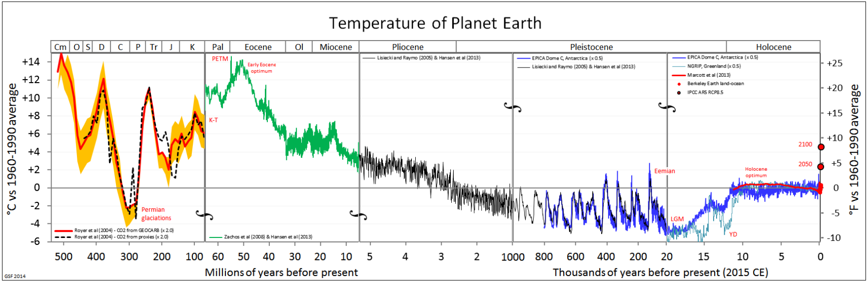
Effects of a Rogue Star on Earth's Climate

Harini Chandramouli
with Richard McGehee
University of Minnesota

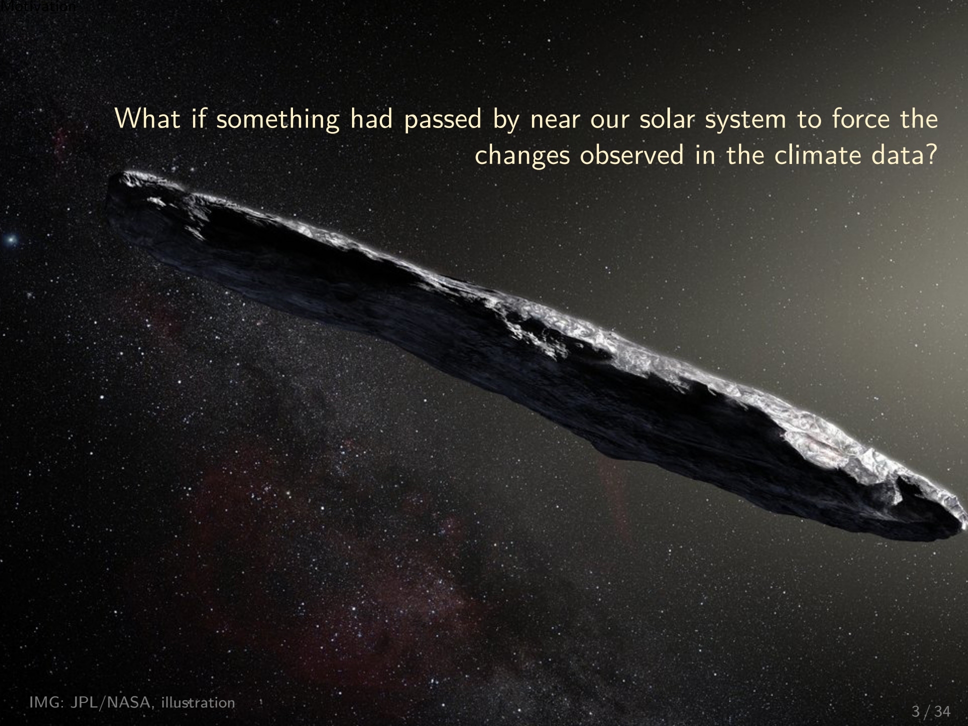
SIAM Conference on Applications of Dynamical Systems
May 22, 2019

Climate Record



Global average temperature estimates for the last 540 My. This shows estimates of global average surface air temperature over the 540 My of the Phanerozoic Eon, since the first major proliferation of complex life forms on our planet. Because many proxy temperature reconstructions indicate local, not global, temperature – or ocean, not air, temperature – substantial approximation may be involved in deriving these global temperature estimates. As a result, the relativities of some of the plotted estimates are approximate, particularly the early ones. Credit: Glen Fergus.

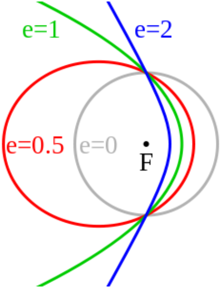
What if something had passed by near our solar system to force the changes observed in the climate data?



IMG: JPL/NASA, illustration

Milankovitch Cycles

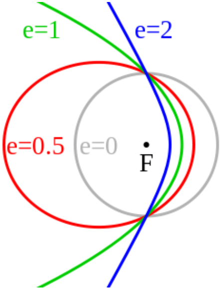
eccentricity



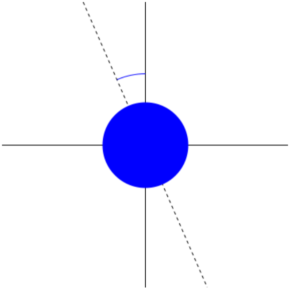
eccentricity img: `https://kids.kiddle.co/Orbital_eccentricity`

Milankovitch Cycles

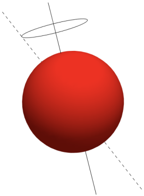
eccentricity



obliquity



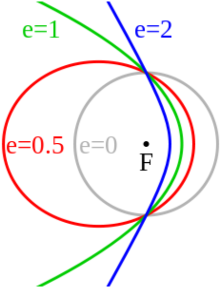
precession



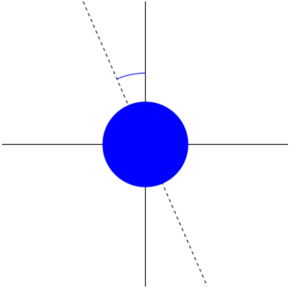
eccentricity img: `https://kids.kiddle.co/Orbital_eccentricity`

Milankovitch Cycles

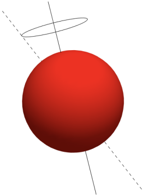
eccentricity



obliquity



precession

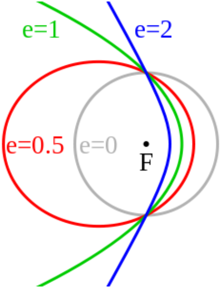


Milankovitch cycles affect our climate because the amount of insolation varies according to the cycles of these three elements.

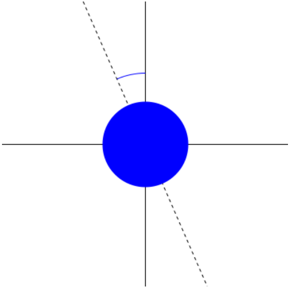
eccentricity img: `https://kids.kiddle.co/Orbital_eccentricity`

Milankovitch Cycles

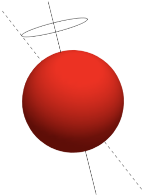
eccentricity



obliquity



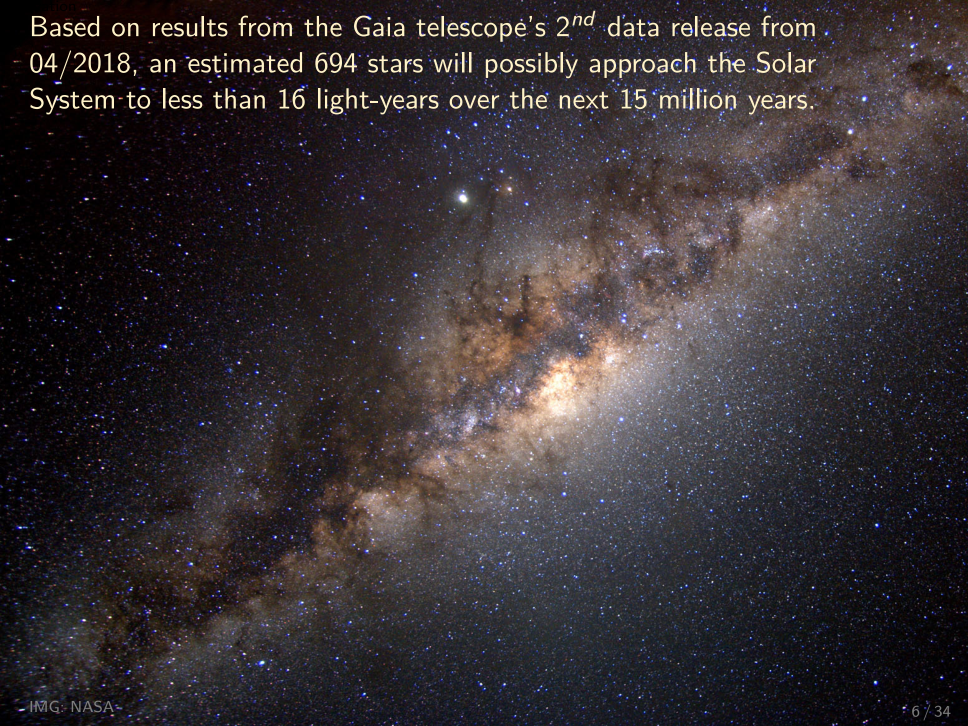
precession



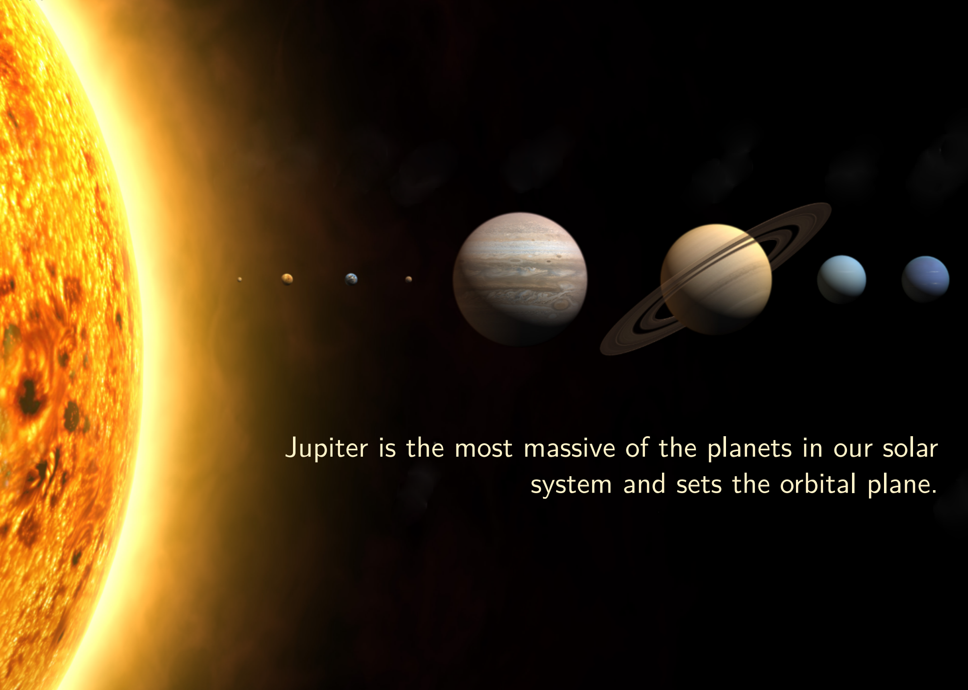
Milankovitch cycles affect our climate because the amount of insolation varies according to the cycles of these three elements.

eccentricity img: `https://kids.kiddle.co/Orbital_eccentricity`

Based on results from the Gaia telescope's 2^{nd} data release from 04/2018, an estimated 694 stars will possibly approach the Solar System to less than 16 light-years over the next 15 million years.



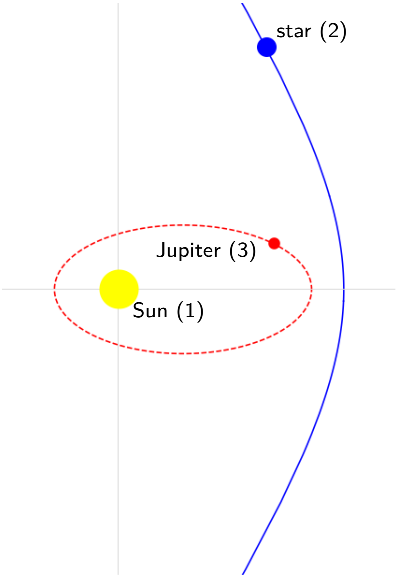
IMG: NASA



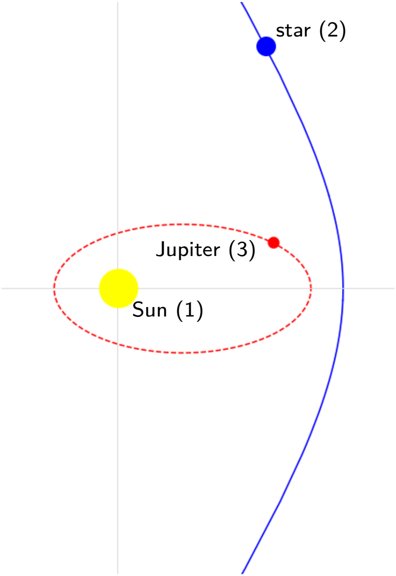
Jupiter is the most massive of the planets in our solar system and sets the orbital plane.

IMG: NASA

Hyperbolic Restricted 3-Body Problem

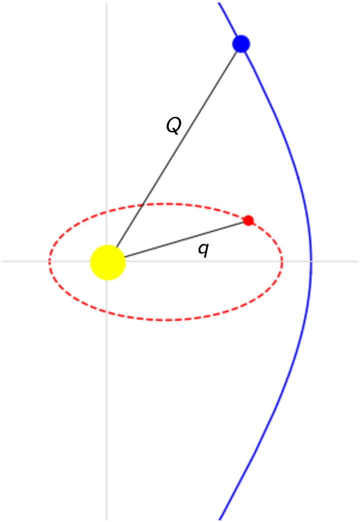


Hyperbolic Restricted 3-Body Problem



$$
\begin{cases}
\ddot{x}_1 &= -Gm_2 \dfrac{x_1 - x_2}{|x_1 - x_2|^3} \\[2em]
\ddot{x}_2 &= -Gm_1 \dfrac{x_2 - x_1}{|x_2 - x_1|^3} \\[2em]
\ddot{x}_3 &= -Gm_1 \dfrac{x_3 - x_1}{|x_3 - x_1|^3} - Gm_2 \dfrac{x_3 - x_2}{|x_3 - x_2|^3}
\end{cases}
$$

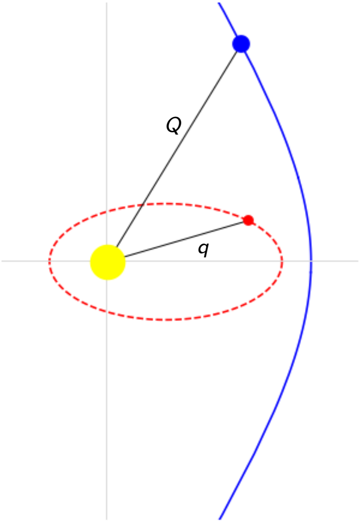
Hyperbolic Restricted 3-Body Problem



Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

$$\begin{cases} \ddot{Q} &= -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} &= -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

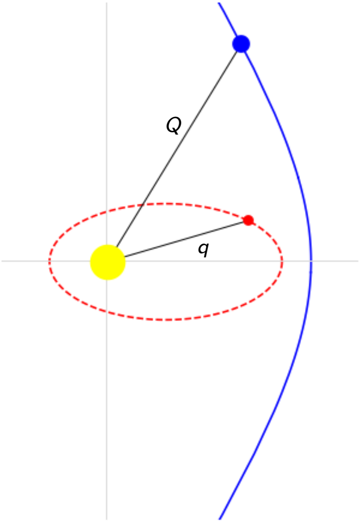
Hyperbolic Restricted 3-Body Problem



Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

$$\begin{cases} \ddot{Q} & = -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} & = -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

Hyperbolic Restricted 3-Body Problem

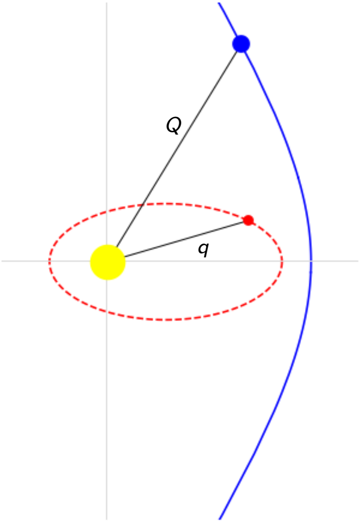


Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

$$\begin{cases} \ddot{Q} & = -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} & = -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

This is the Kepler problem!

Hyperbolic Restricted 3-Body Problem

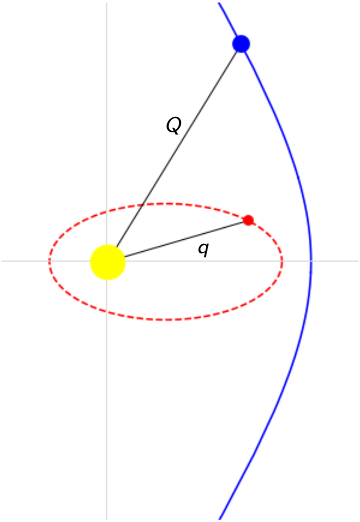


Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

$$\begin{cases} \ddot{Q} &= -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} &= -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

This is the Kepler problem!

Hyperbolic Restricted 3-Body Problem



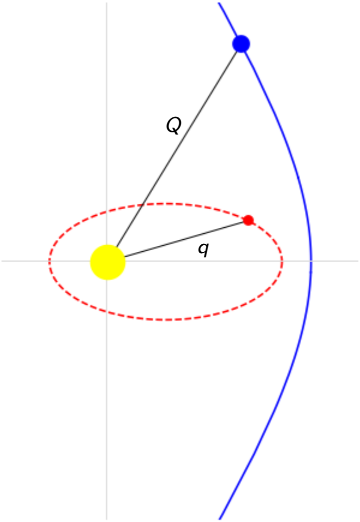
Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

$$\begin{cases} \ddot{Q} &= -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} &= -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

This is the Kepler problem!

Perturbation from the Kepler problem.

Hyperbolic Restricted 3-Body Problem



Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

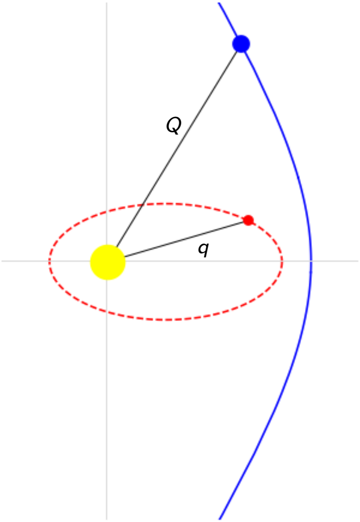
$$\begin{cases} \ddot{Q} &= -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} &= -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

This is the Kepler problem!

Perturbation from the Kepler problem.

- Q describes the relative motion of our two stars (hyperbolic)

Hyperbolic Restricted 3-Body Problem



Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

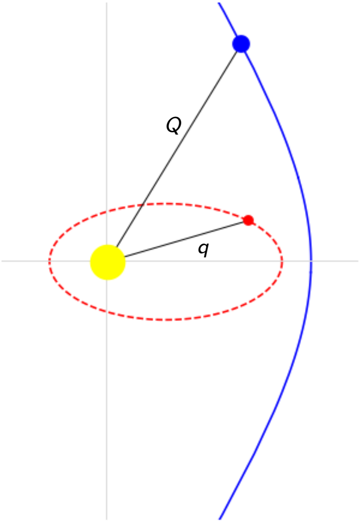
$$\begin{cases} \ddot{Q} &= -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} &= -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

This is the Kepler problem!

Perturbation from the Kepler problem.

- Q describes the relative motion of our two stars (hyperbolic)
- We can plug Q into the second equation, which describes the motion of Jupiter

Hyperbolic Restricted 3-Body Problem



Let $Q = x_2 - x_1$ and $q = x_3 - x_1$, then...

$$\begin{cases} \ddot{Q} & = -\dfrac{(m_1 + m_2)\, Q}{|Q|^3} \\ \ddot{q} & = -\dfrac{m_1\, q}{|q|^3} - \dfrac{m_2\,(q - Q)}{|q - Q|^3} - \dfrac{m_2\, Q}{|Q|^3} \end{cases}$$

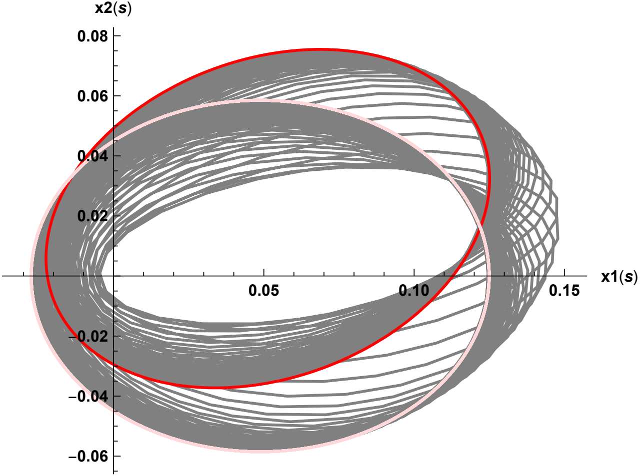
This is the Kepler problem!

Perturbation from the Kepler problem.

- Q describes the relative motion of our two stars (hyperbolic)
- We can plug Q into the second equation, which describes the motion of Jupiter
- Use Levi-Civita regularization to deal with collisions

Observed Motion

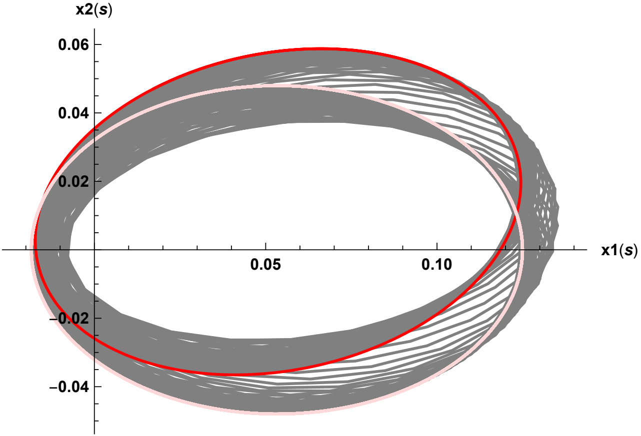
$m_1 = 0.5$ (Sun), $m_2 = 0.5$ (passing star)



The light red path is the original orbit the planet was on before the star passed by, and the right path is the final orbit.

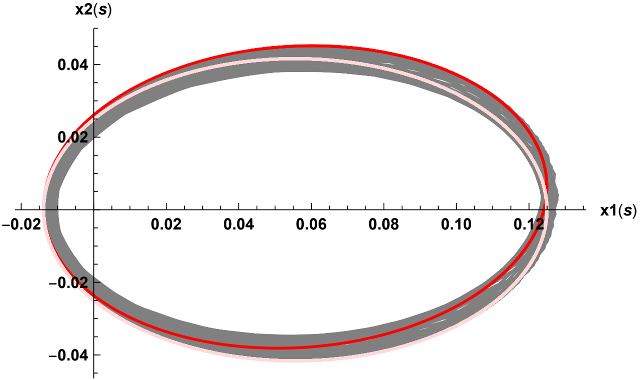
Observed Motion

$m_1 = 0.7, m_2 = 0.3$



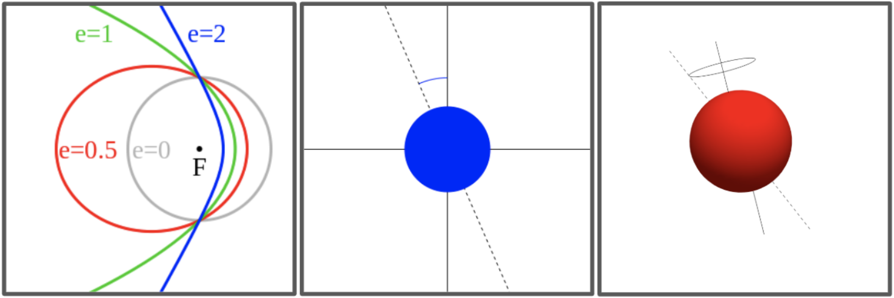
Observed Motion

$m_1 = 0.9, m_2 = 0.1$



Orbital Elements

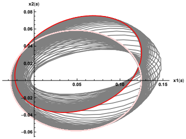
We want to observe how the orbital elements are changed from these perturbations in motion.

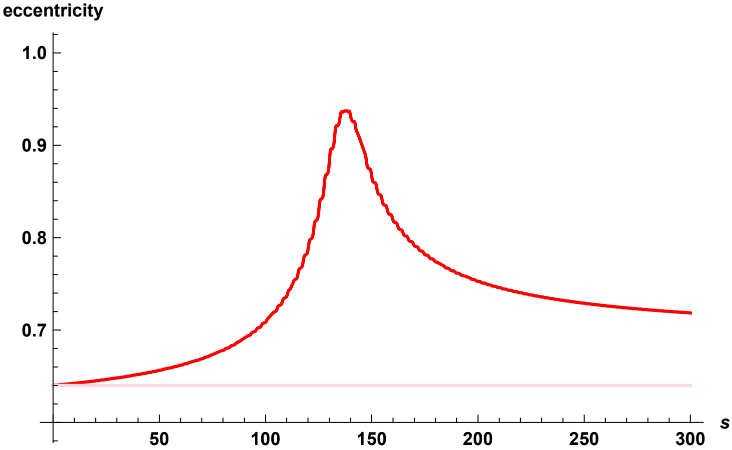


Orbital elements often studied in climate mathematics: eccentricity (left), obliquity or axial tilt (middle), and precession (right). Once again, the eccentricity image is from `https://kids.kiddle.co/Orbital_eccentricity`.

Eccentricity

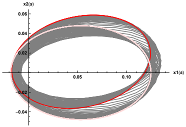
$m_1 = 0.5,\ m_2 = 0.5$

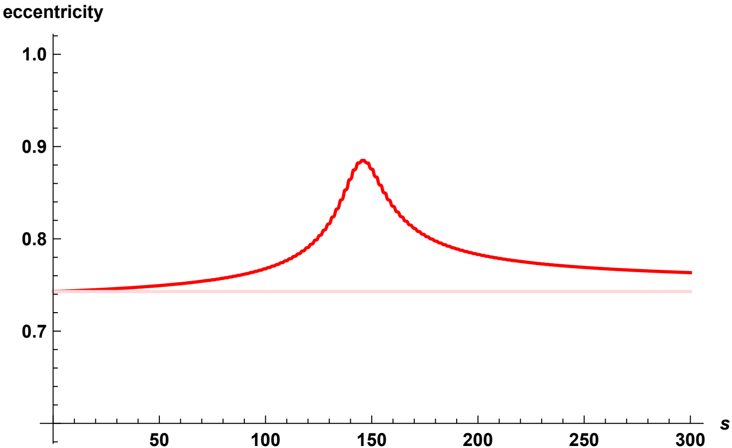




Eccentricity

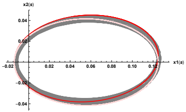
$m_1 = 0.7,\ m_2 = 0.3$

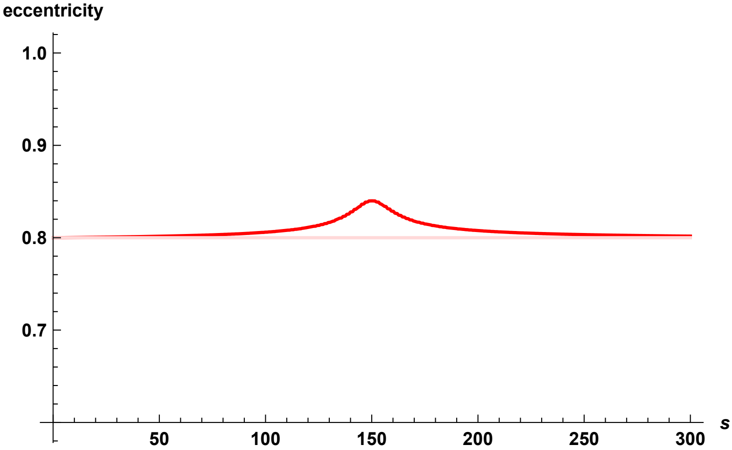




Eccentricity

$m_1 = 0.9, \ m_2 = 0.1$





Eccentricity (e) and Semi-Major Axis (a)

$$\text{mean annual solar intensity} = \frac{\overbrace{K}^{\text{solar output}} \quad a^2}{\sqrt{1-e^2}}$$

- For Earth, a is fairly constant and assumed so in models
- If we change e, there will be a change in a as well

Eccentricity (e) and Semi-Major Axis (a)

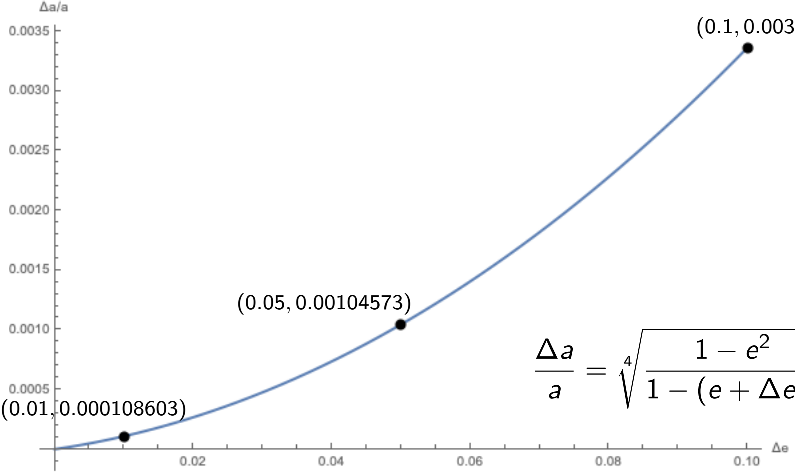
$$\text{mean annual solar intensity} = \frac{\overbrace{K}^{\text{solar output}} \qquad a^2}{\sqrt{1-e^2}}$$

- For Earth, a is fairly constant and assumed so in models
- If we change e, there will be a change in a as well

Assuming the mean annual solar intensity remains constant,

$$\Delta a = a\left(\sqrt[4]{\frac{1-e^2}{1-(e+\Delta e)^2}} - 1\right)$$

Eccentricity (e) and Semi-Major Axis (a)

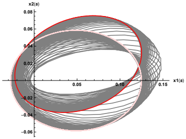


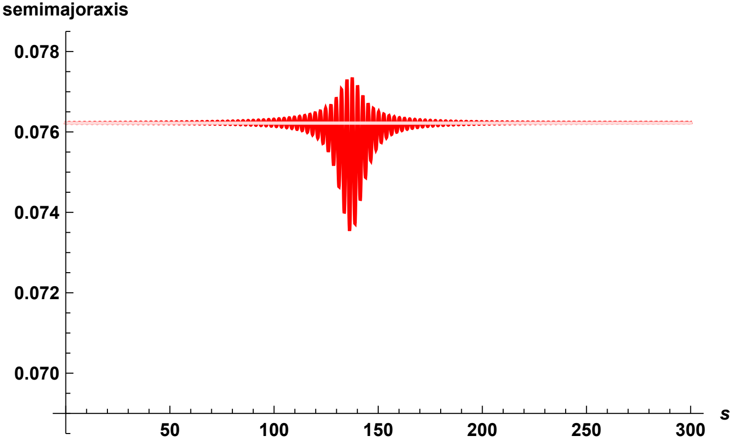
$$\frac{\Delta a}{a} = \sqrt[4]{\frac{1-e^2}{1-(e+\Delta e)^2}} - 1$$

Fix $e = 0.0167086$ and $a = 149.60 \times 10^6$ km, the eccentricity and semi-major axis length of Earth's current orbit. $\frac{\Delta a}{a}$ represents the percent change from Earth's semi-major axis length.

Semi-Major Axis

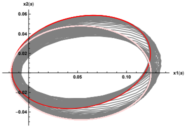
$m_1 = 0.5,\ m_2 = 0.5$

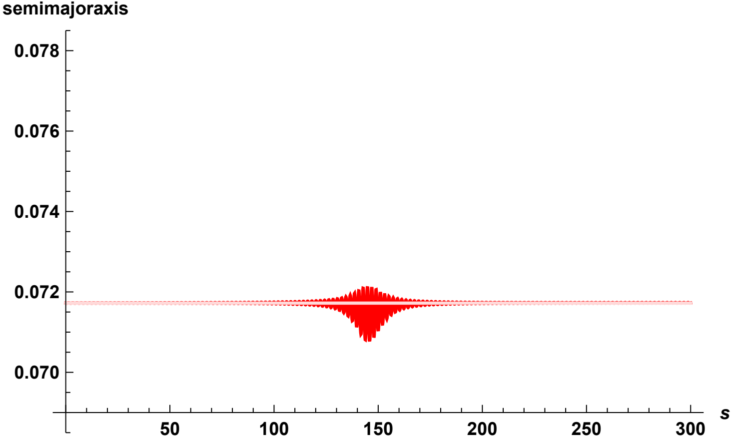




Semi-Major Axis

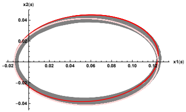
$m_1 = 0.7,\ m_2 = 0.3$

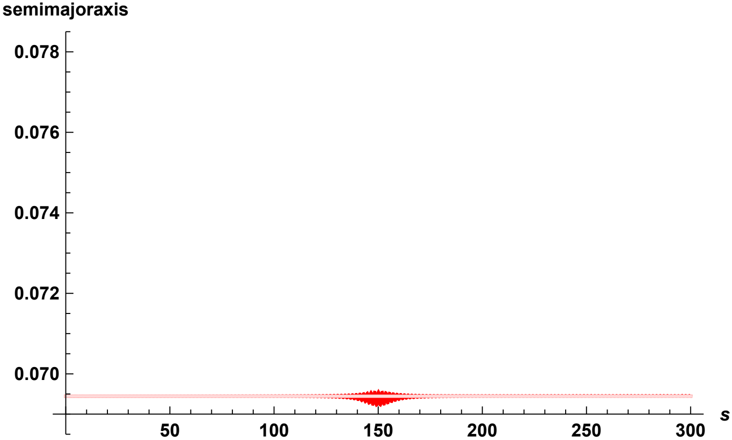




Semi-Major Axis

$m_1 = 0.9, \; m_2 = 0.1$

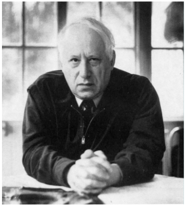




IMG: NASA

How do these changes affect temperature?

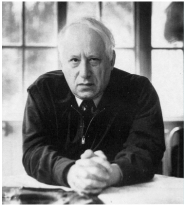
The Budyko-Widiasih Model



The Budyko-Widiasih Model

$$R\frac{\partial T}{\partial t} = \underbrace{Qs(y)\left(1 - \alpha(\eta, y)\right)}_{\text{incoming radiation}} - \underbrace{(A + BT(t, y))}_{\text{OLR}} - \underbrace{C\left(T(t, y) - \overline{T}(t)\right)}_{\text{heat transport}}$$

$$\frac{d\eta}{dt} = \varepsilon(T(\eta) - T_c)$$



The Budyko-Widiasih Model

$$
\begin{aligned}
R\frac{\partial T}{\partial t} &= Qs(y)\left(1-\alpha(\eta,y)\right)-\left(A+BT(t,y)\right)-C\left(T(t,y)-\overline{T}(t)\right)\\
\frac{d\eta}{dt} &= \varepsilon(T(\eta)-T_c)
\end{aligned}
$$

$$
Q = Q(e) = \frac{Q_0}{\sqrt{1-e^2}}
$$

The Budyko-Widiasih Model

$$
\begin{aligned}
R\frac{\partial T}{\partial t} &= Qs(y)\,(1-\alpha(\eta,y)) - (A + BT(t,y)) - C\left(T(t,y) - \overline{T}(t)\right) \\
\frac{d\eta}{dt} &= \varepsilon(T(\eta) - T_c)
\end{aligned}
$$

$$
Q = Q(e) = \frac{Q_0}{\sqrt{1-e^2}}
$$

$$
\begin{aligned}
A &= 202\,\mathrm{Wm}^{-2} \\
B &= 1.9\,\mathrm{Wm}^{-2}(^\circ\mathrm{C})^{-1} \\
C &= 3.04\,\mathrm{Wm}^{-2}(^\circ\mathrm{C})^{-1} \\
Q_0 &= 342.95 \\
\alpha(\eta,y) &= \begin{cases} \alpha_{H_2O} = 0.32, & y < \eta \\ \alpha_{ice} = 0.64, & y > \eta \end{cases}
\end{aligned}
$$

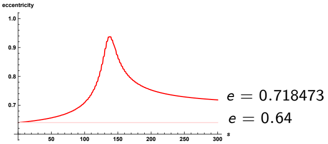
The Budyko-Widiasih Model

$$R\frac{\partial T}{\partial t} = Qs(y)\,(1-\alpha(\eta,y)) - (A+BT(t,y)) - C\left(T(t,y)-\overline{T}(t)\right)$$
$$\frac{d\eta}{dt} = \varepsilon(T(\eta)-T_c)$$

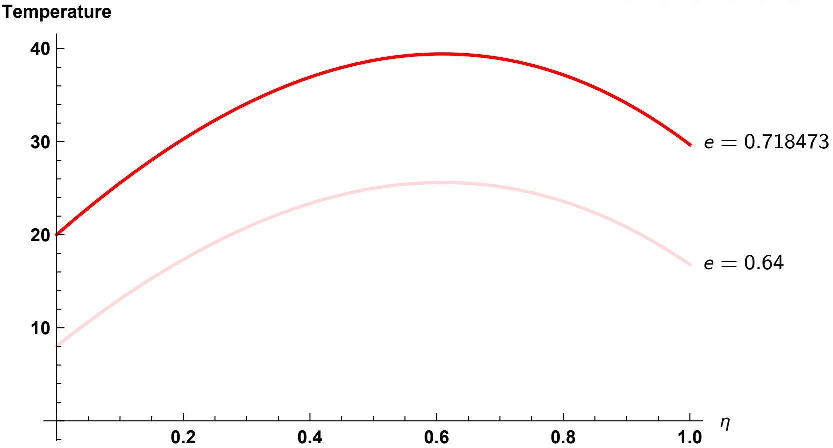
Equilibrium Temperature Profile

$$T^*_\eta(y) = \frac{1}{B+C}\left(Q(e)s(y)\,(1-\alpha(y,\eta)) - A + \frac{C}{B}\,(Q(1-\overline{\alpha}(\eta)) - A)\right)$$
$$\overline{\alpha}(\eta) = \int_0^\eta \alpha_{H_2O}s(y)\,dy + \int_\eta^1 \alpha_{ice}s(y)\,dy$$

Equilibrium Temperature Profiles

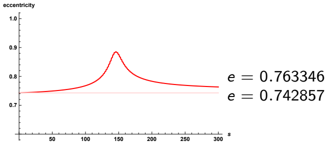


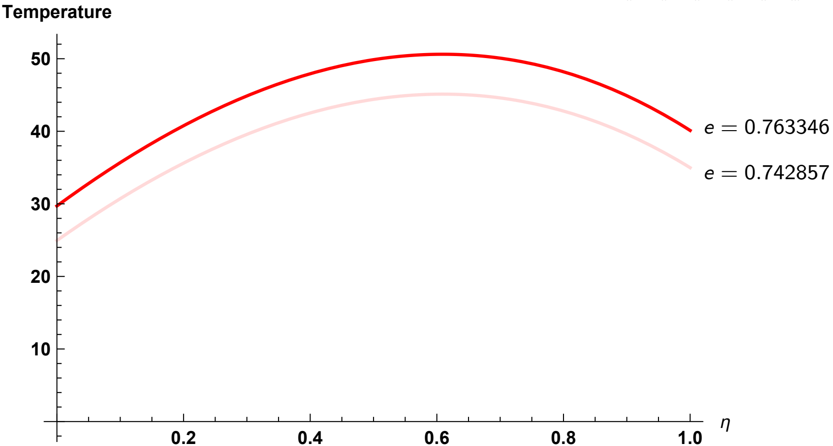
$m_1 = 0.5,\ m_2 = 0.5$



Equilibrium Temperature Profiles

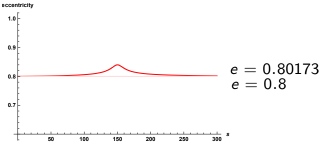
$m_1 = 0.7,\ m_2 = 0.3$

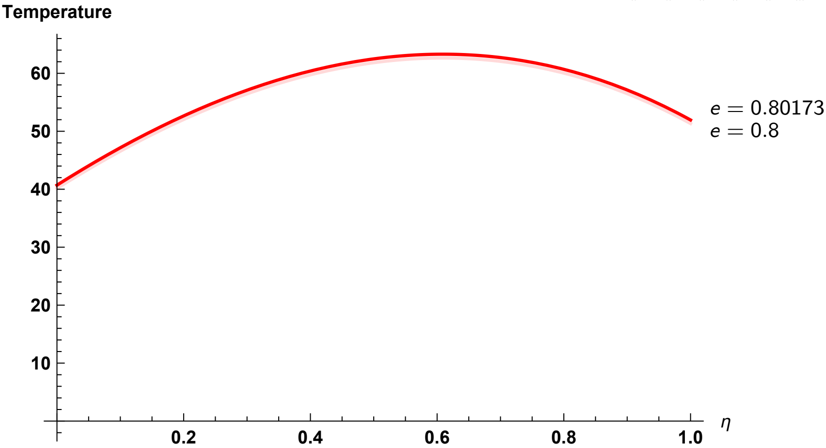




Equilibrium Temperature Profiles

$m_1 = 0.9,\ m_2 = 0.1$





Summary

- Used the HR3BP to model a scenario where a rogue star could pass near our solar system
- Studied its effects in 2D on eccentricity and noticed that it if a passing star was large enough, we could see persistent changes in the eccentricity
- Looked at how those sustained changes resulted in changes in the equilibrium temperature profile of an Earth-like planet

Future Work

- Apply this analysis with initial conditions which reflect the orbits of Jupiter and/or Earth
- Look at how the initial position of the third body changes the system

Thank You!

References

[1] C.A.L. Bailer-Jones, J. Rybizki, R. Andrae, and M. Fouesneau. *New stellar encounters discovered in the second Gaia data release*, A & A, 616 (2018), doi: 10.1051/0004-6361/201833456
[2] M. Budyko. *The effect of solar radiation on the climate of the earth*, Tellus, 21 (1969), pp. 611 - 619.
[3] T. Greicius. *Small asteroid or comet 'visits' from beyond the solar system*, http://www.nasa.gov/feature/jpl/small-asteroid-or-comet-visits-from-beyond-the-solar-system, (October 26, 2017).
[4] R. McGehee and C. Lehman. *A paleoclimate model of ice-albedo feedback forced by variations in earth's orbit*, SIAM J. Appl. Dyn. Syst., 11 (2012), pp. 684 - 707.
[5] R. McGehee. *A Stable Manifold Theorem for Degenerate Fixed Points with Applications to Celestial Mechanics*, J. on Diff. Eqs., 14 (1973), pp. 70 - 88.
[6] J. Moser. *Regularization of kepler's problem and the averaging method on a manifold*, Comm. on Pure and Appl. Math., 23 (1970), pp. 609 - 636.
[7] H. Pollard. *Mathematical introduction to celestial mechanics.*, Prentice-Hall, (1966).
[8] J.L. Simon, P. Bretagnon, J. Chapront, M. Chapront - Touze, G. Francou, and J. Laskar. *Numerical expressions for precession formulae and mean elements for the Moon and planets*, A & A, 282 (1994), pp. 663 - 683.
[9] E. Widiasih. *Dynamics of Budyko's energy balance model*, SIAM Appl. Dyn. Syst., 12 (2013), pp. 2068 - 2092.
[10] D. Williams. *Earth Fact Sheet*, https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html, (May 5, 2019).
[11] D. Williams. *Jupiter Fact Sheet*, https://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html, (May 5, 2019).