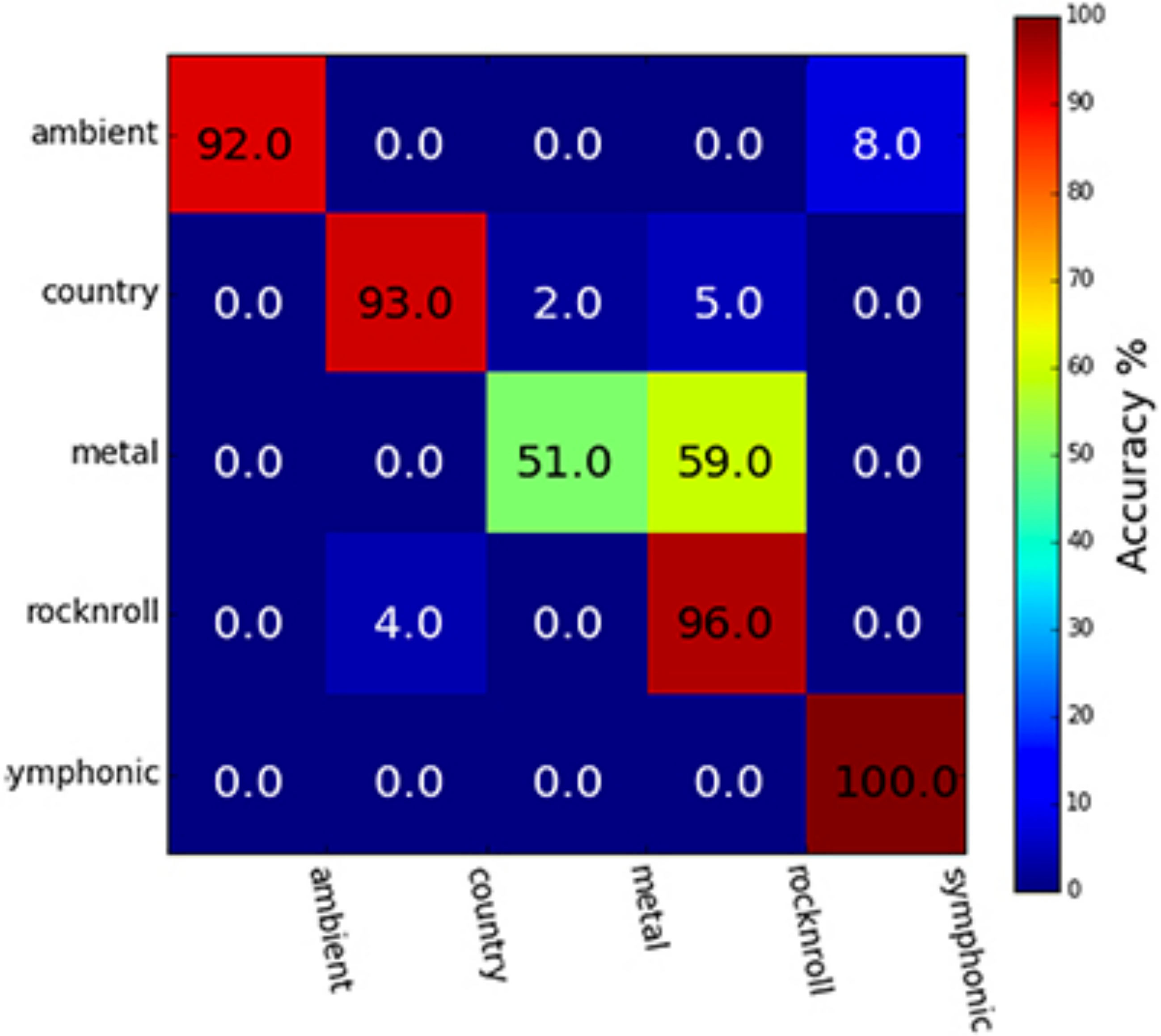
- Perhaps look to neuronal models
- Perhaps models of information processing in the brain/cognition
- Need melody and musical form

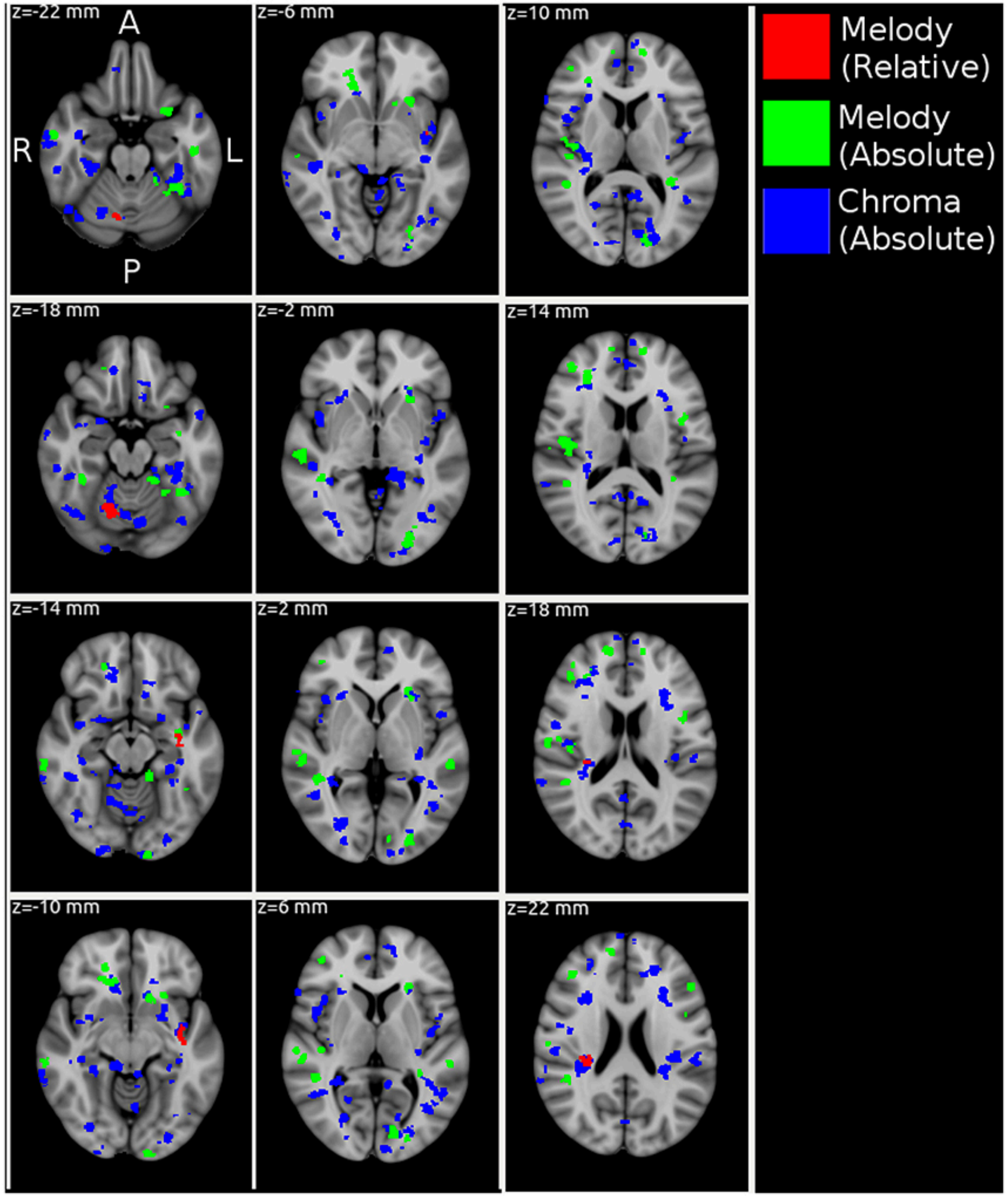
- Far away from automatically generating Bach works

Using fMRI to decode music

- Subjects go into a fMRI machine while listening to music
- From the activation of different brain regions, one can decipher what an individual is listening to.
- This uses machine learning

- Michael Casey “Music of the 7Ts…” Frontiers in Psychology 2017


ambient
country
metal
rocknroll
ymphonic
92.0
0.0
0.0
0.0
8.0
0.0
93.0
2.0
5.0
0.0
0.0
0.0
51.0
59.0
0.0
0.0
4.0
0.0
96.0
0.0
0.0
0.0
0.0
0.0
100.0
ambient
country
metal
rocknroll
symphonic
Accuracy %
100
90
80
70
60
50
40
30
20
10
0


z=-22 mm
A
R
L
P
z=-6 mm
z=10 mm
Melody (Relative)
Melody (Absolute)
Chroma (Absolute)
z=-18 mm
z=-2 mm
z=14 mm
z=-14 mm
z=2 mm
z=18 mm
z=-10 mm
z=6 mm
z=22 mm

The Mathematical Bach

SIAM Conference on the Life Sciences Off–Site Event
A Concert/Discussion on Math, Music and the Brain
Daniel Forger, University of Michigan, Ann Arbor

7 - 8 pm // Monday, August 6th

Chapel of Westminster Presbyterian Church
1200 S Marquette Ave
Across from Orchestra Hall and a short walk down Nicollet Mall or S. Marquette

Program:
- **Great Fantasia in G Minor BWV 542**
- **Trio Sonata in C Minor BWV 526**
- **Trio Sonata in E Minor BWV 528**
- **Trio Sonata in C Major BWV 529**
- **Toccata in F Major BWV 540**

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