Rare Event Computation and Large Deviations for Turbulent Flows and Climate Dynamics

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Works in collaboration with:
E. Simonnet (abrupt climate changes: bistability for turbulent jets) and F. Ragone and J. Wouters (extreme heat waves)

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Outline

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1. Rare transitions and Jupiter's abrupt climate changes
   - Jupiter's abrupt climate changes
   - Rare transitions for zonal jets
   - Averaging and large deviations for zonal jet slow dynamics
2. The Eyring-Kramers formula for irreversible dynamics
   - The irreversible Eyring-Kramers formula
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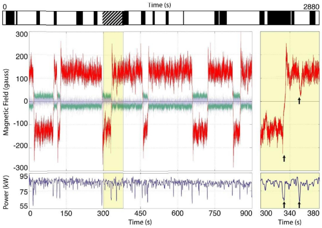
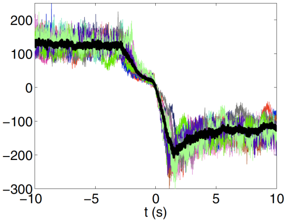
   - Importance sampling of extreme heat waves and teleconnection patterns

Random Transitions in Turbulence Problems

Magnetic Field Reversal (Turbulent Dynamo, MHD Dynamics)



Magnetic field timeseries



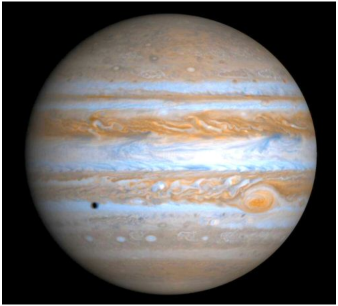
Zoom on reversal paths

(VKS experiment)

In turbulent flows, transitions from one attractor to another (reactive paths) often occur through a predictable path.

Jupiter's Zonal Jets

An example of a geophysical turbulent flow (Coriolis force, huge Reynolds number, ...)

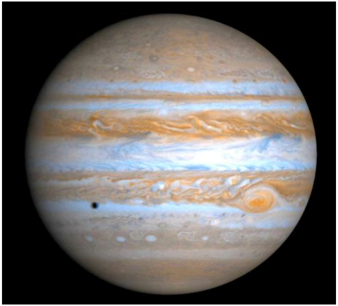


Jupiter's troposphere

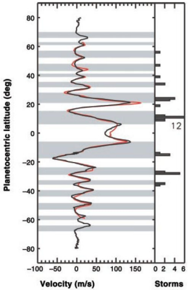
Jupiter's motions (Voyager)

Jupiter's Zonal Jets

We look for a theoretical description of zonal jets



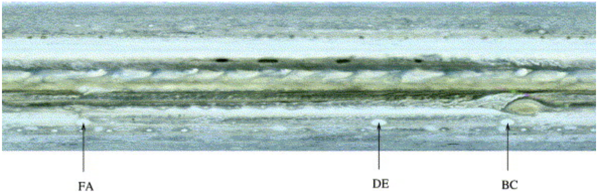
Jupiter's troposphere

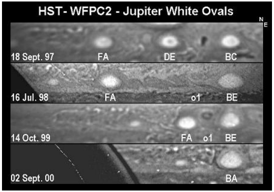


Jupiter's zonal winds (Voyager and Cassini, from Porco et al 2003)

Jupiter's Abrupt Climate Changes

Have we lost one of Jupiter's jets ?



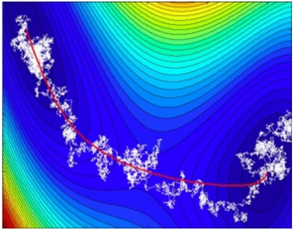


Jupiter's white ovals (see Youssef and Marcus 2005)

The white ovals appeared in 1939-1940 (Rogers 1995). Following an instability of one of the zonal jets?

Freidlin–Wentzell Theory

- In the weak noise limit, most transition paths follow the most probable path (instanton)



$$\frac{d\mathbf{x}}{dt} = \mathbf{b}(\mathbf{x}) + \sqrt{2\varepsilon}\eta(t).$$

Figure by Eric Van den Eijnden

- Arrhenius law then follows, for both gradient (reversible) and non gradient (irreversible) dynamics

$$\lambda \underset{\varepsilon \to 0}{\asymp} e^{-\frac{\Delta V}{\varepsilon}}.$$

The Main Scientific Issues

- How to characterize and predict the attractors of turbulent geophysical flows?
- Can we compute the transition paths and the transition rates?
- For most geophysical problems, an approach through direct numerical simulations is impossible (trade off between realistic turbulence representation and physical time - here one needs both).
- Can we devise new theoretical and numerical tools to tackle these issues?

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The Barotropic Quasi-Geostrophic Equations

- The simplest model for geostrophic turbulence.
- Quasi-Geostrophic equations with random forces

$$\frac{\partial q}{\partial t}+\mathbf{v}.\nabla q=\nu\Delta\omega-\alpha\omega+\sqrt{2\alpha}f_s,$$

where $\omega=(\nabla\wedge\mathbf{v}).\mathbf{e}_z$ is the vorticity, $q=\omega+\beta y$ is the Potential Vorticity (PV), β is the Coriolis parameter, f_s is a random Gaussian field with correlation $\langle f_S(\mathbf{x},t)f_S(\mathbf{x}',t')\rangle=C(\mathbf{x}-\mathbf{x}')\delta(t-t')$.

- Relation between dimensional and non-dimensional parameters:

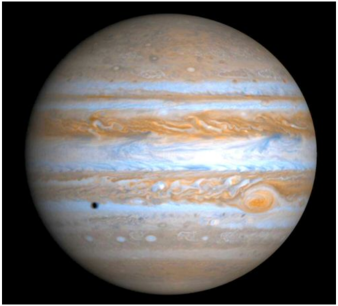
$$\alpha=L\sqrt{\frac{\lambda^3}{\varepsilon}}\text{ and }\beta=\left(\frac{L}{L_R}\right)^2=\beta_d L^2\sqrt{\frac{\lambda}{\varepsilon}}$$

- A reasonable model for Jupiter's zonal jets.

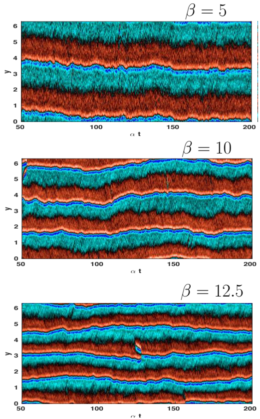
Dynamics of the Barotropic Quasi-Geostrophic Equations

Top: Zonally averaged vorticity (Hovmöller diagram and red curve) and velocity (green). Bottom: vorticity field

Multistability for Quasi-Geostrophic Jets



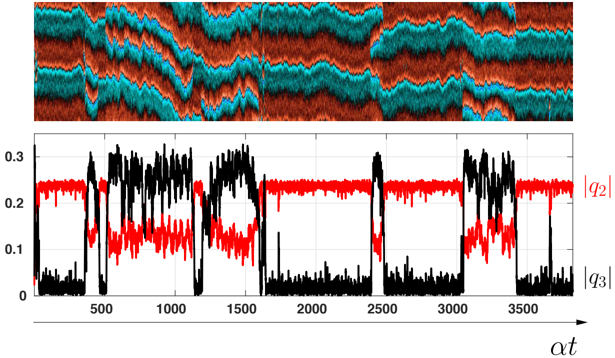
Jupiter's atmosphere



QG zonal turbulent jets

- Multiple attractors had been observed previously by B. Farrell and P. Ioannou.

Rare Transitions Between Quasigeostrophic Jets



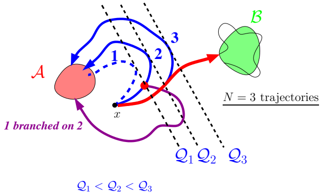
Rare transitions for quasigeostrophic jets (with E. Simonnet)

- This is the first observation of spontaneous transitions.
- How to predict those rare transitions? What is their probability? Which theoretical approach?

Rare Events and Adaptive Multilevel Splitting (AMS)

AMS: an algorithm to compute rare events, for instance rare transition paths

- Rare event algorithms: Kahn and Harris (1953), Chandler, Vanden-Eijnden, Schuss, Del Moral, Dupuis, ...
- The adaptive multilevel splitting algorithm:



AMS algorithm

Strategy: selection and cloning.
Probability estimate:

$$\hat{\alpha} = (1 - 1/N)^K, \text{ where}$$

N is the clone number and K the iteration number.

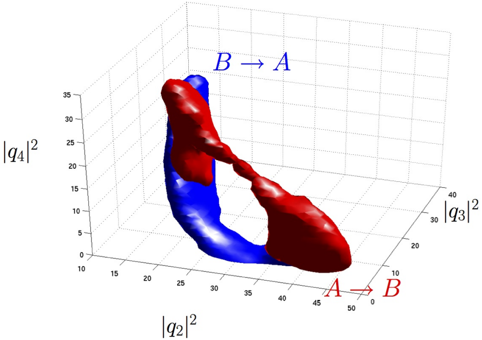
Cérou, Guyader (2007). Cérou, Guyader, Lelièvre, and Pommier (2011).

A Transition from 2 to 3 Jets

Top: Zonally averaged vorticity (Hovmöller diagram and red curve) and velocity (green). Bottom: vorticity field

Atmosphere Jet "Instantons" Computed using the AMS

AMS: an algorithm to compute rare events, for instance rare reactive trajectories

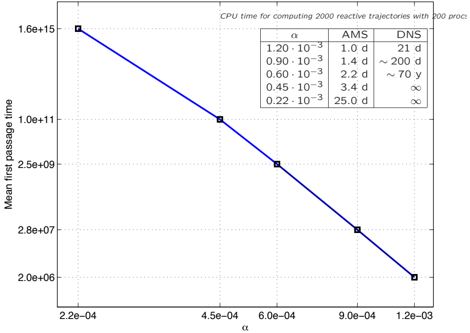


Transition trajectories between the 2 and 3 jet states

- The dynamics of turbulent transitions is predictible.
- Asymmetry between forward and backward transitions.

Transition Rates for Unreachable Regimes Through DNS

With the AMS we can estimate huge average transition times



α	AMS	DNS
$1.20 \cdot 10^{-3}$	1.0 d	21 d
$0.90 \cdot 10^{-3}$	1.4 d	$\sim$ 200 d
$0.60 \cdot 10^{-3}$	2.2 d	$\sim$ 70 y
$0.45 \cdot 10^{-3}$	3.4 d	∞
$0.22 \cdot 10^{-3}$	25.0 d	∞

Average transition time versus α

- With the AMS algorithm, we study transitions that would require an astronomical computation time using direct numerical simulations.

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Which Mathematical Framework for the Inertial Limit?

- Quasi-Geostrophic equations with random forces

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = -\alpha\omega + \sqrt{2\alpha} f_s.$$

- Inertial limit: spin up or spin down time $= 1/\alpha \gg 1 =$ jet inertial time scale (a relevant assumption for Jupiter).
- This is an averaging problem for an Hamiltonian system perturbed by weak non Hamiltonian forces.
- The Hamiltonian system is an infinite dimensional one with an infinite number of conserved quantities.
- We will need to consider large deviations for the slow process.

Decomposition Between Zonal Jets and Turbulence: A Slow/Fast Dynamical System

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = -\alpha\omega + \sqrt{2\alpha} f_s \text{ with } \alpha \ll 1$$

- Time scale separation. We decompose into slow (zonal flows) and fast (eddy turbulence) variables

$$U(y)\mathbf{e}_x = \langle \mathbf{v} \rangle \equiv \frac{1}{2\pi}\int_{\mathscr{D}} \mathrm{d}x\, \mathbf{v} \text{ and } \mathbf{v} = U(y)\mathbf{e}_x + \sqrt{\alpha}\mathbf{v}_t.$$

- Stochastic averaging using the time scale separation.

$$\frac{\partial U}{\partial t} = F(U).$$

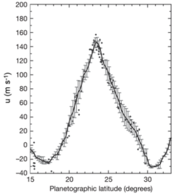
- We identify SSST by Farrell and Ioannou (JAS, 2003); quasilinear theory by Bouchet (PRE, 2004); CE2 by Marston, Conover and Schneider (JAS, 2008); Sreenivasan and Young (JAS, 2011).

An Explicit and "Universal" Formula for the Reynolds Stress

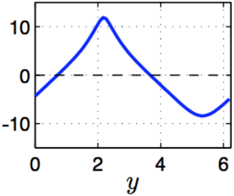
$$\frac{\partial U}{\partial t} = F(U)$$

- An explicit expression for $F(U)$ (see Laurie et al, PRL, 2015)

$$F(U) = -\frac{\partial}{\partial y}\left(\frac{\varepsilon}{\partial U/\partial y}\right) - \alpha U,$$ where ε is the energy injection rate.



Jupiter's velocity profile



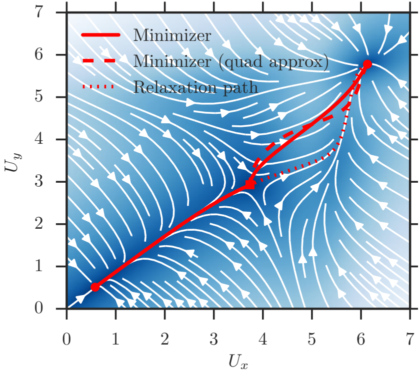
Theoretical velocity profile

- Full justification and more general formula

E. Woillez and F. Bouchet, arXiv:1609.00603.

Gaussian Fluctuations Do Not Describe Rare Transitions

$$\frac{\partial U}{\partial t} = F(U) + \sqrt{\alpha}\sigma(U,t)$$



(Figure from F. Bouchet, T. Grafke, T. Tangarife, and E. Vanden-Eijnden, J. Stat. Phys. 2016)

Zonal Jet Conclusions

- We have computed rare transitions between zonal jets, similar to Jupiter's abrupt climate changes, that can not be computed using direct numerical simulations (with E. S.).
- We have partial results for the justification of averaging (ergodicity, etc ...), (with C.N., and T.T.).
- For small scale forces, the average Reynolds stress can be computed explicitly and is universal. We have a good qualitative agreement with Jupiter's jets. (with E.W.).
- The rare transitions involve non-Gaussian fluctuations of the Reynolds stress.
- Work in progress: a theory based on large deviations can be derived for the computation of transition rates and transition paths between zonal jets (with T.G., B..M., T.T., and E. V-E).

http://perso.ens-lyon.fr/freddy.bouchet/

Outline

Transition Rates for Irreversible (Non-Gradient) Dynamics

$$\frac{d\mathbf{x}}{dt} = \mathbf{b}(\mathbf{x}) + \sqrt{2\varepsilon}\eta(t).$$

- We assume that there exists a transverse decomposition in the instanton neighborhood

$$\mathbf{b}(\mathbf{x}) = -\nabla V(\mathbf{x}) + \mathbf{G}(\mathbf{x}) \text{ with for all } \mathbf{x},\ \nabla V(\mathbf{x}).\mathbf{G}(\mathbf{x}) = 0.$$

- The transition rate then reads

$$\lambda \underset{\varepsilon\to 0}{\sim} \frac{|\lambda_*|}{2\pi}\sqrt{\frac{\det \text{Hess}\, V(x_1)}{|\det \text{Hess}\, V(x_*)|}} \exp\left(-\frac{\Delta V}{\varepsilon}\right) \exp\left\{-\int_{-\infty}^{+\infty} \mathrm{d}t\, [\nabla.G(X(t))]\right\},$$

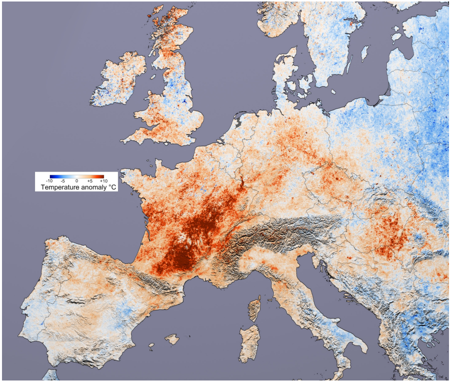
where λ_* is the negative eigenvalue corresponding to the unstable direction at the saddle point, for the dynamics (and not for V) and $\{X(t)\}$ is the instanton.

F. Bouchet and J. Reygner, AHP 2016

Outline

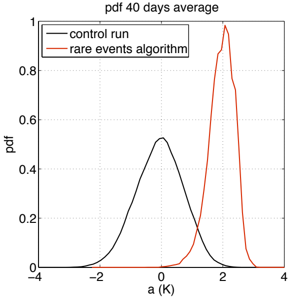
Extreme Heat Waves

Example: the 2003 heat wave over western Europe

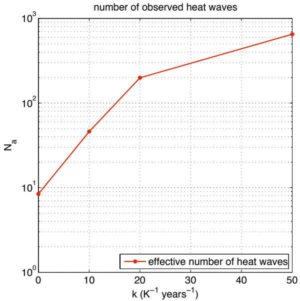


July 20 2003-August 20 2003 land surface temperature minus the average for the same period for years 2001, 2002 and 2004 (TERRA MODIS).

Importance Sampling of Extreme Heat Waves in a Climate Model



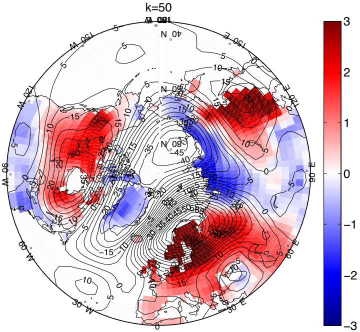
PDF of time averaged temperature



Heat wave number

- At fixed numerical cost, we get hundreds more heat waves with the large deviation algorithm than with the control run.
- We can consider interesting dynamical studies.

Heat Wave Conditional Statistics and Teleconnection Patterns



500 HPa geopotential height anomalies and temperature anomalies

Heat wave statistics defined as statistics conditioned with $\frac{1}{T}\int_0^T Temp(X(t))\,\mathrm{d}t > 2°\mathrm{C}$, with $T = 40\,\mathrm{days}$.

Summary and Perspectives

- Large deviation theory can be applied to geophysical turbulence and climate.
- This is the main approach for non-equilibrium statistical mechanics applied to climate dynamics.
- With rare event algorithms, we can compute probability of rare events that can not be sampled using direct numerical simulations. This should have a huge impact on future computations of climate extremes and rare transitions.
- The dynamics leading to rare events is usually predictable, even for turbulent flows.

http://perso.ens-lyon.fr/freddy.bouchet/