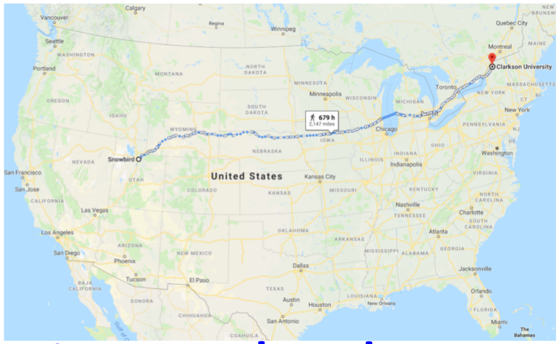
Clarkson University



A Nondestructive Damage Detection Method for BridgeStructures using Information-Theoretic Methods

Amila Sudu Ambegedra, Jie Sun, Kerop Janoyan, Erik Bollt

bolltem@clarkson.edu, http://www.clarkson.edu/~bolltem

C^3S^2 Clarkson Center for Complex Systems Science

“Loosened” Bridge, Damage Detection/Health Monitoring

Bridge Location

New York State Route 345 over big sucker brook in the town of Waddington , NY. Constructed in 1957

Data from Kerop Janoyan’s research group, Department of Civil Engineering, Clarkson University.

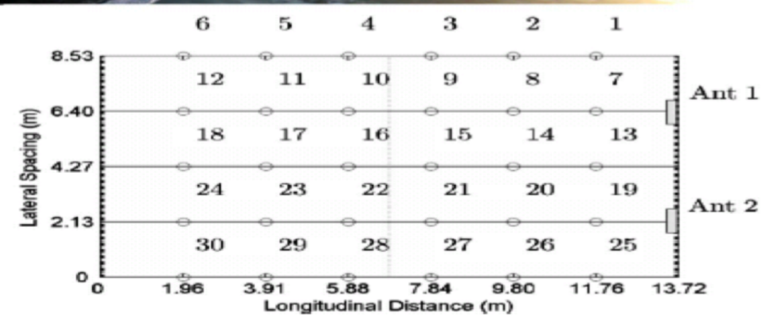
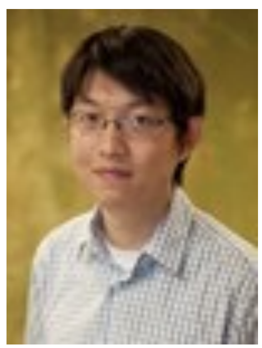


Figure 2: physical locations of the accelerometers (This figure is taken from ” In-Service Diagnostic of a Highway Bridge from a Progressive Damage Case Study, Matthew J, Whelan, S.M.ASCE, and Kerop D. Janoyan, P.E.,M.ASCE)

Amila Ambegedara

Kerop Janoyan

Jie Sun



Given: time series from sensors placed on a bridge
Problem: to infer effective structural connections among the sensor locations

Publication: Amila Sudu Ambegedara, JS, Kerop Janoyan, and Erik Bollt, *Information-theoretical noninvasive damage detection in bridge structures,* Chaos (2016).

Engineering structures…..sometimes they fail.

Clarkson UNIVERSITY

A Damaged bridge in Northridge, Canada

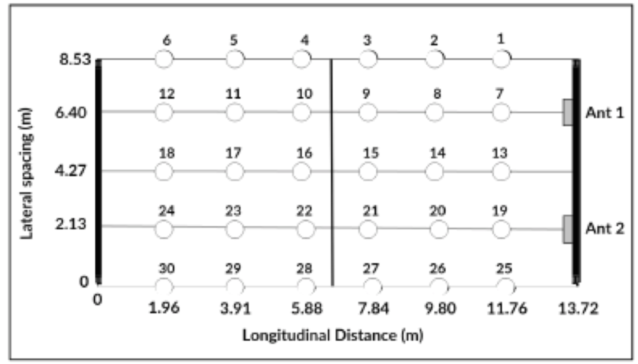
A damaged wind turbine at the centre of the Lincolnshire

Collapsed Cypress Freeway in Oakland after the 1989 Loma Prieta earthquake

Damage Detection In Bridge Structures

Physical Problem - I:


(a)



(b)

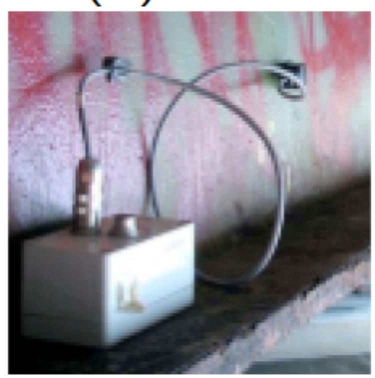

(c)

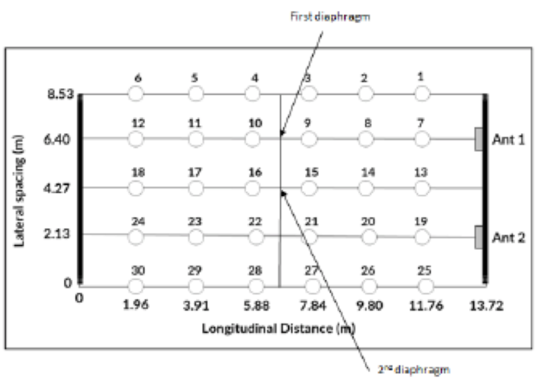
Figure: (a) The Waddington bridge, NY route 345 (b) (Top view) Physical spatial layout of the indexed accelerometers (c) Accelerometer [1]

[1] Mathew J. et.al, 2010

Field Testing and Damage Introduction

Clarkson UNIVERSITY

(a)



(b)

Figure: (a) A truck passes back and forth (b) Diaphragm connection

Scenarios

- Base line - Healthy Structure
- Damage 1 - Removal of 4 out of 6 bolts
- Damage 2 - Removal of all 6 out of 6 bolts

Special Thank: Prof. Kerop Janoyan - Data

Information flow - **Transfer Entropy answers a question:** *Does process x depend just on x, or does it also depend on y.*

WHICH reality is true: $x_{n+1} = f(x_n)$ - versus - $x_{n+1} = f(x_n, y_n)$

The main idea leading to transfer entropy will be to measure the **deviation from the Markov property,** which would presume

$$p(x_{n+1}|x_n^{(k)}) = p(x_{n+1}|x_n^{(k)}, y_n^{(l)}),$$

Decide by **Kullback-Leibler divergence**

$$T_{y\to x} = D_{KL}(p(x_{n+1}|x_n^{(k)}, y_n^{(l)})||p(x_{n+1}|x_n^{(k)})),$$

$$= H(x_{n+1}|x_n^{(l)}) - H(x_{n+1}|x_n^{(l)}, y_n^{(k)}).$$

$$= [H(x_{n+1}, x_n) - H(x_n)] - [H(x_{n+1}, x_n, y_n) - H(x_n, y_n)],$$

$$T_{x\to y} = H(y_{n+1}|y_n^{(l)}) - H(y_{n+1}|x_n^{(l)}, y_n^{(k)}),$$

$$T_{x\to y} \neq T_{y\to x}.$$

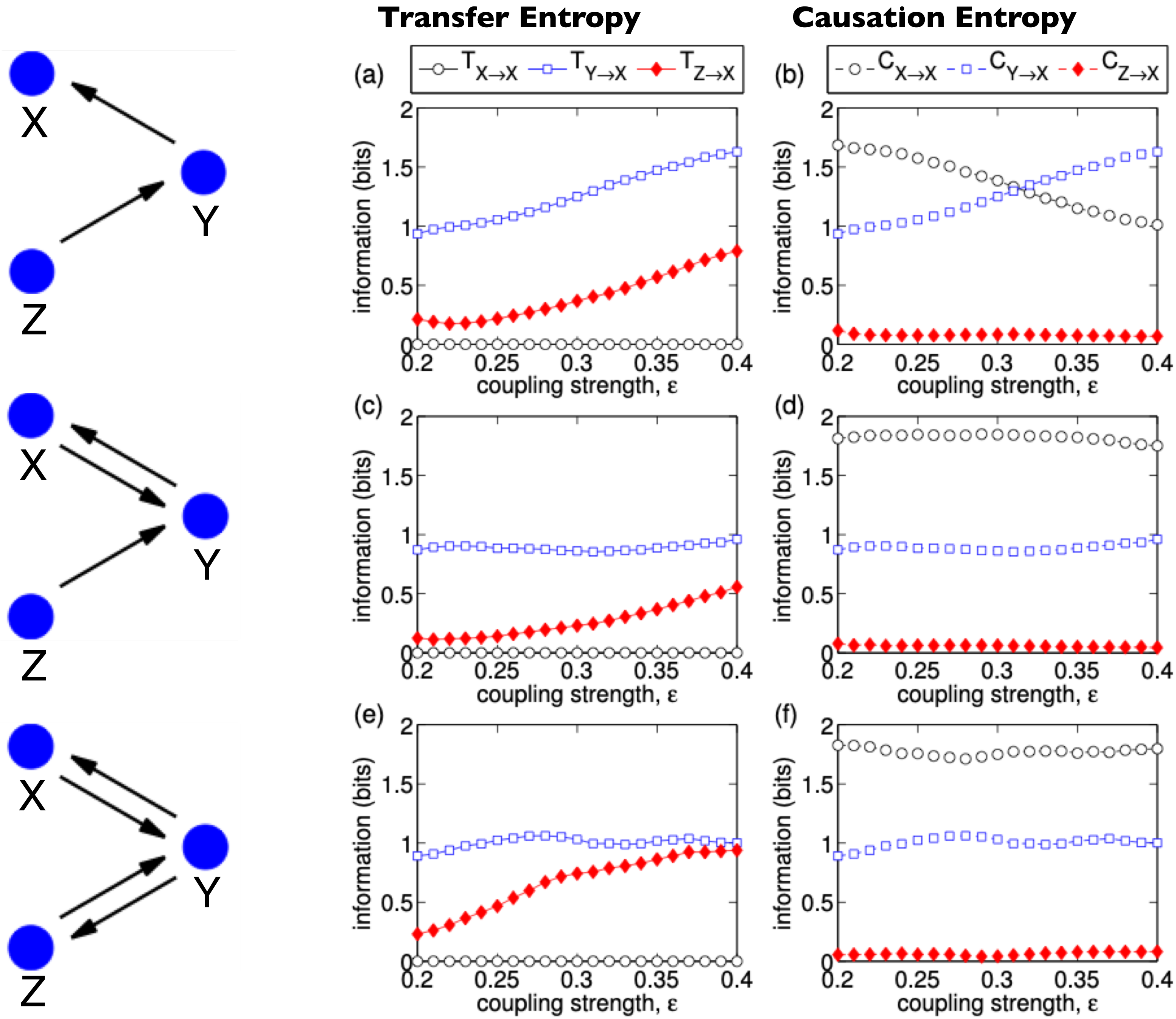
-**Widely popular applications: Stock Market, Financial Markets, Genomics, Bioinformatics, neural spike trains**

VOLUME 85, NUMBER 2 PHYSICAL REVIEW LETTERS 10 JULY 2000

Measuring Information Transfer

Thomas Schreiber

Transfer Entropy (T) vs. Causation Entropy (C)



Causation entropy correctly identifies the causal network structure.

Causation Entropy: Measure of Causality in Networks

Definition 1 (Causation Entropy). *The causation entropy from process $\mathcal{Q}$ to process $\mathcal{P}$ conditioned on the set of processes $\mathcal{S}$ is defined as*

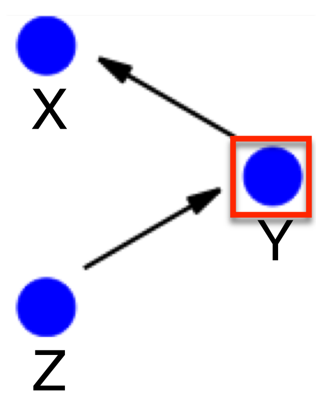
$$C_{\mathcal{Q}\to\mathcal{P}|(\mathcal{S})} = H(\mathcal{P}_{t+1}|\mathcal{S}_t) - H(\mathcal{P}_{t+1}|\mathcal{S}_t, \mathcal{Q}_t).$$

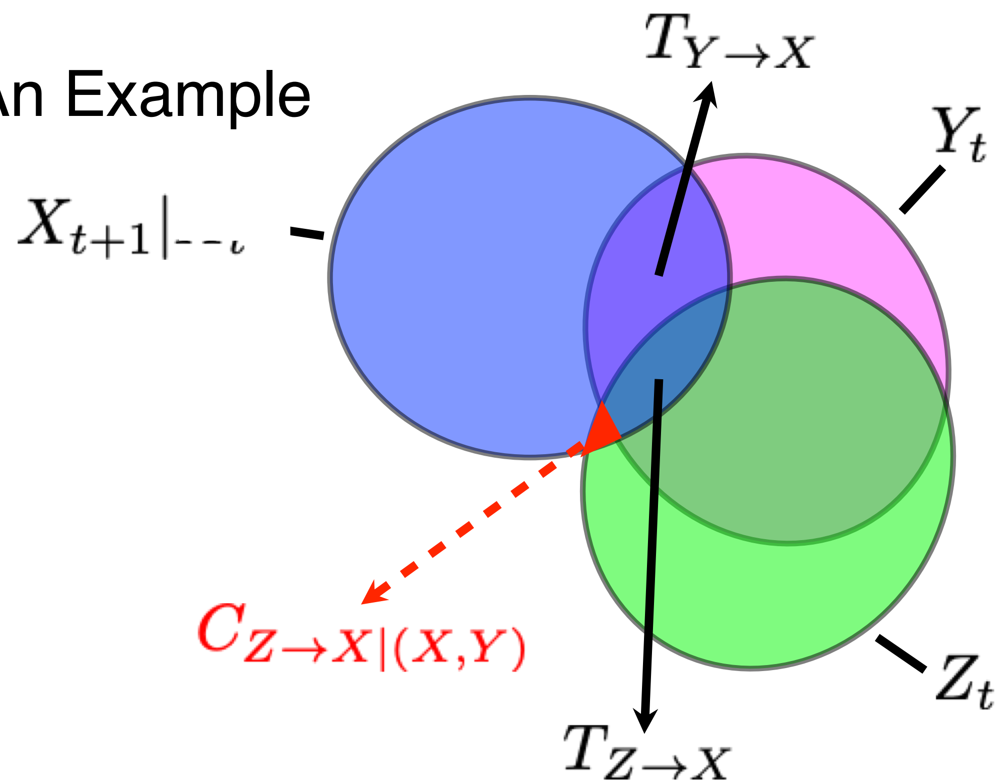
uncertainty of P's future given S

uncertainty of P's future given S and Q

JSun, E. Bollt (Physica D, 2013).
JSun, D Taylor, E. Bollt, SIADS (2015)

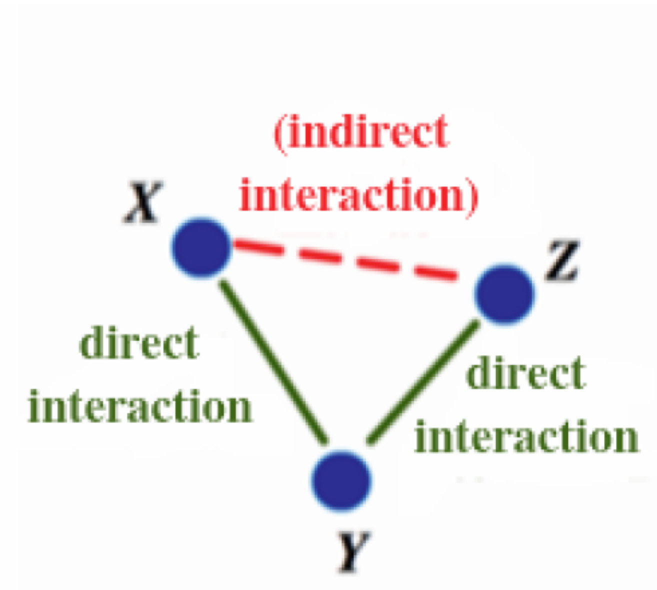
An Example





1. CSE itself does not "solve" the causal inference problem.
2. Definition simply emphasizes the fact that cause-and-effect involves all three parts (***cause***, ***effect***, and ***conditioning***).

Basic Information Measures



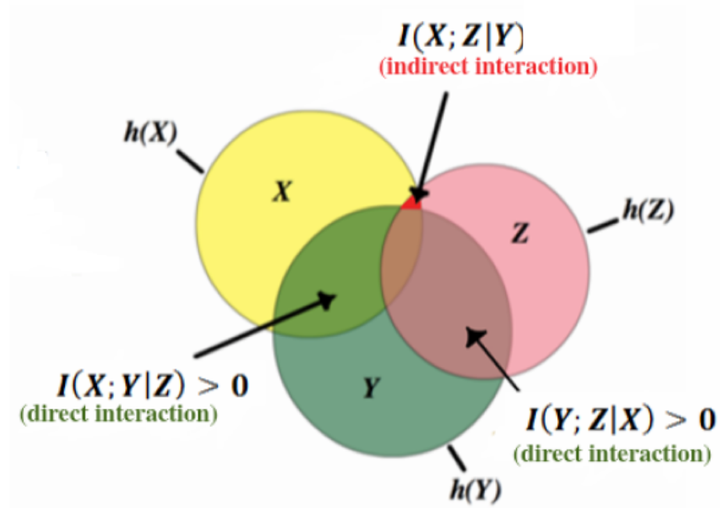


Figure: Direct and indirect mutual information interactions of continuous random variables

Is X independent of Y given Z?

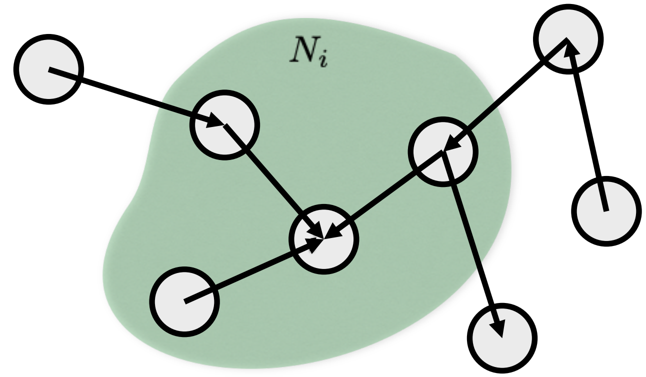
$$H_0: \quad X \perp\!\!\!\perp Y \mid Z$$

$$H_1: \quad X \not\perp\!\!\!\perp Y \mid Z\,.$$

$$I_{X:Y|Z} = 0 \text{ if and only if } X \perp\!\!\!\perp Y|Z$$

Solving the Causal Network Inference Problem

The Causal Network Inference Problem
Given data samples of the process, $\{X_t^{(i)}\}$
Goal: infer N_i ("causal parents") for each i.



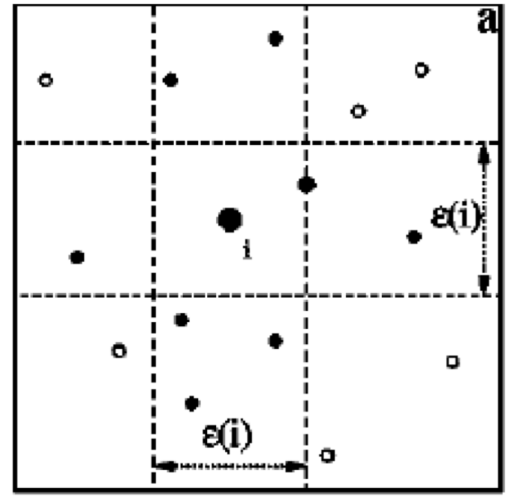
Optimal Causation Entropy (oCSE) Principle: The set of causal "parents" is the *minimal* set of nodes which *maximizes* causation entropy.

$$\begin{cases} C_{\max} = \max_K I(X_{t-1}^{(K)}; X_t^{(i)}) \\ N_i = \arg\min_{I(X_{t-1}^{(K)}; X_t^{(i)}) = C_{\max}} |K| \end{cases}$$

Optimal Causation Entropy (oCSE) Algorithm:
— (Iterative, incremental) local search
— Forward discovery + Backward removal
Model-free, fast (computationally efficient), and quick convergence (data efficient).

Jie Sun, Dane Taylor, and Erik Bollt
Causal network inference by optimal causation entropy
SIAM Journal on Applied Dynamical Systems (2015)

On nonParametric Estimation of the entropies



PHYSICAL REVIEW E **69**, 066138 (2004)

Estimating mutual information

Alexander Kraskov, Harald Stögbauer, and Peter Grassberger
John-von-Neumann Institute for Computing, Forschungszentrum Jülich, D-52425 Jülich, Germany

Rather than use bins, estimate invariant density in those bins, compute the various joint and conditional densities

– and then entropies from there

Make the estimation based on sampled points, and the densities and so **entropies are related to sizes of neighborhoods, of "kth" nearest neigbors**.

"knn methods" also benefit from kdtree searches for near neighbors – knnsearch

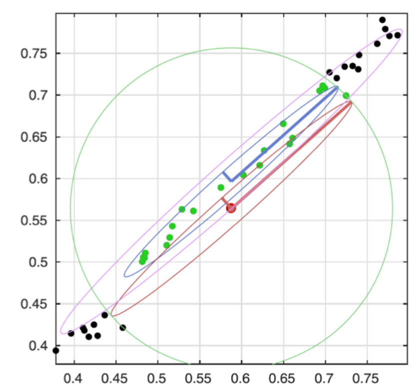
The "jist of it"

$c_d = \pi^{d/2} / \Gamma(1+d/2)/2^d$ for the Euclidean

$\psi(x)$ is the digamma function

$p_i(\epsilon) \approx c_d \epsilon^d \mu(x_i)$

$$\hat{H}(X) = -\psi(k) + \psi(N) + \log c_d + \frac{d}{N}\sum_{i=1}^{N} \log \epsilon(i)$$



Lord, Sun, Bollt, (2018)

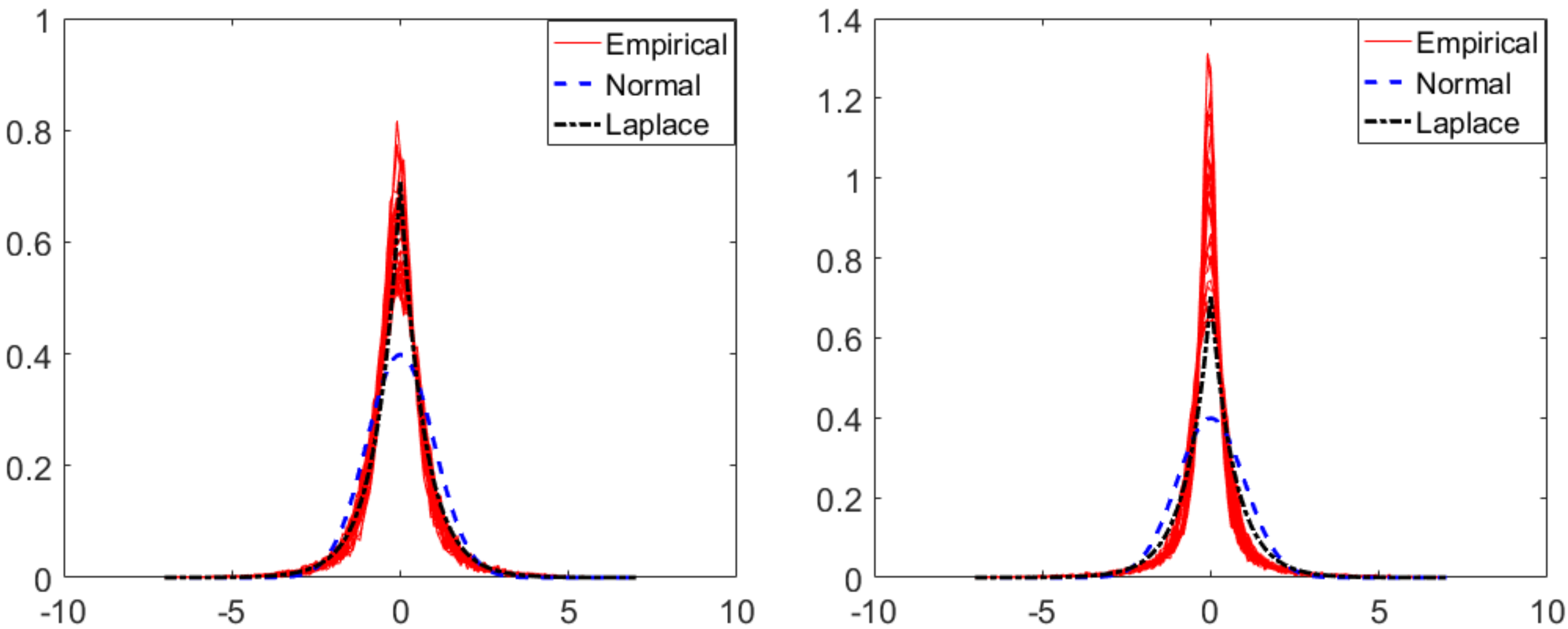
-Kozachenko and Leonenko (1987), Kraskov et al. (2004)

-For CMI: Frenzel and Pompe, 2007; Ve- jmelka and Paluˇs, 2008

A parametric estimator

Normal pdf: $f(x) \propto \exp(-\alpha x^2)$

Laplace pdf: $f(x) \propto \exp(-\beta |x|)$



Accelerations at most of the sensor locations are better captured by the Laplace distribution than the Normal (Gaussian) distribution.

Publication: Amila Sudu Ambegedara, JS, Kerop Janoyan, and Erik Bollt, *Information-theoretical noninvasive damage detection in bridge structures,* Chaos (2016).

Basic Statistical Findings

Sensor data follow a Laplace distribution rather than normal distribution:

$$f_X(\mathbf{x}) = \frac{1}{2\pi^{(d/2)}} \frac{2}{\lambda} \frac{K_{(d/2)-1}\left(\sqrt{\frac{2}{\lambda}q(\mathbf{x})}\right)}{\left(\sqrt{\frac{\lambda}{2}q(\mathbf{x})}\right)^{(d/2)-1}} \quad (1)$$

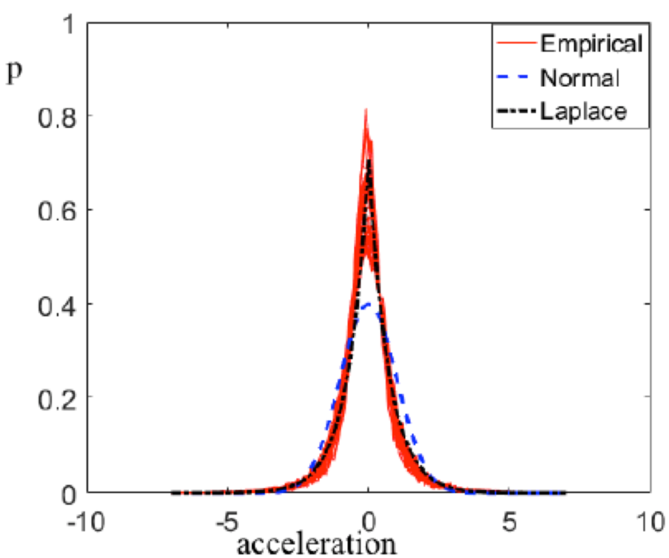


Figure: Distribution of the sensor data

$\mathbf{x} \in \mathbb{R}^d$, $K_{(d/2)-1}$ - modified Bessel function of the second kind with order $(d/2) - 1$ evaluated at $\mathbf{x}$,

$$q(\mathbf{x}) = \lambda (\mathbf{x} - \mu)^\top \Sigma^{-1} (\mathbf{x} - \mu),$$

μ - mean vector, Σ - covariance matrix, $\lambda = \sqrt{\det(\Sigma)}$.

Discovery Stage

- Identify the maximal MI pairwise interaction between sensors
- Use shuffle test to confirm its influence as information transference
- Algorithm 1: Discovery Stage

Input: time series $X_t = \{x_t^{(i)}\}_{i=1,\ldots,N;t=1,\ldots,T}$ and component i

Output: K_i

Initialize: $K_i \leftarrow \{\emptyset\}$, $p \leftarrow \phi$, $x \leftarrow 1$

while $x > 0$ **do**

    $p \leftarrow \arg\max_{j \neq \{i,K_i\}} I(X_t^{(i)}; X_t^{(j)} | X_t^{(K_i)})$

    if $(X_t^{(i)}; X_t^{(p)}; X_t^{(K_i)})$ passes the Shuffle Test (Algorithm 3) **then**

        $K_i \leftarrow K_i \cup \{p\}$

    else

        $x \leftarrow 0$

    end if

end while

Removal Stage

- Remove the indirect nodes in the set K_i
- Use shuffle test to check the direct influence
- Algorithm 2: Removal Stage

Input: time series $X_t = \{x_t^{(i)}\}_{i=1,\ldots,N;t=1,\ldots,T}$, component i, and set K_i

Output: $\hat{K}_i$

for every $j \in K_i$ **do**
    if $(X_t^{(i)}; X_t^{(j)}; X_t^{(K_i/\{j\})})$ fails the Shuffle Test (Algorithm 3) **then**
        $K_i \leftarrow K_i/\{j\}$
    end if
end for
$\hat{K}_i \leftarrow K_i$

Shuffle Test

- A method for stoping criteria
- Algorithm 3: Shuffle Test

Since no finite size sample behavior for CMI

Is X independent of Y given Z?

$$H_0: \quad X \perp\!\!\!\perp Y \mid Z$$

$$H_1: \quad X \not\!\perp\!\!\!\perp Y \mid Z\,.$$

$I_{X:Y|Z} = 0$ if and only if $X \perp\!\!\!\perp Y|Z$

Input: time series $(X_t^{(i)} = \{x_t^{(i)}\}; X_t^{(j)} = \{x_t^{(j)}\}; X_t^{(K)} = \{x_t^{(K)}\}, t = 1, \ldots, T)$, threshold θ and number of shuffles N_s

Output: pass/fail

```
for ℓ = 1, ..., N_s do
    generate a random permutation: σ : {1, ..., T} → {1, ..., T}
    use σ to obtain a shuffled time series, Y_t = {y_t}, where y_t ← x^{(j)}_{σ(t)}
    compute I_ℓ ← I(X_t^{(i)}, Y_t | X_t^{(K)})
end for
S ← the ⌊(1 − θ)N_s⌋th largest value from {I_1, ..., I_{N_s}}
if I(X_t^{(i)}; X_t^{(j)} | X_t^{(K)}) > S then
    output: pass
else
    output: fail
end if
```

Idea: to simulate independence, randomly permute x-values In $\{x_i, y_i, z_i\}_{i=1}^{n}$

Pair Wise Mutual Interactions

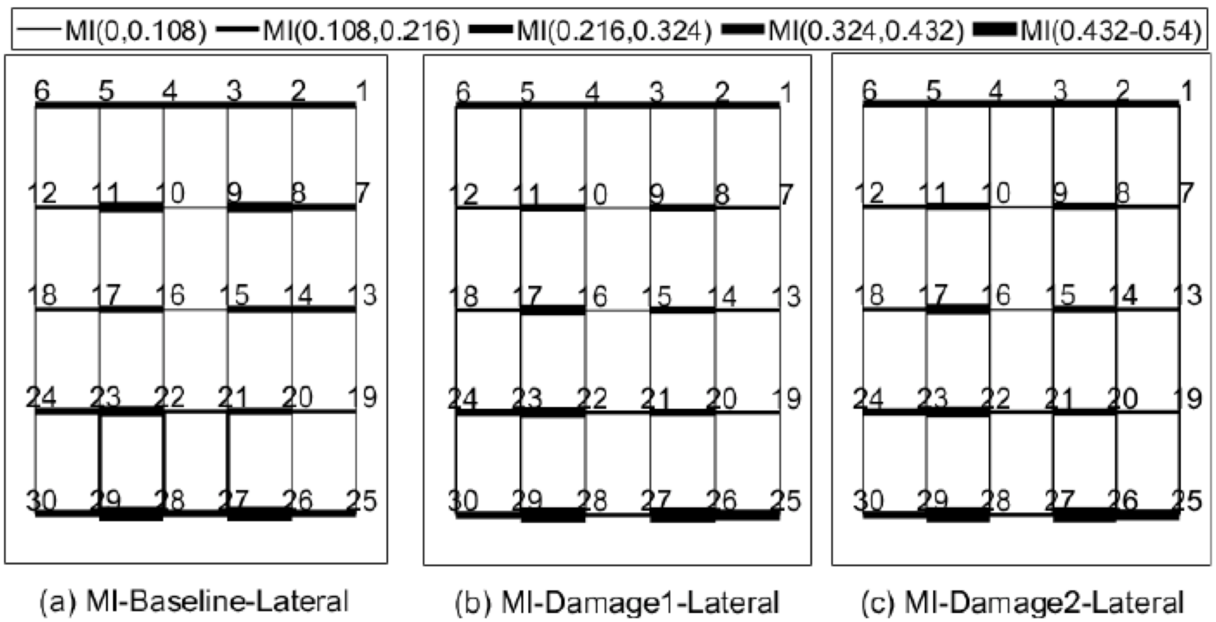
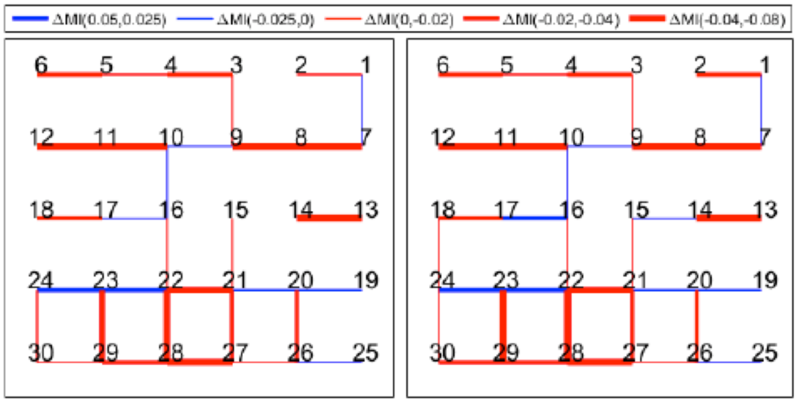


Figure: Pairwise mutual information

- Categorize MI into five ranges
- Thickness of the line is proportional to magnitude of the MI

Difference of the Pairwise MI



(a) MI(Damage1-Baseline)-Lateral (b) MI(Damage2-Baseline)-Lateral

Figure: Difference of the pairwise mutual information

- Red - negative changes after the damage is introduced
- Blue - positive changes the damage is introduced
- Some connections entirely vanished after the damage is introduced

- Damage to the structure seems to generally lower the value of mutual information in the lateral direction between spatially nearby sites (a lower coupling)
- Such changes can be improved with further structural damage

oMII Interactions

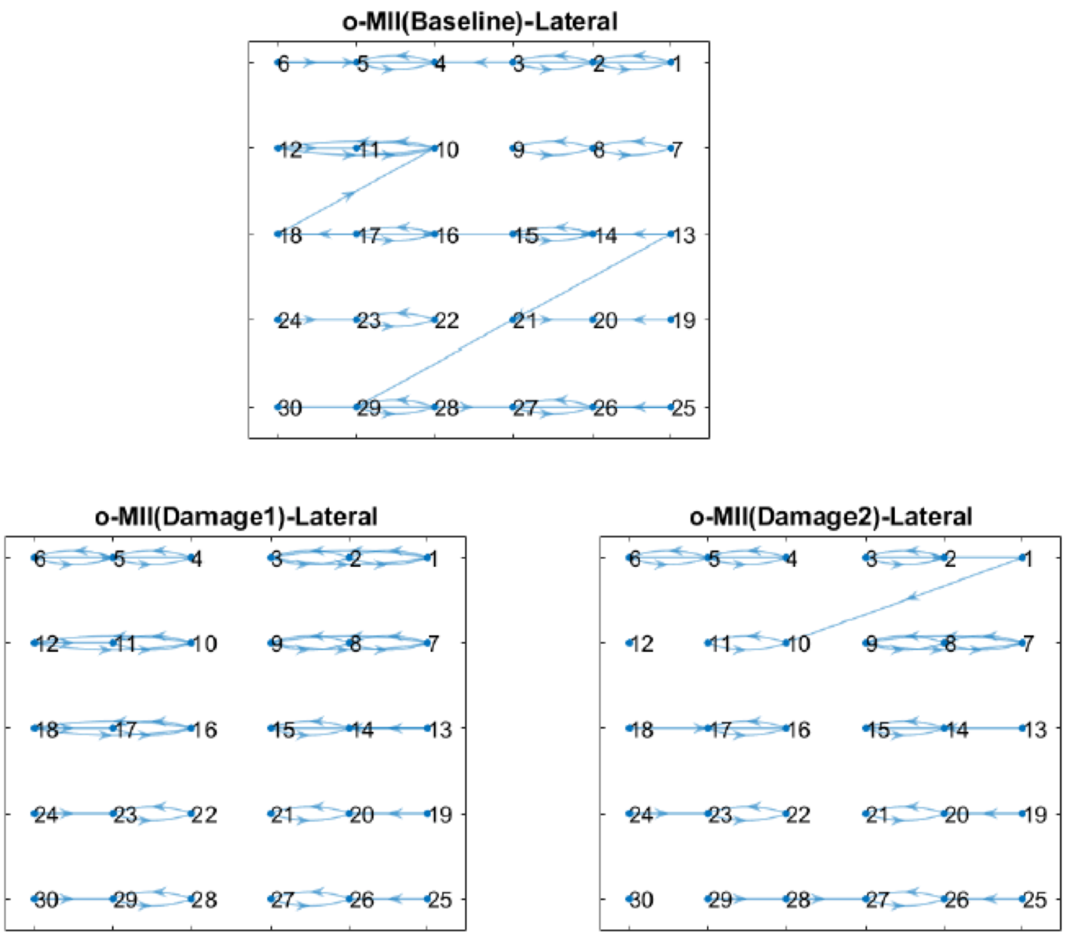


Figure: The optimal mutual information interaction between baseline and damaged bridges

- The bridge structure supports more information flow in the same direction as the truck lanes
- Exceptions are near the center -Due to the first diaphragm

Difference of the oMII Interactions

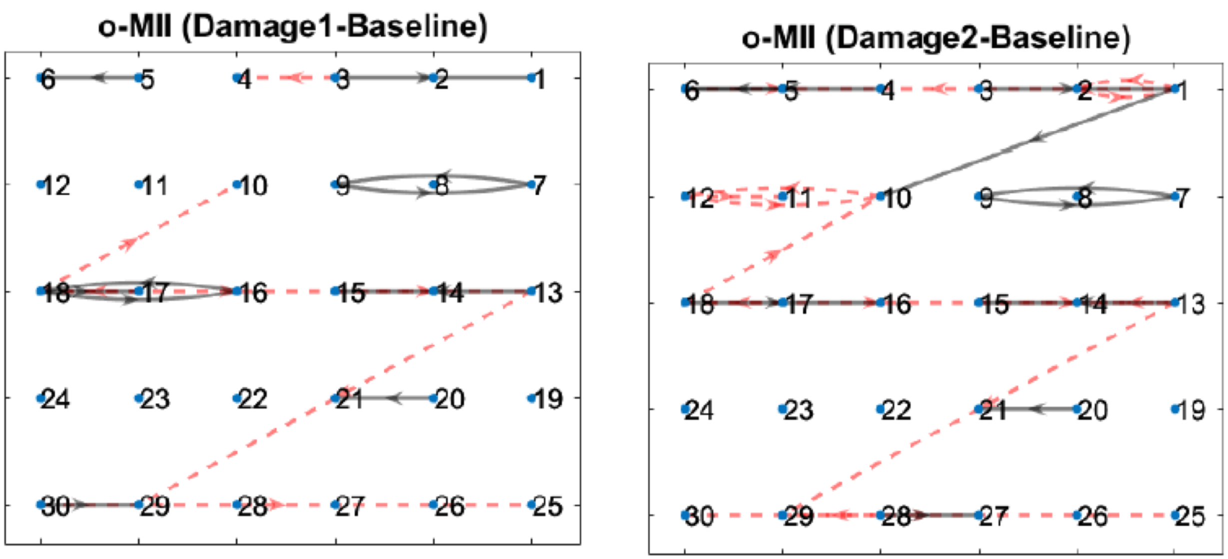
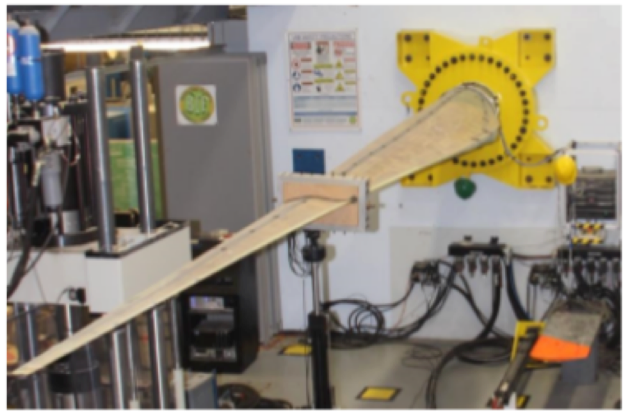


Figure: Difference of the optimal mutual information interaction between baseline and damaged bridges

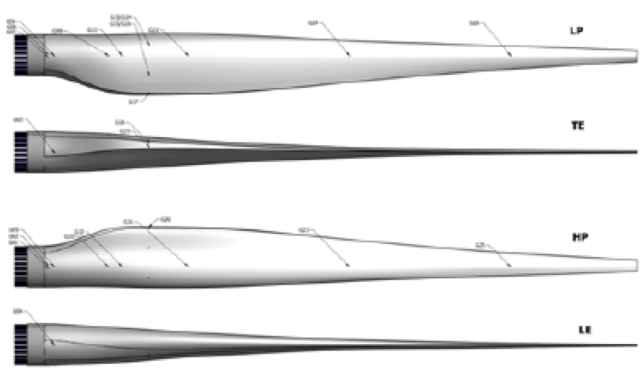
- Dashed red - lost connections
- Black - new connections
- Significant changes in the information transfer

- Sudu Ambegedara, Amila, et. al. "Information-theoretical noninvasive damage detection in bridge structures." Chaos: An Interdisciplinary Journal of Nonlinear Science 26.11 (2016): 116312.

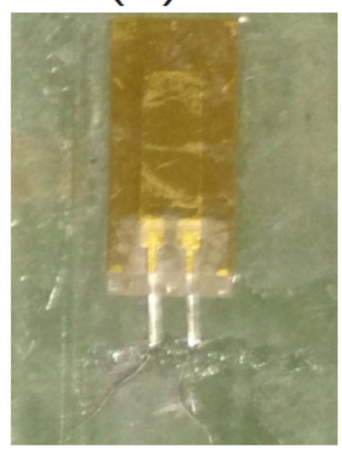
A Fatigue Test for Wind Blades Using Information Theoretical Measures



(a)



(b)



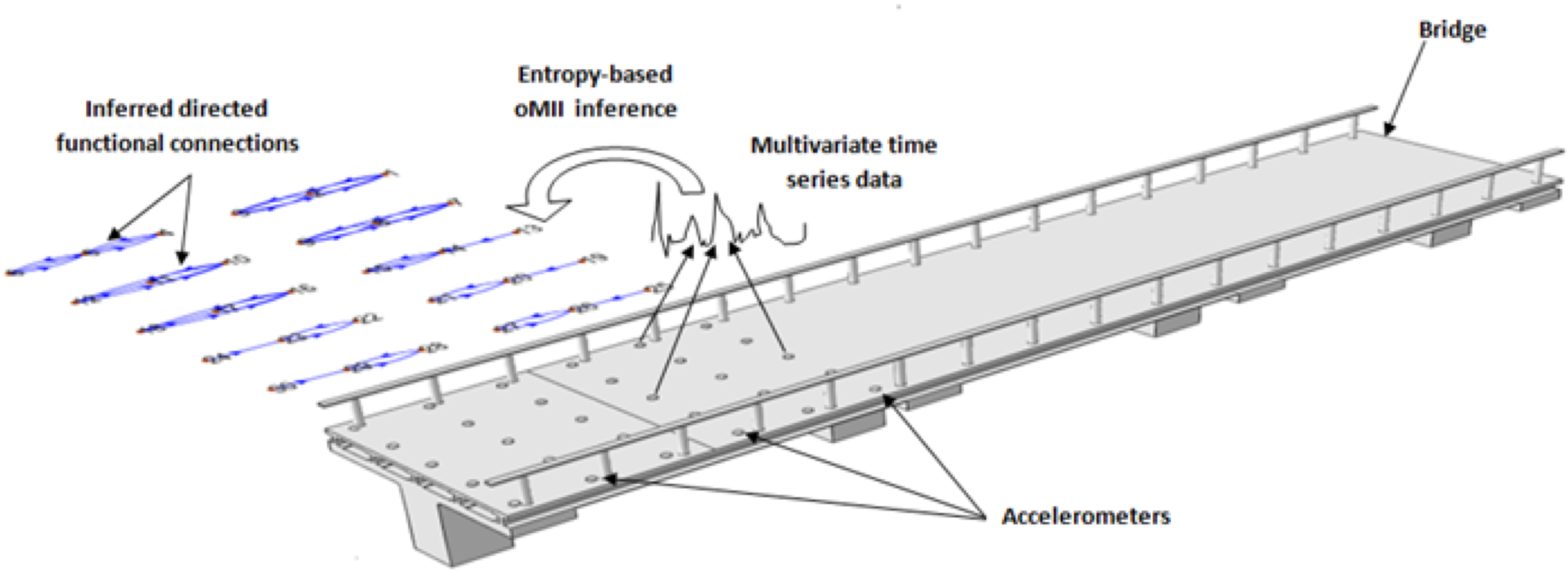
(c)

Figure: (a) Blade System Design Studies blade mounted for fatigue test (b) Strain gauges locations on the wind blade (c) A Strain gauge

Noninvasive Damage Detection

Publication: Amila Sudu Ambegedara, JS, Kerop Janoyan, and Erik Bollt,
Information-theoretical noninvasive damage detection in bridge structures, Chaos (2016).

(1) Directed connections are mostly in the direction of traffic flow
(2) The identified vertical "gap" corresponds to a structural "boundary"



SIAM NEWS BLOG

Research | January 17, 2017

Print

An Information Theory-Based 'Thermometer' to Uncover Bridge Defects

By Lakshmi Chandrasekaran