Queuing Models for Human Task Management

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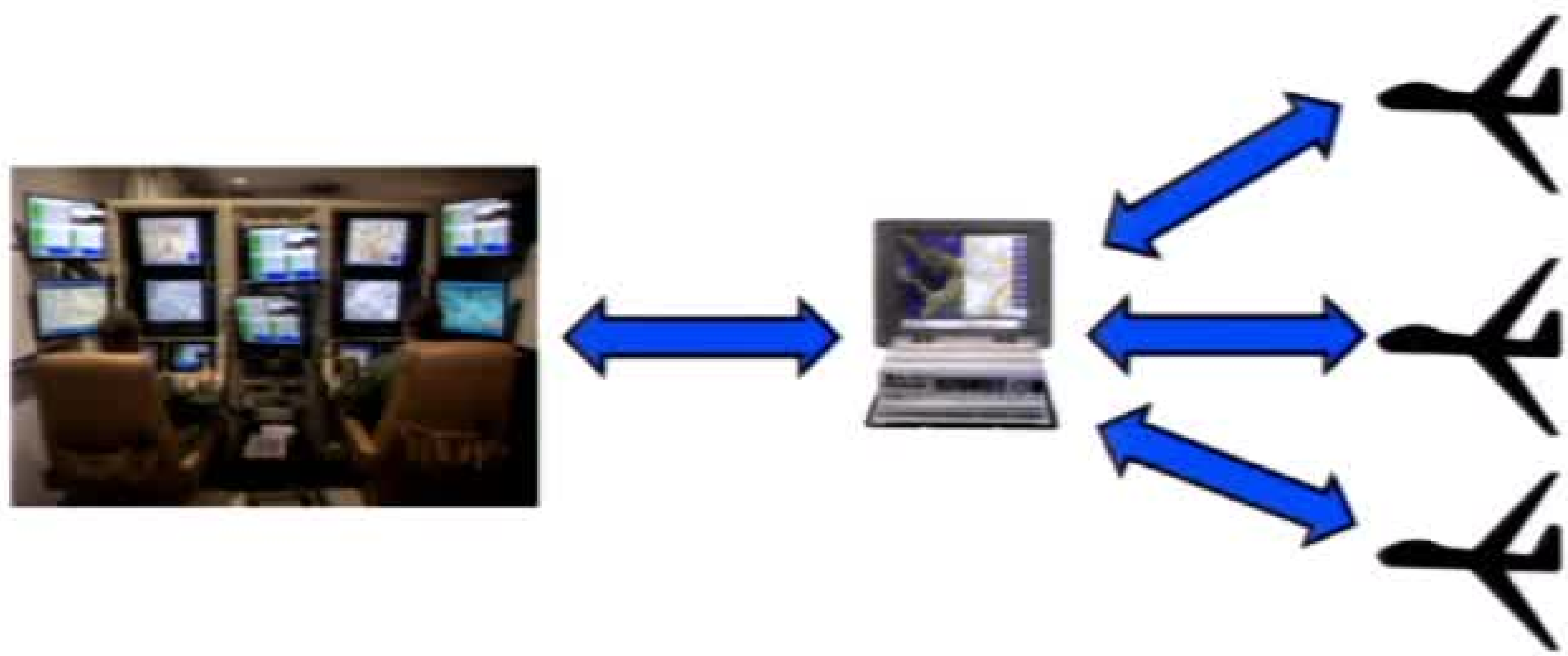
Motivation: Optimizing human-machine collaboration

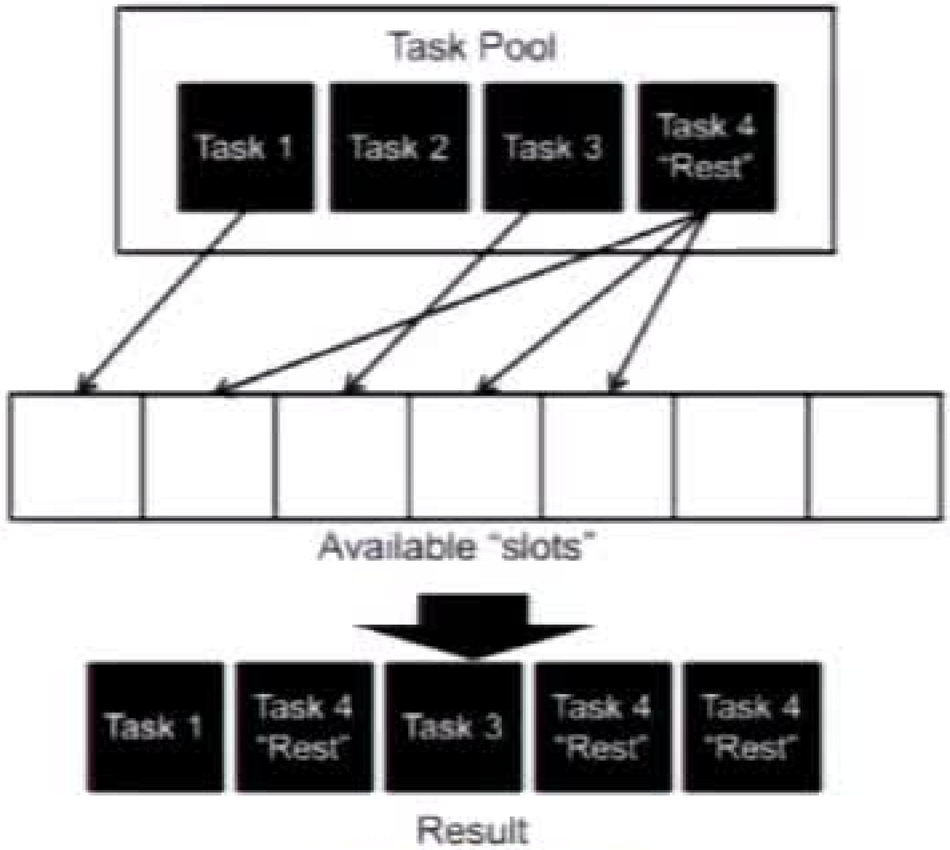
- Even the most autonomous vehicles require some human operators to make decisions, e.g. managing UAVs for surveillance
- Tasks come in regularly and are similar in nature, e.g. image processing for UAV management



Want to schedule tasks to accumulate the maximum reward

Past approaches: Mixed Integer Linear Programming

- *Peters J, Bertucelli L (2016), "Robust scheduling strategies for collaborative human-UAV missions"*
- Assumptions:
  - Fixed time horizon, need to know number of tasks
  - Know processing time of each task, and reward
- Advantages: Robust and adaptive
- Drawbacks
  - Restrictive assumptions: doesn't allow streaming tasks
  - Human aspects treated only through task processing time



Yerkes-Dodson Law

Increase in stress applied to a worker will increase the processing rate up to a point, where too much stress decreases performance

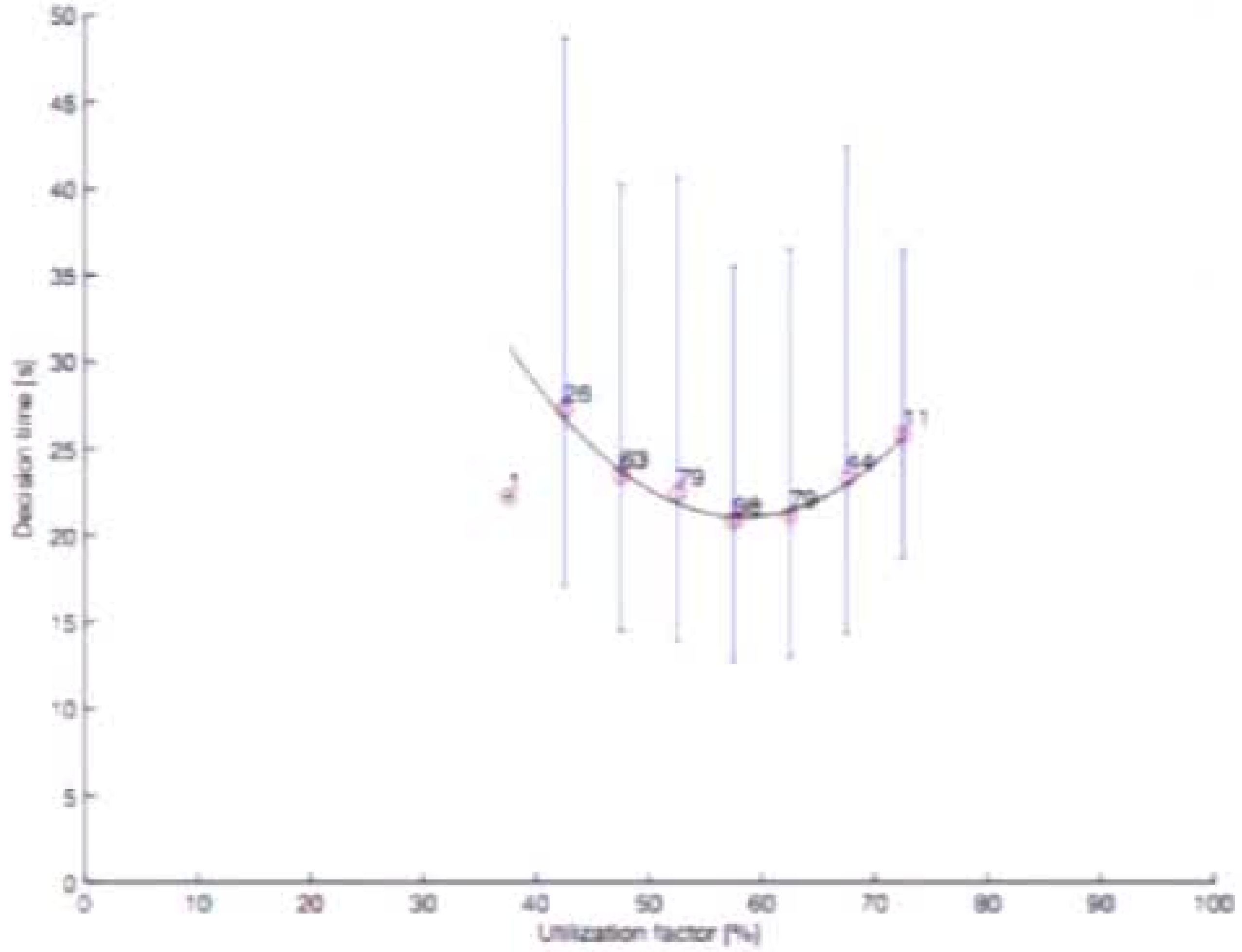


Figure: Image processing time against utilization ratio

Questions

- What do we want from our model?
- What is the fraction of time in which there are more than a certain number of tasks in the queue in the long run? (Asymptotic behavior)
- What is the fraction of time in which server is busy? (Asymptotic behavior)
- What is the probability of loitering for more than some time period (short term or transient behavior).
- **In this talk, we focus on asymptotics.**

Limiting distributions

Are the following well defined?

$$p_X(x) = \lim_{t \to \infty} p_X(t, x)$$

$$p_k(x) = \lim_{t \to \infty} p_k(t, x)$$

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Theorem

A continuous time Markov chain is **ergodic** (has a stationary distribution) if and only if it is **positive recurrent** (every state is visited infinitely often) and **irreducible** (possible to get to any state from any other state)

Answer

Yes if and only if $\lambda < \mu(1)$. Intuition: the maximum sustainable rate at which one can work is the rate when one is most stressed, so the maximum rate tasks can arrive should match that rate.

Action: Make sure $\lambda < \mu(1)$.

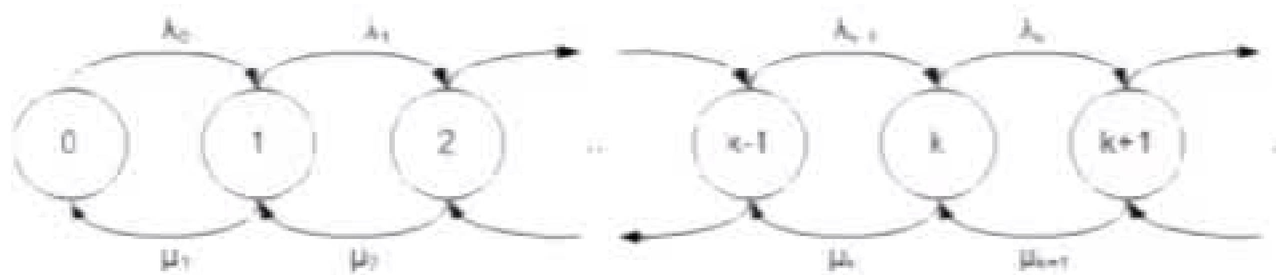
Equivalence to queues with length dependent service times

Question: is there a relation between this queue and queues where the service rate is only dependent on the number of tasks to be processed?

Theorem

There exists a birth death process with arrival rate λ and a sequence of service rates $\{\mu_k\}_{k=1}^{\infty}$ whose equilibrium distribution is the same as the $M/M_C/1$ queue.

Takeaway: in equilibrium, how fast an operator can work is related to how much work one has



Proof sketch: Use mean value theorem, to show there is a x_{k+1} such that

$$\lambda P_k = \mu(x_{k+1}) \int_0^1 p_{k+1}(x)dx = \mu_{k+1} P_{k+1}$$