# Linking Compositional Properties and Epeirogenic Movement in Mantle Flow Models

or

Thermo-chemical Adjoint Equations

Sia Ghelichkhan

Many Thanks to H.-P. Bunge, R. Pail, L. Colli, J. Oeser

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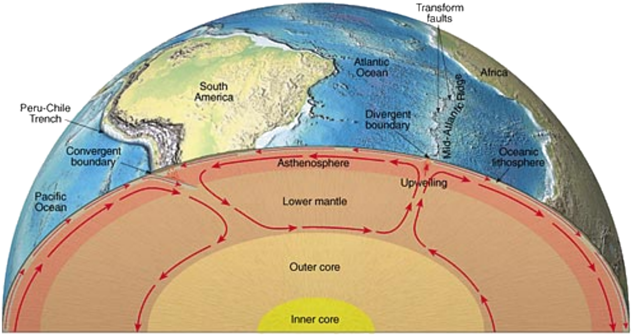
Geophysics Section
Ludwig-Maximilians-Universität München

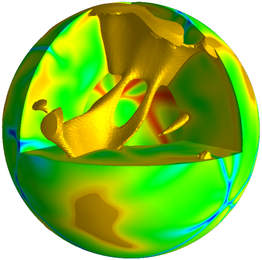
SIAM GS19

# Earth's Mantle

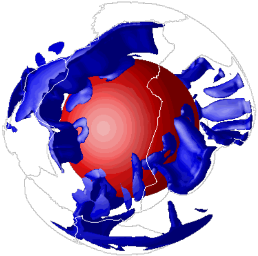



- **3000 km** deep layer (half-way to the Earth's center)
- made out of sillicates (known as Rocks)
- Mantle Convection: Slowly deforming by *creep*
- providing forces to maintain **Plate Tectonics**

# Mantle Convection Codes



Terra



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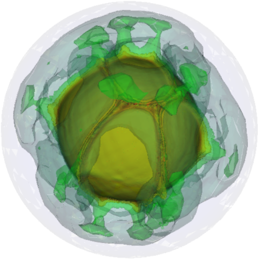
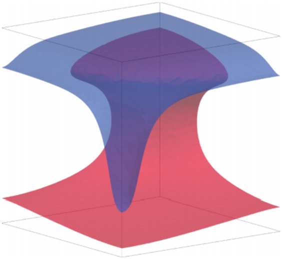
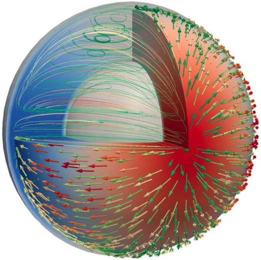


Aspect



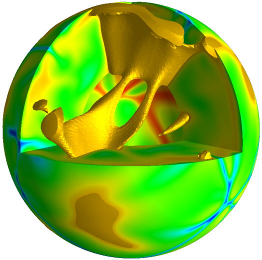
Fluidity



Terra Neo

# Mantle Convection Codes

- Today: Many (Community) Codes available



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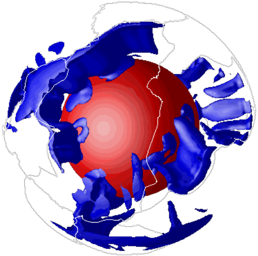

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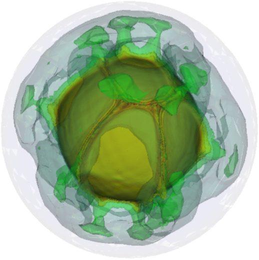

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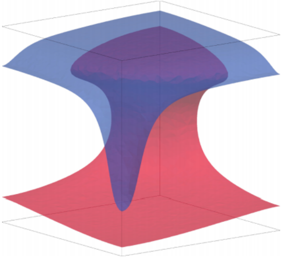

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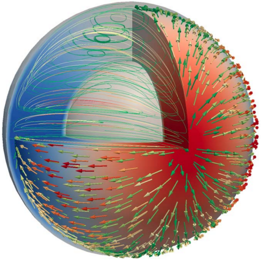

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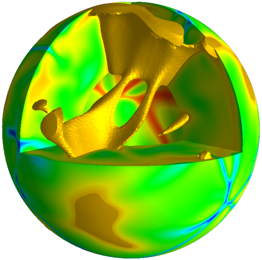

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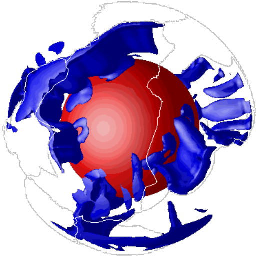
- Today: Many (Community) Codes available
- Many complex features: non-linear rheology, thermochemical flow



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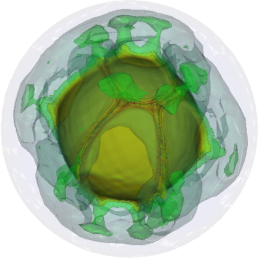

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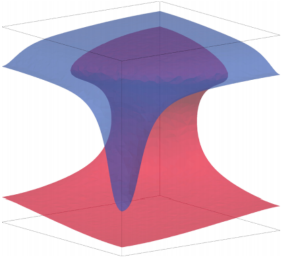

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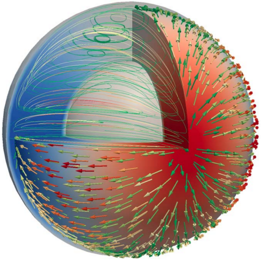

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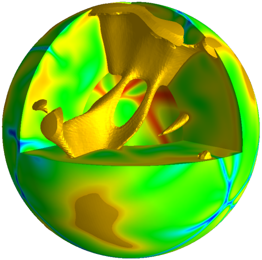
# Mantle Convection Codes

- Today: Many (Community) Codes available
- Many complex features: non-linear rheology, thermochemical flow
- Lack of *First Principle Physics*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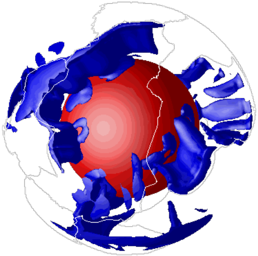
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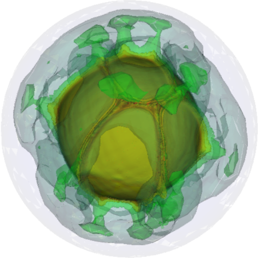
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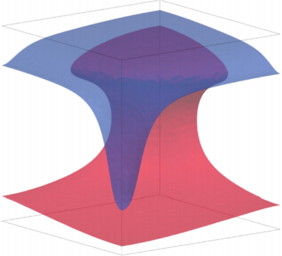
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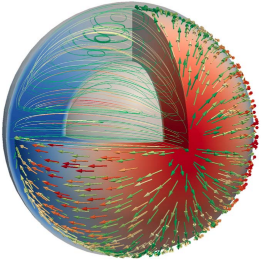
Terra Neo

# Forward Problem

**Governing Equations:** (Boussinesq approximation)

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho v) = 0$$

$$\varrho \frac{Dv}{Dt} = -\nabla P + \eta \nabla^2 v + F$$

$$\frac{\partial T}{\partial t} = -v \cdot \nabla T + k \nabla^2 T + H$$

# Forward Problem

## Governing Equations: (Boussinesq approximation)

$$\underbrace{\frac{\partial \varrho}{\partial t}}_{\text{No acoustic wave}} + \nabla \cdot (\underbrace{\varrho}_{\varrho=\text{const}} v) = 0$$

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**Governing Equations:** (Boussinesq approximation)

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$$0 = -\nabla P + \eta \nabla^2 v + F$$

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## Governing Equations: (Boussinesq approximation)

$$\nabla \cdot v = 0$$

$$\underbrace{-\eta \nabla^2 v}_{\textit{Resisting}} = \underbrace{-\nabla P + F}_{\textit{Driving}}$$

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Resisting — Driving

Buoyancy:

$$F = \bar{\varrho} g \alpha \Delta T$$

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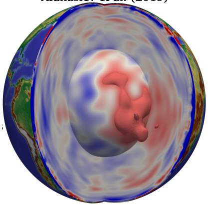
Afanasiev et al. (2015)



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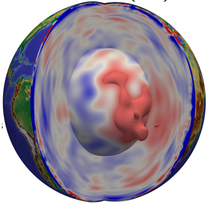

Forward, $\Delta t > 0$ →

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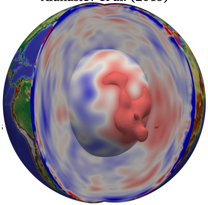

Forward, $\Delta t > 0$ →

No Observation

Large Time-Scales

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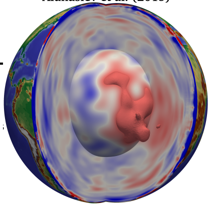
Backward, $\Delta t < 0$



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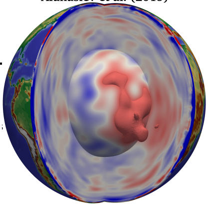

Geology ... ← Backward, $\Delta t < 0$

Forward, $\Delta t > 0$ → No Observation

Large Time-Scales

# Forward Problem

**Governing Equations:** (Boussinesq approximation)

$$\nabla \cdot v = 0$$

$$\underbrace{-\eta \nabla^2 v}_{\color{red}{Resisting}} = \underbrace{-\nabla P + F}_{\color{blue}{Driving}}$$

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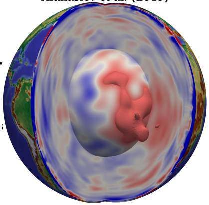


Geology
...

Backward, $\Delta t < 0$

Unstable!

Forward, $\Delta t > 0$

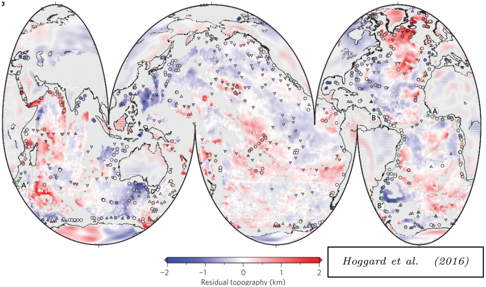
No Observation

Large Time-Scales

# Testing Models Against Observations

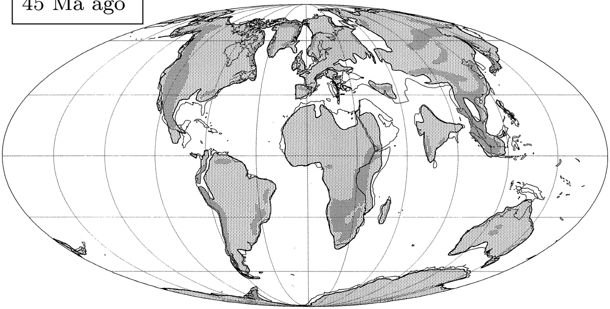
## Dynamic Topography

- Deformation of the Earth's surface due to convection currents in the Earth's mantle.
- Equipotential Figure of the Earth (geoid).




*Hoggard et al.* (2016)

45 Ma ago



## Observations

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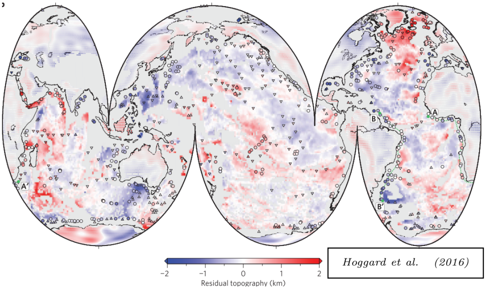


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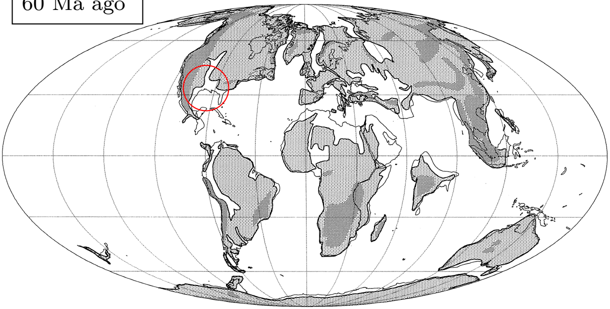

## Observations

- Opening of the North-American interior seaway

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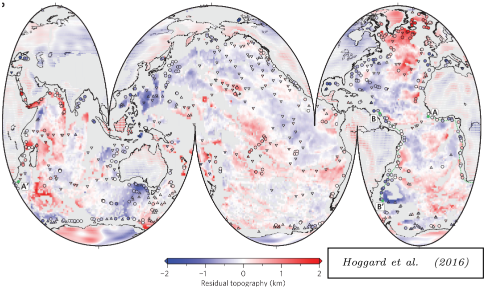
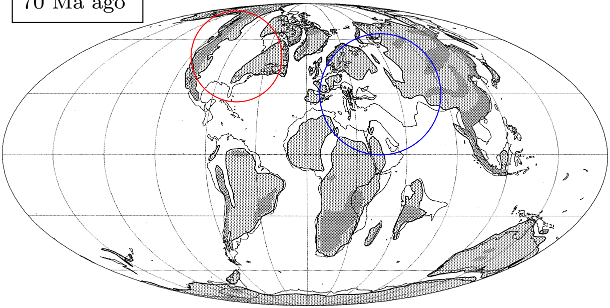



*Hoggard et al. (2016)*

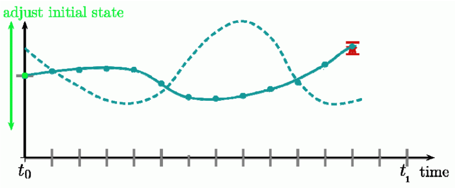
70 Ma ago



## Observations

- Opening of the North-American interior seaway
- Subsidence in the Tethys-Realm

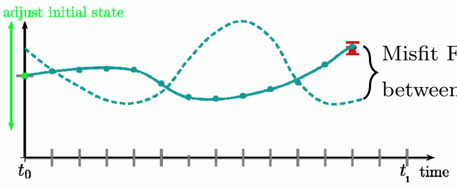
# Adjoint Method




*Fournier et al., (2012)* similar to other approaches in oceanography, meteorology

## Thermal$_{\text{(Boussinesq)}}$ Adjoint Equations

# Adjoint Method

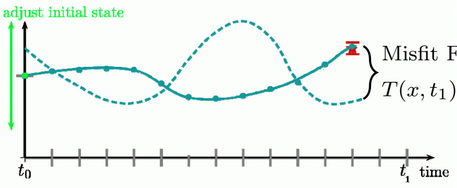



Misfit Function: Difference between model and observation

*Fournier et al., (2012)* similar to other approaches in oceanography, meteorology

## Thermal(Boussinesq) Adjoint Equations

# Adjoint Method




Misfit Function:

$T(x, t_1) - T_E$

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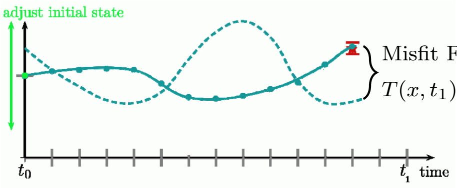
## Thermal(Boussinesq) Adjoint Equations

# Adjoint Method




Misfit Function:

$T(x, t_1) - T_E$

$$\mathbf{M} = 1/2 \int_{x^3} \left(T\left(x, t_1\right) - T_E\right)^2 \, dx$$

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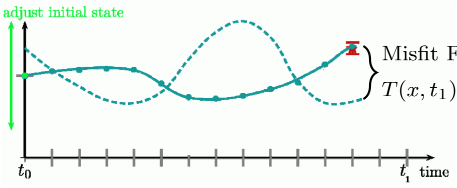
## Thermal(Boussinesq) Adjoint Equations

# Adjoint Method

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$$\partial_{T_0}\mathbf{M} = \Psi(x, t_0)$$
$$T'(x, t_0) = T(x, t_0) - \alpha \partial_{T_0} M$$

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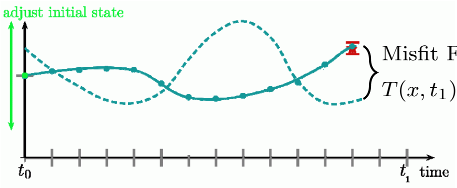
# Adjoint Method

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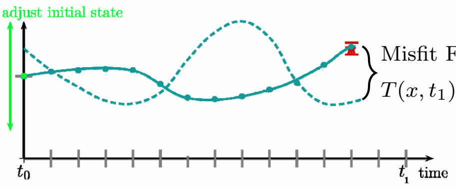
## Thermal$_{\text{(Boussinesq)}}$ Adjoint Equations

$$\begin{aligned}
\nabla \cdot \varphi &= 0 \\
\nabla \cdot [\eta(\nabla\varphi + (\nabla\varphi)^T)] - \varrho_r \nabla\lambda + \Psi\nabla T &= 0 \\
\partial_t \Psi + v\nabla \cdot \Psi + k\nabla^2\Psi - \alpha_r g \cdot \varphi &= 0 \\
\Psi(x,t_1) &= T(x,t_1) - T_E
\end{aligned}$$

# Adjoint Method




Misfit Function:

$T(x, t_1) - T_E$

*Fournier et al., (2012)* similar to other approaches in oceanography, meteorology

$$\mathbf{M} = 1/2 \int_{x^3} (T(x, t_1) - T_E)^2 \, dx$$

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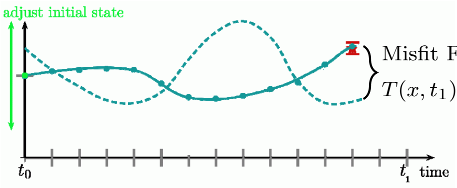


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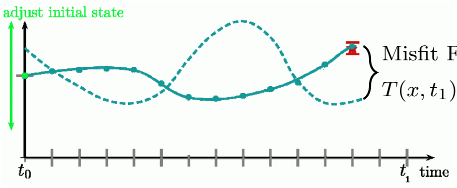
$$\mathbf{M} = 1/2 \int_{x^3} \left(T\left(x,t_1\right) - T_E\right)^2 \, dx$$

$$\begin{aligned} \partial_{T_0}\mathbf{M} &= \Psi(x,t_0) \\ T'(x,t_0) &= T(x,t_0) - \alpha\partial_{T_0}M \end{aligned}$$

## Thermal(Boussinesq) Adjoint Equations

$$\begin{aligned} \nabla\cdot\varphi &= 0 \\ \nabla\cdot[\eta(\nabla\varphi + (\nabla\varphi)^T)] - \varrho_r\nabla\lambda + \Psi\nabla T &= 0 \\ \partial_t\Psi + v\nabla\cdot\Psi + k\nabla^2\Psi - \alpha_r g\cdot\varphi &= 0 \\ \Psi(x,t_1) &= T(x,t_1) - T_E \end{aligned}$$

# Adjoint Method




Misfit Function:

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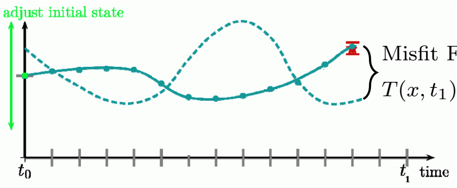


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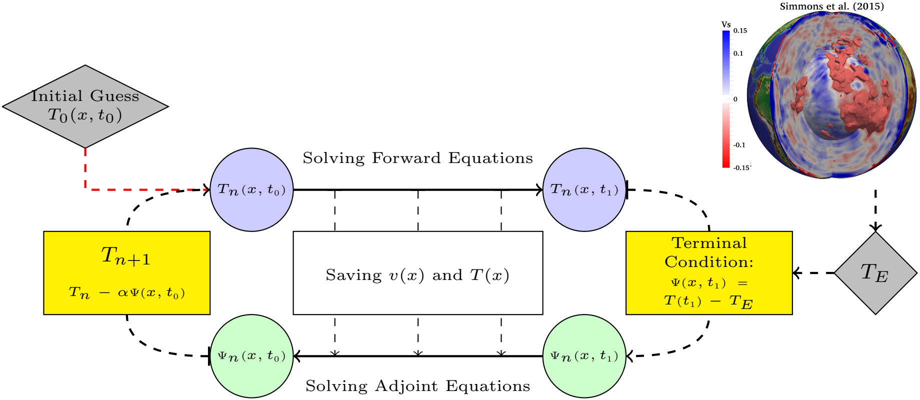
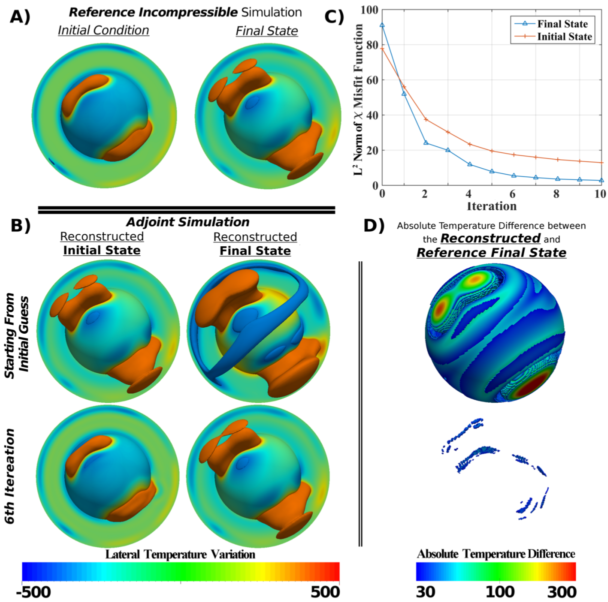
Ghelichkhan & Bunge (2016), *Int. Journal of Geomathematics*

Compressible Adjoint Equations in Geodynamics

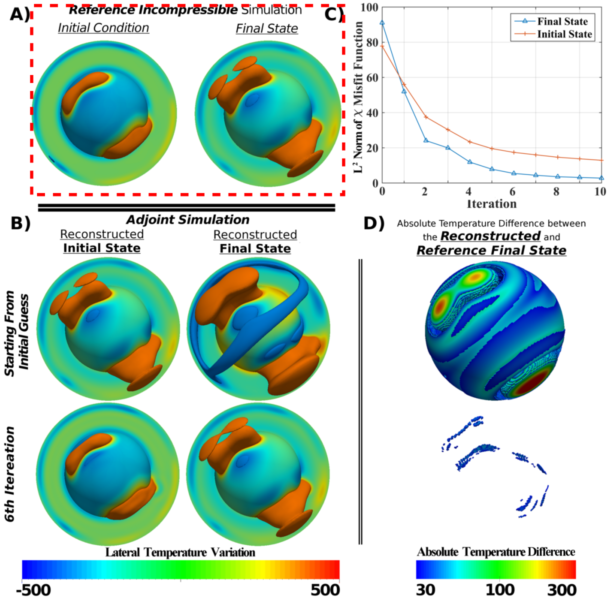
# How?


Simmons et al. (2015)
Vs
0.15
0.1
0
-0.1
-0.15
Initial Guess
$T_0(x, t_0)$
Solving Forward Equations
$T_n(x, t_0)$
$T_n(x, t_1)$
$T_{n+1}$
$T_n - \alpha\Psi(x, t_0)$
Saving $v(x)$ and $T(x)$
Terminal Condition:
$\Psi(x, t_1) = T(t_1) - T_E$
$T_E$
$\Psi_n(x, t_0)$
$\Psi_n(x, t_1)$
Solving Adjoint Equations




## Twin Experiment

- Goal: To test performance of the method
- **Reference Twin**: A ref simulation with known initial and final conditions
- **Reconstructed Twin**: Applying the adjoint method to restore the evolution
- Measurable misfit both for the final state and **initial state**

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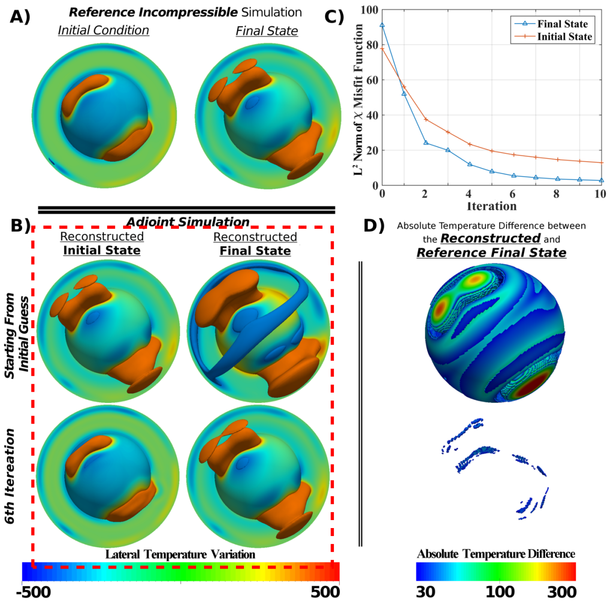


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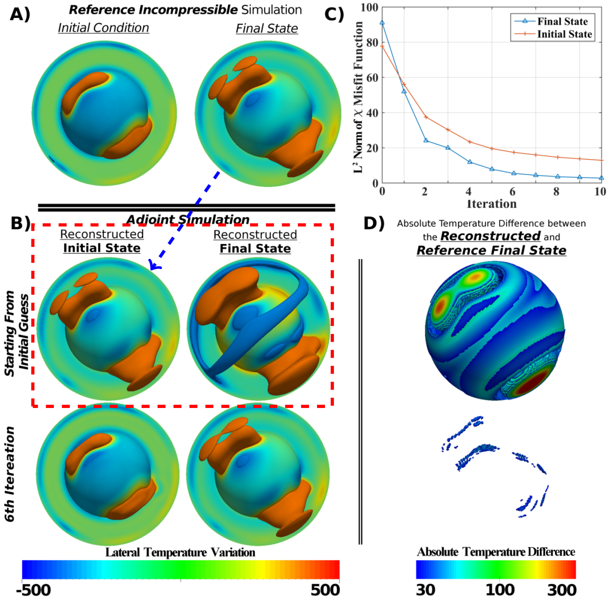


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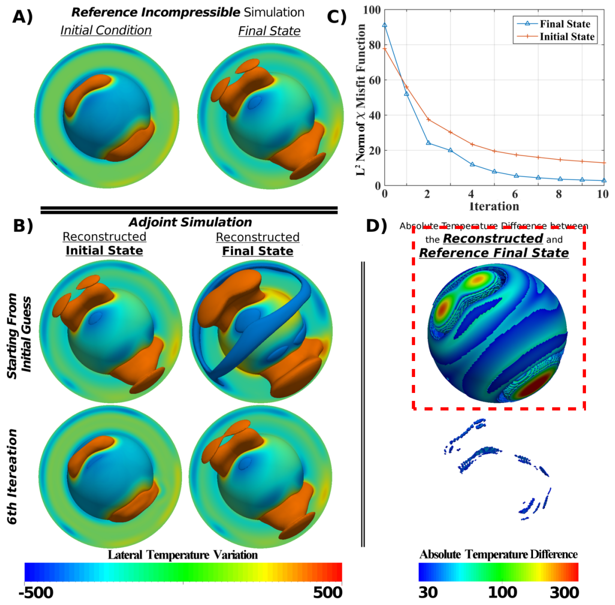


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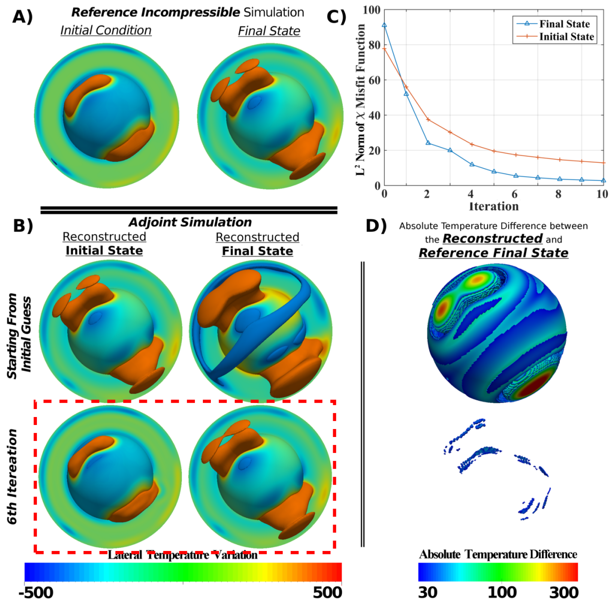


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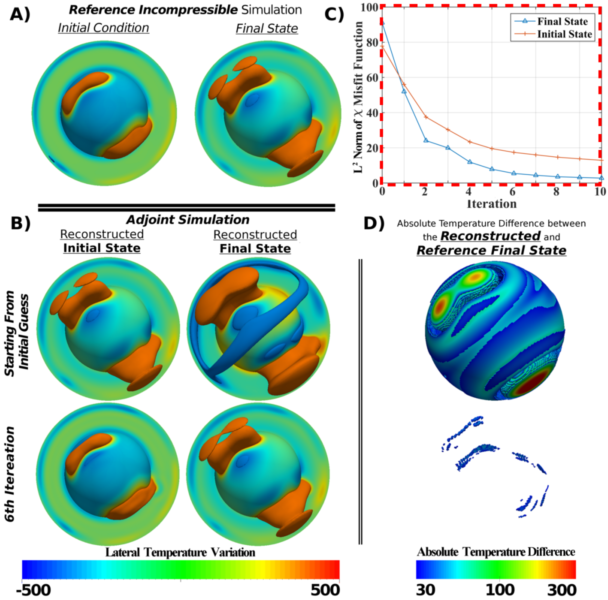


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**Publications:**

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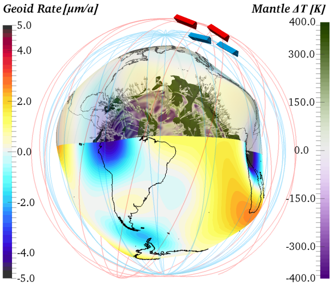


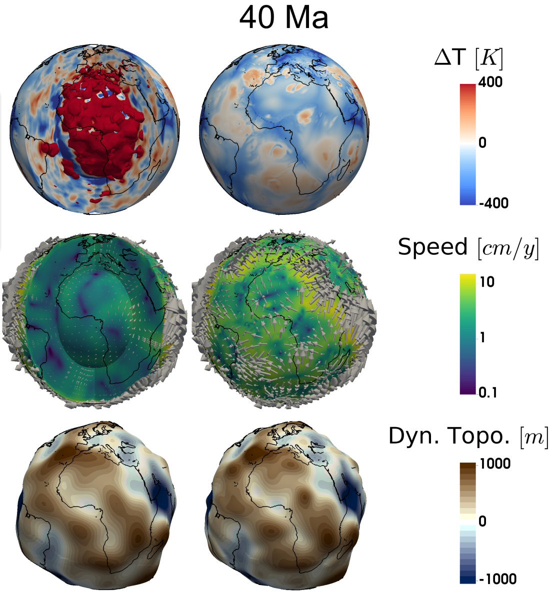

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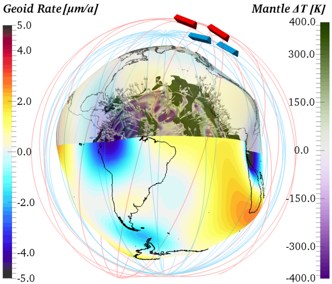


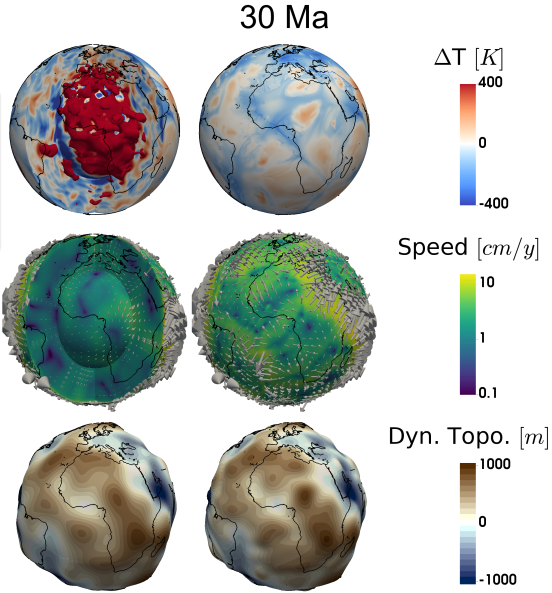

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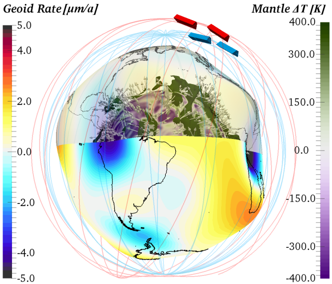


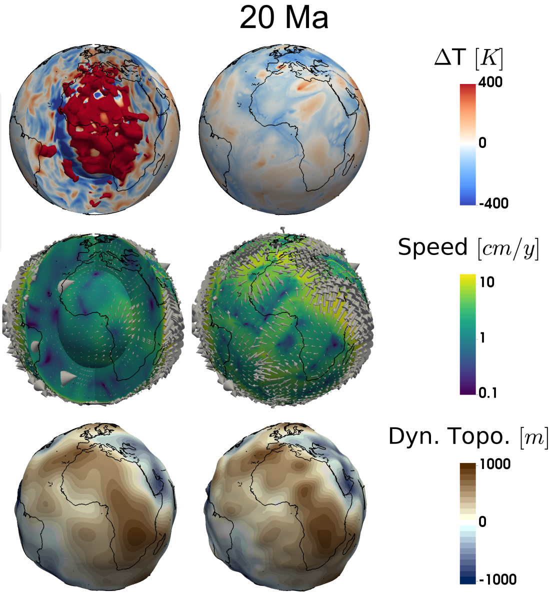

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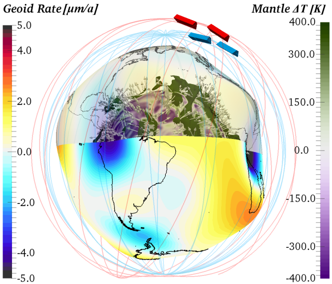


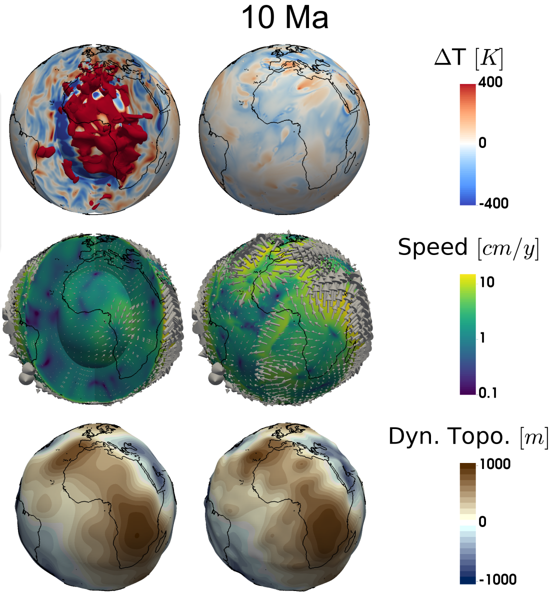

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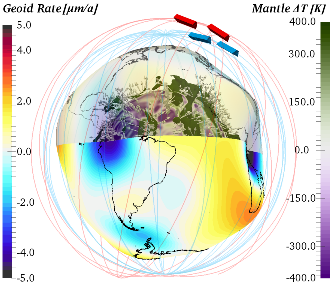
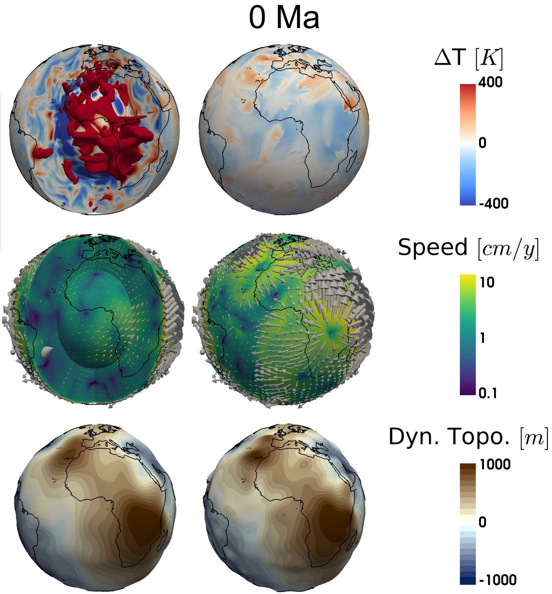


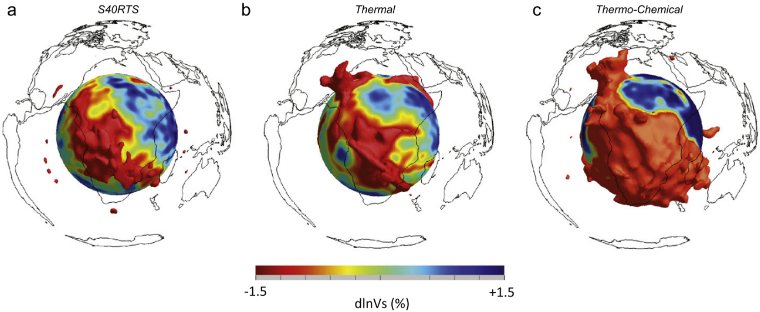



# Large Scale Chemical Heterogeneity?

**LLSVPs**

"**L**arge **L**ow **S**hear **V**elocity **P**rovince" s

| Observation (among others): | TH | TCH |
|---|---|---|
| LLSVP morphology | ✓ | ✓ |
| shear-wave velocity amplitudes and gradients | ✓ | ✓ |
| (relative) variation of shear, compressional and bulk-sound speeds | ✓ | ✓ |
| . . . | ✓ | ✓ |




Davies et al. (2015), *The Earth's Heterogeneous Mantle*

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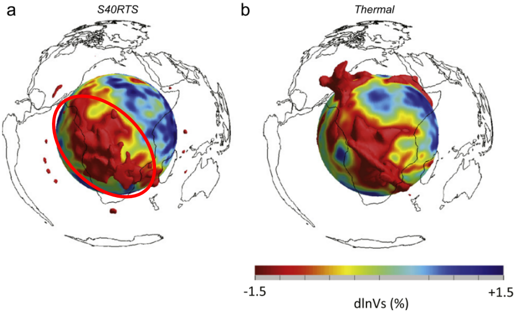


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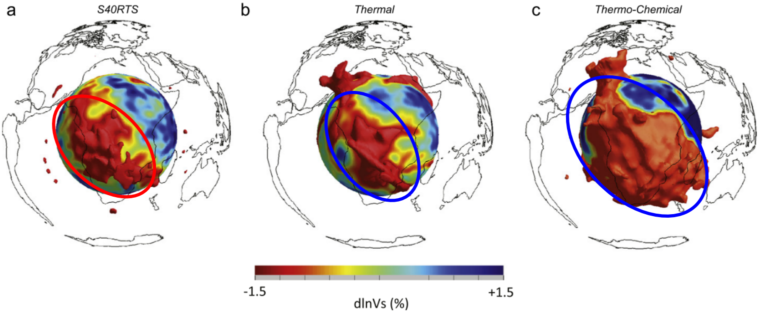


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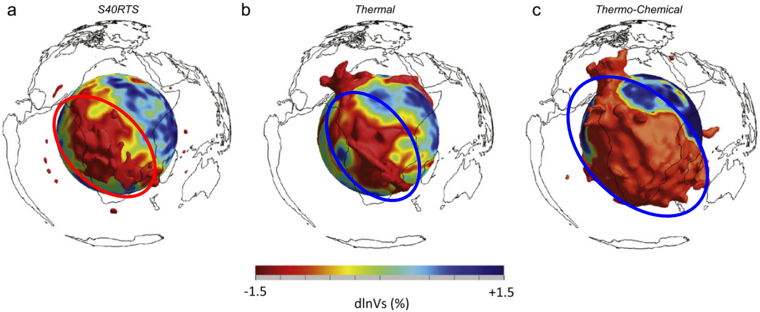


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# Large Scale Chemical Heterogeneity?

## LLVPs

"**L**arge **L**ow **V**elocity **P**rovince" s
In absence of sensitive evidence, people on both sides of the aisle:
**Thermochemical** Piles $\Longrightarrow$ recycle of oceanic plates, primordial material

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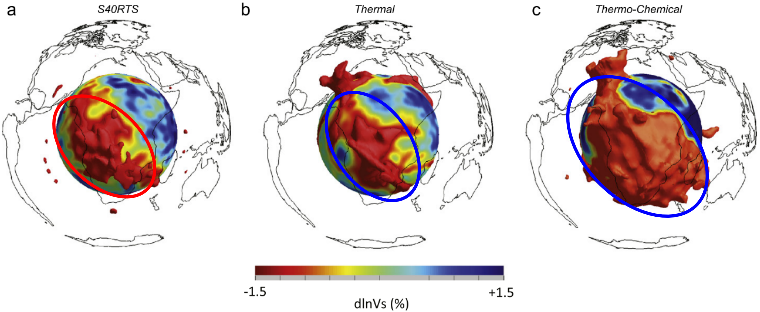


Davies et al. (2015), *The Earth's Heterogeneous Mantle*

# Large Scale Chemical Heterogeneity?

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**Thermochemical** Piles ⟹ recycle of oceanic plates, primordial material
**Thermal** Piles ⟹ Let's avoid unnecessary degree of freedom

| Observation (among others): | TH | TCH |
|---|---|---|
| LLSVP morphology | ✓ | ✓ |
| shear-wave velocity amplitudes and gradients | ✓ | ✓ |
| (relative) variation of shear, compressional and bulk-sound speeds | ✓ | ✓ |
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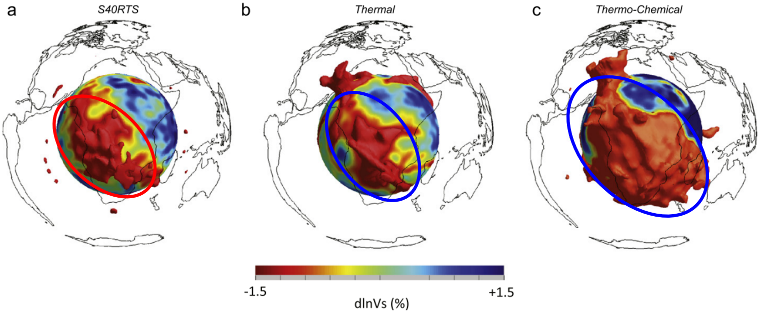


Davies et al. (2015), *The Earth's Heterogeneous Mantle*

## (Anelastic Liquid) Thermochemical Convection

$$\begin{aligned}
\nabla \cdot (\varrho_r v) &= 0 \\
\nabla \cdot \left[\eta\left(\nabla v\right) + \left(\nabla v\right)^T - \tfrac{2}{3}\nabla \cdot v\mathbb{I}\right] - \nabla P &= \mathrm{F} \\
\partial_t T + \gamma T \nabla \cdot v + v \cdot \nabla T - \frac{1}{\varrho_r c_v}\left[\nabla \cdot (k \nabla T) + \tau : \nabla v\right] &= 0 \\
\partial_t C + v \cdot \nabla C &= 0
\end{aligned}$$

## (Anelastic Liquid) Thermochemical Adjoint Equations

## (Anelastic Liquid) Thermochemical Convection

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\partial_t C + v \cdot \nabla C &= 0 \quad C(x,t) = C(x,t_0)
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$$
M = 1/2 \int_{x^3} \left( (T\,(x,t_1) - T_E)^2 + (C\,(x,t_1) - C_E)^2 \right) dx^3
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$$\nabla \cdot [\eta(\nabla \varphi + (\nabla \varphi)^T)] - \varrho_r \nabla \lambda + \Psi \nabla T = 0$$

$$\partial_t \Psi + v \cdot \nabla \Psi - (\gamma - 1)\Psi \nabla \cdot v + \nabla \cdot (k \nabla (\frac{\Psi}{\varrho_r c_v})) - \alpha_r g \cdot \varphi = (T - T_E)\delta(t - t_1)$$

$$\partial_t \Gamma + \nabla \cdot (\Gamma v) + \beta \varrho_r g \cdot \varphi = (C - C_E)\delta(t - t_1)$$

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$$\partial_t\Psi + v \cdot \nabla\Psi - (\gamma - 1)\Psi\nabla \cdot v + \nabla \cdot \left(k\nabla\left(\frac{\Psi}{\varrho_r c_v}\right)\right) - \alpha_r g \cdot \varphi = (T - T_E)\delta(t - t_1)$$

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## (Anelastic Liquid) Thermochemical Convection

$$\begin{aligned}
\nabla \cdot (\varrho_r v) &= 0 \\
\nabla \cdot \left[\eta\,(\nabla v) + (\nabla v)^T - \tfrac{2}{3}\nabla \cdot v\mathbb{I}\right] - \nabla P &= \boxed{\varrho_r g[-\alpha\Delta T + K_T^{-1}\Delta P + \beta\Delta C]} \\
\partial_t T + \gamma T \nabla \cdot v + v \cdot \nabla T - \frac{1}{\varrho_r c_v}\left[\nabla \cdot (k\nabla T) + \tau : \nabla v\right] &= 0 \\
\partial_t C + v \cdot \nabla C &= 0
\end{aligned}$$

$$T(x,t) = T(x,t_0)$$
$$C(x,t) = C(x,t_0)$$

$$M = 1/2 \int_{x^3} \left( (T\,(x,t_1) - T_E)^2 + (C\,(x,t_1) - C_E)^2 \right) dx^3$$

$$\partial_{T_0,C_0} M = \Psi(x,t_0)\Delta T + \Gamma(x,t_0)\Delta C$$

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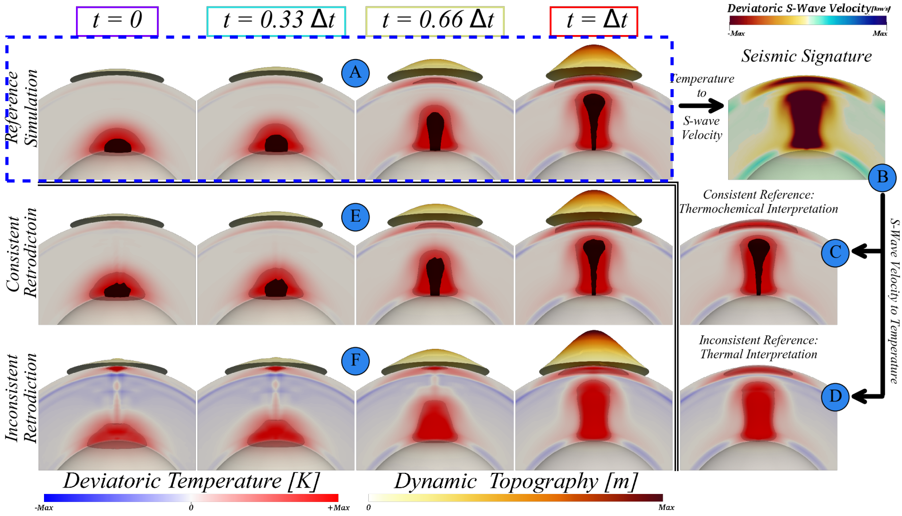
# Thermochemical Twin Experiment



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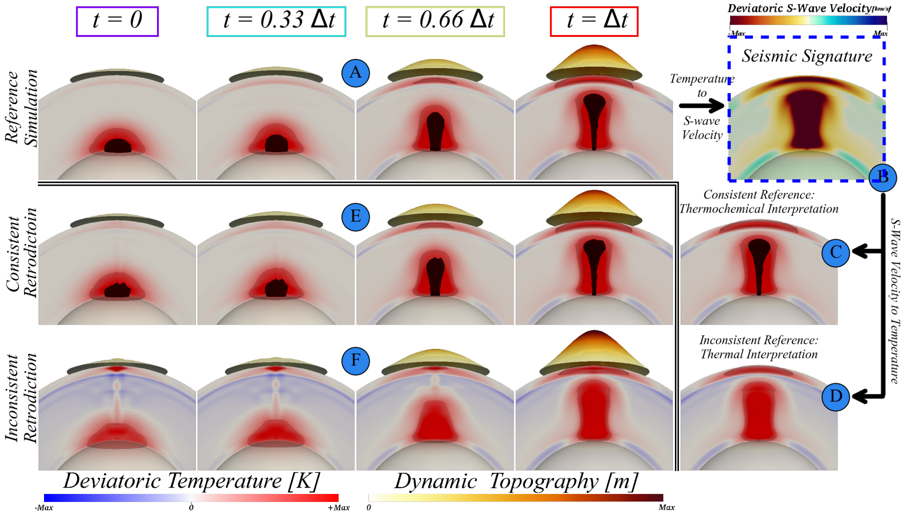

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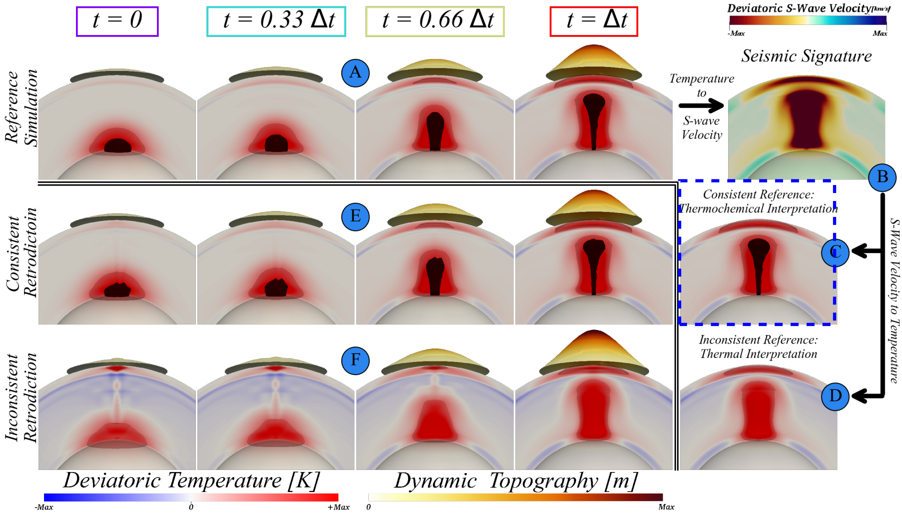

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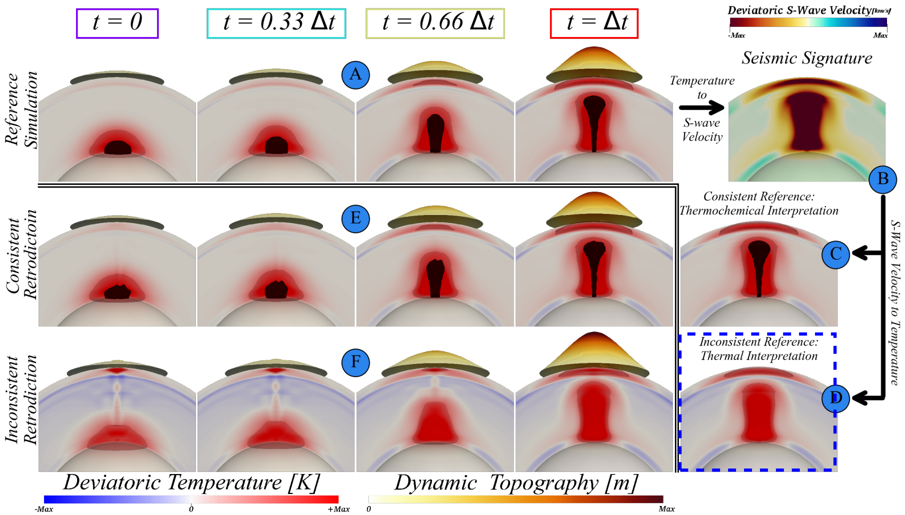

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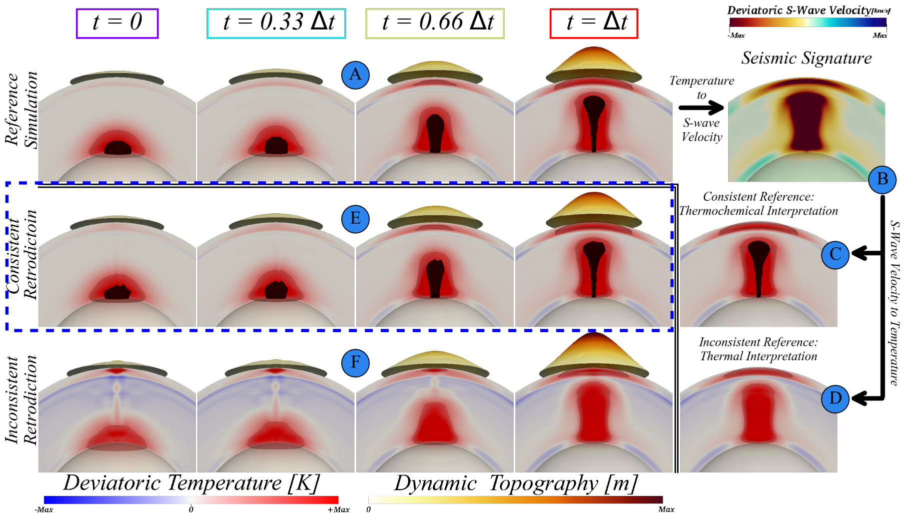

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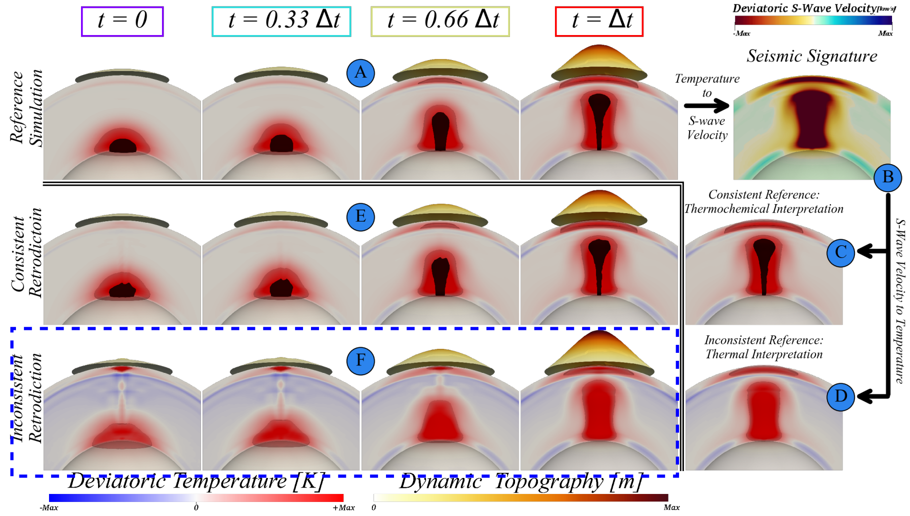

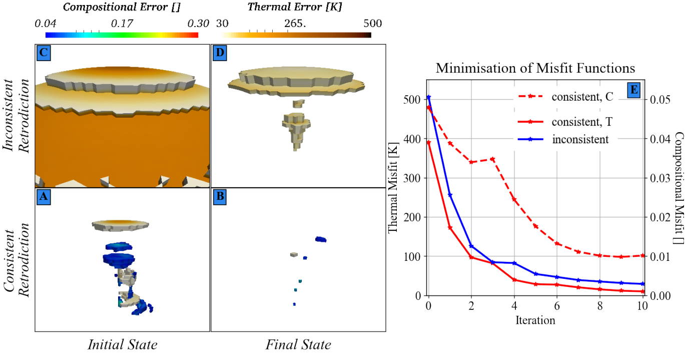


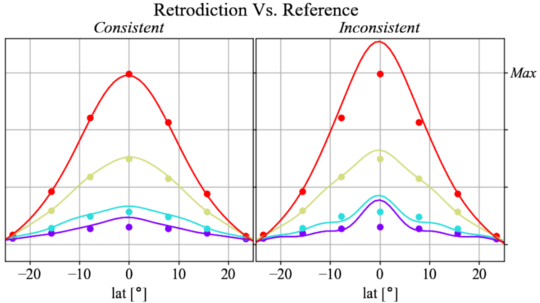


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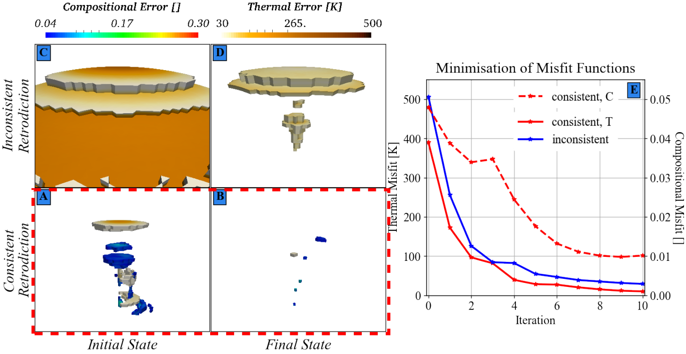
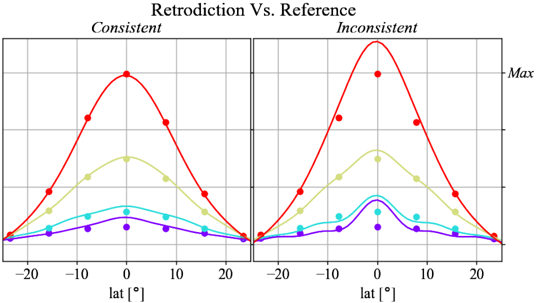
- Time-trajectories based on different hypotheses of the heterogeneities in the mantle can be built.
- Observations of dynamic motion of the Earth's surface.

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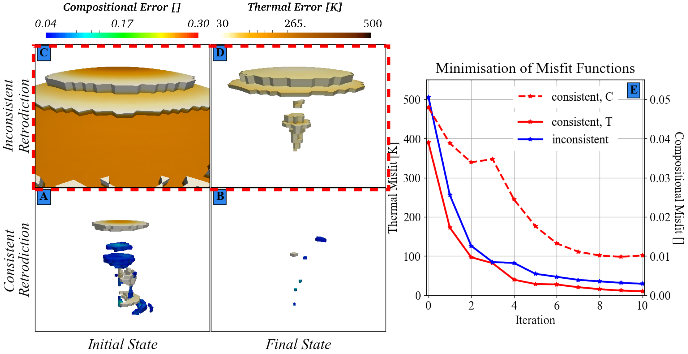


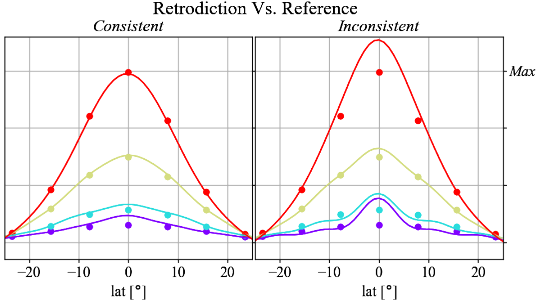


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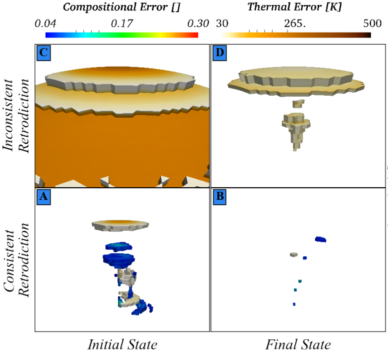


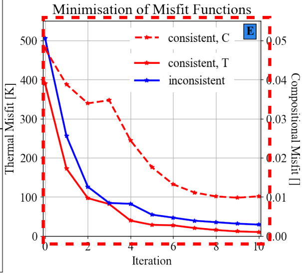


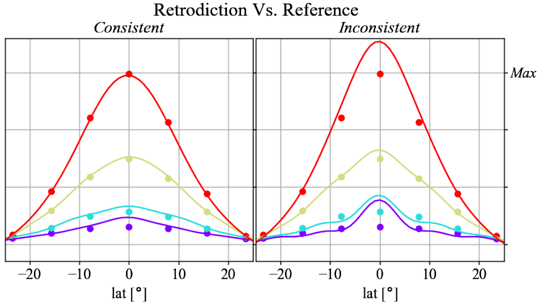


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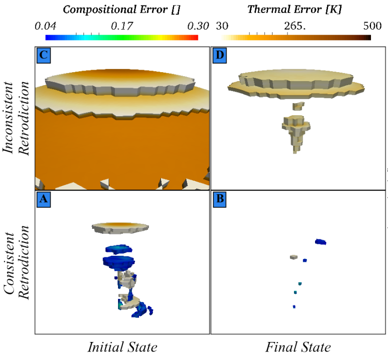


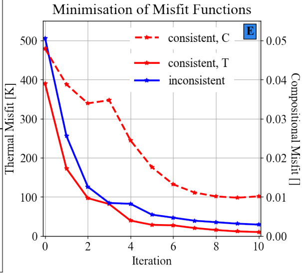


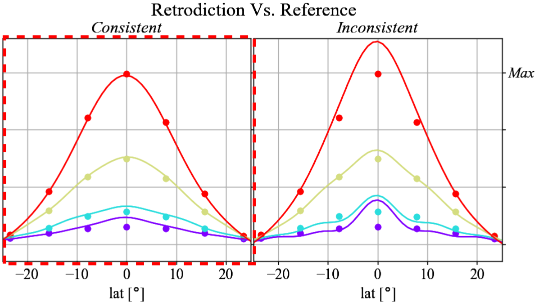


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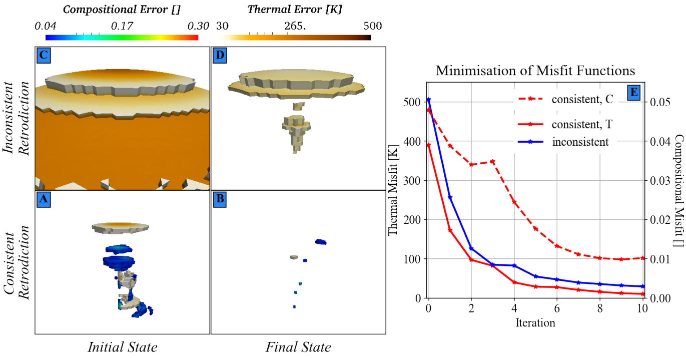


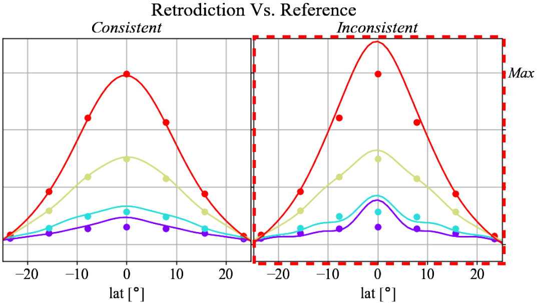


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# Issues and Outlook

### Thermochemical Adjoint

- It works!
- The main obstacle: how to interpret **seismic tomographies**. (temperature, composition, and density)
- Develop consistent model parameters.
- New realm where hypotheses on the role and extent of compositional heterogeneities can be tested.

### Uncertainty

- Input to retrodictions are often results of an inverse problem themselves: **Seismology**, **Plate Reconstructions**, **Mineral Physics**
- Current definitions of the **misfit function** are simplistic.
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Thank You!