Semi-Implicit Convolutional Neural Networks

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Outline

1) **Introduction**
    1) High vs Low Dimensional Output
    2) Receptive Field
    3) Neural Network Stability
2) **IMEXnet**
    1) Formulation
    2) Implementation
3) **Numerical Experiments**
    1) Q-tips
    2) NYU Depth

Classification

Low Dimensional Output

$\rightarrow$

Class	Probability
Truck	0.01
Mouse	0.85
Dog	0.05
Cat	0.09

$$R^{h \mathrm{x} w \mathrm{x} d} \quad \rightarrow \quad R^k$$

where h, w, and d are the image dimensions, and k is the number of target classes.

Semantic Segmentation

High Dimensional Output

$\rightarrow$

$$R^{h\mathrm{x}w\mathrm{x}d} \rightarrow R^{h\mathrm{x}w\mathrm{x}k}$$

where h, w, and d are the image dimensions, and k is the number of target classes.

Semantic Segmentation

High Dimensional Output

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$$R^{h\text{x}w\text{x}d} \rightarrow R^{h\text{x}w\text{x}k}$$

where h, w, and d are the image dimensions, and k is the number of target classes.

$$k \geq d$$

Receptive Field

IMEXnet

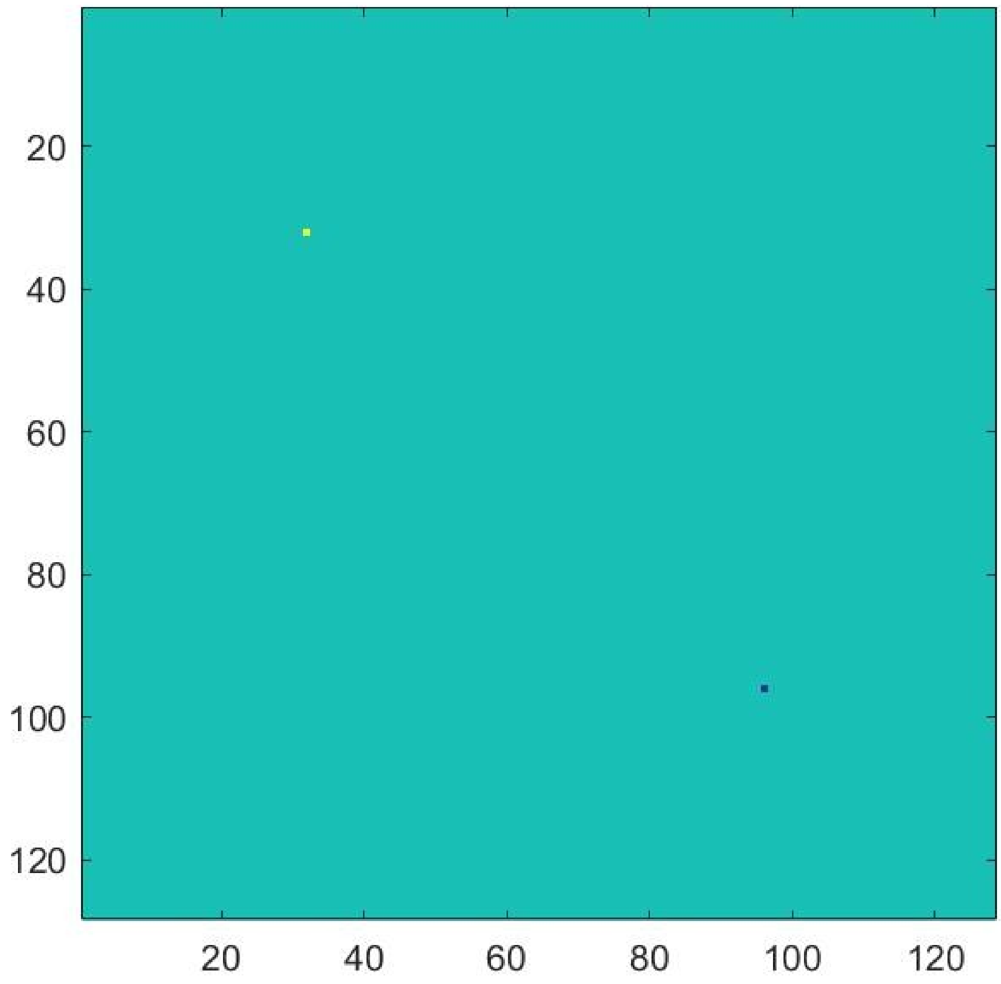
ResNet:

$$\mathbf{Y}^{n+1} = \mathbf{Y}^{n} + \mathbf{K}_2 \sigma(N_{\alpha,\beta}(\mathbf{K}_1 \mathbf{Y}^{n})) \quad \text{for} \quad n = 0, ..., N-1$$

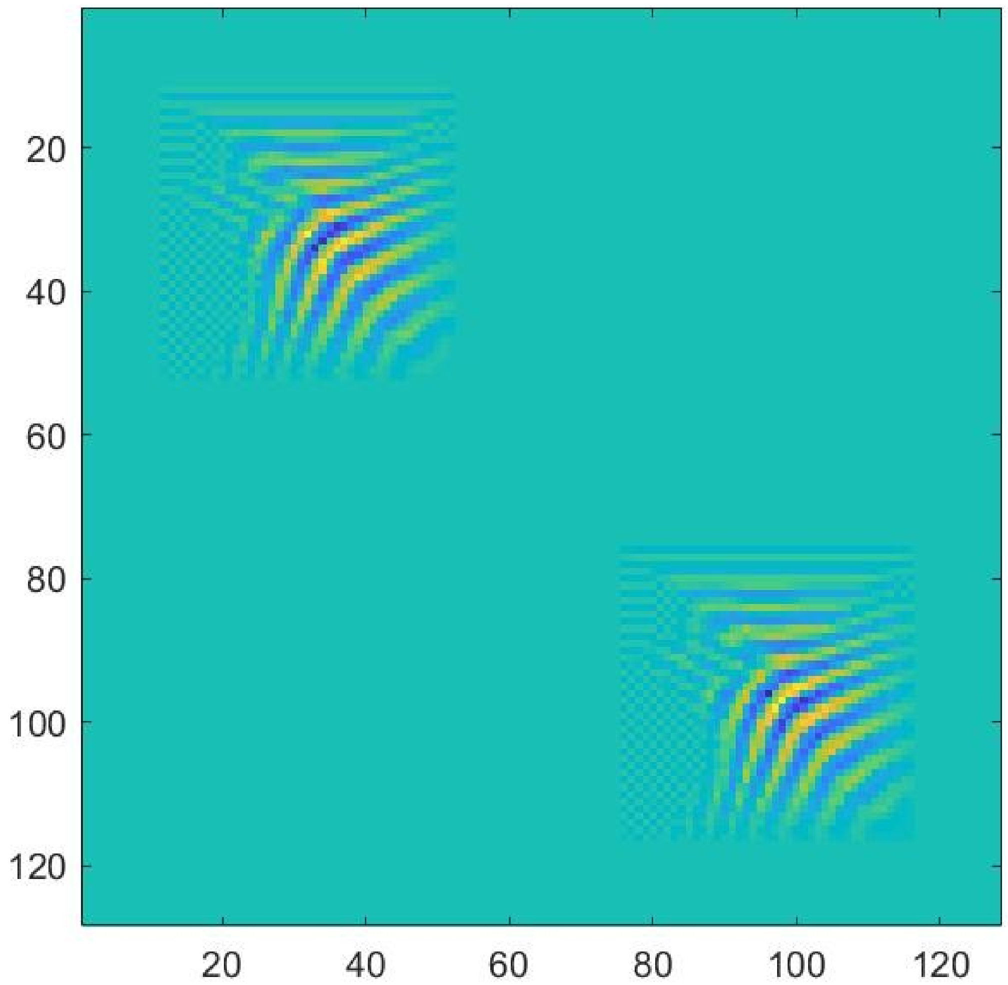
where $\mathbf{Y}^0$ is the input image, $\mathbf{K}_{1,2}$ are convolutions, σ is a non-linear activation function, and N is a normalization function that depends on α and β

K. He et al. (2015)

Receptive Field



Receptive Field



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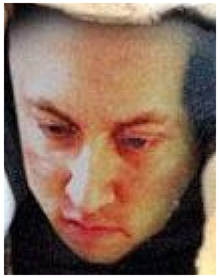

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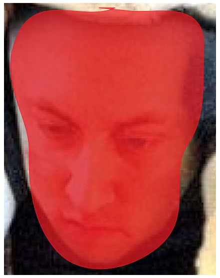
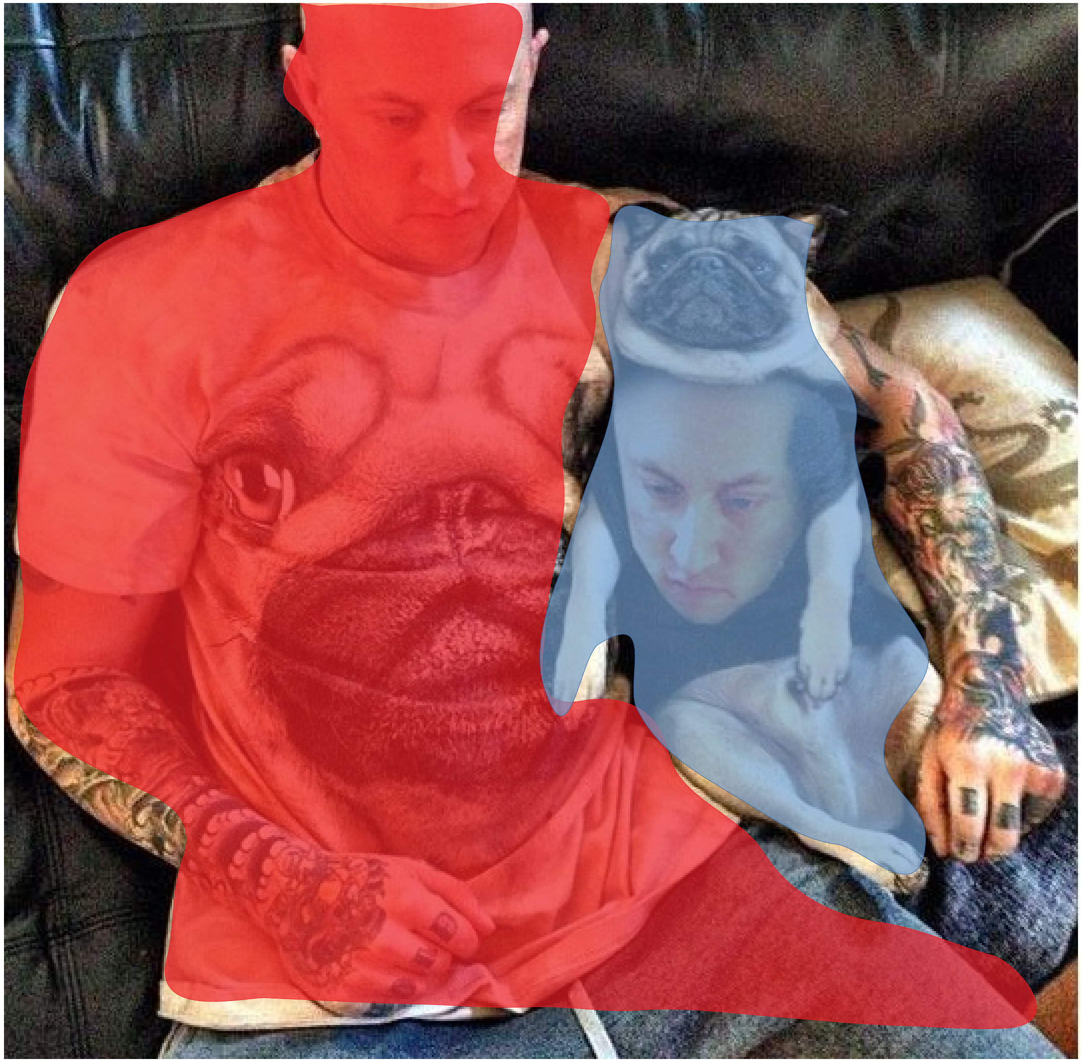


Receptive Field

Non-local information is often necessary.

Receptive Field



Receptive Field

Receptive Field

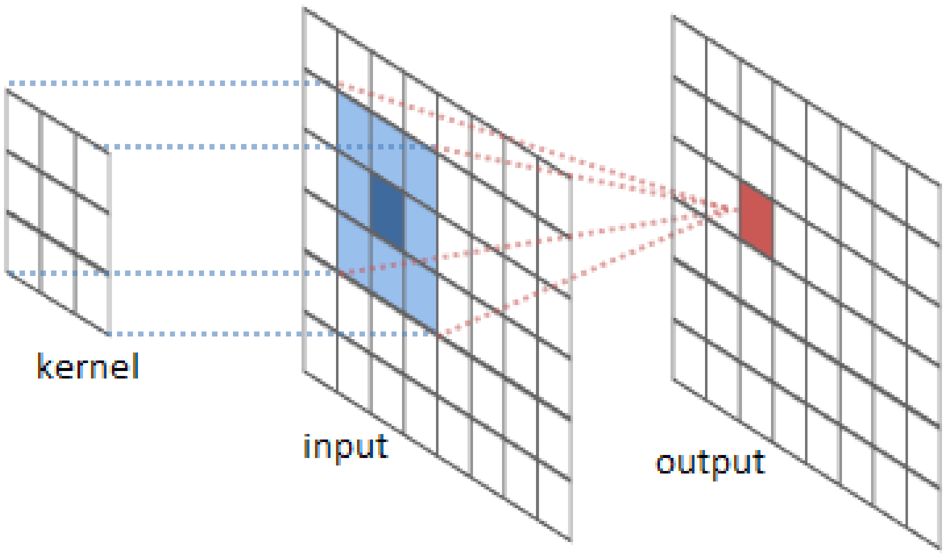
Receptive Field

How can we increase a network's receptive field?

Receptive Field

How can we increase a network's receptive field?

1) Convolutions



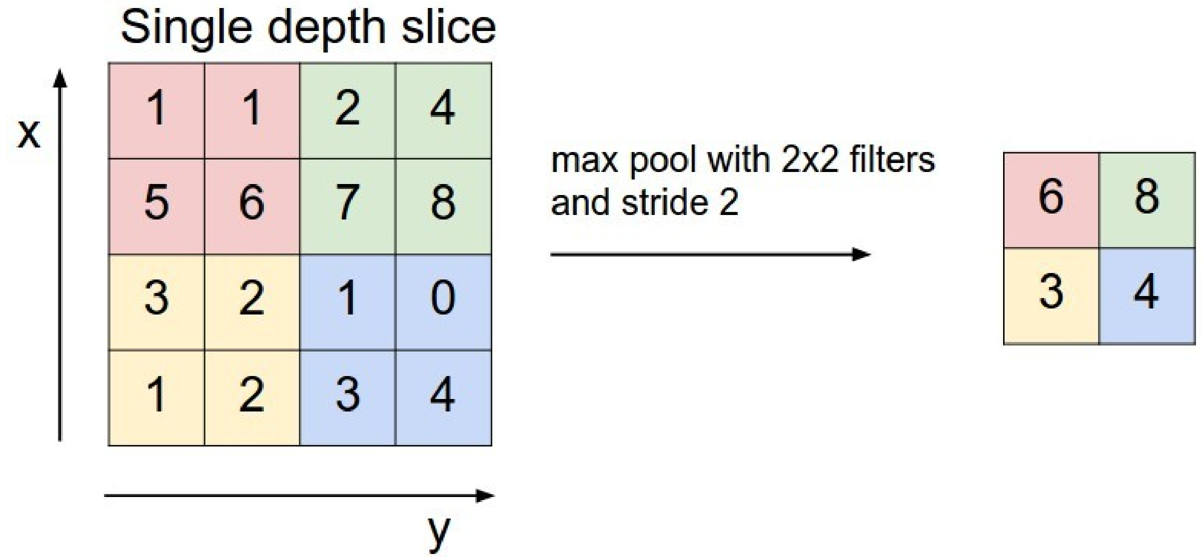
http://colah.github.io/posts/2014-07-Understanding-Convolutions/

Receptive Field

How can we increase a network's receptive field?

1) Convolutions

2) Coarsening

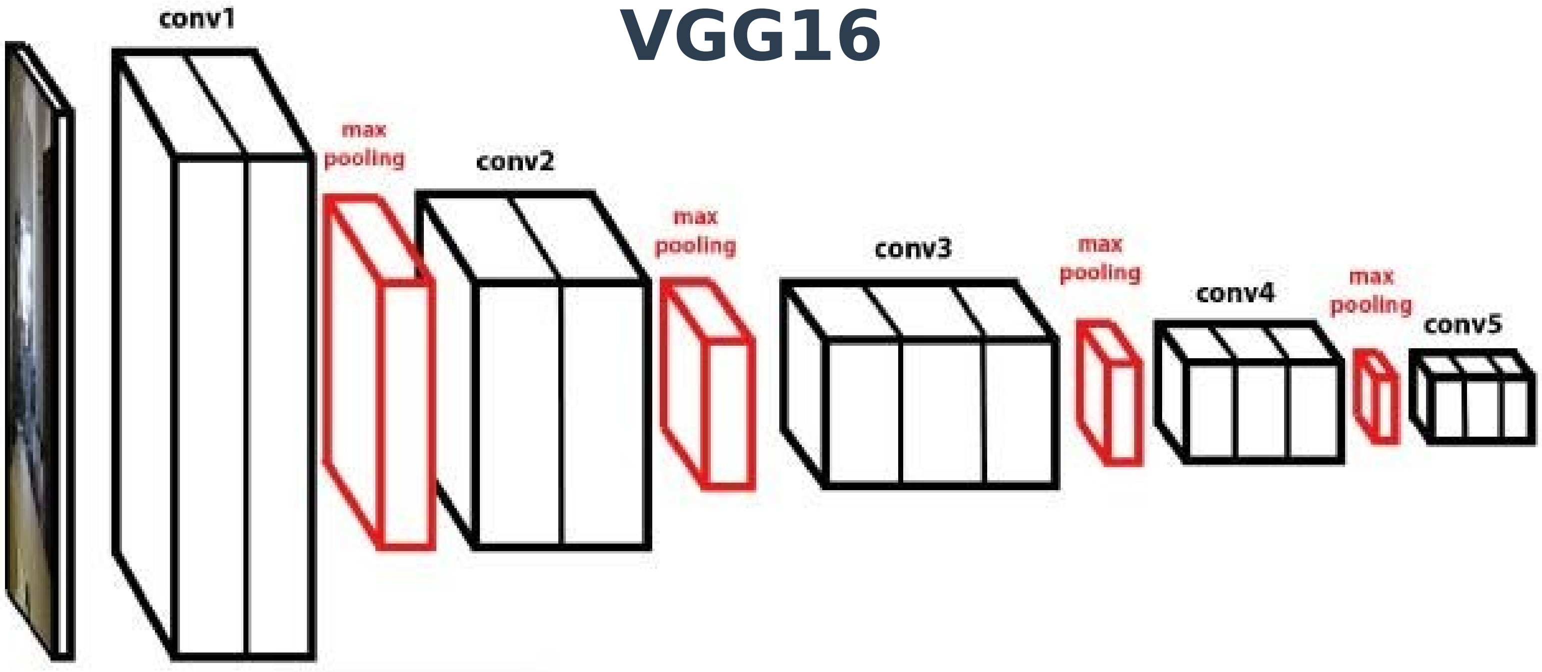


http://cs231n.github.io/convolutional-networks/

Receptive Field

In practice, a combination of both

VGG16

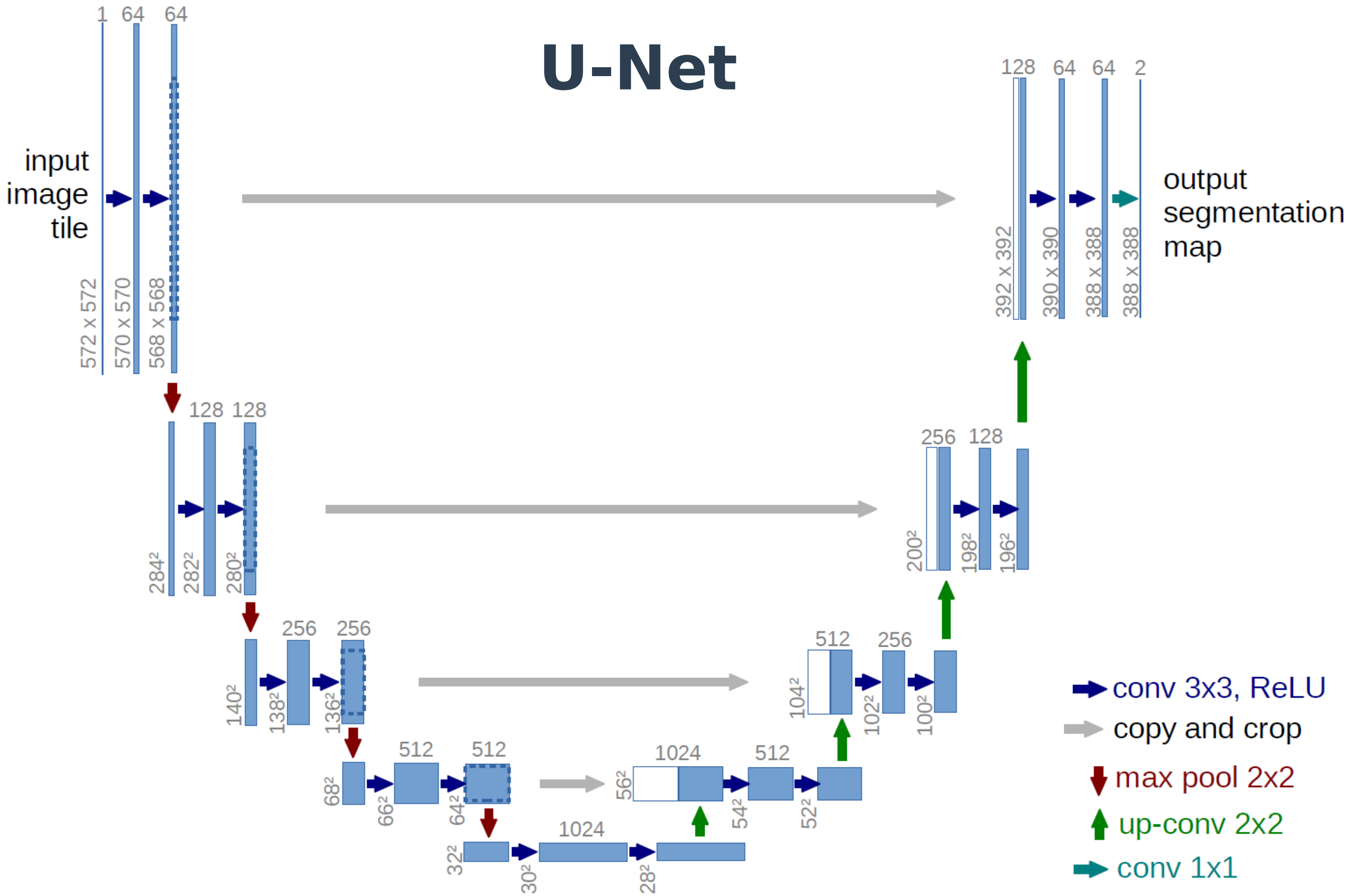


https://www.cs.toronto.edu/~frossard/post/vgg16/

Receptive Field

In practice, a combination of both

U-Net



O. Ronneberger and P.Fischer and T. Brox (2015)

Receptive Field

How can we increase a network’s receptive field?

1) Convolutions
   - Pros: Increase model complexity, increase receptive field
   - Cons: More expensive, less stable

2) Coarsening
   - Pros: Reduce dimensionality, increase receptive field
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Receptive Field

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3) Implicit Steps
   - Pros: Increase stability, cheap, couple all pixels
   - Cons: Additional hyperparameters, periodicity (depending on implementation)

Stability

Forward Stability:

- Small changes to the input result in small changes to the output

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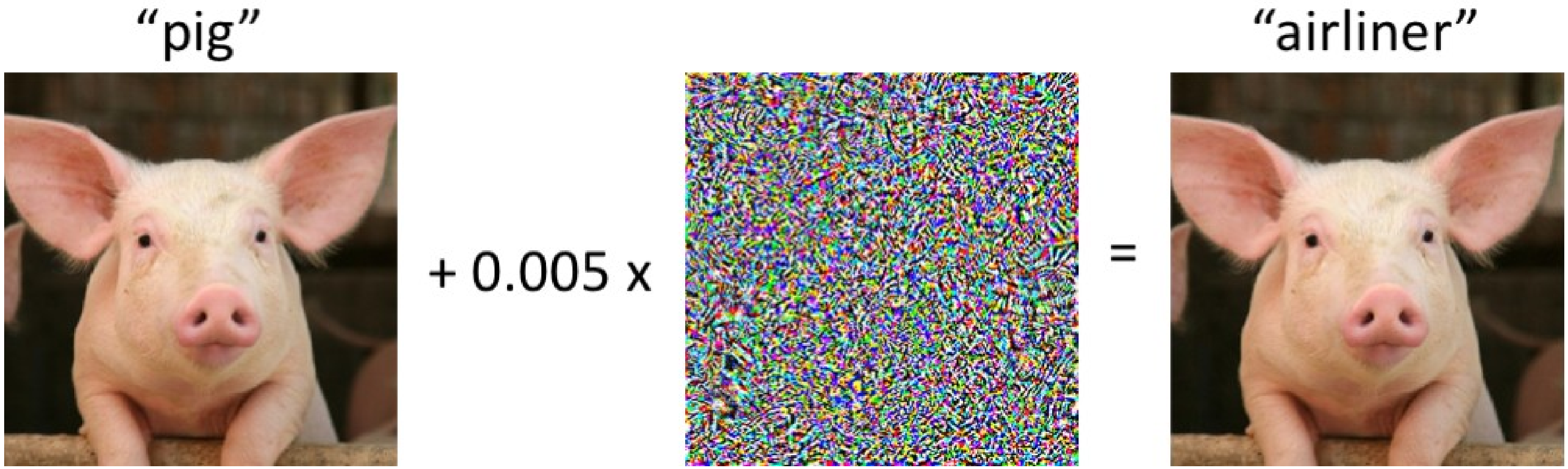
Forward Stability:

- Small changes to the input result in small changes to the output

- Small pertubations do not grow at depth, which could lead to vanishing/exploding gradients, making deep networks difficult to train

Adversarial Examples

Exploit a network's instability inorder to fool a classifier



MIT CSAIL

Adversarial Examples

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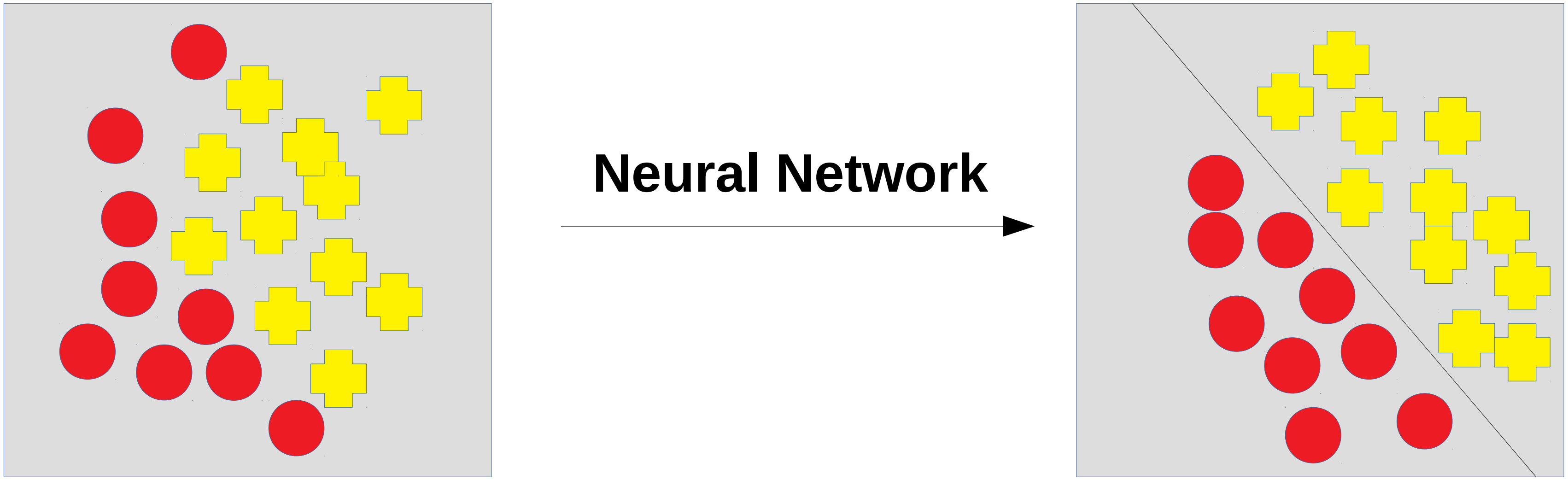

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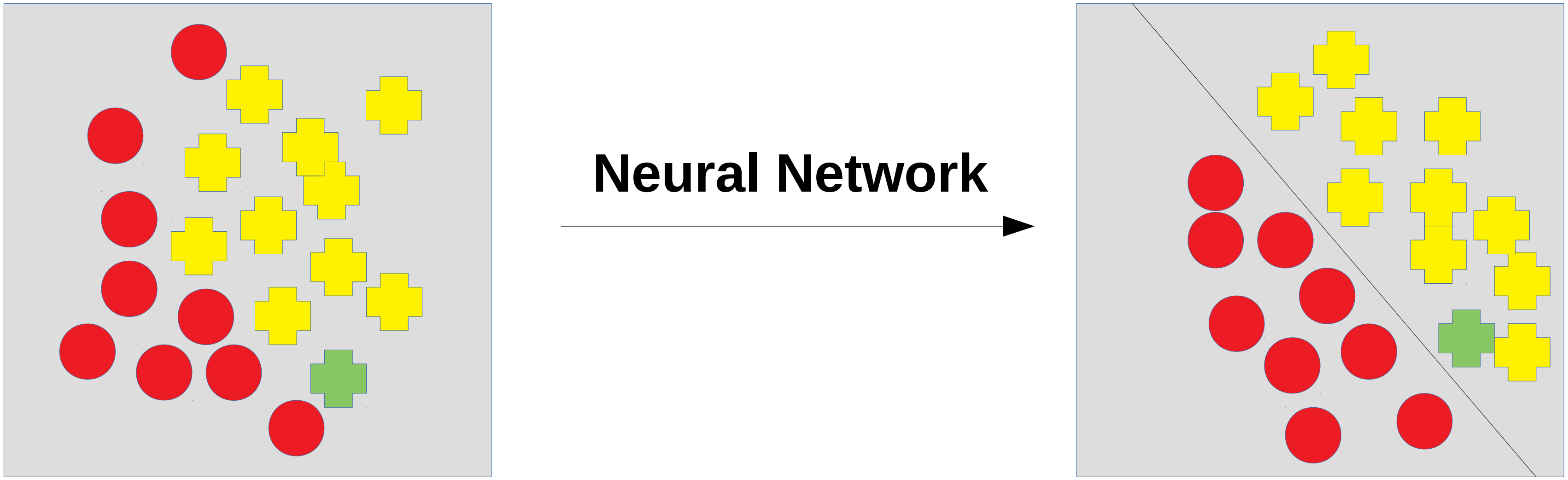

Adversarial Examples

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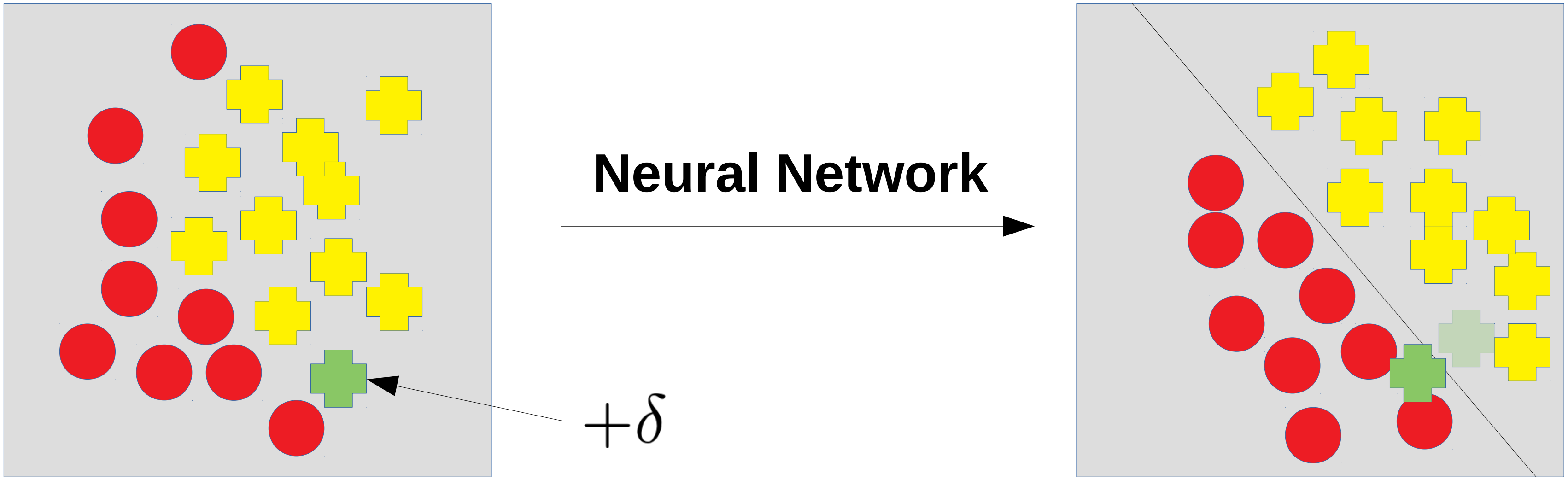

IMEXnet

Start with ResNet:

$$\mathbf{Y}^{n+1} = \mathbf{Y}^{n} + \mathbf{K}_2 \sigma(N_{\alpha,\beta}(\mathbf{K}_1 \mathbf{Y}^{n})) \quad \text{for} \quad n = 0, ..., N-1$$

where $\mathbf{Y}^0$ is the input image, $\mathbf{K}_{1,2}$ are convolutions, σ is a non-linear activation function, and N is a normalization function that depends on α and β

K. He et al. (2015)

IMEXnet

Start with ResNet:

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$$\mathbf{Y}^{n+1} = \mathbf{Y}^n + f(\mathbf{Y}^n, \theta^n) \quad \text{for} \quad n = 0, ..., N-1$$

Which can be seen with as a forward Euler discretization of the non-linear ODE (Haber & Ruthotto, 2017).

$$\dot{\mathbf{Y}}(\mathbf{t}) = f(\mathbf{Y}(t), \theta(t))$$

IMEXnet

We can view using a ResNet as solving the following initial value problem using forward Euler and a step size $h = 1$.

$$\dot{\mathbf{Y}}(\mathbf{t}) = f(\mathbf{Y}(t), \theta(t)), \quad \mathbf{Y}(0) = \mathbf{Y}_0$$

We are not limited to forward Euler, for example midpoint methods have been used, and RK methods have been proposed (Haber & Ruthotto, 2017; Chen et al., 2018).

IMEXnet

Recap of ResNet issues:

- Often unstable.
- Information takes many layers to travel across the computational grid.

One way to accelerate the communication of information across all pixels is to use an implicit method (Ascher & Petzold, 1998).

IMEXnet

Backward Euler

$$\mathbf{Y}^{n+1} = \mathbf{Y}^n + hf(\mathbf{Y}^{n+1}, \theta^{n+1}) \quad \text{for} \quad n = 0, ..., N-1$$

Treating the non-linear term implicilty requires solving the above equation at evey time step.

In order to avoid this expensive step, we instead consider an implicit-explicit (IMEX) method.

IMEX Method

Treat non-linear part of the RHS explicitly, and the linear part of it implicitly (Ascher et al, 1995; 1997).

Commonly used in fluid dynamics, surface formation, and image denoising.

IMEX Method

Treat non-linear part of the RHS explicitly, and the linear part of it implicitly.

There is no natural division in this context.

$$\dot{\mathbf{Y}}(t) = \underbrace{f(\mathbf{Y}(t), \boldsymbol{\theta}(t)) + \mathbf{LY}(t)}_{\text{explicit term}} - \underbrace{\mathbf{LY}(t)}_{\text{implicit term}}$$

where $\mathbf{L}$ is a linear invertible matrix

IMEX Method

We use forward Euler for the explicit terms, and backward Euler for the implicit term.

$$(\mathbf{I} + h\mathbf{L})\mathbf{Y}^{n+1} = \mathbf{Y}^{n}(\mathbf{I} + h\mathbf{L}) + hf(\mathbf{Y}^{n}, \theta^{n})$$

IMEX Method

We use forward Euler for the explicit terms, and backward Euler for the implicit term.

$$\mathbf{Y}^{n+1} = (\mathbf{I} + h\mathbf{L})^{-1}(\mathbf{Y}^n(\mathbf{I} + h\mathbf{L}) + hf(\mathbf{Y}^n, \theta^n))$$

IMEX Method

We use forward Euler for the explicit terms, and backward Euler for the implicit term.

$$\mathbf{Y}^{n+1} = (\mathbf{I} + h\mathbf{L})^{-1}(\mathbf{Y}^n(\mathbf{I} + h\mathbf{L}) + hf(\mathbf{Y}^n, \theta^n))$$

Note that the inverted term is dense, so it couples the entire computational grid in each step.

Absolute Stability

ResNet

$$\mathbf{Y}_{j+1} = (1 + h\lambda)\mathbf{Y}_j$$

$$\text{Stable if and only if } |1 + \lambda h| \leq 1$$

IMEXnet

$$\mathbf{Y}_{j+1} = \frac{1 + h\lambda + h\alpha}{1 + h\alpha}\mathbf{Y}_j \text{ when } \mathbf{L} = \alpha\mathbf{I}$$

$$\text{Stable if and only if } \left|\frac{1 + h\lambda + h\alpha}{1 + h\alpha}\right| \leq 1$$

Implementation

In the implicit step we solve the linear system

$$(\mathbf{I} + h\mathbf{L})\mathbf{Y} = \mathbf{B},$$

where **L** is a group convolution and **B** is the collection of explicit terms.

To compute this efficiently, we represent the convolution in the Fourier domain.

Implementation

Consider the convolution

$$\mathbf{A}\mathbf{Y} = \mathbf{B}, \ \text{where } \mathbf{A} = (\mathbf{I} + h\mathbf{L}).$$

By choosing **L** to be a convolution, we can use it's form in the Fourier domain

$$\mathbf{A} * \mathbf{Y} = \mathbf{F}^{-1}((\mathbf{F}\mathbf{A}) \odot (\mathbf{F}\mathbf{Y}))$$

to compute the product of the inverse

$$\mathbf{A}^{-1} * \mathbf{Y} = \mathbf{F}^{-1}((\mathbf{F}\mathbf{Y}) \oslash (\mathbf{F}\mathbf{A}))$$

$$\text{where } \oslash \text{ is element wise division.}$$

Implementation

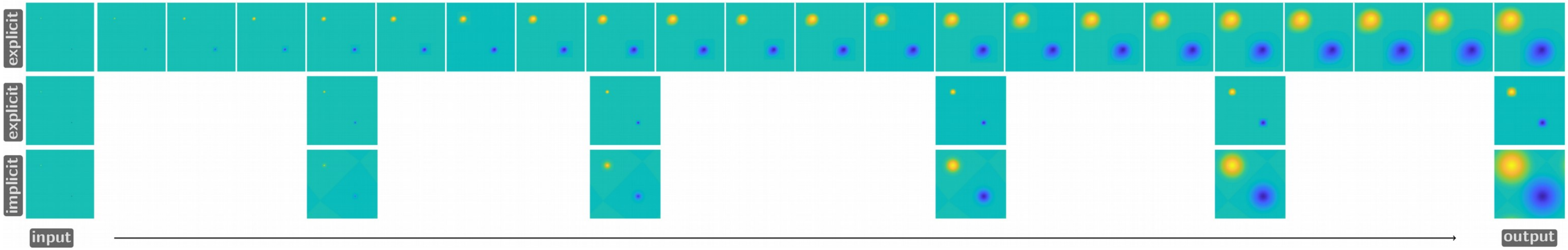
In order to ensure that **A** is invertible, we define **L** to be positive semi-definite in the form

$$\mathbf{L} = \mathbf{C}^T \mathbf{C},$$

where **C** is a groupwise convolution operator.

This implementation is CUDA enabled and supported by AutoGrad packages.

IMEXnet



Numerical Experiments

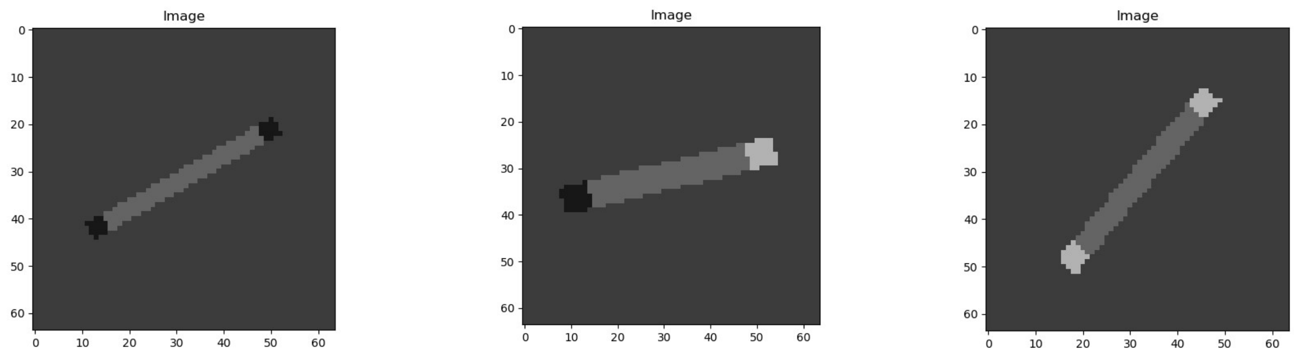
Q-tips Dataset

- Sythentic semantic segmentation dataset
- 1024 training examples, 64 valdiation examples
- Single object images randomly sampled from a unifrom distribution of lengths, widths, and orientations.
- 3 object classes
- Requires a large receptive field

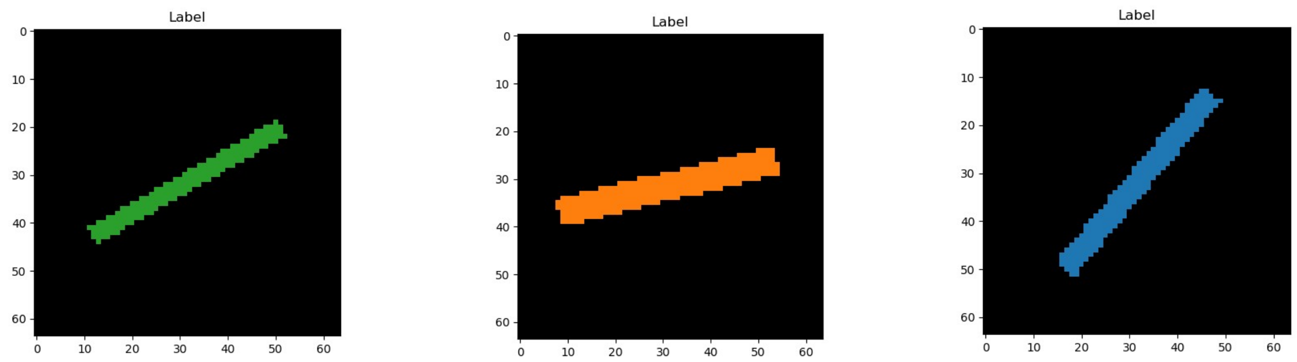
Numerical Experiments

Q-tips Dataset

Image



Label



Numerical Experiments

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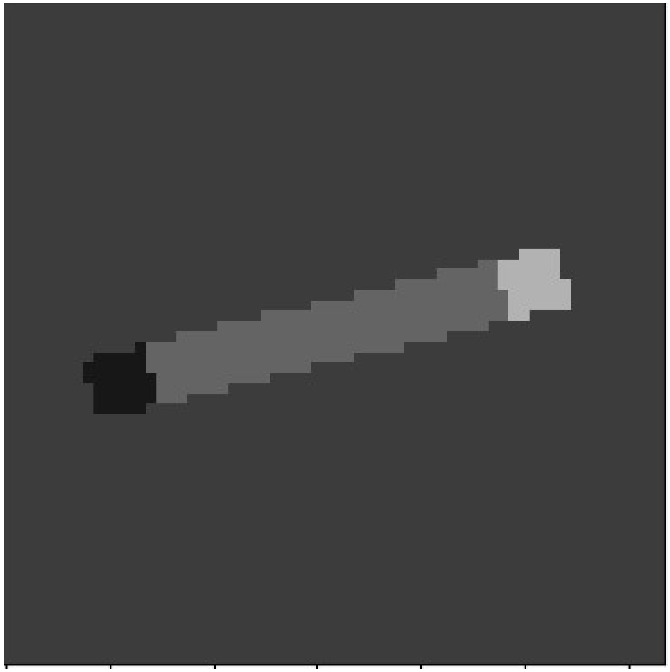

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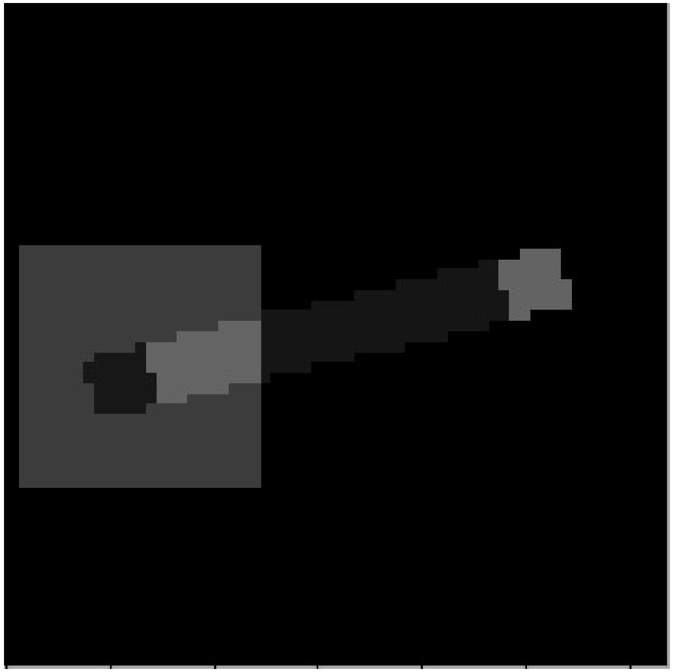

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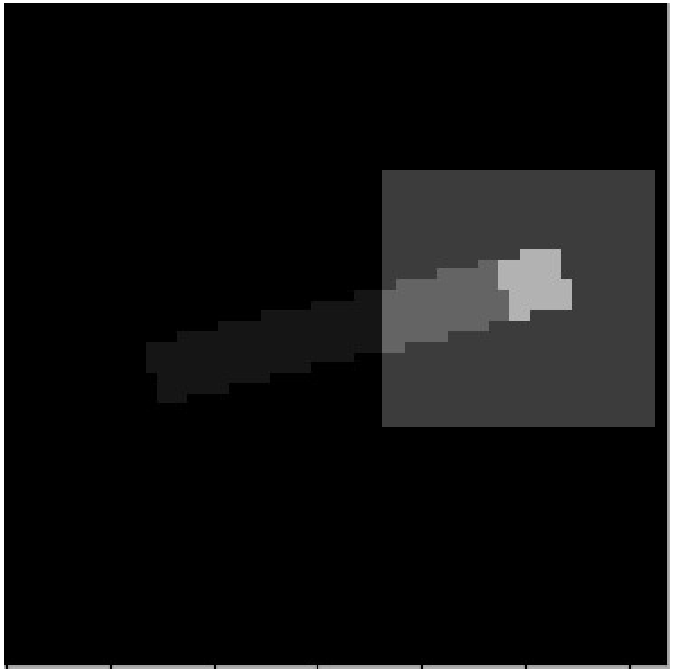
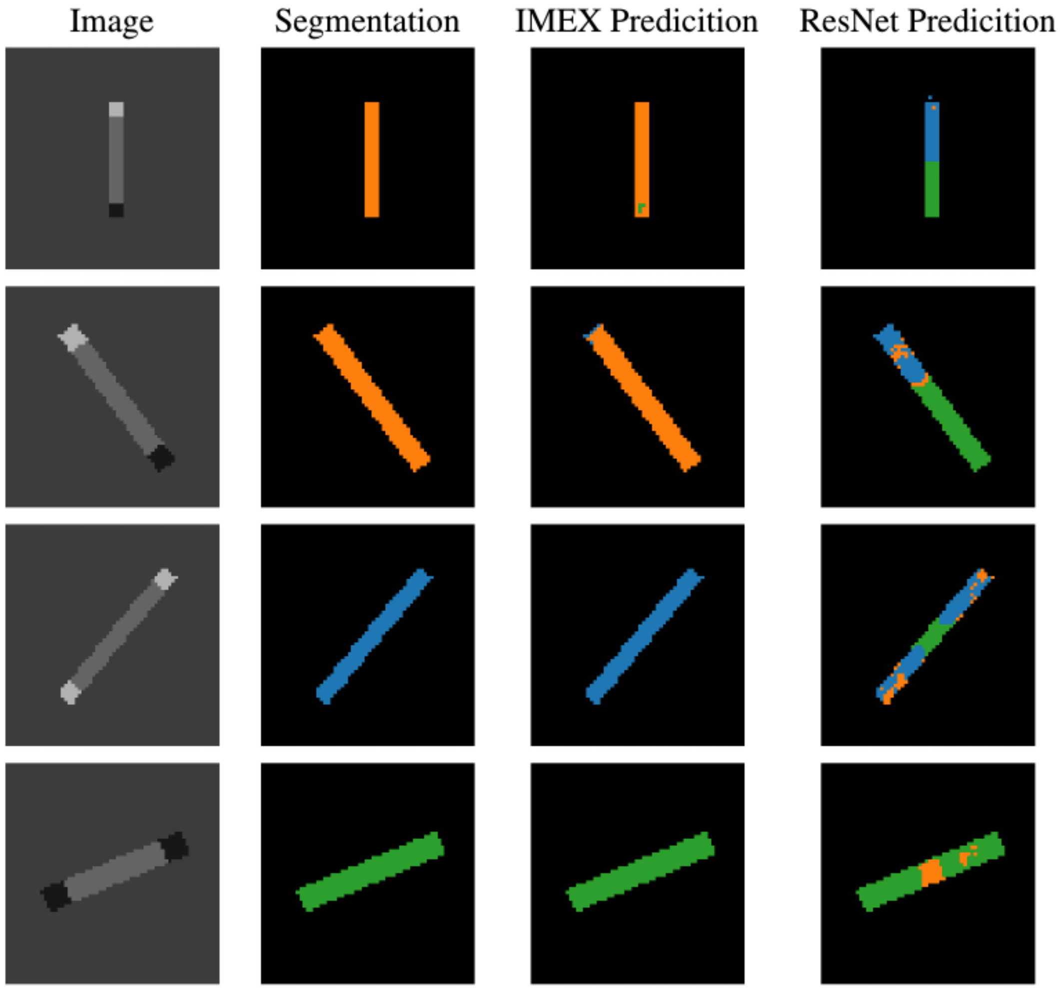


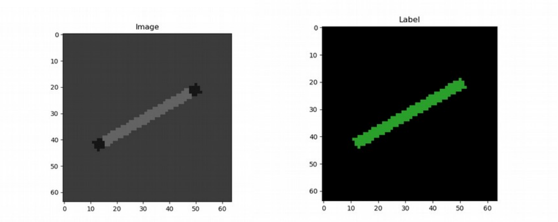
Numerical Experiments

Results - Predictions

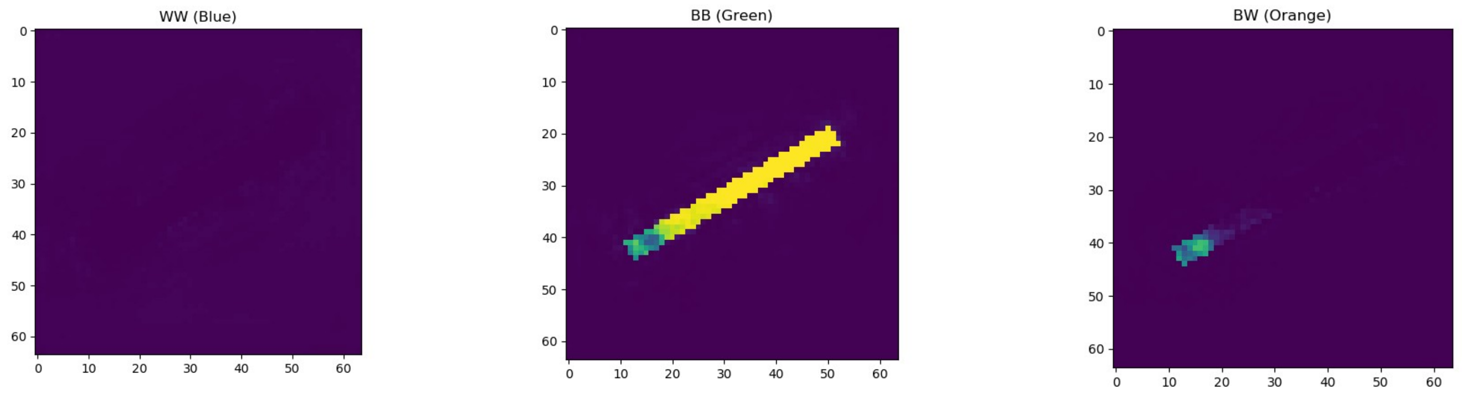


Numerical Experiments

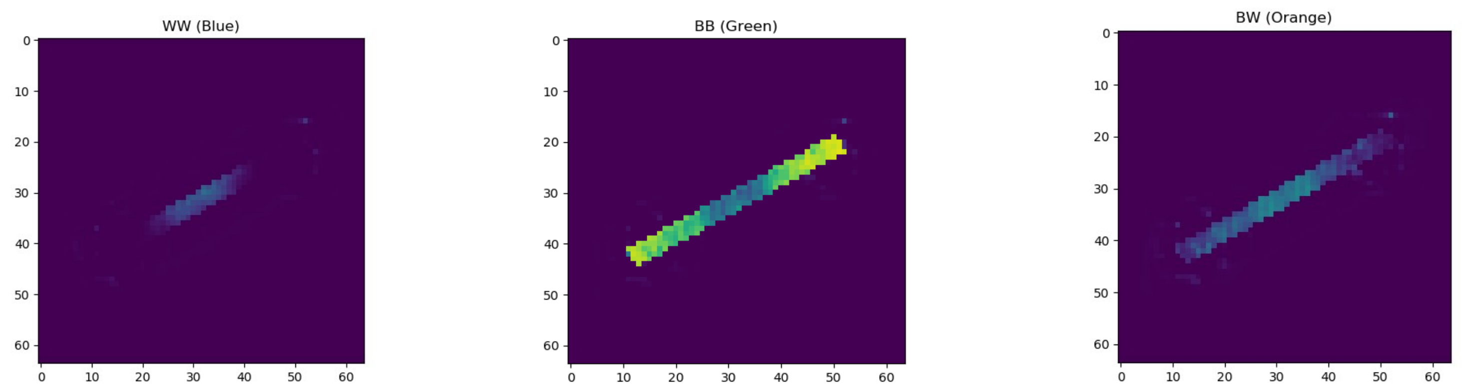
Results – Probability Maps



IMEXnet



ResNet



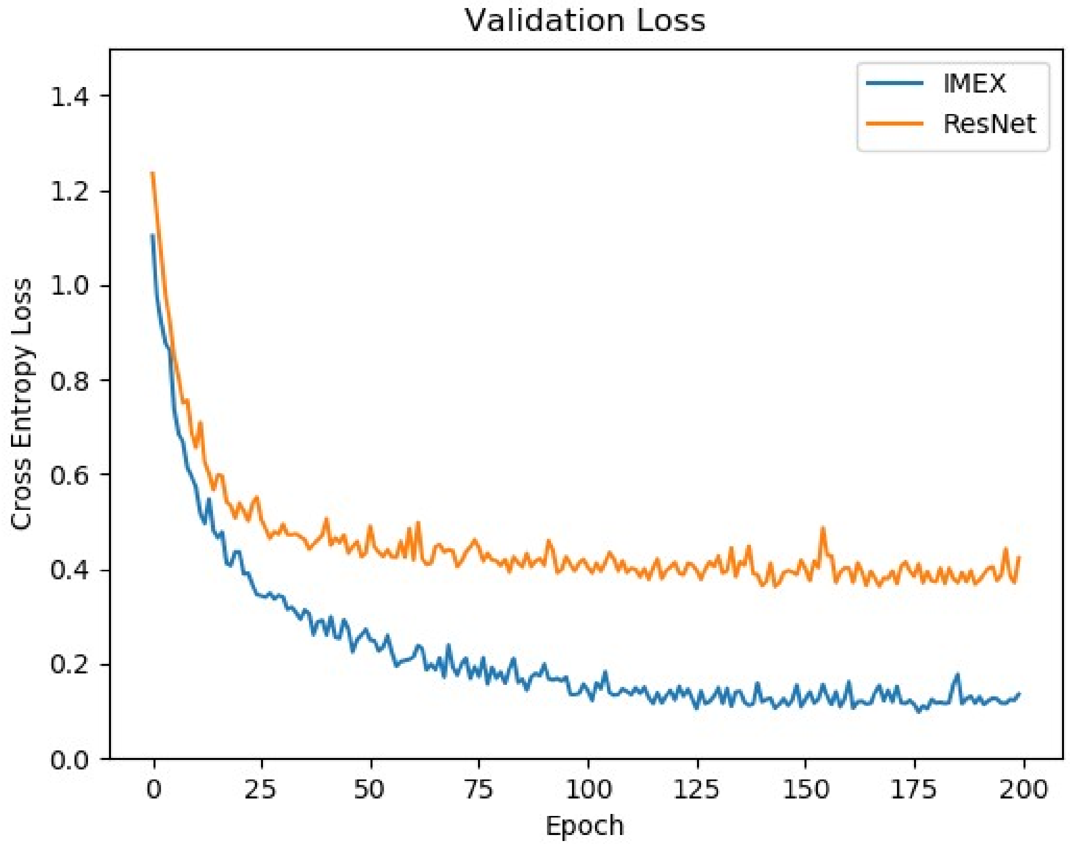
Numerical Experiments

Q-tips Results

Network	Parameters	IOU	Loss	Accuracy
IMEXnet	2701440	0.926	0.0982	99.56
ResNet	2691648	0.741	0.3332	98.18

Numerical Experiments

Q-tips Results



NYU Depth Results

Kitchen scene

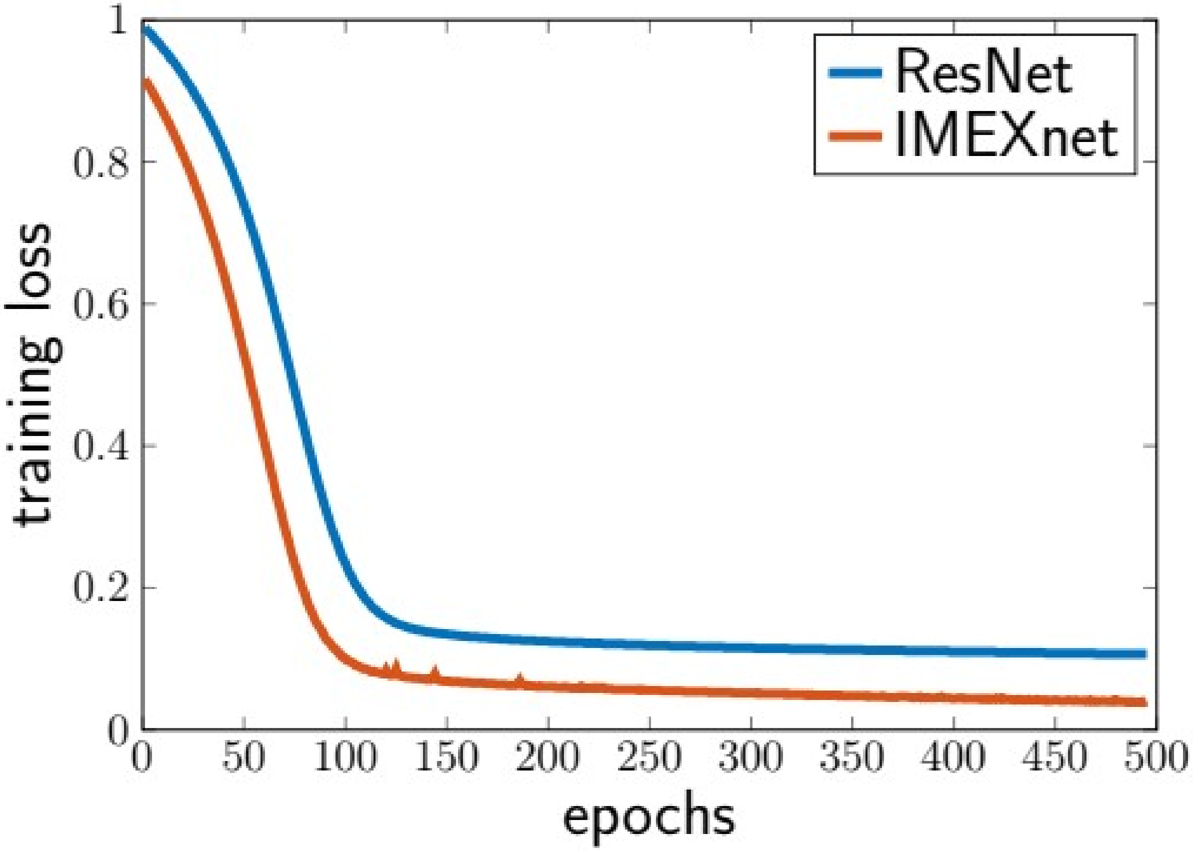
Depth map

ResNet recovery

Implicit net recovery

Numerical Experiments

NYU Depth Results



Receptive Field

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   - Pros: Increase model complexity, increase receptive field
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Acknowledgments

Paper:

Haber, E., Lensink, K., Triester, E., Ruthotto, L. IMEXnet – A forward stable deep neural network. 2019

Code:

github.com/HaberGroup/SemiImplicitDNNs

Acknowledgments

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