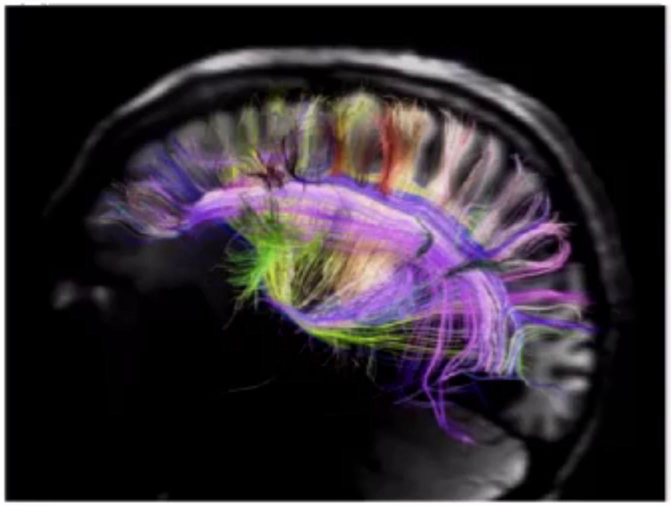
SIAM DS 2015

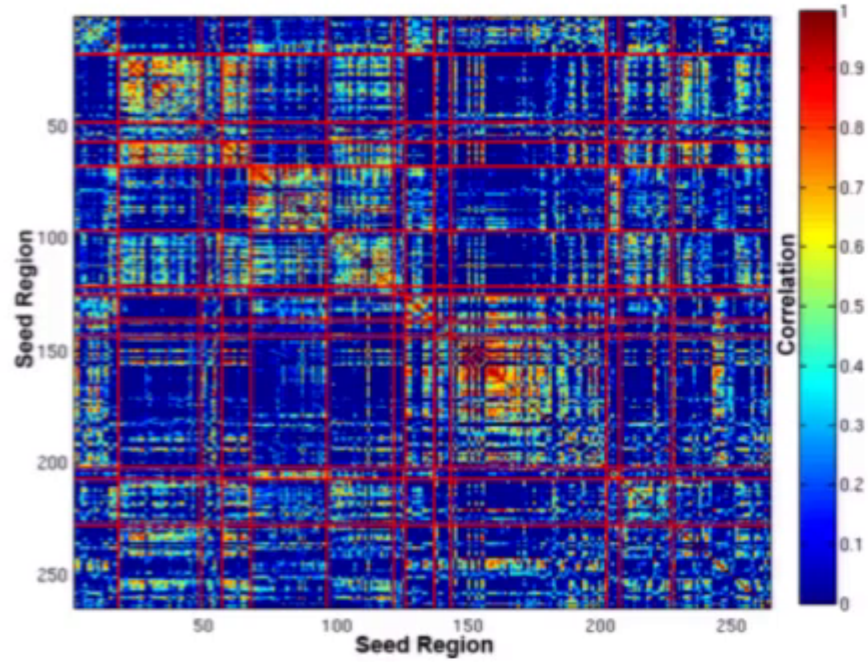
Featured Minisymposium

Applications of Algebraic Topology to Neuroscience

The Obama BRAIN Mapping Initiative, the Human Connectome Project and the Human Brain Project



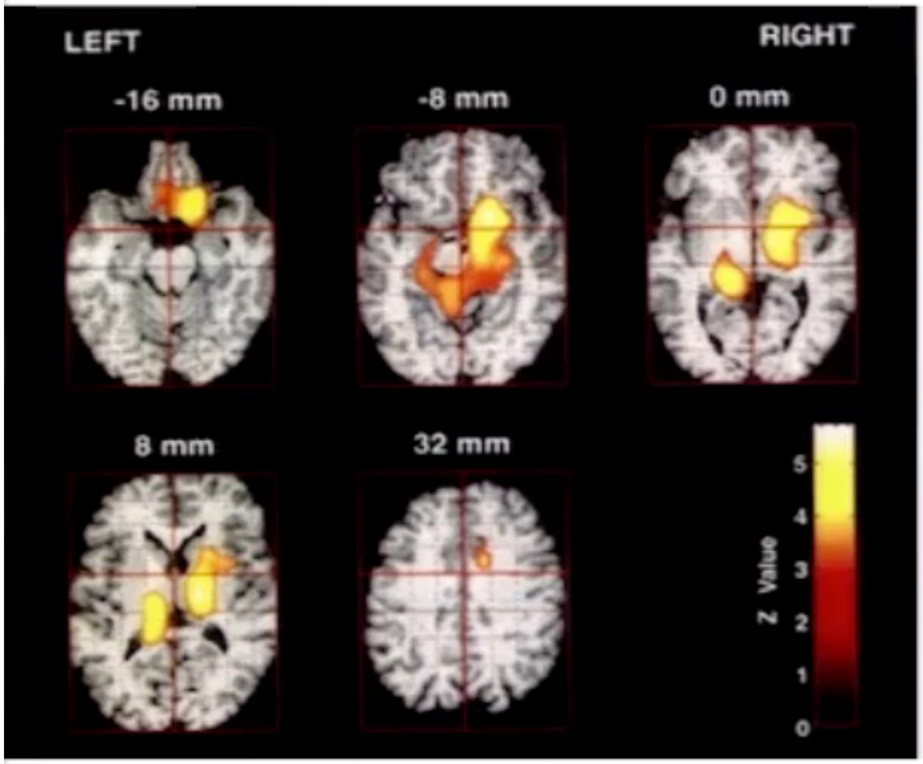
MGH-UCLA Human Connectome Project



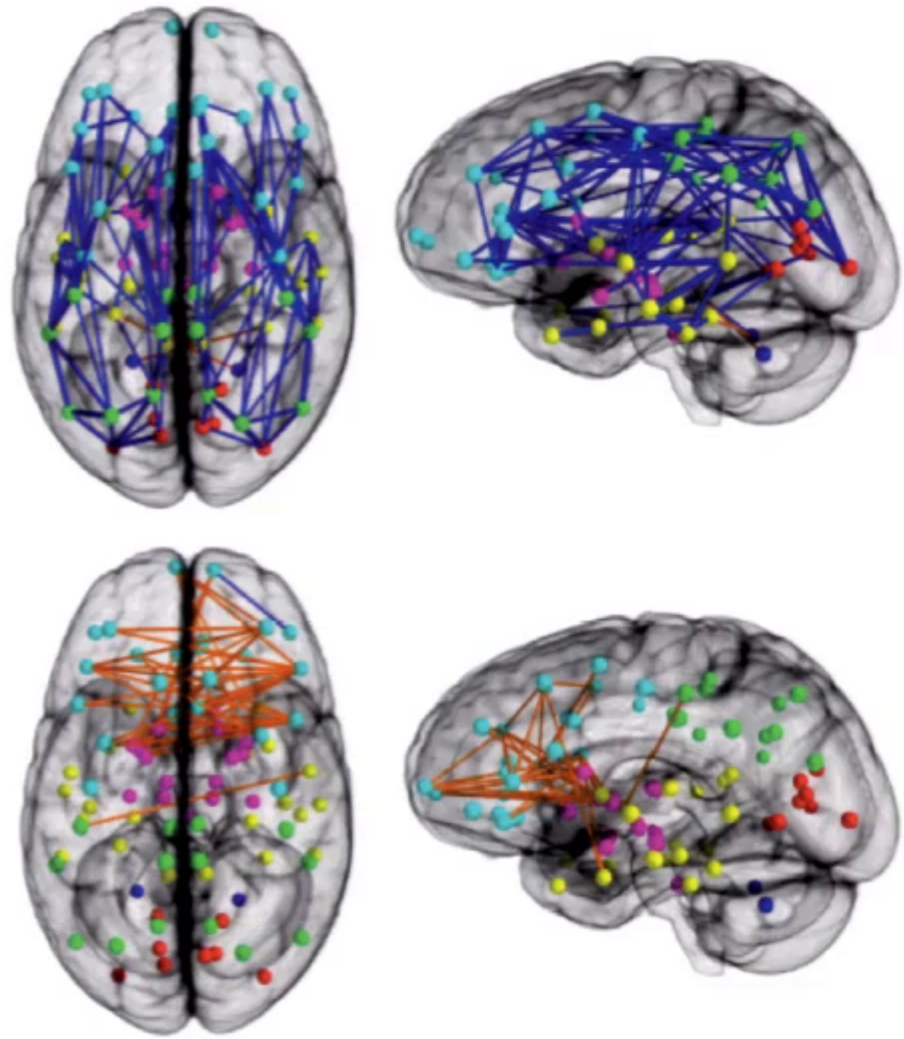
Functional connectivity correlation matrix
WU/Minn Consortium

http://www.neuroscienceblueprint.nih.gov/connectome/
https://www.humanbrainproject.eu/

Massive clinical brain imaging studies



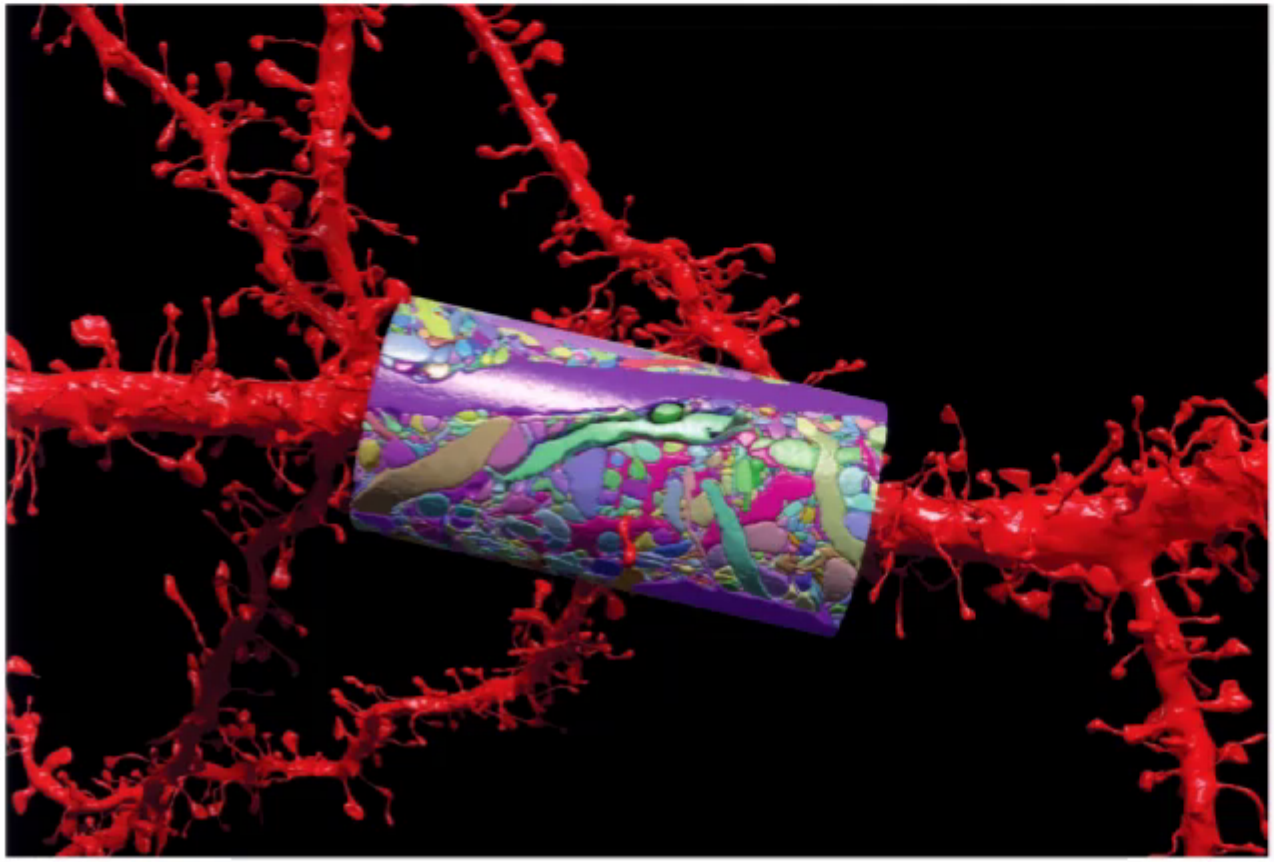
Harvard Functional Neuroimaging Laboratory



Philadelphia Neurodevelopmental Cohort (data from over 9500 individuals)

Microcircuit Structure and Connectomics

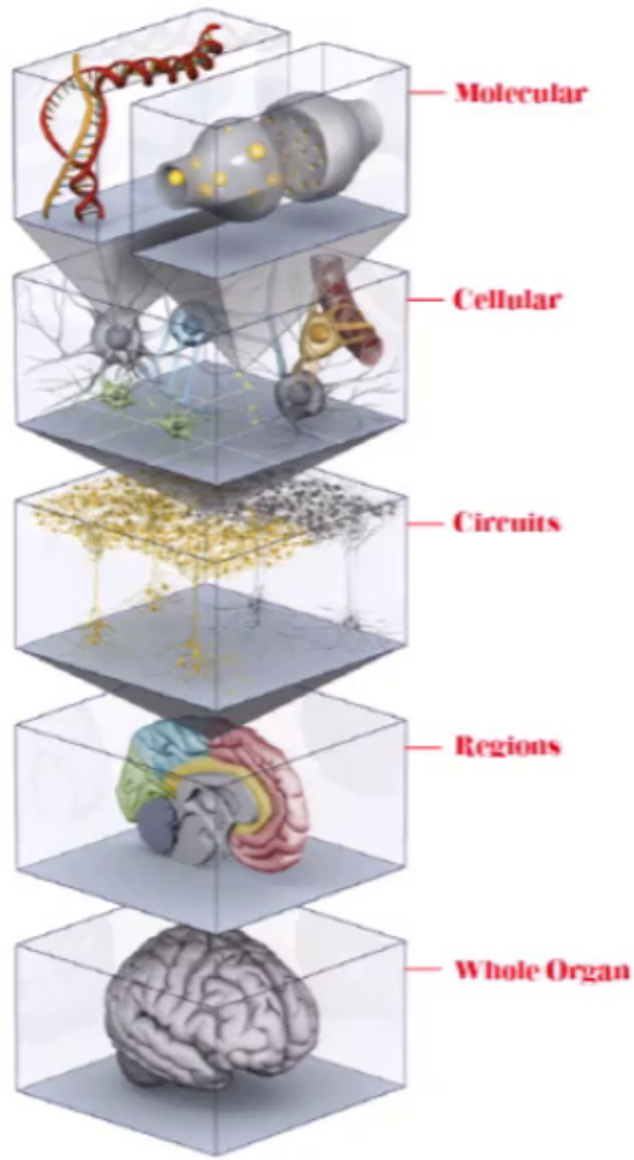
Fine structure. A composite of artificially colored EM images reveals details within a cylinder of mouse brain tissue, smaller than a grain of sand, that contains 680 nerve fibers and 774 synapses.



www.sciencemag.org **SCIENCE** VOL 342 22 NOVEMBER 2013

Lichtman lab (Harvard)

Further, none of this data is independent!



This massive data is already being produced. What are are we going to do with all of it?

The development of tools for detecting structure in and reasoning about these huge, noisy, nonlinear datasets is one of the fundamental challenges in modern neuroscience.

Opinion: algebraic topology is an untapped source of exactly these sorts of tools.

Topological detection of structure in neural activity recordings

Chad Giusti
Warren Center for Network and Data Sciences, University of Pennsylvania
Carina Curto and Vladimir Itskov
Department of Mathematics, Pennsylvania State University

SIAM Dynamical Systems 2015
Featured Minisymposium on Applications of Algebraic Topology to Neuroscience
Sunday, May 17th

Motivating Question:

How do we detect structure or absence of structure in a correlation matrix?

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How do we detect structure or absence of structure in a correlation matrix?

LETTER

doi:10.1038/nature12063

Random convergence of olfactory inputs in the *Drosophila* mushroom body

Sophie J. C. Caron[1], Vanessa Ruta[2], L. F. Abbott[1,3] & Richard Axel[1,4,5]

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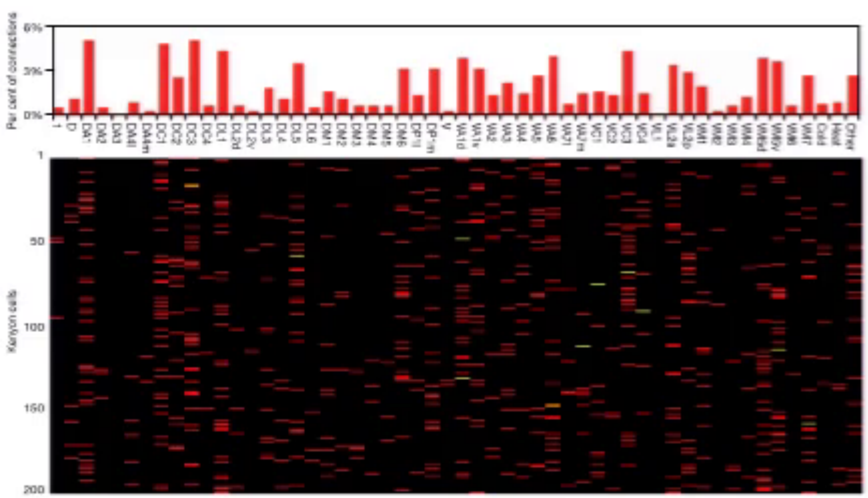


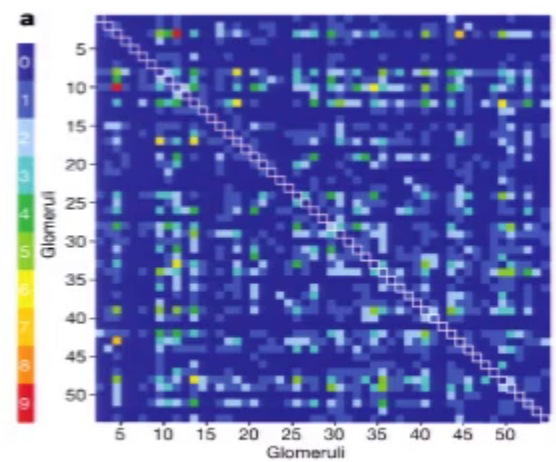
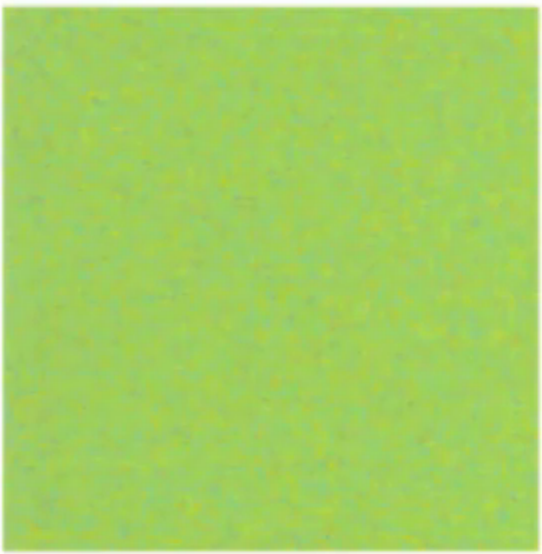


Figure 4 | KCs do not receive structured input. **a**, Two glomeruli projecting to the same KC are considered a connected pair. All possible connected pairs are depicted as squares in a 53 × 53 matrix (51 AL glomeruli and 2 pseudoglomeruli), coloured according to their observed frequency in the data (white outlined squares along the diagonal depict the frequency of identical pairs where a glomerulus is paired with itself). **b**, The frequency of KCs

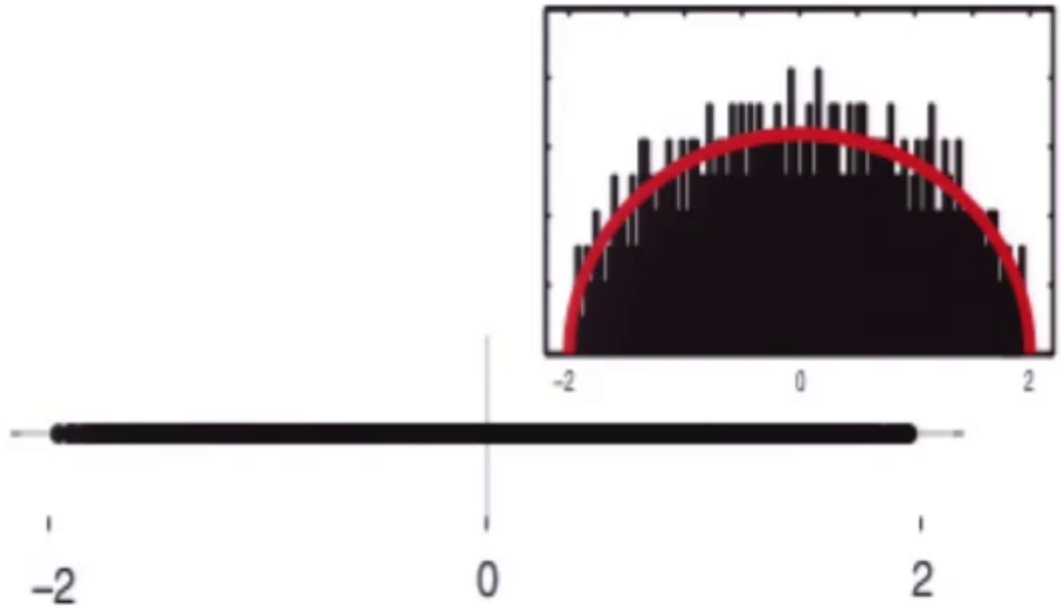
Often, the answer is: compute spectra!

A



1000 x 1000 random matrix
with entries sampled
independently from N(0,1)

Spectrum of A



Eigenvalues are real, lie in [-2, 2]
and are distributed along a semicircle
(Wigner's semicircle law)

Is this the right tool for the job?

Wigner's semicircle law is a *limiting* theorem, so not a stable signature for small matrices ...

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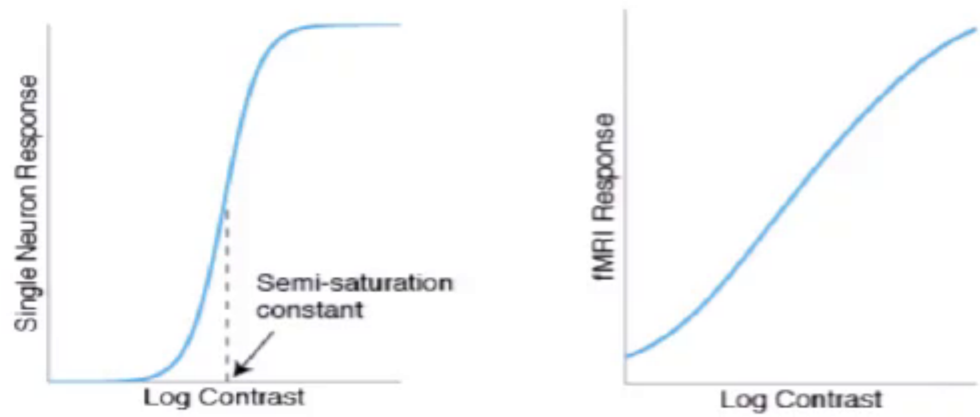
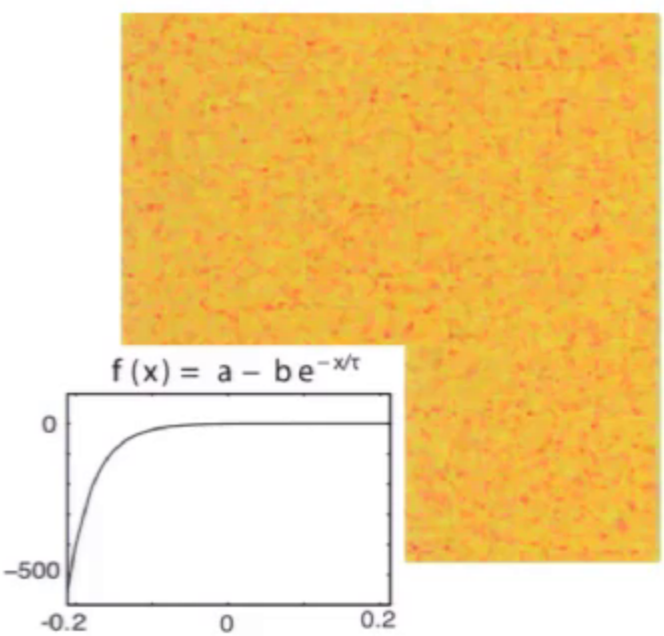


... and, biological responses (and measurements thereof) to stimuli often have monotonic *nonlinearities*, under which the spectrum is not invariant!

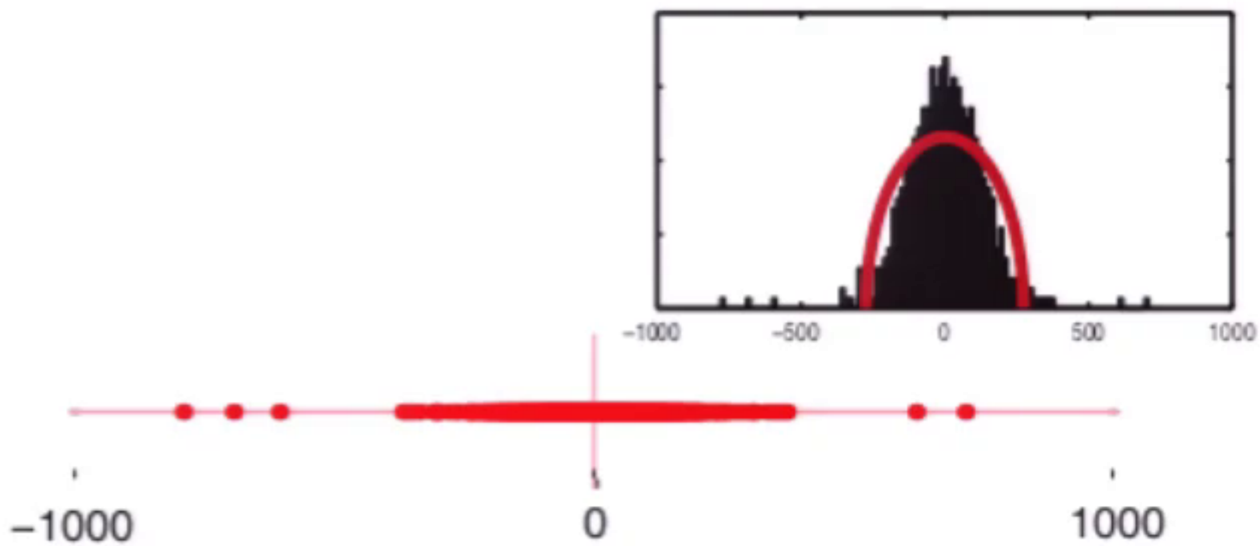
This might lead to “detection” of structure where none exists!

f(A)



1000 x 1000 random matrix with entries sampled independently from f(N(0,1))

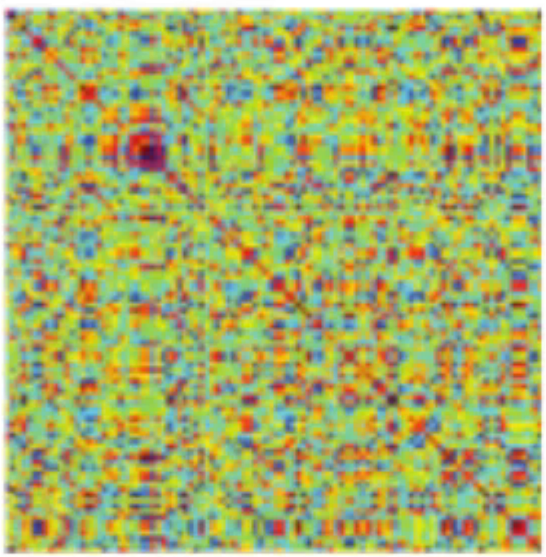
Spectrum of f(A)



Eigenvalues are real, lie... hmm.

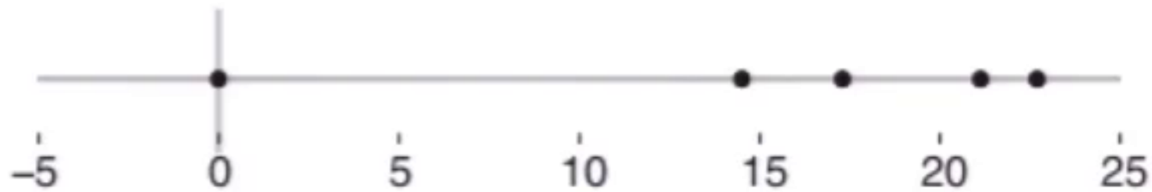
How do signatures of extant structure fare?

B



rank 4 matrix
(product of a 1000x4 matrix
and a 4x1000 matrix)

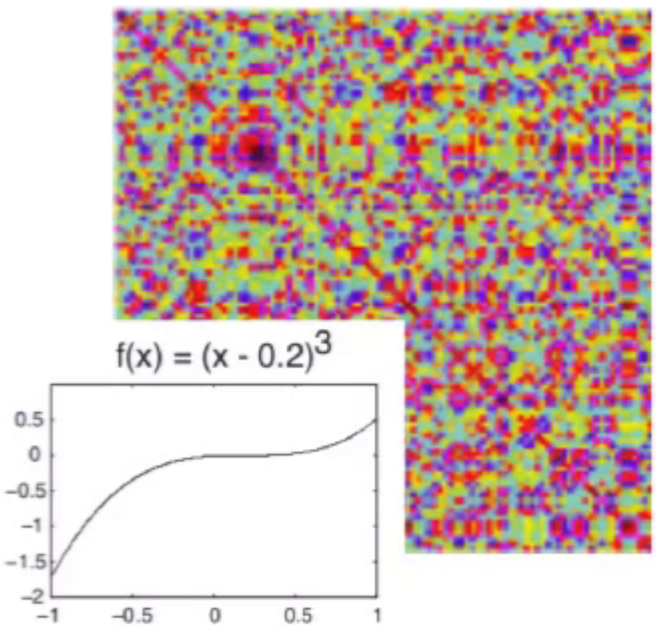
Spectrum of B



Four non-zero eigenvalues

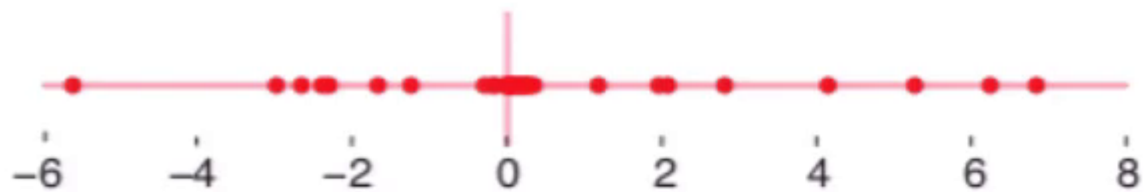
Signatures of structure are destroyed, too.

f(B)



nonlinear transform of a rank 4 matrix (product of a 1000x4 matrix and a 4x1000 matrix)

Spectrum of f(B)



Again, hmm.

Goal:
Find a tool that recognizes pertinent structure, or lack thereof, in matrices, which is robust to monotonic nonlinearities and is reliable for small matrices.

The “Correct” Question

What structure is *invariant* when we apply a monotonic (increasing) transformation to the elements of a matrix?

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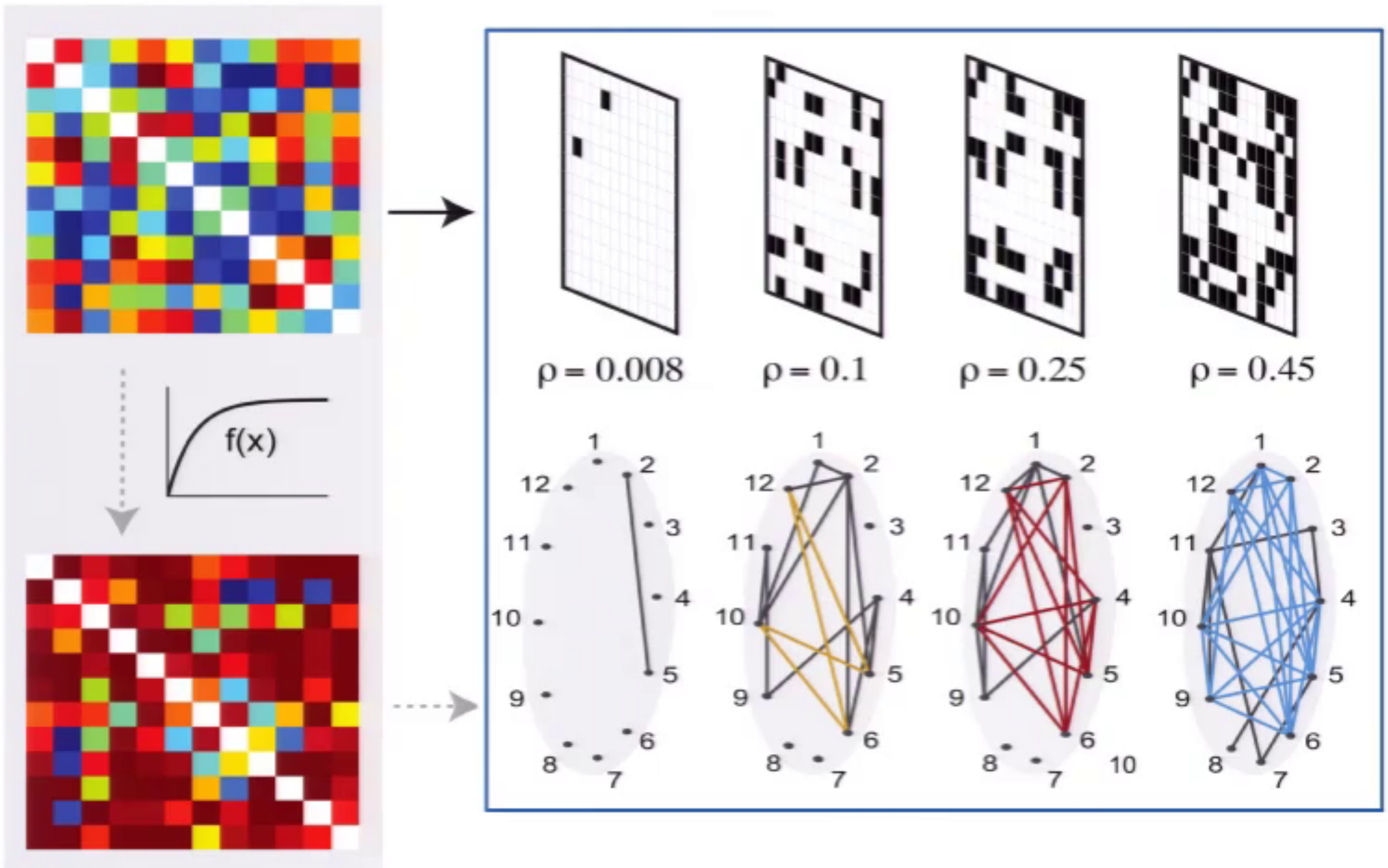
What structure is *invariant* when we apply a monotonic (increasing) transformation to the elements of a matrix?

$f(x)$ is *monotonic increasing* if

$x < y$ implies $f(x) < f(y)$

So, the *order* of the elements is preserved.

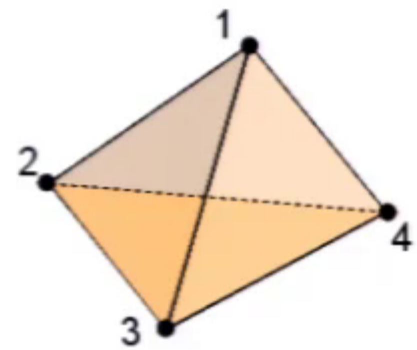
“Order Complex” of a Symmetric Matrix



How do we use this to detect structure?
Compute persistent homology of clique complexes.

What is persistent homology?

A measure of the evolving structure of cliques in the graphs…



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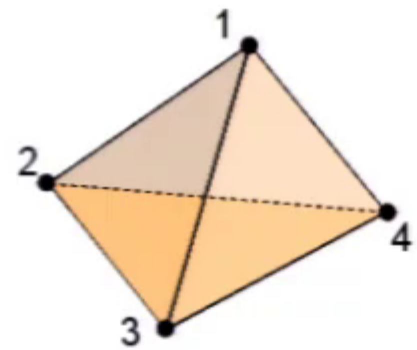
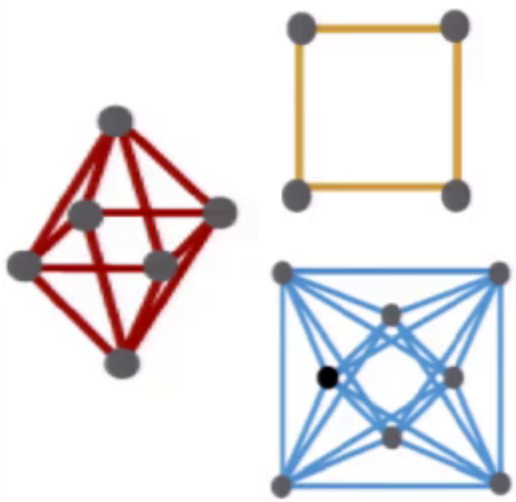
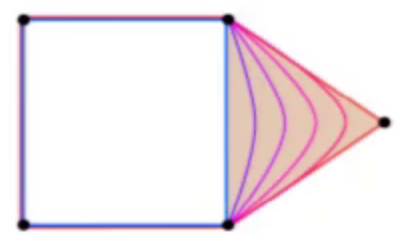
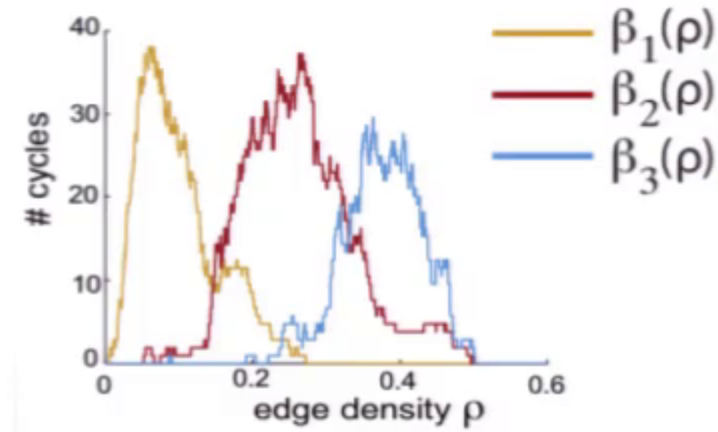




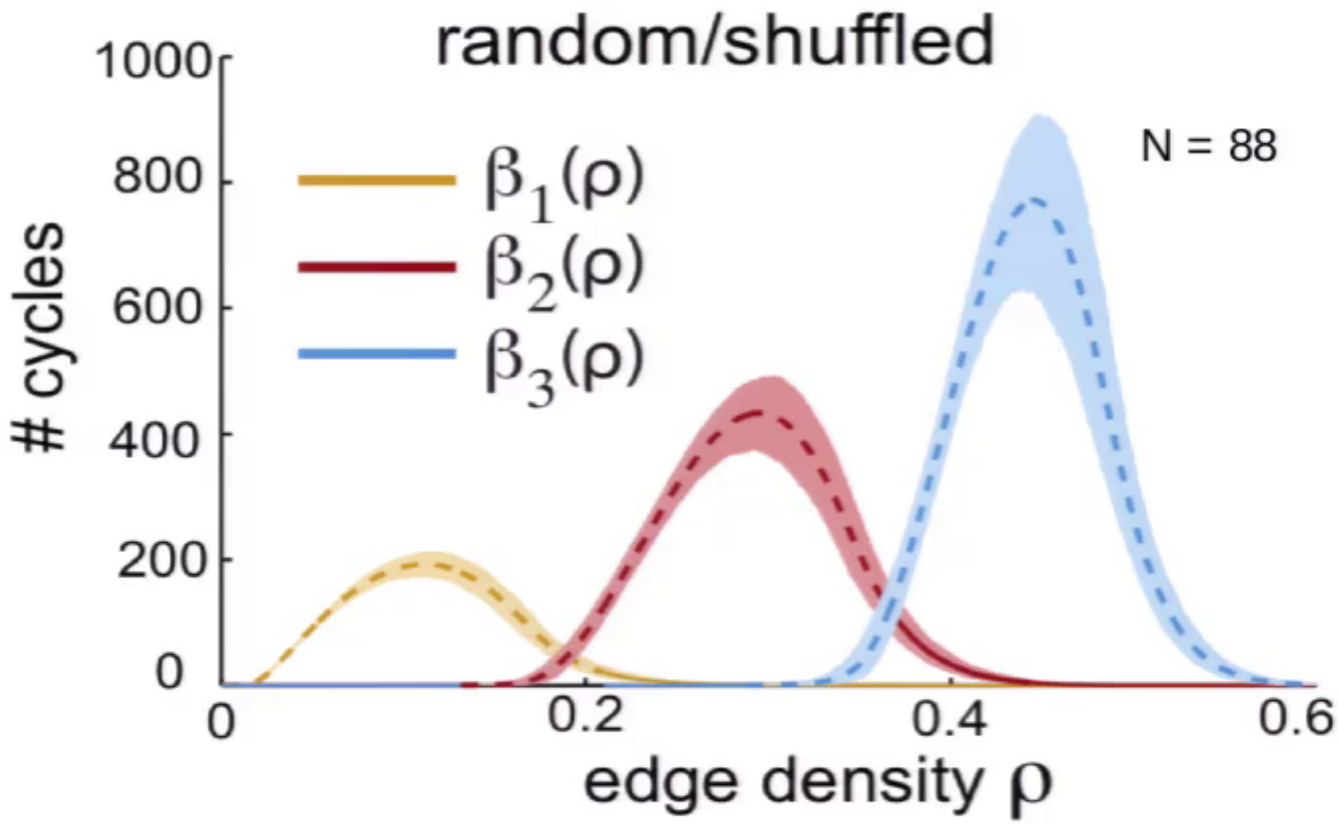
... obtained by counting cycles of various dimensions, up to some equivalence relation ...



... summarized as “Betti curves” indexed by graph edge density.

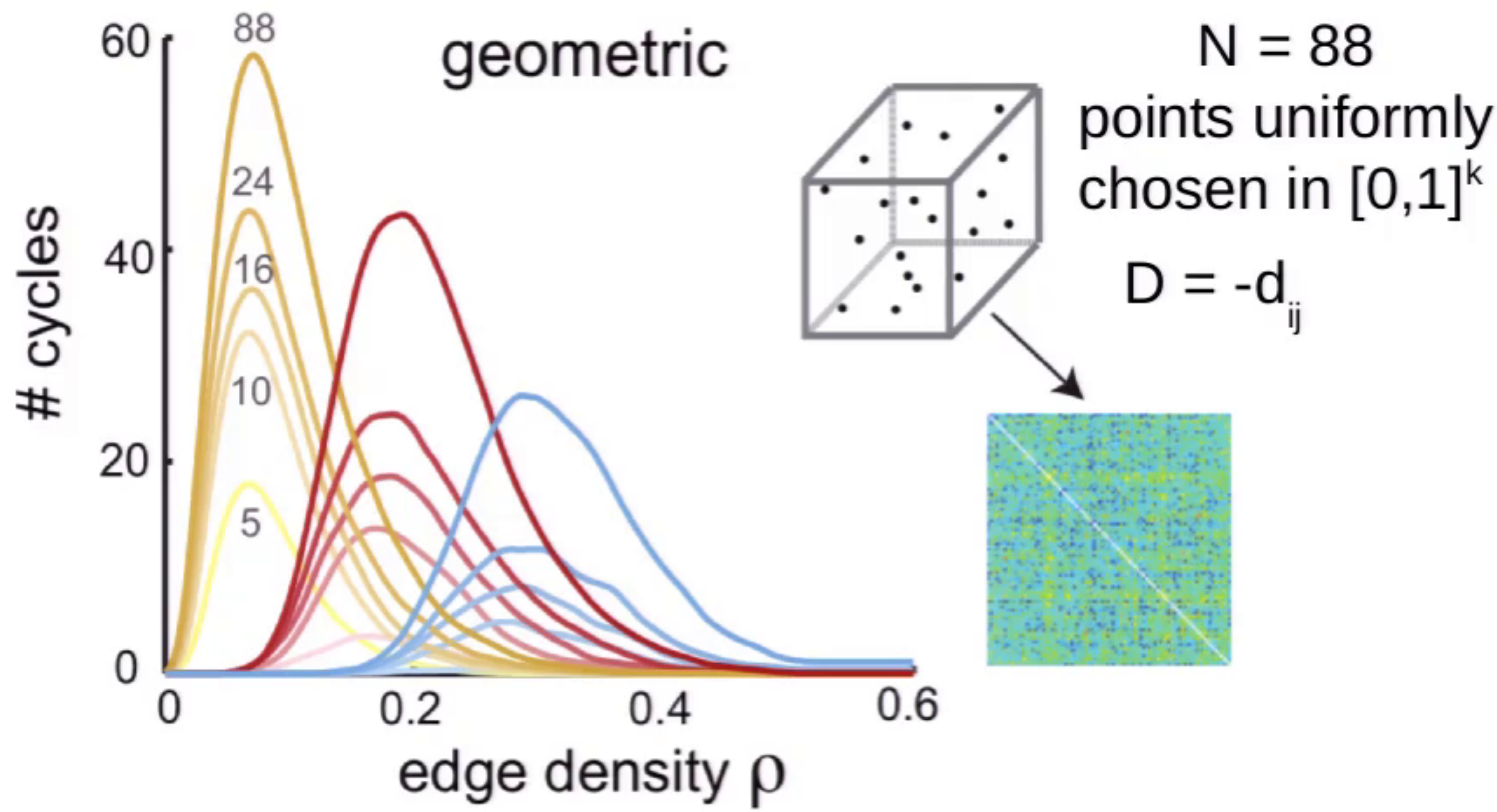


This method provides a robust signature of random (iid) matrices.



These *Betti curves* are stereotyped for matrices with entries drawn iid *from any distribution*.

Are there matrices with different signatures?



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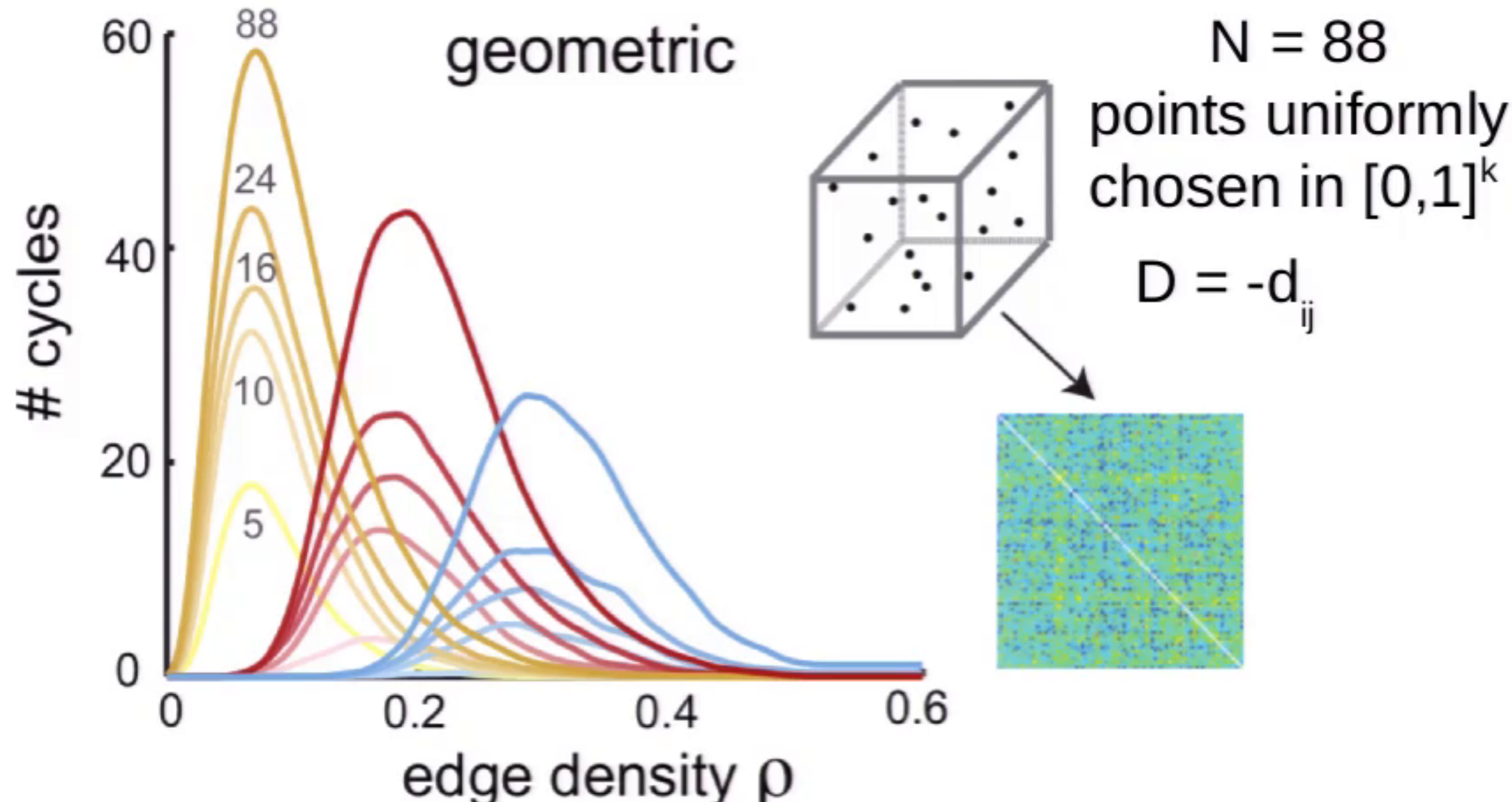

The triangle inequality introduces (roughly) an upper bound on the lifetime of a cycle.

Where might we expect to find “geometric structure”?

“Place cells” in hippocampus have receptive fields that correspond to physical location. However, they are *not* correspondingly physically organized, so we can only hope to see this structure in activity.

Illustration of a Hippocampus

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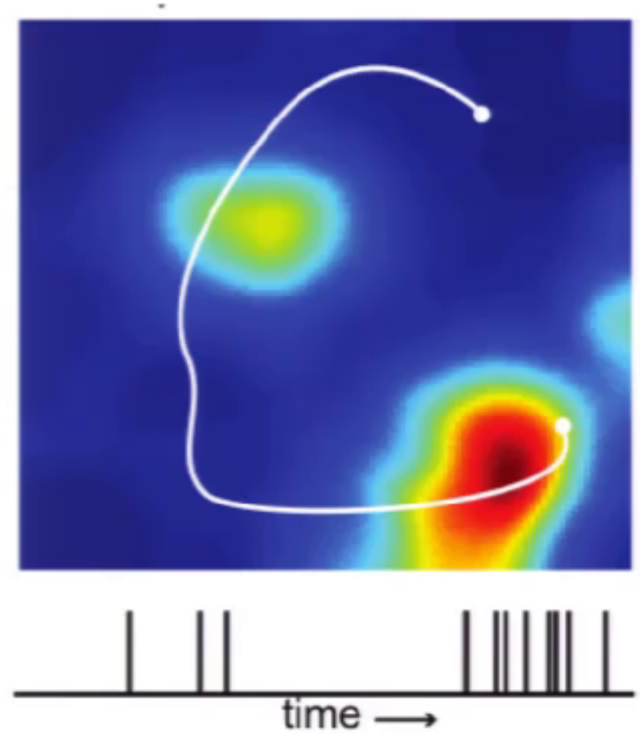
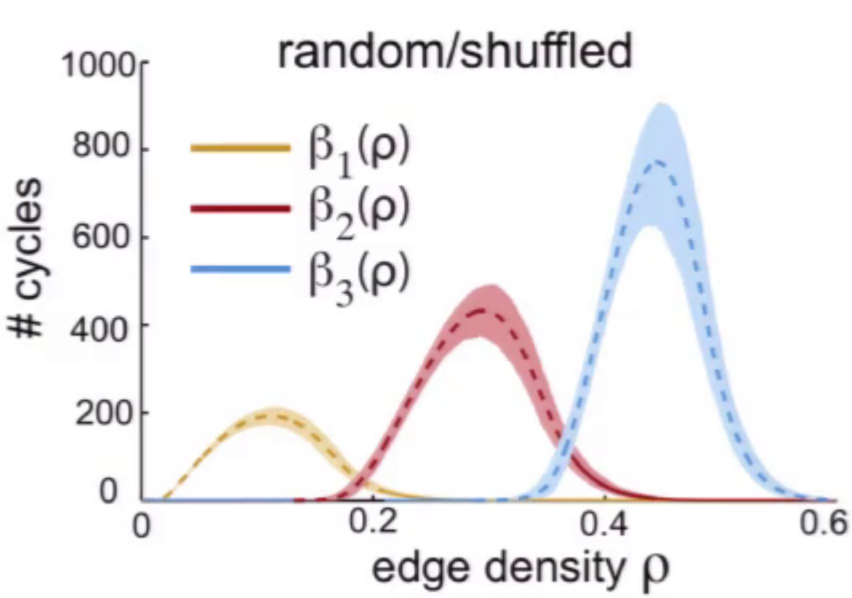
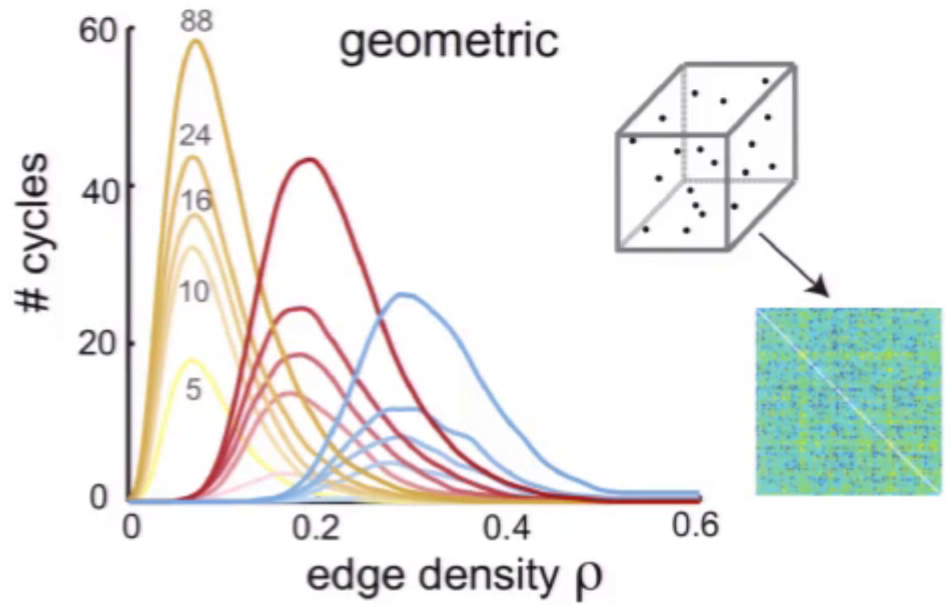
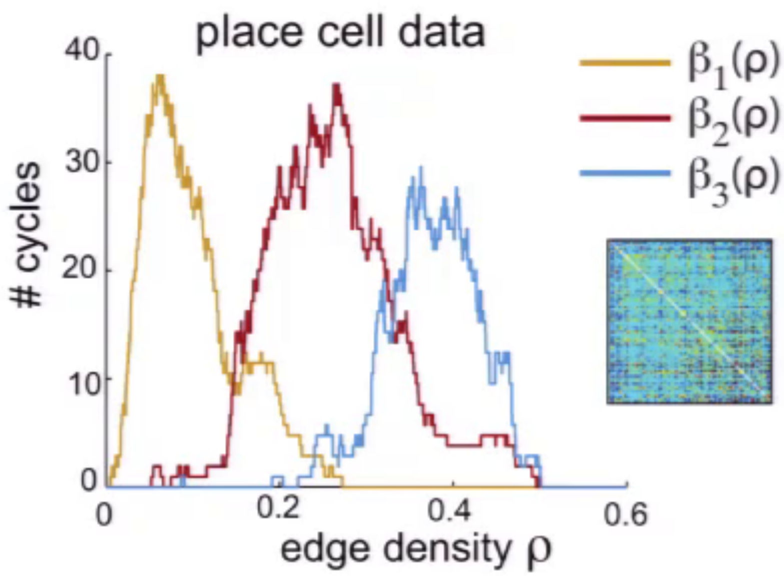
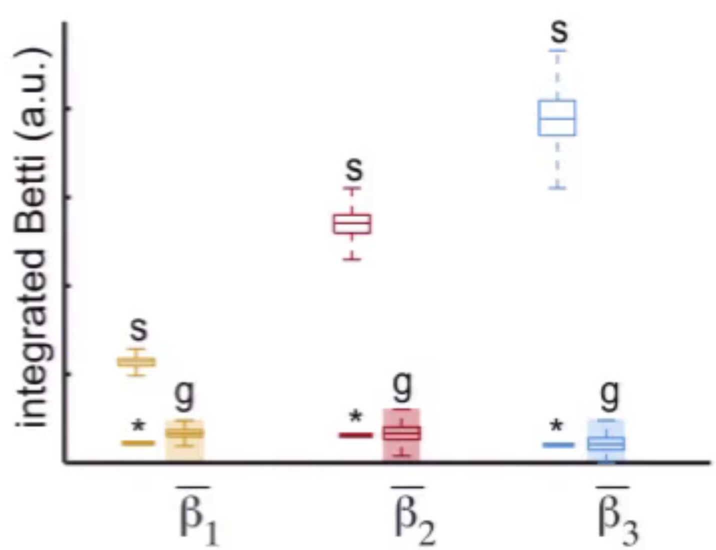


How do the Betti curves compare?

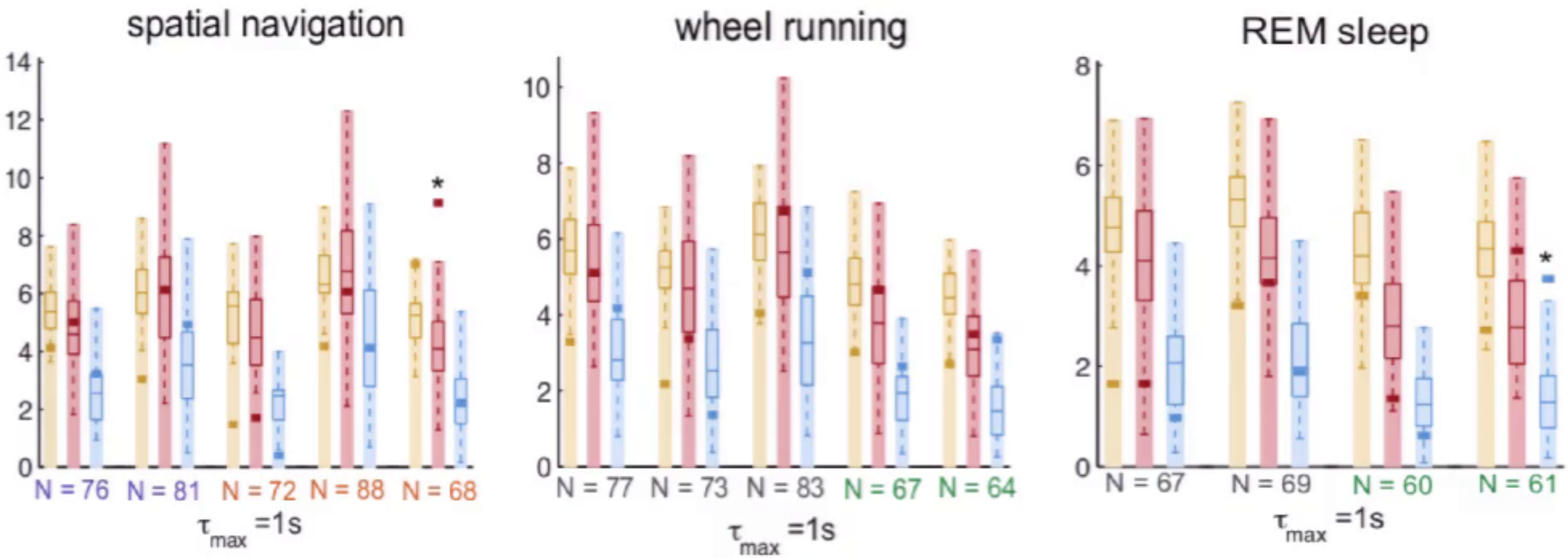








Surprisingly, this structure persists across behaviors!



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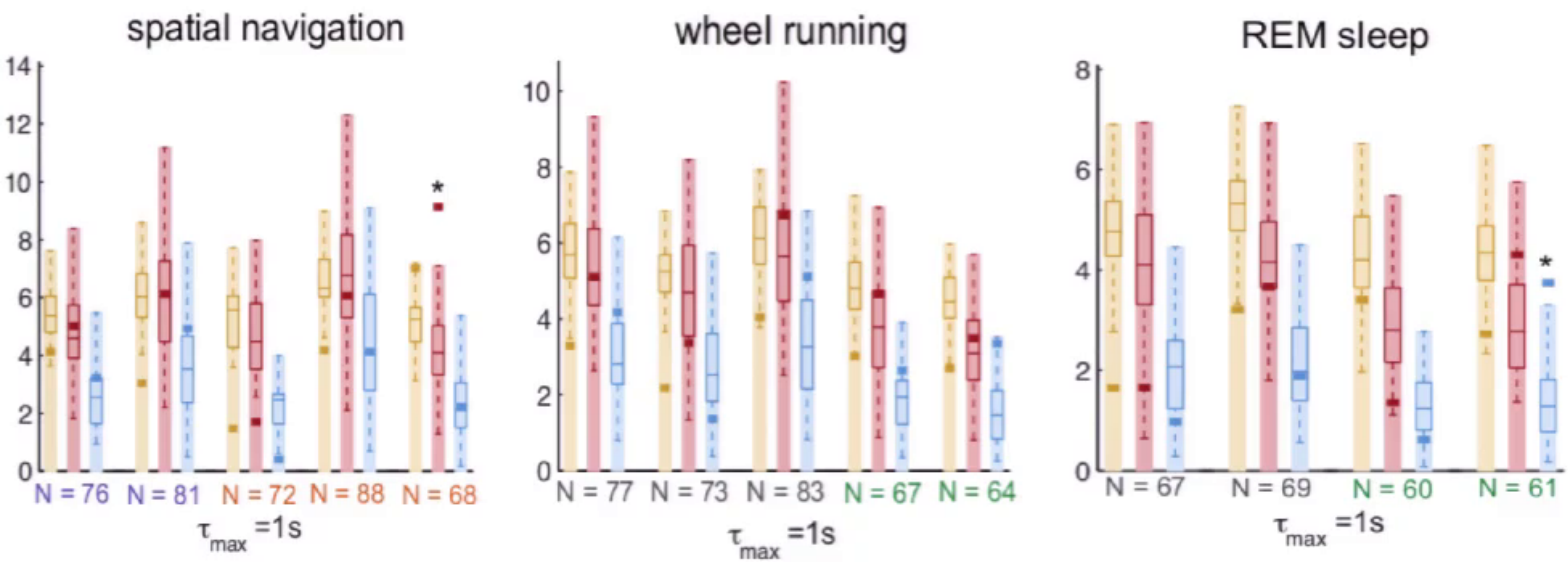

Conclusion: “geometric” structure is a fundamental property of the functional connectivity of the hippocampus, and not a result of stimulus or state.