Lorentz force impedance tomography in 2D: Theory and Experiments

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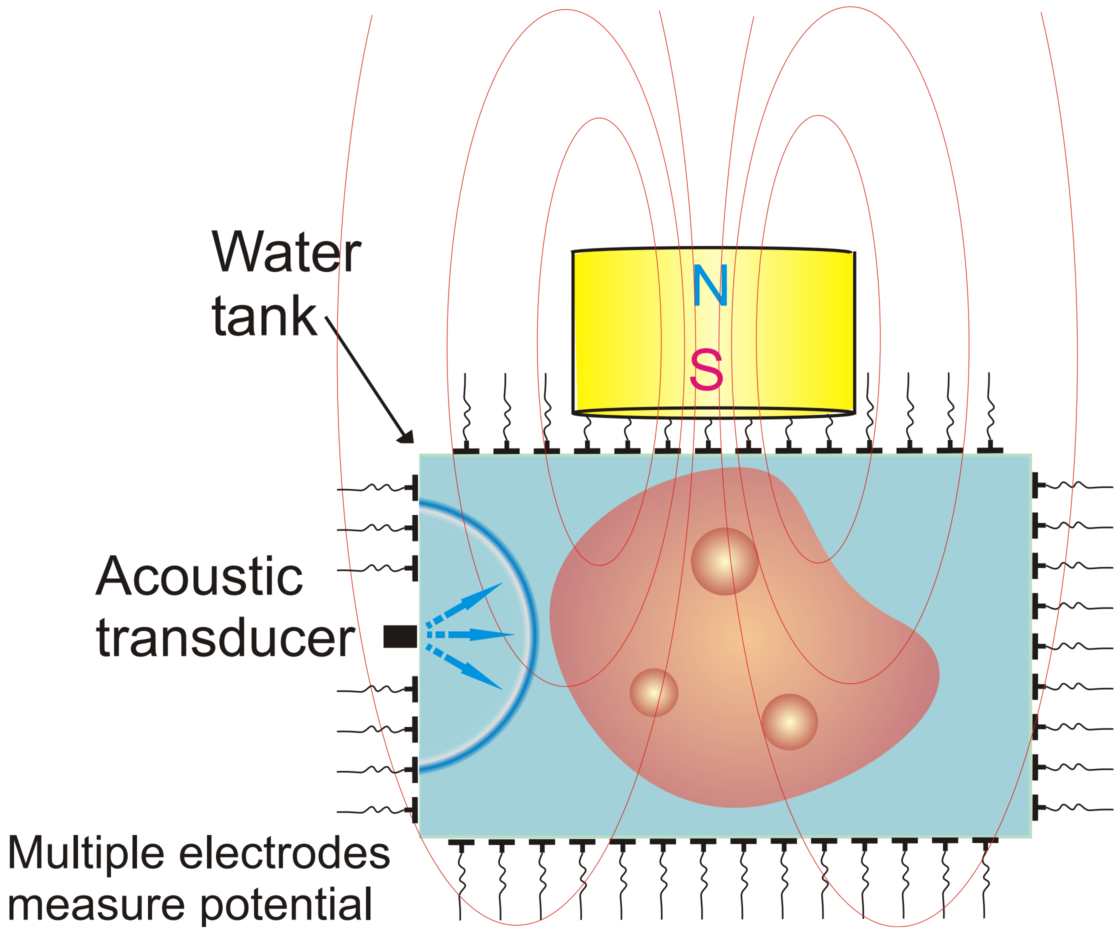
Lorentz Force Tomography (a.k.a MAET)



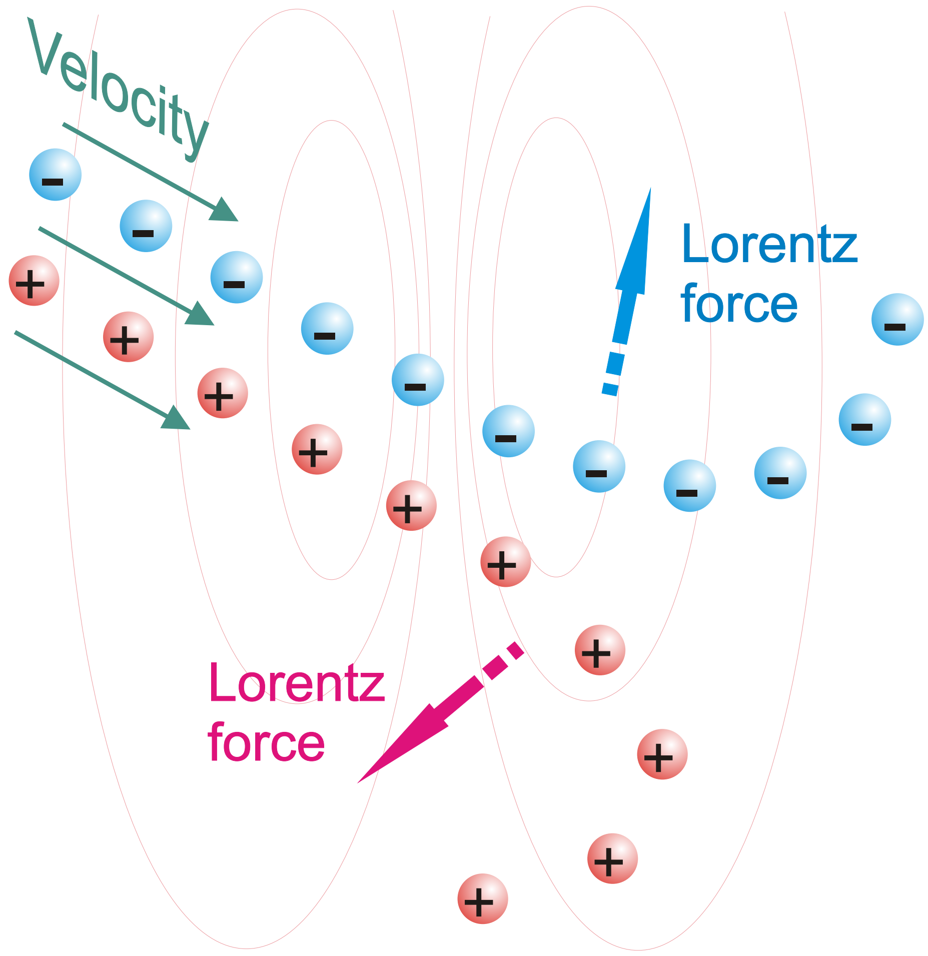
Ultrasound makes electrones and ions vibrate.

As a result, moving electrons and ions are separated by the Lorentz force.

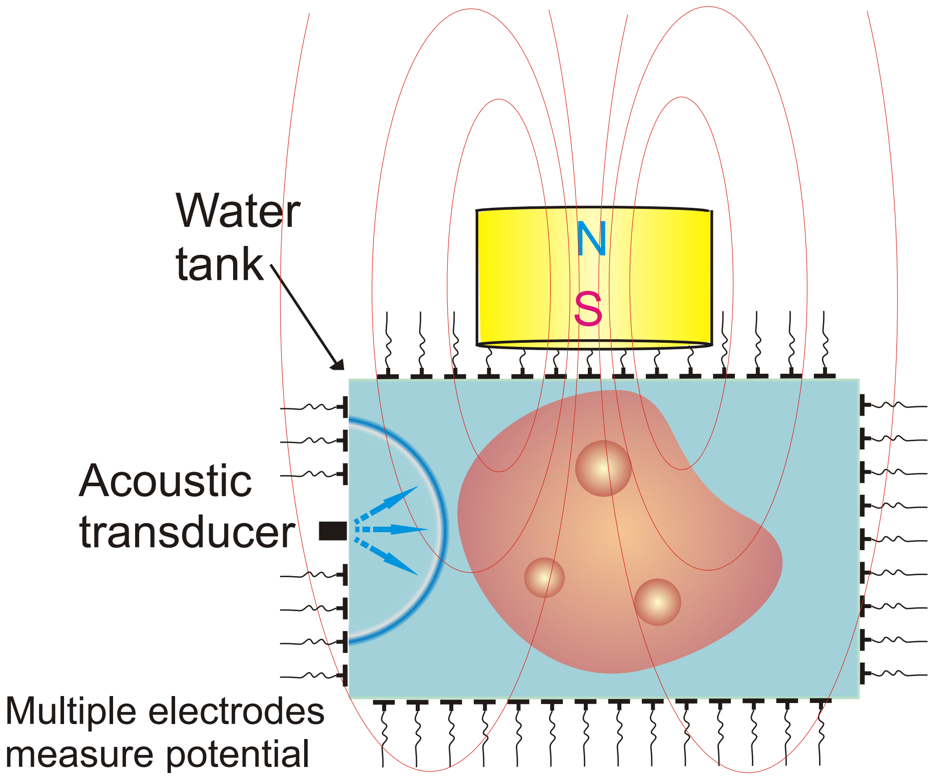
What's the Lorentz Force?

In a magnetic field the Lorentz force pushes moving charges sideways

Positive and negative particles are pushed in the opposite directions



MAET (a.k.a Lorentz Force Tomography)



Separated charges create an electric field that's picked up by the electrodes

With some clever mathematics one can reconstruct an image

Previous work on MAET

The mathematics of MAET (partially explaind below) is very promising

MAET signal has been demonstrated only in one-directional measurements

No truly tomographic MAET images have been obtained before

Our goal: to demonstrate the feasibility of a full-scale MAET

Physics & mathematics of MAET

Tissue moving with velocity $V(x,t)$ produces Lorentz currents $J_L(x,t)$:

$$J_L(x,t) = \sigma(x) B \times V(x,t)$$

There will also be Ohmic currents satisfying Ohm's law

$$J_O(x,t) = \sigma(x) \nabla u(x,t).$$

There are no sinks or sources, the total current is divergence-free

$$\nabla \cdot (J_L + J_O) = 0.$$

Thus

$$\nabla \cdot \sigma \nabla u = -\nabla \cdot (\sigma B \times V).$$

BC: the normal component of the total current $J_L(x,t) + J_O(x,t)$ vanishes:

$$\left. \frac{\partial}{\partial n} u(z) \right|_{\partial \Omega} = -(B \times V(z)) \cdot n(z)$$

Measuring functionals

At any given time t we measure potential $u(z,t)$ at all $z \in \partial\Omega$.

Integrate boundary values of u with weight $I(z)$ and get a functional $M(t)$:

$$M(t) = \int_{\partial\Omega} I(z)u(z,t)dA(z),$$

Introduce lead currents = virtual currents

Consider lead potential $w_I(x)$ and lead current $J_I(x) = \sigma(x)\nabla w_I(x)$:

$$\nabla \cdot \sigma \nabla w_I(x) = 0,$$

$$\left.\frac{\partial}{\partial n} w_I(z)\right|_{\partial\Omega} = I(z).$$

Then, using the second Green's identity (= **reciprocity** principle):

$$M(t) = \int_{\Omega} B \cdot J_I(x) \times V(x,t)dx$$

Analyzing the velocity field

Assume that speed of sound c and density ρ are constant.

Then, velocity is the gradient of the velocity potential $\varphi(x,t)$:

$$V(x,t) = \frac{1}{\rho}\nabla\varphi(x,t),$$

where velocity potential $\varphi(x,t)$ is the time anti-derivative of pressure $p(x,t)$:

$$p(x,t) = \frac{\partial}{\partial t}\varphi(x,t).$$

Substitute into equation for $M(t)$ and integrate by parts:

$$M(t) = \frac{1}{\rho}B\cdot\left[\int_{\partial\Omega}\varphi(z,t)J_I(z)\times n(z)dA(z) + \int_{\Omega}\varphi(x,t)\nabla\times J_I(x)dx\right]$$

Volumetric part shows that we measure components of **curl** $J_I(x)$!

$$\mathbf{curl}\ J_I(x) = \nabla\times[\sigma(x)\nabla w_I(x)] = \nabla\sigma(x)\times\nabla w_I(x) = \nabla\ln\sigma(x)\times J_I(x)$$

Notice: in the regions where $\sigma(x)$ is constant, **curl** $J_I(x) = 0$. No signal!

Reconstruction procedure

If $\varphi(x,t)$ could be focused into a point, i.e. $\varphi(x,0)=\delta(x-x_0)$, then

$$M_{x_0}(0)=\frac{1}{\rho}B\cdot\left[\int_\Omega \delta(x-x_0)\text{curl}J_I(x)dx\right]=\frac{1}{\rho}B\cdot\text{curl}J_I(x_0).$$

If three differenent directions of B are used, we have $C(x_0)$ = $\text{curl}J_I(x_0)$!

Chain of equtions to solve:

Curl C -> Current I -> $\nabla\ln\sigma(x)$ -> Conductivity $\sigma(x)$.

The second step comes from:

$$\nabla\ln\sigma\times J=C.$$

If we have two currents $J^{(j)}(x)$, $j=1,2$, then solve for $\nabla\ln\sigma$ at each x

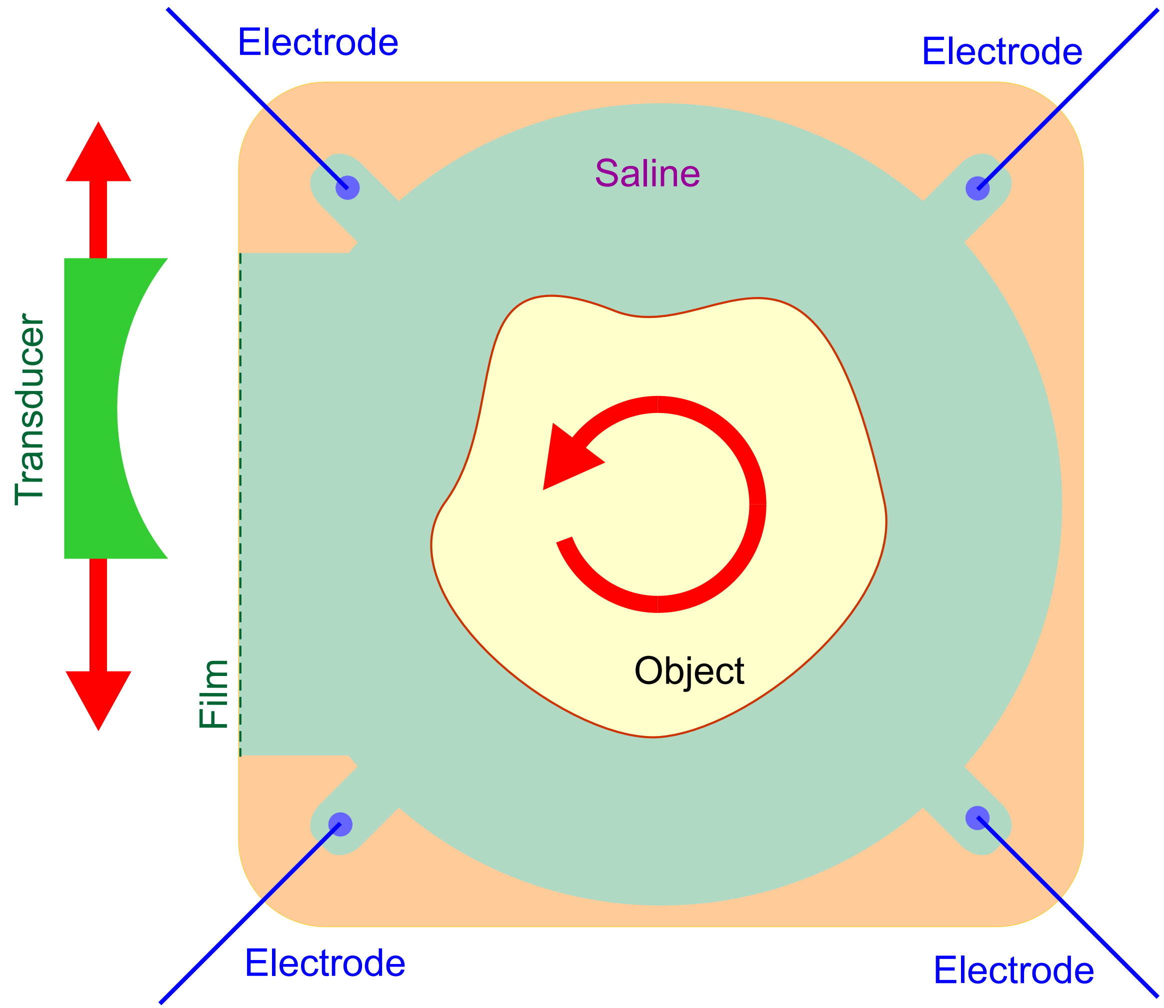
$$\begin{cases}\nabla\ln\sigma(x)\times J^{(1)}(x)=C^{(1)}(x)\\ \nabla\ln\sigma(x)\times J^{(2)}(x)=C^{(2)}(x)\end{cases}.$$

Something simpler: 2D MAET

Full 3-D scanner for MAET is difficult to build

We want to demonstrate the feasibility of MAET in a 2D setting

A simple 2D MAET scanner, top view:



2D MAET: assumptions and an approximate solution

Reasonable assumptions:

Everything is constant in the vertical direction ($\vec{e}_z$).

Magnetic induction $B = b\vec{e}_z$ (vertical and constant).

All the objects have vertical boundaries (genereralized cylinders)

Electrodes are vertical lines

Then, all curls are vertical and parallel to B and we measure $\frac{b}{\rho}\mathrm{curl}_z J$

Crude and unreasonable approximations:

Lead currents $J^{(1)}$ and $J^{(2)}$ are slightly perturbed orthogonal vectors:

$$J^{(k)}(x) = \vec{e}^{(k)} + \varepsilon j^{(k)}(x), \qquad \vec{e}^{(1)} \cdot \vec{e}^{(2)} = 0, \ \varepsilon \text{ is very small.}$$

Then

$$\Delta \ln \sigma = \frac{\partial}{\partial e_2} C_z^{(1)} - \frac{\partial}{\partial e_1} C_z^{(2)}.$$

Synthetic transducer and synthetic currents

Add measurements along one-angle scan -> synthetic flat transducer

Synthetic flat transducer -> integral over a line = Radon transform

Invert the Radon transform to get the curl

Also, synthetic currents, since the object rotates! (Explain?)

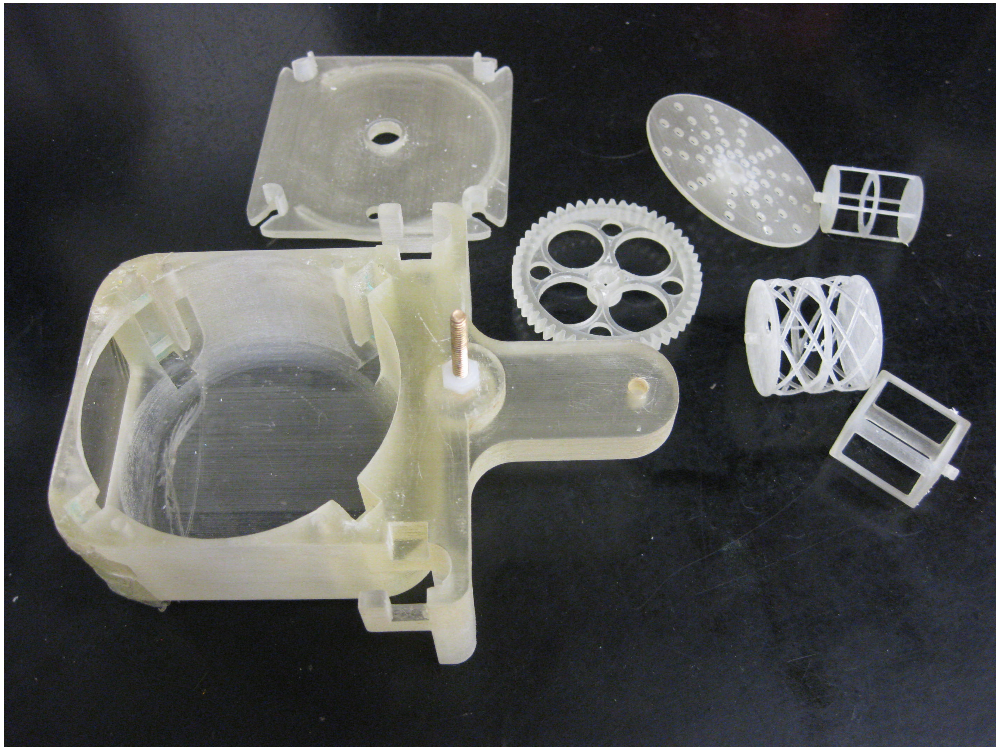
The practical side of the story

Joint work with R. Witte and P. Ingram, Medical Imaging Department, UA

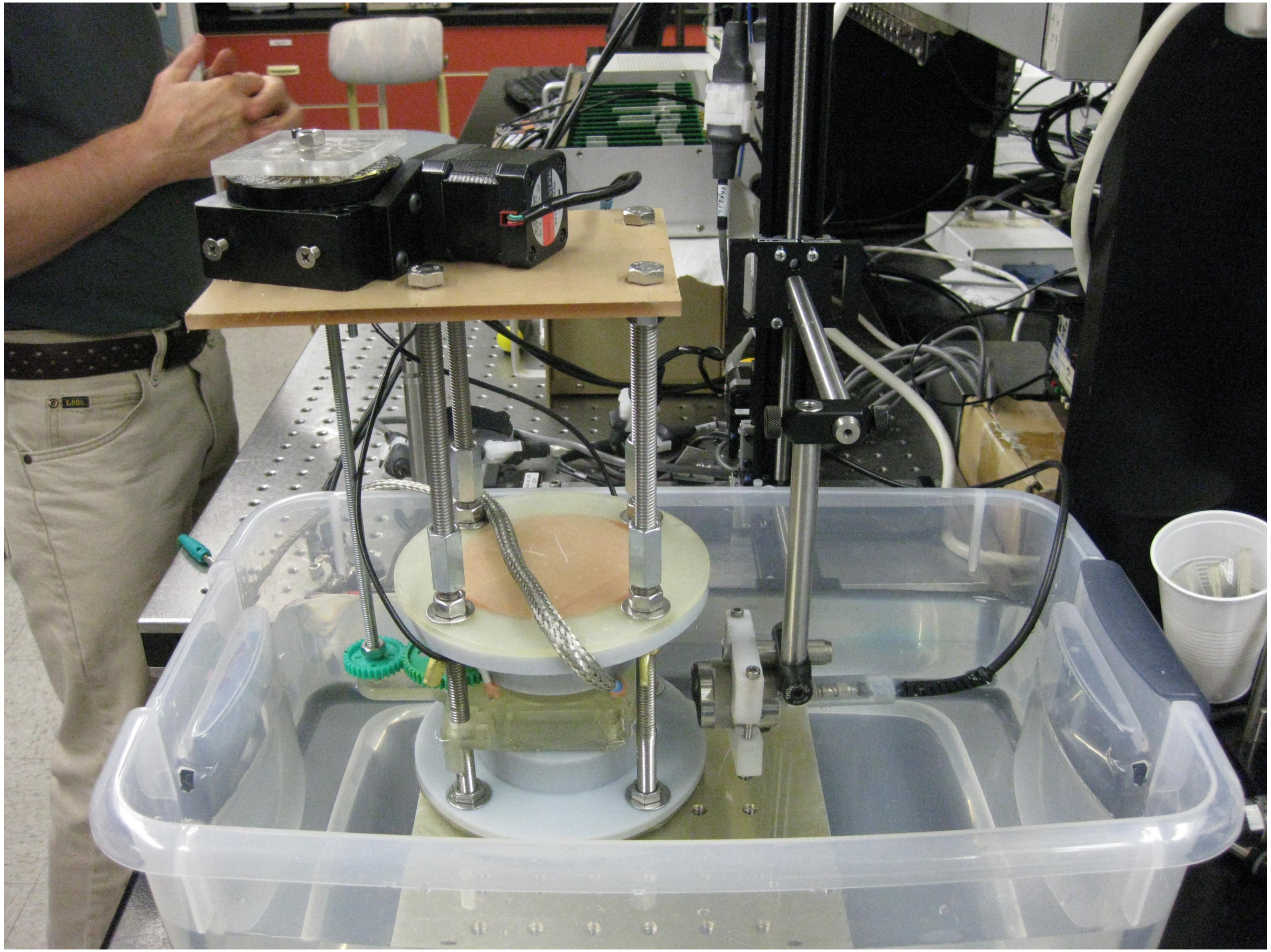
Supported by a BIO5 fellowship, but no money for hardware :(

Goal: build the first MAET scanner, get first MAET images

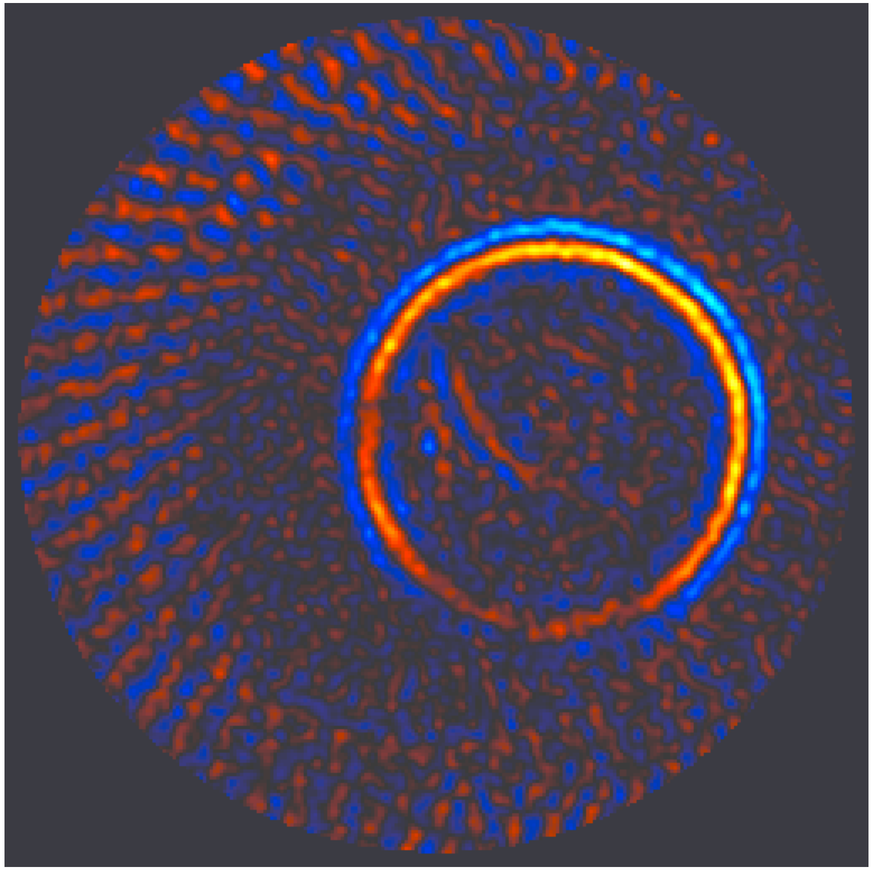
Parts were designed in SolidWorks and 3D-printed

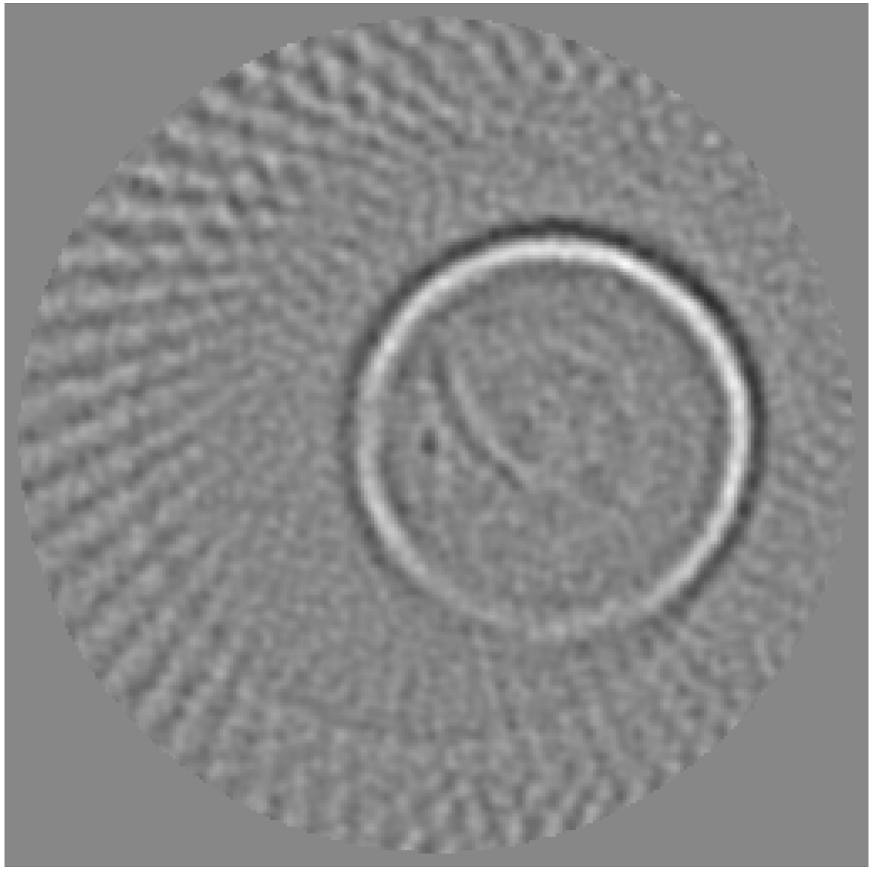


Fully assembled, in a tank, with a transducer

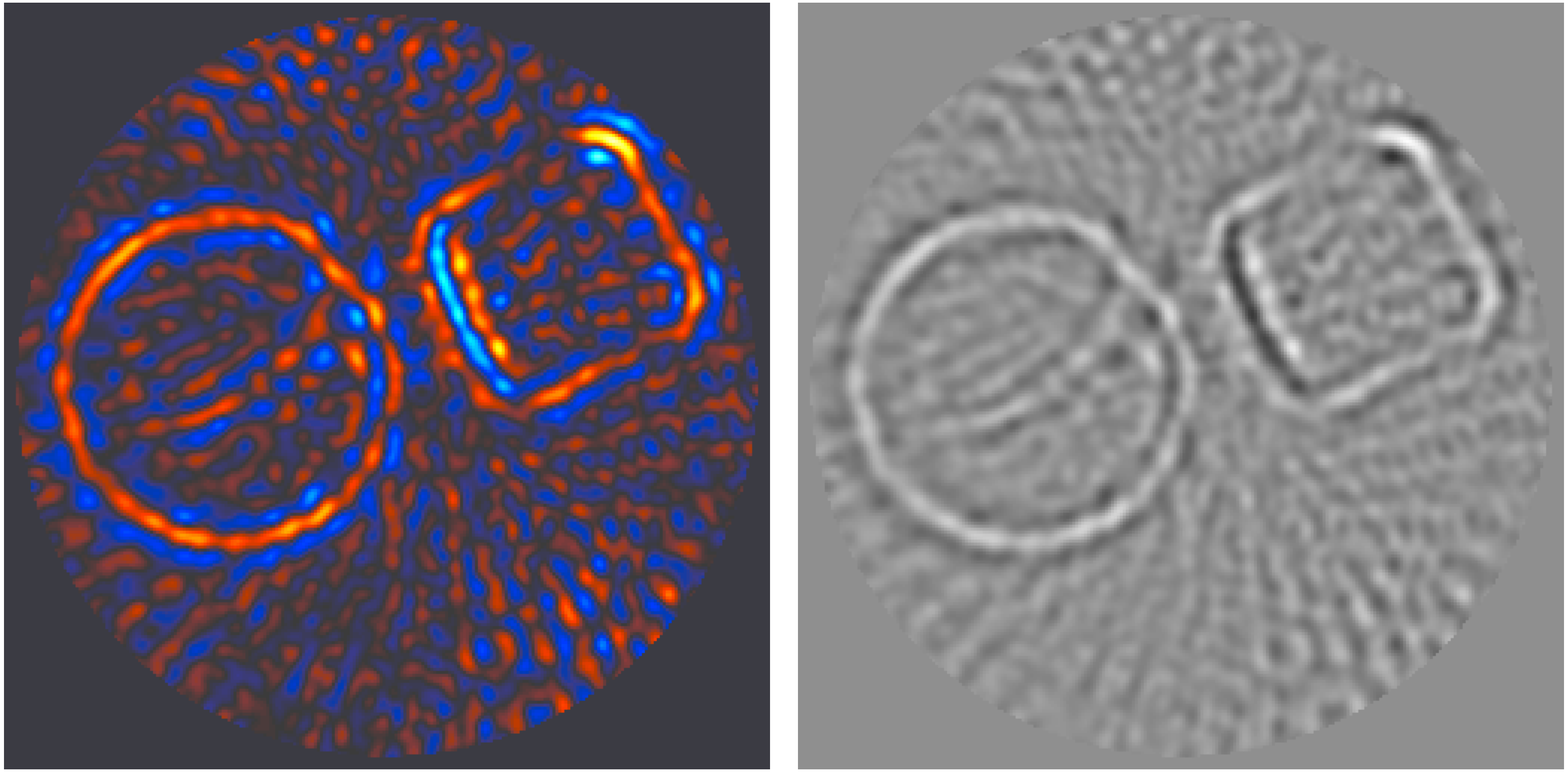


First reconstruction: round Tagaderm holder

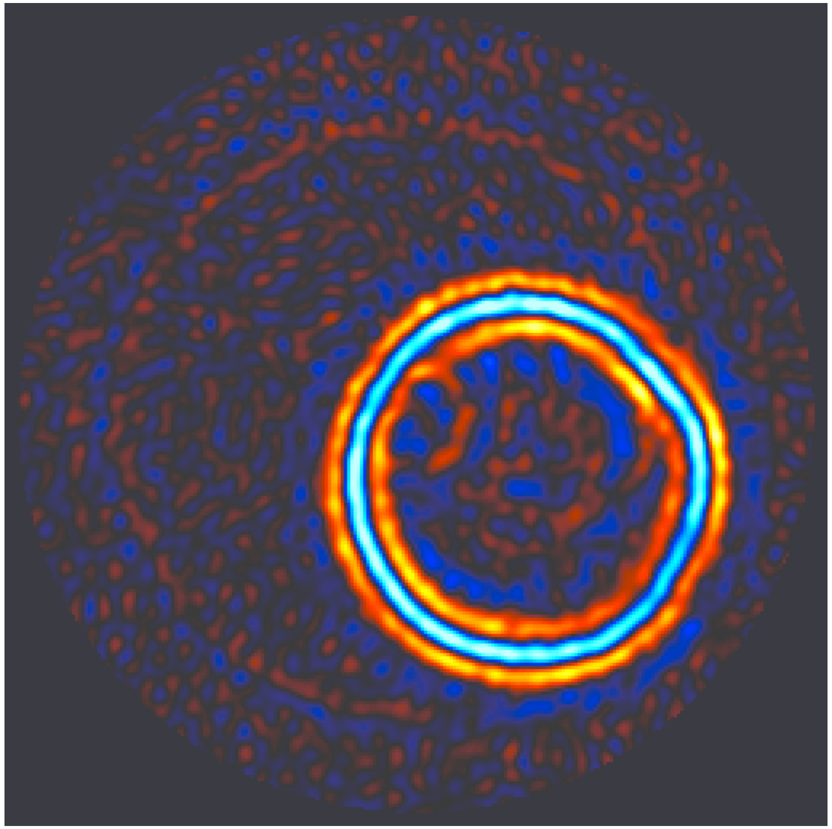


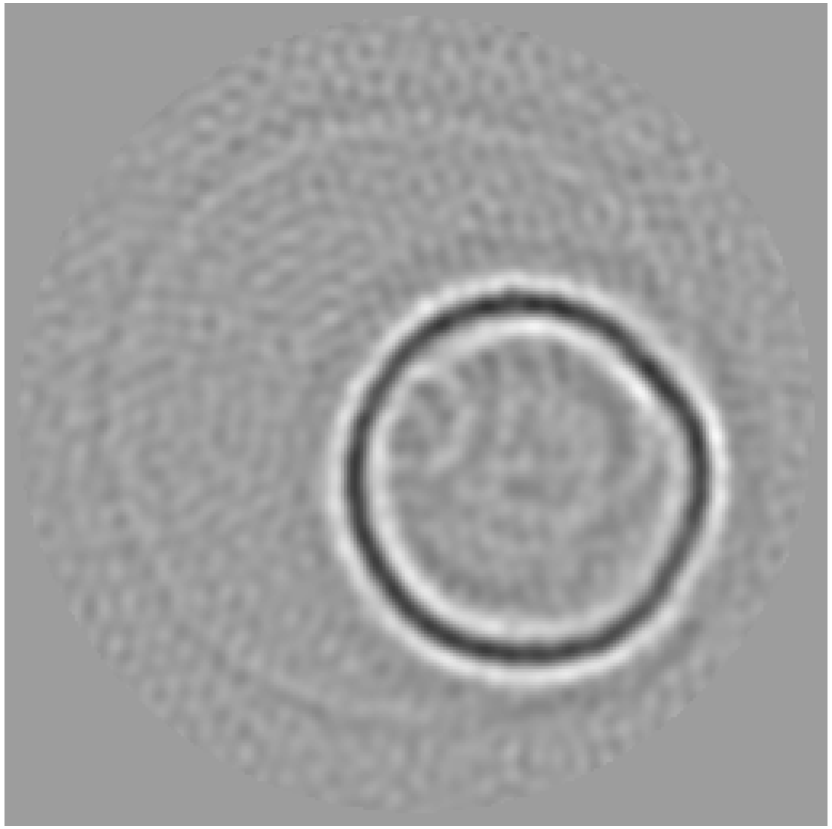


Round and square Tagaderm holders

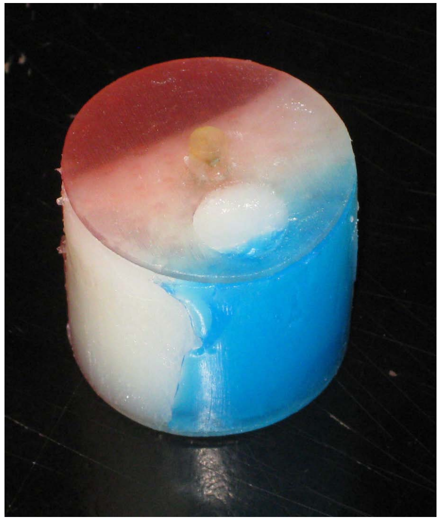


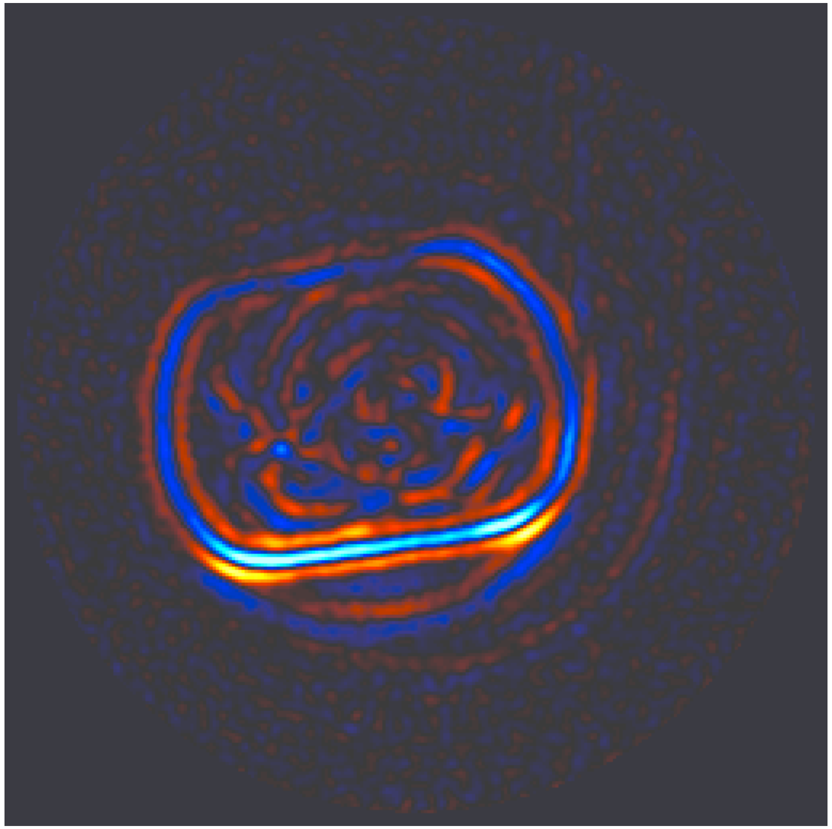
Round lard column, 30mm in diameter

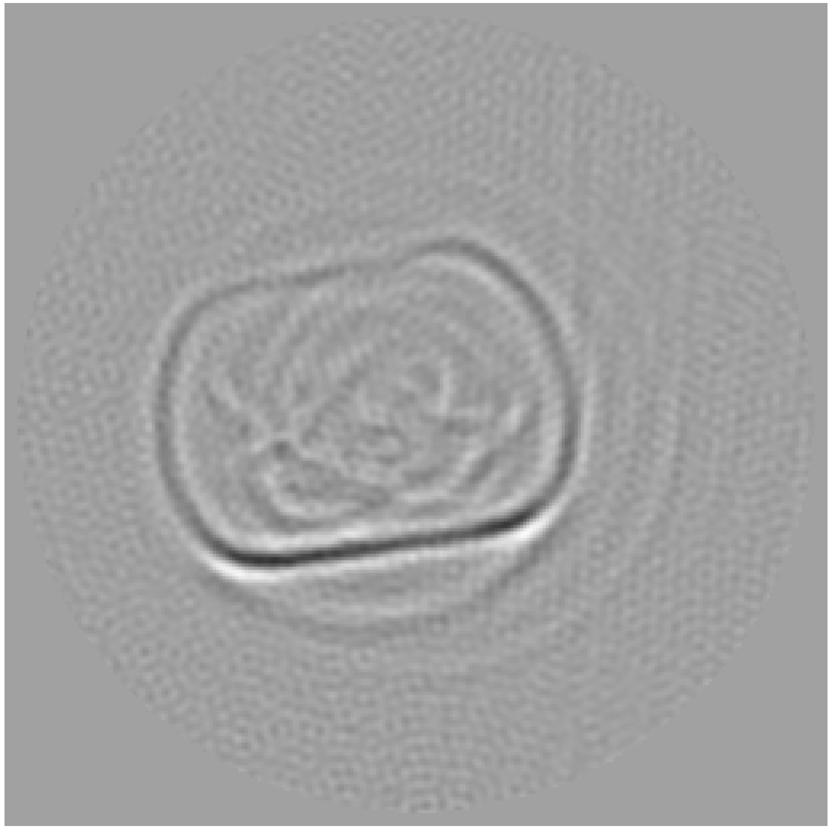




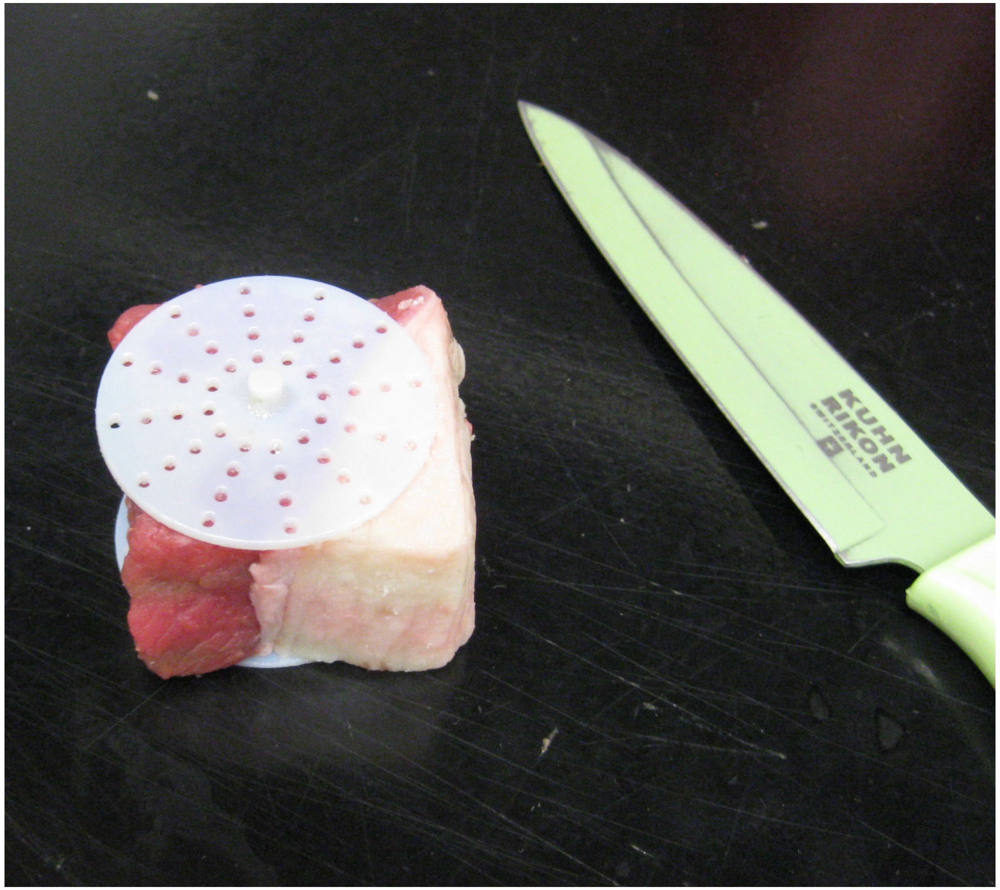
Layered gel-lard-gel phantom







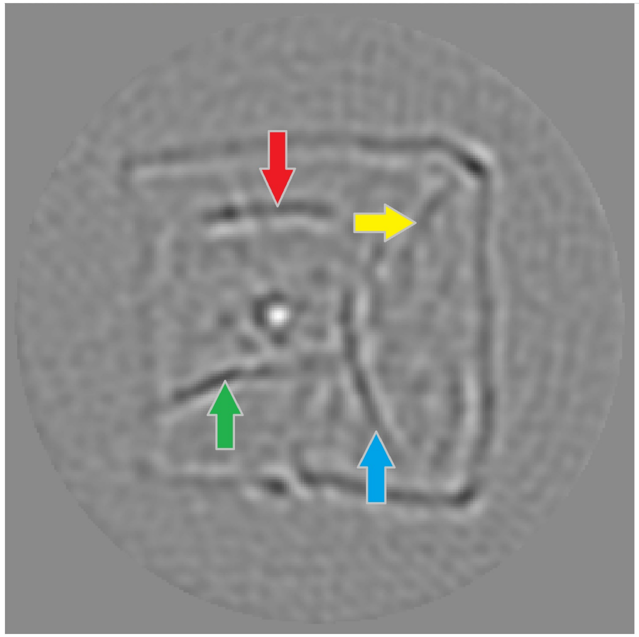
A beef sample



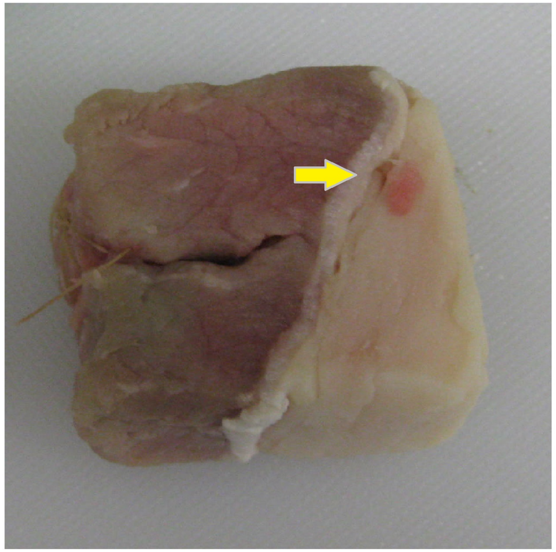
This is the first truly tomographic MAET image of a biological tissue.

Are the details in the image real?

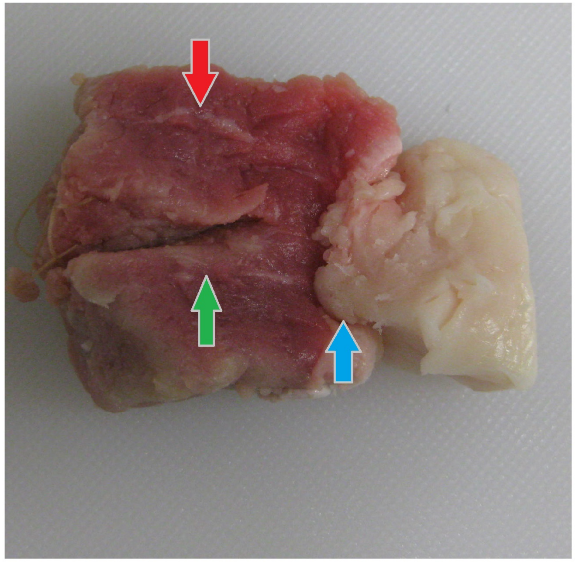
Image



Sample



Sample cut in half



What’s next?

MAET in a bore of an MRI scanner?

Photoacoustic generation of ultrasound waves?

Electromagnetic generation of ultrasound waves?

The end