# Spatio-Spectral Background Estimation in Remote Sensing Imagery

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#### Motivation: detect weak signals in cluttered backgrounds





#### Hunts Needle in a Haystack

How LONG does it take to find a needle in a haystack? Jim Moran, Washington, D. C., publicity man, recently dropped a needle into a convenient pile of hay, hopped in after it, and began an intensive search for (a) some publicity and (b) the needle. Having found the former, Moran abandoned the needle hunt.



■ To solve **Remote Sensing** problems

• To detect targets, first estimate the target-free background

Implanting

Symmetry

- Can we use Image Processing tools?
  - To reduce noise, need to estimate noise-free pixel values
  - · In-painting, need to estimate missing pixel values

Imagery



#### Inspired by recent developments in image processing

- Patched-based methods: filtering, regression, etc.
- eg, noise reduction using "non-local means"



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Nigilar Olgori Mentiller 20.1308/007-2011.20202 Julie of publication: I December 2012

Milanfar, IEEE Signal Processing Magazine (Jan 2013), 106-128

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The proposed framework is used to arrive at new insights and methods, both practical and theoretical. In particular, sevent powel optimally properties of adjorithms in wide use such as block-matching and three-dimensional (3-D) filtering (BCEO) and methods for their iterative improvement (or nonesistence three of are discussed.



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RX Regression ~ Imagery

## Target and anomaly detection



- Detect weak signals in cluttered backgrounds
- Characterize cluttered backgrounds

« RX Regression Implanting Imagery

## Target and anomaly detection



- Detect weak signals in cluttered backgrounds
- Characterize cluttered backgrounds
- Step 1: estimate the target-free background
  - eg, global Gaussian model: mean and covariance from data
  - or: mixtures, subspaces, endmembers, manifolds....
  - or: local mean, local or global covariance
  - or: local regression
- Step 2: compare estimated to observed
  - disagreement implies anomalousness

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#### RX = Mahalanobis distance



RX



- Step 1: estimate the target-free background
  - average pixels in surrounding annulus
  - Estimate:  $\hat{y} = (x_1 + x_2 + \dots + x_M)/M$

 $\ll$ 

#### RX = Mahalanobis distance



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- Step 1: estimate the target-free background
  - average pixels in surrounding annulus
  - Estimate:  $\hat{y} = (x_1 + x_2 + \dots + x_M)/M$
- Step 2: compare estimated to observed
  - using Mahalanobis distance
  - Difference:  $e = y \hat{y}$
  - Covariance:  $R = \langle ee^{\mathsf{T}} \rangle$
  - Anomalousness:  $A = e^{T} R^{-1} e$



Imagery

# Regression

- Estimate target-free pixel value based on the pixels in the surrounding annulus
- eg, local mean:

$$\hat{y} = \frac{1}{M}(x_1 + \cdots + x_M)$$

• Seek function f, learned from the entire image, such that

$$\hat{y} = f(\mathbf{x}) = f(x_1, \ldots, x_M)$$

provides a good estimate of y





Imagery

Implanting

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Aside: Linear  $f(\mathbf{x})$  corresponds to convolutional filter

RX Regression Imagery Implanting Symmetry
Regression: multiple (multi-/hyper-spectral) bands

When we say:



RX Regression Imagery Implanting Symmetry
Regression: multiple (multi-/hyper-spectral) bands

When we say:

$$\hat{\mathbf{y}} = f(\mathbf{x})$$

What we mean is:



•  $\hat{y} = f(\mathbf{x})$ 

RX Regression Imagery Implanting Symmetry
Regression for target and anomaly detection

$$\hat{\Box} = f( \Box)$$

This study:

- local estimate of central pixel:  $\hat{y} = f(\mathbf{x}) = f(x_1, \dots, x_M)$
- global estimate of covariance:  $R = \langle (y \hat{y})(y \hat{y})^{\mathsf{T}} \rangle$
- Additive target detection  $(y \text{ vs. } y + \epsilon \mathbf{t})$ :
  - local matched filter:  $\mathcal{T}(\mathbf{x}, y) = \mathbf{t}^{\mathsf{T}} R^{-1}(y f(\mathbf{x}))$
- Anomaly detection
  - RX-like:  $\mathcal{A}(\mathbf{x}, y) = (y f(\mathbf{x}))^{\mathsf{T}} R^{-1} (y f(\mathbf{x}))$

RX

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• Aim is to find f such that  $y \approx \hat{y} = f(\mathbf{x})$ 

Regression

■ *k*NN - (weighted) *k*-nearest neighbors (in **x**-space)

Imagery

Given annulus x around the pixel under test:
 Find locations i<sub>1</sub>, i<sub>2</sub>, ..., i<sub>k</sub> such that annuli x<sub>i1</sub>, x<sub>i2</sub>, ..., x<sub>ik</sub> are close to x. That is: ||x - x<sub>i</sub>|| is small.

Implanting

- Assign weights  $w_1, \ldots, w_k$  so closer points have more weight: eg,  $w_i \sim \exp(\|\mathbf{x} - \mathbf{x}_i\|^2 / \sigma^2)$
- Average the associated y values:  $\hat{y} = f(\mathbf{x}) = \sum_{j=1}^{k} w_j y_{i_j}$



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Implanting

Symmetry

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- Average the associated y values:  $\hat{y} = f(\mathbf{x}) = \sum_{j=1}^{k} w_j y_{i_j}$
- Global least-squares linear fit

Regression

- Let  $f(\mathbf{x}) = f(x_1, \dots, x_M) = \alpha_1 x_1 + \dots \alpha_M x_M$
- Fit  $\alpha$ 's globally over the whole image: minimize  $\sum_{i} [y_i - f(\mathbf{x}_i)]^2$

RX

Imagery

Implanting

Symmetry

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#### Images









#### WorldView-2



- 2 Photographs
- 2 Remote sensing
- Bands: 3,3,8,126



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Two strategies

- **Band-by-band:** for each band *b*, estimate *b*th component of *y* as function of *b*th components of annulus pixels *x*<sub>1</sub>,...,*x*<sub>M</sub>.
- Bands-together: treat y and x<sub>1</sub>,..., x<sub>M</sub> as vectors; this mostly affects choice of nearest neighbors
- For local mean, two strategies are equivalent
- Results: kNN outperforms local mean

SNR(dB)	Local	kNN	kNN
	Mean	band-by-band	bands together
Parrots	20.13	26.48	25.60
Boat	12.05	18.10	17.62
WV-2 (bands 1,2,3)	12.94	17.24	16.06
HyMap (bands 1,2,3)	16.04	19.66	19.49

Symmetry

 $\ll RX \qquad Regression \qquad Imagery \qquad Implanting \qquad Symmetry \qquad \gg$ Higher SNR  $\implies$  Improved target detection performance?



N. Hasson, S. Asulin, S. R. Rotman, and D. Blumberg, "Evaluating backgrounds for subpixel target detection: when closer isn't better." *Proc. SPIE* **9472**, p. 94720R, 2015.

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#### Symmetry

## Implanting targets: misplaced pixel anomalies



## Implanting targets: misplaced pixel anomalies



■ Select some pixel locations at random (red squares)

Symmetry

#### Implanting targets: misplaced pixel anomalies



- Select some pixel locations at random (red squares)
- Select some more locations at random (yellow squares)

#### Implanting targets: misplaced pixel anomalies



- Select some pixel locations at random (red squares)
- Select some more locations at random (yellow squares)
- Copy pixel values at red locations to yellow locations

Symmetry

 $\gg$ 

#### Anomaly detection with implanted targets

- Choose a small number of locations at random
- Replace pixels with anomalous targets
  - eg,  $\mathbf{y} \leftarrow (1 \epsilon)\mathbf{y} + \epsilon \mathbf{t}$
  - with  $\mathbf{t} \in \mathcal{U}$
- Use regression to learn f
- Apply f to compute background estimates:  $\hat{\mathbf{y}}_i = f(\mathbf{x}_i)$
- **•** Make residual map:  $\mathbf{e} = \mathbf{y} \hat{\mathbf{y}}$
- Compute covariance of residuals:  $\Phi = \langle ee^T \rangle$
- Use RX-like anomaly detection:  $\mathcal{A}(\mathbf{e}) = \mathbf{e}^T \Phi^{-1} \mathbf{e}$
- $\blacksquare$  Implanted targets should have high values of  $\mathcal A$

RX Regression Imagery Implanting
Implanted targets (HyMap - Cooke City)
Mean



Anomalousness

744 false alarms

Symmetry



Anomalousness

233 false alarms





Anomalousness

167 false alarms

# RX Regression Imagery Implanting Implanted targets (HyMap - Cooke City) Direct band-by-band





Symmetry

# RX Regression Imagery Implanting Implanted targets (HyMap - Cooke City) PCA band-by-band





Symmetry

RX Regression Imagery

Implanting

Symmetry

 $\gg$ 

## Aside: symmetrized regression

 $\hat{y} = f(\mathbf{x})$ 

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RX Regression Imagery

Implanting

Symmetry

 $\gg$ 

#### Aside: symmetrized regression

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Introduce symmetry-preserving features:

 $\Phi_p(x_1, x_2, \ldots, x_k)$ 

Symmetry

 $\gg$ 

#### Aside: symmetrized regression



Introduce symmetry-preserving features:

 $\Phi_p(x_1, x_2, \ldots, x_k)$ 

Make regression a function of features:

$$\hat{y} = f(\Phi_1, \ldots, \Phi_P)$$

« RX Regression Imagery Implanting S

Symmetry

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#### Aside: symmetrized regression



Introduce symmetry-preserving features:

 $\Phi_p(x_1, x_2, \ldots, x_k)$ 

Make regression a function of features:

$$\hat{y} = f(\Phi_1, \ldots, \Phi_P)$$

■ Features preserve symmetry, so choice of function is arbitrary:

- (weighted) k-nearest neighbors
- Linear regression
- Support vector regression, deep neural network, etc, etc.

RX Regression Imagery

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# Dihedral group $(D_4)$ symmetry

$$D_4 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$$



- Group has two generators: s, r
- s: reflection sa = b, sb = a, sc = h, sd = g, etc.
- r: rotation ra = c, rb = d, rc = e, rd = f, etc.
- Composition rules:  $s^2 = 1$ ,  $r^4 = 1$ ,  $rs = sr^3$

e.g., 
$$r^2 \times sr^2 = r(rs)r^2 = r(sr^3)r^2 = (rs)r^5 = (rs)r = (sr^3)r = sr^4 = s$$
.

« ~·		Regression
Sigma	( <i>D</i> 4と) †	eatures

	a	b	
h			с
g			d
	f	e	

#### $\Phi_{\Sigma\Sigma\Sigma} = a + b + e + f + c + d + g + h$

Implanting

Symmetry

 $\gg$ 

• Features invariant to group operations:  $g\Phi = \Phi$  for  $g \in D_4$ 

Imagery

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## Sigma-Delta $(D_4 \Sigma \Delta)$ features



$$\begin{split} \Phi_{\Sigma\Sigma\Sigma} &= a + b + e + f + c + d + g + h \\ \Phi_{\Sigma\Sigma\Delta} &= \left| (a + b + e + f) - (c + d + g + h) \right| \\ \Phi_{\Sigma\Delta\Sigma} &= \left| (a + b) - (e + f) \right| + \left| (c + d) - (g + h) \right| \\ \Phi_{\Sigma\Delta\Delta} &= \left| |(a + b) - (e + f)| - |(c + d) - (g + h)| \right| \\ \Phi_{\Delta\Sigma\Sigma} &= |a - b| + |e - f| + |c - d| + |g - h| \\ \Phi_{\Delta\Sigma\Delta} &= \left| (|a - b| + |e - f|) - (|c - d| + |g - h|) \right| \\ \Phi_{\Delta\Delta\Sigma} &= \left| |a - b| - |e - f| \right| + \left| |c - d| - |g - h| \right| \\ \Phi_{\Delta\Delta\Delta} &= \left| ||a - b| - |e - f| \right| - \left| |c - d| - |g - h| \right| \end{split}$$

• Features invariant to group operations:  $g\Phi = \Phi$  for  $g \in D_4$   $e.g., r\Phi_{\Sigma\Sigma\Delta} = \left| (ra + rb + re + rf) - (rc + rd + rg + rh) \right|$  $= \left| (c + d + g + h) - (a + b + e + f) \right| = \Phi_{\Sigma\Sigma\Delta}$ 

Imagery

Implanting

Symmetry

 $\gg$ 

## Various symmetries



**Diamond Rings** 





Square Rings



Dihedral D<sub>4</sub>

Mean

Using various symmetries

Regression

RX

 $\ll$ 

 Symmetry provides comparable performance with fewer features

Imagery



Implanting

Symmetry

Using various symmetries

Regression

RX

 $\ll$ 

 Symmetry provides comparable performance with fewer features

Imagery



Implanting

Symmetry

Using various symmetries

Regression

RX

 $\ll$ 

 Symmetry provides comparable performance with fewer features

Imagery



Implanting

Symmetry



## Conclusions

Background non-stationarity is a great problem



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Background non-stationarity is a great problem opportunity

- Better background estimation  $\rightarrow$  better target detection



#### $\gg$

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- Local mean is not the only way to estimate central pixel



- Background non-stationarity is a great problem opportunity
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- Although motivated by modern image processing, we saw improvement just using ordinary regression
  - kNN, simple linear fit: both are better than local mean
  - and better still for larger annulus size



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  - Better background estimation  $\rightarrow$  better target detection
- Local mean is not the only way to estimate central pixel: we can do better!



- Although motivated by modern image processing, we saw improvement just using ordinary regression
  - kNN, simple linear fit: both are better than local mean
  - and better still for larger annulus size
- What we did was simple you can do better!



- Apply "real" machine learning tools
- Look at band-by-band variants
  - Consider transformations besides PCA
- Segment image; learn separate  $f(\mathbf{x})$  for each segment

Imagery

• Use k-means-like approach to learn  $f(\mathbf{x})$  and segments together

Implanting

Symmetry

- Use a separate covariance matrix R for each segment
- Identify appropriate regularization of R
- Extend to known (and variable) target detection scenarios
- Optimize direct measures of performance (vs least squares)
  - Remember: closer isn't always better