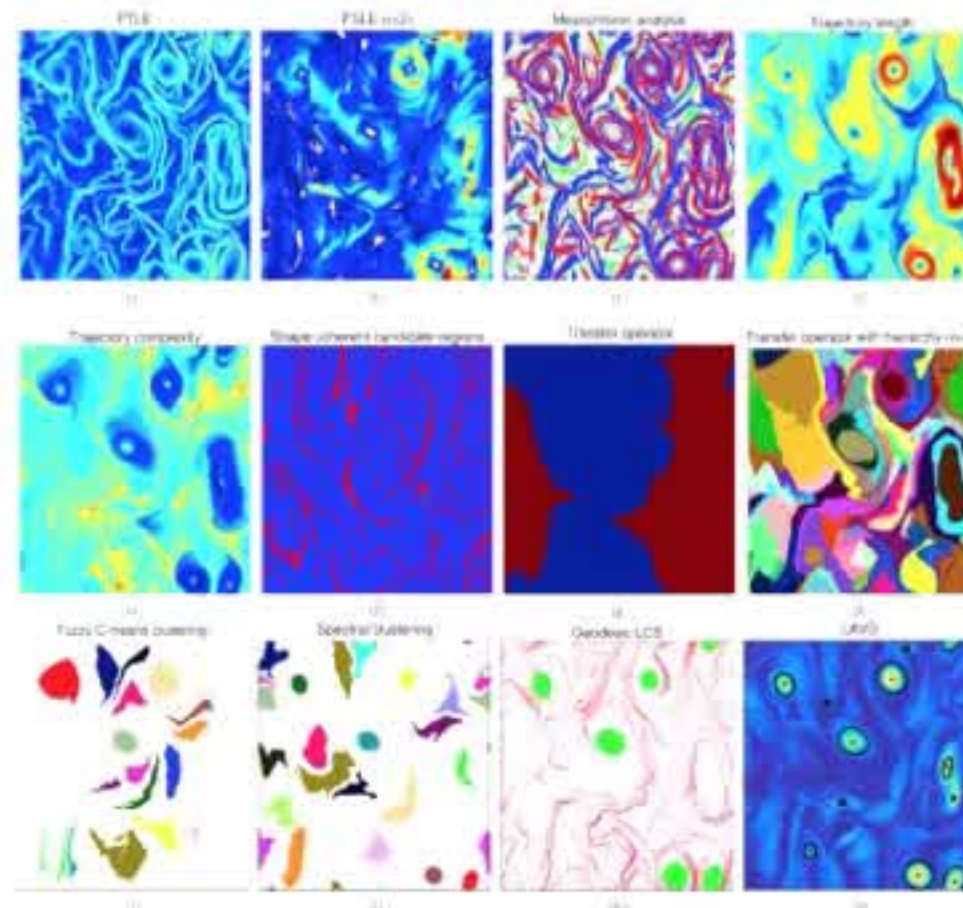


A (Critical) Comparison of Lagrangian Methods for Coherent Structure Detection

Alireza Hadjighasem (MIT),

Mohammad Farazmand (MIT), Daniel Blazevski (Insight Data Sci.),

Gary Froyland (UNSW) and George Haller (ETH)



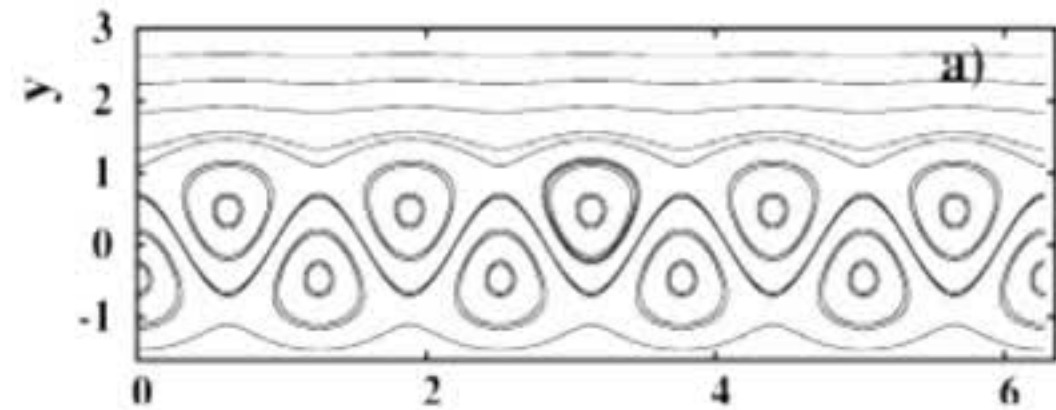
Three benchmark problems

1. Quasi-periodic Bickley jet [Beron-Vera et al. (2010)]

$$\psi(x, y, t) = \psi_0(y) + \psi_1(x, y, t),$$

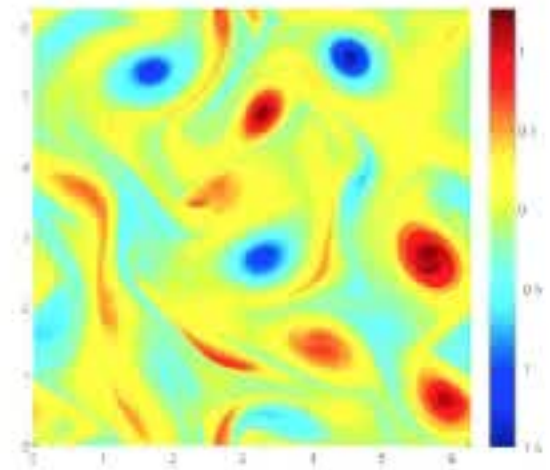
$$\psi_0(y) = -UL \tanh\left(\frac{y}{L}\right),$$

$$\psi_1(x, y, t) = UL \operatorname{sech}^2\left(\frac{y}{L}\right) \operatorname{Re}\left[\sum_{n=1}^3 f_n(t) \exp(ik_n x)\right].$$



2. 2D turbulence simulation on periodic domain (M. Farazmand)

$$\partial_t v + v \cdot \nabla v = -\nabla p + \nu \Delta v + f, \quad \nabla \cdot v = 0$$



Two main types of methods

1. Diagnostic methods

- visually inspect scalar field derived from intuition
- generally no precise definition of coherence

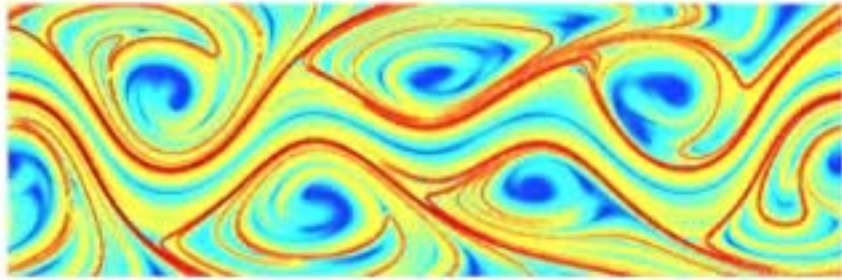
2. Mathematical methods

- solve mathematically formulated coherence principle for a precisely defined coherent structure

Diagnostic methods

Results for quasiperiodic Bickley jet

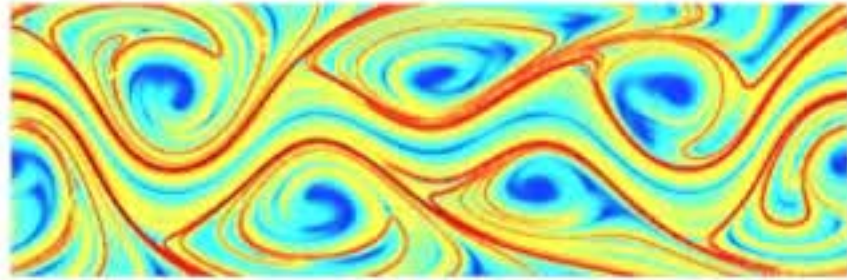
FTLE



(a)

Results for quasiperiodic Bickley jet

FTLE



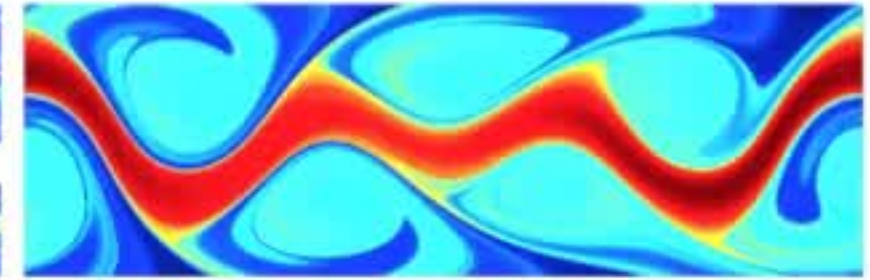
(a)

FSLE ($r=5$)

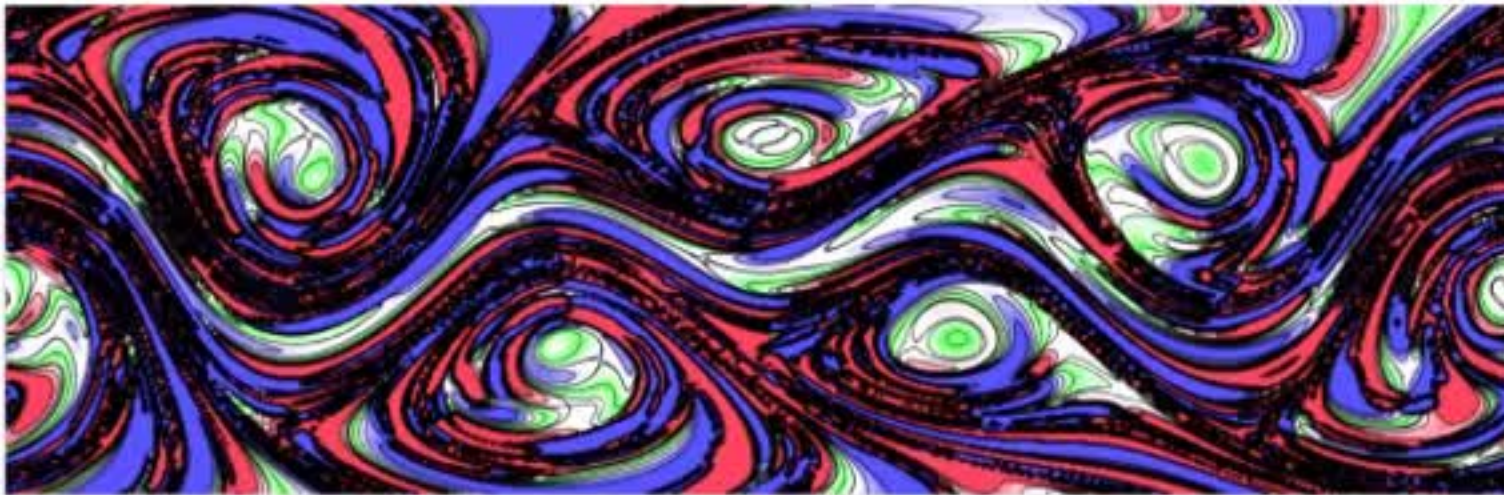


(b)

M function

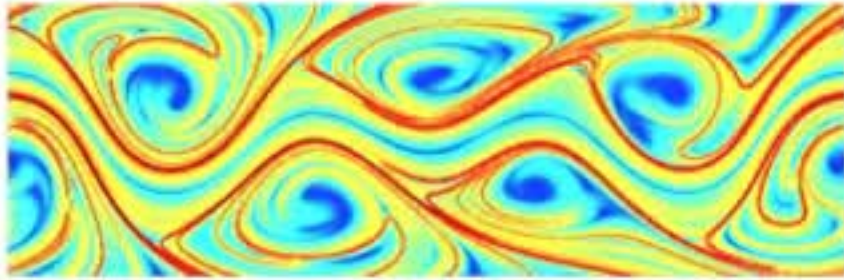


(c)



Results for quasiperiodic Bickley jet

FTLE



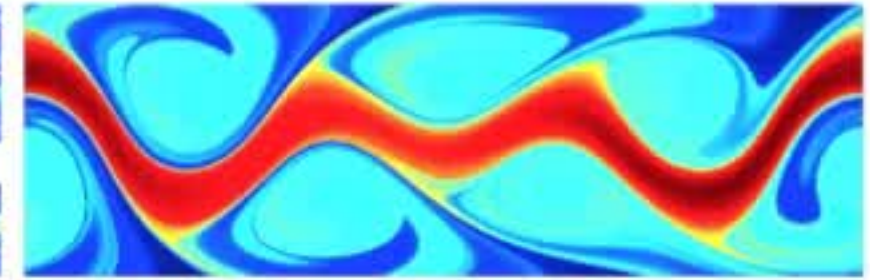
(a)

FSLE ($r=5$)



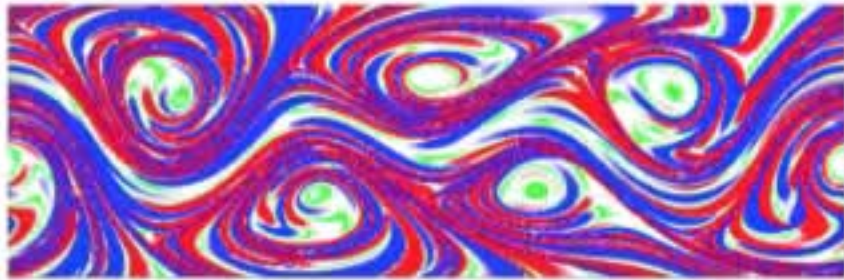
(b)

M function



(c)

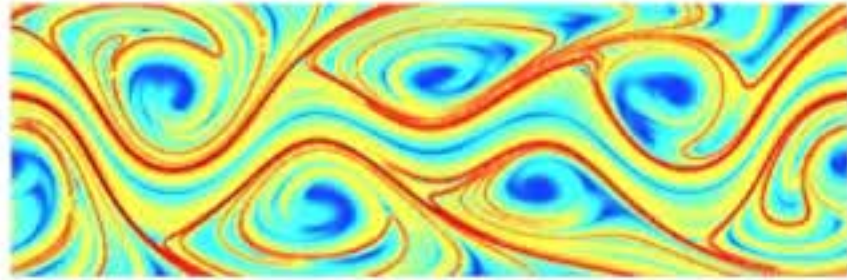
Mesochronic analysis



(d)

Results for quasiperiodic Bickley jet

FTLE



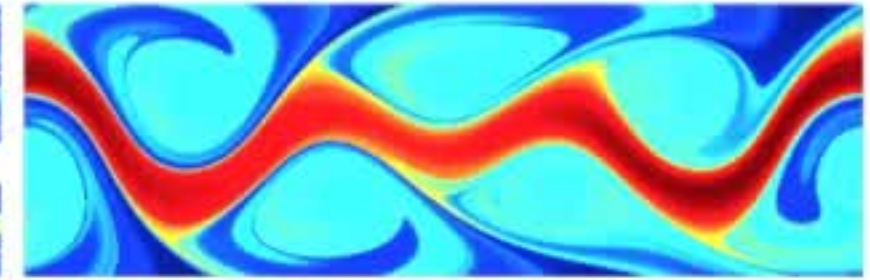
(a)

FSLE ($r=5$)



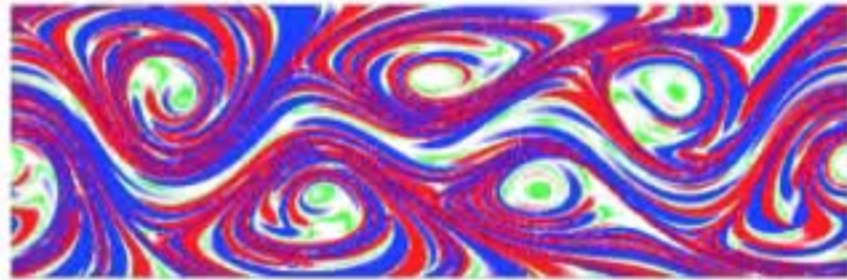
(b)

M function



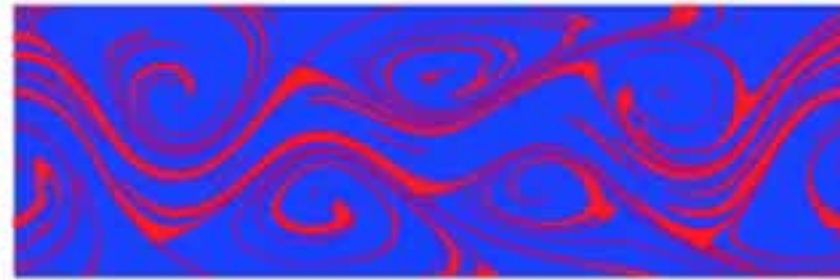
(c)

Mesochronic analysis



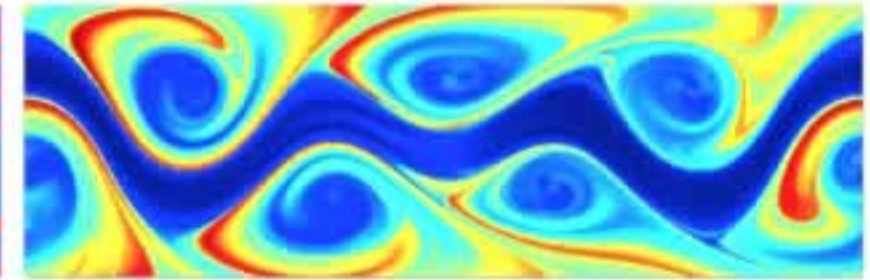
(d)

Shape coherent candidate regions



(e)

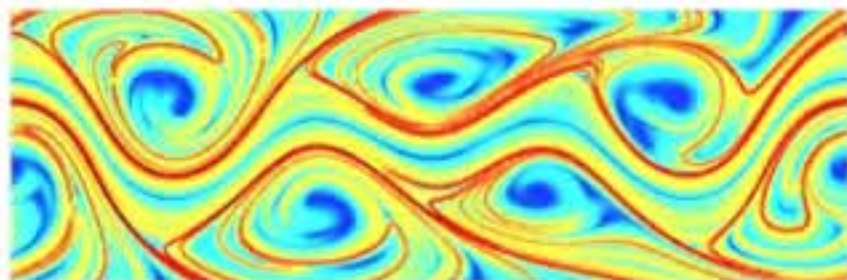
Complexity Method



(f)

Results for quasiperiodic Bickley jet

FTLE



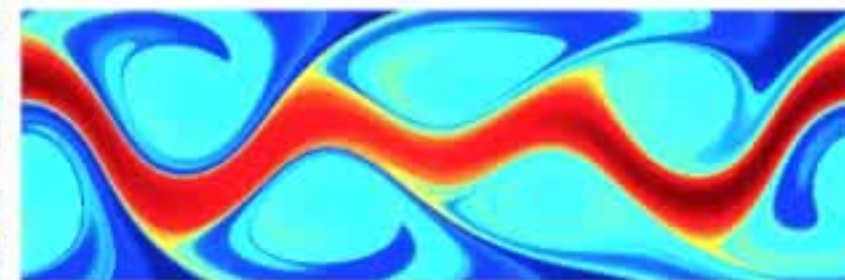
(a)

FSLE ($r=5$)



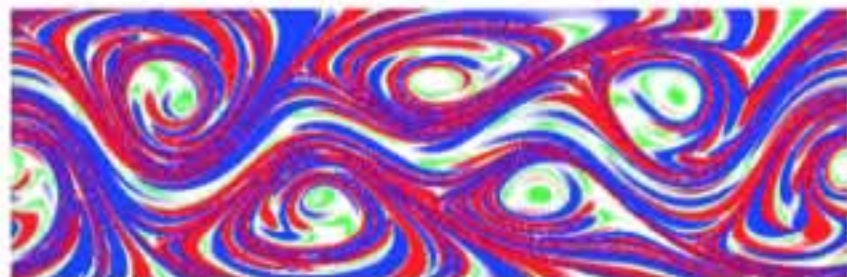
(b)

M function



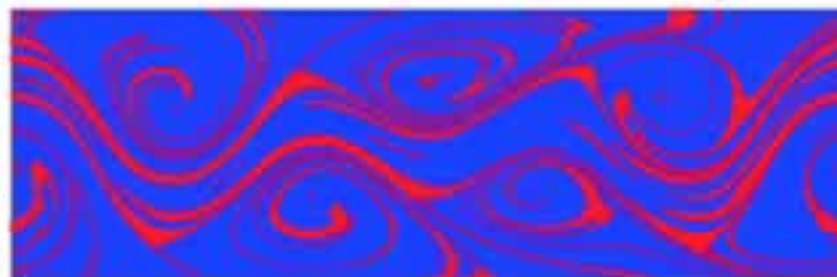
(c)

Mesochronic analysis



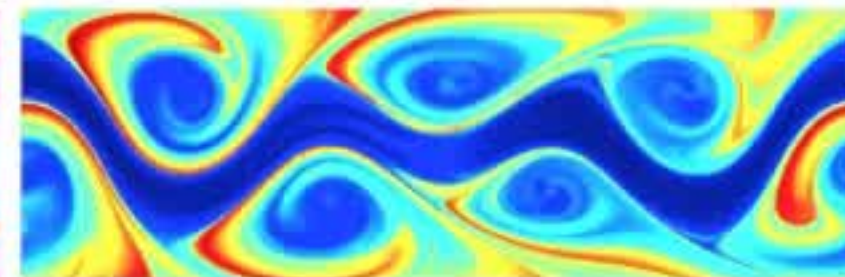
(d)

Shape coherent candidate regions



(e)

Complexity Method



(f)

Transfer operator



(g)

Transfer operator with hierarchy ($n=5$)



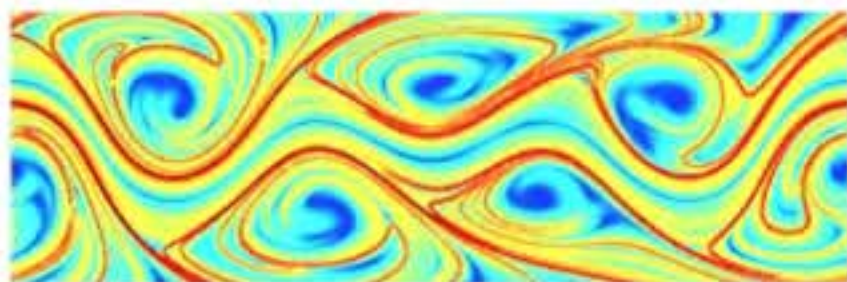
(h)

Fuzzy C-means clustering



Results for quasiperiodic Bickley jet

FTLE



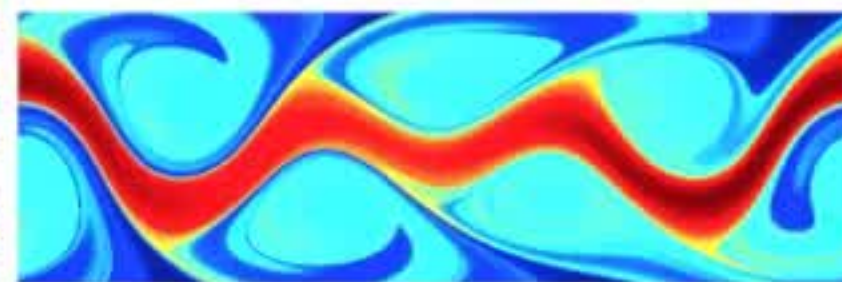
(a)

FSLE ($r=5$)



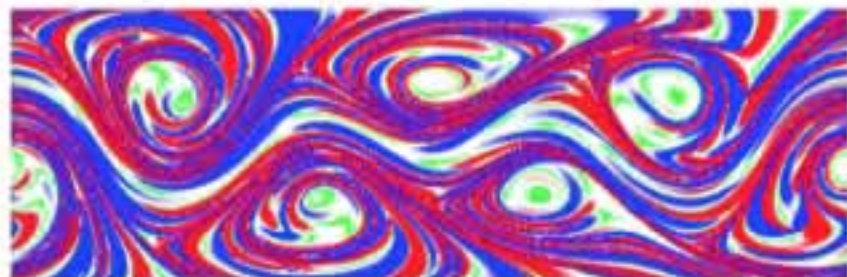
(b)

M function



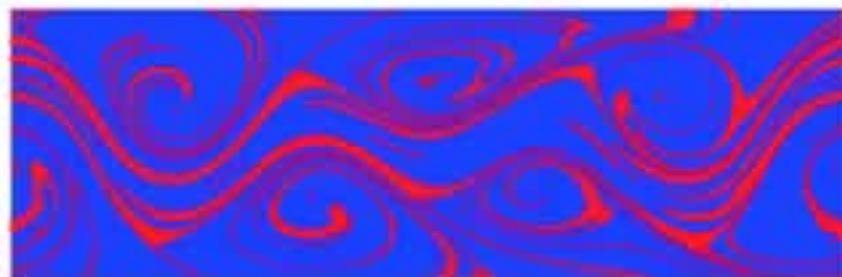
(c)

Mesochronic analysis



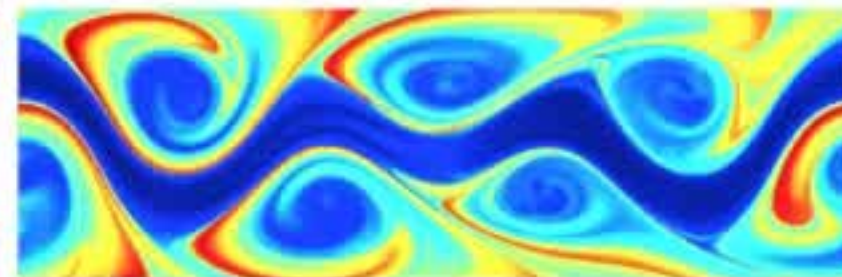
(d)

Shape coherent candidate regions



(e)

Complexity Method



(f)

Transfer operator



(g)

Transfer operator with hierarchy ($n=5$)



(h)

Fuzzy C-means clustering



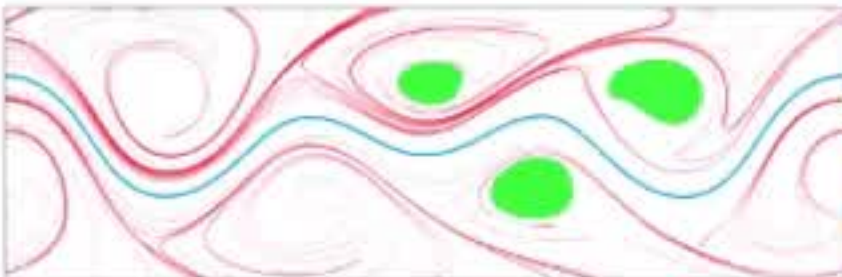
(i)

Spectral clustering



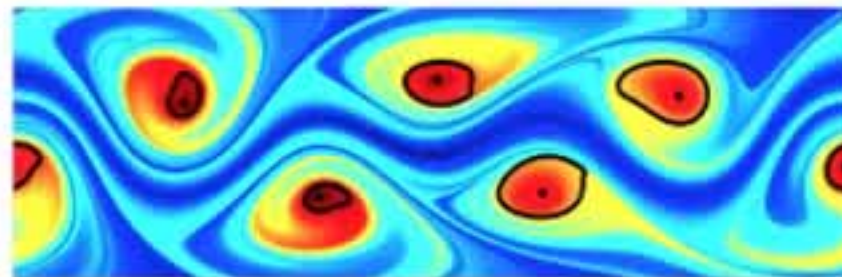
(j)

Geodesic LCS



(k)

LAVD

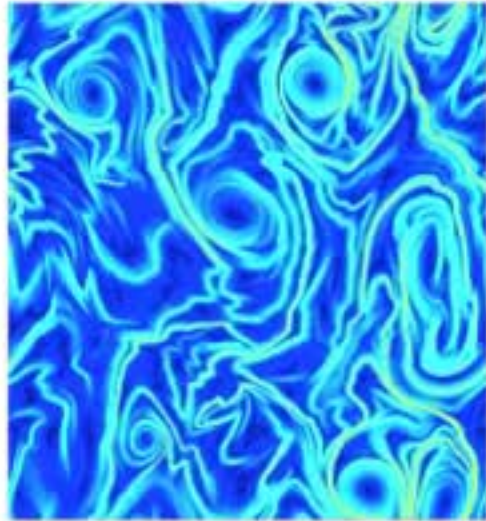


(l)

Results for 2D turbulence

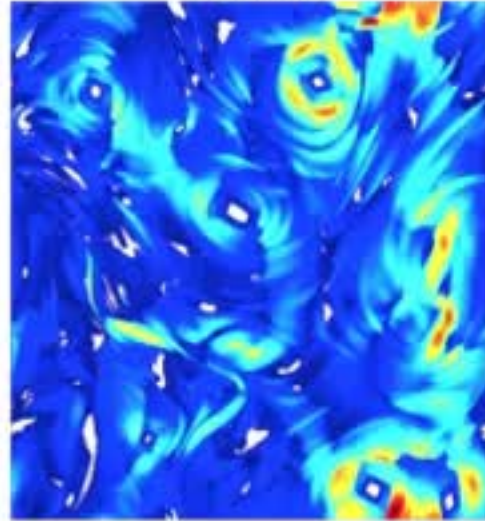
Results for 2D turbulence

FTLE



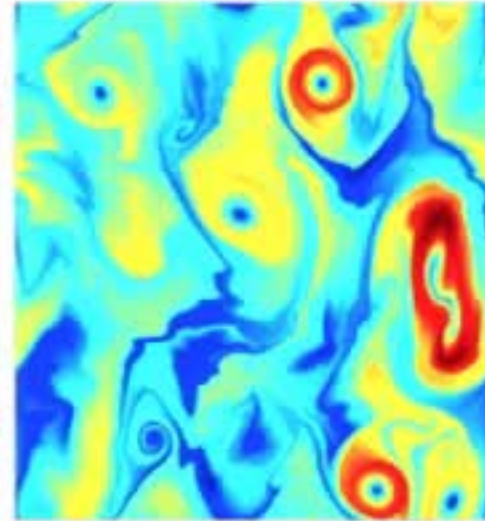
(a)

FSLE (r=2)



(b)

M function



(c)

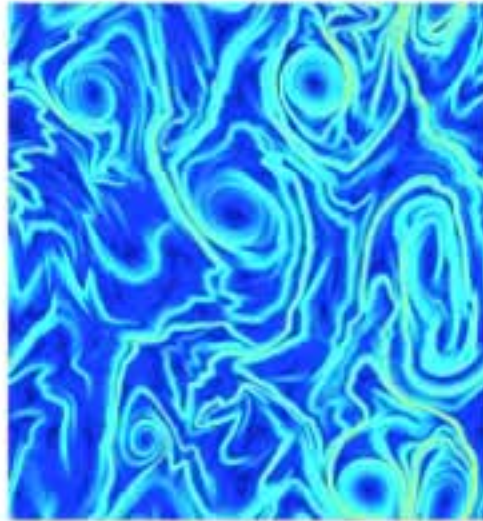
Mesochronic analysis



(d)

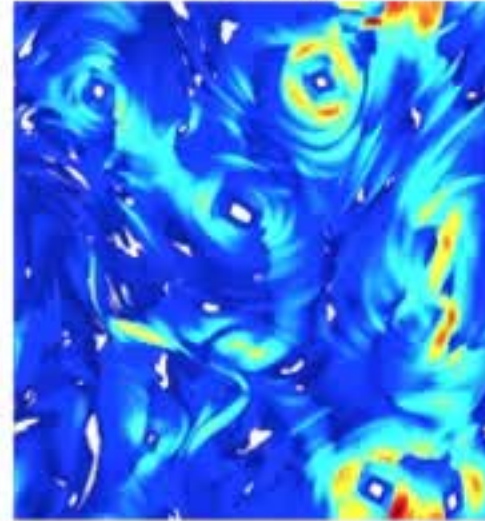
Results for 2D turbulence

FTLE



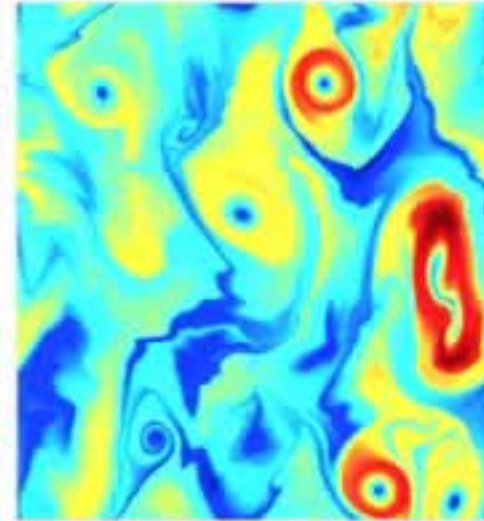
(a)

FSLE ($r=2$)



(b)

M function



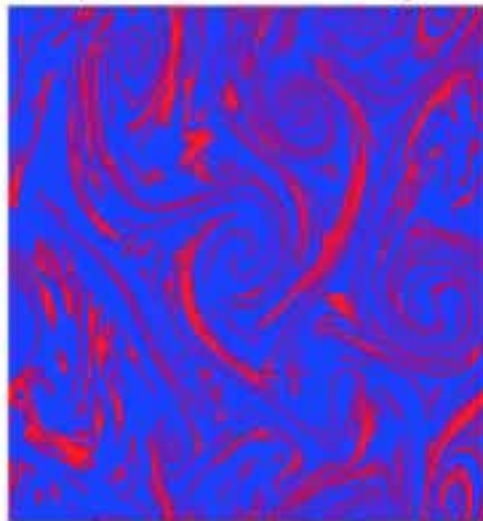
(c)

Mesochronic analysis



(d)

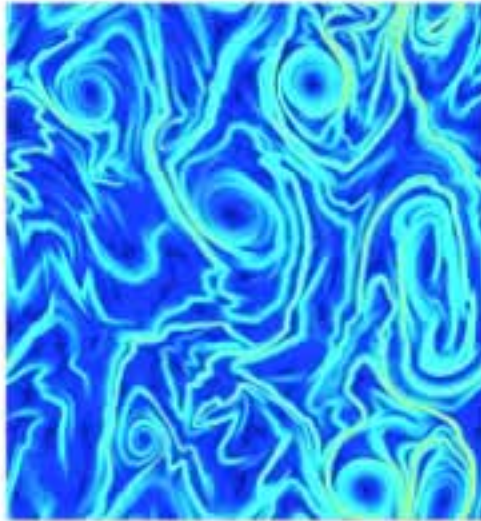
Shape coherent candidate regions



(e)

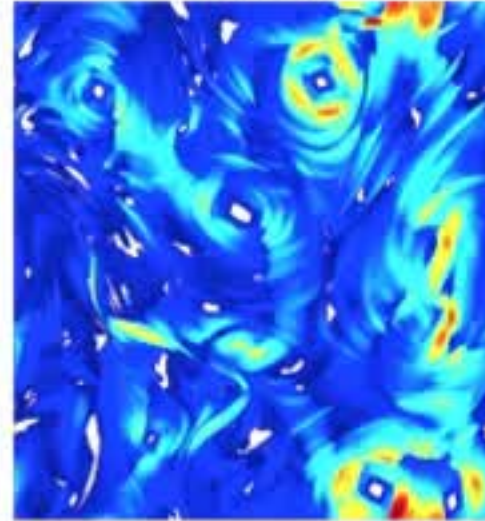
Results for 2D turbulence

FTLE



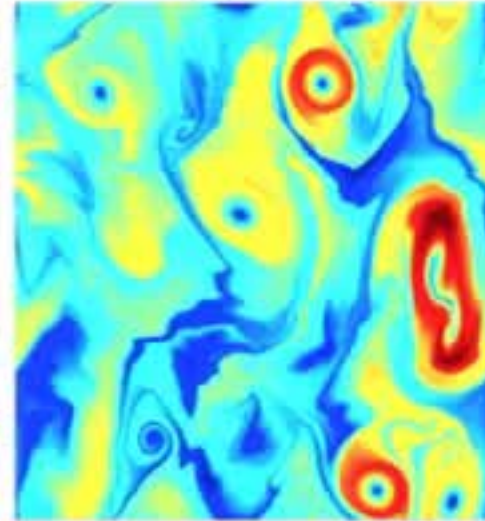
(a)

FSLE (r=2)



(b)

M function



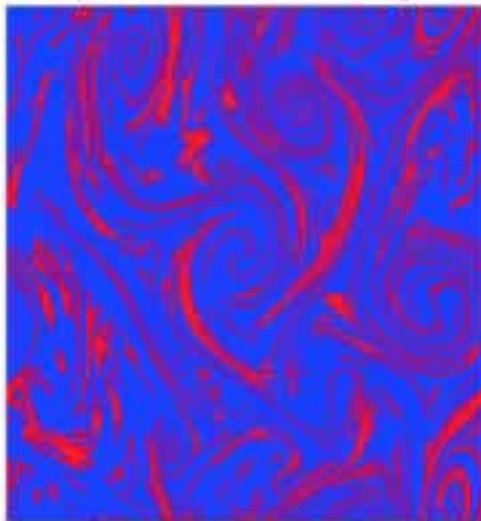
(c)

Mesochronic analysis



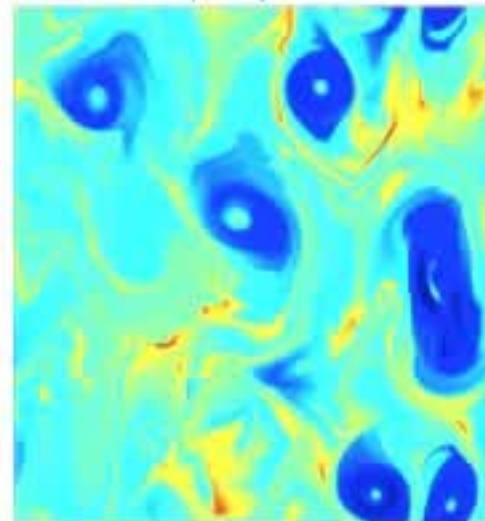
(d)

Shape coherent candidate regions



(e)

Complexity Method



(f)

Transfer operator



(g)

Transfer operator with hierarchy (n=8)

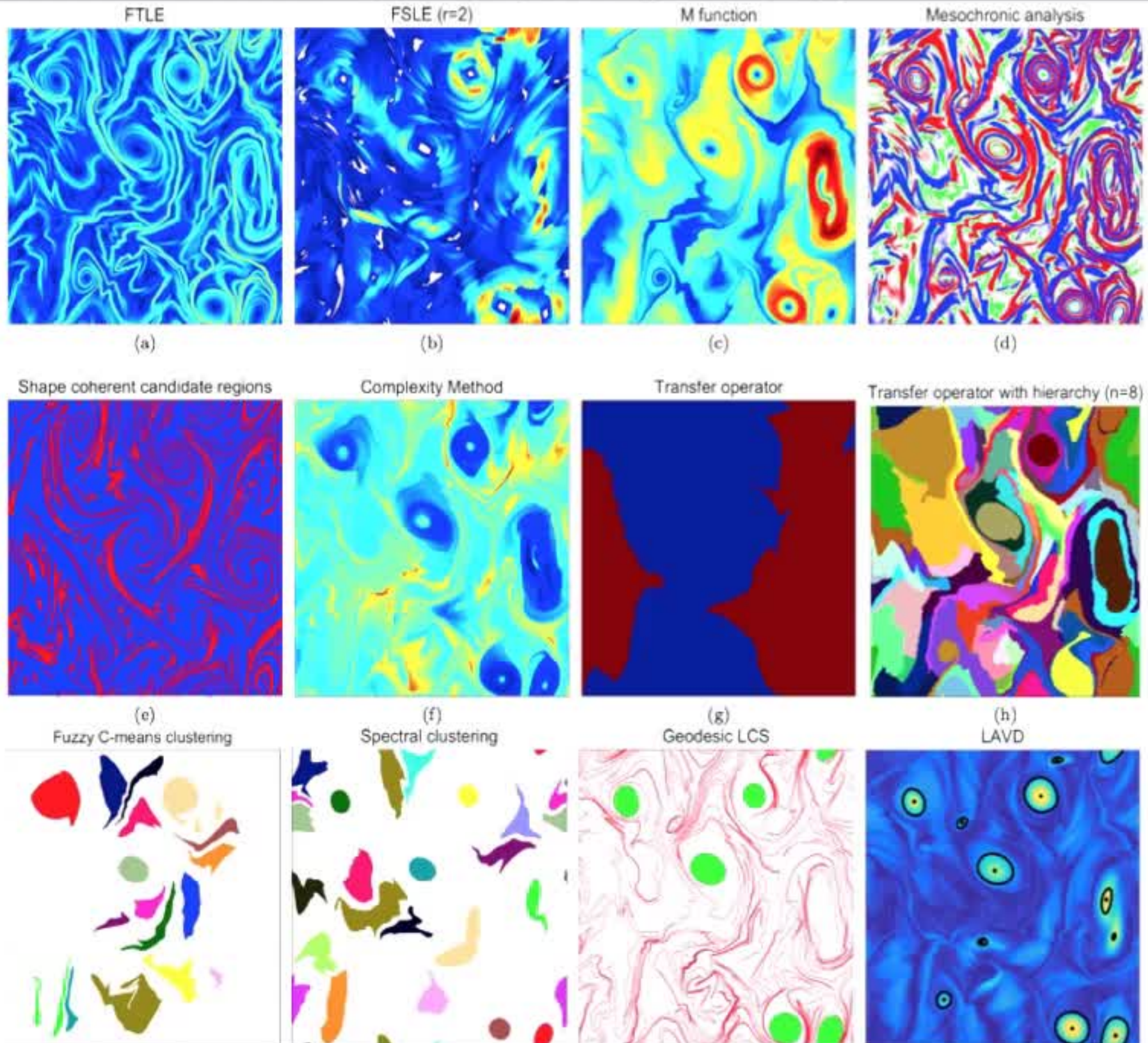


(h)

Fuzzy C-means clustering



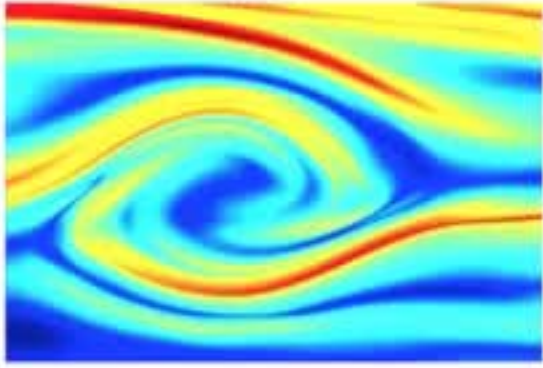
Results for 2D turbulence



Results for Jupiter velocity data

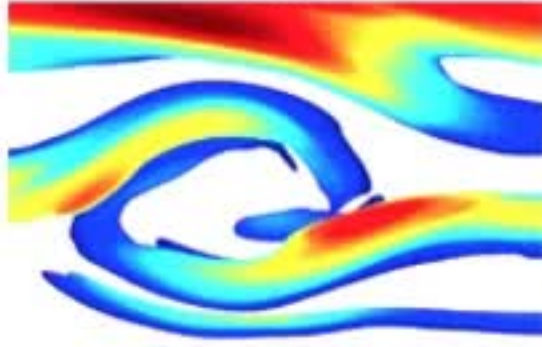
Results for Jupiter velocity data

FTLE



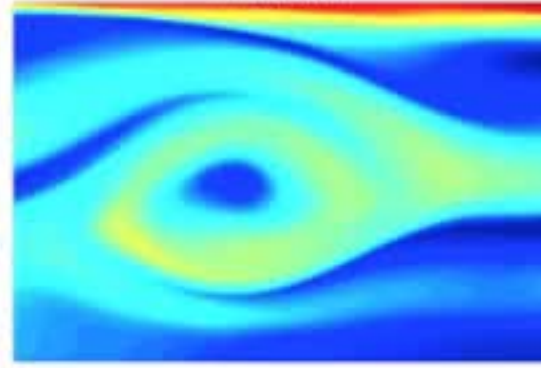
(a)

FSLE



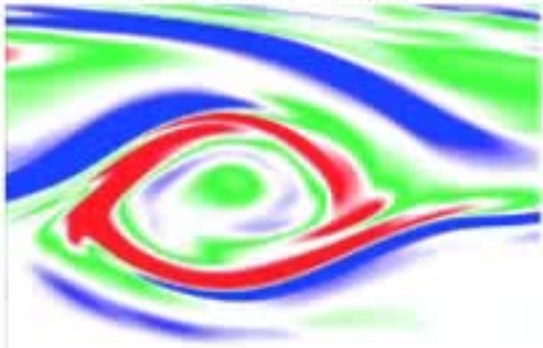
(b)

M function



(c)

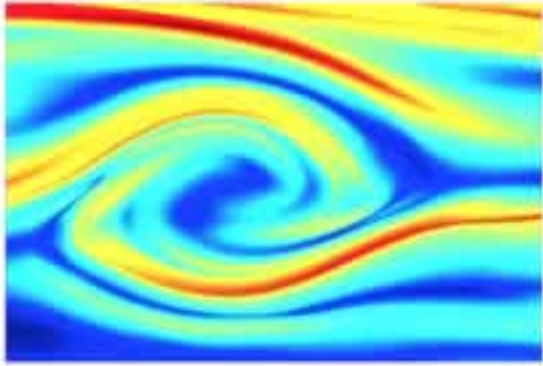
Mesochronic analysis



(d)

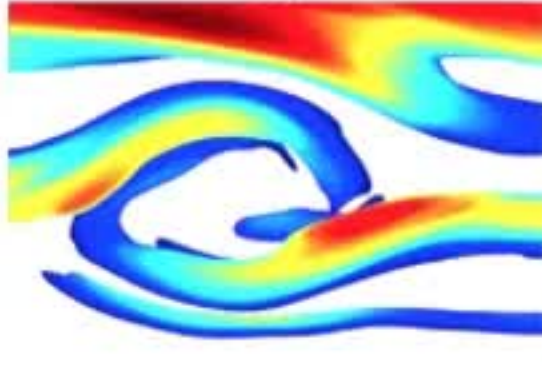
Results for Jupiter velocity data

FTLE



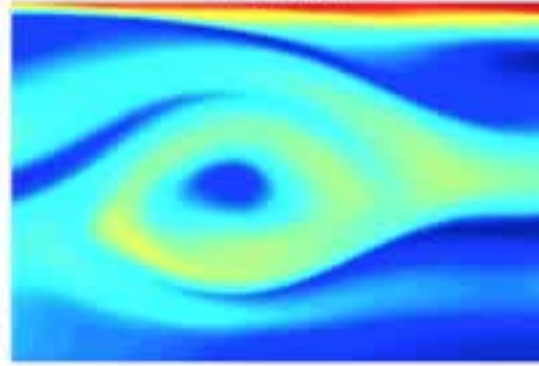
(a)

FSLE



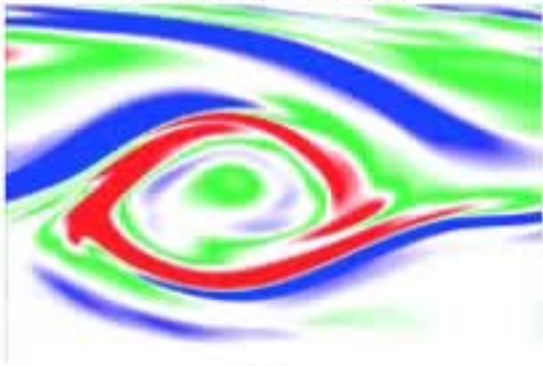
(b)

M function



(c)

Mesochronic analysis



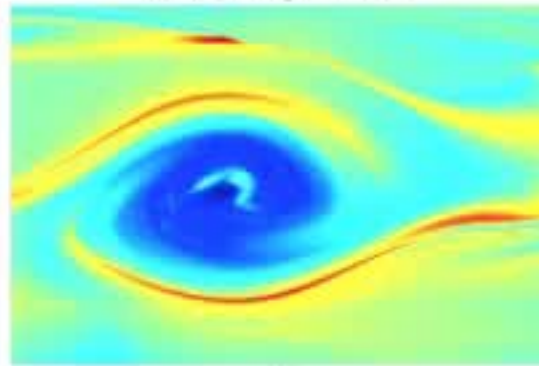
(d)

Shape coherent candidate regions



(e)

Complexity Method



(f)

Transfer operator



(g)

Transfer operator with hierarchy (n=3)



(h)

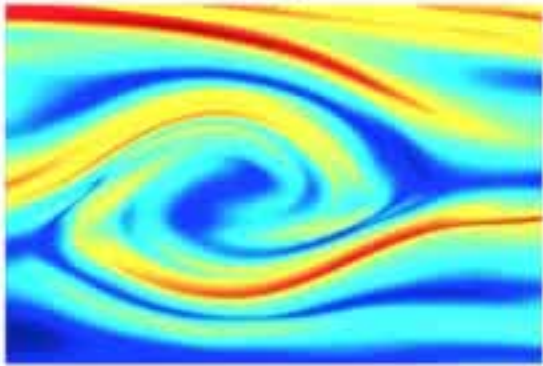
Fuzzy C-means clustering



(i)

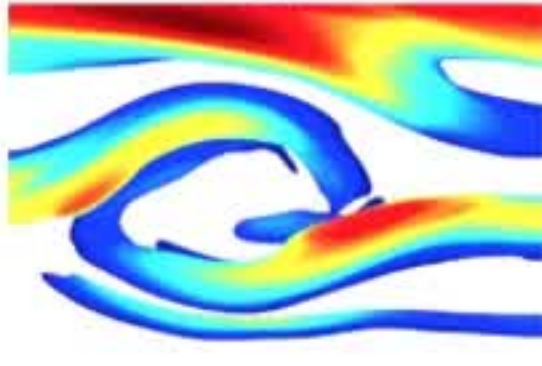
Results for Jupiter velocity data

FTLE



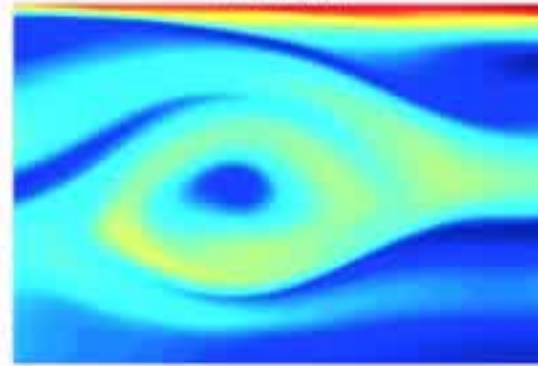
(a)

FSLE



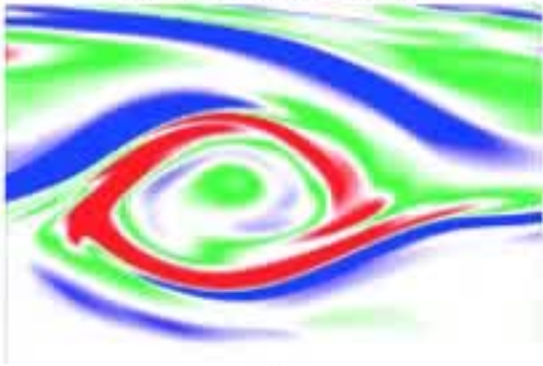
(b)

M function



(c)

Mesochronic analysis



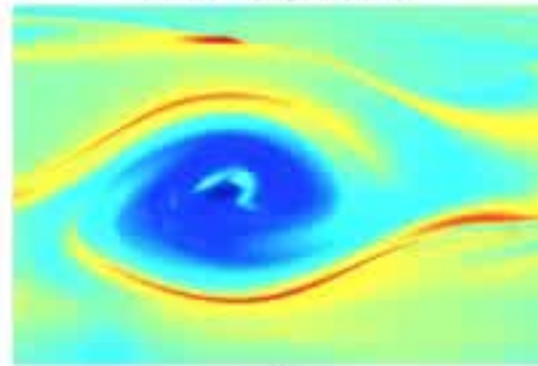
(d)

Shape coherent candidate regions



(e)

Complexity Method



(f)

Transfer operator



(g)

Transfer operator with hierarchy (n=3)



(h)

Fuzzy C-means clustering



(i)

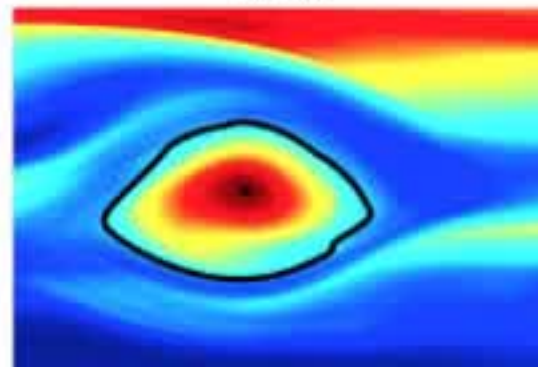
Spectral clustering



Geodesic LCS

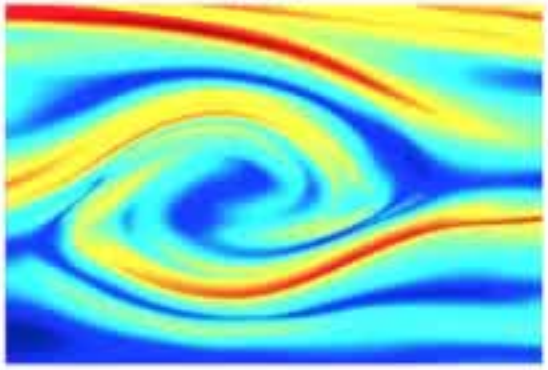


LAVD



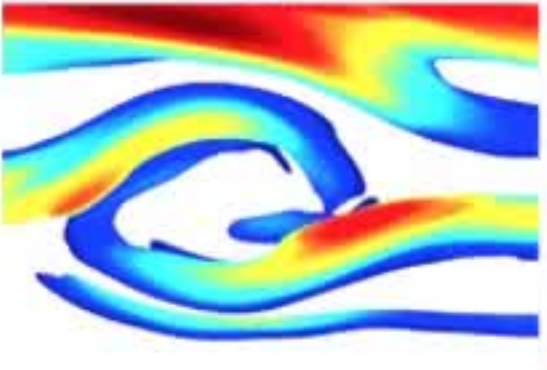
Results for Jupiter velocity data

FTLE



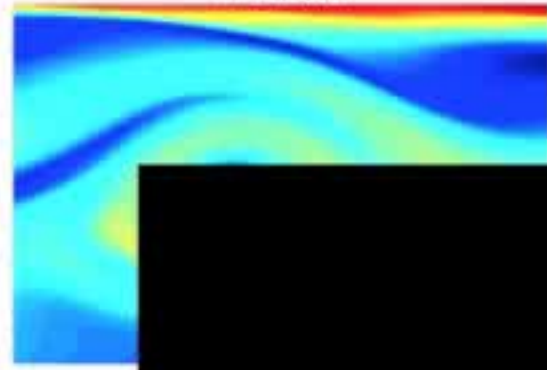
(a)

FSLE

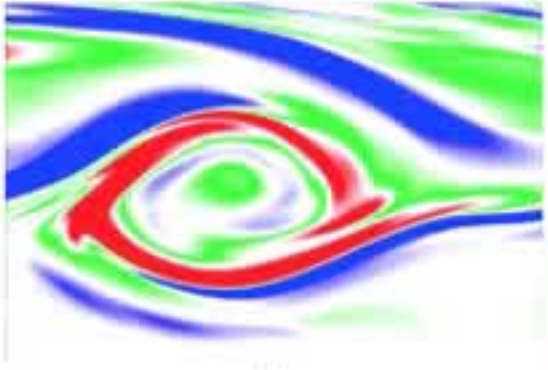


(b)

M function



Mesochronic analysis

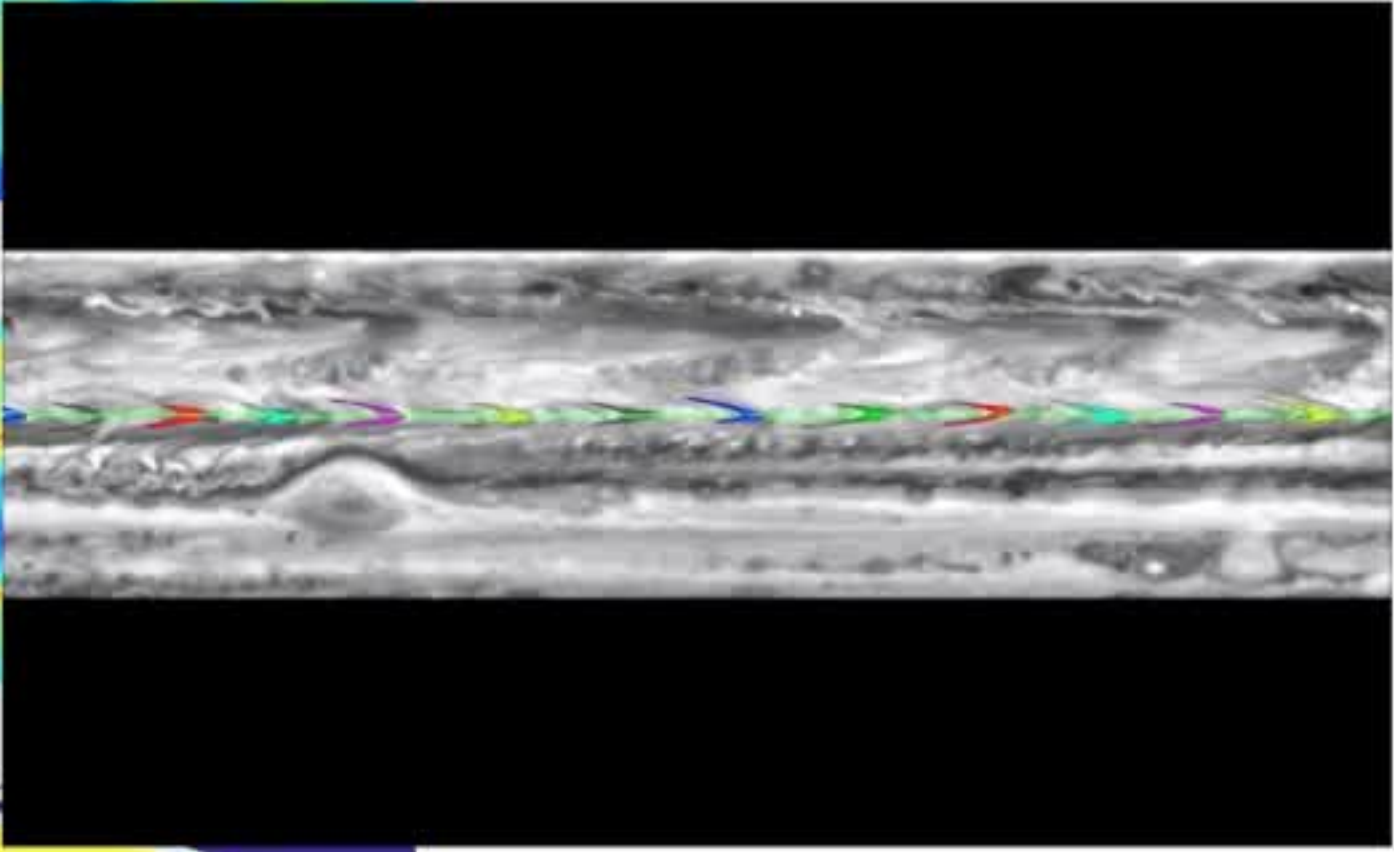
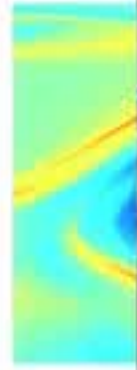


(d)

Shape coherent candidate regions



(e)



Transfer operator



(g)

Transfer operator with hierarchy (n=3)



(h)

Fu



(i)

Spectral clustering



Geodesic LCS



LAVD

