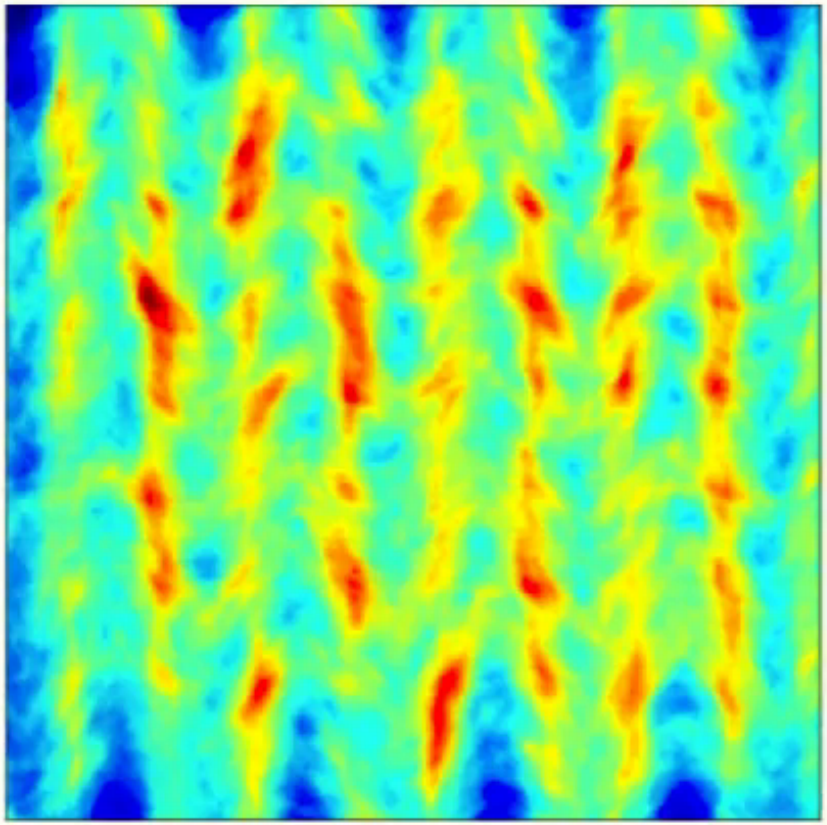
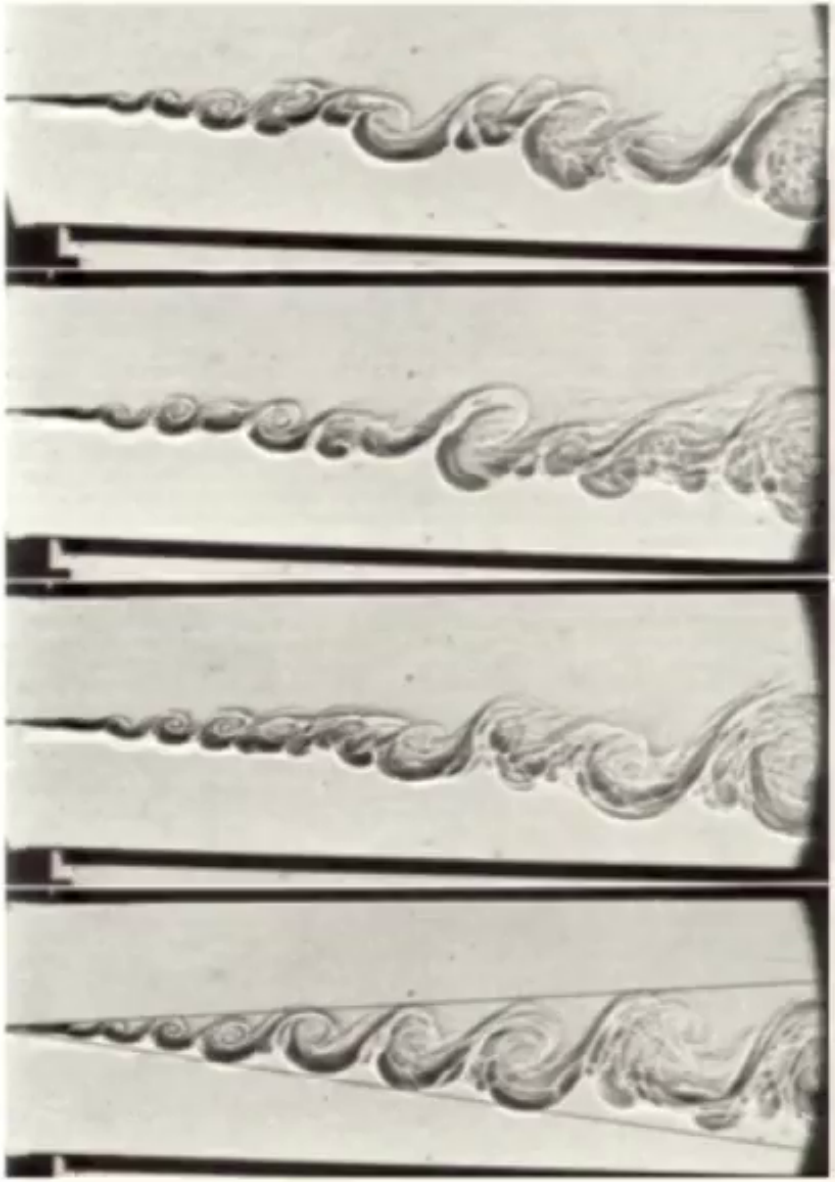


Correlating Dynamical Structures with Forcing in 2D Flow

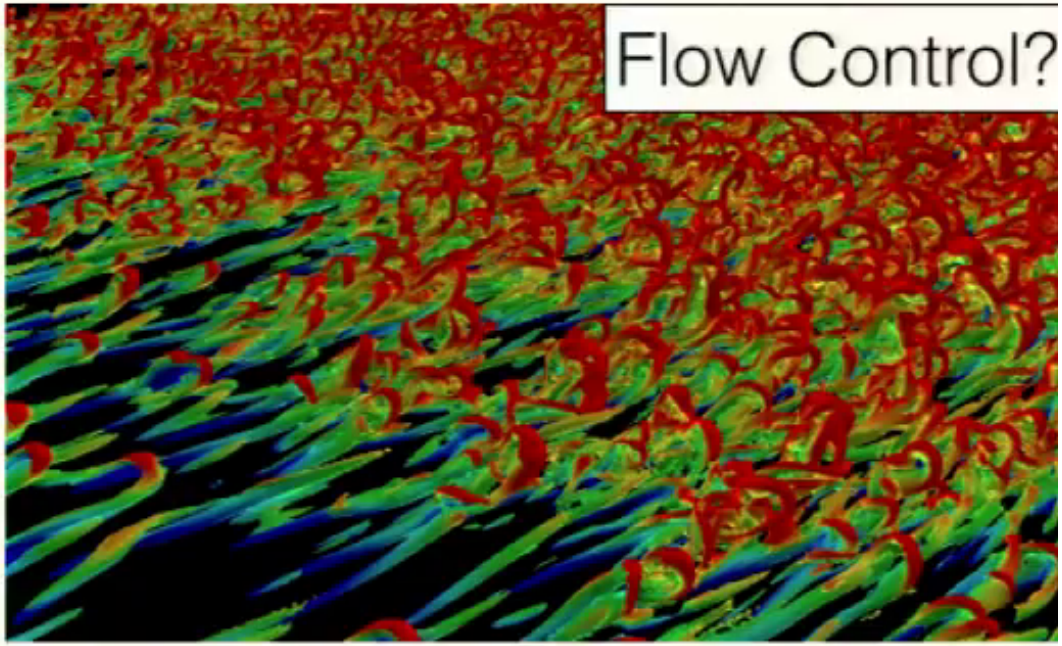
N.T. Ouellette

C. Hogg, Y. Liao

Coherent Structures in Turbulence



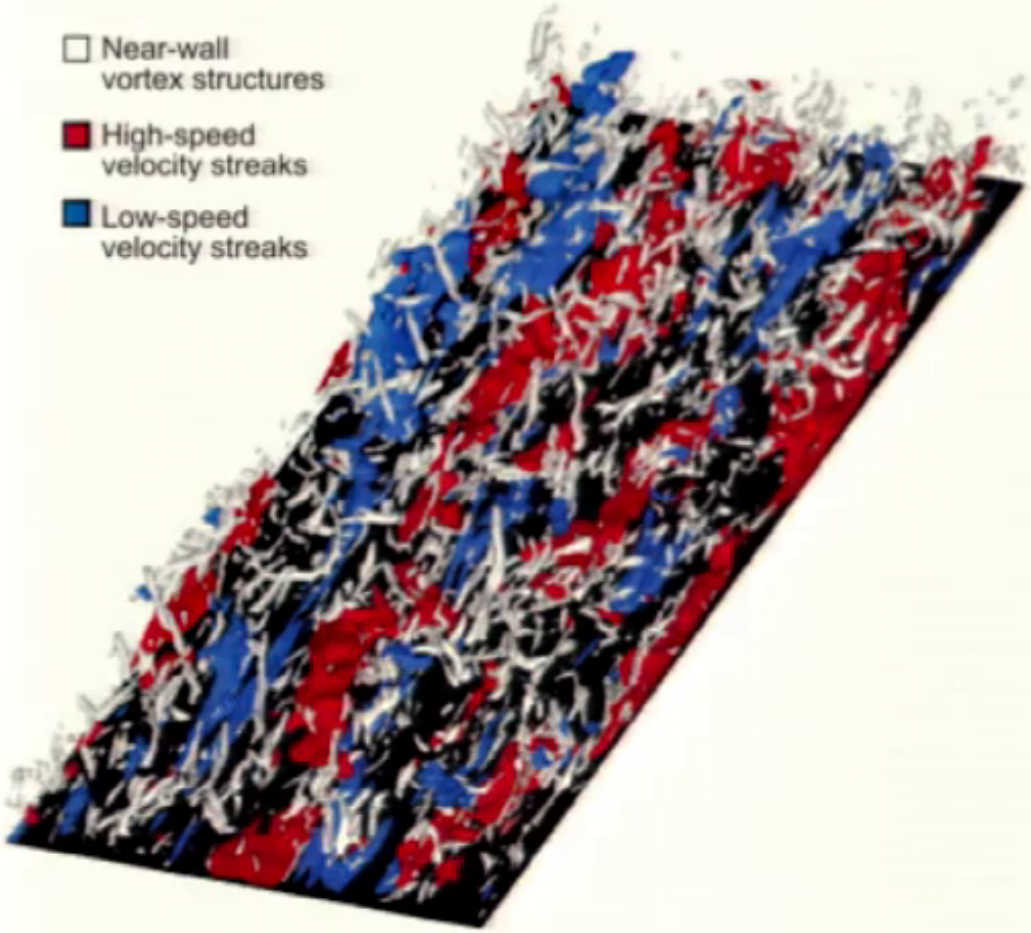
Brown & Roshko, *J. Fluid Mech.* (1974)



Flow Control?

Schlatter *et al.* (2011)

Example: Boundary-layer Flow Control



C.M. White & M.G. Mungal, *Annu. Rev. Fluid Mech.* (2008)

Interaction of structures leads to persistence of turbulence

Modify structures to disrupt regeneration cycle

Active control: momentum sources, sinks

Passive control: boundary conditions

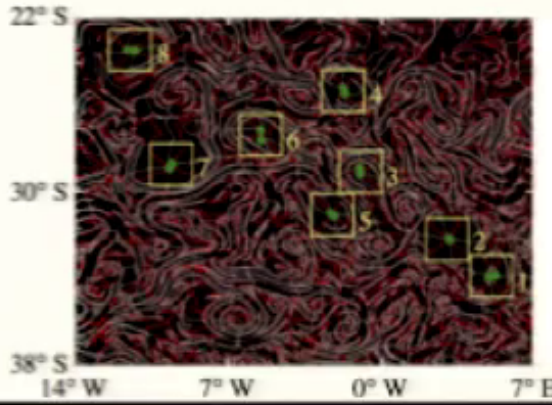
Lagrangian Coherent Structures

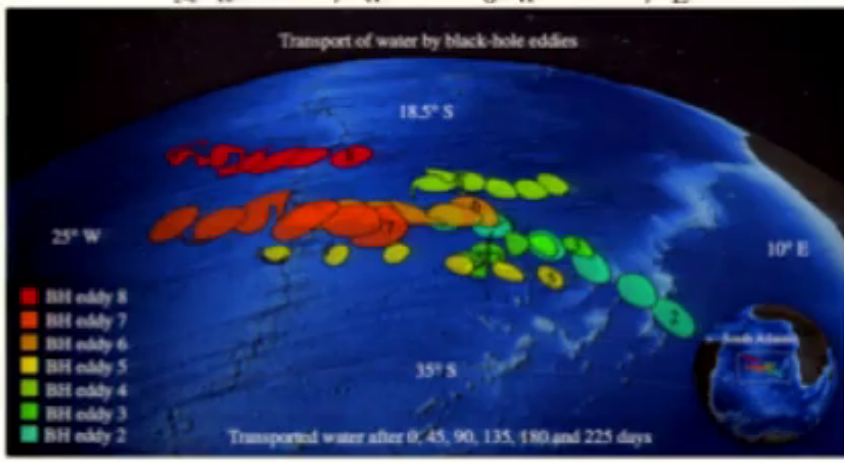
Maximally attracting/repelling material lines

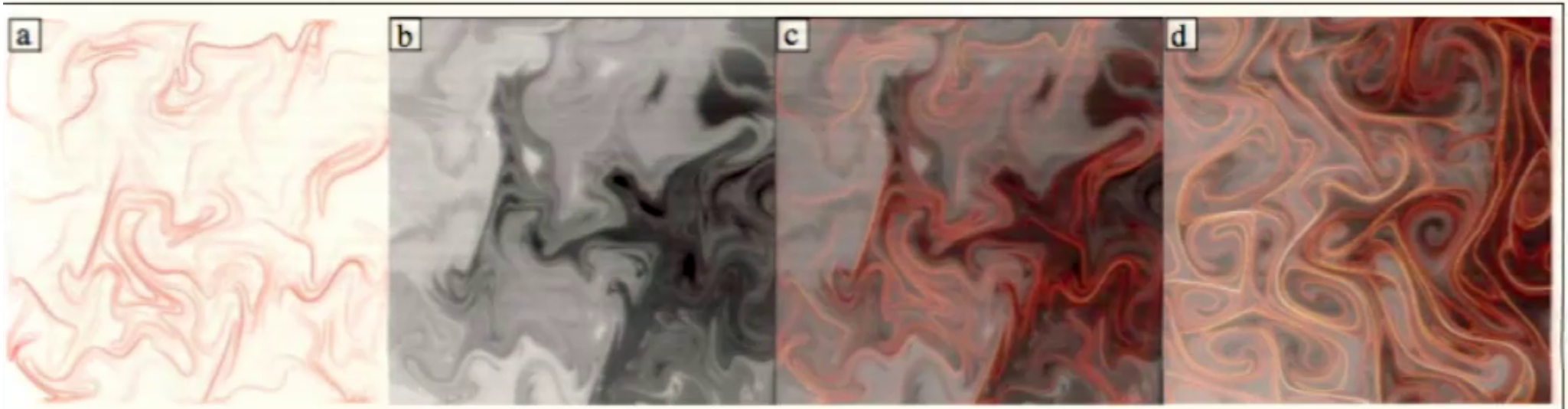
Most important transport barriers: mixing 'skeleton'

G. Haller, *Annu. Rev. Fluid Mech.* (2015)

G. Haller & F.J. Beron-Vera, *J. Fluid Mech.* (2013







G.A. Voth *et al.*, *Phys. Rev. Lett.* (2002)

Lagrangian Coherent Structures

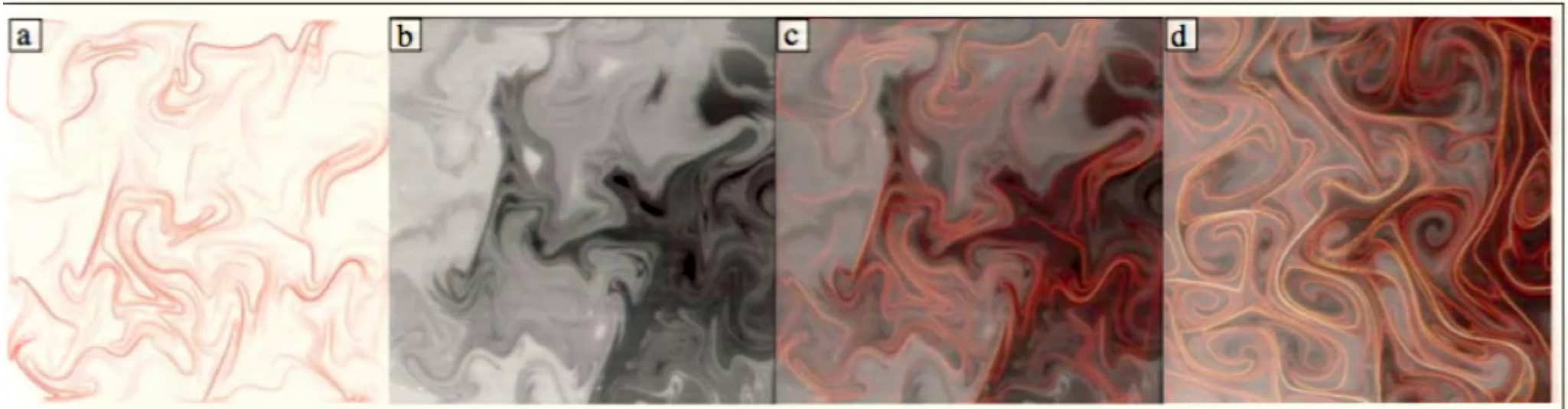
Maximally attracting/repelling material lines

Most important transport barriers: mixing 'skeleton'

G. Haller, *Annu. Rev. Fluid Mech.* (2015)

Useful descriptive tool:
Explains why things happened the way they did

Little predictive capacity:
Requires knowledge of the future to compute



G.A. Voth *et al.*, *Phys. Rev. Lett.* (2002)

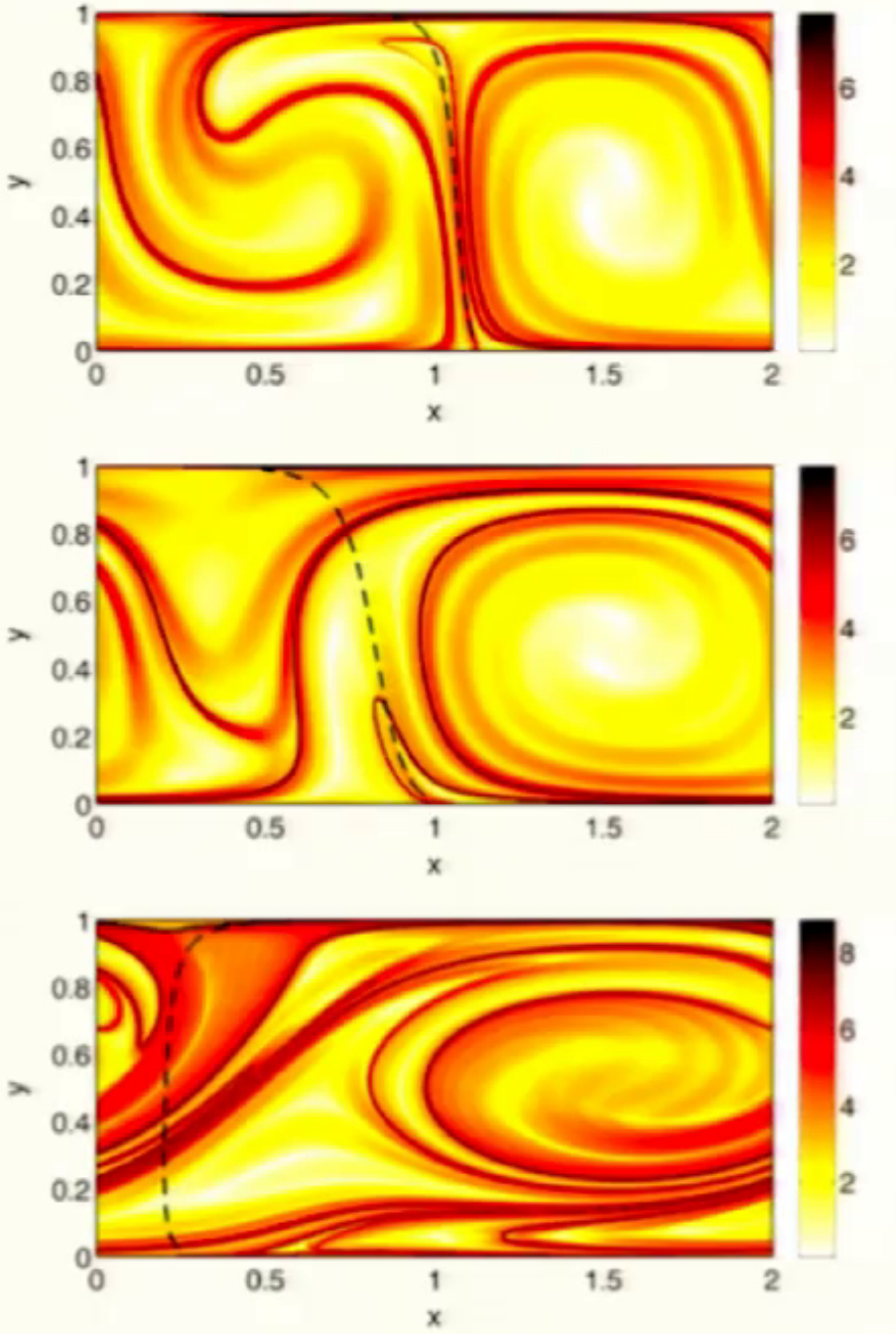
Control of LCS

- Generate transport barrier in desired location
- Move existing barrier somewhere else
- Destroy existing barrier

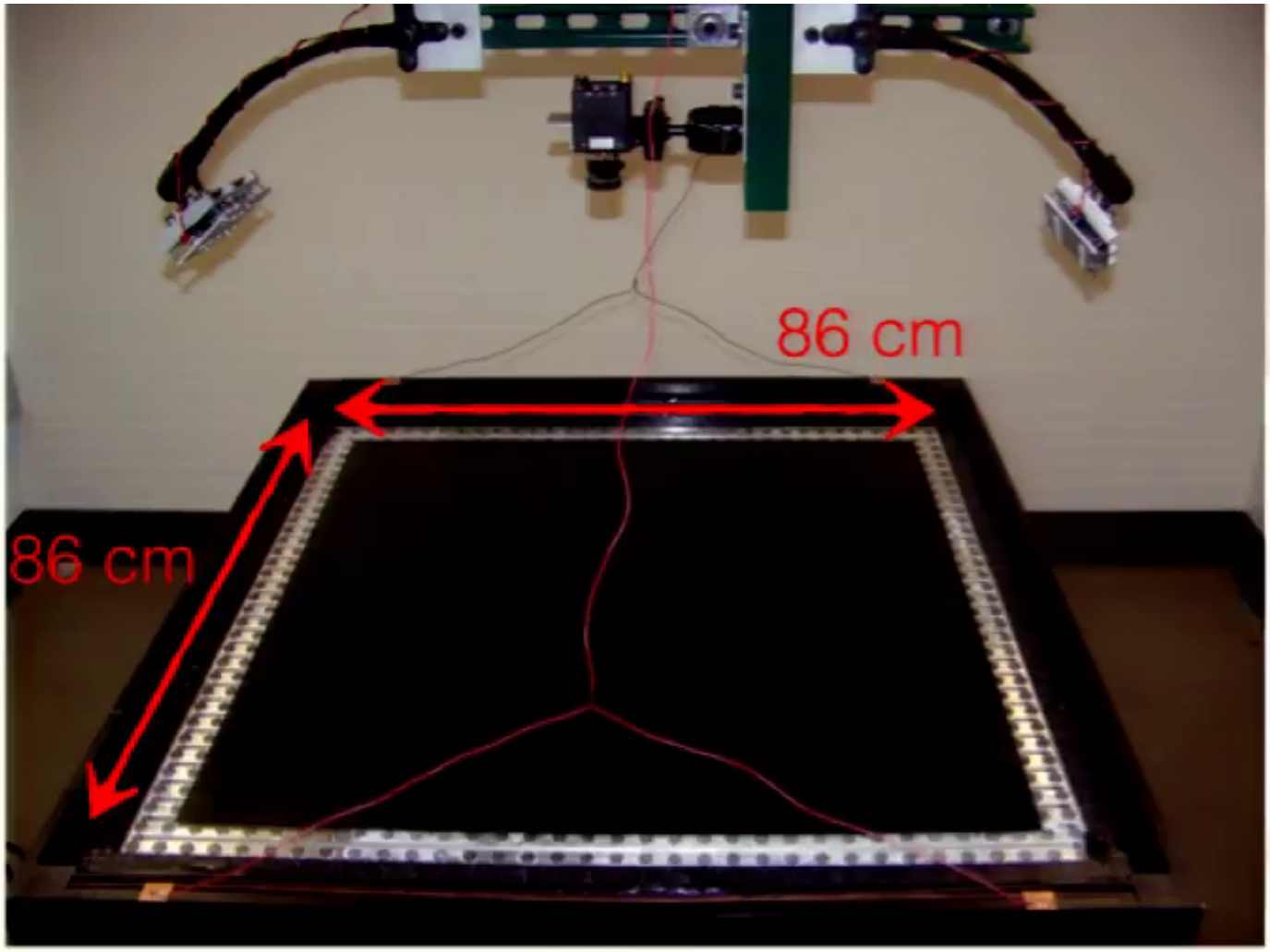
Establish connections between things we can *control* and LCS locations:

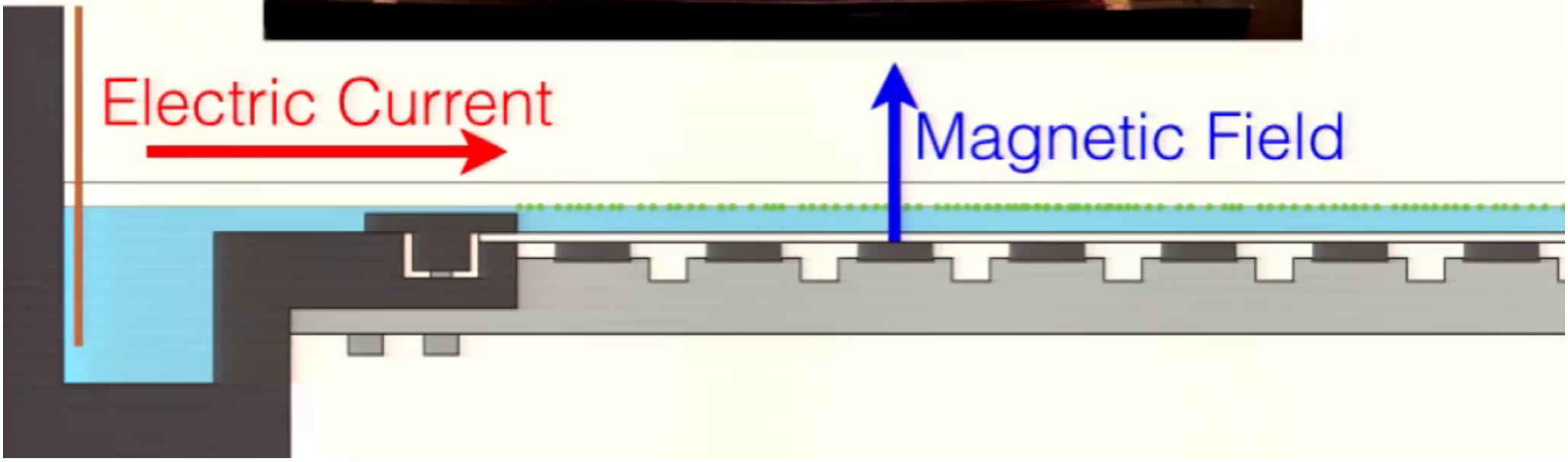
Forcing

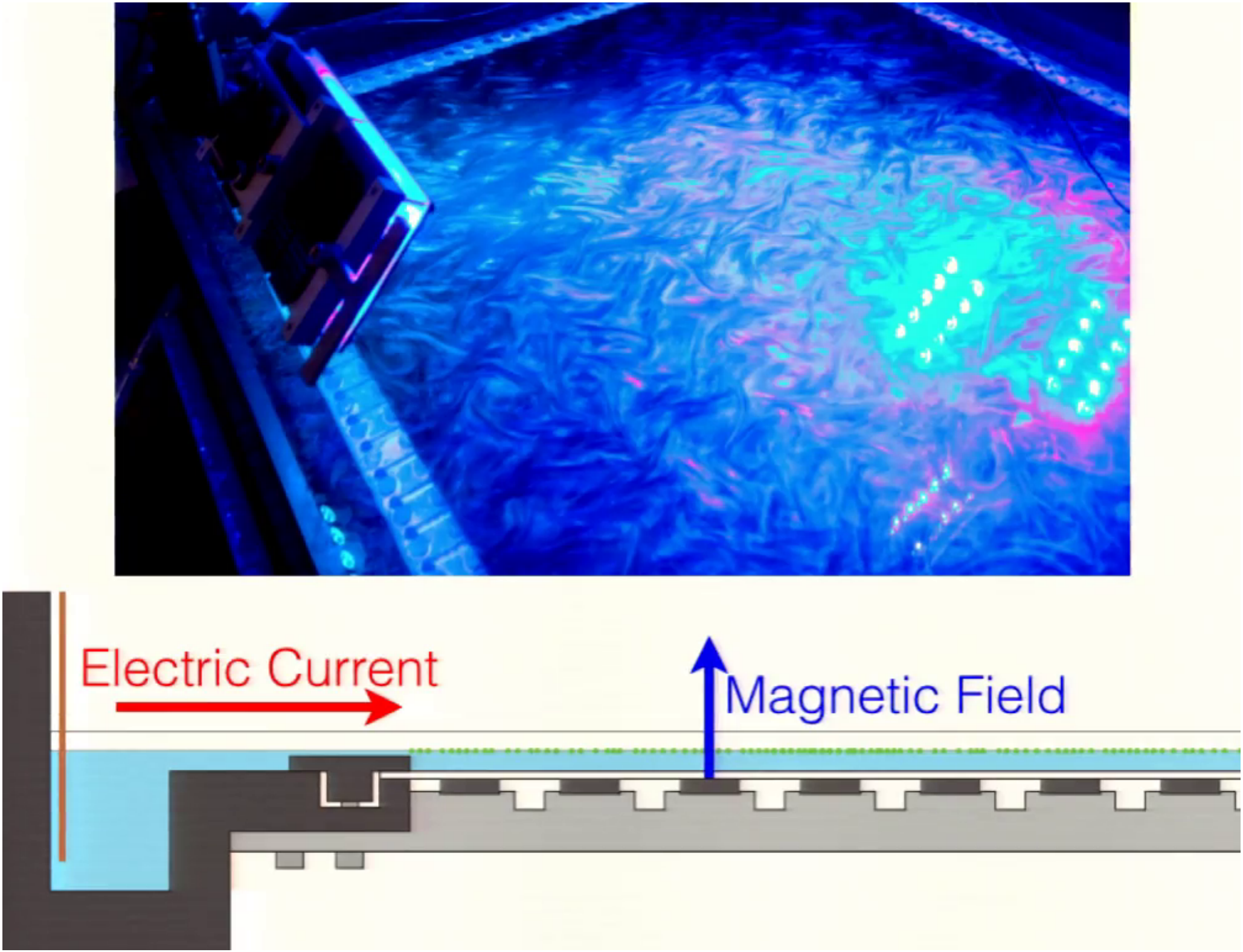
Boundary Conditions

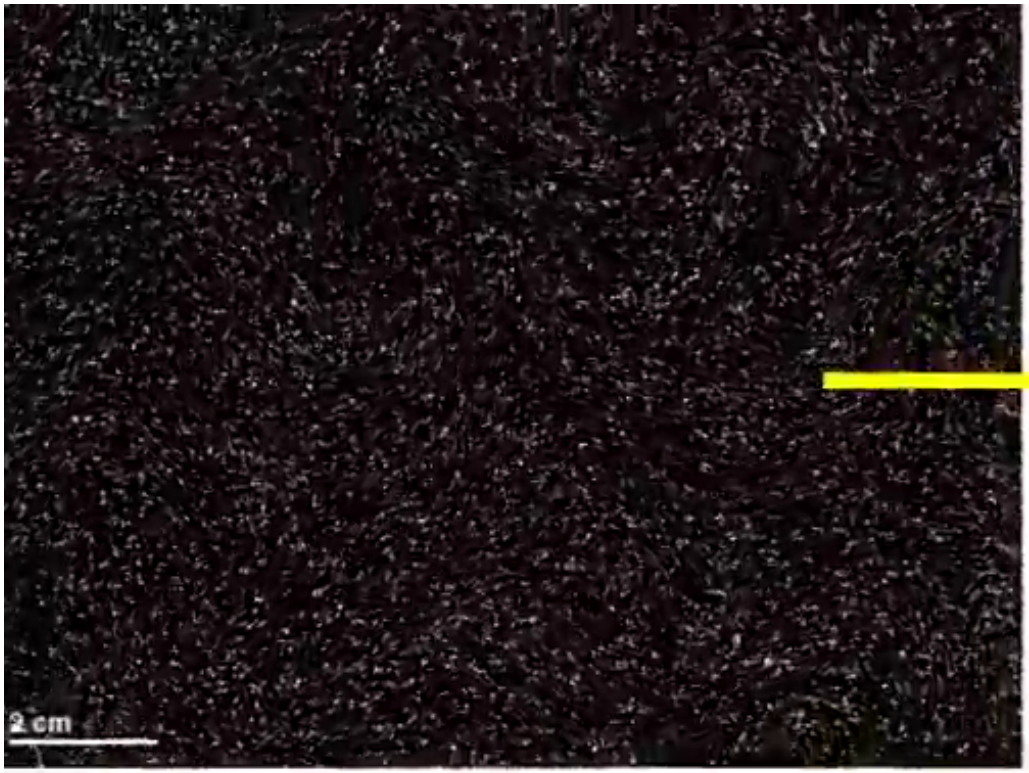


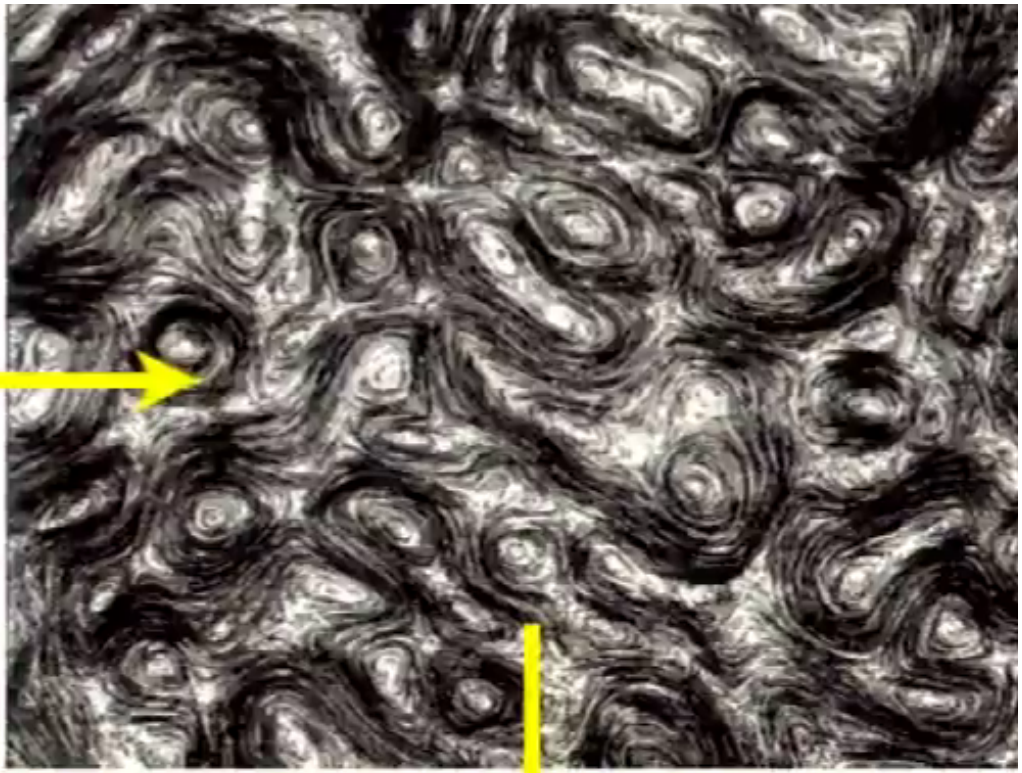
S. Balasuriya & K. Padberg-Gehle,
Physica D (2014)

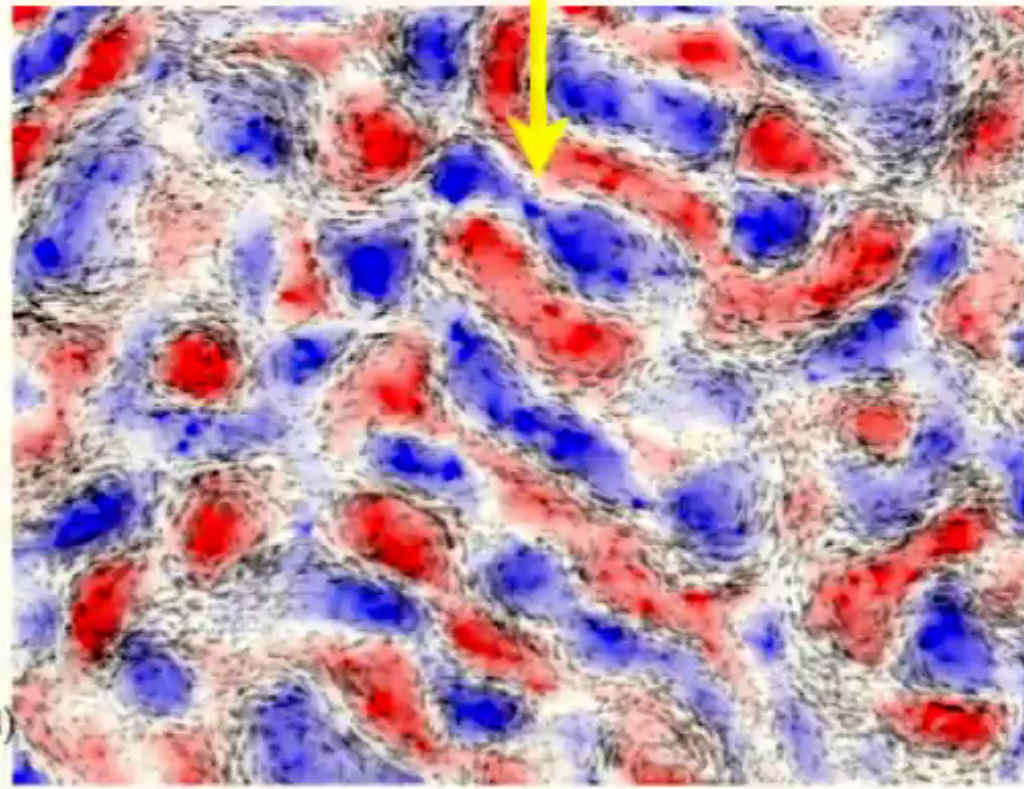

86 cm
86 cm


Electric Current
Magnetic Field


Electric Current
Magnetic Field







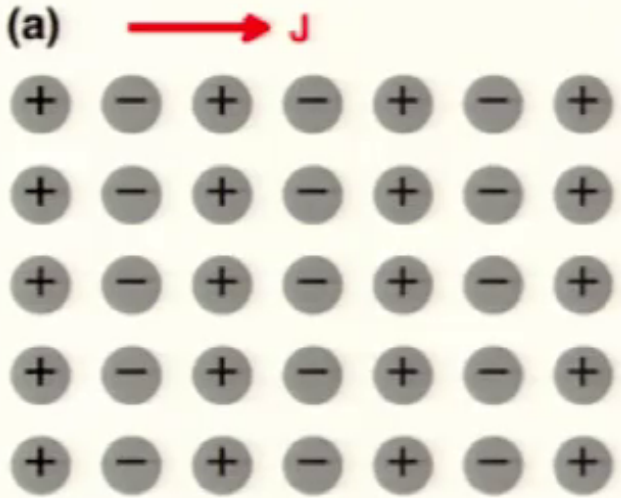
Obtain velocity field with PTV

50 μm particles, ~35k per frame

Advect virtual particles through field

NTO, H. Xu, & E. Bodenschatz, *Exp. Fluids* (2006)
NTO, P.J.J. O'Malley, & J.P. Gollub, *Phys. Rev. Lett.* (2008)
).H. Kelley & NTO, *Phys. Fluids* (2011)

Data Parameters



Kolmogorov flow configuration

Re = 235

DC, static forcing

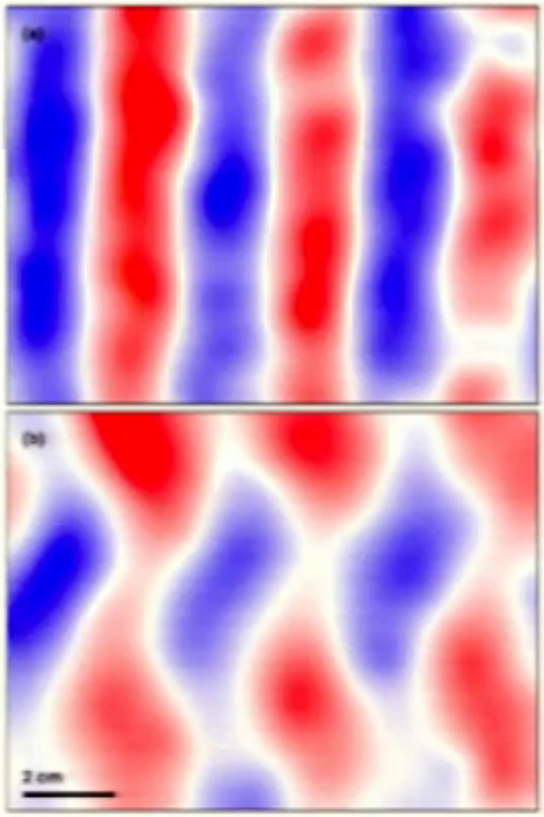
Fluctuations are large:
turbulence intensity is ~250%

Characteristic time scale:

$$T_L = \frac{u'}{L_m} = 1.8 \text{ s}$$

Very long data record:

~700 T_L

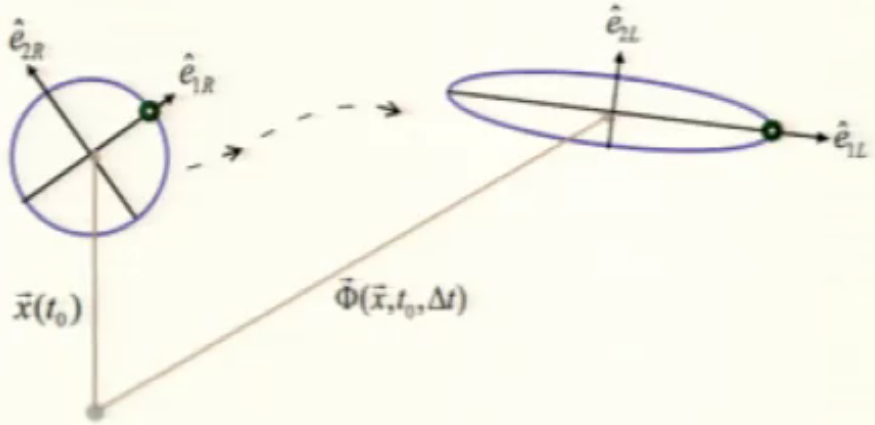


D.H. Kelley & NTO, *Am. J. Phys.* (2011)
Y. Liao, D.H. Kelley, & NTO, *Phys. Rev. E* (2012)

Lagrangian Coherent Structures

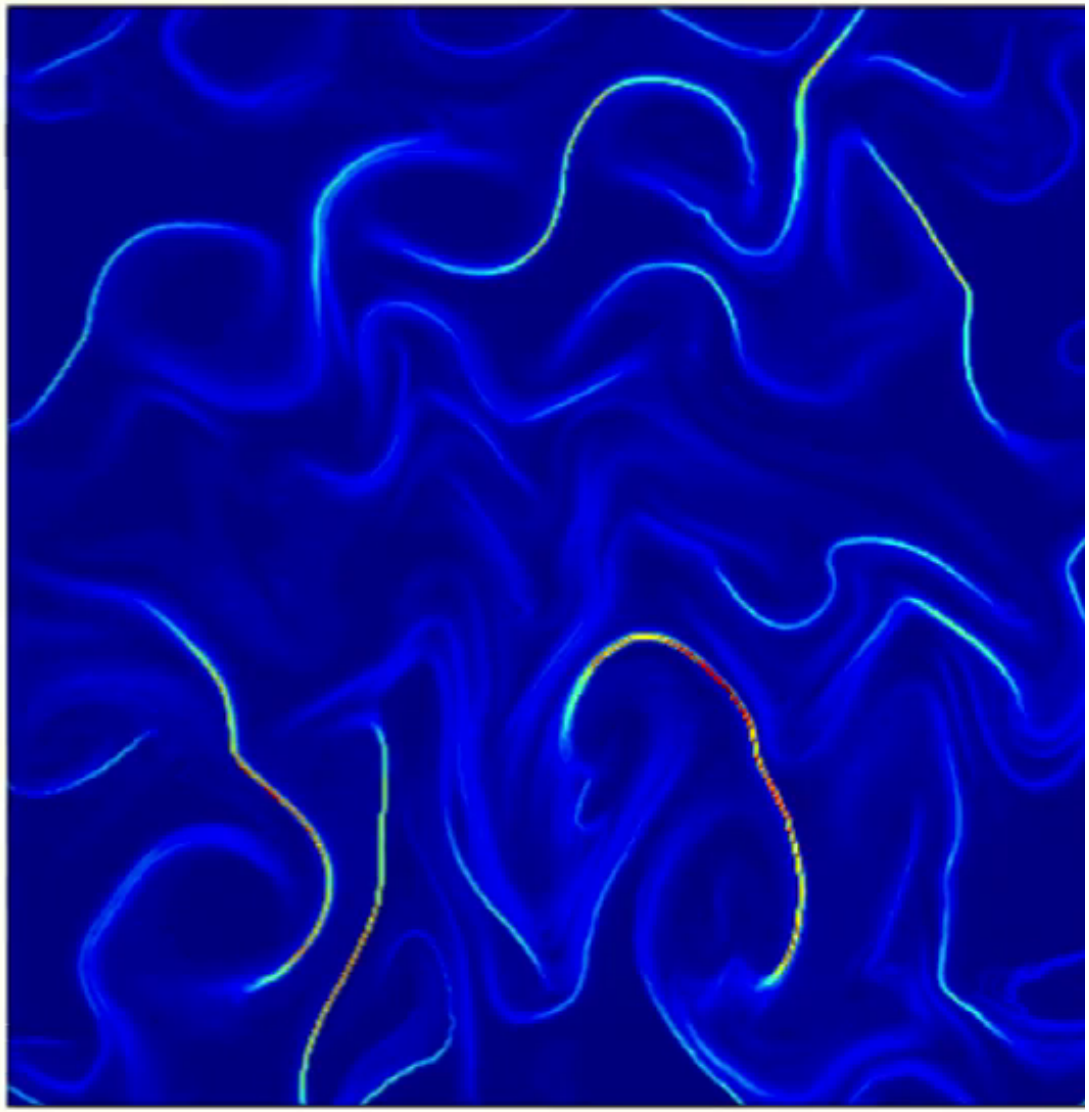
Use stretching and FTLE as proxies for LCSs

$$\vec{x}(t) = \vec{\Phi}(\vec{x}, t, \Delta t)$$



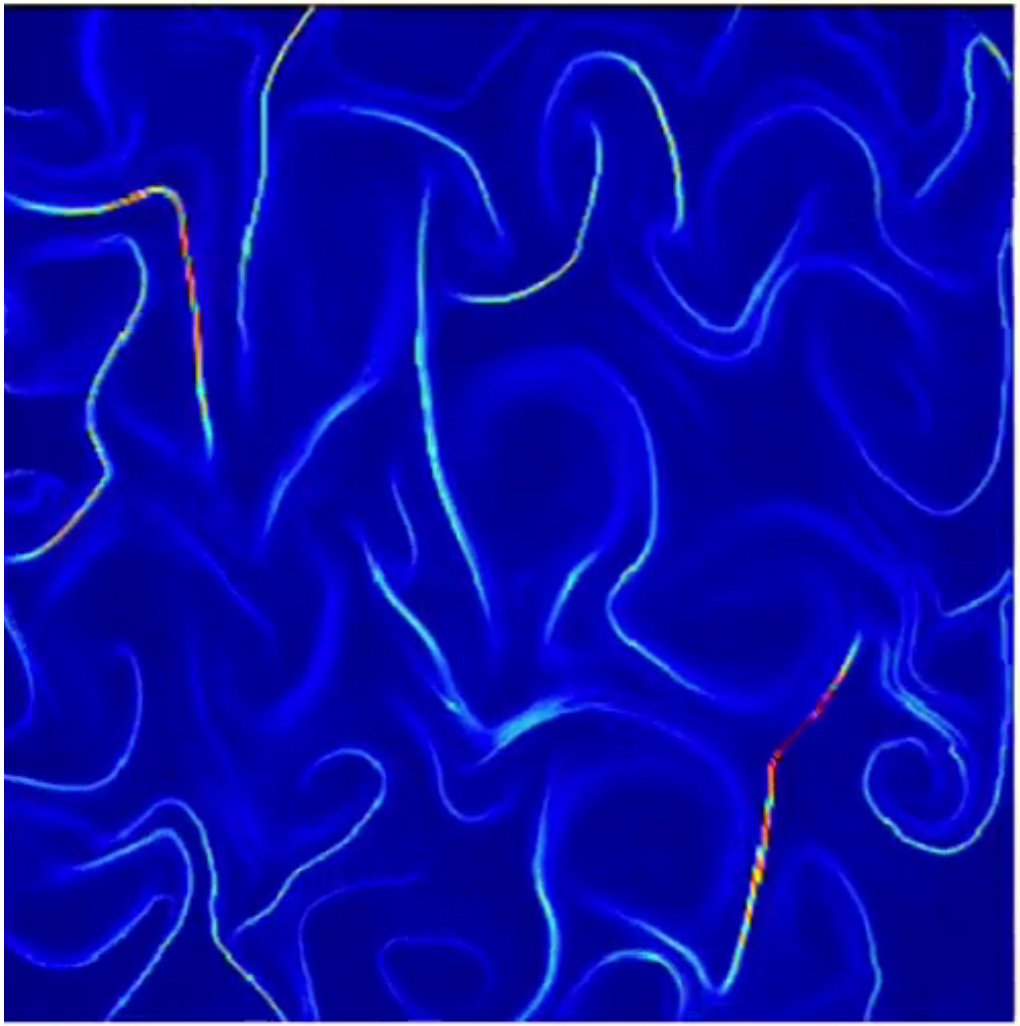
$$C_{ij} = \frac{\partial \Phi_k}{\partial x_i} \frac{\partial \Phi_k}{\partial x_j}$$

$$\sigma(\vec{x}, t, \Delta t) = \log(\sqrt{\lambda_{\max}(\vec{x})})/\Delta t$$

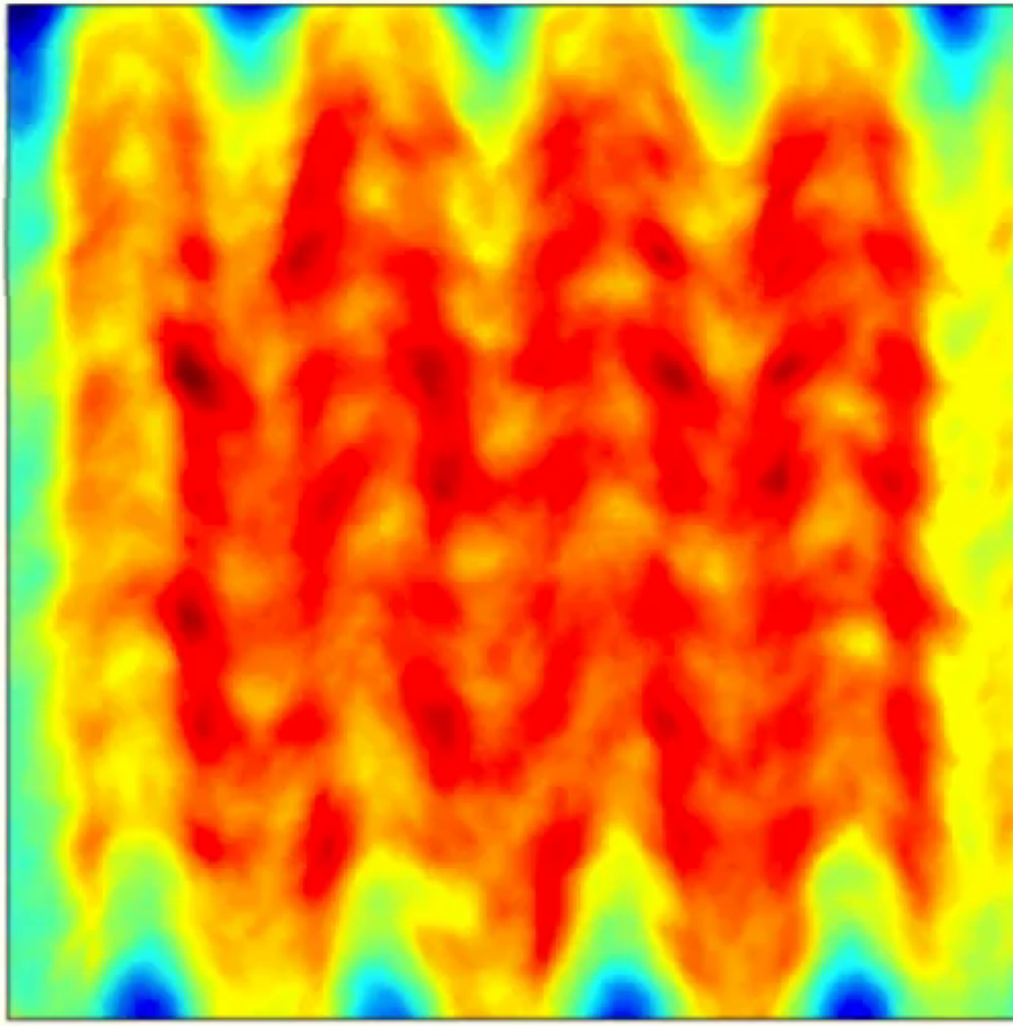


G. Haller & G. Yuan, *Physica D* (2000
G.A. Voth *et al.*, *Phys. Rev. Lett.* (2002

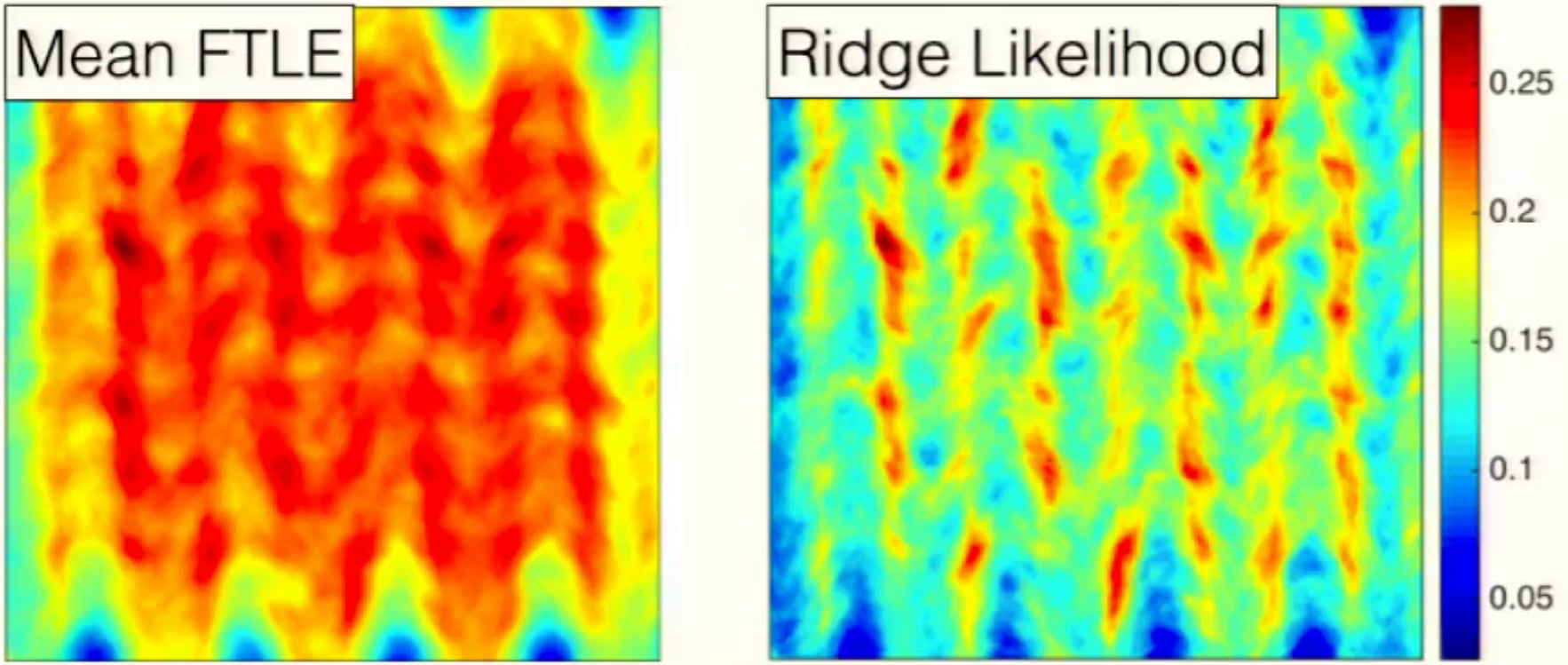
I. LCSs are not random in space



Instantaneous



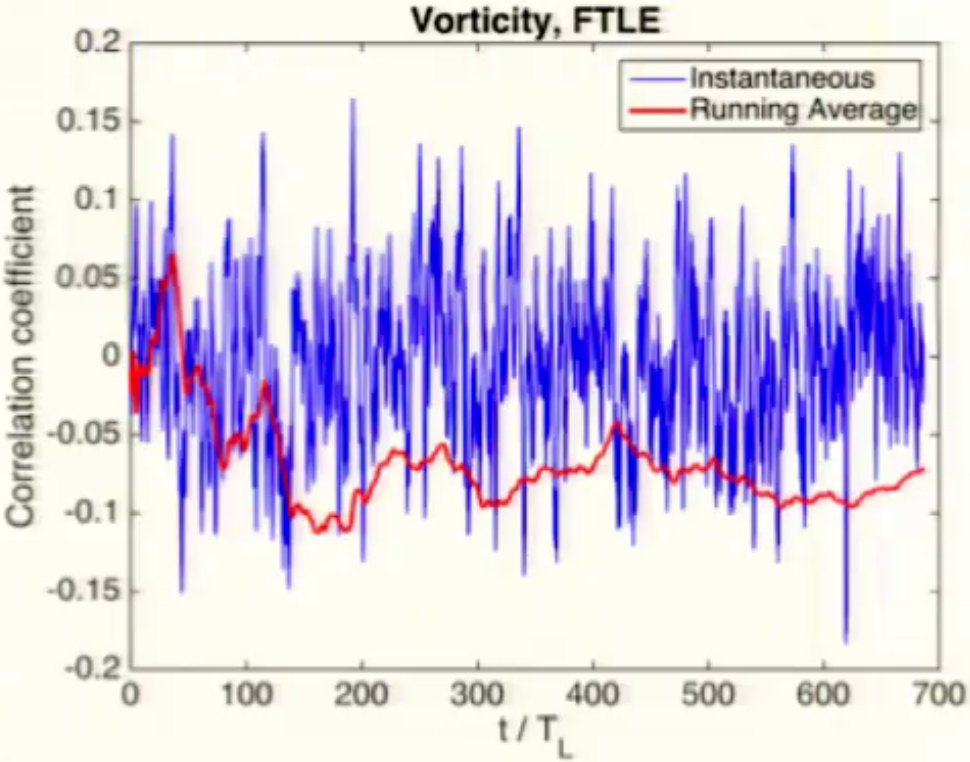
$$\frac{1}{N_f}\sum_{i=1}^{N_f}\sigma(\vec{x},t_i,\Delta t)$$

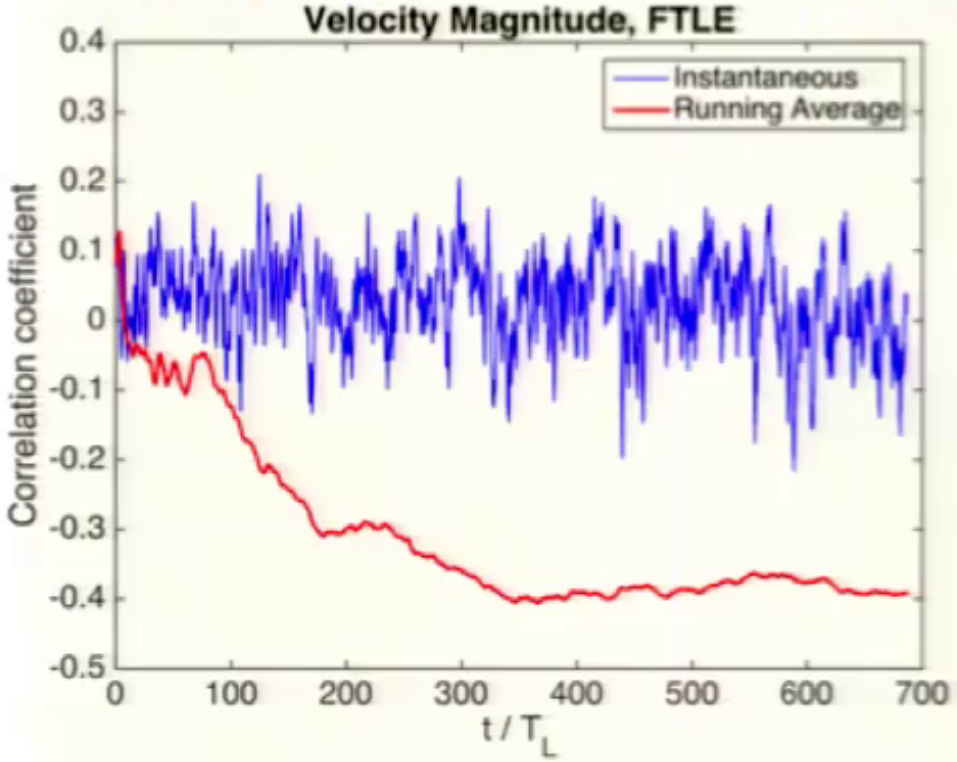

Mean FTLE
Ridge Likelihood
0.25
0.2
0.15
0.1
0.05

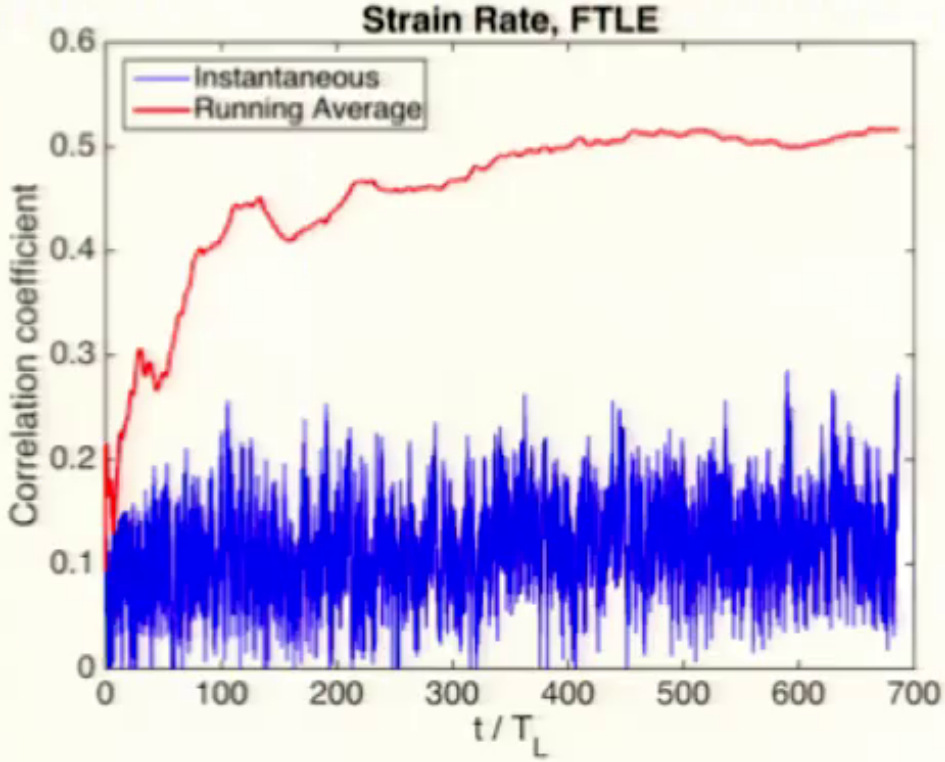
Quantify links between FTLE, mean field via correlation

Instantaneous correlation is weak

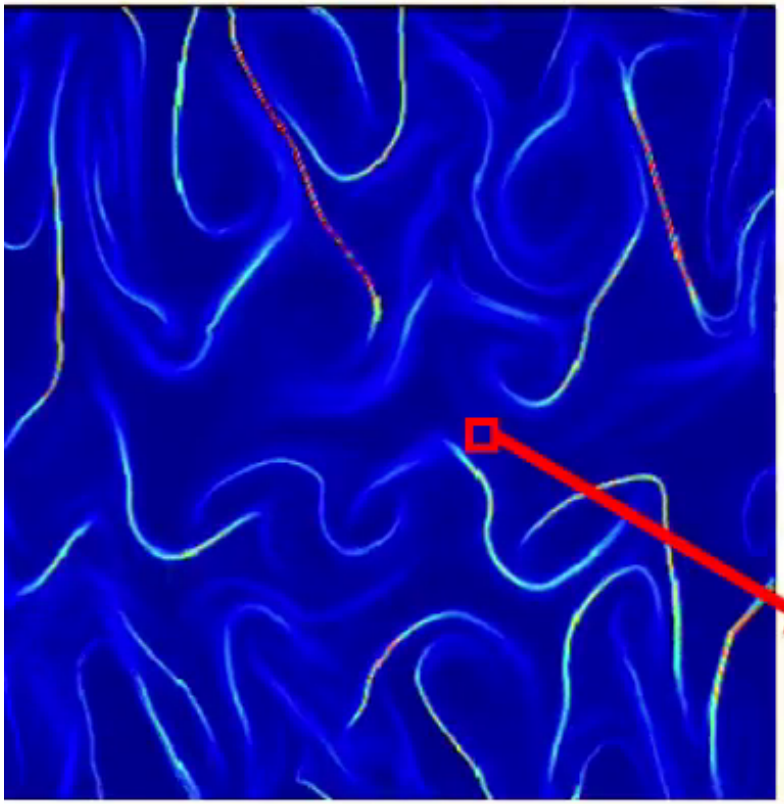
Time-averaged correlation is stronger, but slow to develop



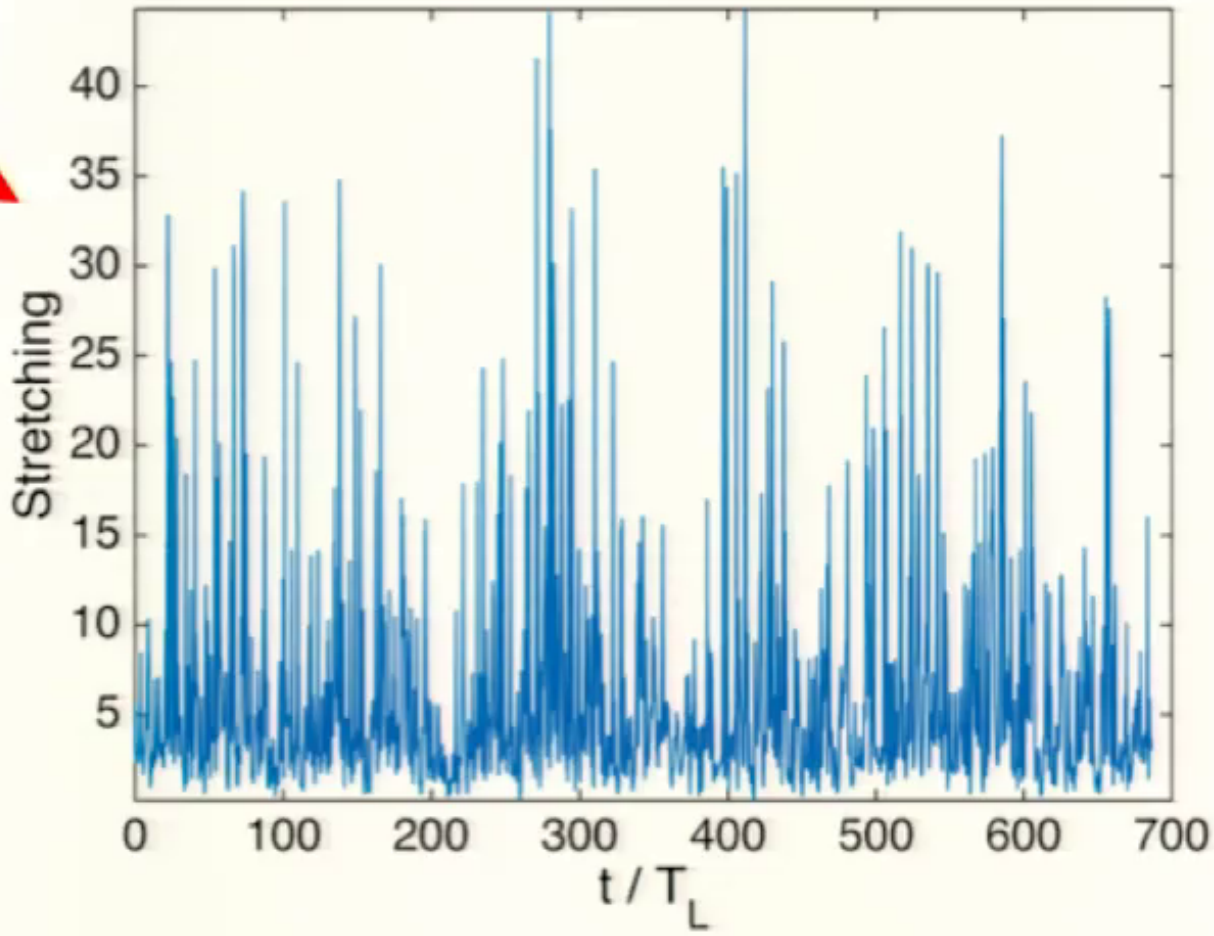


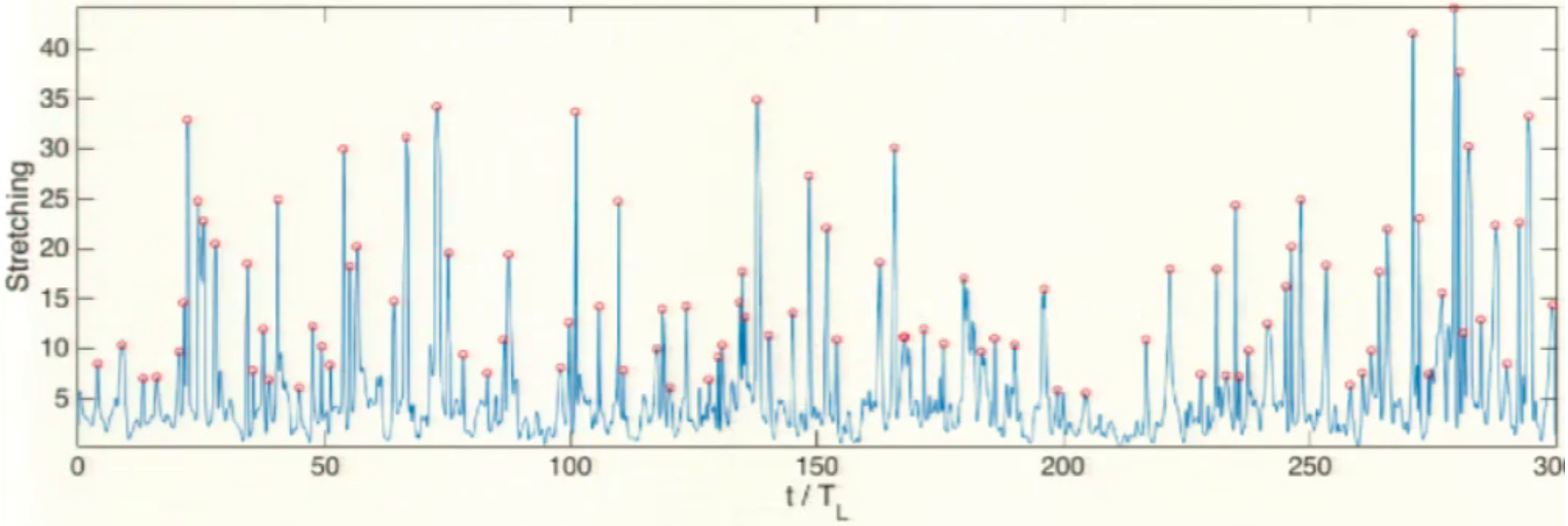


II. LCSs are not random in time



Examine pointwise time series of FTLEs

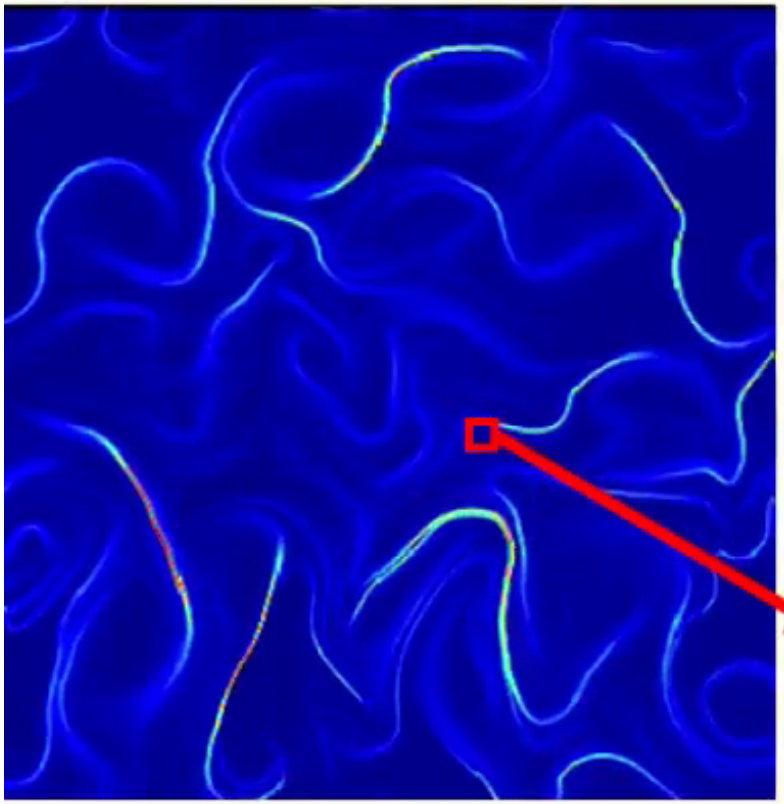




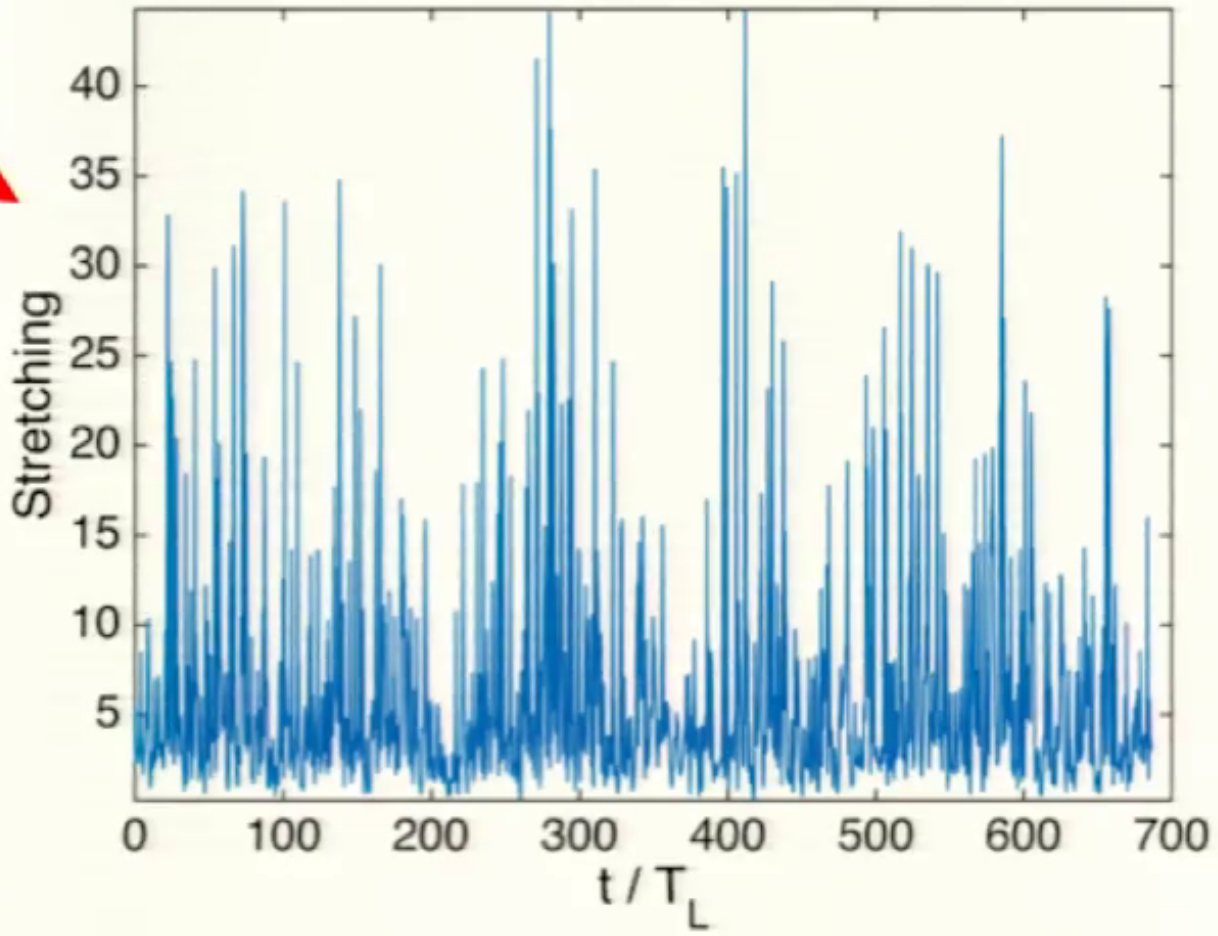
Find peaks in signal; study inter-peak time statistics

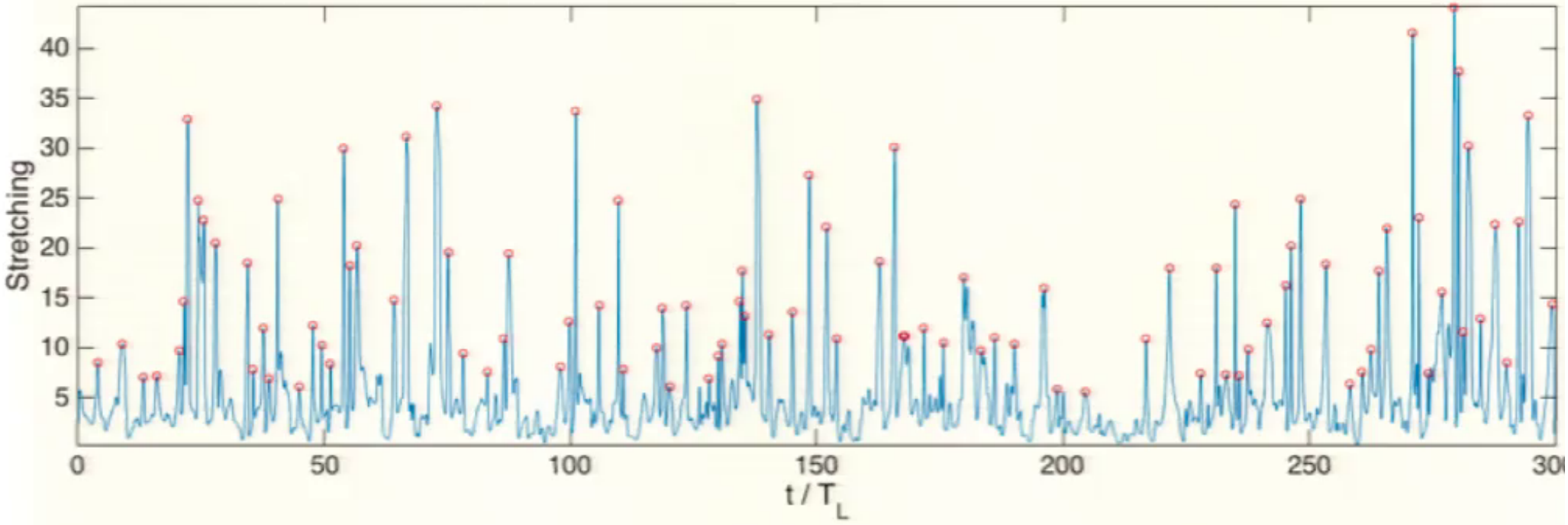
If signal were random and memoryless, should be exponential

II. LCSs are not random in time



Examine pointwise time series of FTLEs





Find peaks in signal; study inter-peak time statistics

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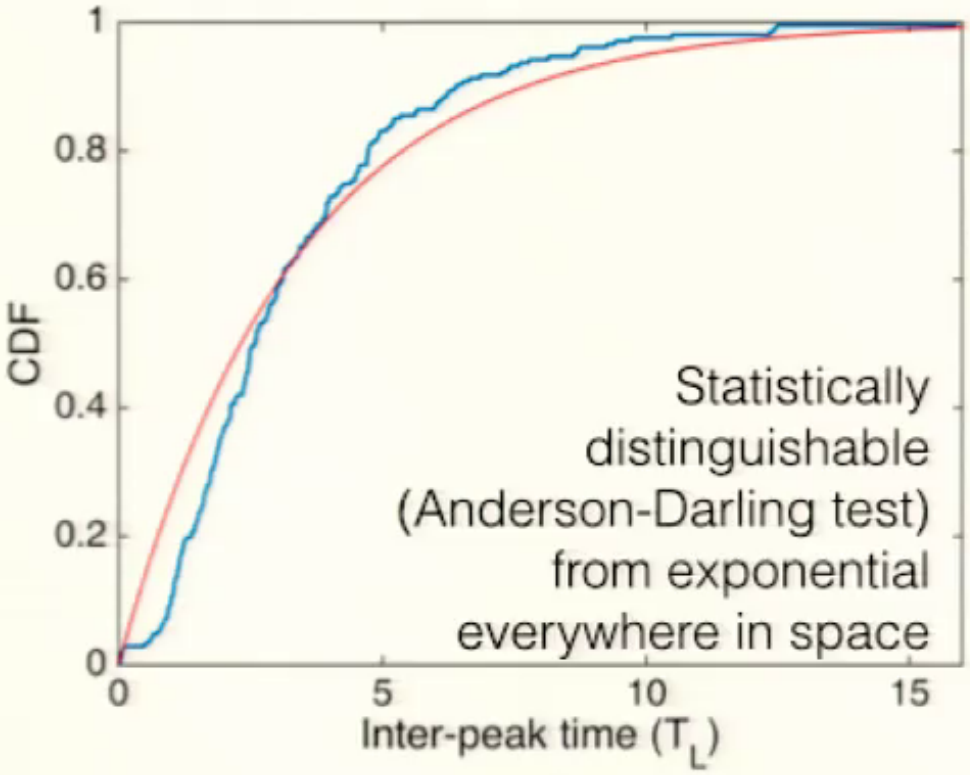
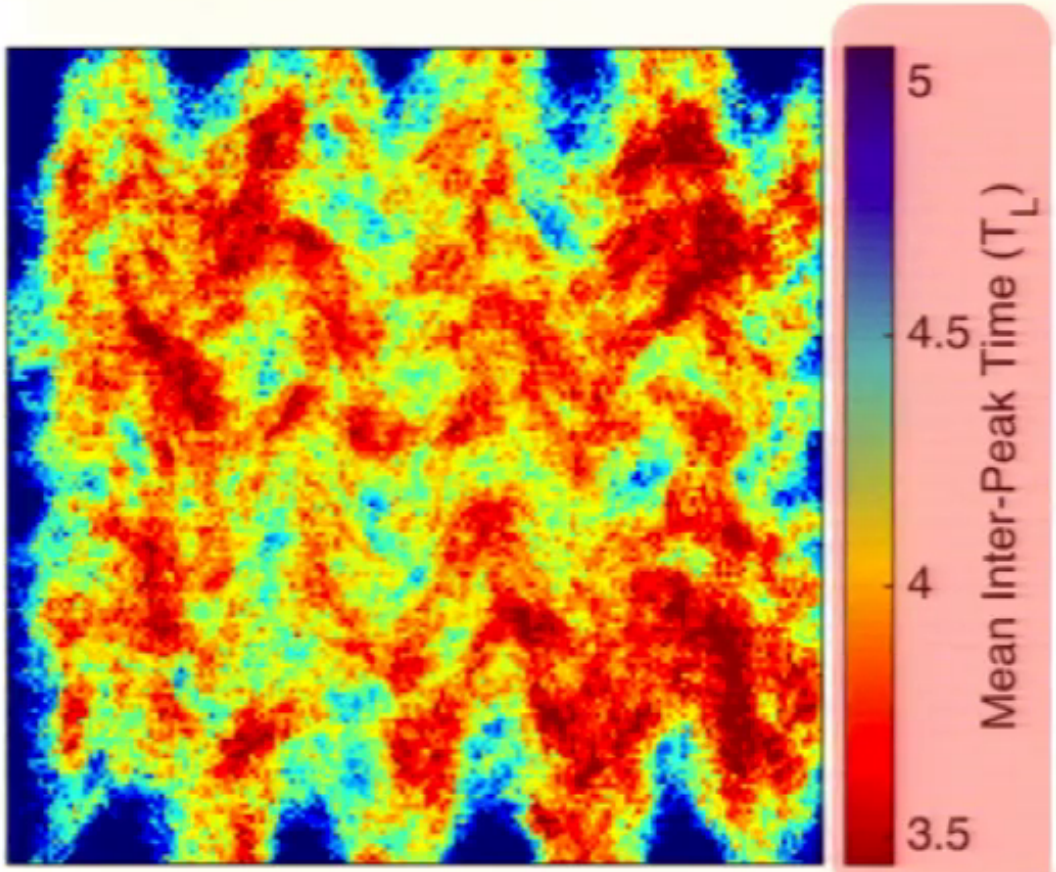


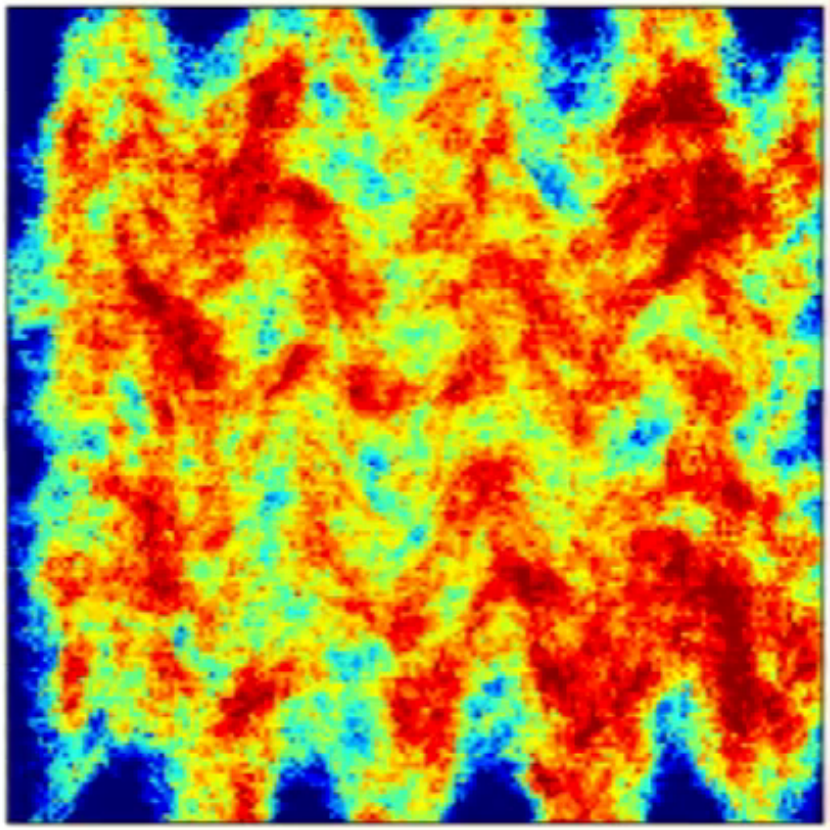
Ridge wait times are not random in space



Inter-peak wait time

Much faster!

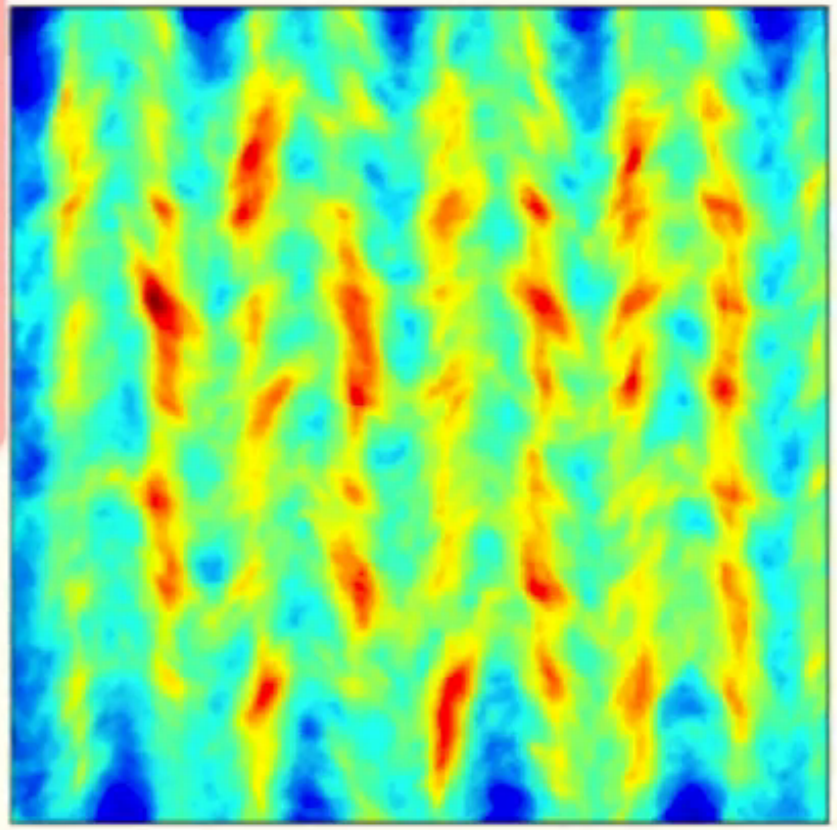
Ridge wait times are not random in space

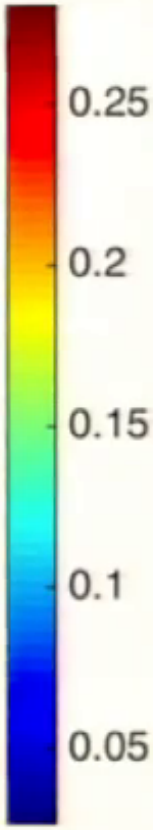


Inter-peak wait time

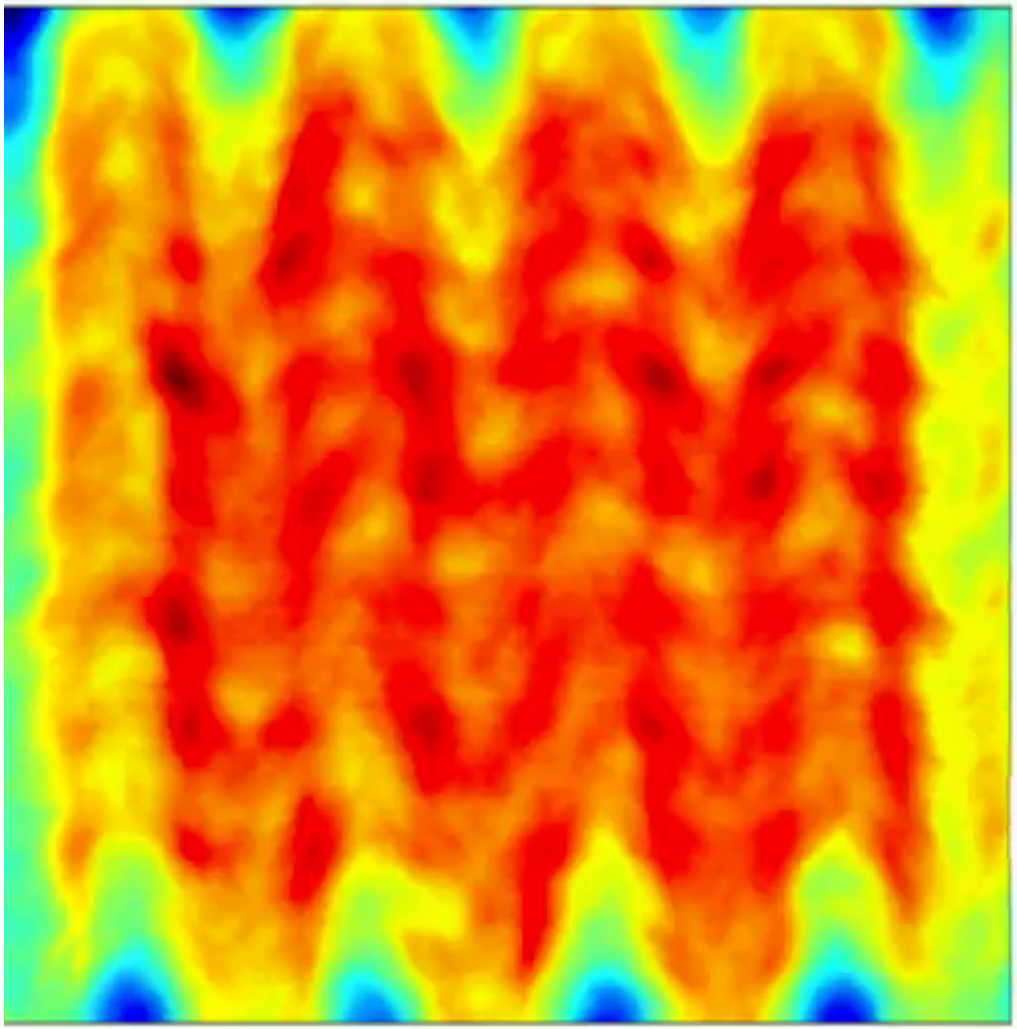
Much faster!

Likelihood of finding an FTLE ridge

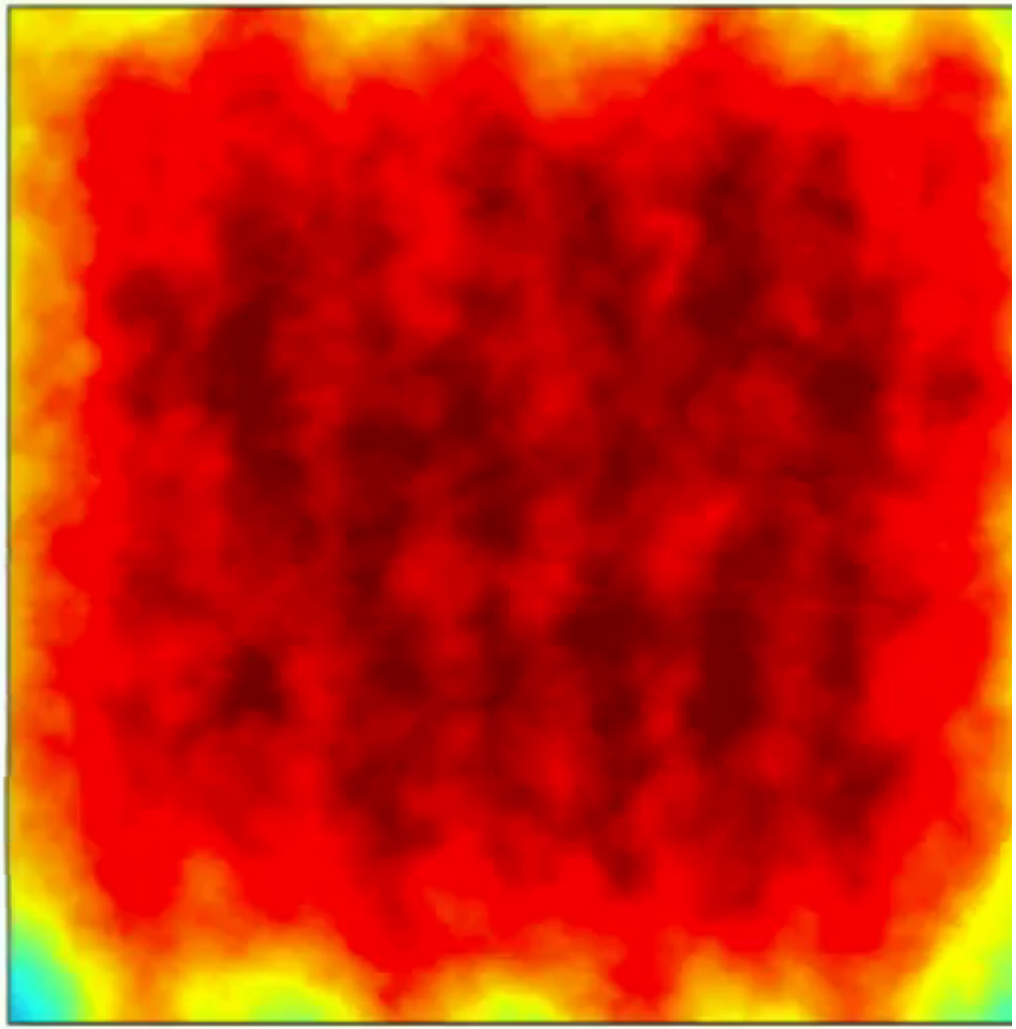




Integration direction?

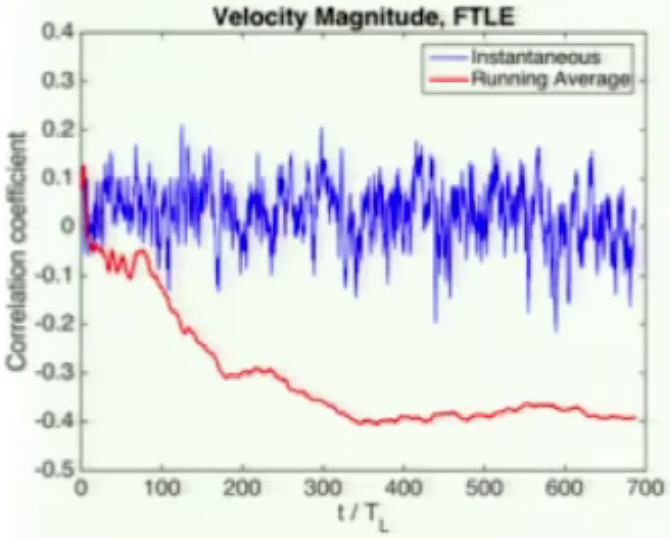


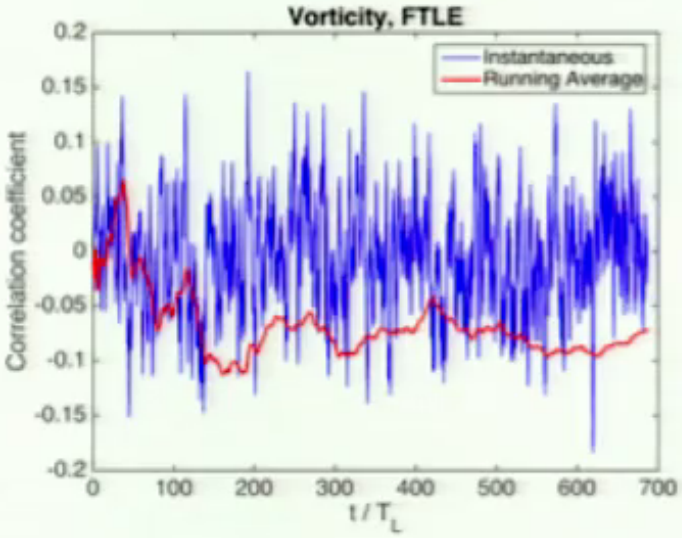
Forward time

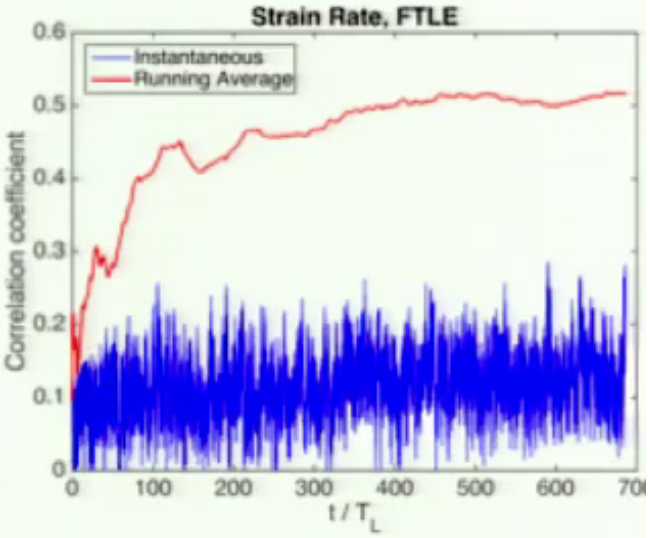


Backward time

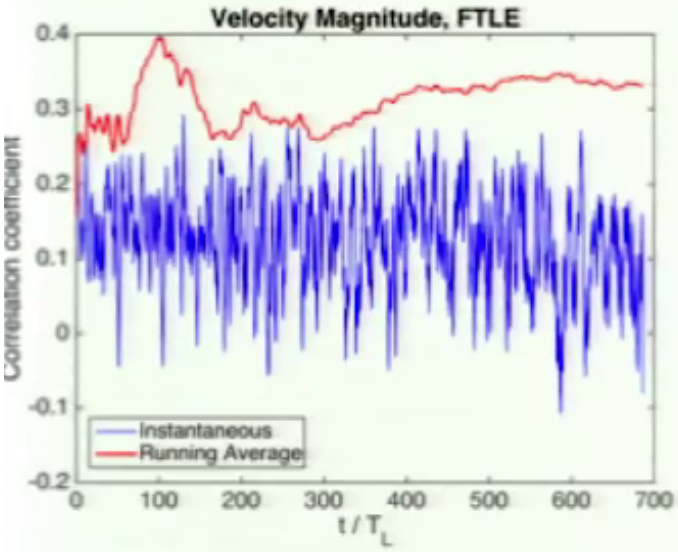
Forward time

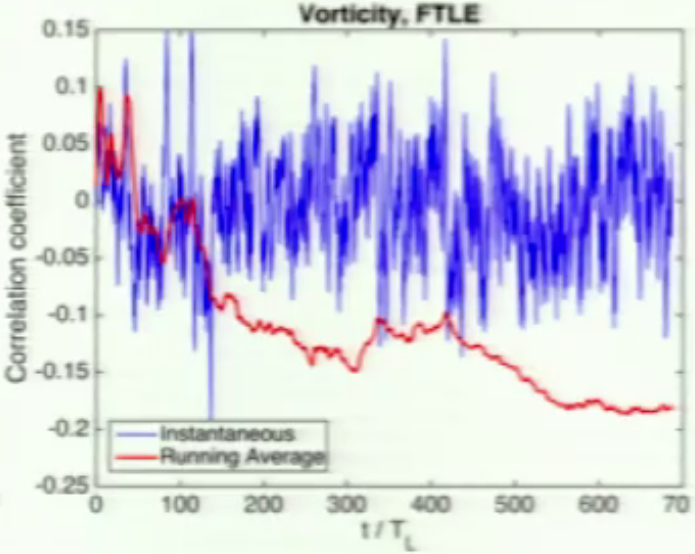


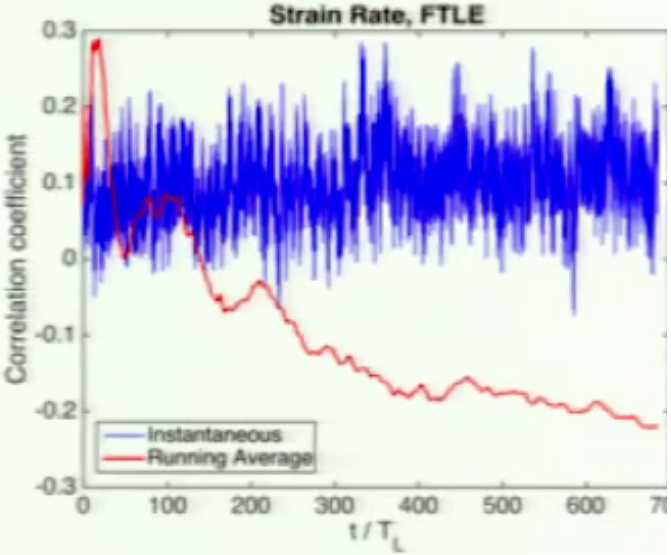




Backward time







Summary (so far)

LCSs are not random in space or time

Location is correlated with mean strain, anti-correlated with mean speed
(slow time scales)

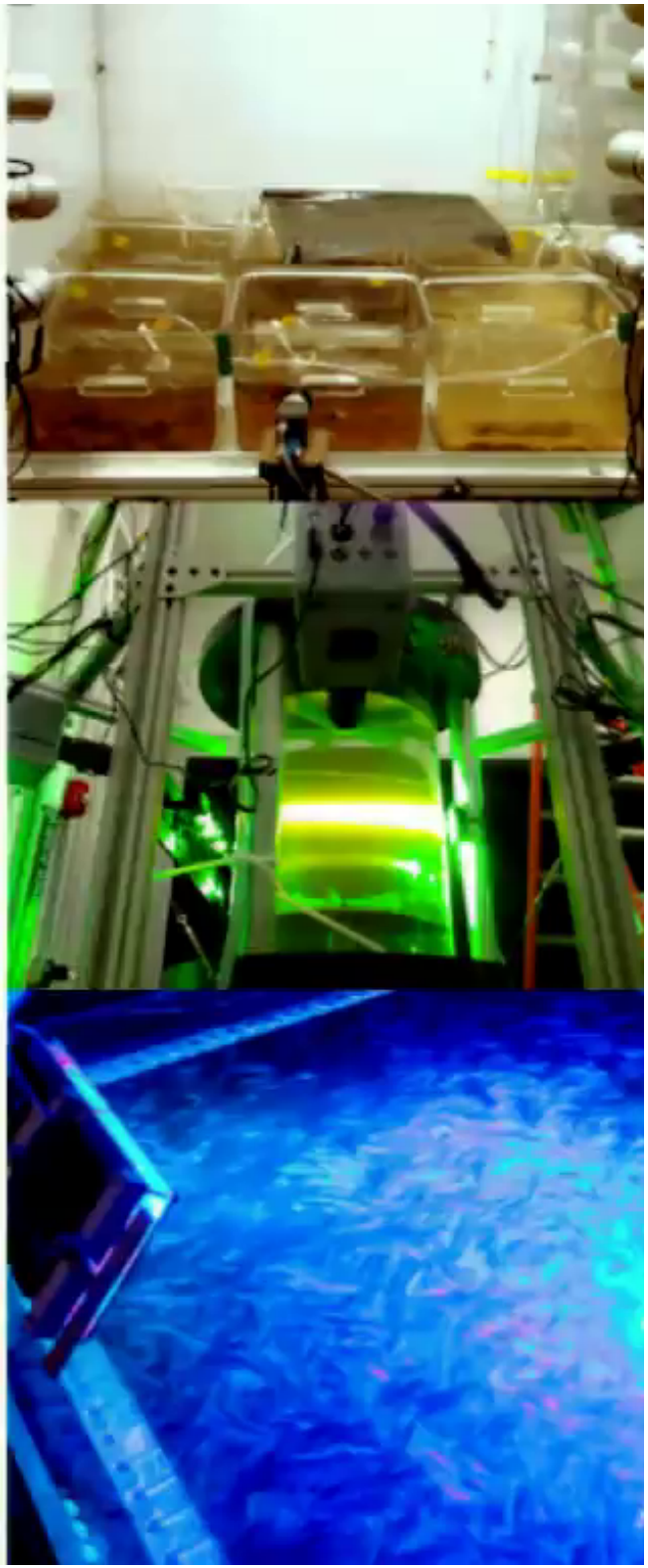
Wait times are shorter in same places
(faster time scales)

Future Directions:

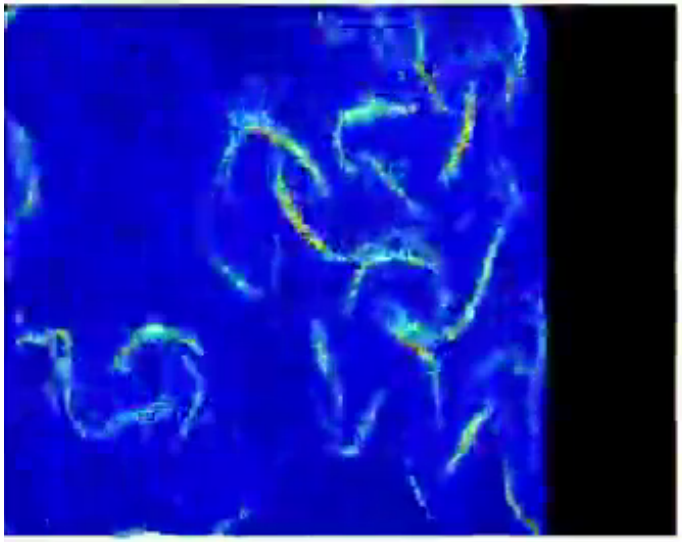
Temporally modulated forcing

Effect of lateral boundaries

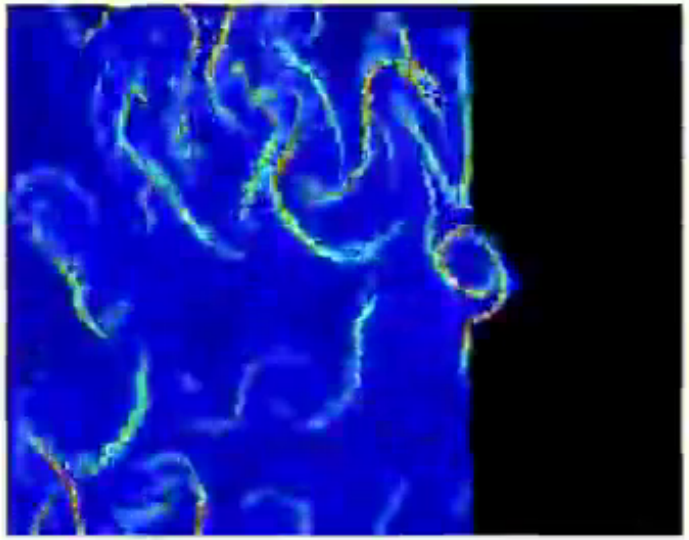
http://leviathan.eng.yale.edu



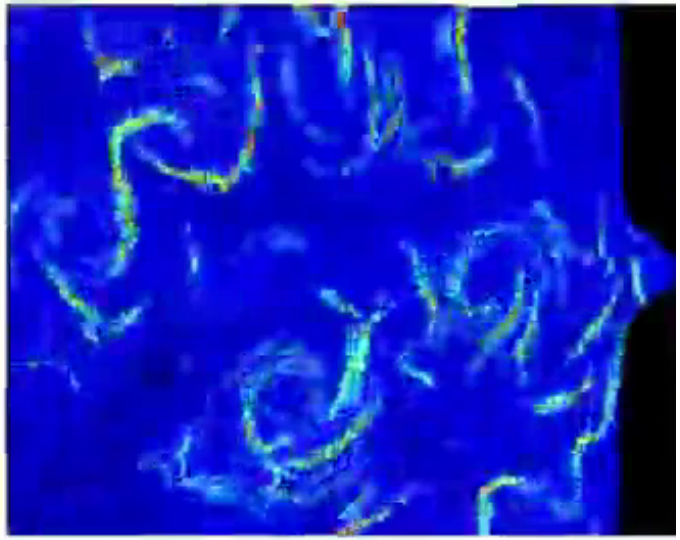
Future work: “coastlines”



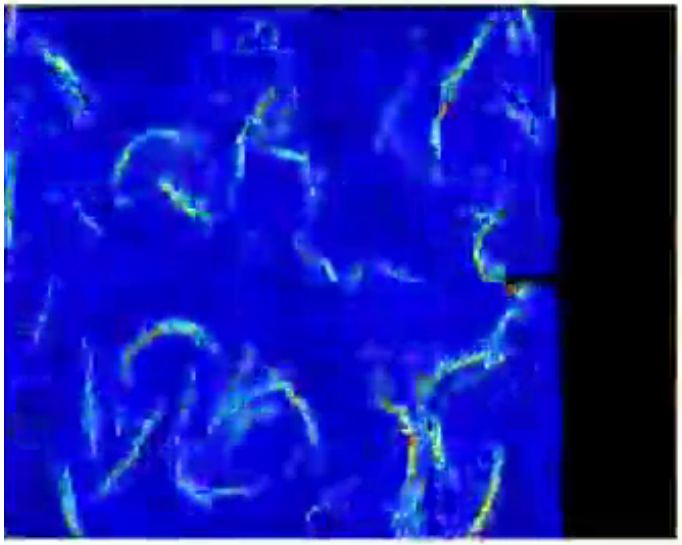
Featureless



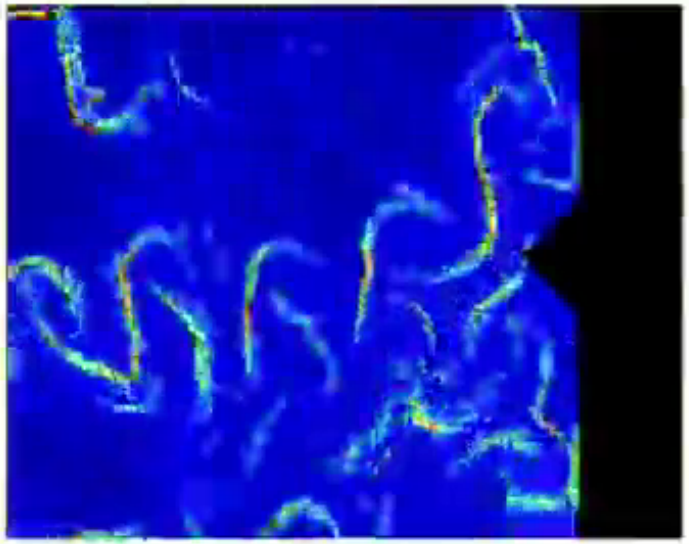
Triangular Bay



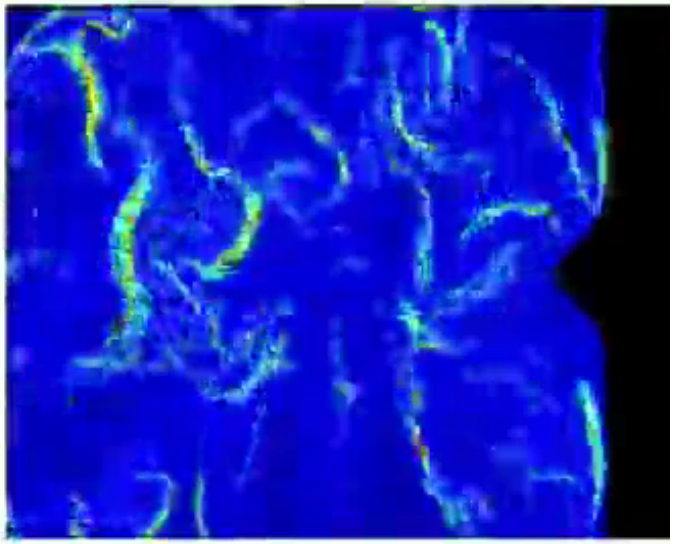
Gaussian Bay



Spike



Triangular Headland



Gaussian Headland