



# SIAM CSE17

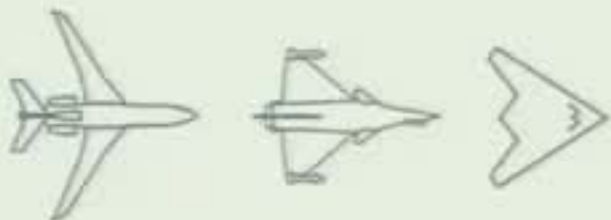
## Computational Science and Engineering Achievements in the Designing of Aircraft

**B. STOUFFLET**

**CTO Dassault Aviation**

**March 2, 2017**

**HIGHER TOGETHER™**



# The Falcon Family, Rafale and nEUROn



FALCON 8X  
6,450 NM - Trijet



RAFALE



FALCON 2000S  
3,350 NM – Twin-jet



nEUROn

# The Falcon Family, Rafale and nEUROn



2007 New wing:  
Aerodynamic efficiency +30%



FALCON 7X  
5,950 NM - Trijet



FALCON 5X  
5,200 NM - Twin-jet



FALCON 8X  
6,450 NM - Trijet



RAFALE



FALCON 2000S  
3,350 NM - Twin-jet



FALCON 2000LXS  
4,000 NM - Twin-jet



FALCON 900LX  
4,750 NM - Trijet



nEUROn



- **Computational Science and Engineering**
  - Mainly Engineering standpoint will be addressed
- **CSE is predominant in Design activities**
- **New fields of CSE applications are however emerging**
  - Stochastic approaches

# The Falcon Family, Rafale and nEUROn



2007 New wing:  
Aerodynamic efficiency +30%



New Wing with Flaperon  
115V AC  
less hydraulic  
more electric



Extended F7X  
Mass optimization  
New Winglets



Multi-role aircraft  
Tens of configurations



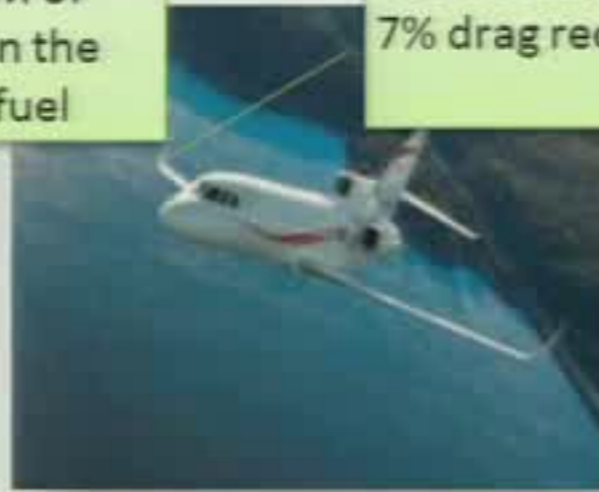
FALCON 7X  
2011 New engine version  
reduces missions 20%



FALCON 5X  
5,200 NM - Twin-jet



FALCON 8X  
2009 Winglets: +200NM of range on the same fuel



2011 Winglets: 7% drag reduction

RAFALE  
Technological demonstrator of stealth UCAV



FALCON 2000S  
3,350 NM - Twin-jet

FALCON 2000LXS  
4,000 NM - Twin-jet

FALCON 900LX  
4,750 NM - Trijet

nEUROn

- **Computational Science and Engineering**
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  - Stochastic approaches



# Computational engineering in the product life-cycle



DESIGN

DEVELOPMENT

SUPPORT

High Fidelity Modeling  
MDO  
Multiphysics

System engineering  
Safety analysis  
Embedded software

Predictive maintenance  
Fleet analysis

Numerical Analysis  
Scientific Computation  
HPC  
Optimization  
Uncertainty quantification  
Robust Design

Automatics  
Formal methods  
Static analysis of codes  
Rare events probabilities

Data Analytics  
System Identification



## Design

- Industrial state-of-the art of CFD
- Automatic shape optimization
- Multiphysics: example of Aeroelasticity
- Computational Electromagnetics
- Surrogate models
- Uncertainty quantification – Robust design
- Challenges of next generation HPC (towards Exascale)

## Development

- An example of rare-event probability evaluation

## Support

- First attempts in Data Analytics



# Multidisciplinary Design Loop



## Global options

- Architectures
- Technologies



## Design per discipline and Optimization

- Aerodynamics
- Structure
- Acoustics
- Propulsive integration
- Vehicle systems

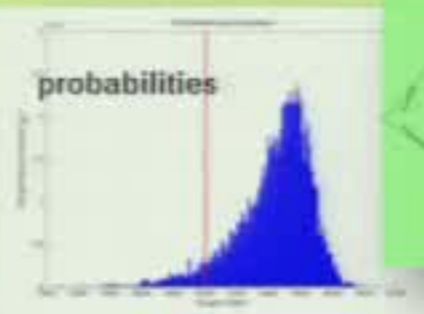


## Requirements (market, regulation)

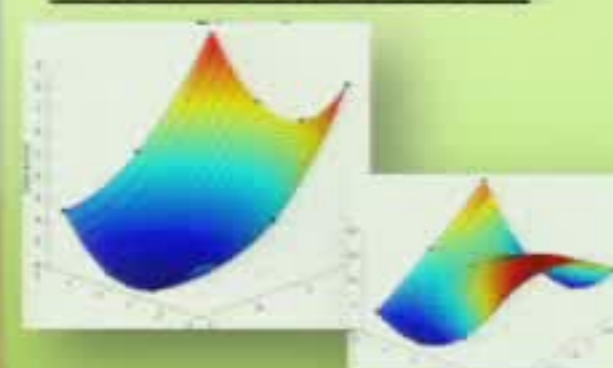
- Range
- Fields length
- Cruise speed
- Comfort
- Environmental objectives
- Costs

## Global synthesis

- Exploration of design space
- Global sensitivities
- Risks evaluation



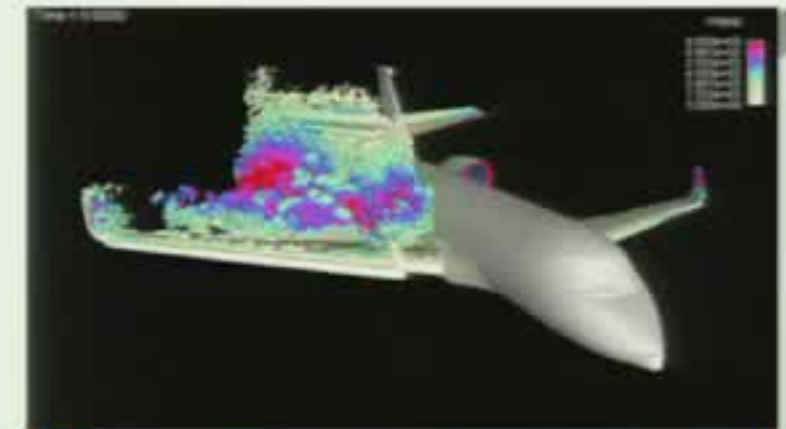
## Parametric models



# A tremendous evolution of computational fluid dynamics codes (CFD)

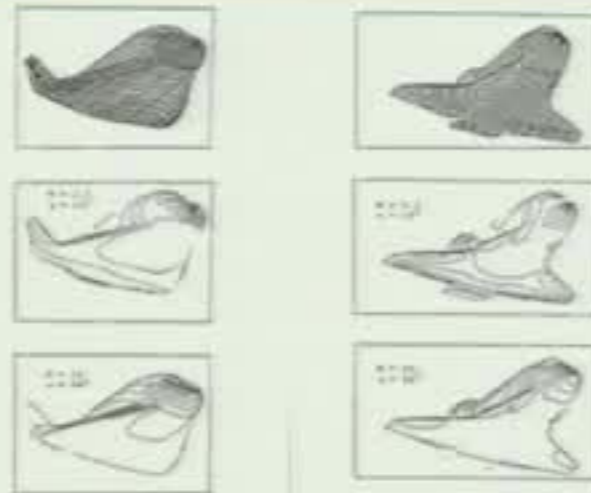


**MID 2000s**  
**Navier-Stokes solvers**  
**10 Million nodes**



Navier-Stokes equations  
Unstructured meshes  
Petrov-Galerkin formulation  
Implicit methods  
GMRES

**MID 80s**  
**Euler solvers**  
**10 000 nodes for a half geometry**



Euler equations  
Unstructured meshes  
Finite Volume / Finite Element  
Implicit methods  
Geometric Multigrid

**EARLY 80s**  
**A premiere: the first industrial complete aircraft aerodynamics computation**



Full potential equations  
Finite element discretization  
Least-square formulation



- Navier-Stokes solver

- In house development (cooperation with research teams)
- Stabilized Finite-Element method
- Implicit methods : GMRES with BSOR preconditioning
- Turbulence models : two-layer  $k$ - $\epsilon$ ,  $k$ - $k_L$ , w/wo EARSM (and S-A,  $k$ - $\omega$ , DRSM, LES/DES)
- Efficient parallel code architecture (routine use on 2048-core classes)

- Two types of computations:

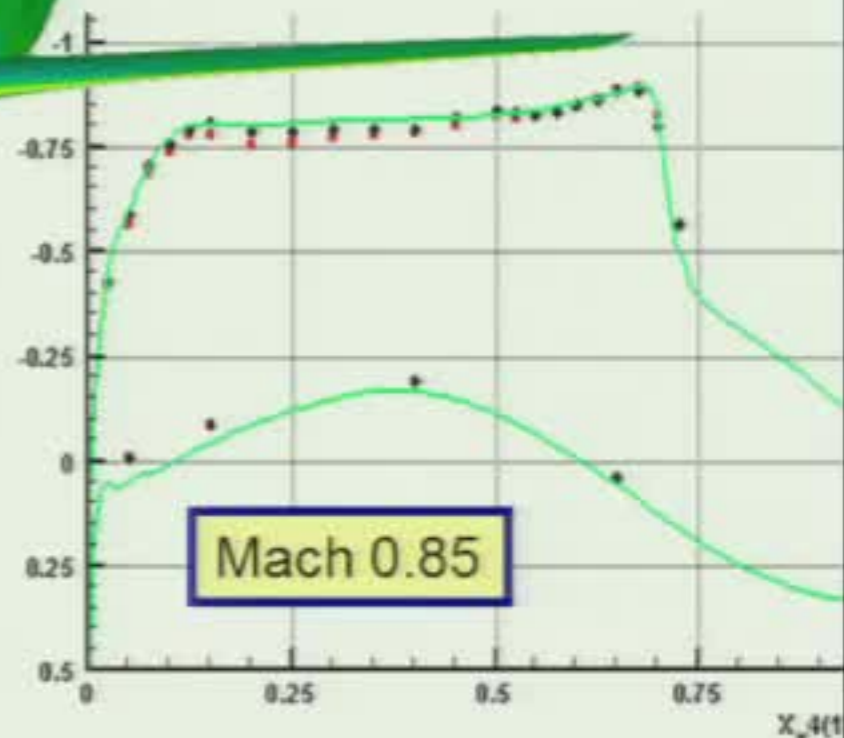
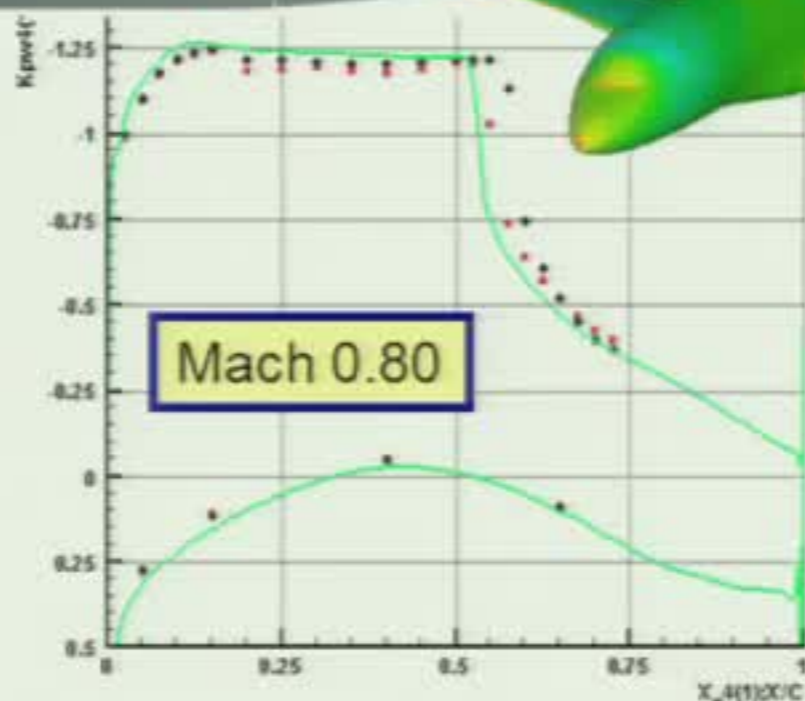
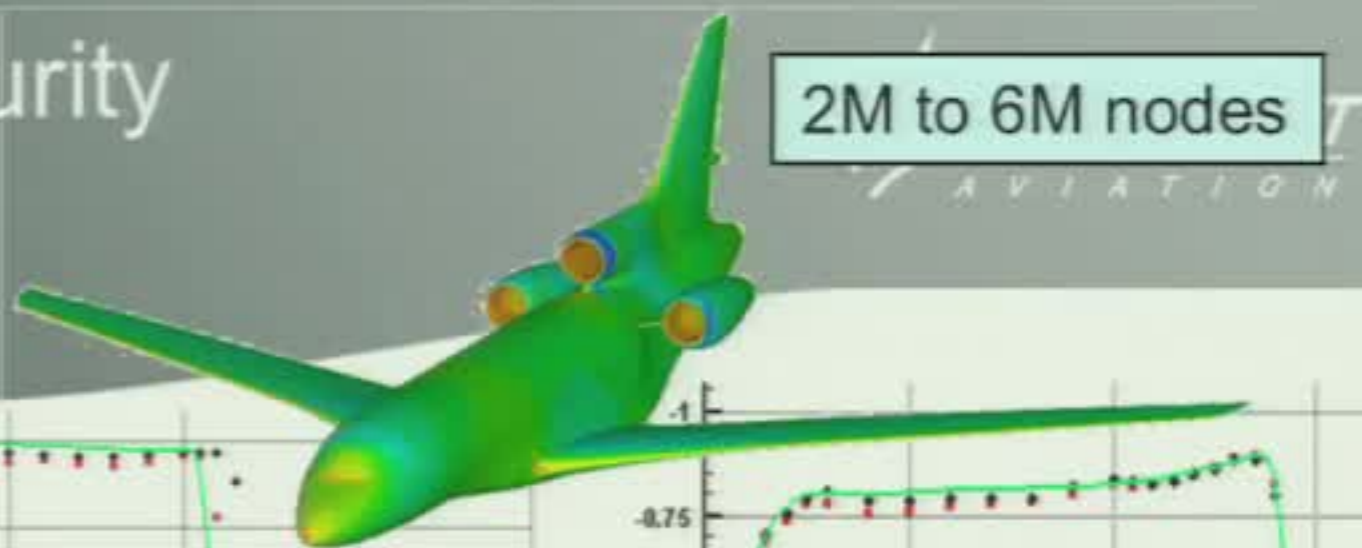
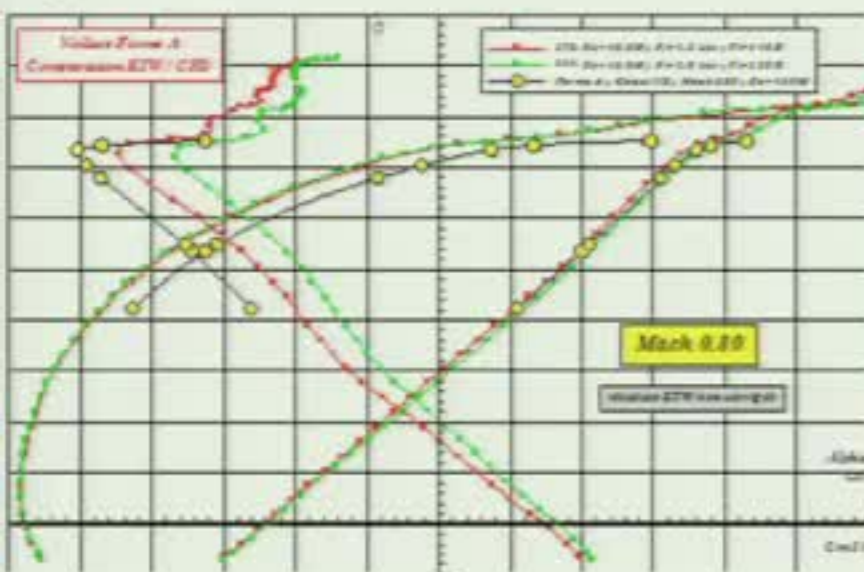
- Reynolds average Navier-Stokes (RANS)      => steady equations
- Direct Eddy-Simulation (DES)                      => unsteady equations



# Mid 2000's: Industrial maturity of CFD codes



Cryotechnic test of generic Falcon shape in ETW



- Full aircraft Navier-Stokes simulations are used at all stages of design
  - Very good validation is obtained at cruise conditions
  - Design for cruise conditions is based on CFD
- Wind tunnel tests can be limited to intermediate and final check-out if sufficient validation is obtained at the actual flight Reynolds number

# CFD: State of the art of RANS codes

## Transonic cruise



*Boundary layer shape factor*

*Pressure*

*Cruise*

2001 – several hours  
1 million grid points



2014 – 15 minutes  
~20 million grid points  
~500 computations possible per day

### Challenges for the future :

- drag accuracy at cruise ~0.5-1% (viscous drag accuracy, corner flows, ...) → improved RANS modeling
- 30 secs as typical computing time



# CFD: State of the art of RANS codes

## Low-speed configurations

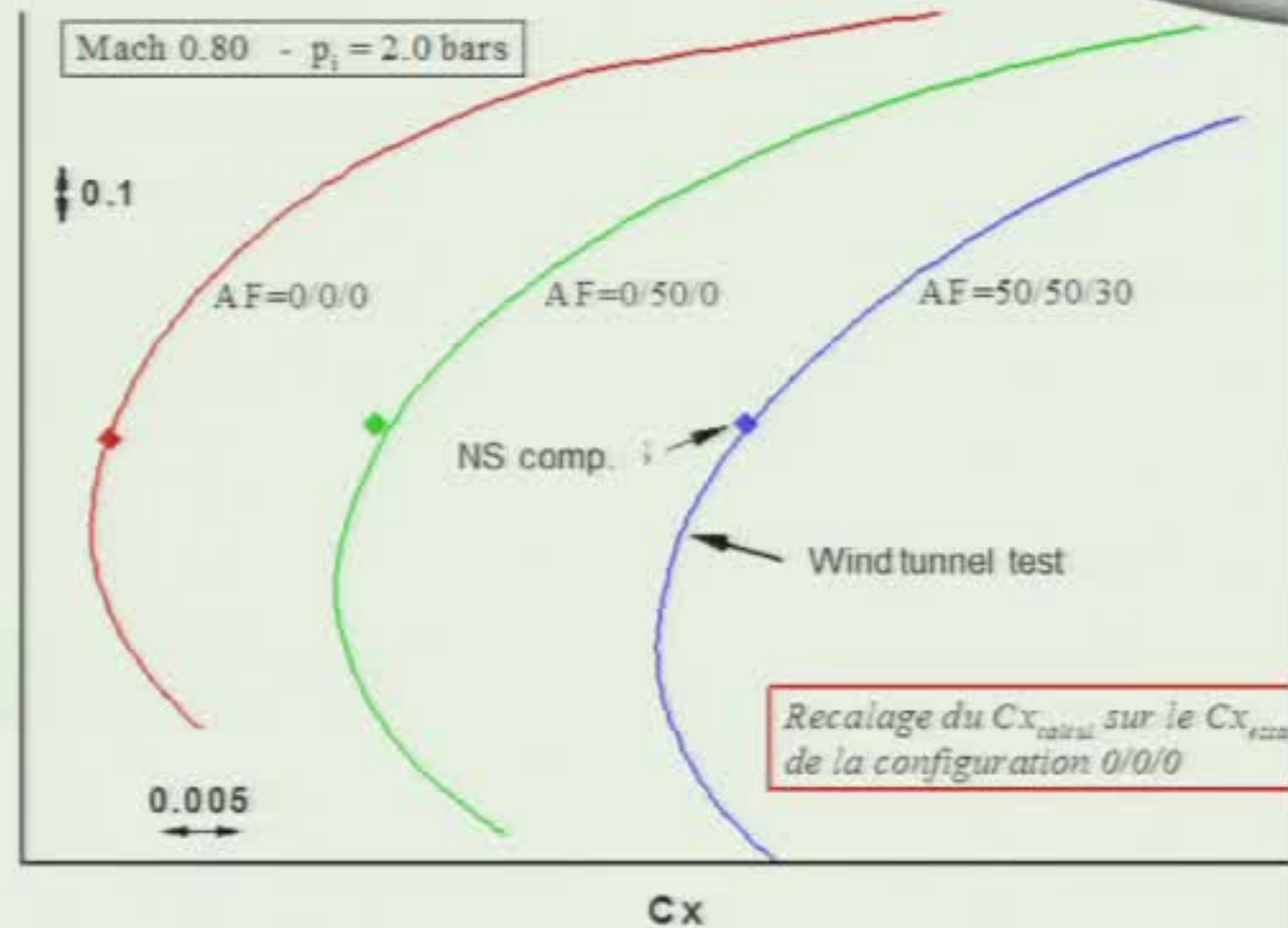
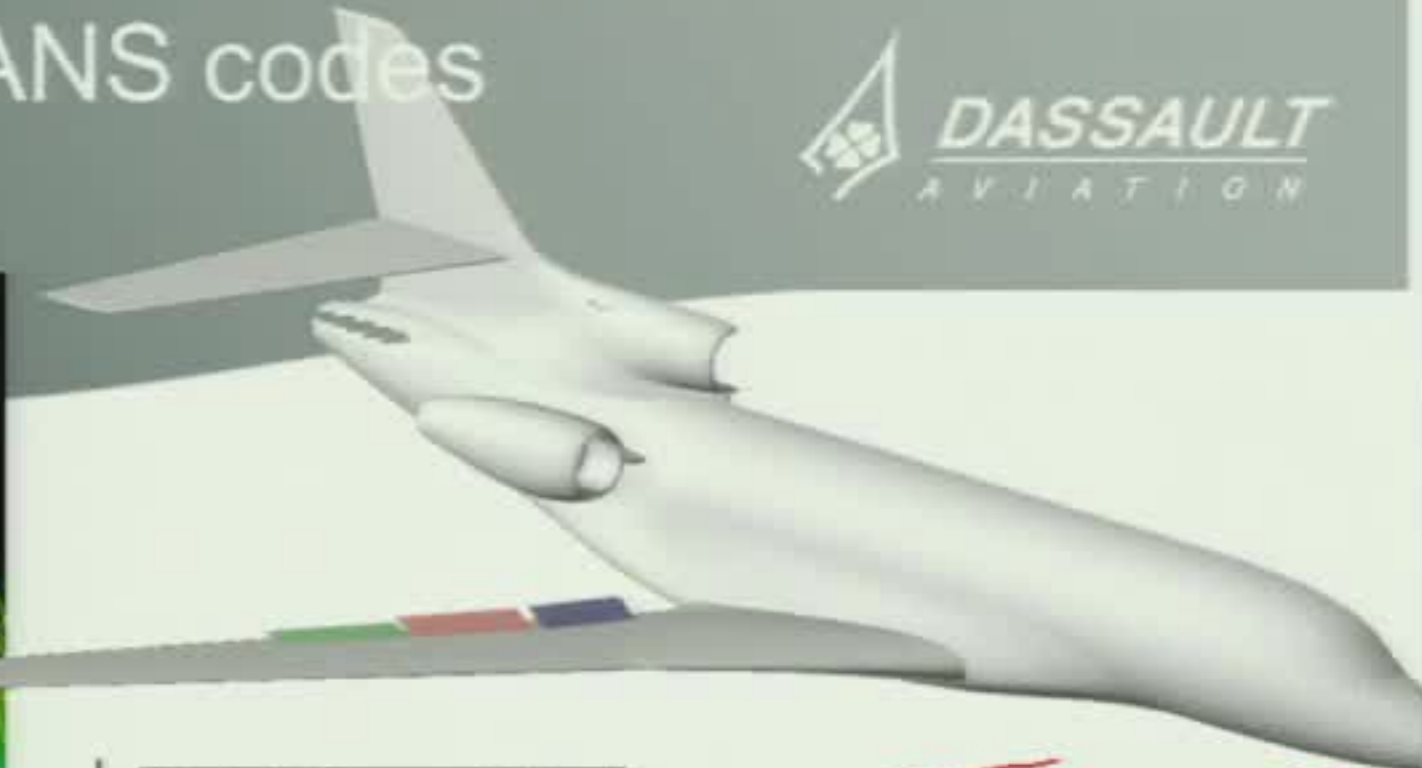
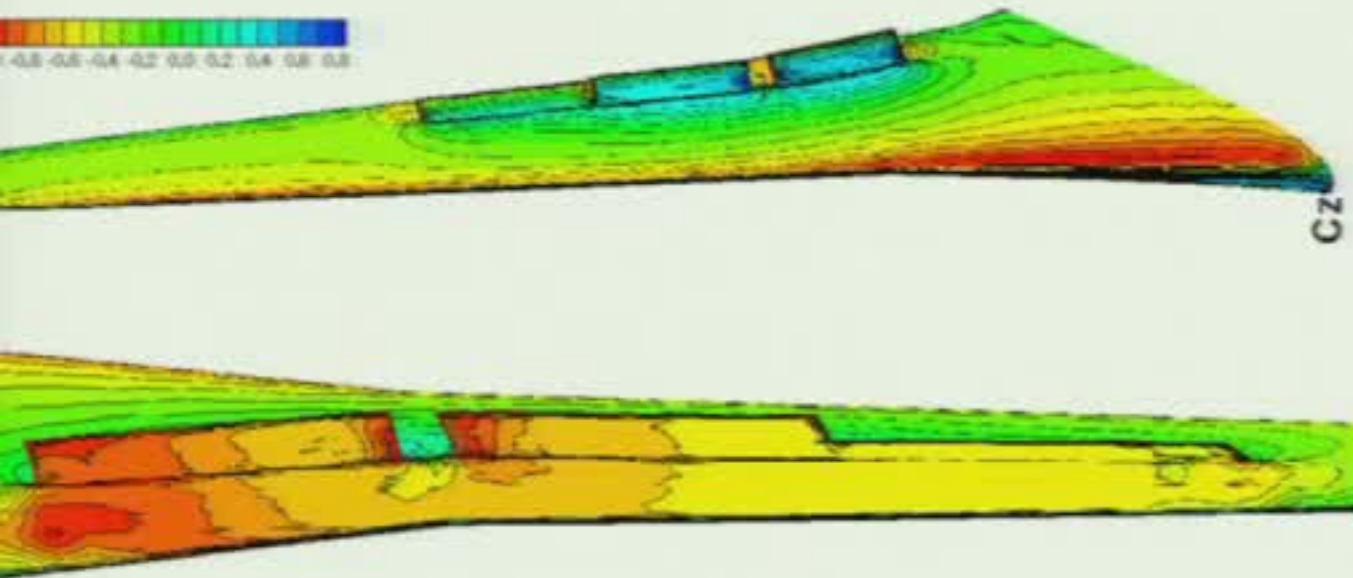
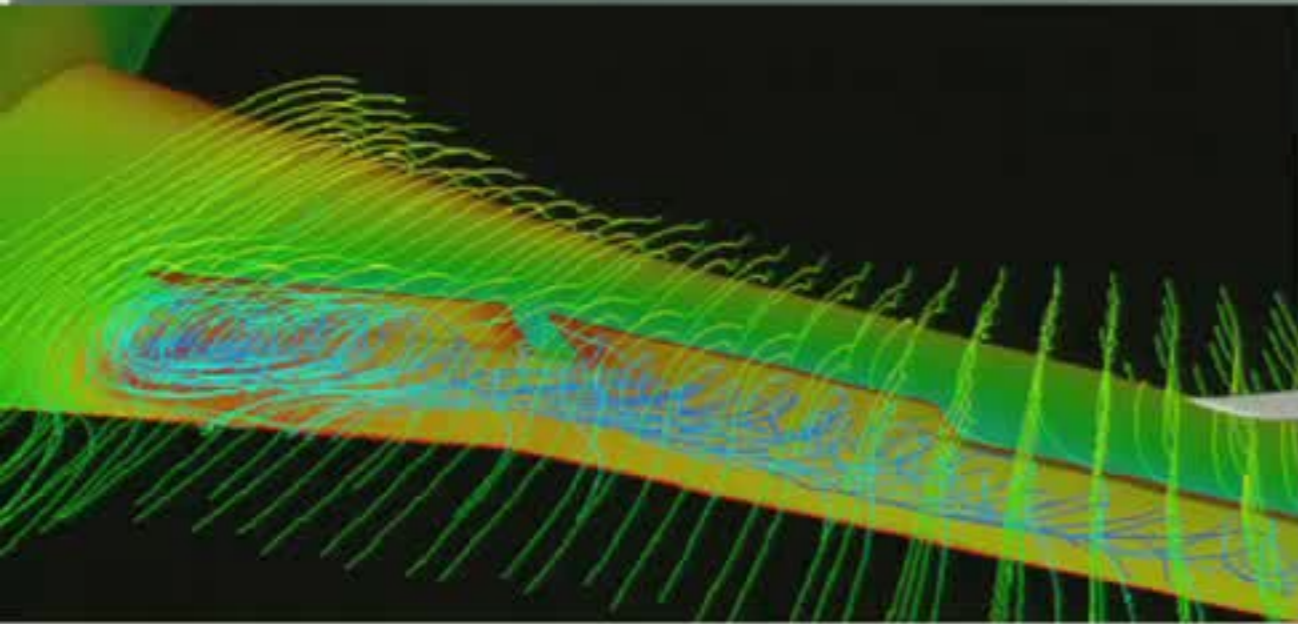


- CFD is not yet reliable enough to predict max lift
  - CFD is a key tool for analysis and understanding of the local flow physics
  - Challenge for the future : accurate max lift (illustrate trend towards use of CFD for limits of flight domain)
- Improved RANS modeling

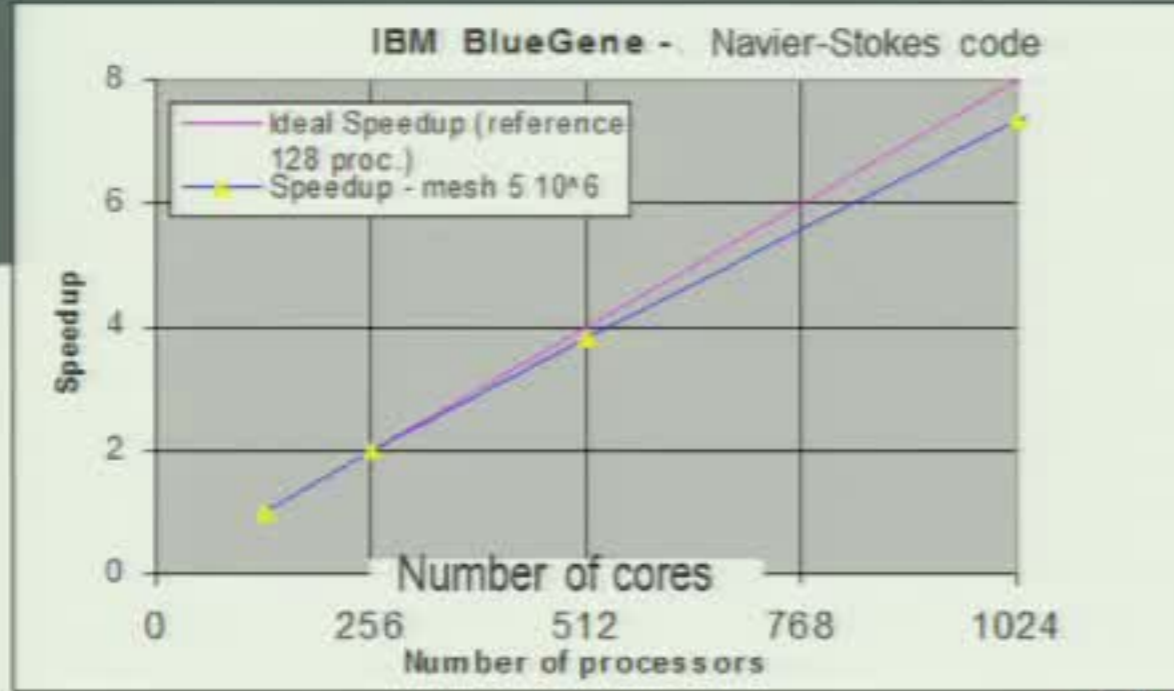


# CFD: State of the art of RANS codes

## Airbrake design



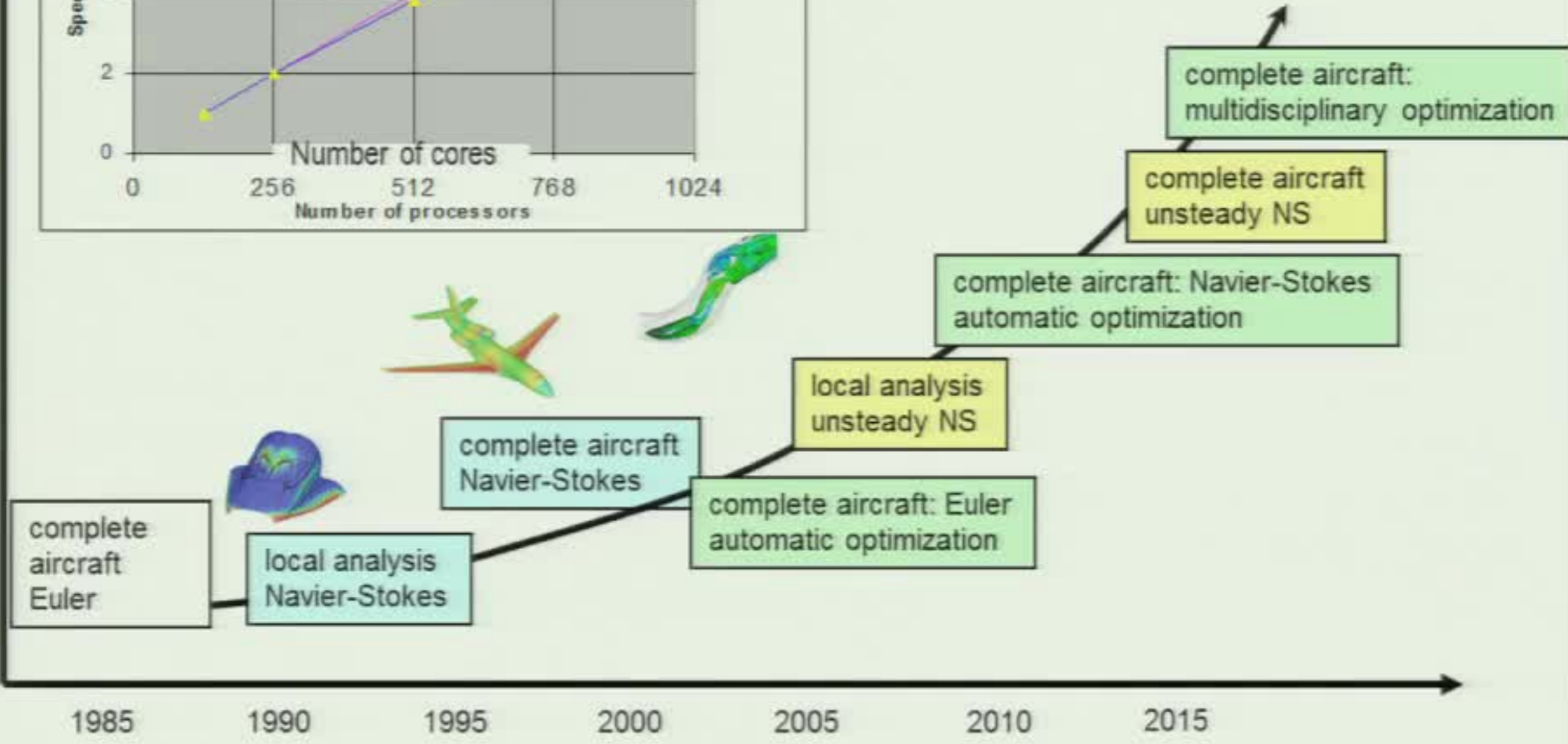
# CFD: Computational capabilities over 30 years



50 Teraflops

1 Teraflops

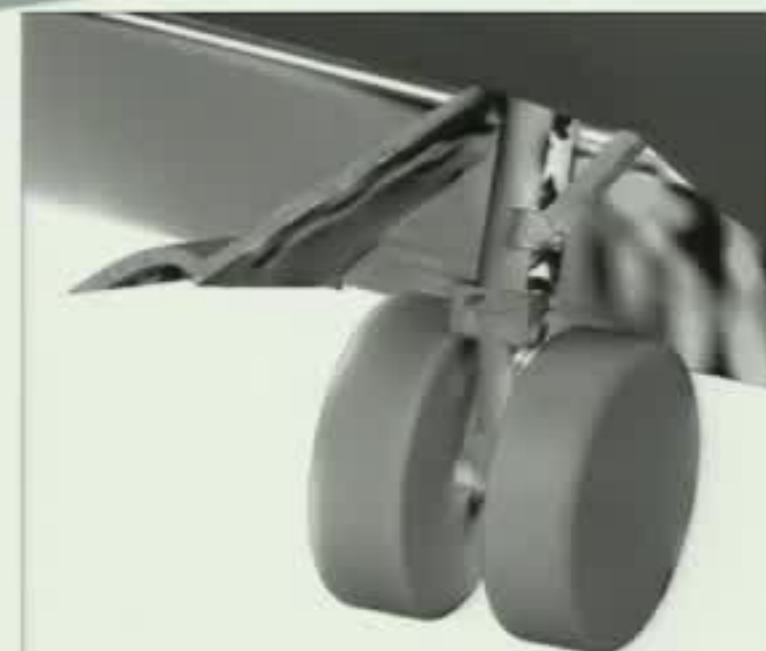
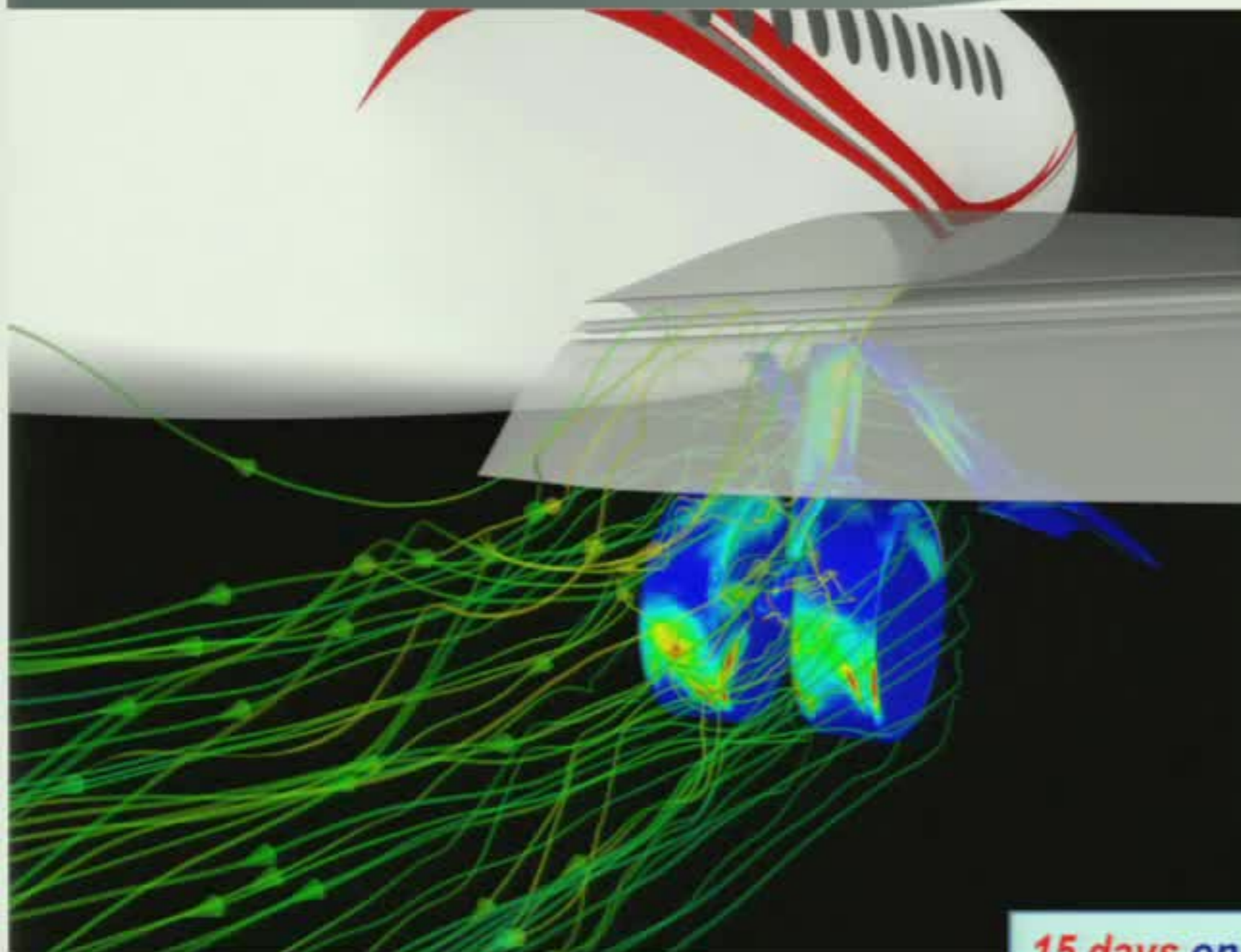
0 MegaFlops





# CFD: DES application to airframe aeroacoustics

## Landing gear noise



**15 days on 2048 BG/P cores**



# CFD: DES application to airframe aeroacoustics

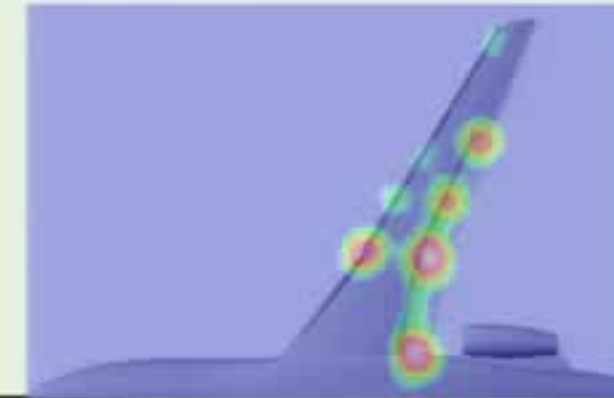


## Landing gear noise

## Influence of landing gear bay

### Example of detailed study: gear bay integration

- Gear bay as a noise source
- Disturbance of mean flow field due to gear bay
- Mixing layer over the bay interacting with gear components

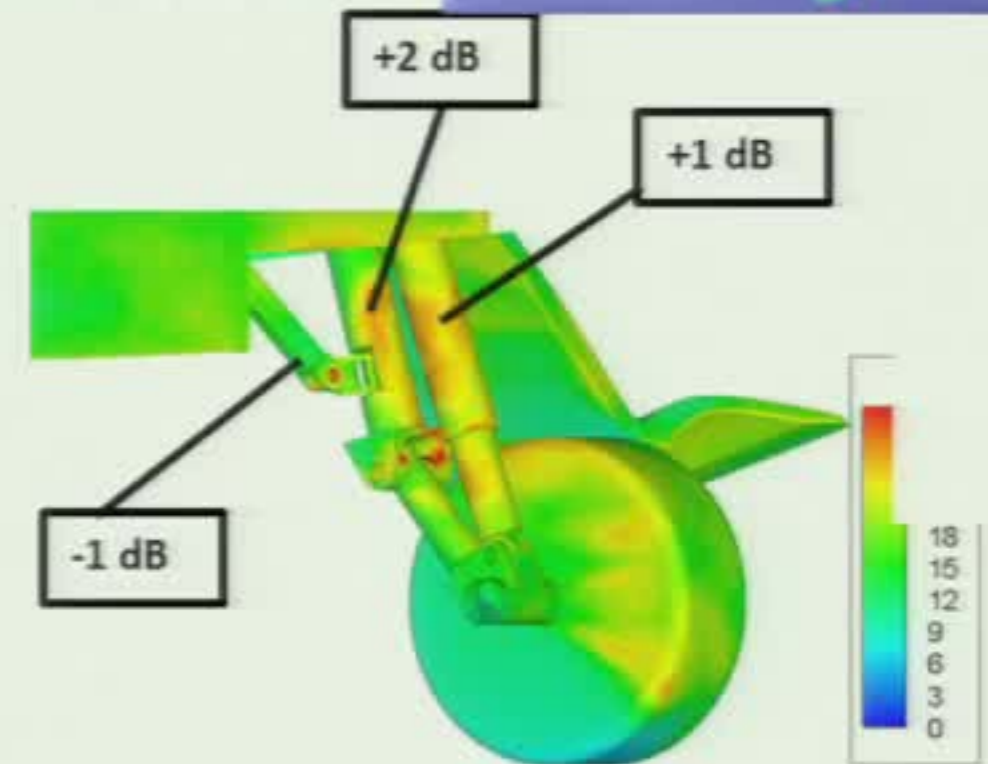


gear bay open



gear bay closed

Acoustic pressure (bottom view of the aircraft)



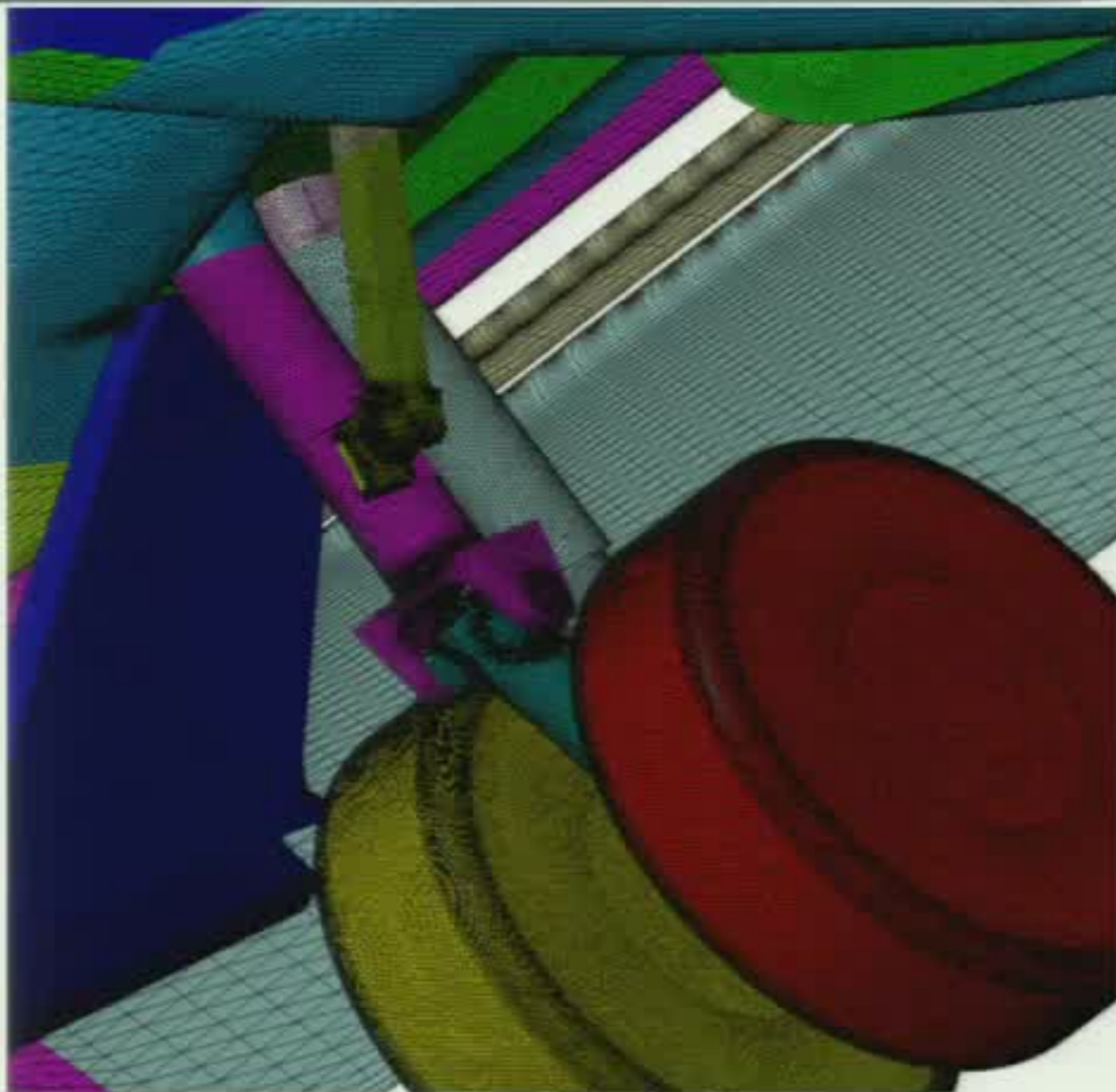
Modification of noise sources intensity due to gear bay opening



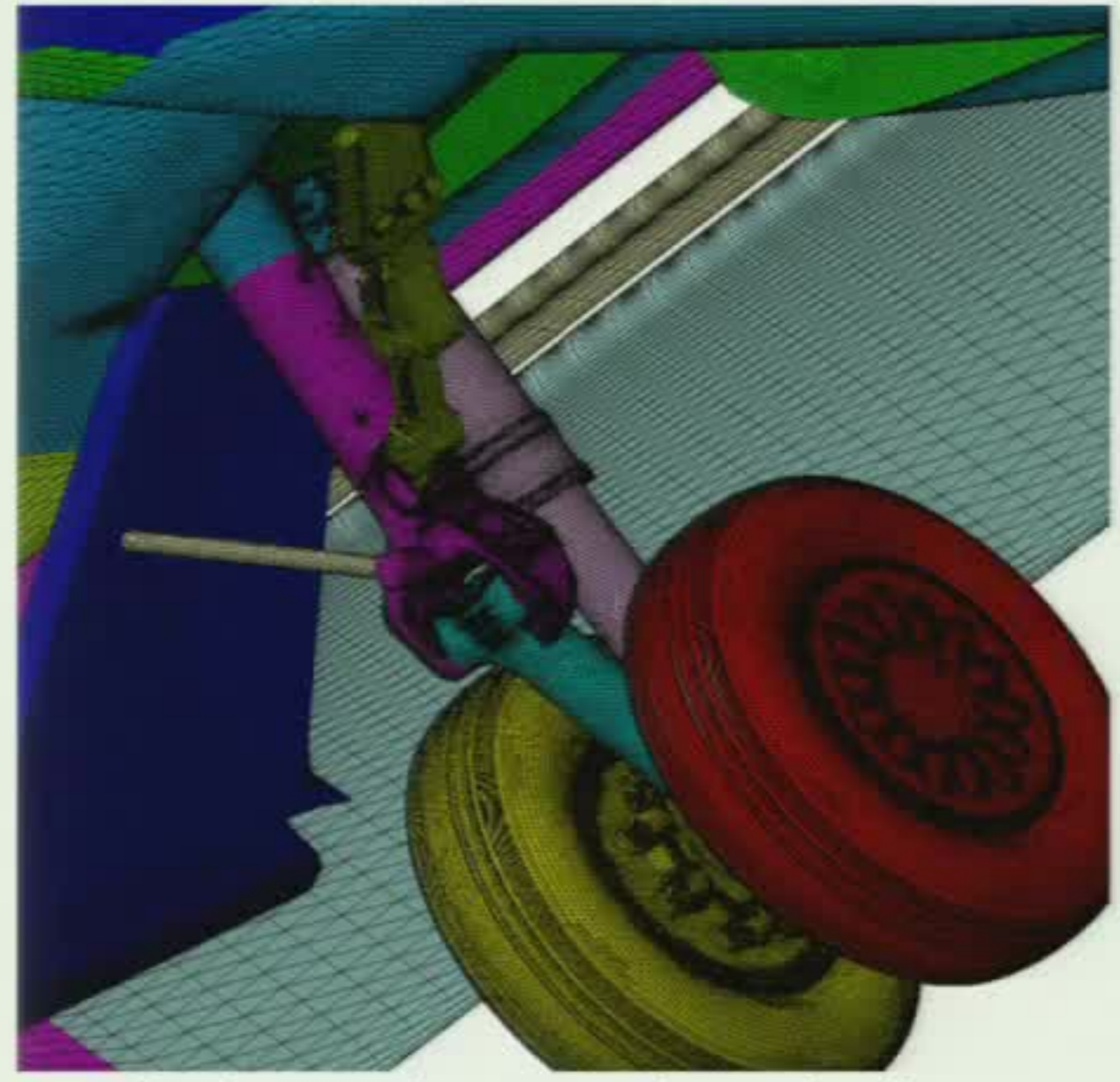
# CFD: DES application to airframe aeroacoustics

## Landing gear noise

### Influence of geometrical details



Surface mesh – « simplified » landing gear  
430 809 nodes



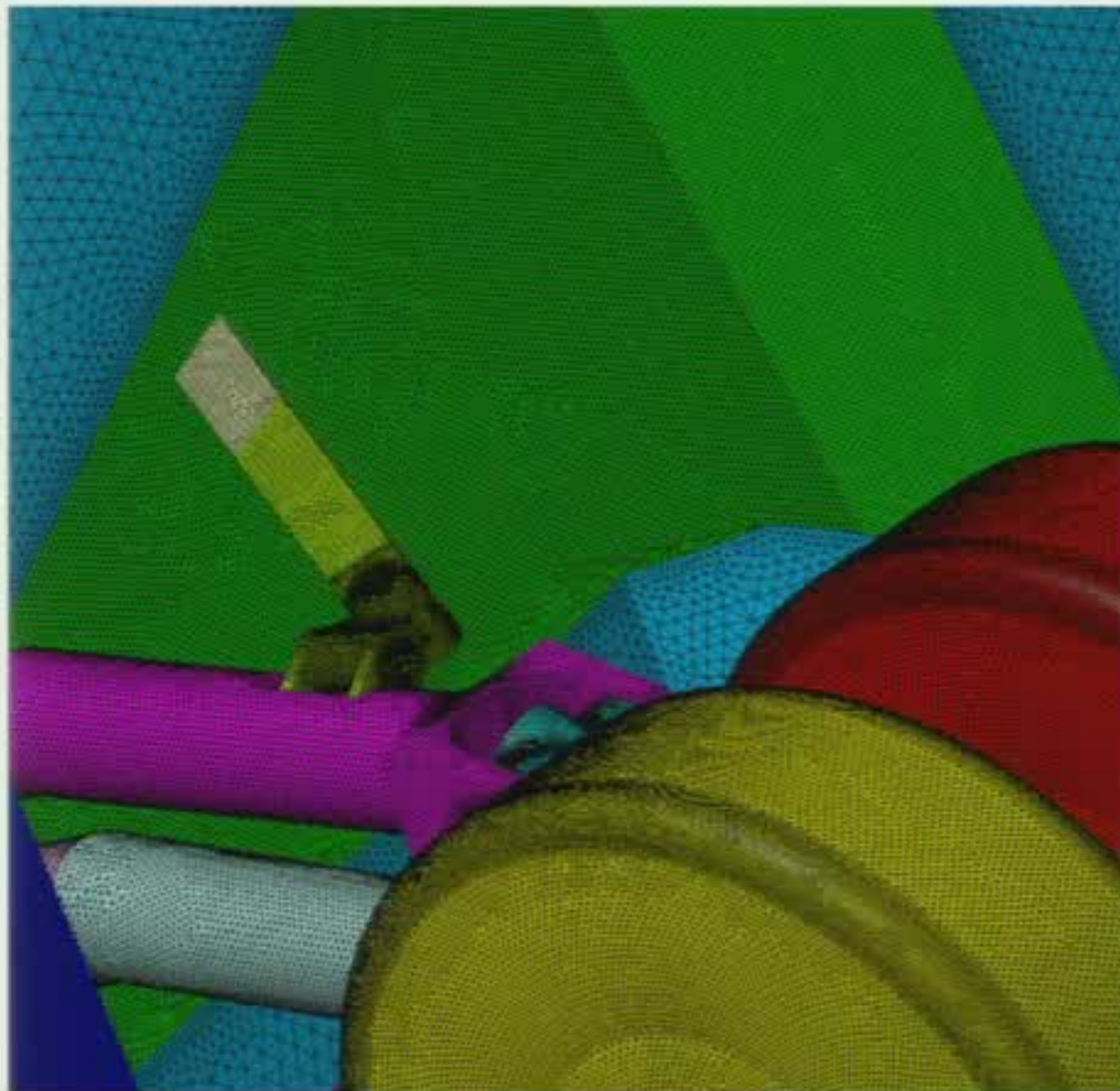
Surface mesh – « complexe » landing gear  
493 445 nodes



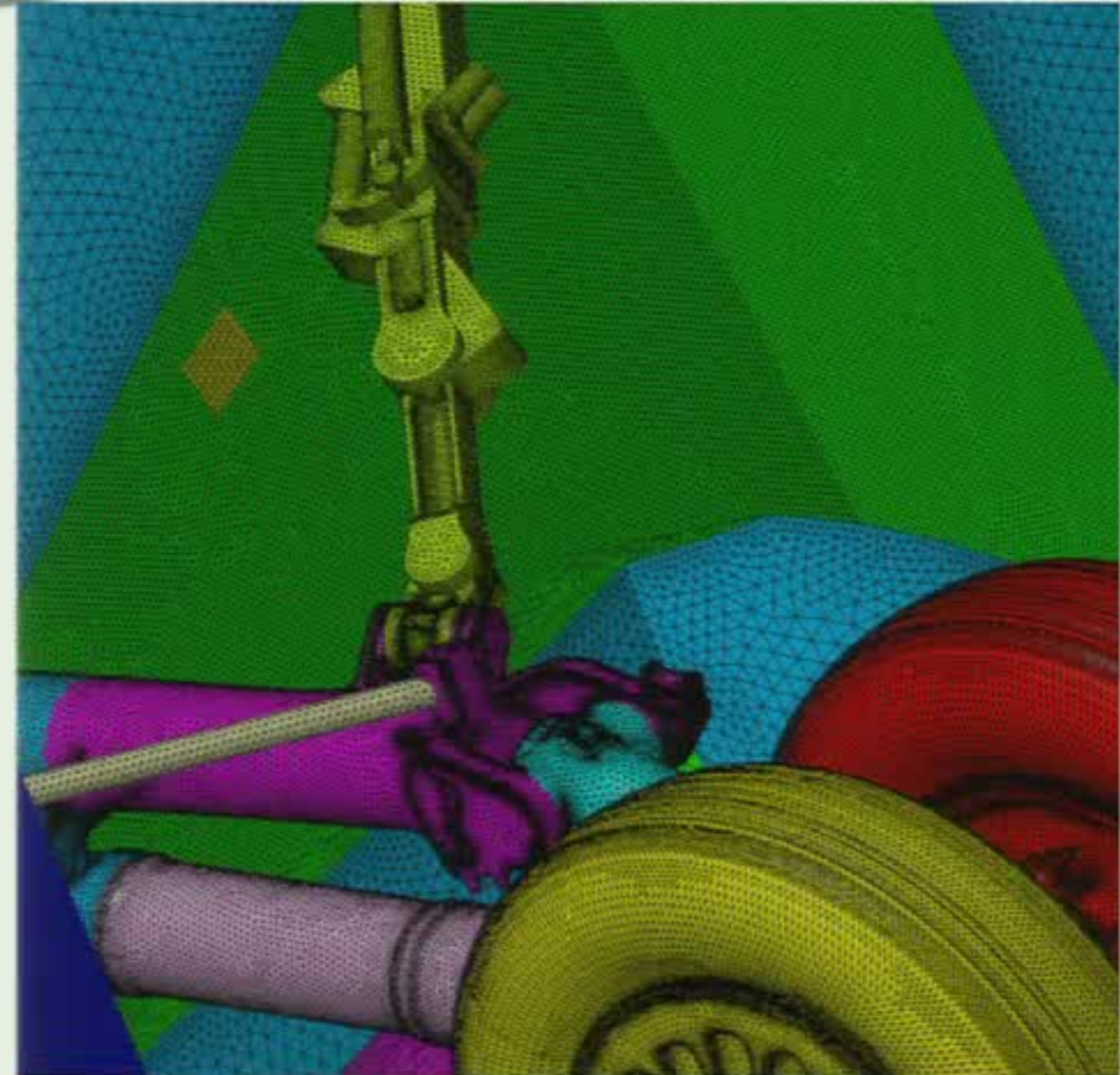
# CFD: Application to airframe aeroacoustics

Landing gear noise

Influence of geometrical details



**Surface mesh – « simplified » landing gear**  
430 809 nodes



**Surface mesh – « complex » landing gear**  
493 445 nodes



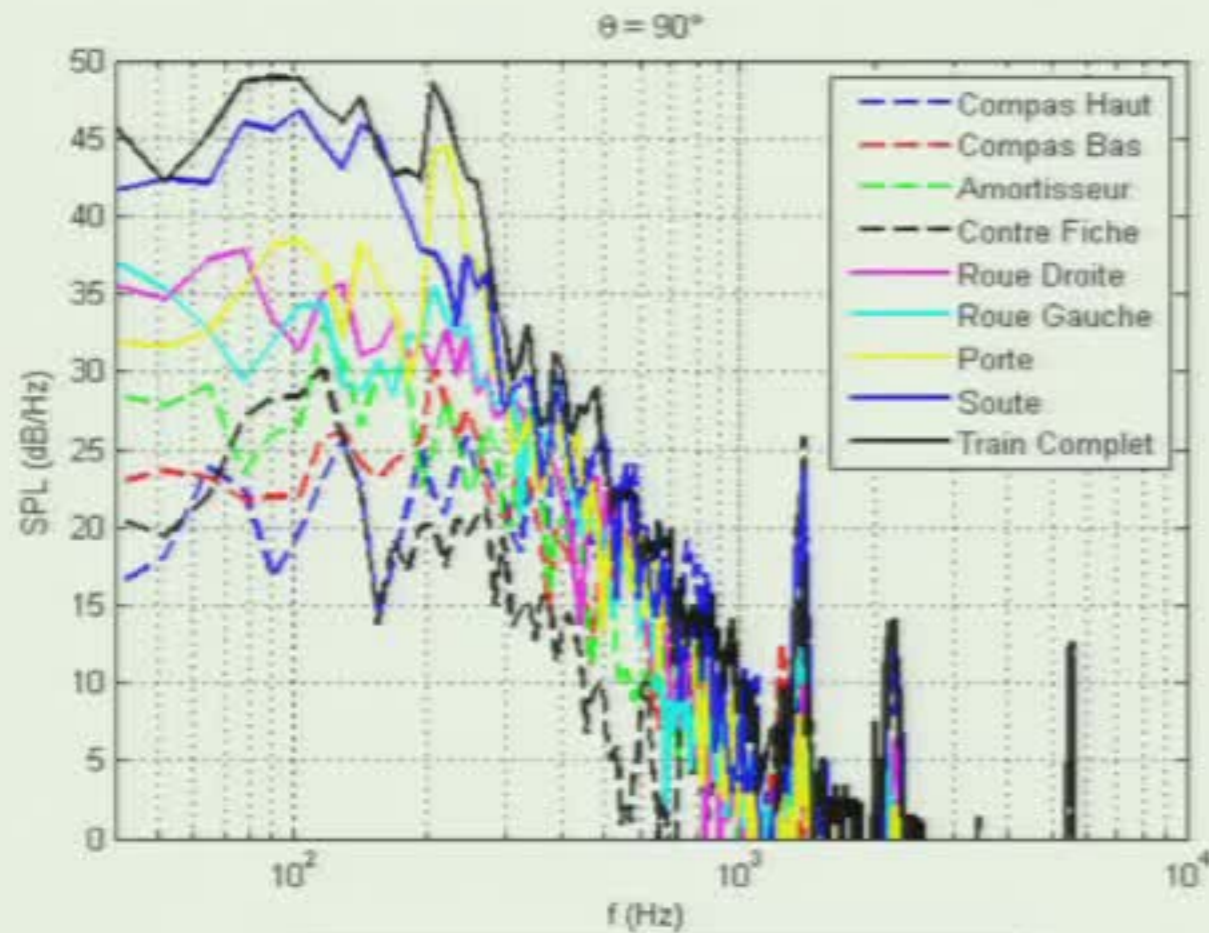
# CFD: Application to airframe aeroacoustics

## Landing gear noise

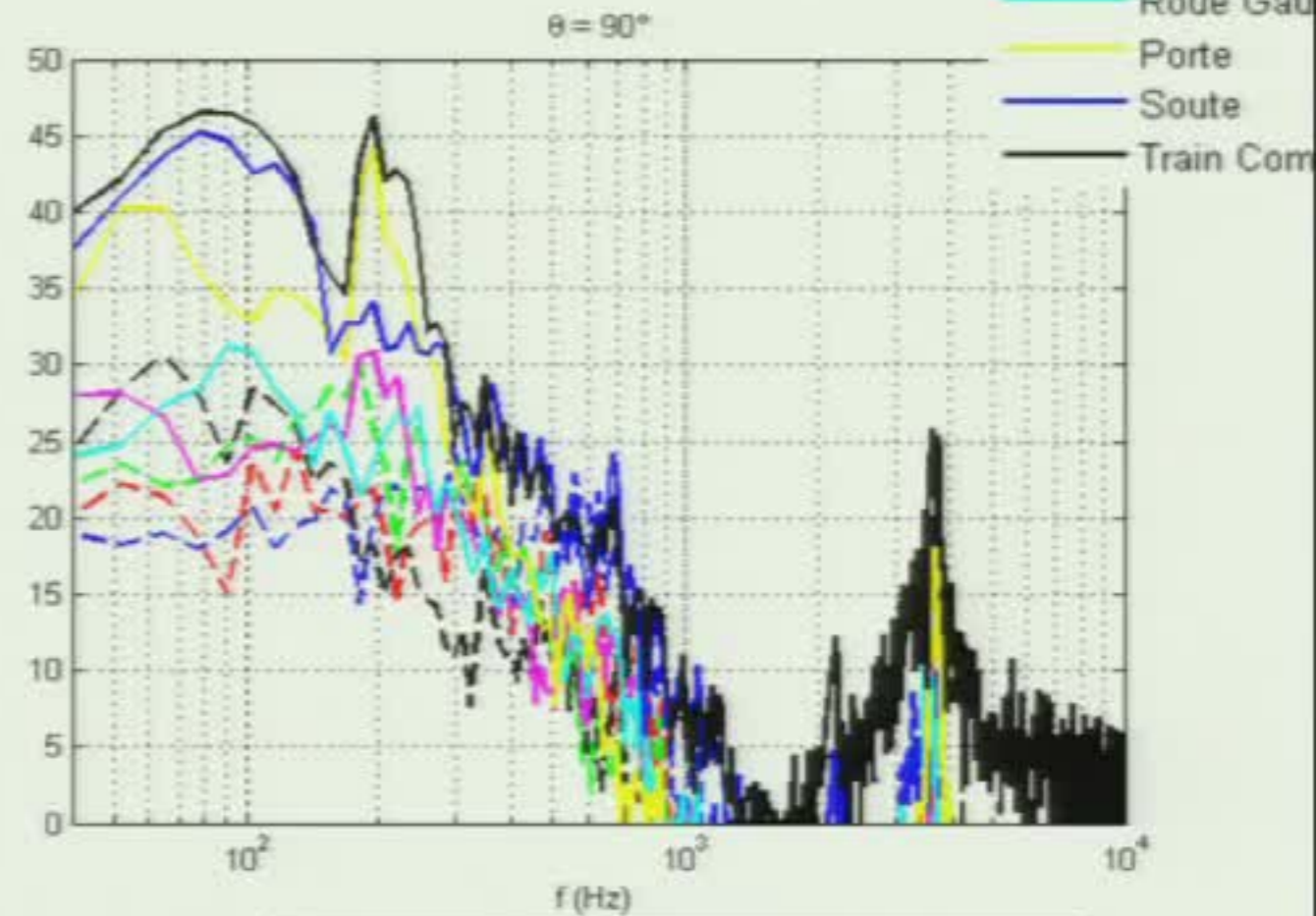
### Influence of geometrical details



## Far field spectra – simple / complex DES



SPL (dB/Hz)  
Simple landing gear / Experiment



SPL (dB/Hz)  
Complex landing gear / Experiment

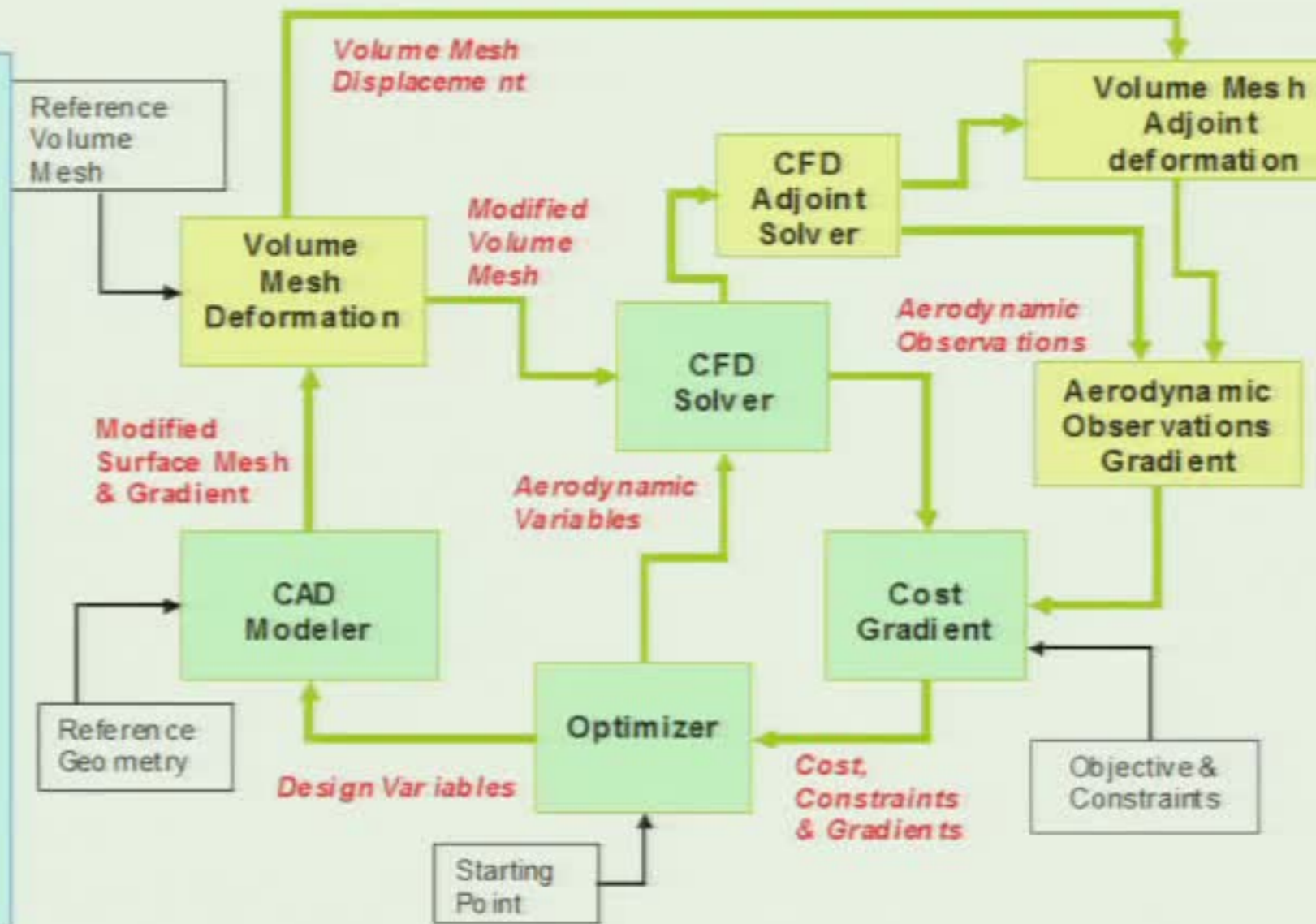


# Automatic shape optimization

## Introduction



- Complex process : large effort to develop and mature
- Key ingredients:
  - Adjoint approach including mesh motion
  - Parameterization (CAD + features)
  - Extensive library of cost functions
- Process progressively applied to many real life design problems
  - Strong interaction with design team to define relevant formulation of the problem



# Automatic shape optimization

## Gradient computation



- *State equation*  $E(\mu, W(\mu)) = 0$
- *Cost function*  $j(\mu) = J(\mu, W(\mu))$
- *Constraints functions*  $g(\mu) = G(\mu, W(\mu))$
- *Minimizing*  $j(\mu)$  *while respecting constraints*  $g_i(\mu) \leq 0$
- *Observation*

$$f(\mu) = F(\mu, W(\mu)) = (J(\mu, W(\mu)), G(\mu, W(\mu)))$$

$\mu = (l, v)$  with  $l$  = aerodynamic parameters and  $v$  = geometric parameters

- *CAD modeler*  $v \rightarrow d(v)$
- *Mesh deformation equation*  $L(d(v), D(v)) = 0$

*PDE control theory*  
J-L Lions  
Dunod, 1968

Collaborations with INRIA



# Automatic shape optimization

## Gradient computation



- To estimate

$$\delta f = \frac{df(\mu)}{d\mu} \cdot \delta\mu = \frac{dF(\mu, W(\mu))}{d\mu} \cdot \delta\mu$$

$$\mu = (l, v)$$

$$\delta f = \frac{\partial F}{\partial W} \frac{\partial W}{\partial l} \cdot \delta l + \frac{\partial F}{\partial W} \frac{\partial W}{\partial v} \cdot \delta v + \frac{\partial F}{\partial l} \cdot \delta l + \frac{\partial F}{\partial v} \cdot \delta v$$

- Thanks to the state equation

$$E(\mu, W(\mu)) = 0$$

- and then

$$\delta E(\mu, W(\mu)) = \frac{\partial E}{\partial W} \cdot \delta W + \frac{\partial E}{\partial \mu} \cdot \delta\mu = 0$$

- Thanks to the mesh deformation equation

$$L(d(v), D(v)) = 0$$

- and then

$$\delta L(d(v), D(v)) = \frac{\partial L}{\partial d} \cdot \delta d + \frac{\partial L}{\partial D} \cdot \delta D = 0$$

# Automatic shape optimization

## Gradient computation



- Evaluate variations of the Lagrangian  $\delta f^* = \delta f - \Psi(\mu)^T \delta E - \Phi(v)^T \delta L$
- with

$$\left( \frac{\partial E}{\partial W}(l, D(v), W(\mu)) \right)^T \Psi(\mu) = \left[ \frac{\partial F}{\partial W}(\mu, W(\mu)) \right]^T$$

$$\frac{\partial F}{\partial D}(l, D(v), W(\mu)) - \Psi^T \frac{\partial E}{\partial D}(l, D(v), W(\mu)) = \Phi^T \frac{\partial L}{\partial D}(d(v), D(v))$$

- to obtain

$$\begin{cases} \frac{dF}{dl} = \frac{\partial F}{\partial l} - \Psi^T \left[ \frac{\partial E}{\partial l} \right] \\ \frac{dF}{dv} = -\Phi^T \begin{bmatrix} \frac{\partial L}{\partial d} & \frac{\partial d}{\partial v} \end{bmatrix} \end{cases}$$



# Automatic shape optimization

## Optimization ingredients



*Automatic Differentiation software Tapenade (INRIA-Sophia-Antipolis)*

*Gradient-based optimization*

*Feasible (direction) Sequential Quadratic Programming*

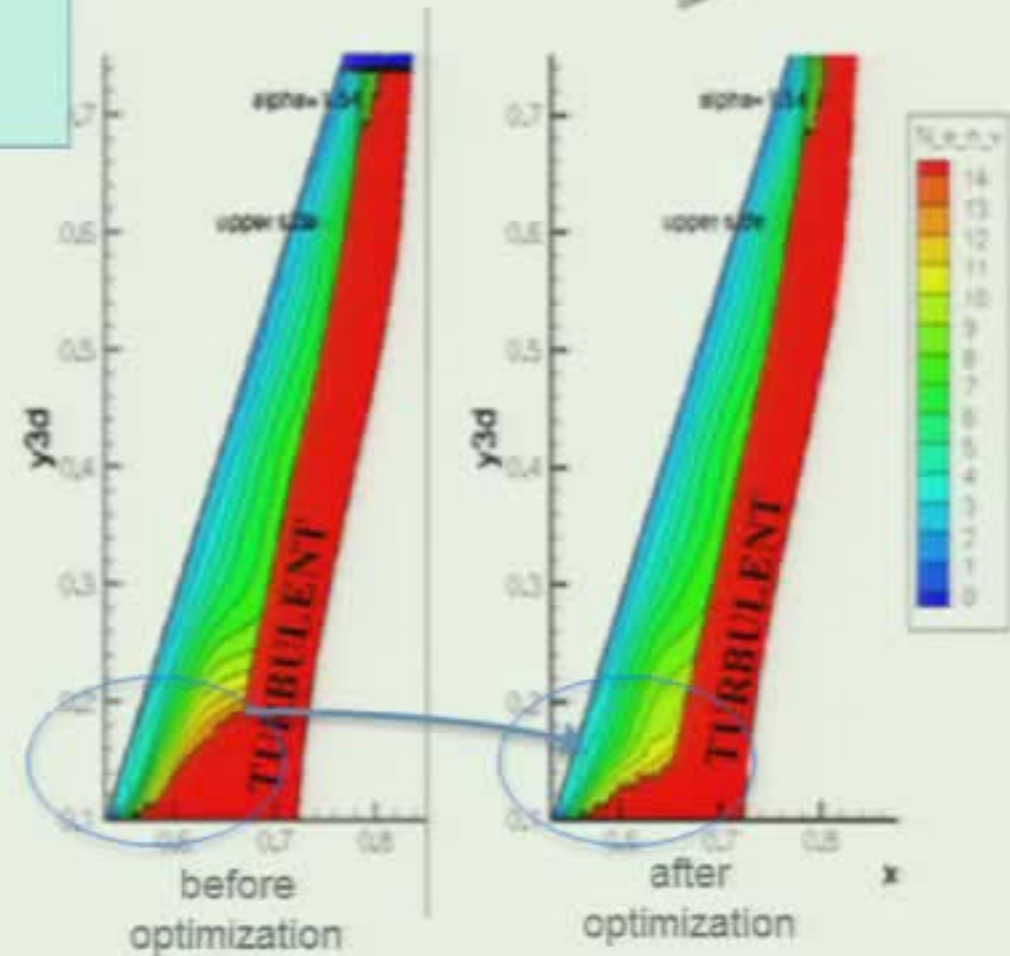
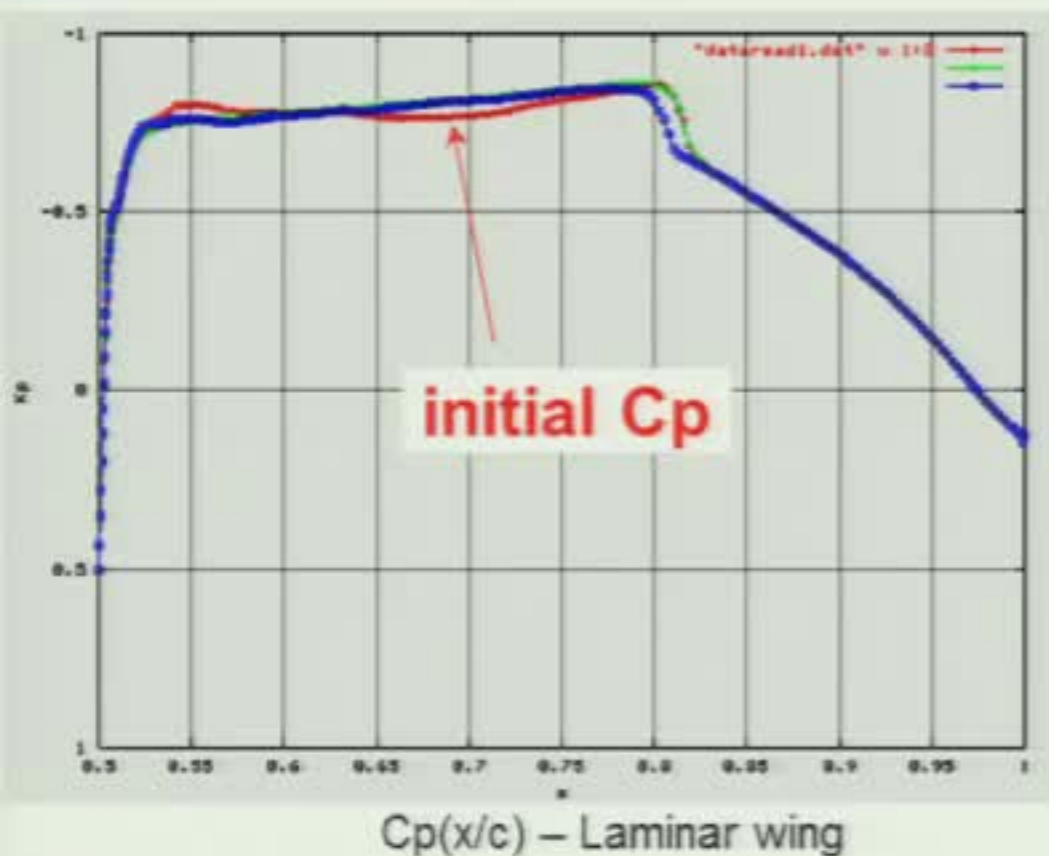
*Feasible Arc Interior Point Algorithm (FAIPA) developed by Prof. J.N. Herskovits & co-workers*

# Automatic shape optimization

## Laminar wing optimization to increase laminar area on wing next to the fuselage



- Laminar wing,  $\Phi = 20^\circ$
- Mach = 0.75, angle of attack =  $3^\circ$
- Objective : increase laminar area on wing next to the fuselage  $\rightarrow C_p$  &  $\delta C_p / \delta x$  target locally
- Variables: Leeward wing section profile
- Navier-Stokes with discrete adjoint





# Automatic shape optimization

Afterbody optimization of innovative configuration

Example of complex objective functions

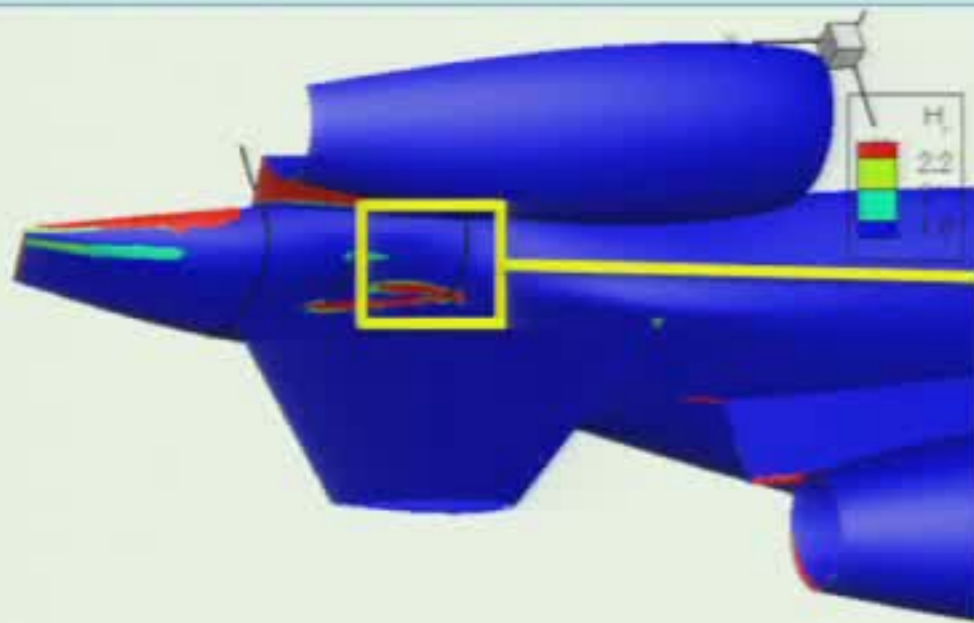
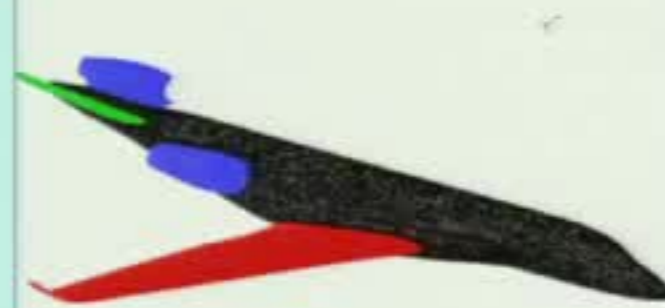


Mach = 0.85, angle of attack = 1.5°

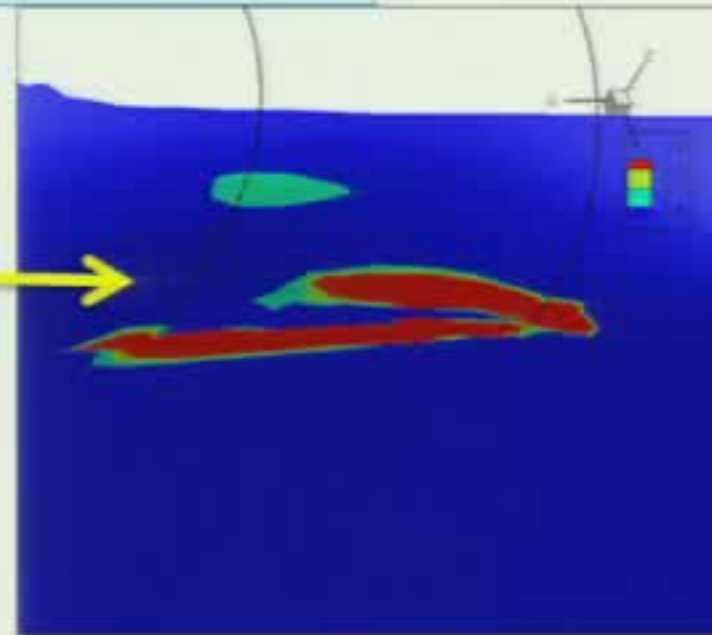
Cost function is based on the boundary layer shape parameter  $H_i$  (ratio of displacement and momentum thickness)

Fuselage shape: 10 variables

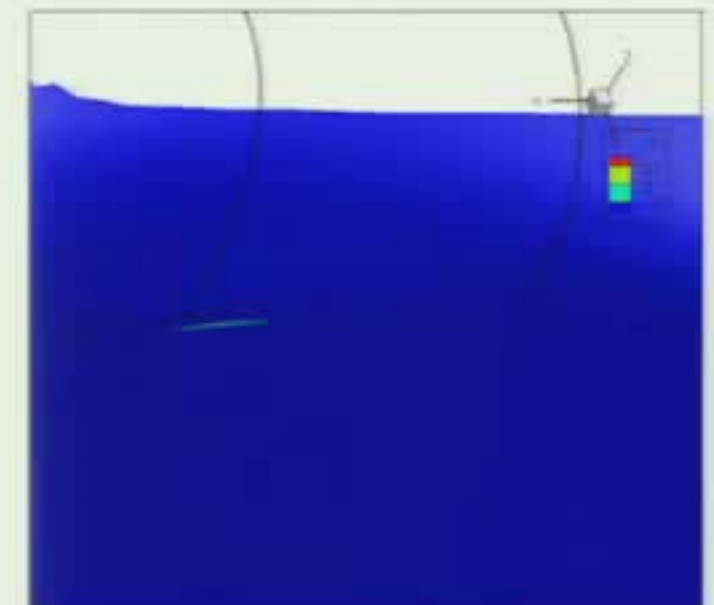
Adjoint approach - Convergence requires about 20 NS computations



Recirculation zone ( $H_i > 2.2$ ) red



$H_i$  before optimization



$H_i$  after optimization

# Automatic shape optimization

## Low speed – high speed wing tip optimization

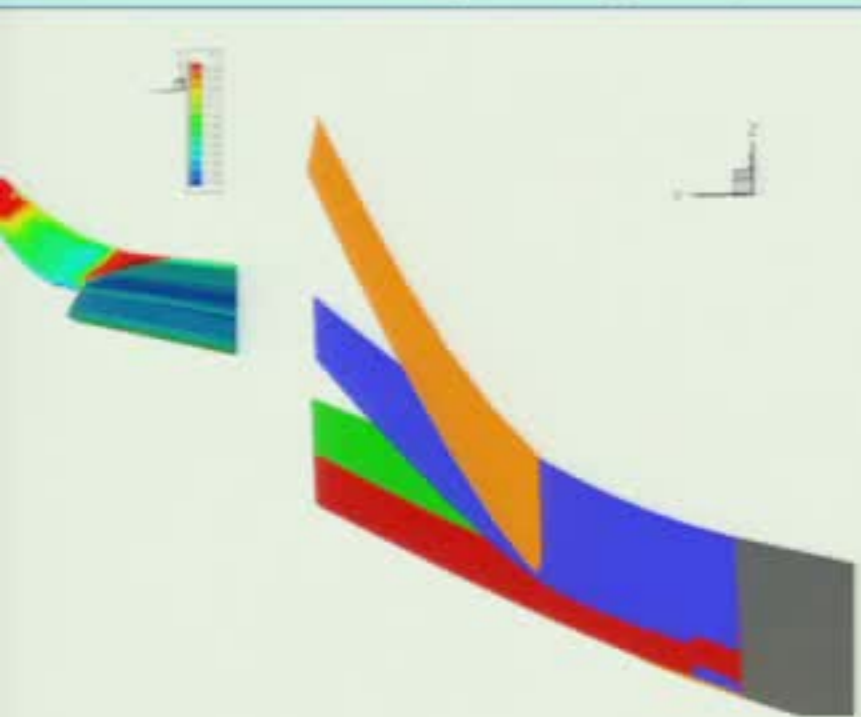


Multipoint optimization : low speed (Mach = 0.18, high lift configuration) – high speed (Mach = 0.8, cruise configuration)

Minimize

- drag at high speed (constraints on lift + bending moment at  $y = 8 \text{ m}$ )
- surface  $H_i > 2$  at low speed

Parameters: aoa (at High speed), twist, sweep angle, dihedral, thickness, span



winglet shape proposed by multipoint optimization

1 % reduction of drag

Boundary layer shape factor on the optimized winglet – high lift configuration

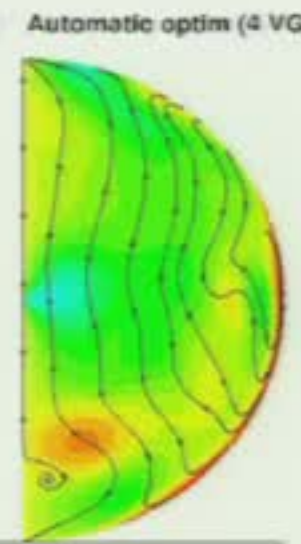
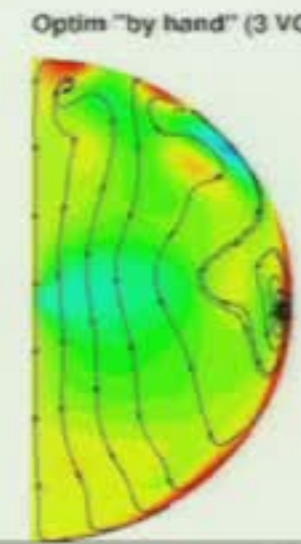
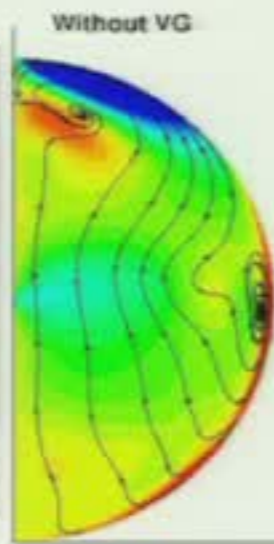
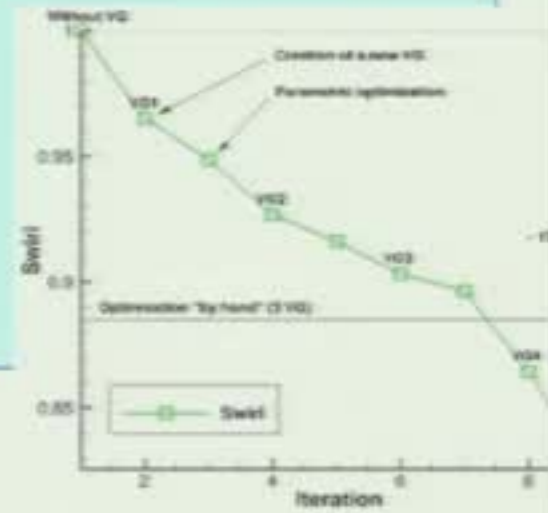


# Automatic shape optimization

## Combination with topological optimization



- Automated methods for the control and the optimization of separated flows
- Application to curved air ducts for UCAV
- Use of mechanical or fluidic vortex generators (VG)
- Optimization: **topological** + shape



Swirl

# Automatic shape optimization Achievements

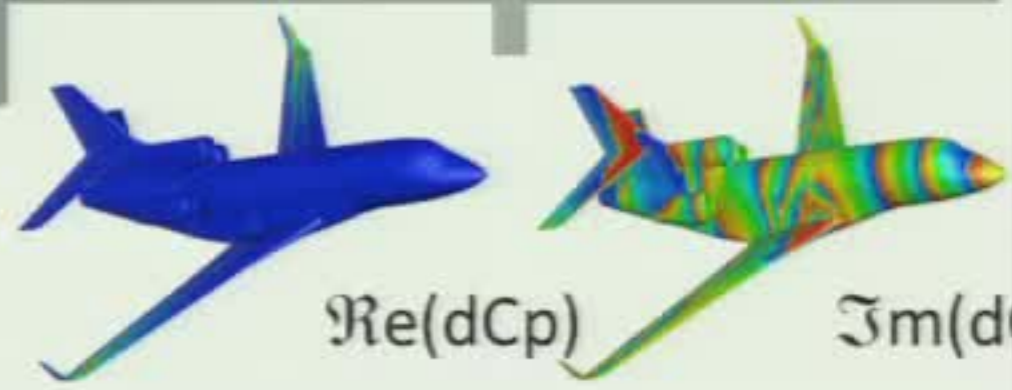


- The total in-house control of the tooled process enables us to develop an optimization chain for aerodynamic design at the industrial level
- Automatic shape optimization accelerates the elementary design cycle and gives access to an enlargement of exploration of potential solutions
- The analysis by engineers remains an essential element of the design cycle
- This optimization chain is currently daily used for industrial design



# Multiphysics: CFD for Aeroelasticity

## Linearized Euler and Navier-Stokes equations



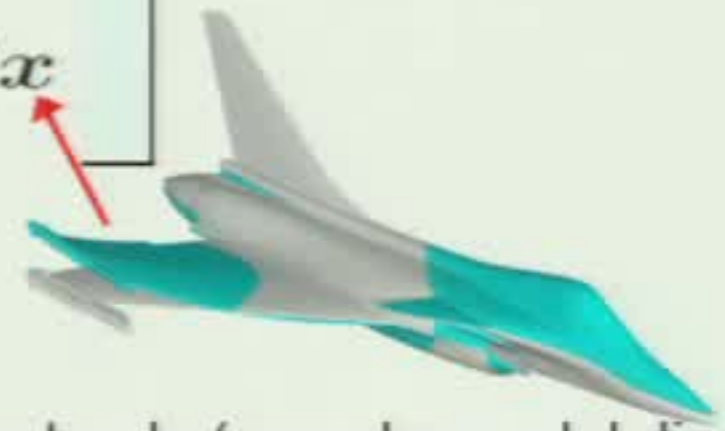
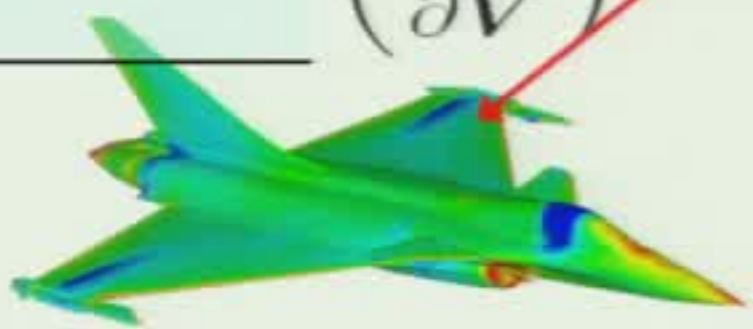
Aerodynamics : Euler or Navier Stokes equations  $E(\mathbf{V}_0, \mathbf{x}_0) = 0$

Linearization:  $\mathbf{V} = \mathbf{V}_0 + d\mathbf{V}$   
 $\mathbf{x} = \mathbf{x}_0 + d\mathbf{x}$

$$dE = \frac{\partial E}{\partial \mathbf{V}} d\mathbf{V} + \frac{\partial E}{\partial \mathbf{x}} d\mathbf{x} = 0$$

Linear problem

$$\left( \frac{\partial E}{\partial \mathbf{V}} \right) d\mathbf{V} = - \left( \frac{\partial E}{\partial \mathbf{x}} \right) d\mathbf{x}$$



Output =  $d\mathbf{V}$  (complex aerodynamic pressure force)

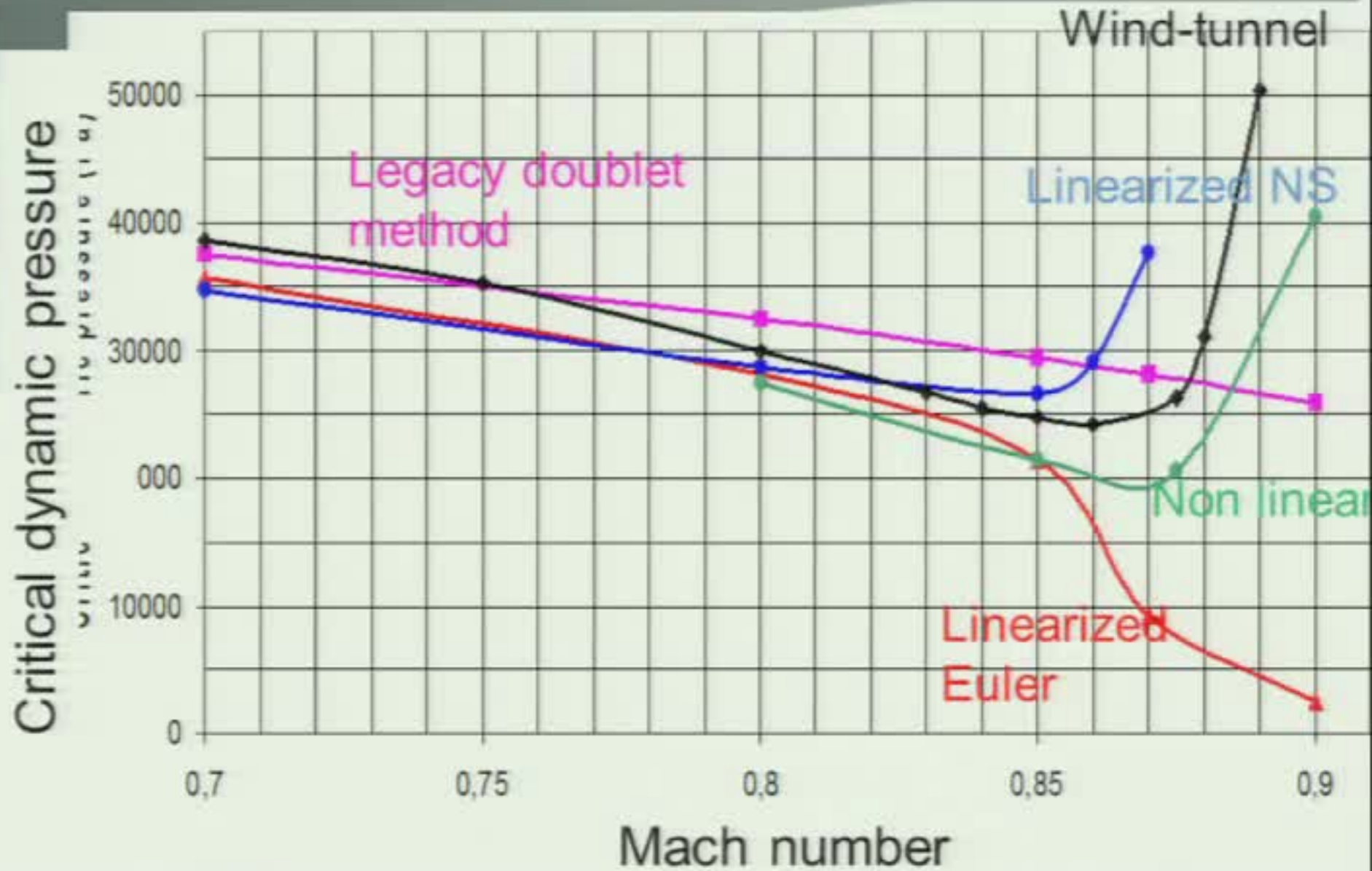
Input =  $d\mathbf{x}$  (complex nodal displacement)

# Multiphysics: CFD for Aeroelasticity

## Linearized CFD - Validation



Transonic wing in ONERA S2MA wind tunnel

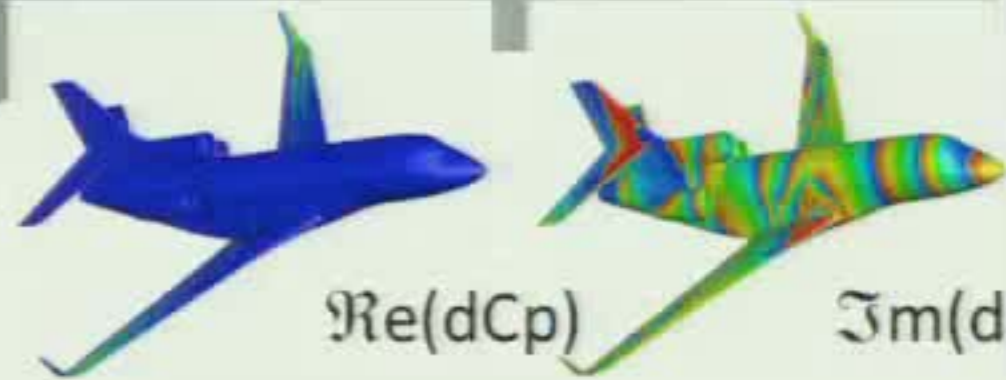


- Linearized CFD can predict “transonic dip”
- Linearized NS leads to improved results compared to linearized Euler



# Multiphysics: CFD for Aeroelasticity

## Linearized Euler and Navier-Stokes equations



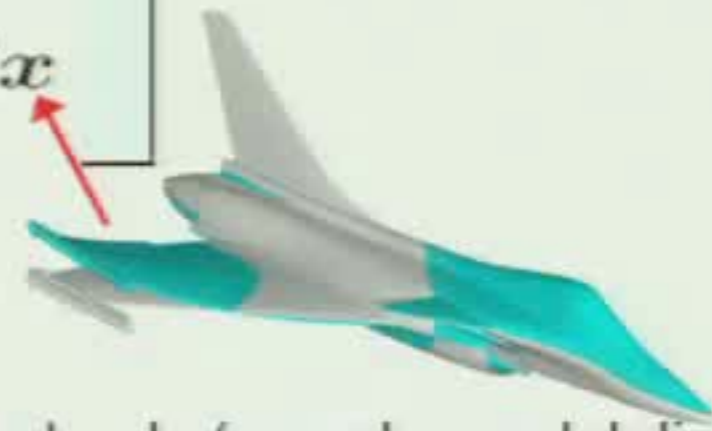
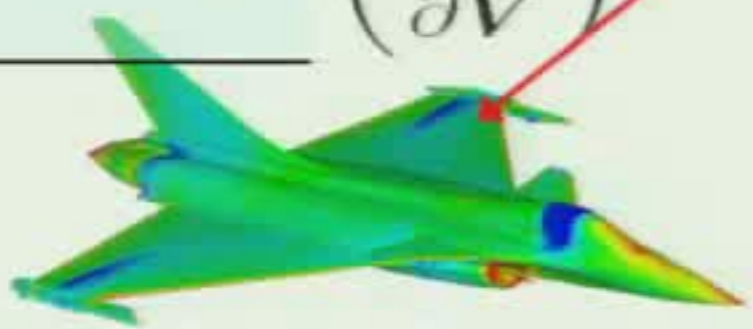
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$$dE = \frac{\partial E}{\partial \mathbf{V}} d\mathbf{V} + \frac{\partial E}{\partial \mathbf{x}} d\mathbf{x} = 0$$

Linear problem

$$\left( \frac{\partial E}{\partial \mathbf{V}} \right) d\mathbf{V} = - \left( \frac{\partial E}{\partial \mathbf{x}} \right) d\mathbf{x}$$



Output = dV (complex aerodynamic pressure force)

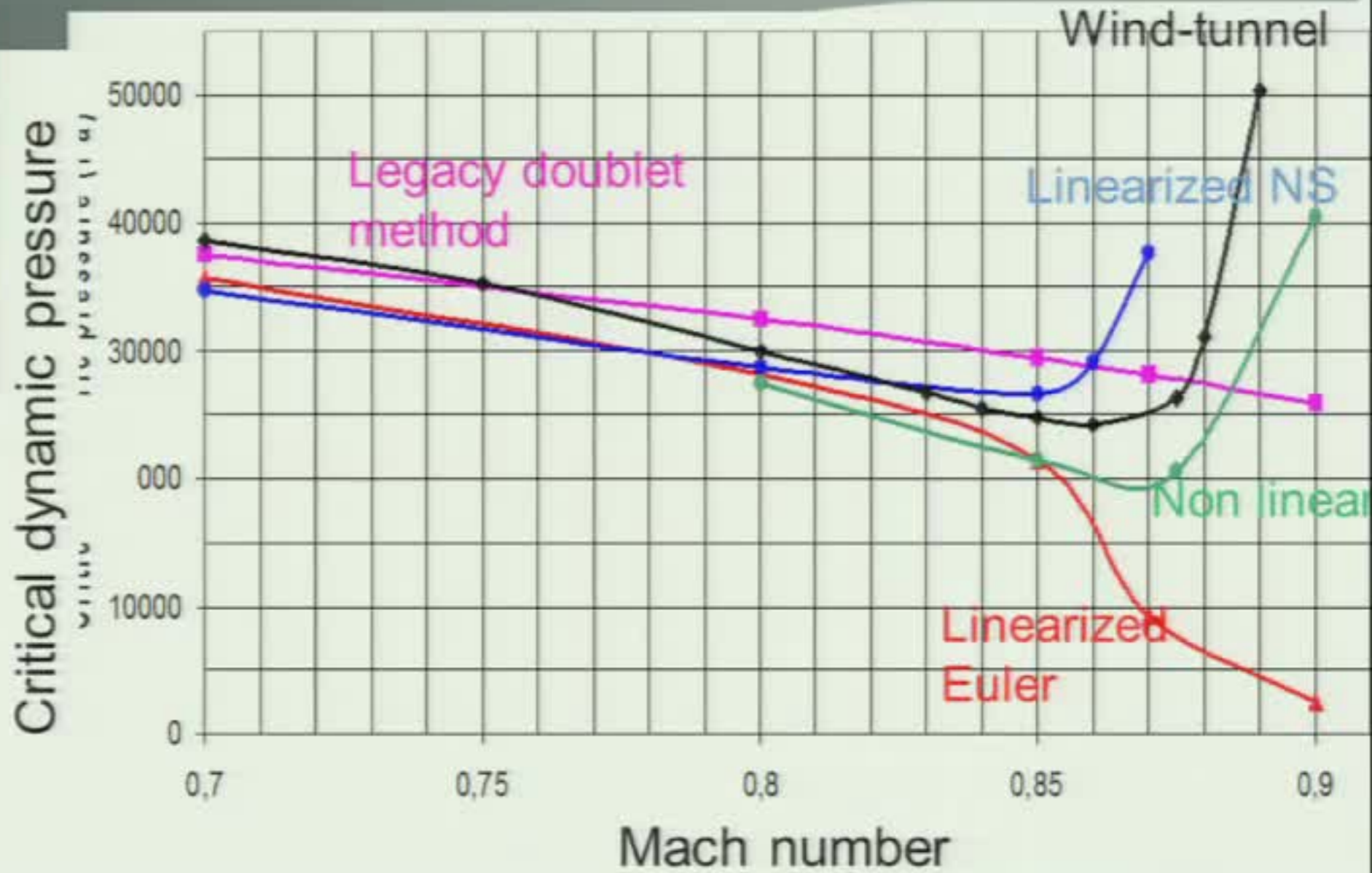
Input = dx (complex nodal displacement)

# Multiphysics: CFD for Aeroelasticity

## Linearized CFD - Validation



Transonic wing in ONERA S2MA wind tunnel



- Linearized CFD can predict “transonic dip”
- Linearized NS leads to improved results compared to linearized Euler

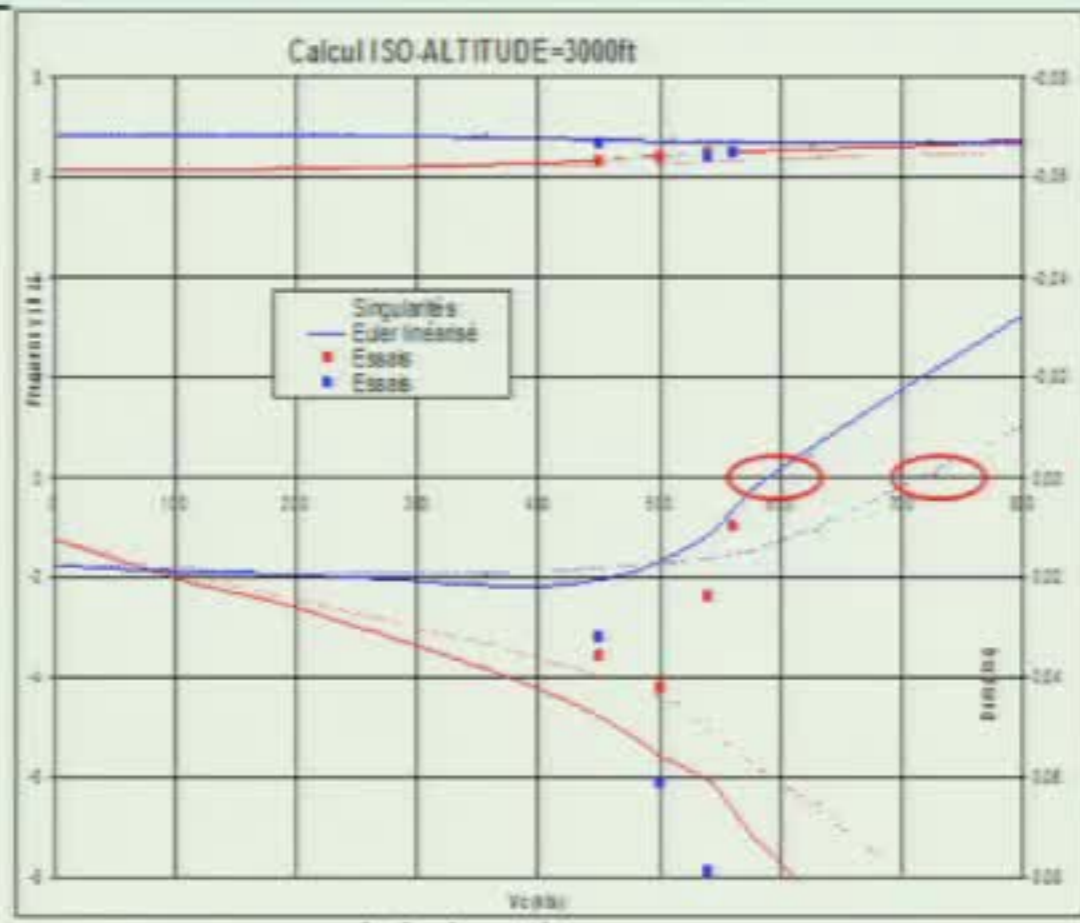
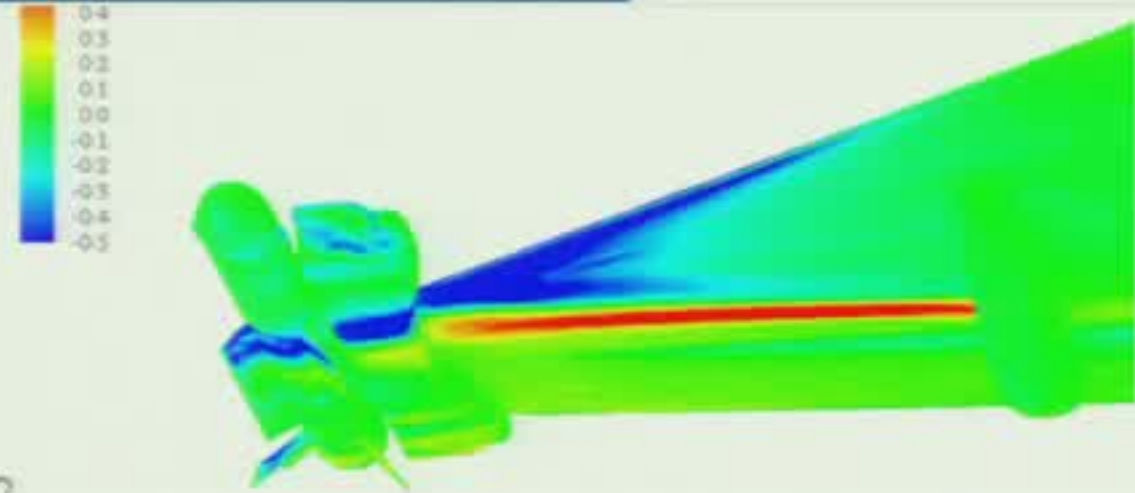


# Multiphysics: CFD for Aeroelasticity

## Application to military aircraft



- Linearized Euler approach applied to various weapon configurations for a combat aircraft (more than 10 000 computations)
- Example : influence of the missile correctly predicted (agreement with flight test)



damping < 0  
unstable  
-----  
damping > 0 stable

Velocity

# Multiphysics: CFD for Aeroelasticity

Linearized Euler and Navier-Stokes equations



## Challenge for the future :

- further increase efficiency and robustness of linear solvers

- very large scale linear problem :100-200 million unknowns
- very sparse ill-conditioned non symmetric matrix
- massively parallel computers and novel architectures

Research need:  
innovative iterative  
solvers in HPC  
environment





Formulations by multidomain equations: for each domain  $j$  write

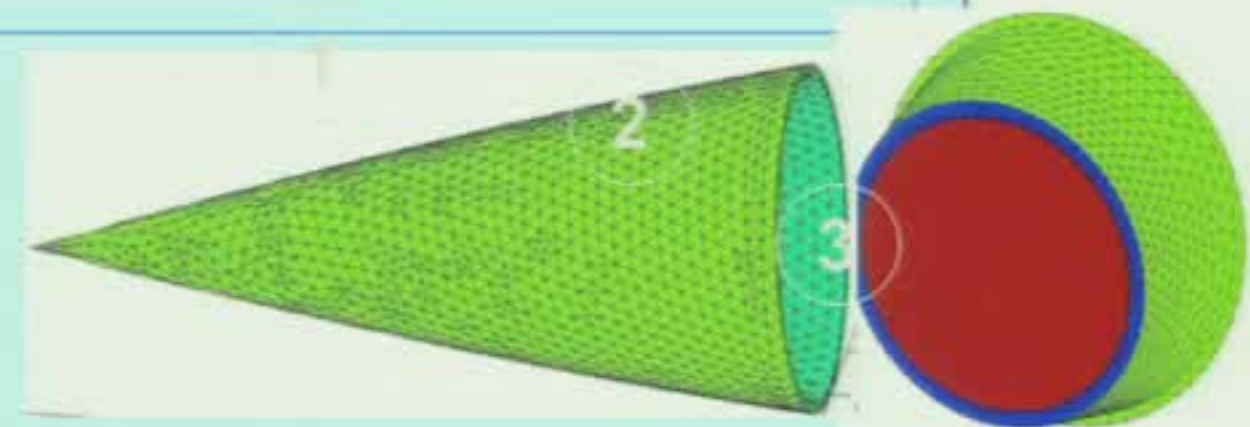
$$\frac{jZ_j}{4\pi} \int_{\Sigma_j} (k_j G_j - \frac{1}{k_j} \nabla \nabla' G_j) J' ds' - \frac{1}{2} (jM \times n) - \frac{j}{4\pi} \int_{\Sigma_j} (\nabla' G_j) \times jM' ds' = E_i$$

$$-\frac{1}{2} (J \times n) - \frac{j}{4\pi} \int_{\Sigma_j} (\nabla' G_j) \times J' ds' + \frac{j}{4\pi Z_j} \int_{\Sigma_j} (k_j G_j - \frac{1}{k_j} \nabla \nabla' G_j) jM' ds' = jH_i$$

Frequency domain

Variational formulation

Finite elements discretization



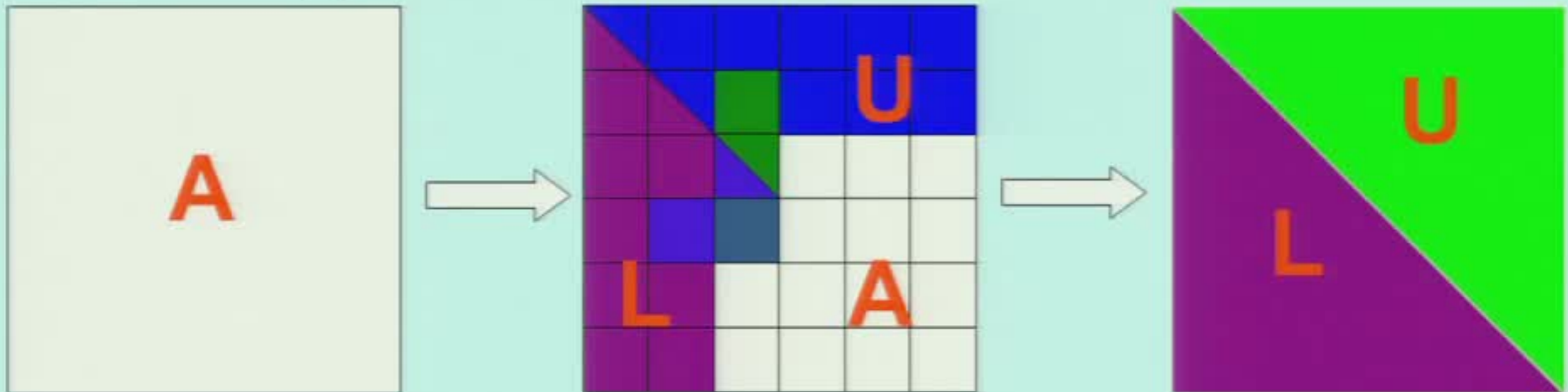
Homogeneous materials, thin materials (impedance conditions)

# Computational Electromagnetics

## Out-of-core linear solvers



Solution of linear systems ( $AX = B$ ) with  $A$  complex full matrix,  $p$  right hand-sides  
 $A$  stored on disk (up to over 7 To)



FMSlib :

- commercial product
- portable (Bull, IBM, PC...), efficient, reliable

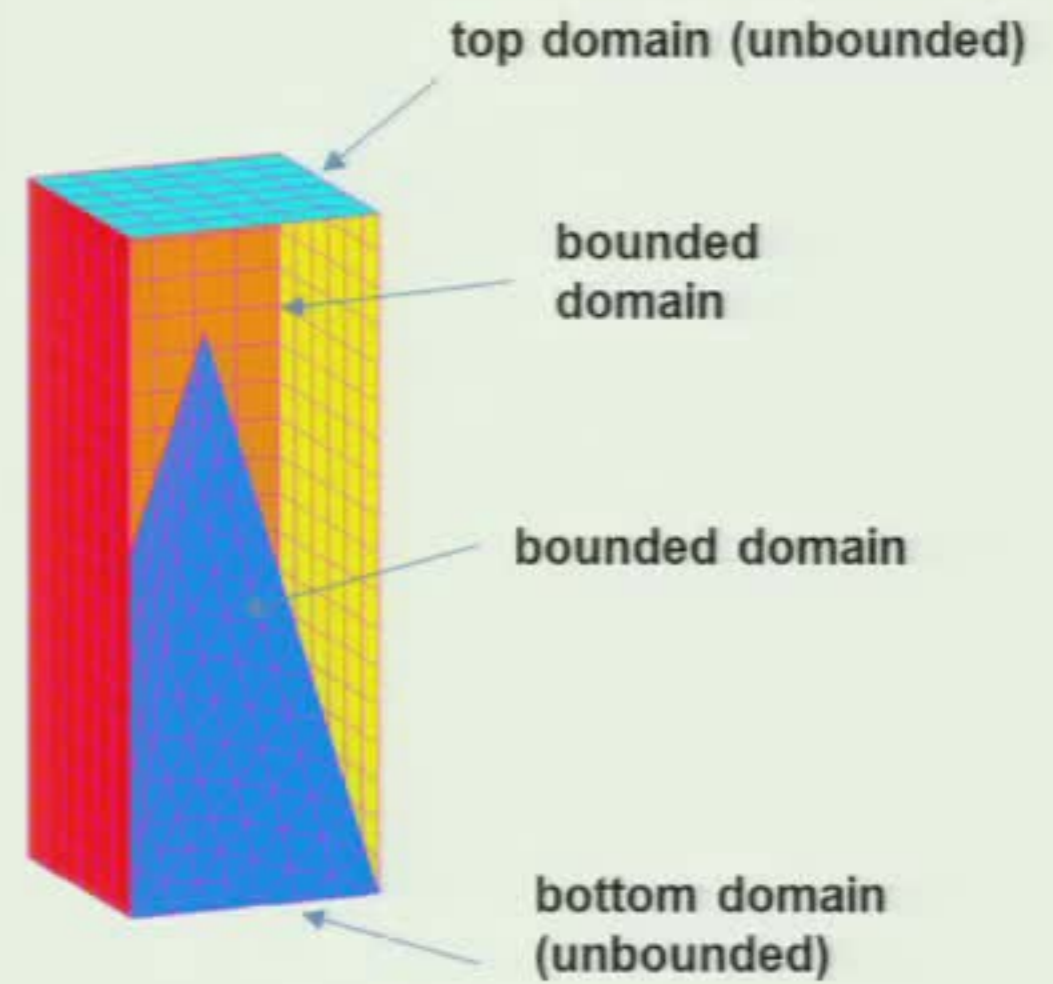
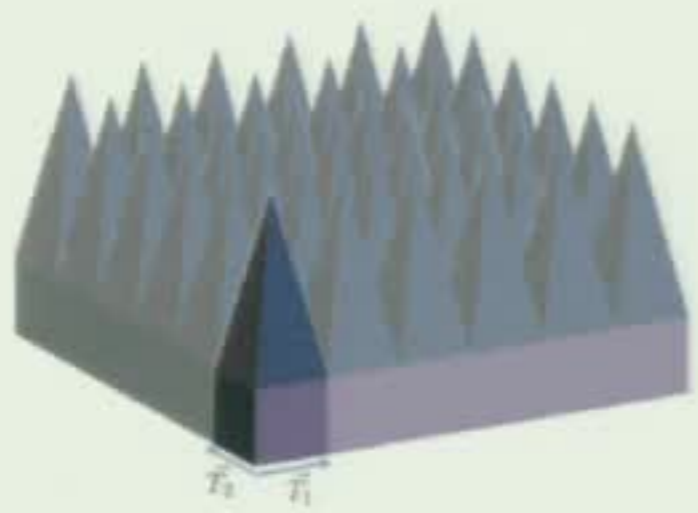


# Computational Electromagnetics

## Complex materials



- **Metamaterials : bi-periodic networks of complex cells**
- **Fictitious surface to use limited domains**
- **Pseudo-periodic conditions at the interfaces**
- **Pseudo-periodic Green function**



**Example of a multi-domain unit cell**

Cooperation with CNRS laboratories, CEA

# Computational Electromagnetics

## Multi-level Fast Multi-Pole Method (MLFMM)

to sustain the frequency increase



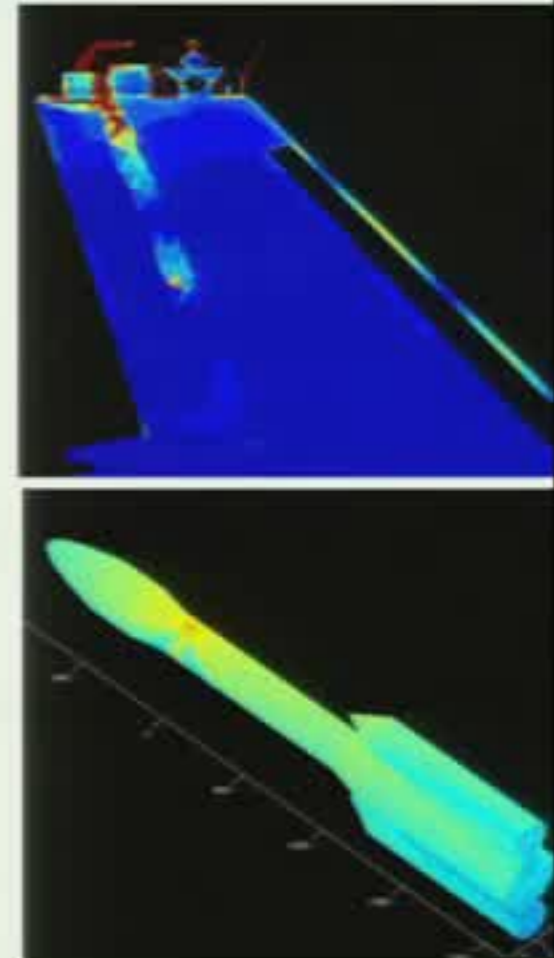
When the direct solution becomes too heavy ( ~ some days to . years) or simply impossible (disk size)

Limited robustness of iterative solver:

- sensitive to local mesh refinements: geometrical details (antennas, ...), materials with high index
- very much dependent upon the complexity of physical phenomena

Limitation in size of iterative solver:

- Computational time directly proportional to the number of incident fields to be evaluated



**the full characterization of a complete aircraft in X band would require years of computation**

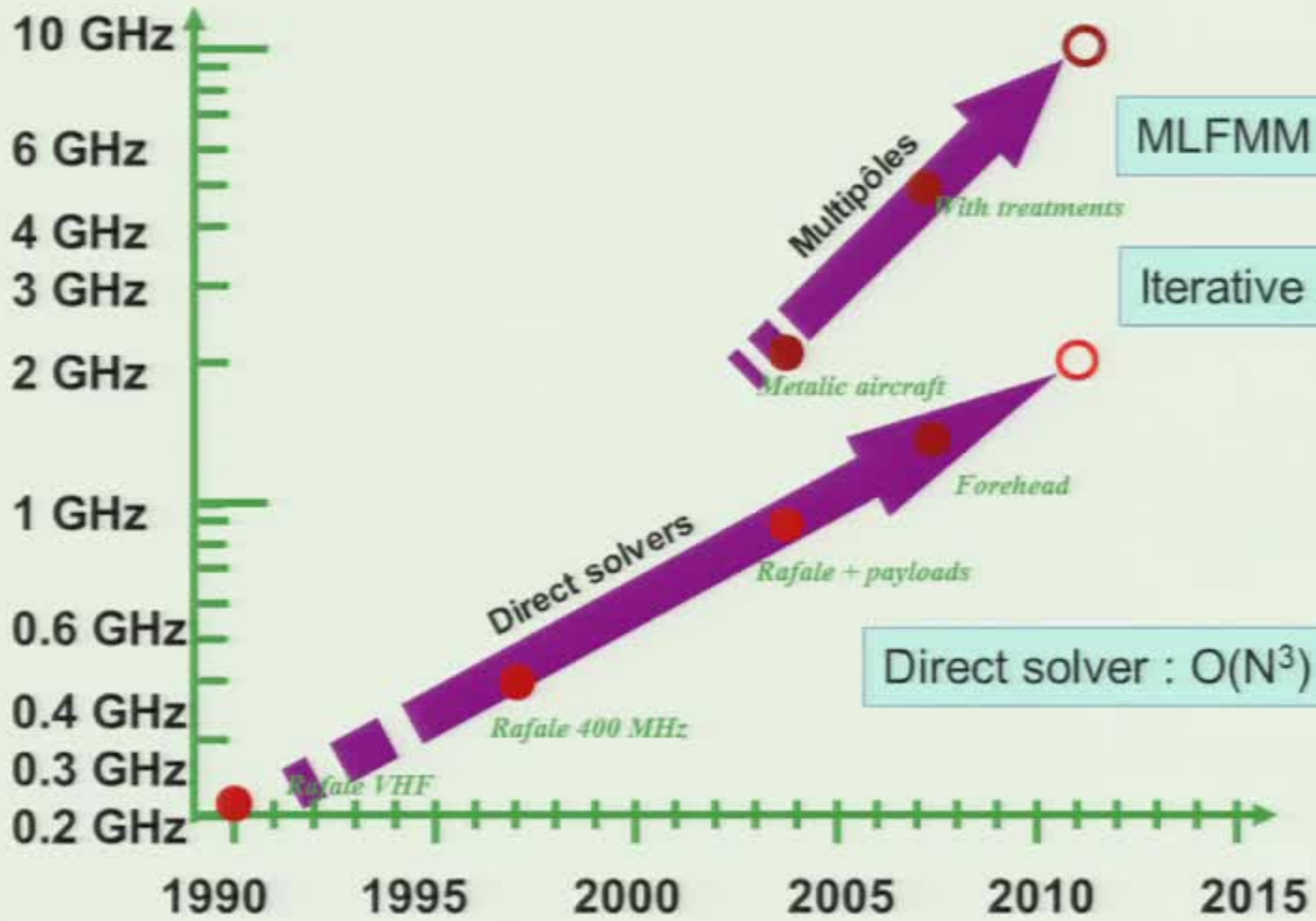
**> MLFMM based on mathematical developments of the Green function (truncature of the series) as an accelerator of an iterative method**



# Computational Electromagnetics Capacities



Computed frequency



MLFMM :  $O(N_{iter} N \log N)$

Iterative solver :  $O(N_{iter} N^2)$

Direct solver :  $O(N^3)$

# Computational Electromagnetics

Collaborative computations by the

multi-domain technique



## Decomposition of the computational domain

I : external domain

II : conduct

III : engine

Coupling interfaces

Reduction of problem sizes

Gain between  $k$  and  $k^2$   
( $k$  number of sub-domains)

Respect of the industrial sharing of responsibility



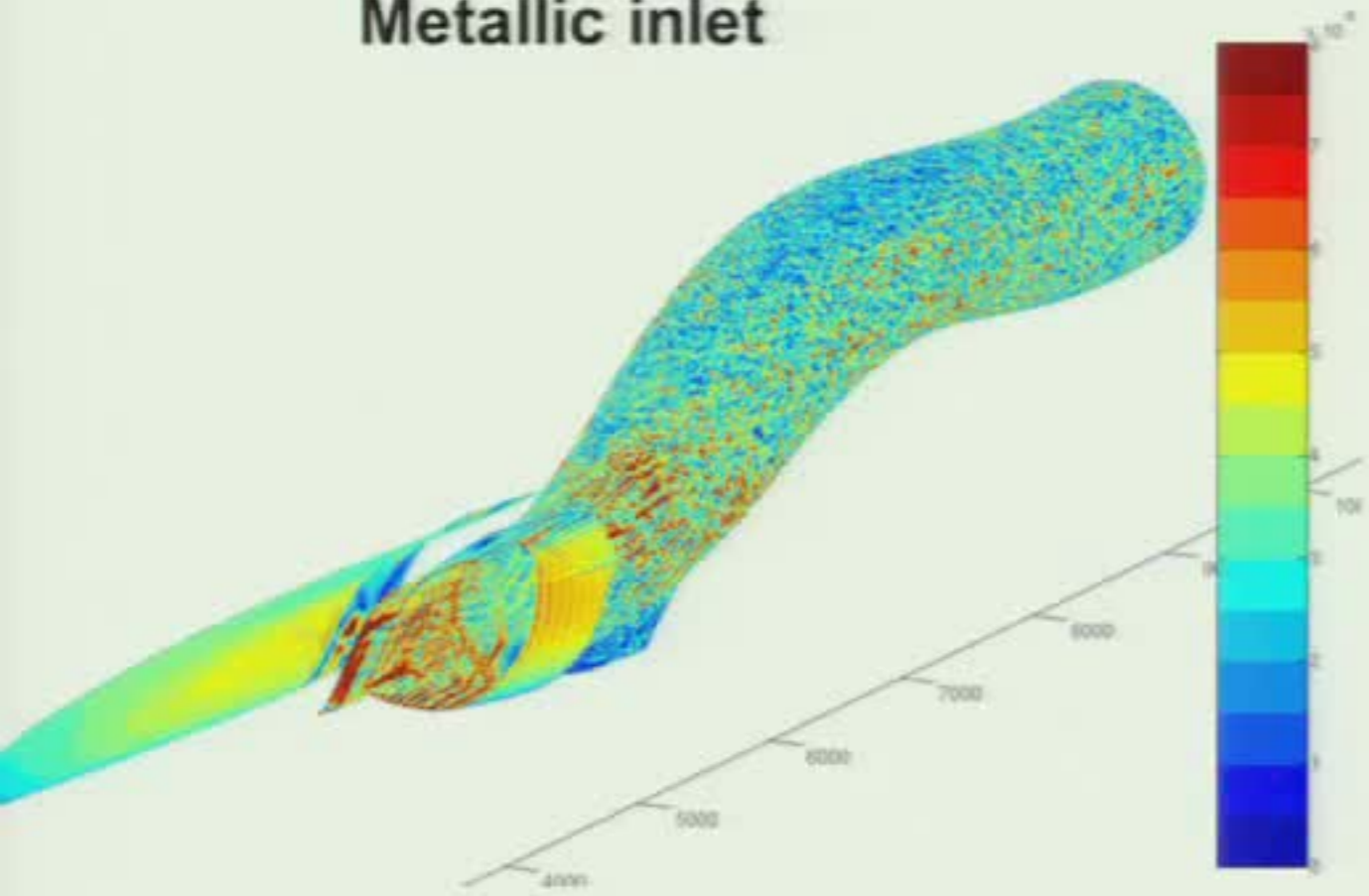
# Computational Electromagnetics

Collaborative computations by the multi-domain technique

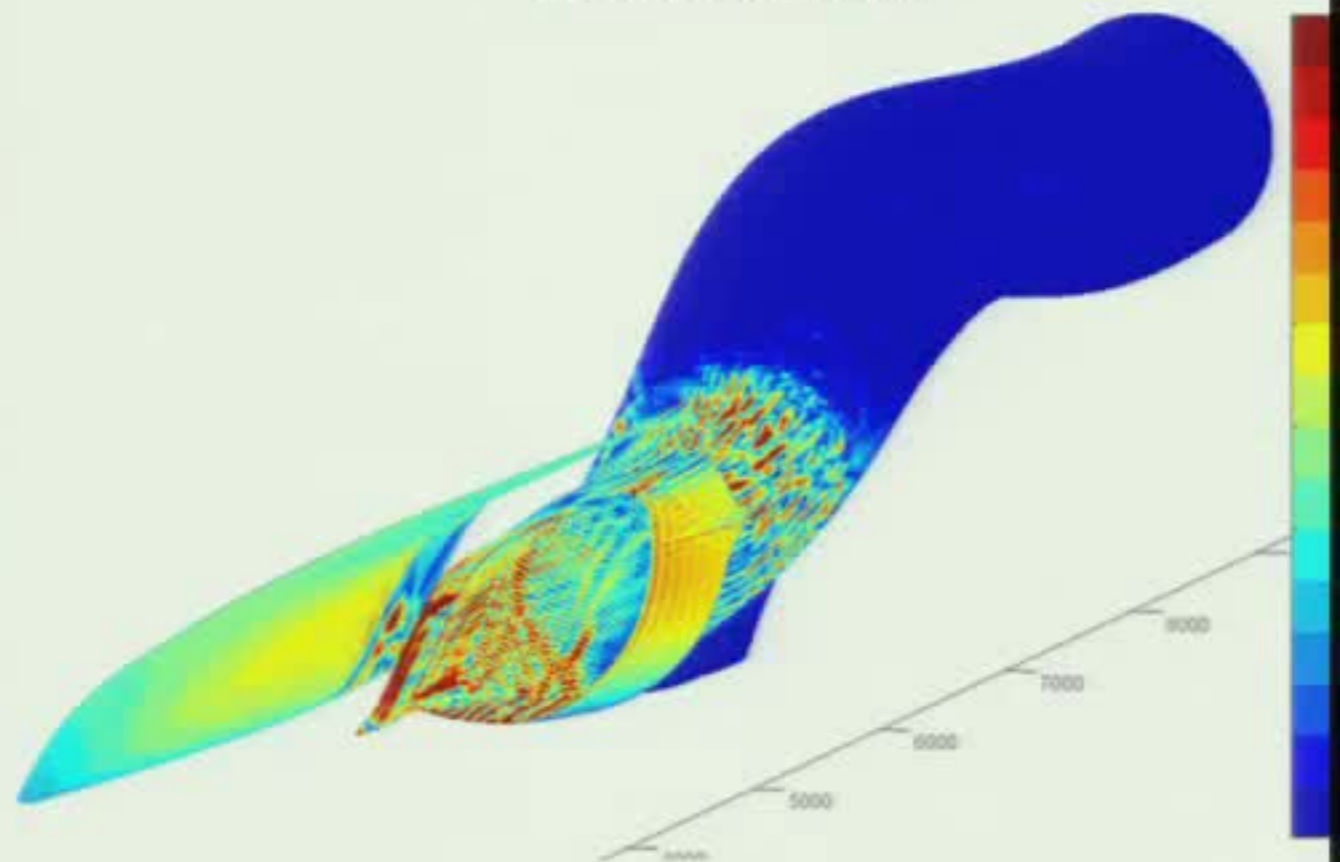


10 GHz

Metallic inlet



Treated inlet

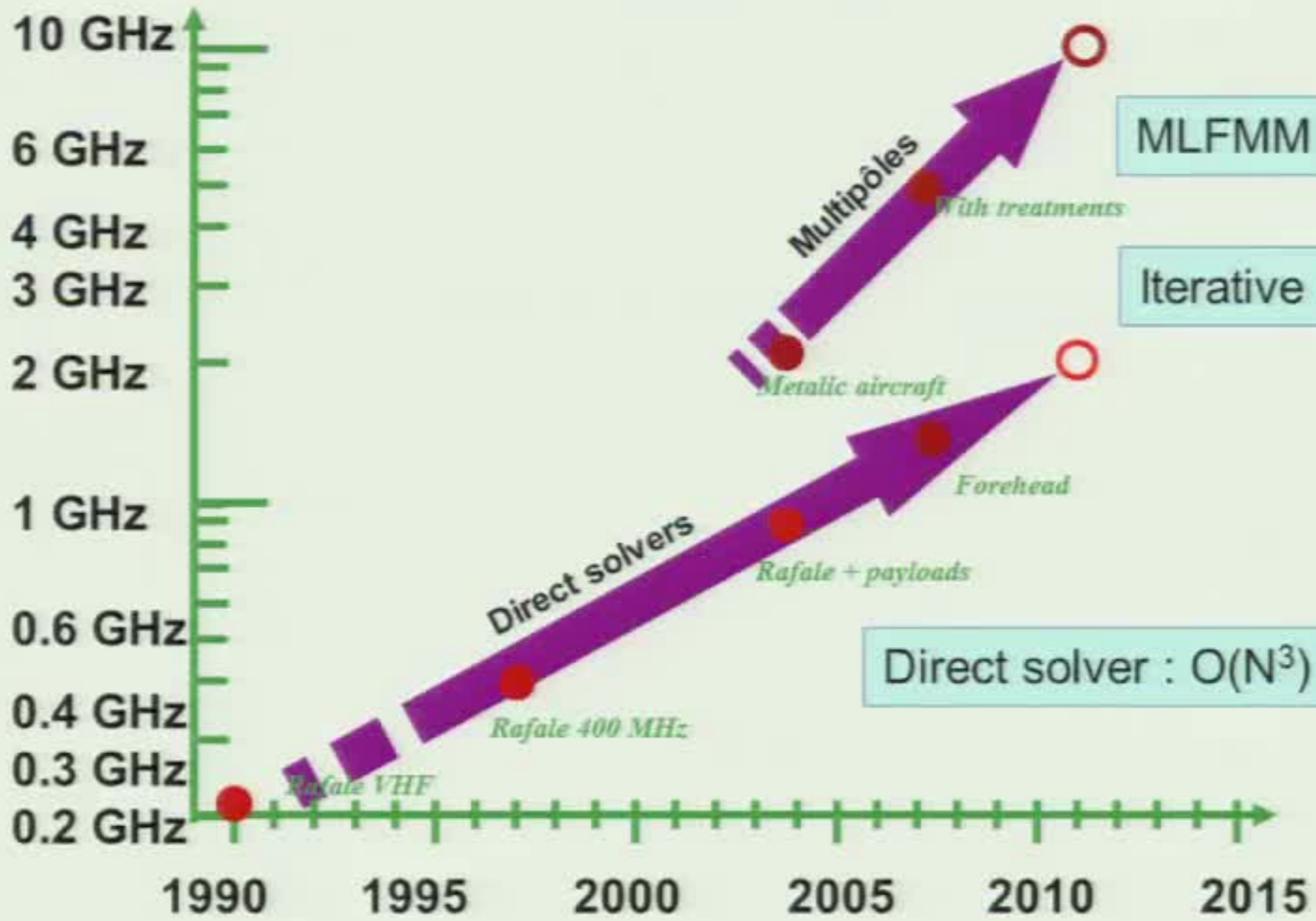


4 500 000 unknowns (FMM)

# Computational Electromagnetics Capacities



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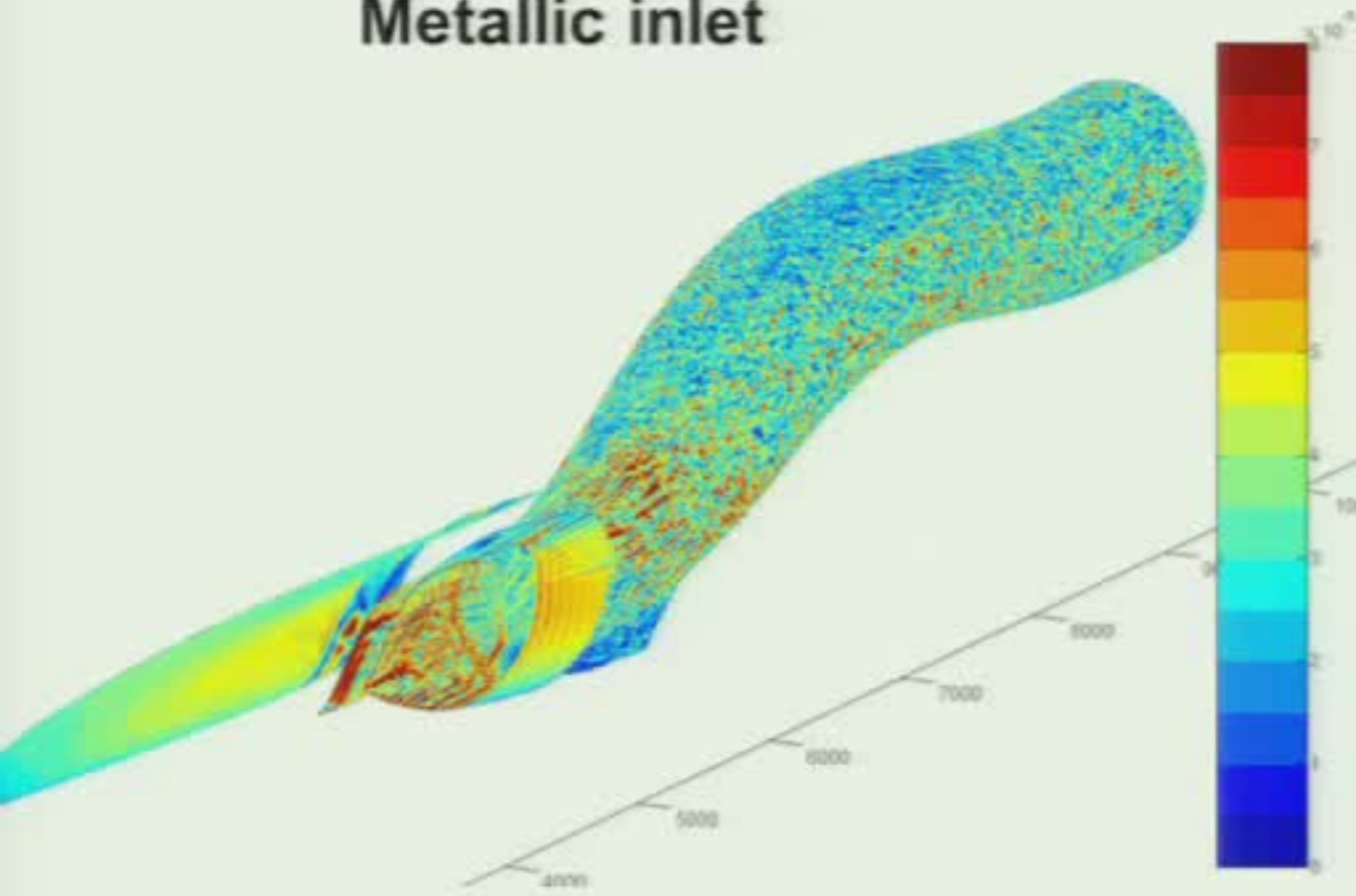
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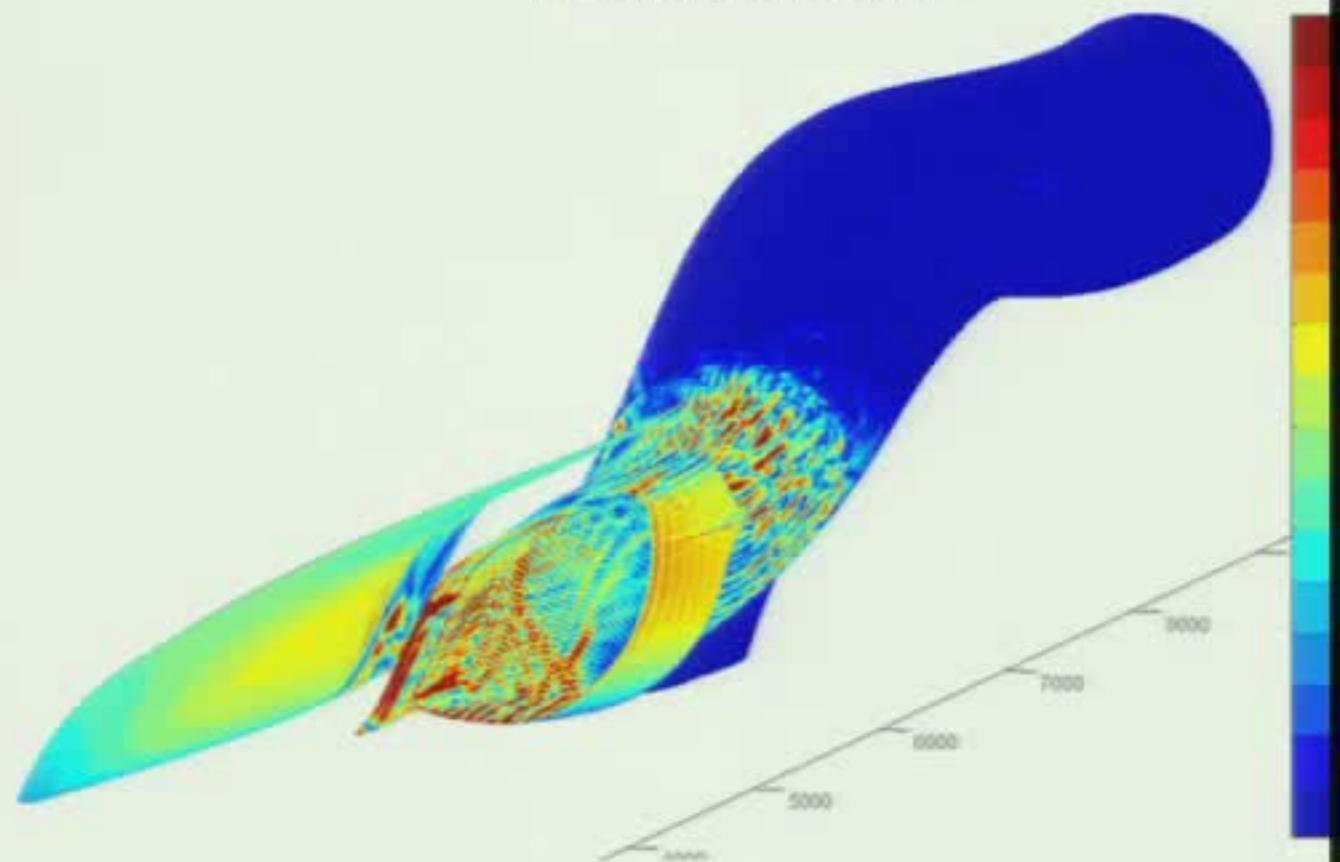


10 GHz

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Treated inlet



4 500 000 unknowns (FMM)



# Multidisciplinary Design Loop



## Global options

- Architectures
- Technologies



## Design per discipline and Optimization

- Aerodynamics
- Structure
- Acoustics
- Propulsive integration
- Vehicle systems

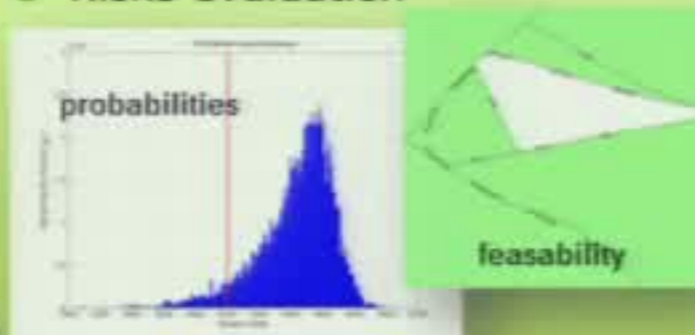


## Requirements (market, regulation)

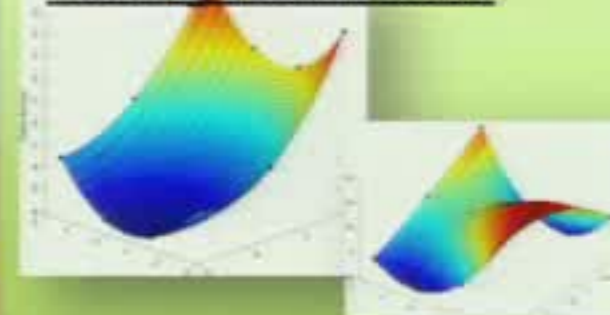
- Range
- Fields length
- Cruise speed
- Comfort
- Environmental objectives
- Costs

## Global synthesis

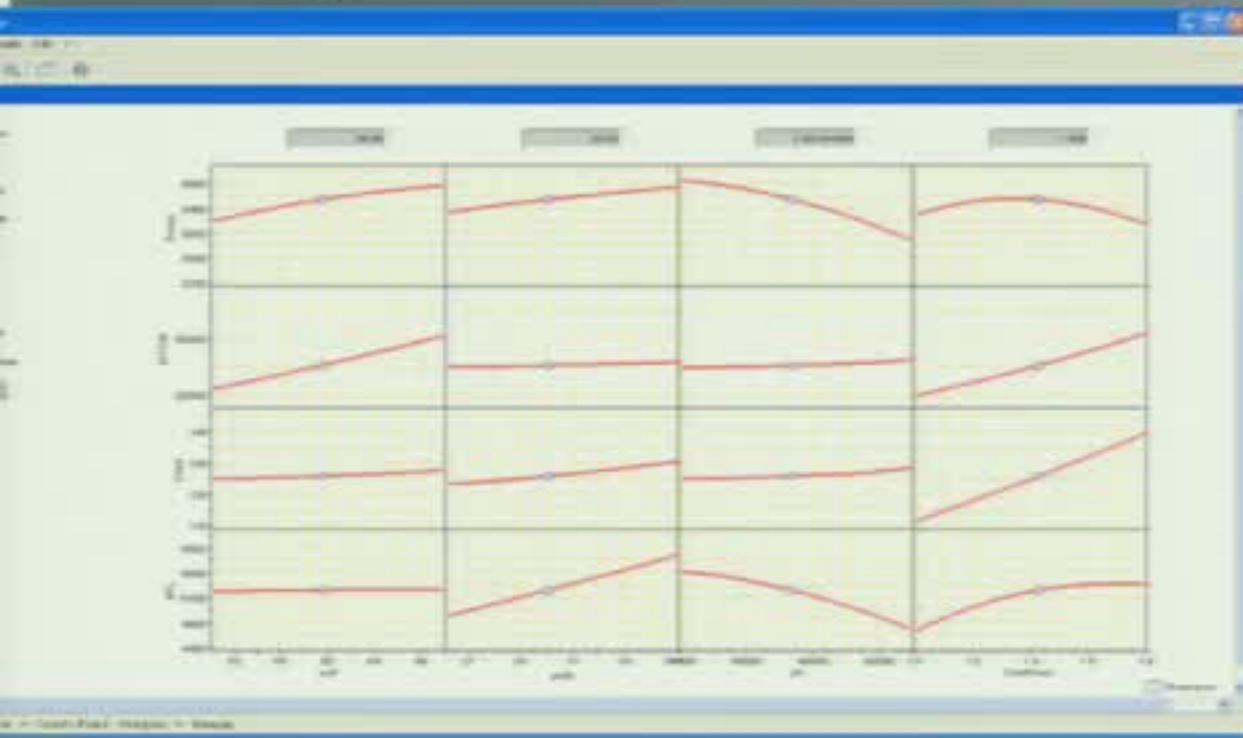
- Exploration of design space
- Global sensitivities
- Risks evaluation



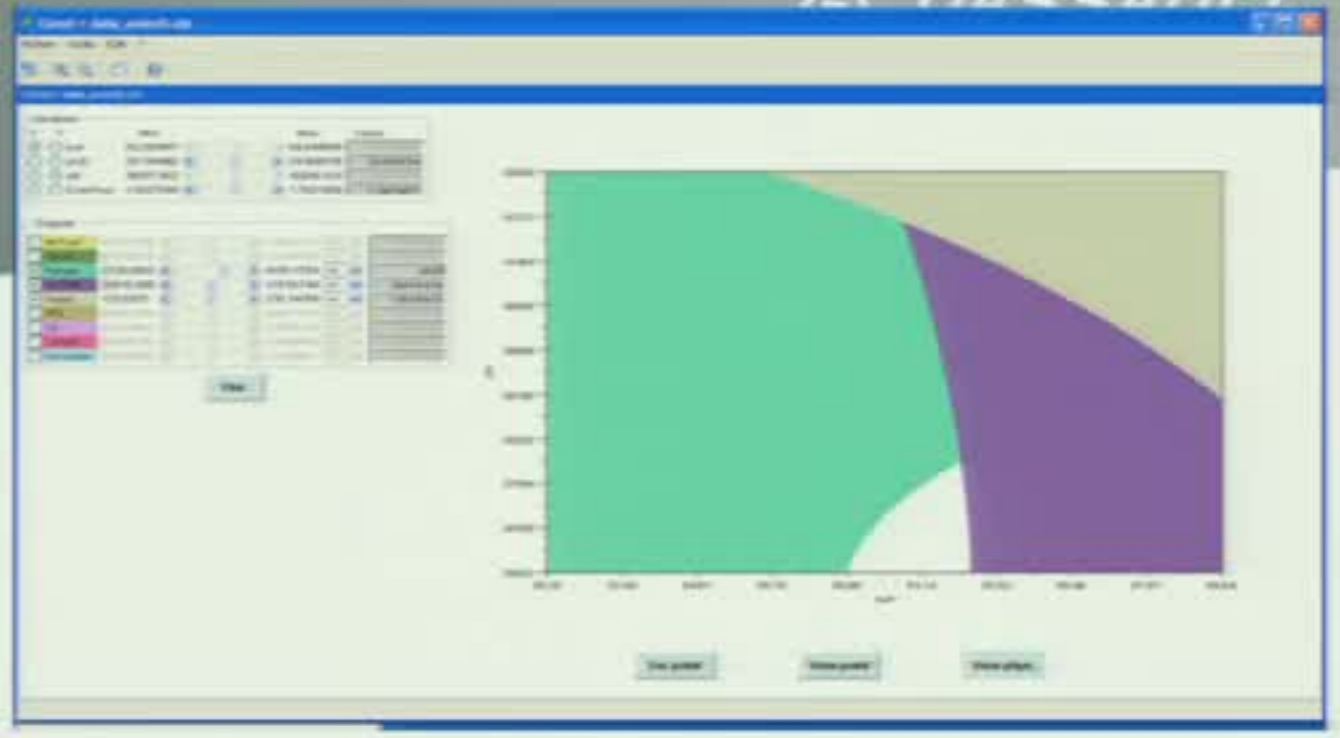
## Parametric models with surrogate models



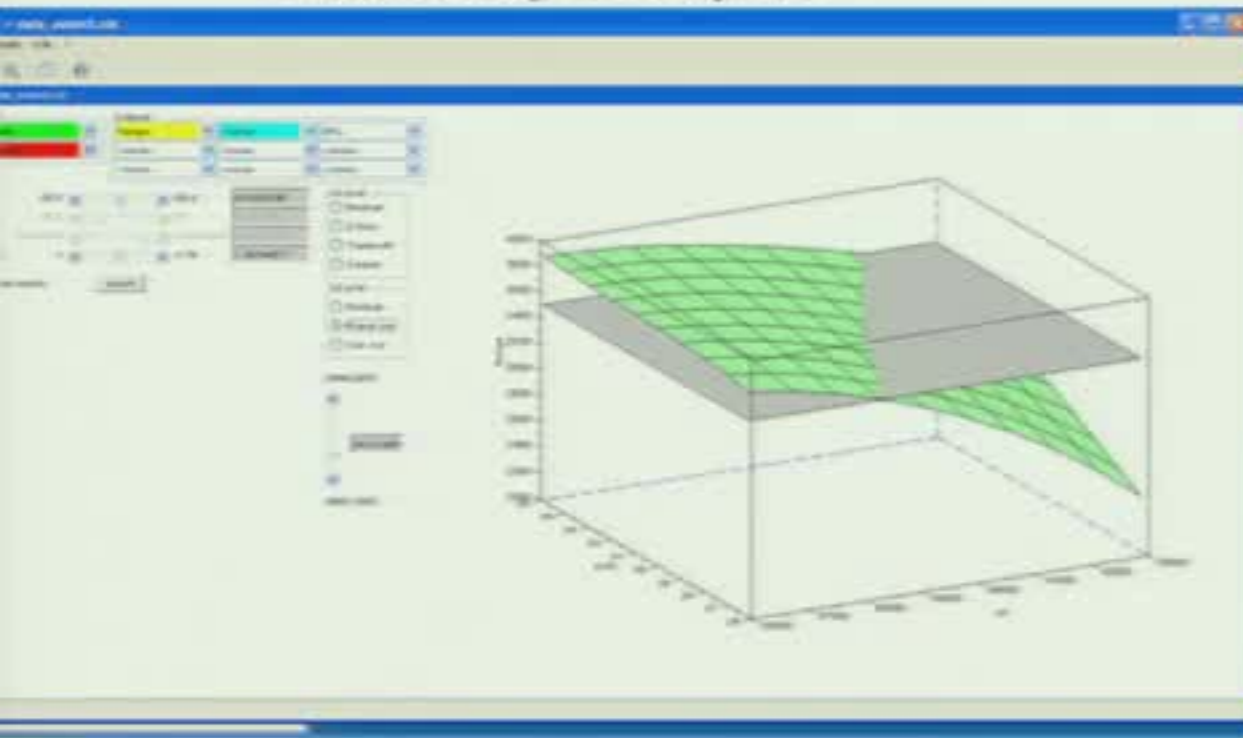
# Surrogate models for interactive exploration



Sensitivity Analysis



Feasible domain evaluation



Data Exploration



Generation of new configurations and filtering

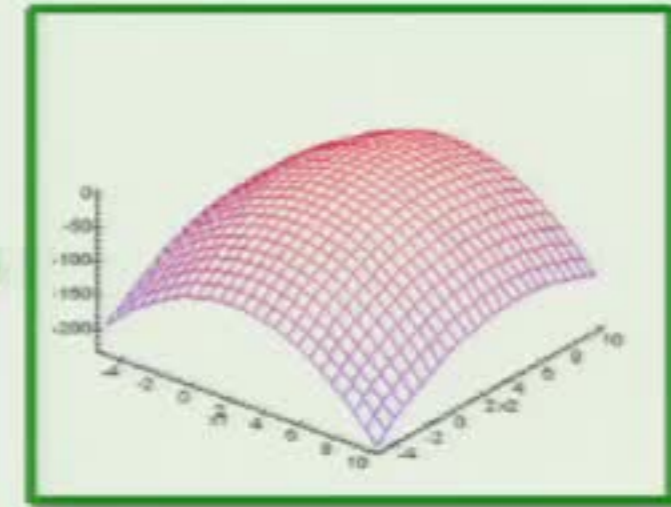
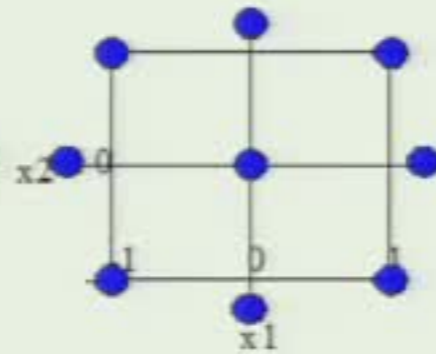
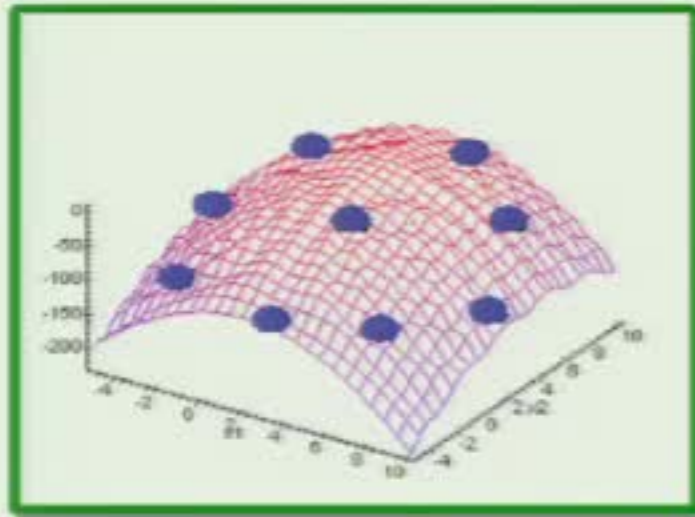


# Surrogate models: ingredients

System to model

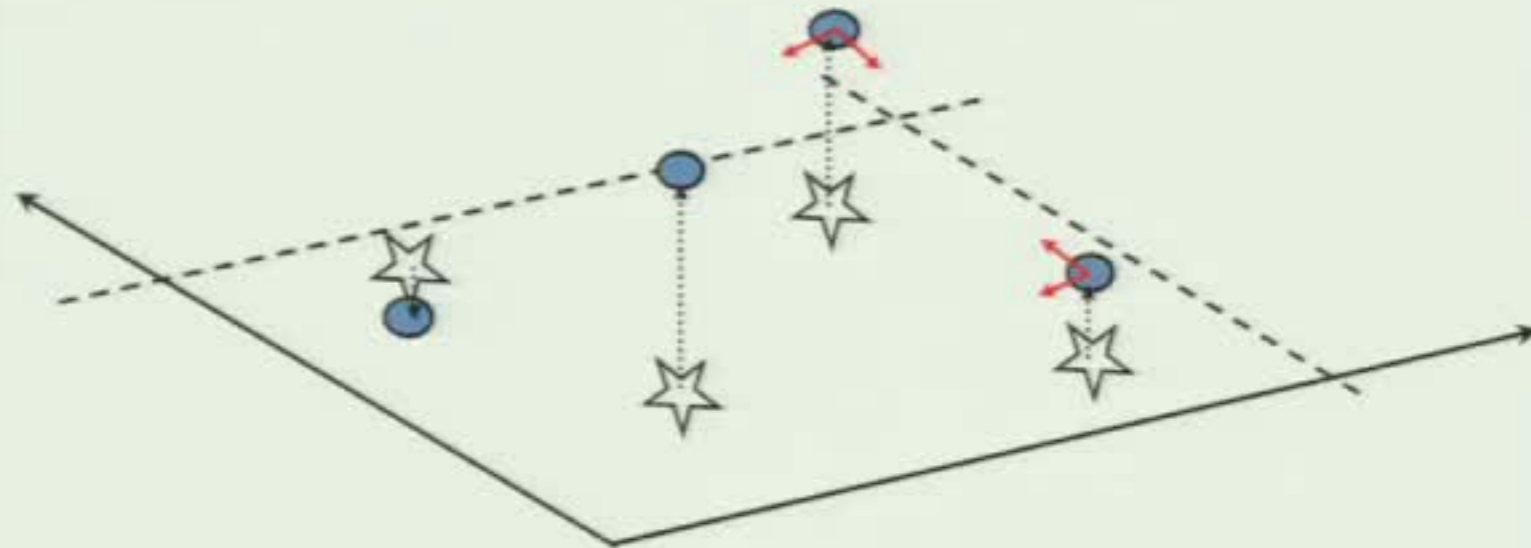
D.O.E

Surrogate



## D.O.E

- Latin Hypercube Sampling
- max(min)
- pseudo MC
- Adapted



**Robust design** : Find a shape which is as less as possible insensib to small variations of uncertain parameters

**Reliability-based design** : Find a shape associated to a probability not realize a target less than a given acceptable value i.e.  $P(X > \text{given value})$



- Objective : “manage uncertainties” instead of adding “margins”
- Need to propagate uncertainties :

Probability density function of uncertain input data / (geometry, inflow ..)

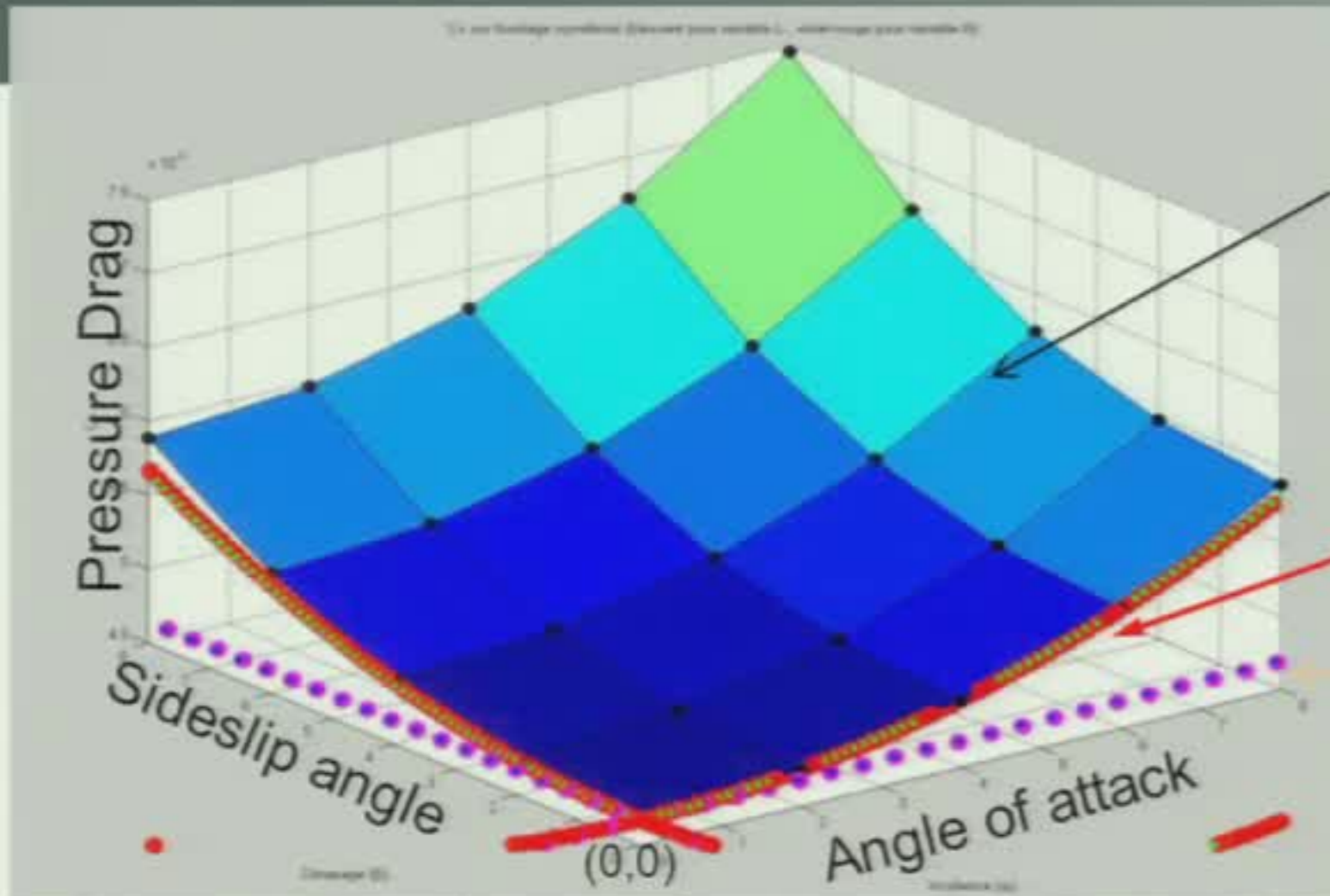
Method to propagate uncertainty:  
- Monte Carlo  
- Polynomial Chaos (~3 input)  
- Perturbation method

Probability density function of output of drag, lift, ...

- Second-order derivatives ( $\delta^2 O / \delta^2 I$ ) are needed both for the Monte Carlo method (response surface) and the Perturbation Method
- Computation of second-order derivatives is feasible using CFD solvers
  - new formulations
  - automatic differentiation tools (ex Tapenade)

# Uncertainty quantification in aerodynamics

## Computation of 2nd order derivatives

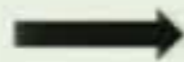


Response surface obtained using 25 non linear aerodynamics computations

2<sup>nd</sup> order derivatives computed at (0,0)

gradient

Pressure Drag( angle of attack, side slip angle)  
generic fuselage

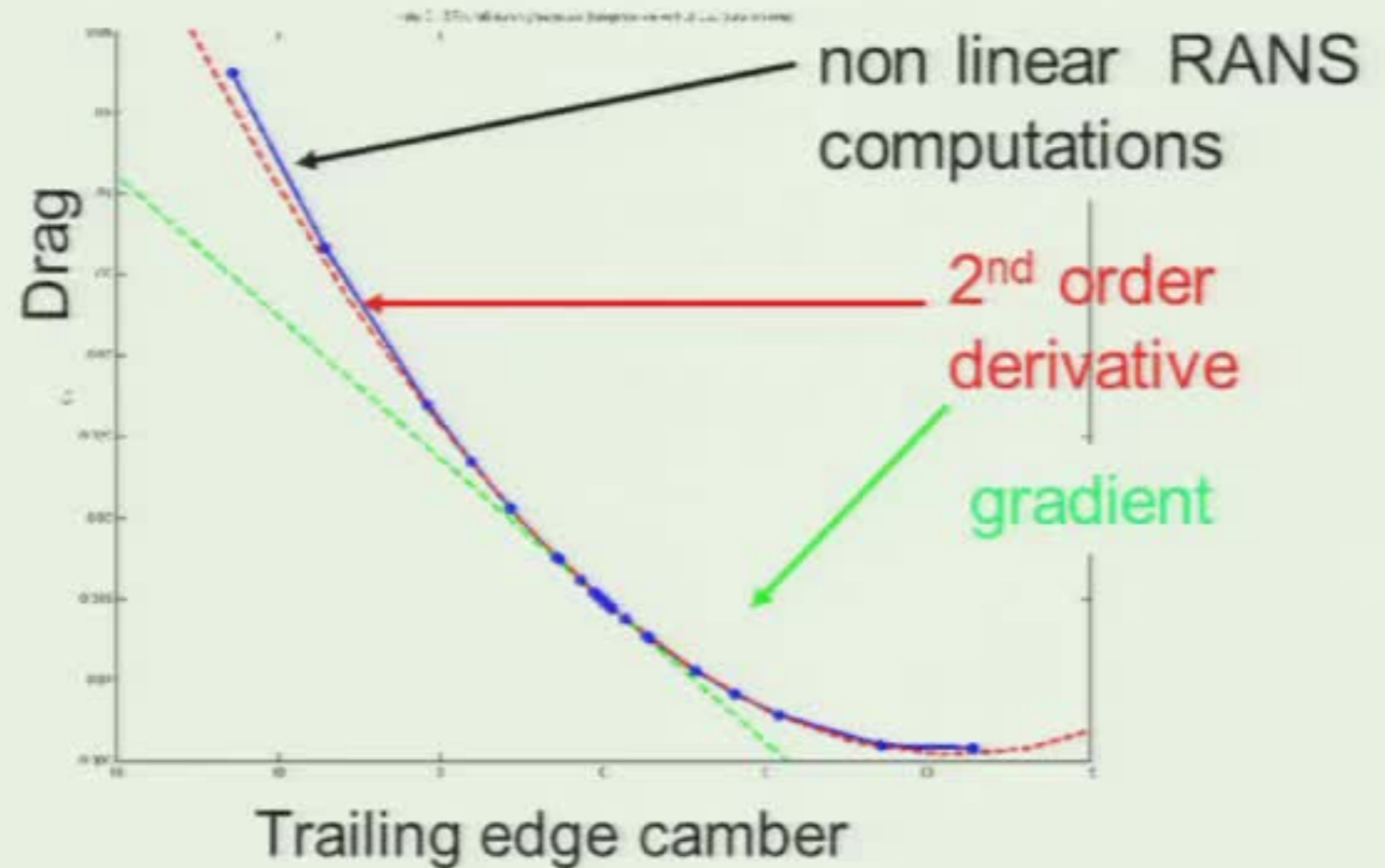
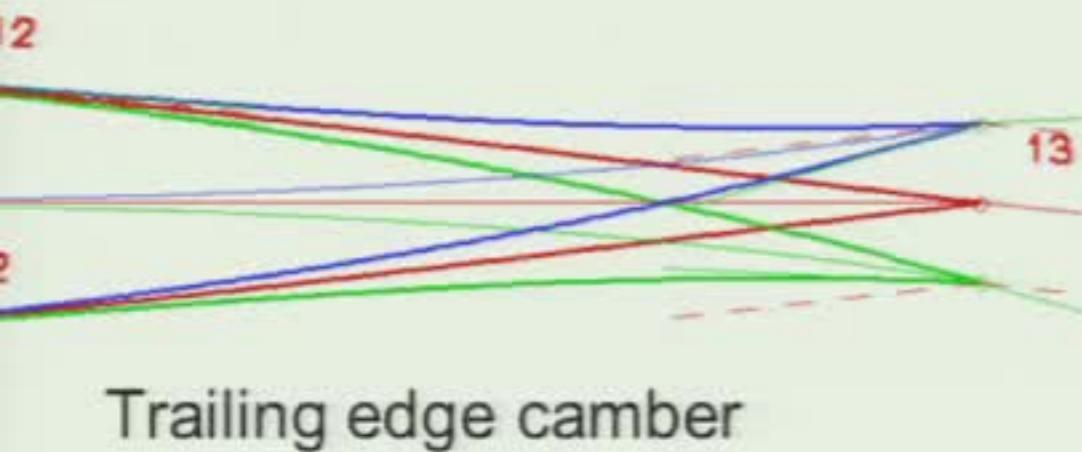


Computed second-order derivatives give a good approximation of the response surface



# Uncertainty quantification in aerodynamics

## Computation of 2nd order derivatives



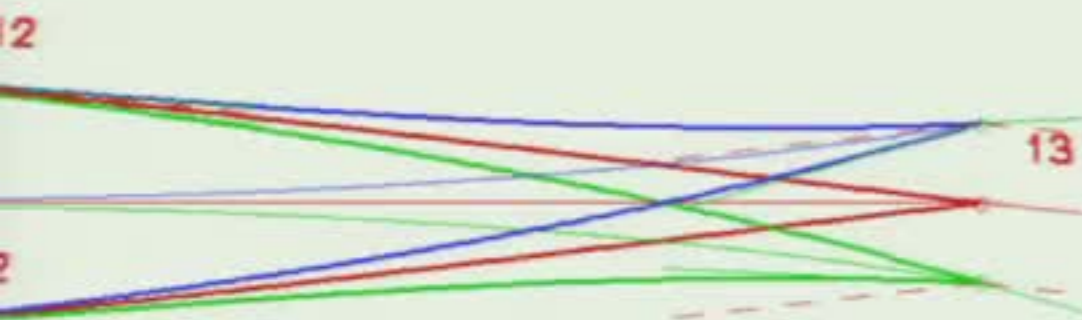
Computed second-order derivatives give a good approximation of the response surface

- **Key ingredients for numerical assessment of robust design**
  - Ability to compute second-order derivatives with CFD tools
  - Perturbation method : ( second-order derivatives )  $\rightarrow$  ( moments of the probability distribution function (pdf) )
  - Pearson's method : ( moments of the pdf )  $\rightarrow$  ( pdf )
- **Uncertainties can be combined in MDO framework**
  - Assess uncertainty in global performance given uncertainty in aero, structure, engine ...

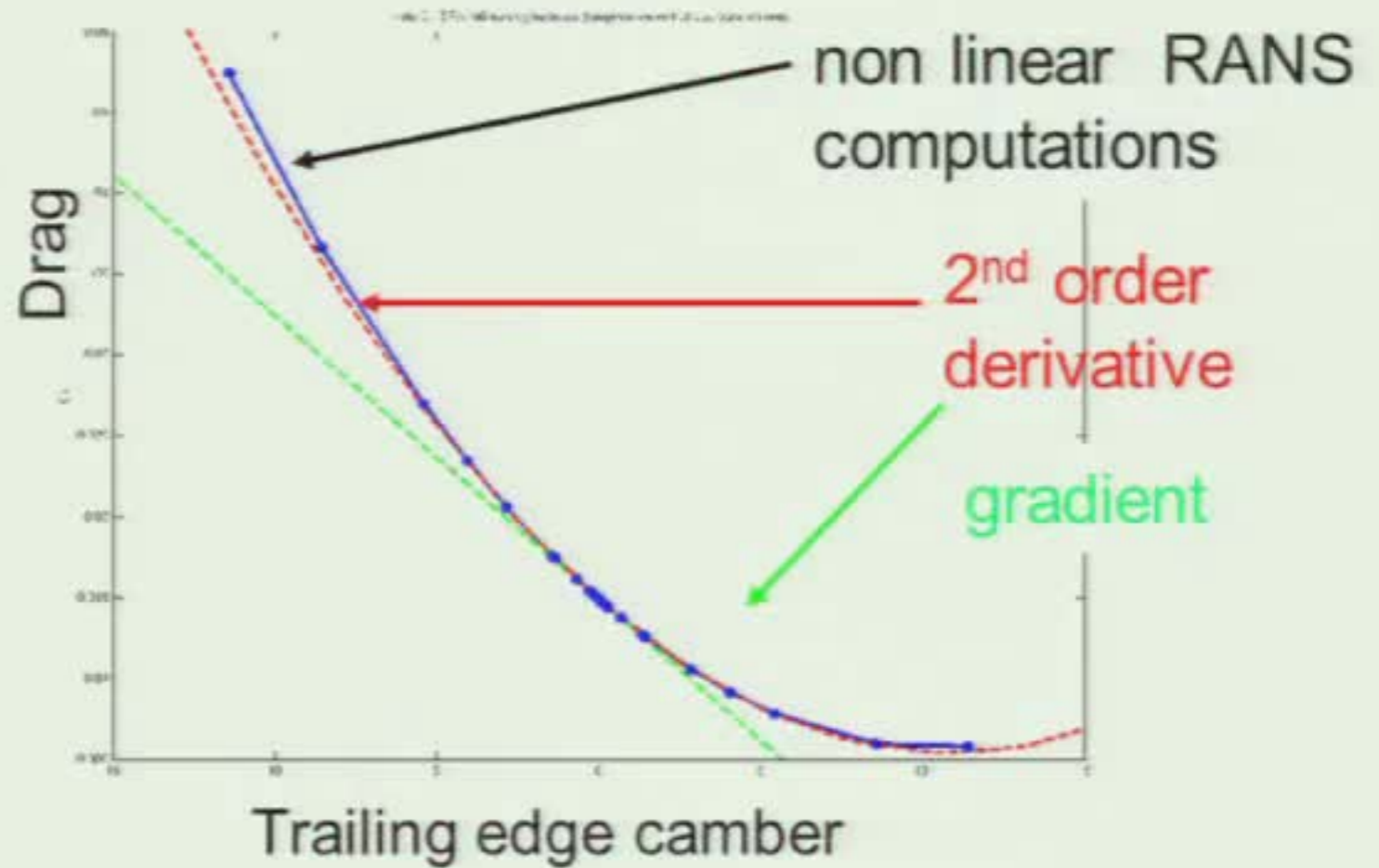


# Uncertainty quantification in aerodynamics

## Computation of 2nd order derivatives



Trailing edge camber



Computed second-order derivatives give a good approximation of the response surface

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# Reliability-Based Design

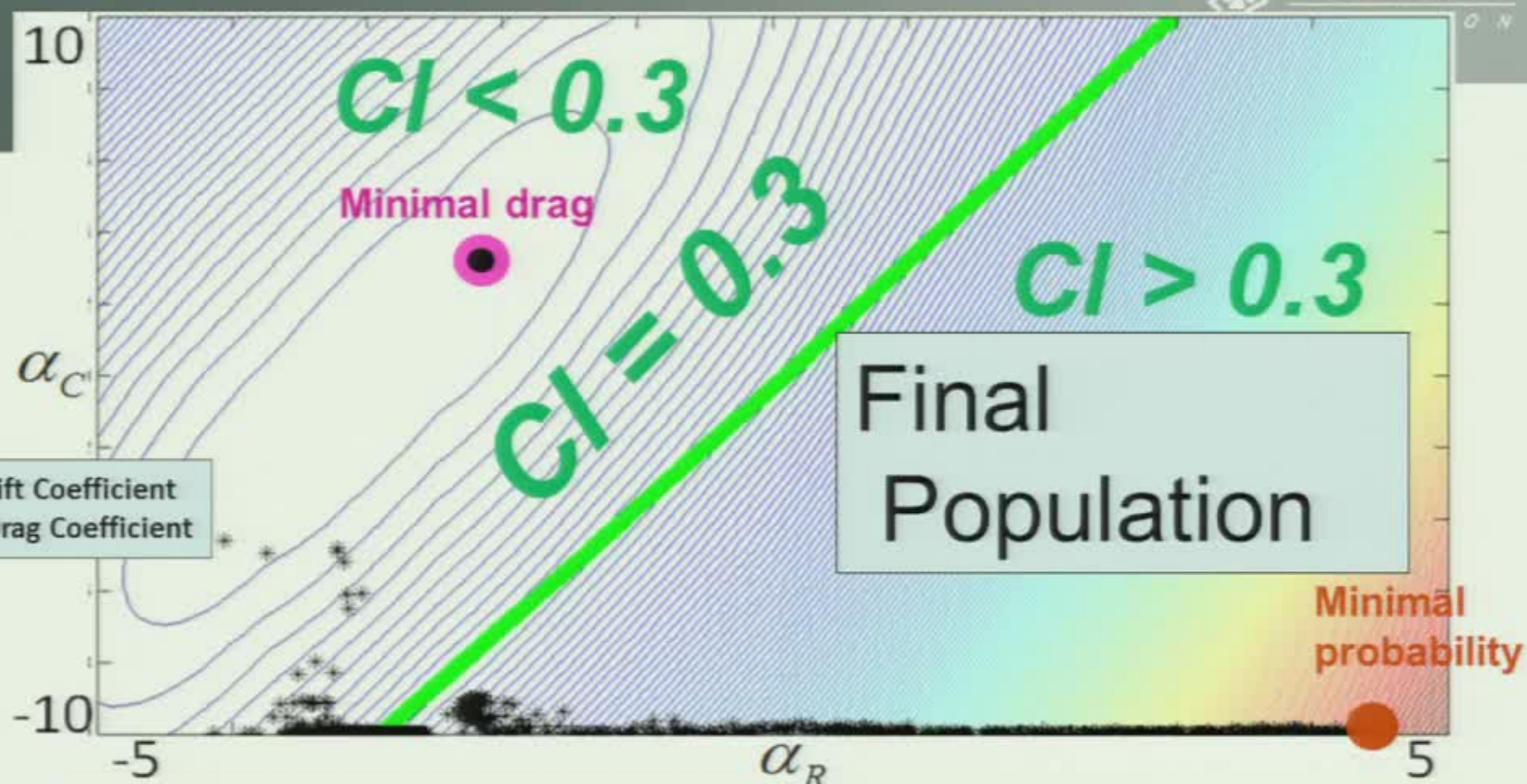
## An example



- Test case: ONERA M6 wing
- CFD solver: 3D Euler solver
- Construction of the surrogate model by Radial Basis Functions by using first and second-order derivatives
- Two design parameters: Twist and Trailing Edge camber angles
- Objectives: Minimize Drag (mean Drag Coefficient  $E(CD)$ ) and probability  $P(CI < 0.3)$
- Optimization is performed by Genetic Algorithm (MOGA)



# Reliability-Based Design: Exploration space



Objectives: Minimize Drag (mean Drag Coefficient  $E(CD)$ ) and probability  $P(Cl < 0.3)$



# Reliability-Based Design

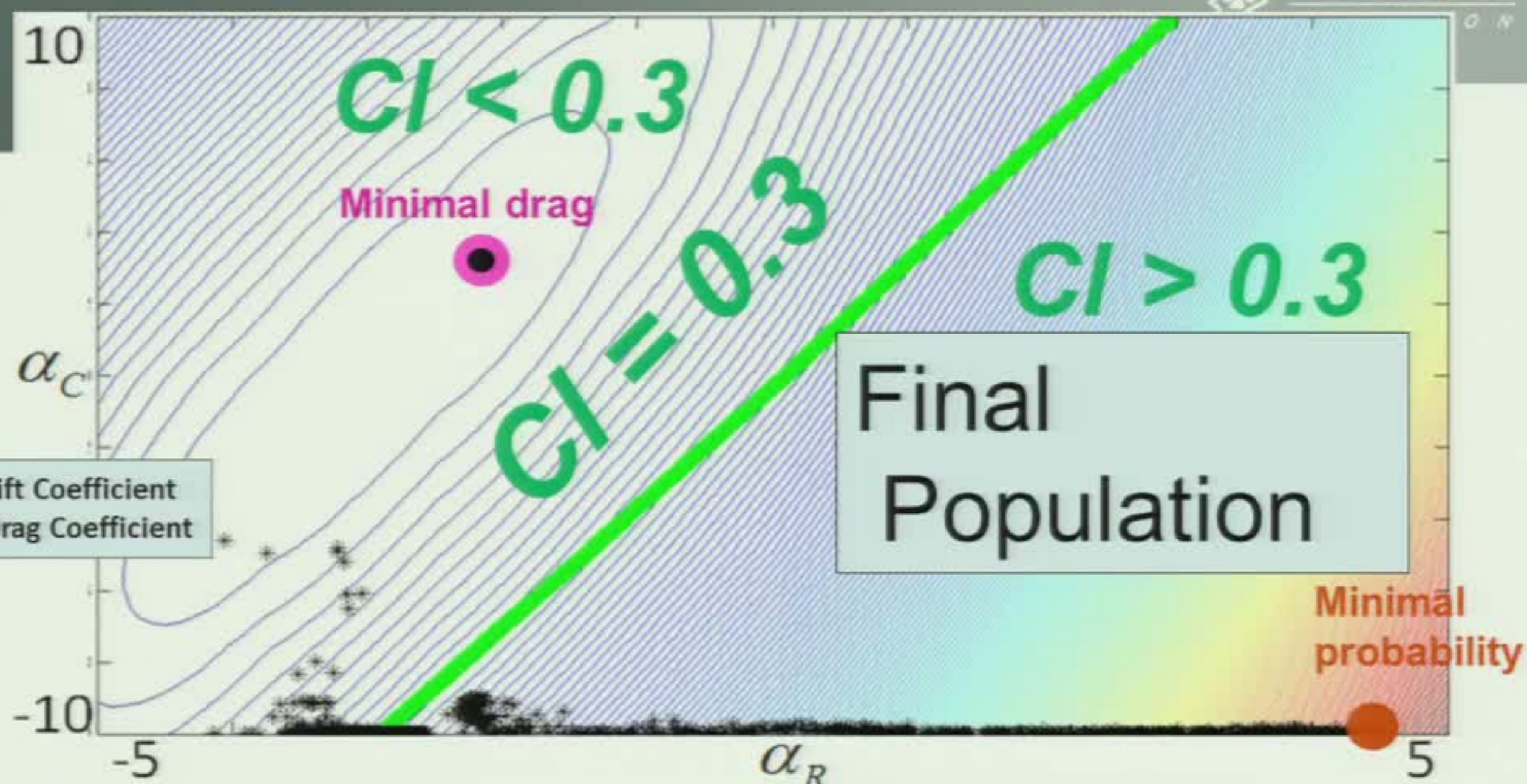
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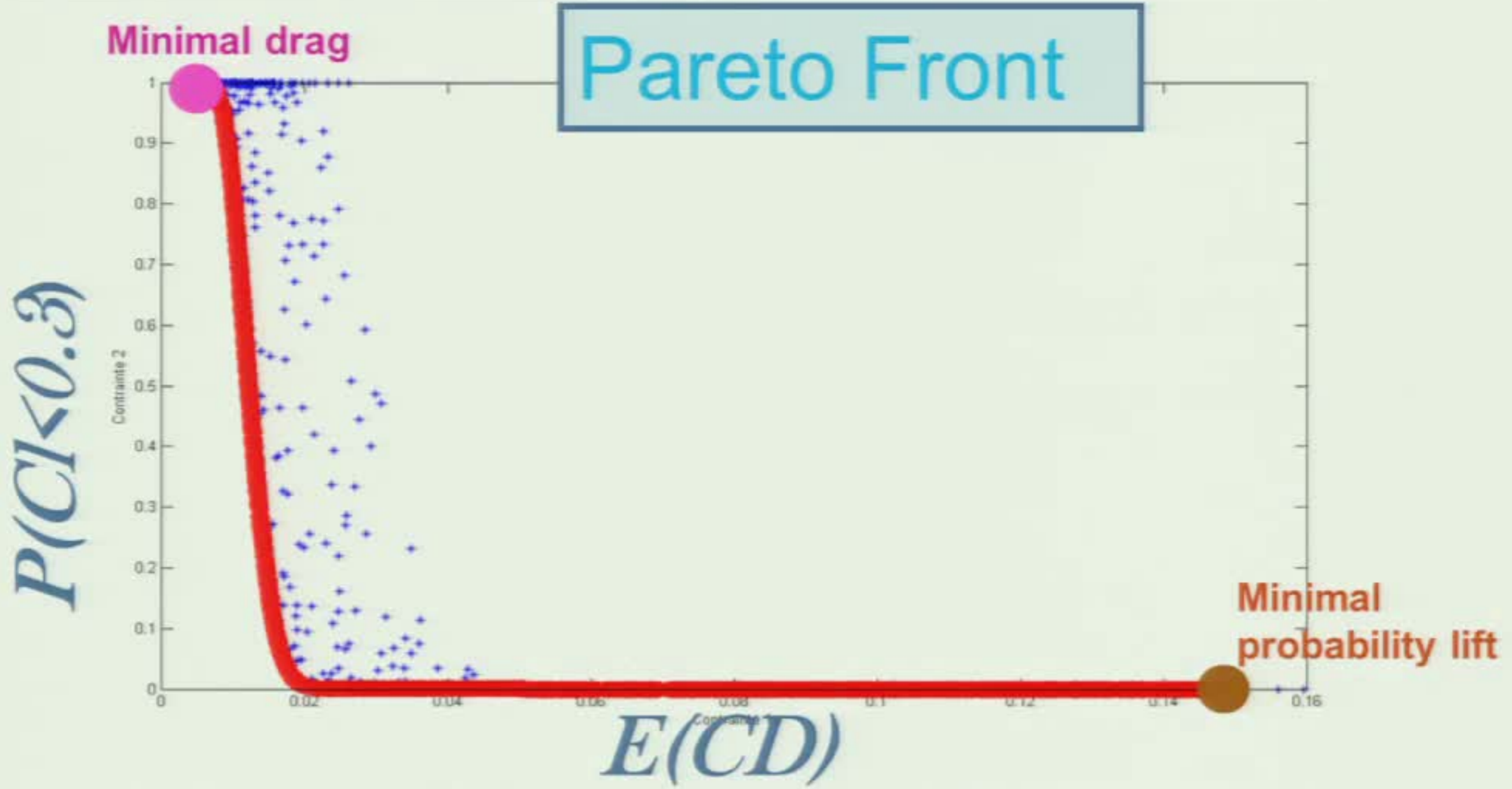
# Reliability-Based Design: Exploration space



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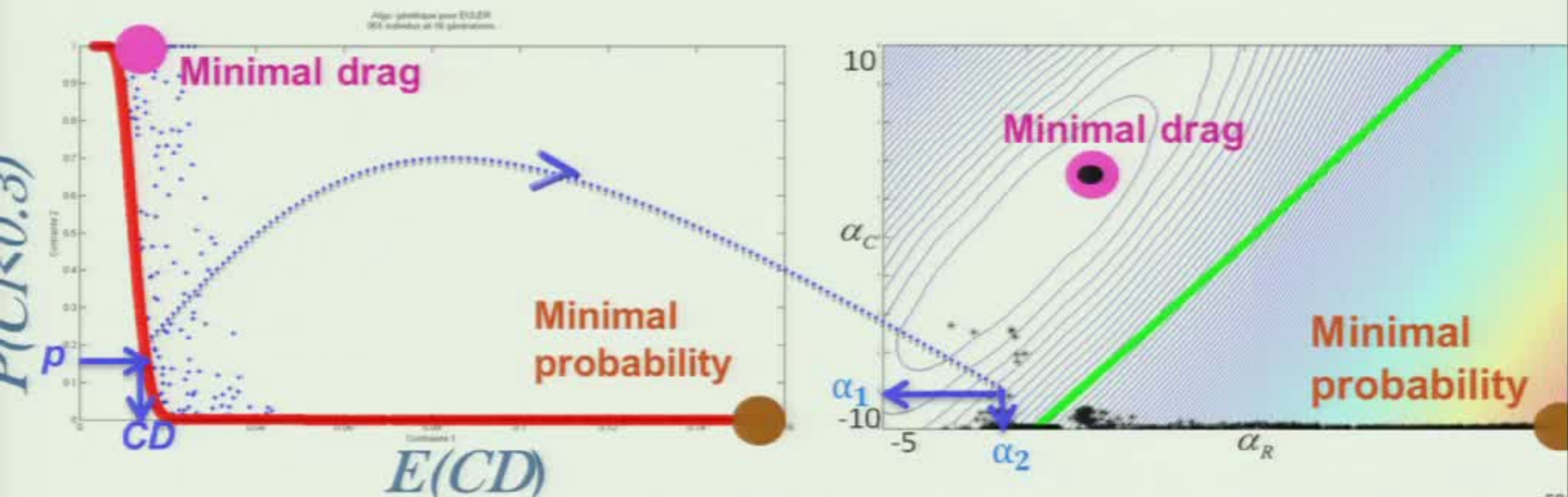


# Reliability-Based Design: Pareto front



Decision: Accept a probability for the lift to be less than a value with minimal drag

- Determination of drag mean  $CD$  (Pareto front)
- Determination of nominal values of geometrical parameters  $\alpha_1$  and  $\alpha_2$  (camber and twist angles)





# Computational trends



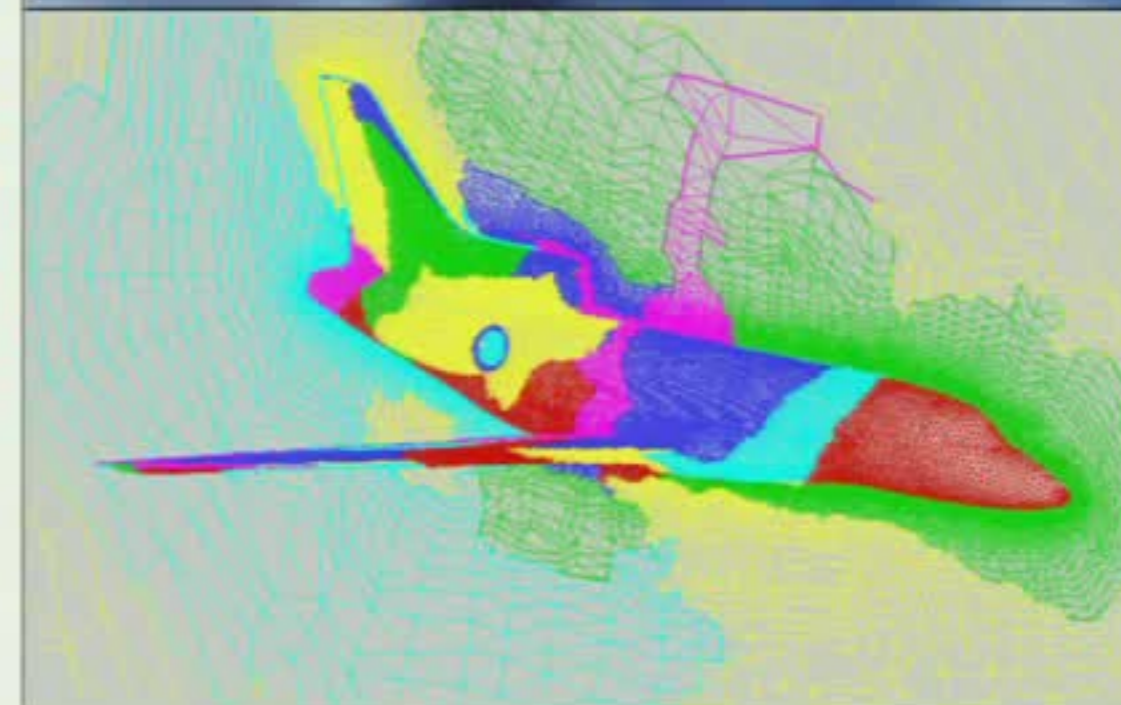
Computers power will increase by a factor ~500  
within the next 10 years:

Week-long computation will be available in 1/2 hour  
Day-long computation will be available for automatic  
optimization

Hour-long computation will be available interactively

Open the way for short cycle Multidisciplinary Design  
Optimization

Computers architecture evolves to an almost  
exponentially increasing number of processors  
The architecture of the codes must fit to the computers' one



- Status : Demonstrated efficiency of domain partitioning using MPI up to 20000 cores
- Challenge :
  - “classical” multiprocessor architecture → 2 levels: multiprocessor / many cores architecture
- Code modernization effort:
  - From a **Bulk Synchronous Model** to a **Multi-Level Asynchronous Model**
- On going R&T effort : combine “classical domain partitioning + MPI” and “local sub-partitionning using a **Divide & Conquer** algorithm + multithreading ”



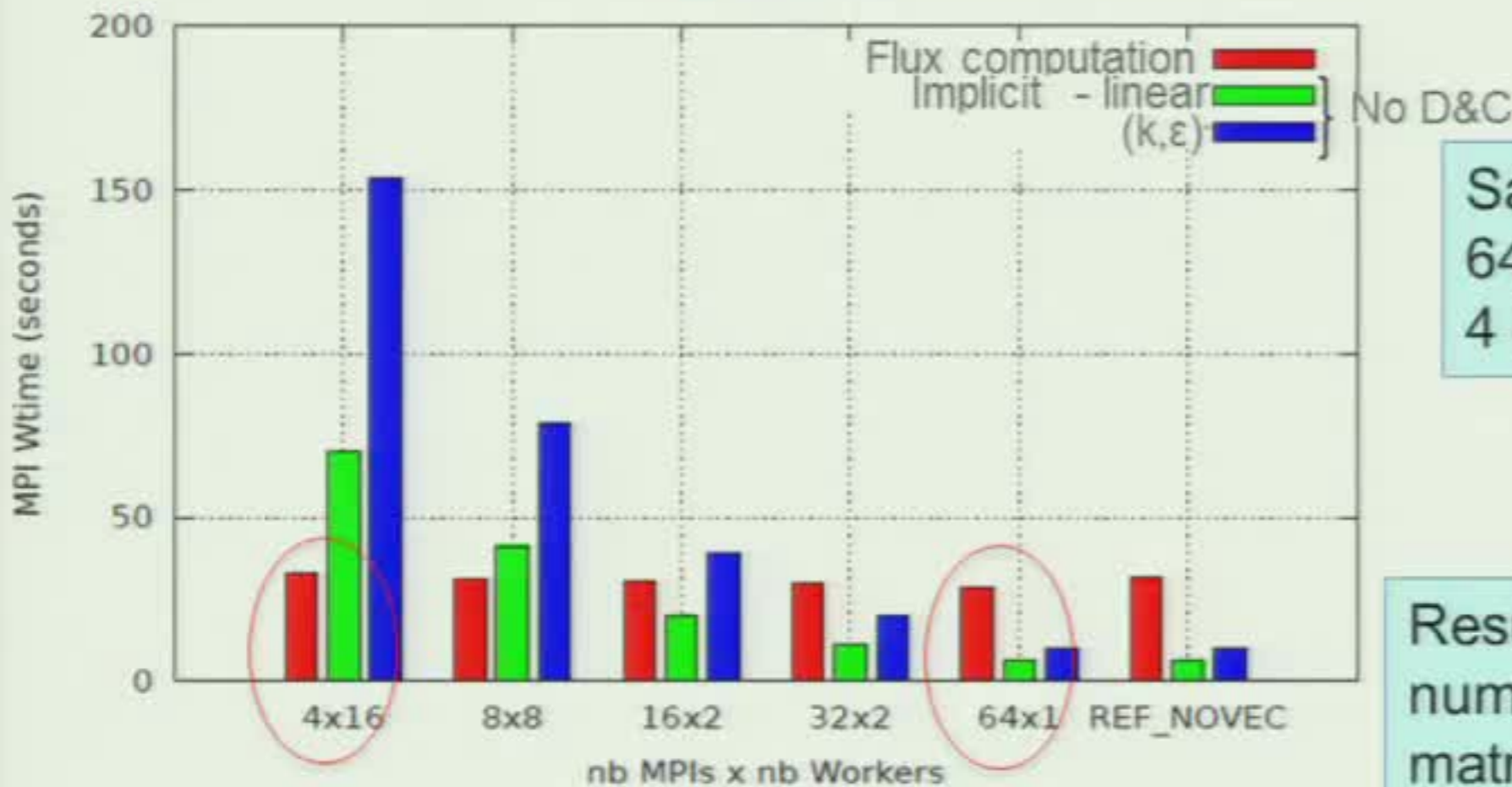
# Next generation HPC

## Result using the Divide & Conquer approach



Test case of Navier-Stokes solver with a mesh of  $5 \cdot 10^6$  grid points on 4 processors with 16 cores each

64 Cores - F7X



Same turn around time with  
64 MPI  
4 MPI + multithreads with D&C

Result must be confirmed on large number of cores and extended to matrix vector product

# Outline



## Design

- Industrial state-of-the art of CFD
- Automatic shape optimization
- Multiphysics: example of Aeroelasticity
- Computational Electromagnetics
- Surrogate models
- Uncertainty quantification – Robust design
- Challenges of next generation HPC (towards Exascale)

## Development

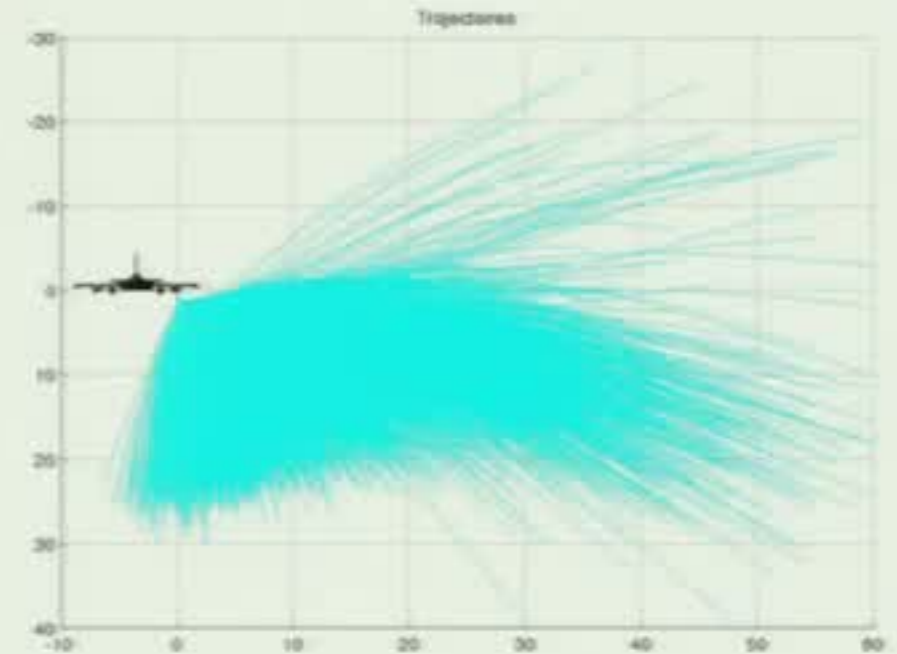
- An example of rare-event probability evaluation

## Support

- First attempts in Data Analytics



# Rare event probability evaluation: risk of collision during a store release



## Store trajectories depend on 2 types of variables

- $x_C \in C$  variables under pilot control like speed, altitude, load factor
- $x_E \in E$  uncontrolled parameters: load dispatch, turbulence, ...

## Envelope clearance problem: find the subset of $C$ where load release is safe

### Means

- Simulator: computes the trajectory when variables values are given
- Budget: maximum simulator runs

### Dangerousness score (« algebraic distance »)

- $f: C \times E \rightarrow R$ : collision iff  $f(x_C, x_E) < 0$

# Rare event probability evaluation: application context



## Formalization

- Uncontrolled parameters values are realizations of a random vector  $X_E$  whose law can be easily simulated
- Risk at  $x_C \in \mathcal{C}$  is  $\pi(x_C) = \mathbb{P}(f(x_C, X_E) < 0)$

## Qualification of the release safety at every point $x_C \in \mathcal{C}$

- Safe if  $\pi(x_C) < p_S$  (typically  $p_S = 10^{-5}$ )
- Dangerous if  $\pi(x_C) \geq p_D$
- Relatively safe if  $p_D < \pi(x_C) < p_S$

## Strategy

- Estimate at a sufficient number of  $x_C \in \mathcal{C}$

## Budget matters (number of affordable runs)



# Rare event probability evaluation: brute force Monte Carlo is unfeasible



- Let  $X_{1:L} = (X_1, \dots, X_L)$  be a  $L$ -sample of  $X$ , the following statistic is a binomial  $\mathcal{B}(L, \pi)$

$$\Gamma(R, X_{1:L}) =_{def} \sum_{k=1}^L \mathbb{1}_{]-\infty, 0]}(f(X_k)) = \sum_{k=1}^L \mathbb{1}_R(X_k)$$

- And so:  $a(\Gamma(R, X_{1:L}), L, \alpha) \leq \pi \leq b(\Gamma(R, X_{1:L}), L, \alpha)$  with confidence level  $1 - \alpha$
- But in most cases,  $\pi \sim 0$ 
  - Which leads to  $\Gamma(R, X_{1:L}) = 0$  with high probability
  - Meaning  $L$  over  $200000$  simulator runs (typically one full day of computations) would be necessary to get  $\pi \leq 10^{-5} = b(0, L, 0.1)$  with confidence level of  $95\%$

# Rare event probability evaluation: principles of the importance sampling approach



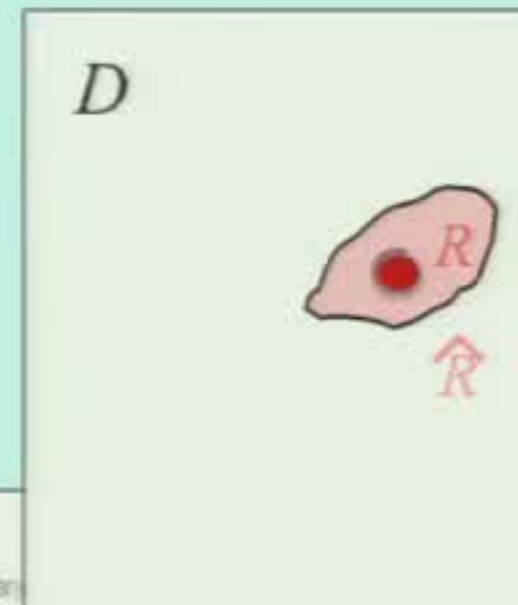
Substitute importance random variable  $Z$  for  $X$  whose law is:

$$\mathbb{P}_Z : A \subset E \mapsto \mathbb{P}(X \in A | X \in \hat{R})$$

Where  $\hat{R} \subset E$  is such that  $R \subset \hat{R}$ ,  $\hat{R}$  close to  $R$  and  $\|\hat{R}$  evaluated at (very) low cost  
based on an approximate function  $\hat{f}$  of  $f$ )

Compute a Monte-Carlo estimation of  $\mathbb{P}(Z \in R) = \frac{\mathbb{P}(X \in R)}{\mathbb{P}(X \in \hat{R})}$  from which is taken an  
estimation of  $\mathbb{P}(X \in R)$

Targeted benefit:  $Z$  is hitting (far) more frequently  $R$  so  
Monte-Carlo estimate of  $\mathbb{P}(Z \in R)$  expected to be (far)  
more precise





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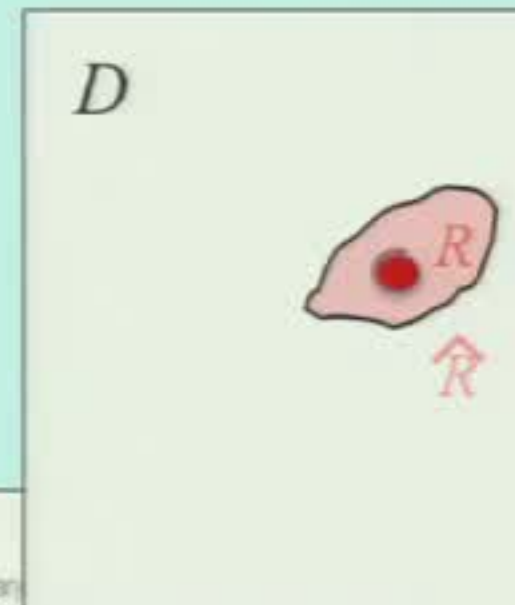
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Targeted benefit:  $Z$  is hitting (far) more frequently  $R$  so Monte-Carlo estimate of  $\mathbb{P}(Z \in R)$  expected to be (far) more precise





# Rare event probability evaluation:

implementation of the importance sampling approach



• A budget of  $N$  runs is given

• Define  $\hat{R} = \{\hat{f}(x) \leq M\}$  and  $\check{R} = \{\hat{f}(x) \leq -M\}$  on the basis of data  $(x_1, f(x_1)), \dots, (x_{\frac{N}{2}}, f(x_{\frac{N}{2}}))$  with  $M$  sufficiently large to ensure that  $\check{R} \subset R \subset \hat{R}$

Step 1 : Sample importance variable  $Z : (Z_1, \dots, Z_L)$

- By sampling original variable  $X : (X_1, \dots, X_K)$  and extract those  $X_i$  that hit  $\hat{R}$  with  $K$  may be  $\gg L$
- Computing confidence bounds  $\check{a}_K(\alpha)$  and  $\hat{b}_K(\alpha)$   
so that  $\check{a}_K(\alpha) \leq \mathbb{P}(X \in \check{R}) \leq \mathbb{P}(X \in \hat{R}) \leq \hat{b}_K(\alpha)$  with confidence level  $1-\alpha$
- Going on sampling  $X$  until one of these conditions is satisfied
  - $\hat{b}_K(\alpha) < p_s$  : point  $x_c$  is safe
  - $p_d \leq \check{a}_K(\alpha)$  : point  $x_c$  is dangerous
  - The number of  $Z$  samples reaches  $\frac{N}{2} \Rightarrow$  go to Step 2

# Rare event probability evaluation: implementation of the importance sampling approach



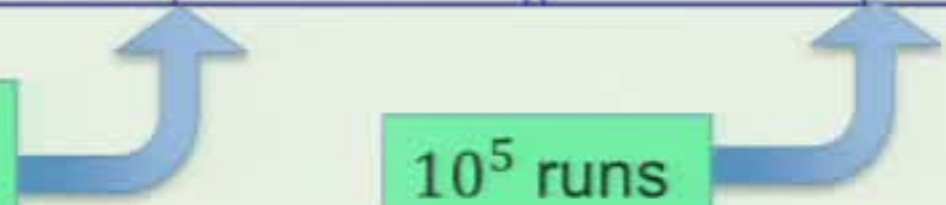
**Step 2: Compute the binomial  $\mathcal{B}\left(\frac{N}{2}, \frac{\pi}{\mathbb{P}(X \in \hat{R})}\right)$  statistic  $\Gamma\left(R, Z_{1:\frac{N}{2}}\right)$  consuming the remaining budget to get the confidence bounds**

$$a(\alpha) =_{def} a\left(\Gamma\left(R, Z_{1:\frac{N}{2}}\right), \frac{N}{2}, \alpha\right) \leq \frac{\pi}{\mathbb{P}(X \in \hat{R})} \leq b(\alpha) =_{def} b\left(\Gamma\left(R, Z_{1:\frac{N}{2}}\right), \frac{N}{2}, \alpha\right) \text{ with confidence level } 1 - \alpha$$

CONTROLLED VARIABLES					IMPORTANCE SAMPLING		MONTE CARLO	
M	Z	Carburant	Nz	Pression bouteille	Borne Inf	Borne Sup	Borne Inf	Borne Sup
0.9	5.0	0.2	1.2	270	$3.53 \cdot 10^{-5}$	$7.5 \cdot 10^{-5}$	$1.79 \cdot 10^{-5}$	$14.5 \cdot 10^{-5}$
0.9	5.0	0.2	1.0	350	$0.045 \cdot 10^{-5}$	$0.146 \cdot 10^{-5}$	0.0	$4.6 \cdot 10^{-5}$
0.9	11.0	0.0	1.0	210	$32.06 \cdot 10^{-5}$	$54.67 \cdot 10^{-5}$	$28.4 \cdot 10^{-5}$	$59.7 \cdot 10^{-5}$
0.9	5.0	0.8	1.0	270	$0.94 \cdot 10^{-5}$	$2.19 \cdot 10^{-5}$	$0.44 \cdot 10^{-5}$	$10.04 \cdot 10^{-5}$

2000 runs  
Confidence level 99%

$10^5$  runs





# Massive Data Analysis: Practical Motivation



Dramatic increase in our ability to collect data from various connected sources

« Standard » large scale Web-based applications

Connected devices – mobile phones, cars, ... – Within the Internet of Things (IoT)

Cyber-Physical Systems, equipped with large sensor networks

Complex systems and data intensive applications that raise the data scale to an unprecedented level

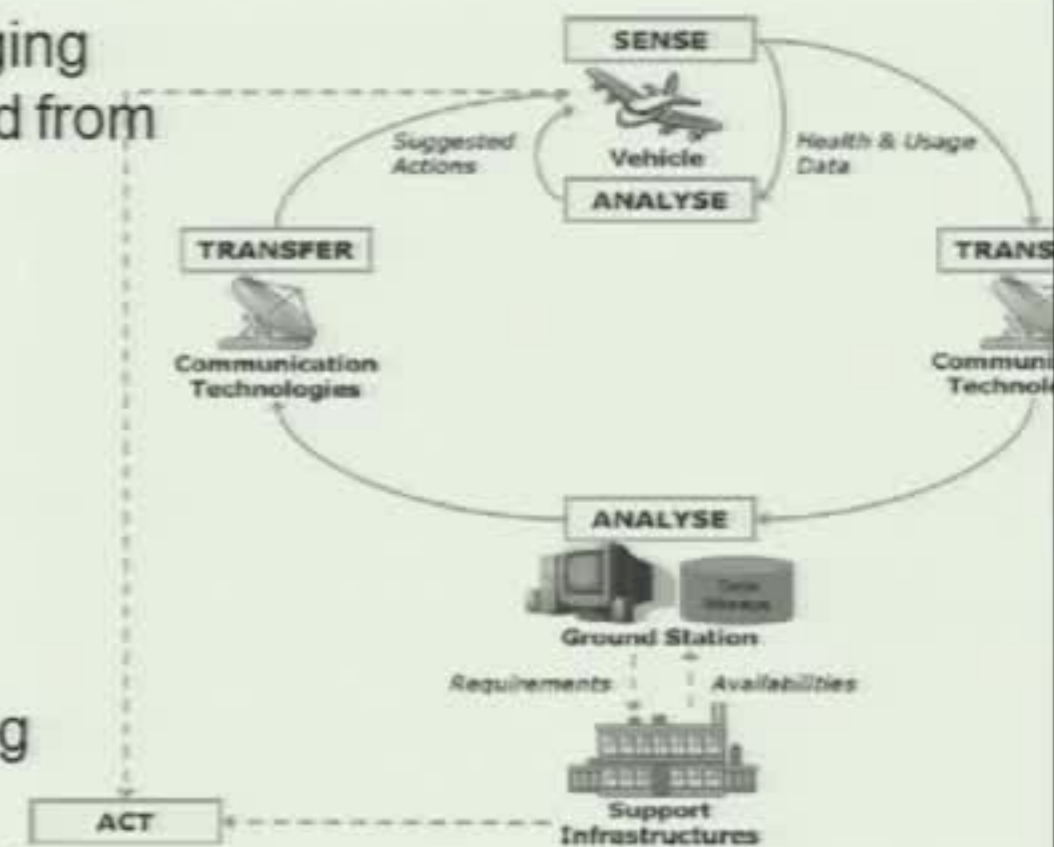
Aerospace is highly concerned! Many applications are emerging that relies on the systematic analysis of sensor data collected from aircraft systems

## Health Monitoring and Predictive Maintenance

- Early anomaly detection/prognostics
- Detection of activity peaks, e.g. significant increases of unscheduled maintenance

## Flight Safety Analysis

- Routine recording and analysis of flight parameters during entire flights
- To support a proactive data-driven approach to flight safety



Source: Benedettini et al 2009

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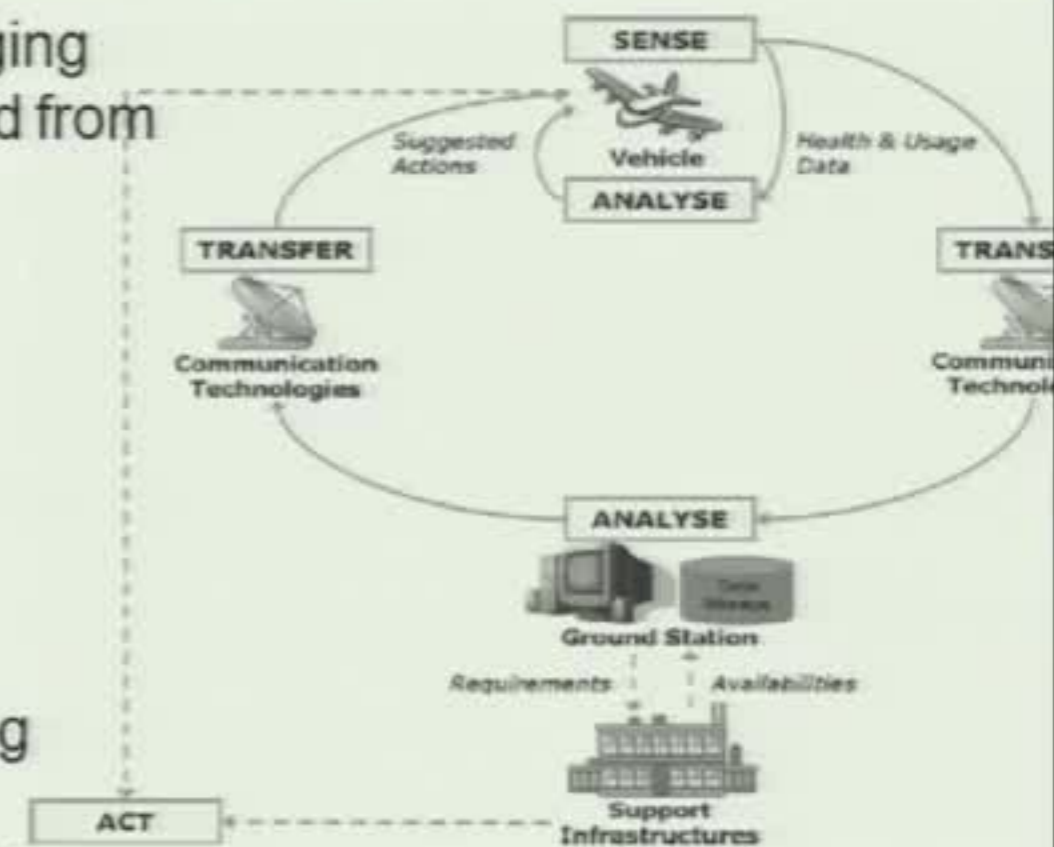
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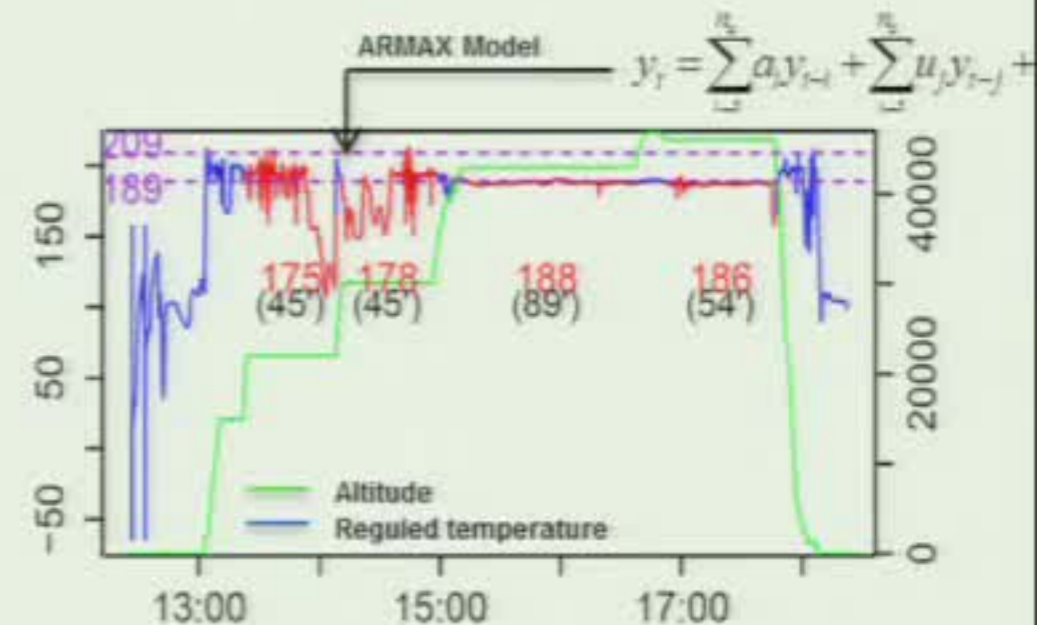
# Data-Driven Statistical-based Methods for Data Analysis



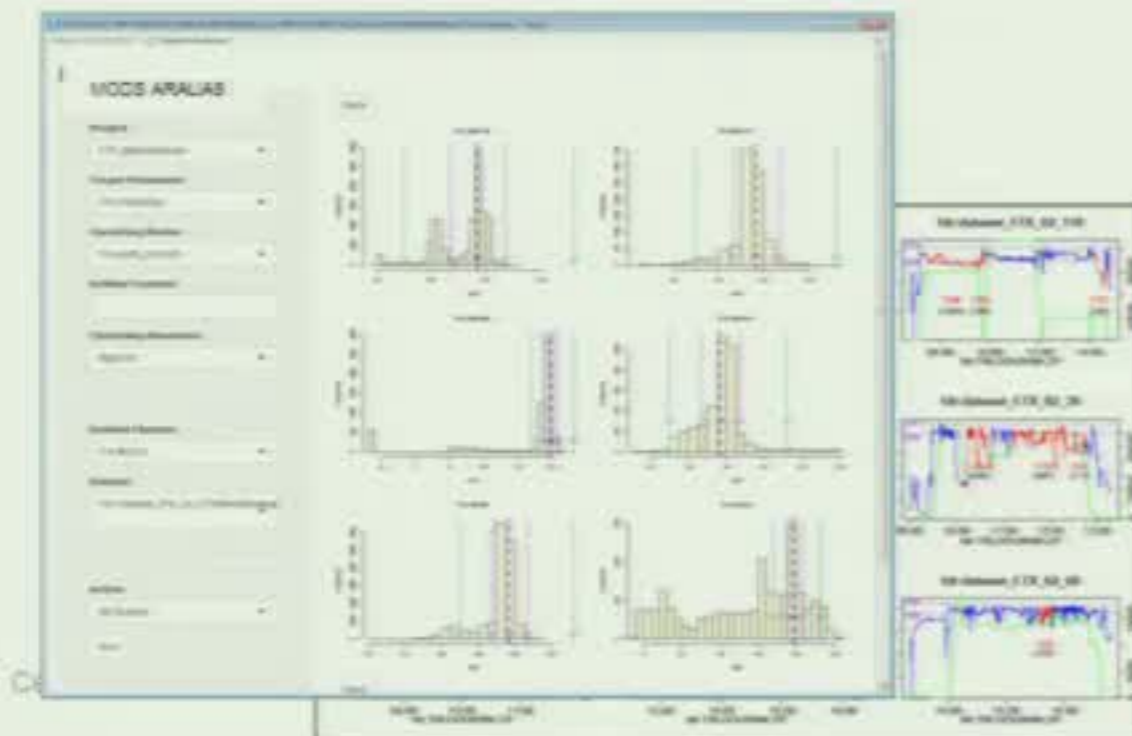
→ Advanced data analysis is required to take advantage of the streams of historical data collected from heterogeneous sources (sensors, ops. databases, ...)

## System Identification and Time Series Analysis

- « Time-aware » parametric models to capture aircraft system underlying dynamics
- Suitable statistics and estimation methods to assess the performance of monitored systems
- Trend analysis and forecasting



## Novel Data analytics and Visualization techniques



- **A Key Challenge**

Evolving from « surgical » analysis of isolated datasets to large-scale model estimation and exploitation

Maintain an unceasing effort to increase the efficiency of the design **process** (engineering time – elementary cycle)

- Algorithmic evolutions identified to reduce the return time
- Taking the maximum advantage from future computer architectures

Exploit the benefits to be provided by UQ

- Robustness regarding a shape degradation (manufacturing tolerance, aging)
- Robustness regarding the jig shape of the aircraft considering the flight point

Increase the confidence of engineers in stochastic approaches

Explore the applications of Data Analytics that bring added-value

- Formulation in scientific language of problems expressed by engineers
- Justification of the correctness and reliability of « Machine Learning »-type algorithms