Transport and Feedback in Models of Self-Organizing Vegetation Patterns

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Committee on Computational & Applied Math
+Department of Statistics
University of Chicago

in collaboration with
Punit Gandhi (MBI) & Sarah Iams (Harvard)
Karna Gowda (UIUC) & Lucien Werner (Caltech)
Sara Bonetti (Duke) & Amilcare Porporato (Princeton)

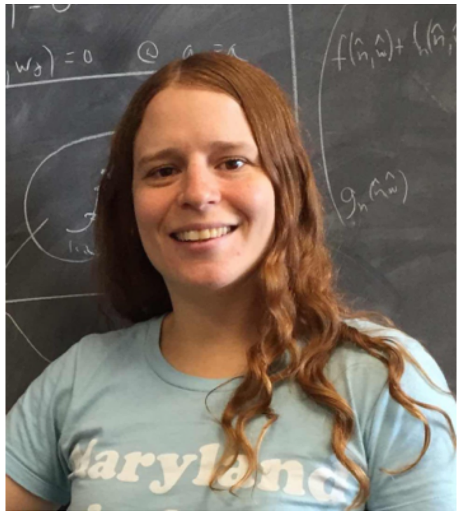
supported by NSF-DMS

Outline

- background: pictures & a movie(!) of vegetation patterns

- background: reaction-advection-diffusion vegetation models

- topographic influence - vegetation band arcing & topographic confinement
  (work with **Punit Gandhi**, Karna Gowda, **Sarah Iams,** Lucien Werner)

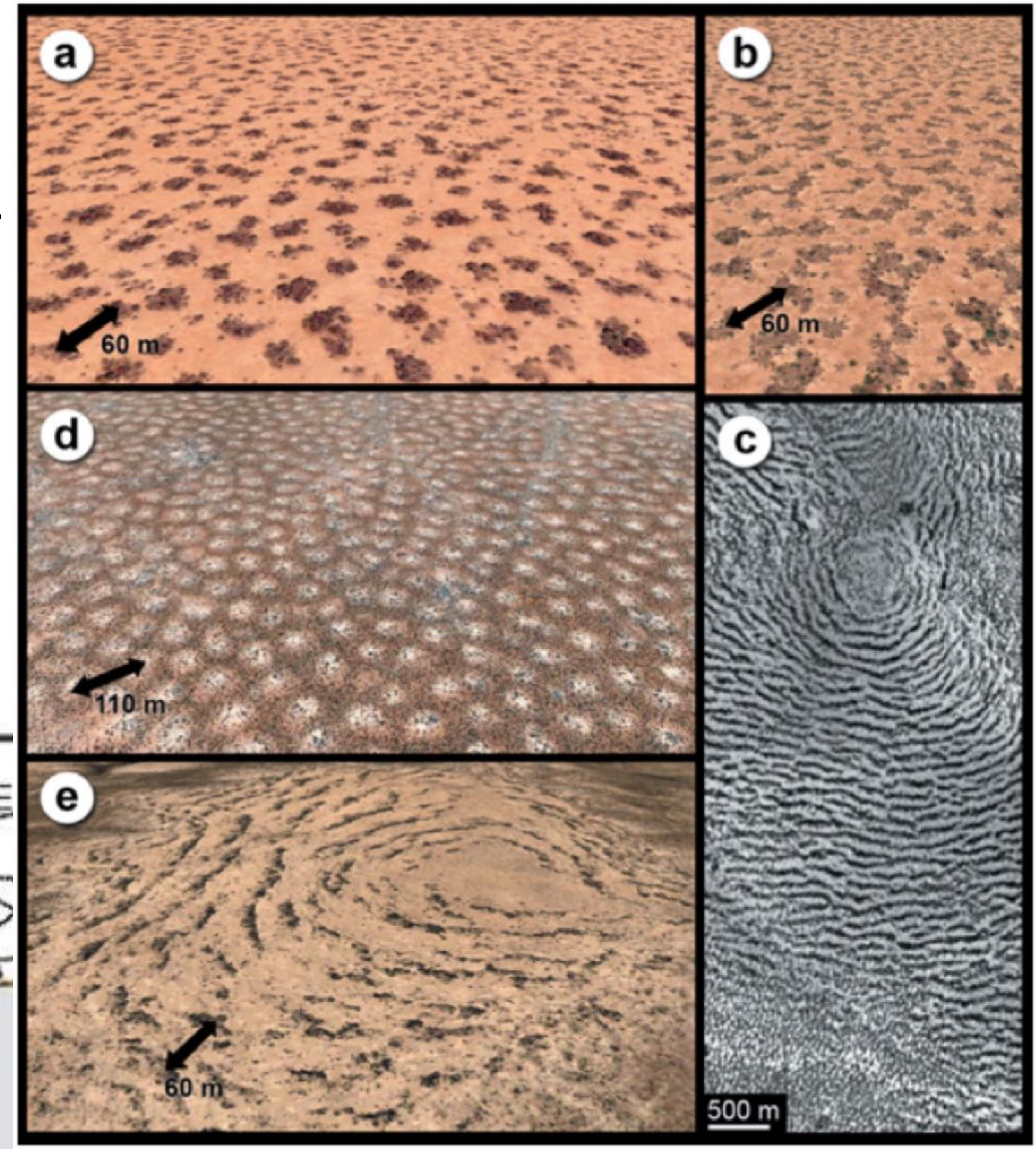


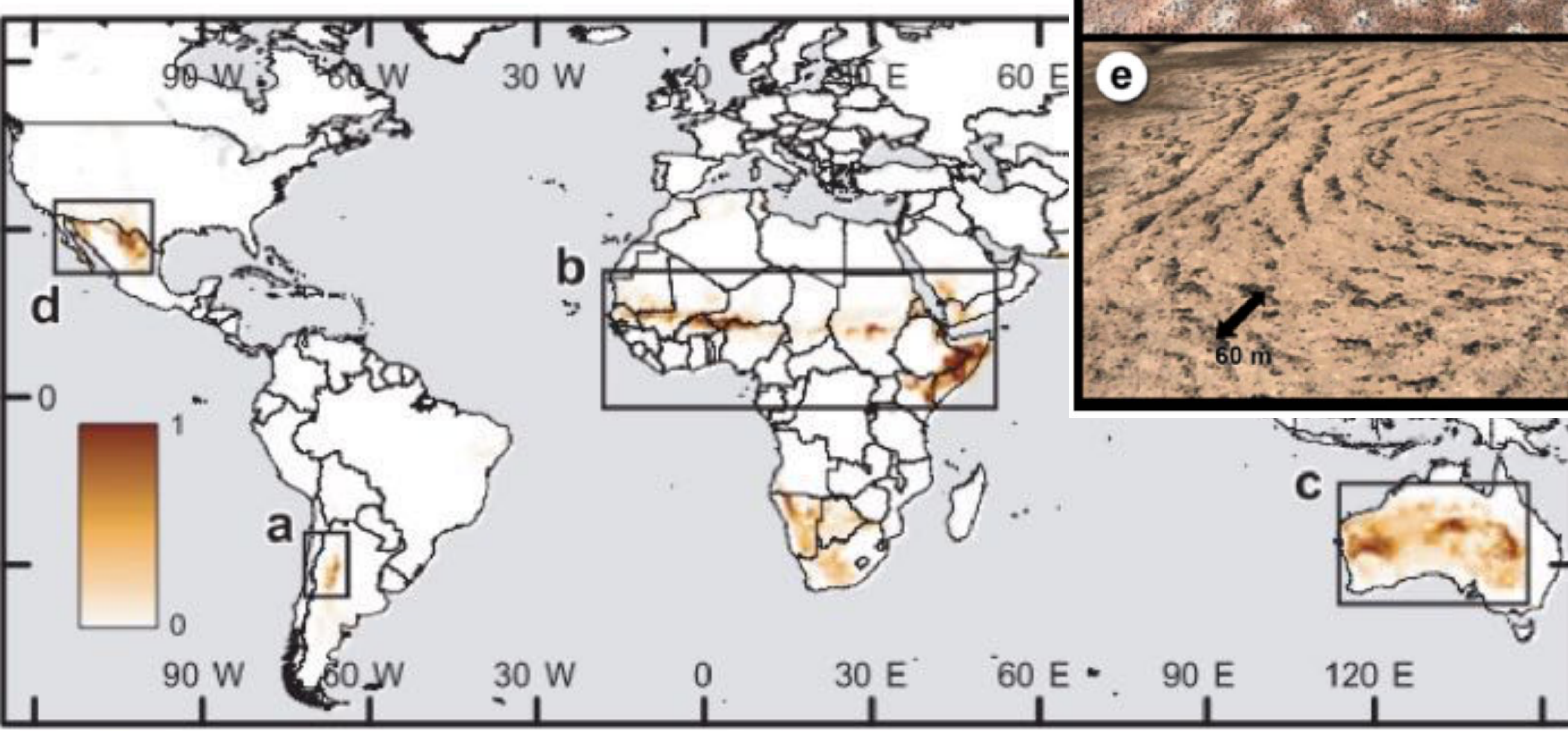
Wed. 4pm MS88

- modeling the biomass feedback on transport - some preliminary results
  (work with **Punit Gandhi**, Sarah Iams, Sara Bonetti, Amilcare Porporato)

Self-organized vegetation

Deblauwe V, Barbier N, Couteron P, Lejeune O, Bogaert J. The global biogeography of semi-arid periodic vegetation patterns. Global Ecology and Biogeography. 2008.

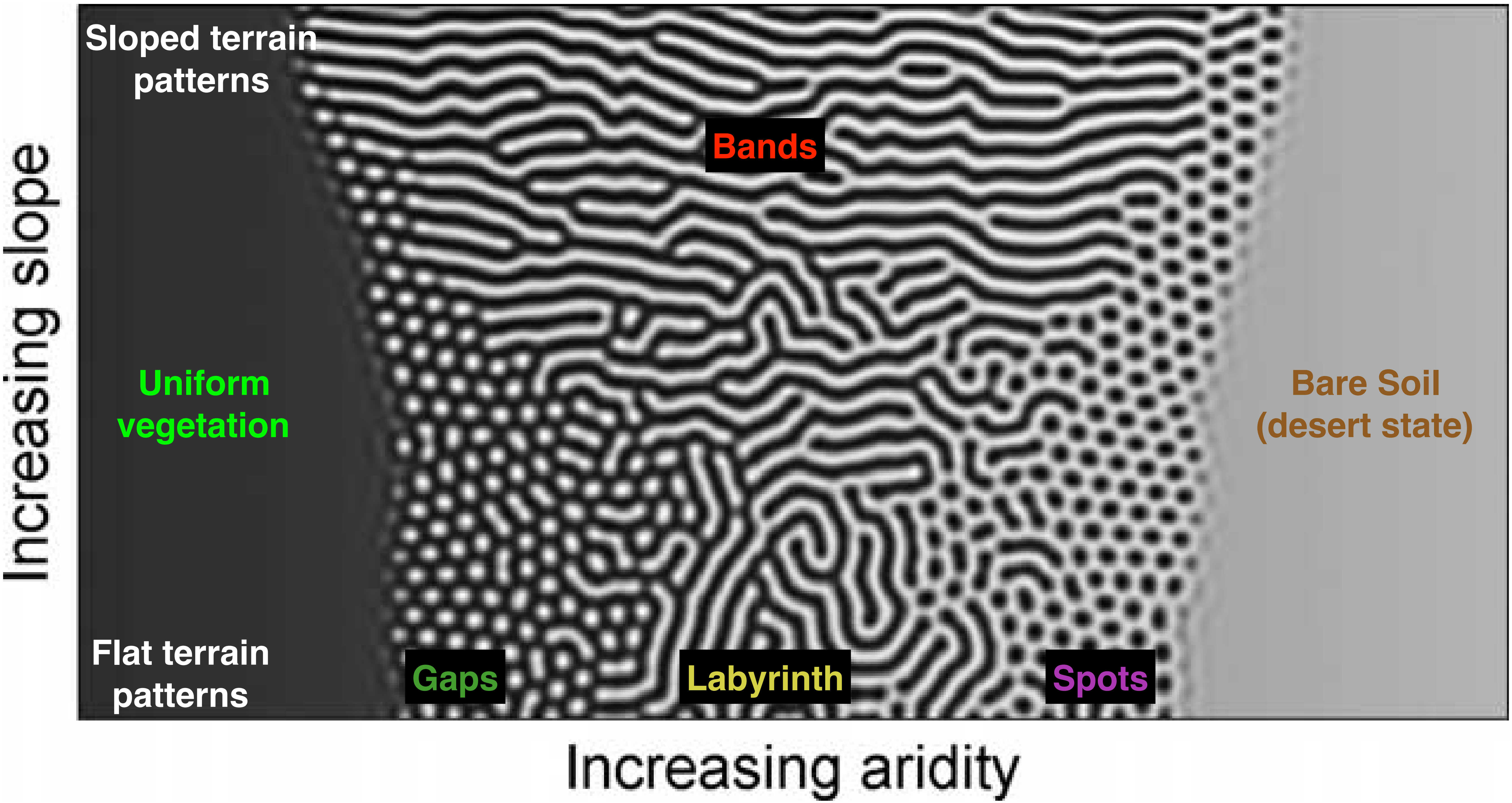




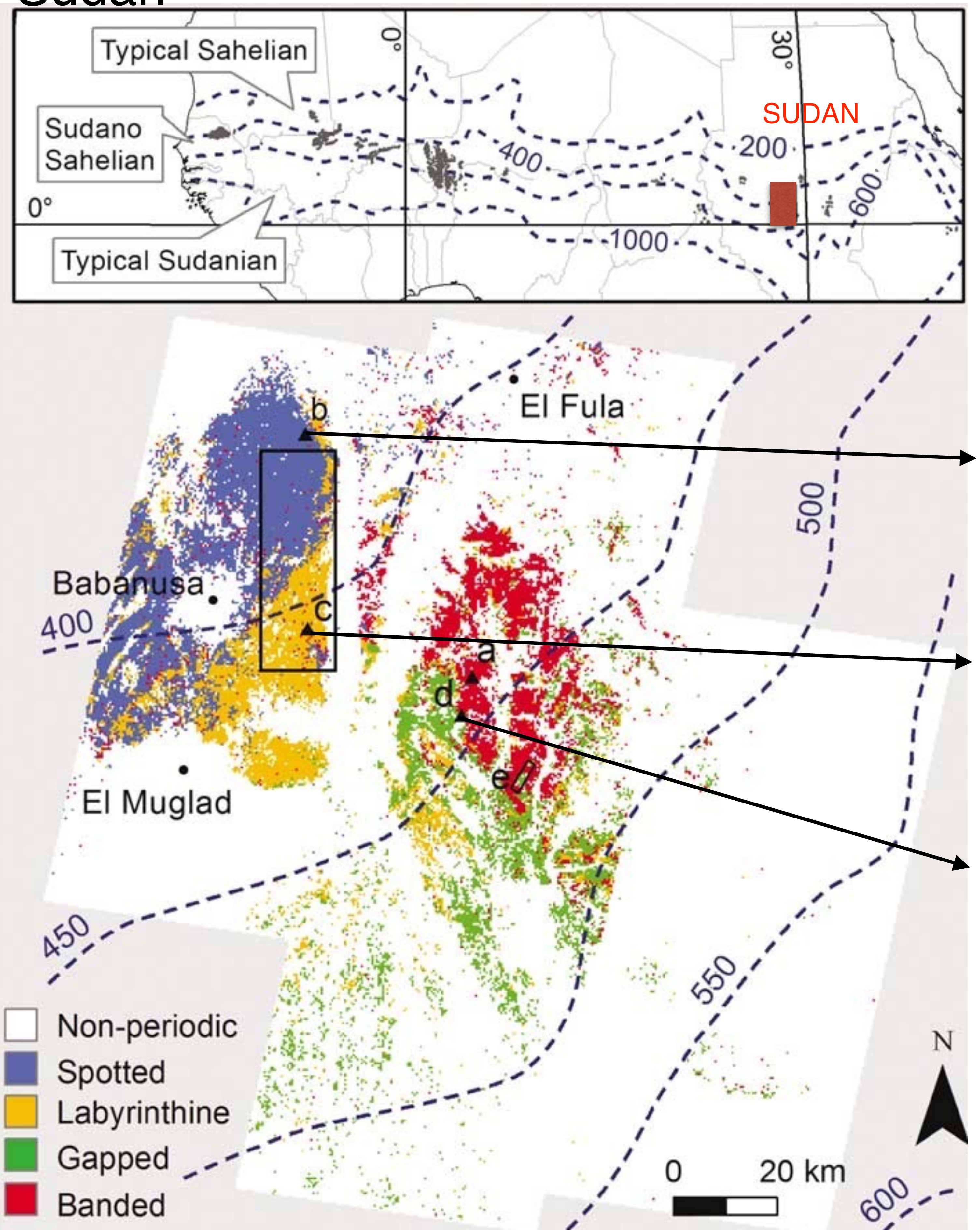
Dry, Hot, Flat

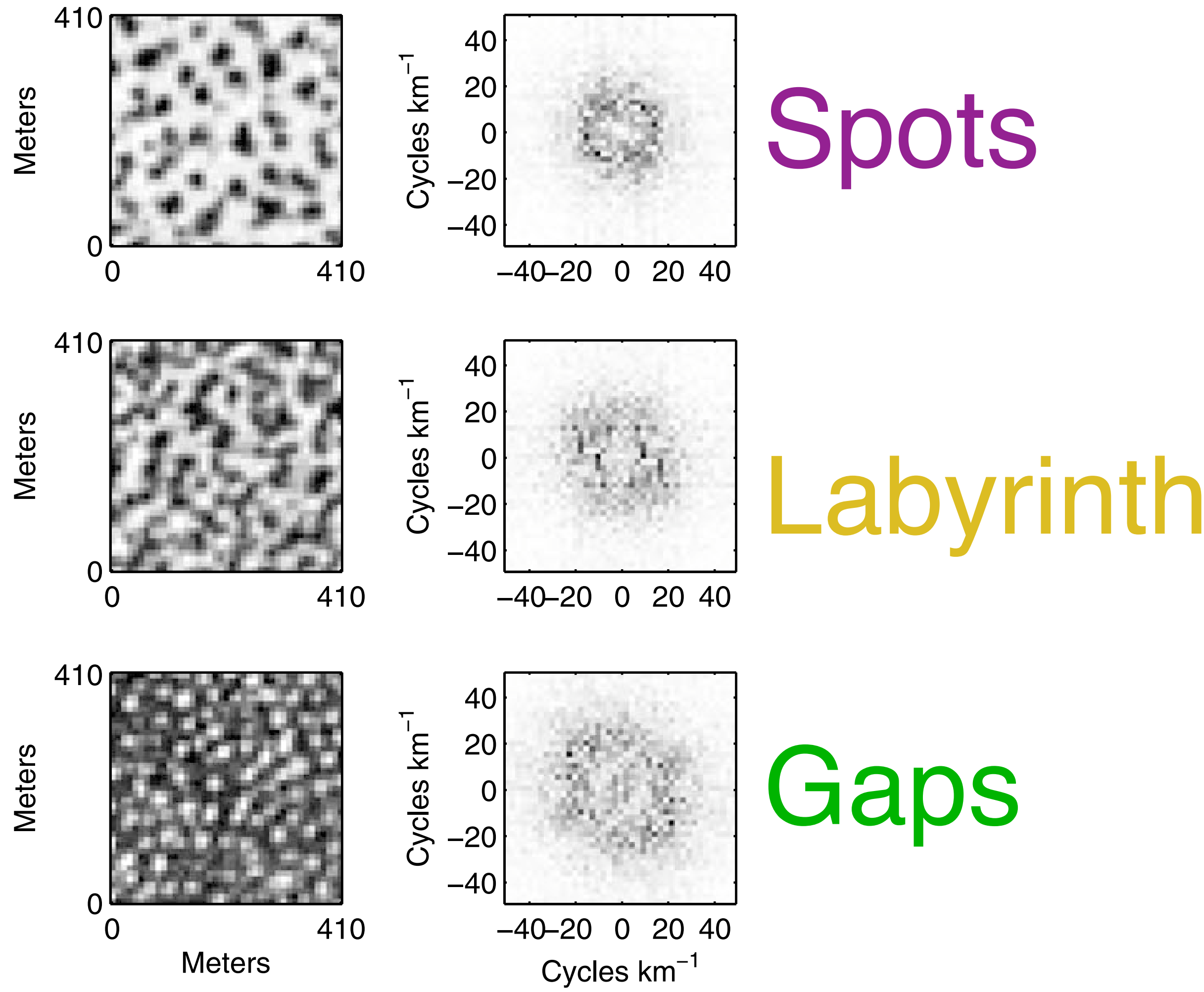
Environmental modulation of self-organized periodic vegetation patterns in Sudan

Vincent Deblauwe, Pierre Couteron, Olivier Lejeune, Jan Bogaert and Nicolas Barbier
Ecography 34: 990–1001, 2011



Deblauwe et al. (2011)
Sudan





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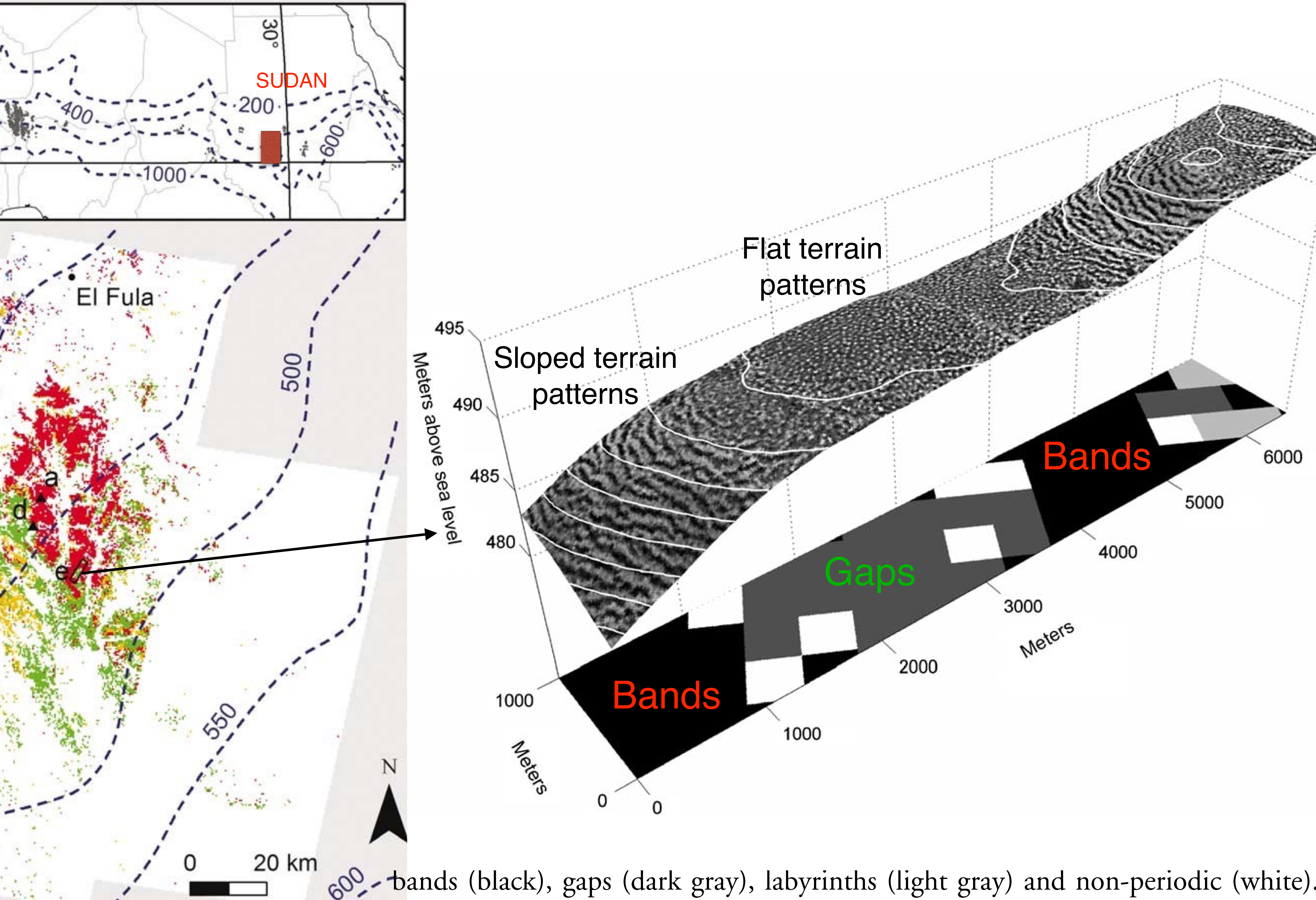
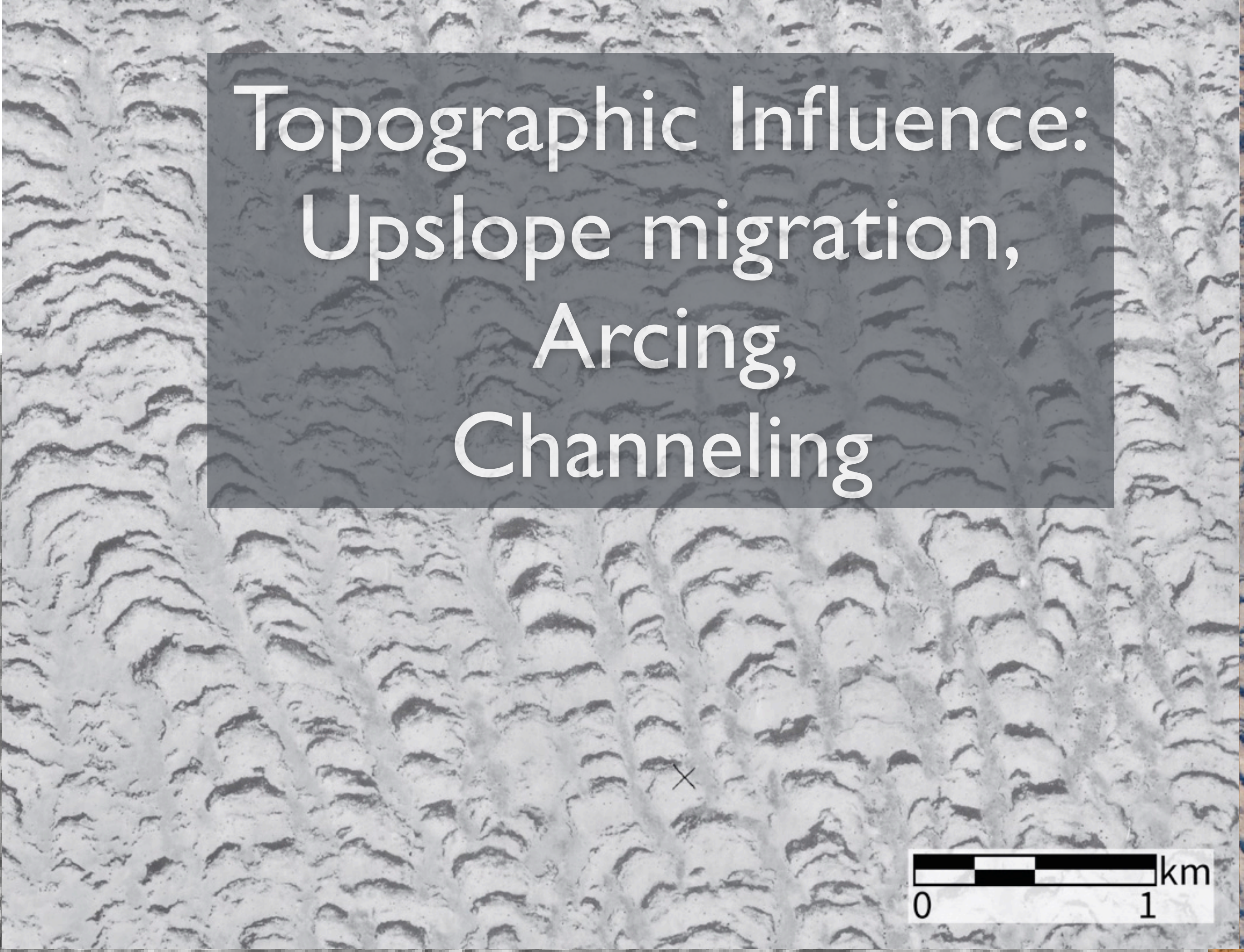
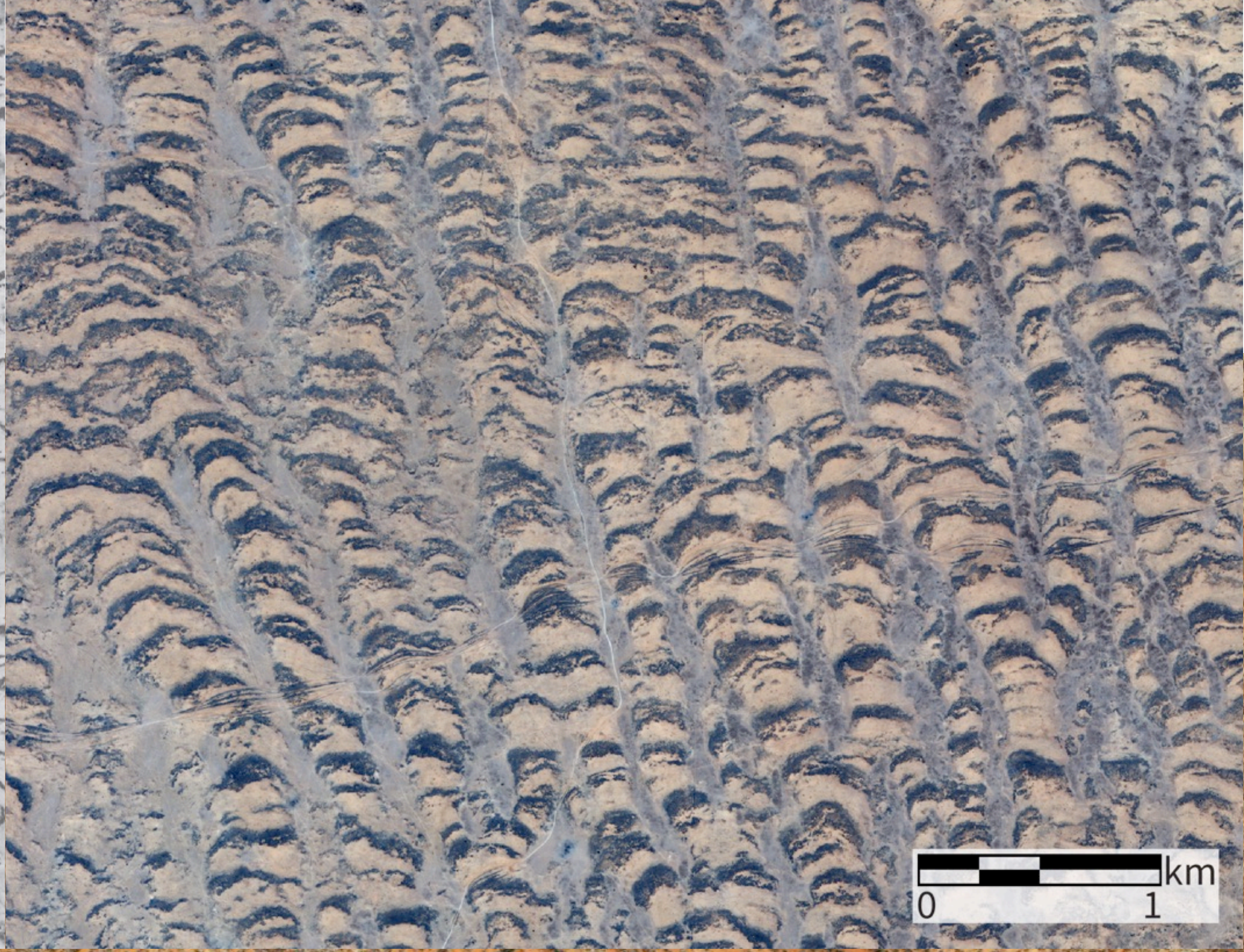
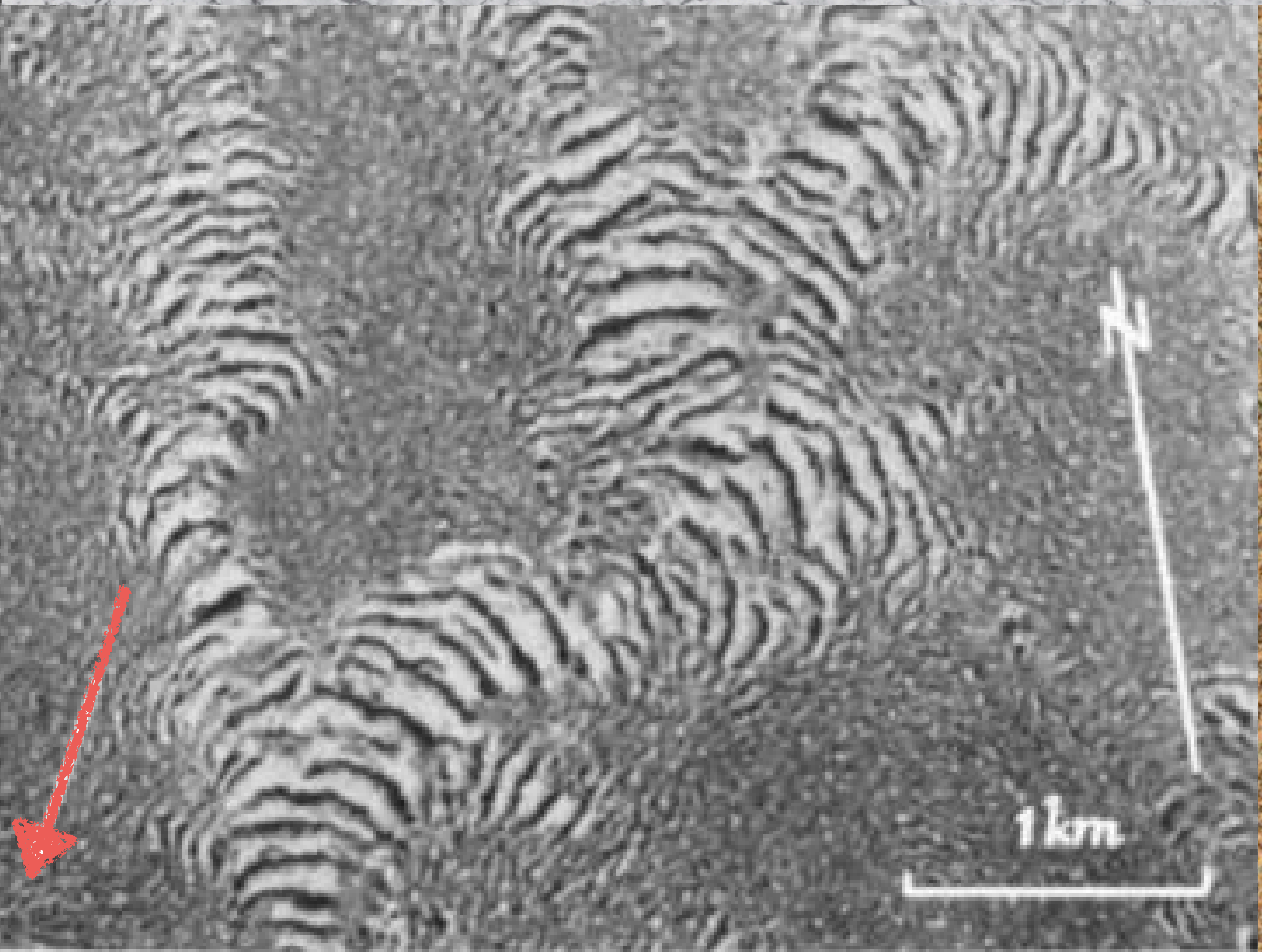


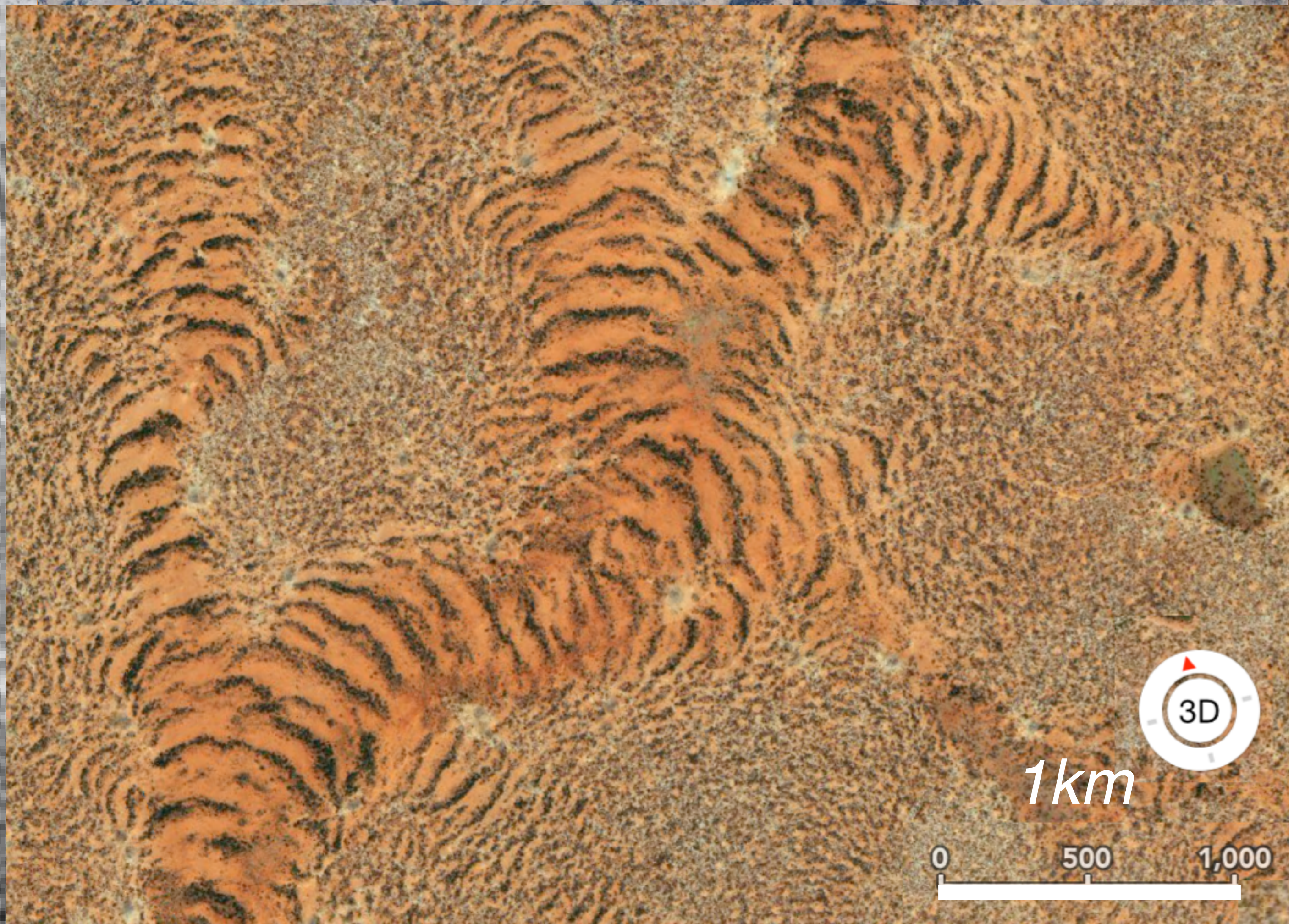
bands (black), gaps (dark gray), labyrinths (light gray) and non-periodic (white).

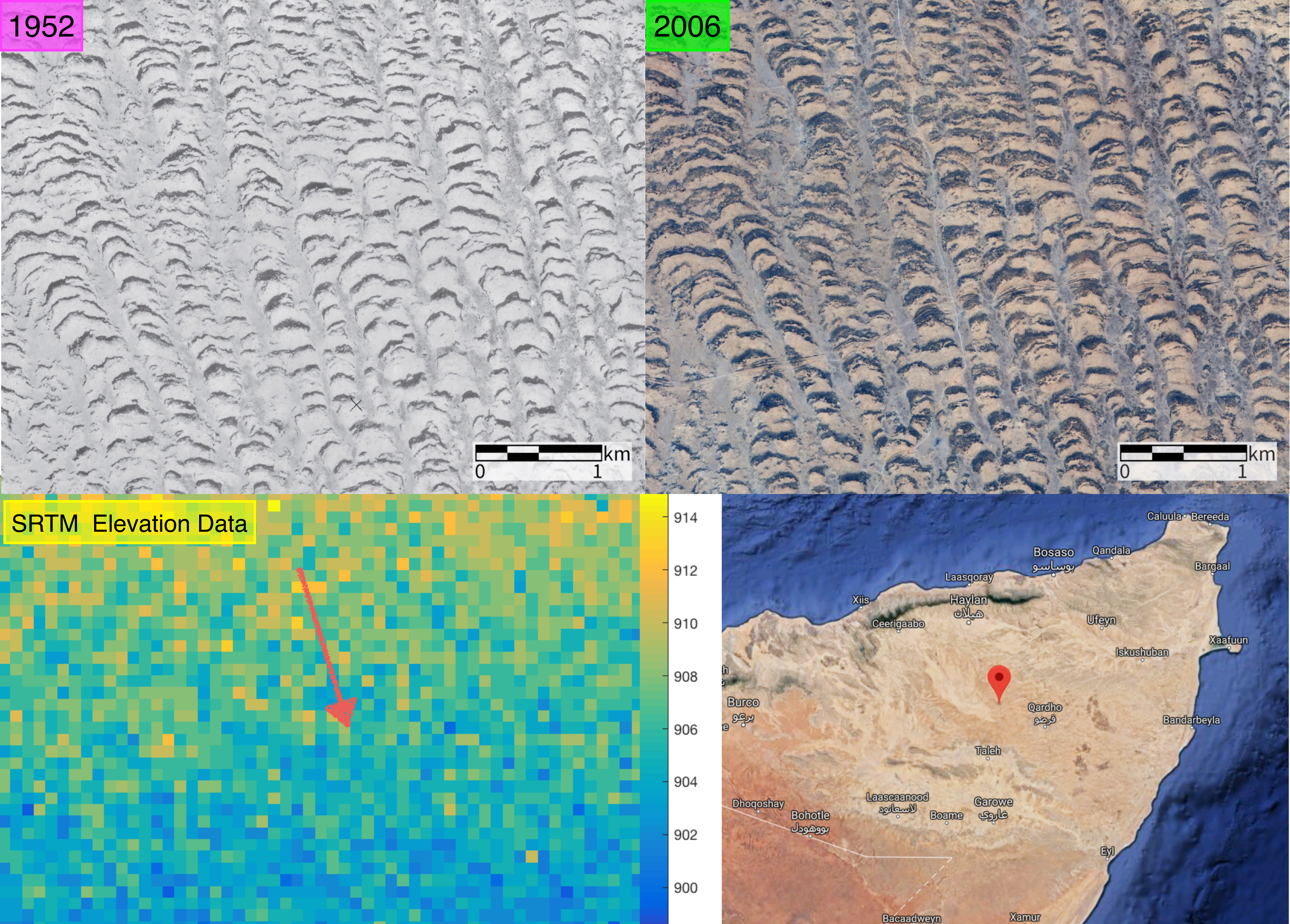
Topographic Influence: Upslope migration, Arcing, Channeling



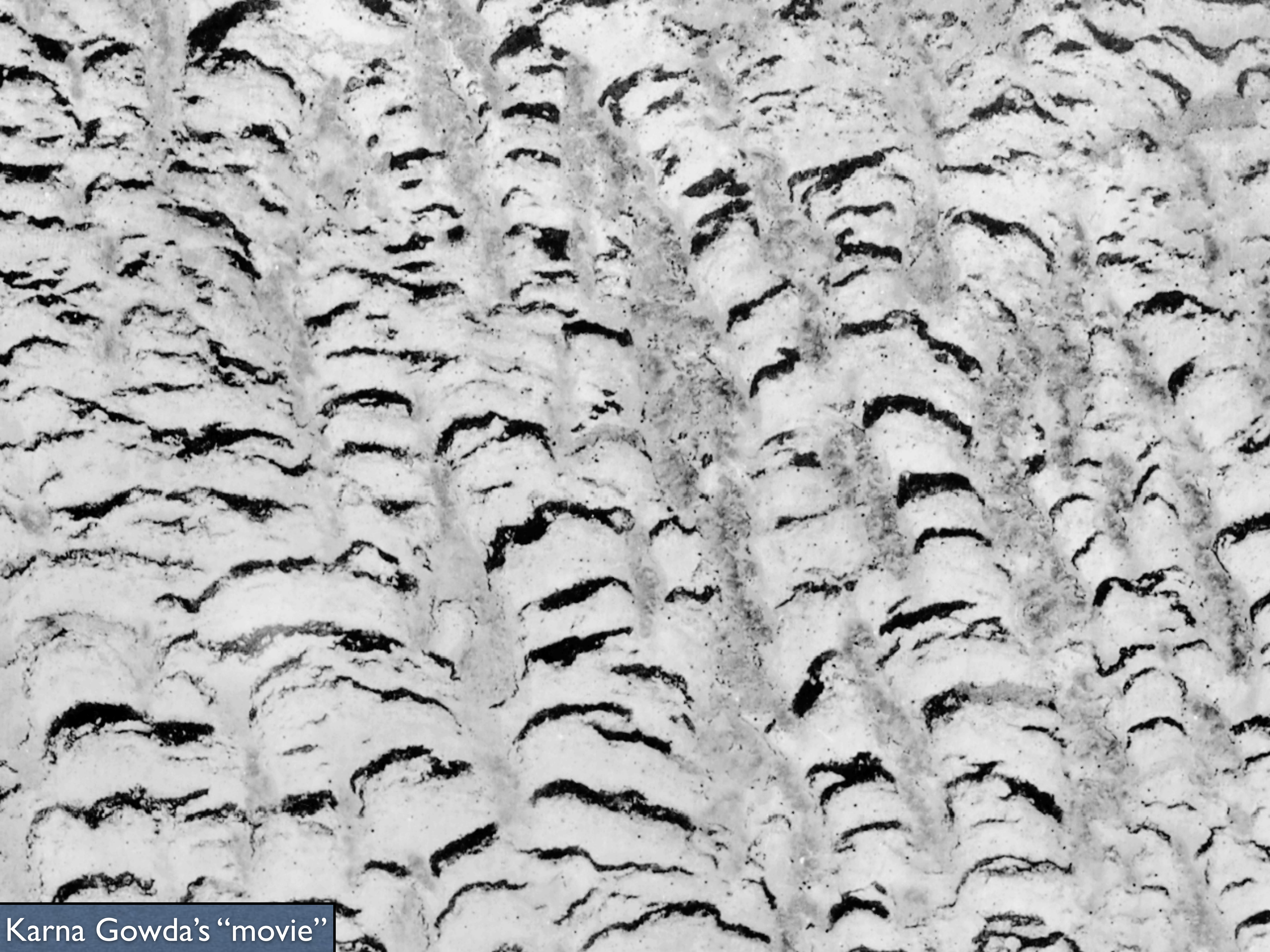








Karna Gowda[1], Sarah Iams[2] & Mary Silber[3] (2018)



Karna Gowda's “movie”

Reaction-Advection-Diffusion Models of Vegetation Patterns

Regular and Irregular Patterns in Semiarid Vegetation

Christopher A. Klausmeier

Science (1999)

Diversity of Vegetation Patterns and Desertification

J. von Hardenberg,[1,4] E. Meron,[1,3] M. Shachak,[2] and Y. Zarmi[1,3]

PRL (2001)

Self-Organization of Vegetation in Arid Ecosystems

Max Rietkerk,[1,2,*] Maarten C. Boerlijst,[3,†] Frank van Langevelde,[2,4,‡] Reinier HilleRisLambers,[3,§] Johan van de Koppel,[5,6,‖] Lalit Kumar,[7,#] Herbert H. T. Prins,[2,**] and André M. de Roos[3,††]

Am. Nat. (2002)

Ecosystem Engineers: From Pattern Formation to Habitat Creation

E. Gilad,[1,2] J. von Hardenberg,[3] A. Provenzale,[3,4] M. Shachak,[5] and E. Meron[2,1]

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Rise and Fall of Periodic Patterns for a Generalized Klausmeier–Gray–Scott Model

Sjors van der Stelt, Arjen Doelman, Geertje Hek, Jens D. M. Rademacher

JNLS (2013)

See Borgogno *et al.*, Reviews of Geophysics (2009)

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Christopher A. Klausmeier

11 JUNE 1999 VOL 284 SCIENCE

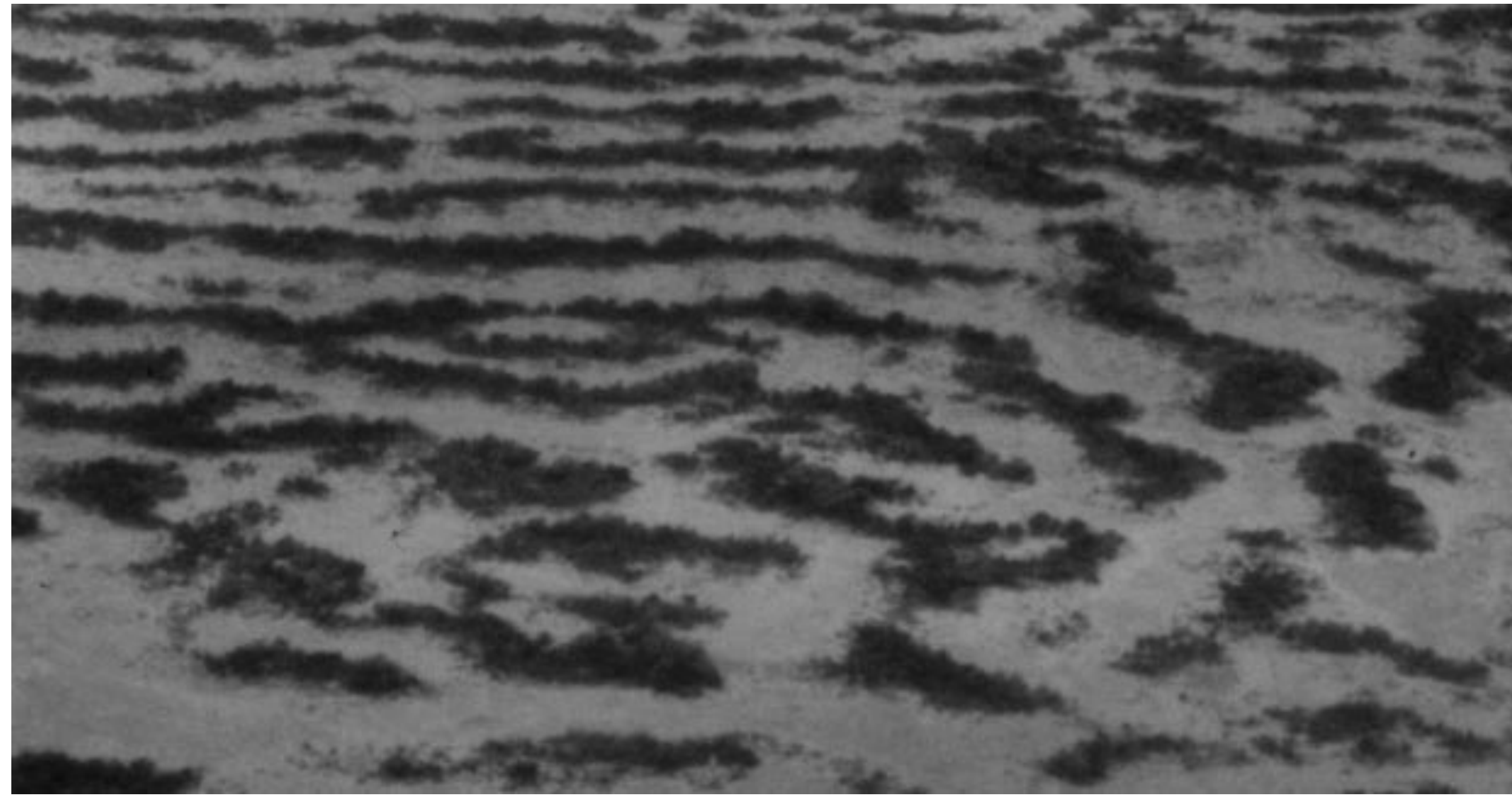
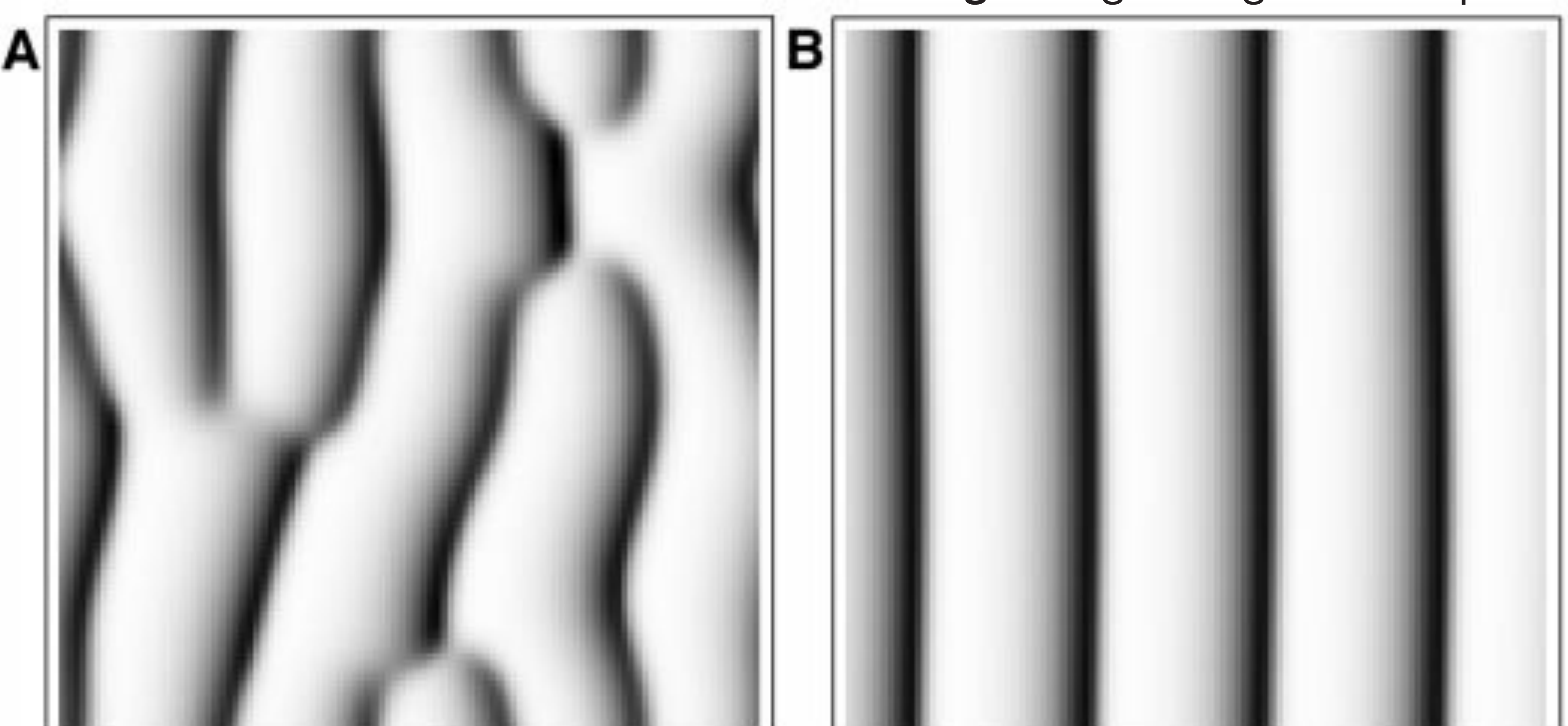


Fig. 1. Regular vegetation stripes near Niamey, Niger.



$$\frac{\partial W}{\partial T} = A - LW - RWN^2 + V\frac{\partial W}{\partial X}$$

$$\frac{\partial N}{\partial T} = RJWN^2 - MN + D\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)N$$

W = "Water"

N = "Biomass"

Reaction-Advection-Diffusion Models of Vegetation Patterns

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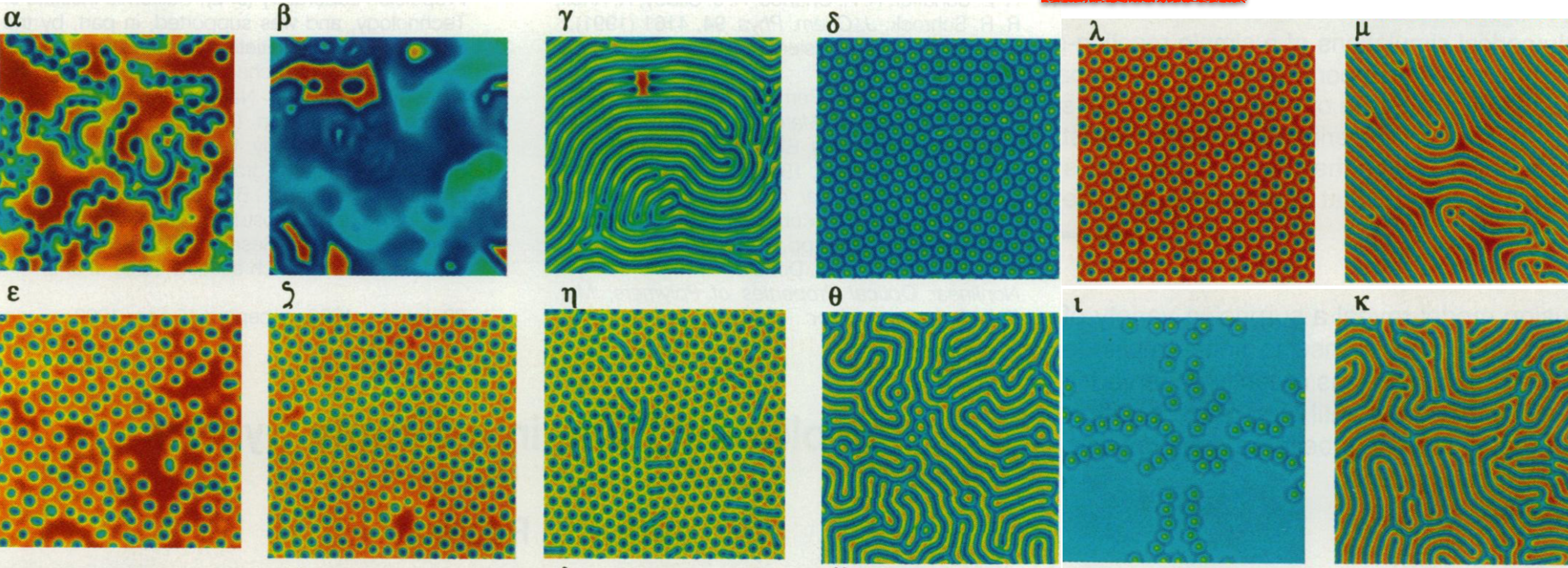
Complex Patterns in a Simple System

John E. Pearson SCIENCE • VOL. 261 • 9 JULY 1993



P. Gray and S. K. Scott, *Chem. Eng. Sci.* **38**, 29 (1983); *ibid.* **39**, 1087 (1984); *J. Phys. Chem.* **89**,

$$\frac{\partial U}{\partial t} = D_u \nabla^2 U - UV^2 + F(1 - U)$$

$$\frac{\partial V}{\partial t} = D_v \nabla^2 V + UV^2 - (F + k)V$$

$$U + 2V \rightarrow 3V$$

$$V \rightarrow P$$

\

Complex Patterns in a Simple System

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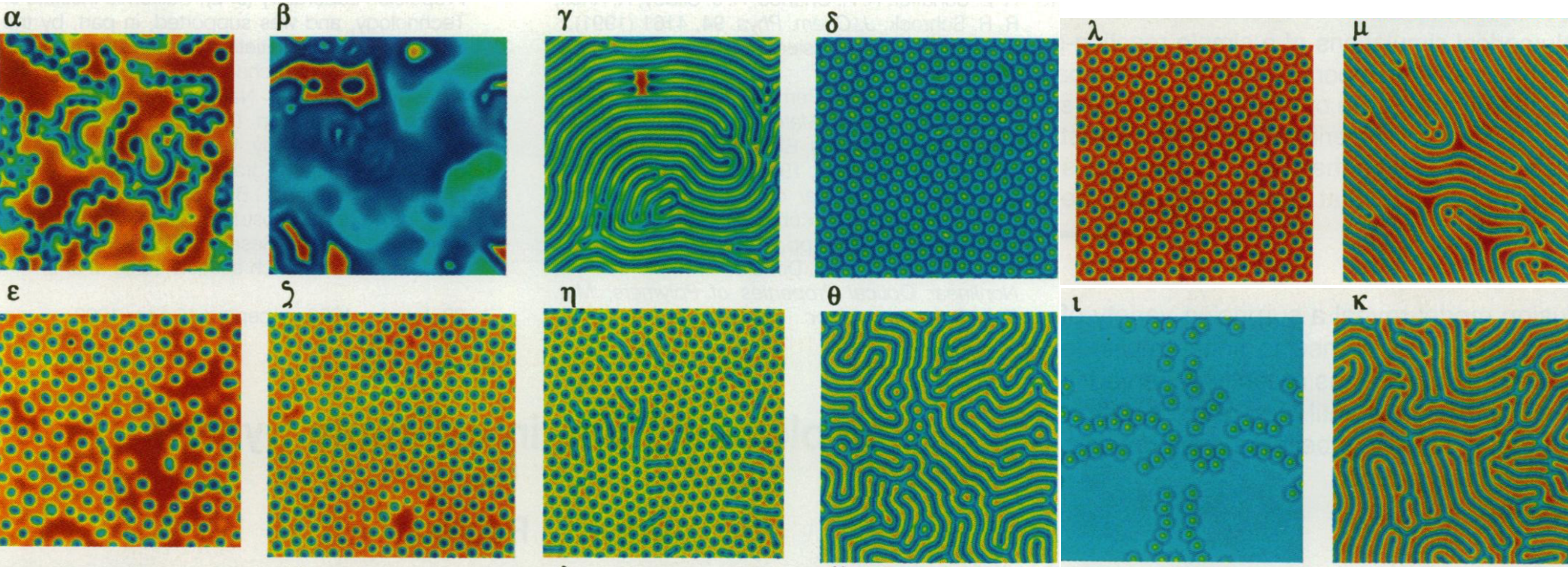

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$$\frac{\partial N}{\partial T} = RJWN^2 - MN + D\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)N$$

$$W = \frac{A}{L}U$$

$$N = \frac{AJ}{L}V$$

$$T = \frac{L^2}{A^2J^2R}\, t$$

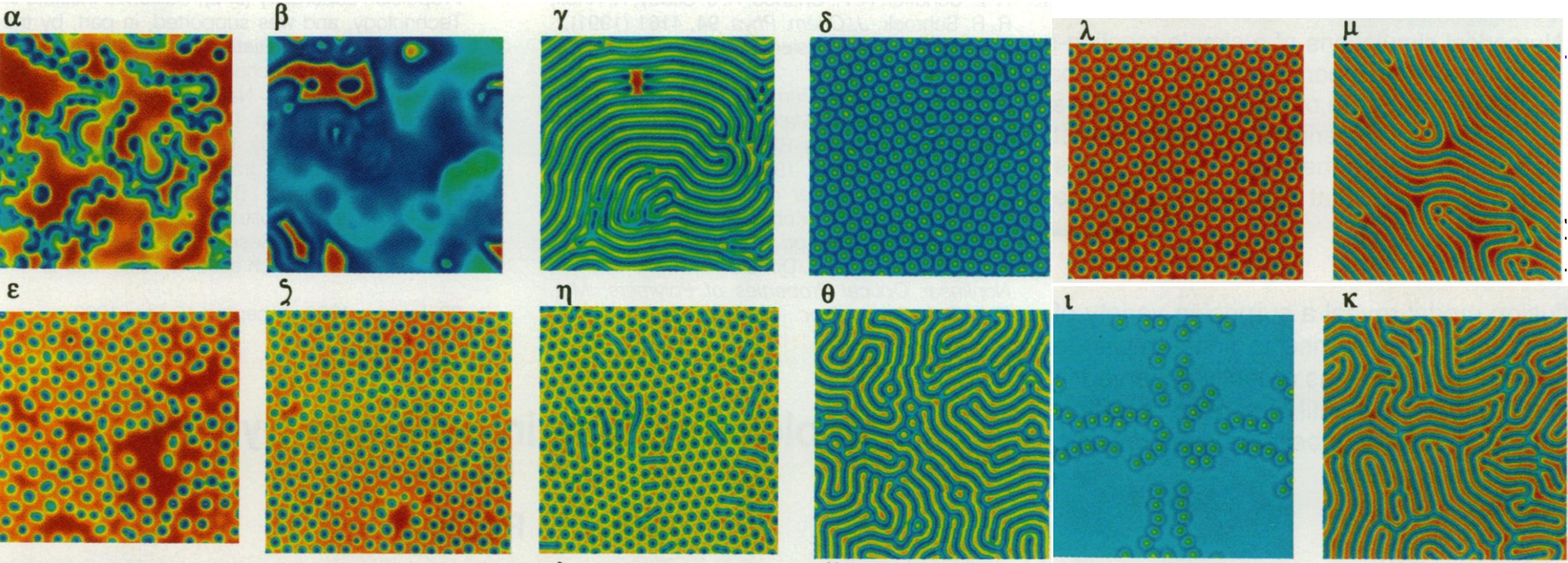
$$\frac{\partial U}{\partial t} = D_u\nabla^2 U - UV^2 + F(1 - U)$$

$$\frac{\partial V}{\partial t} = D_v\nabla^2 V + UV^2 - (F + k)V$$

$$F = \frac{L^3}{A^2J^2R} \qquad k = \frac{L^2(M - L)}{A^2J^2R}$$

Complex Patterns in a Simple System

John E. Pearson SCIENCE • VOL. 261 • 9 JULY 1993



Regular and Irregular Patterns in Semiarid Vegetation

Christopher A. Klausmeier

$$\frac{\partial W}{\partial T} = A - LW - RWN^2 + V\frac{\partial W}{\partial X}$$

Klausmeier's water transport down a "hill slope"

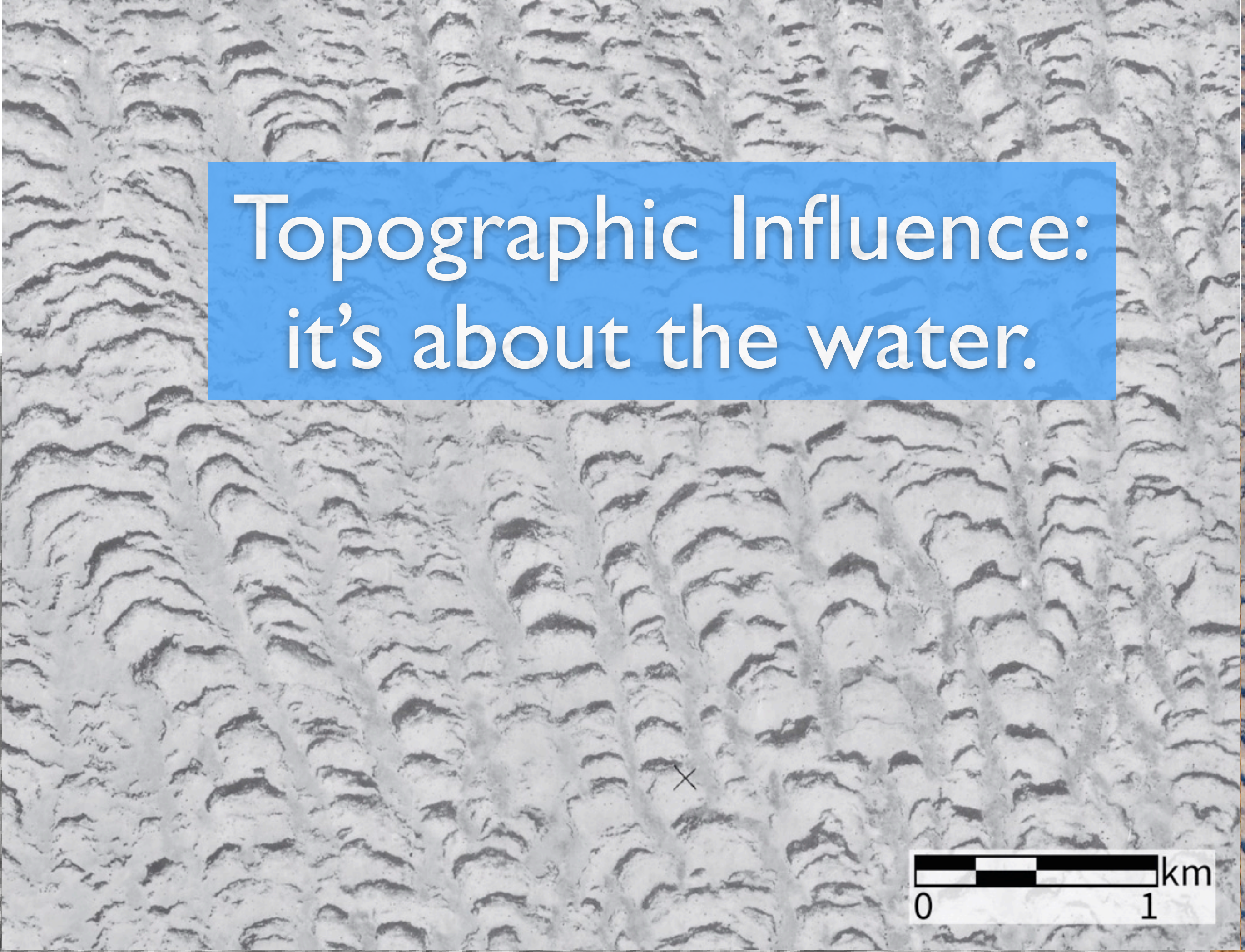
$$\frac{\partial N}{\partial T} = RJWN^2 - MN + D\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)N$$

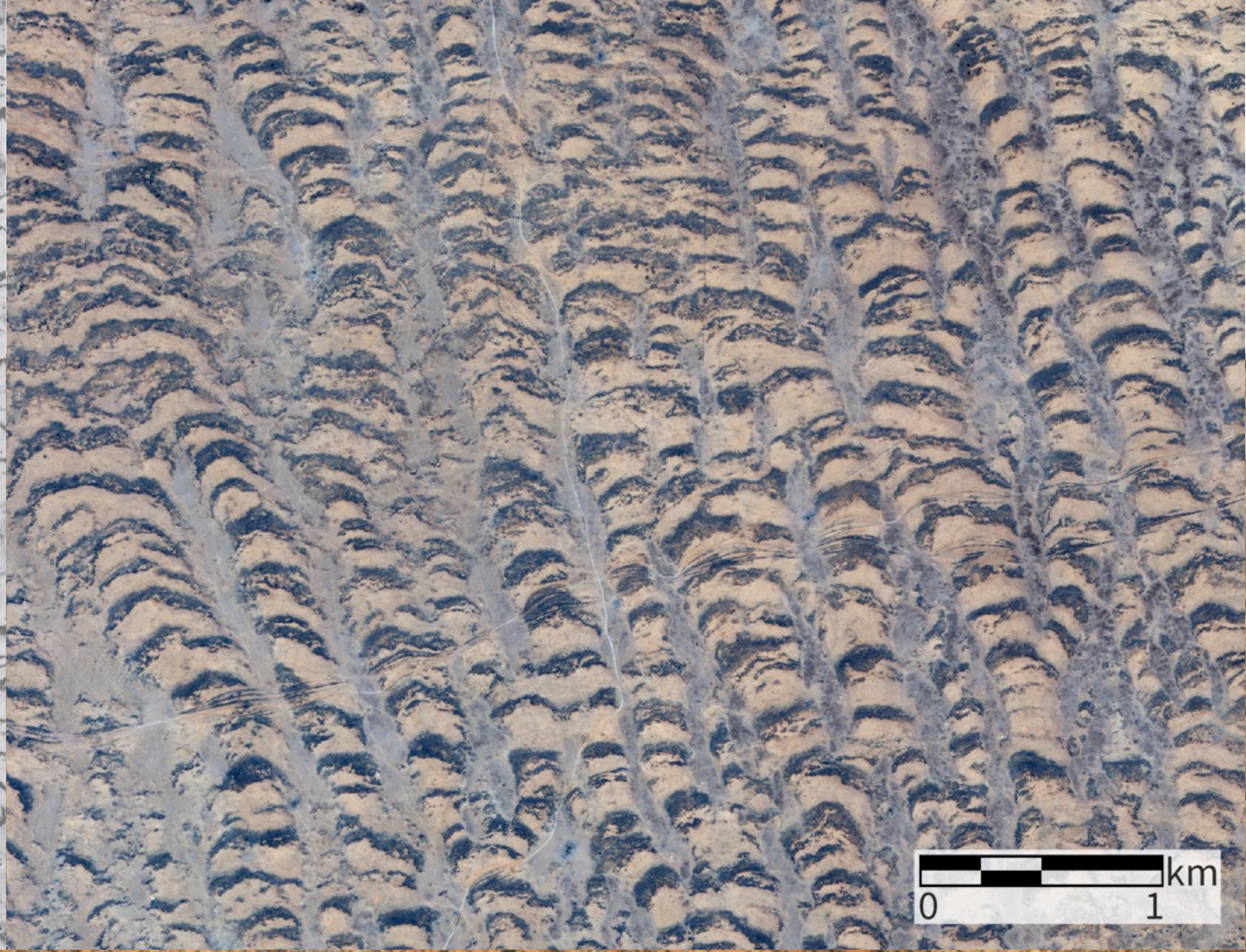
P. Gray and S. K. Scott, *Chem. Eng. Sci.* **38**, 29 (1983); *ibid.* **39**, 1087 (1984); *J. Phys. Chem.* **89**,

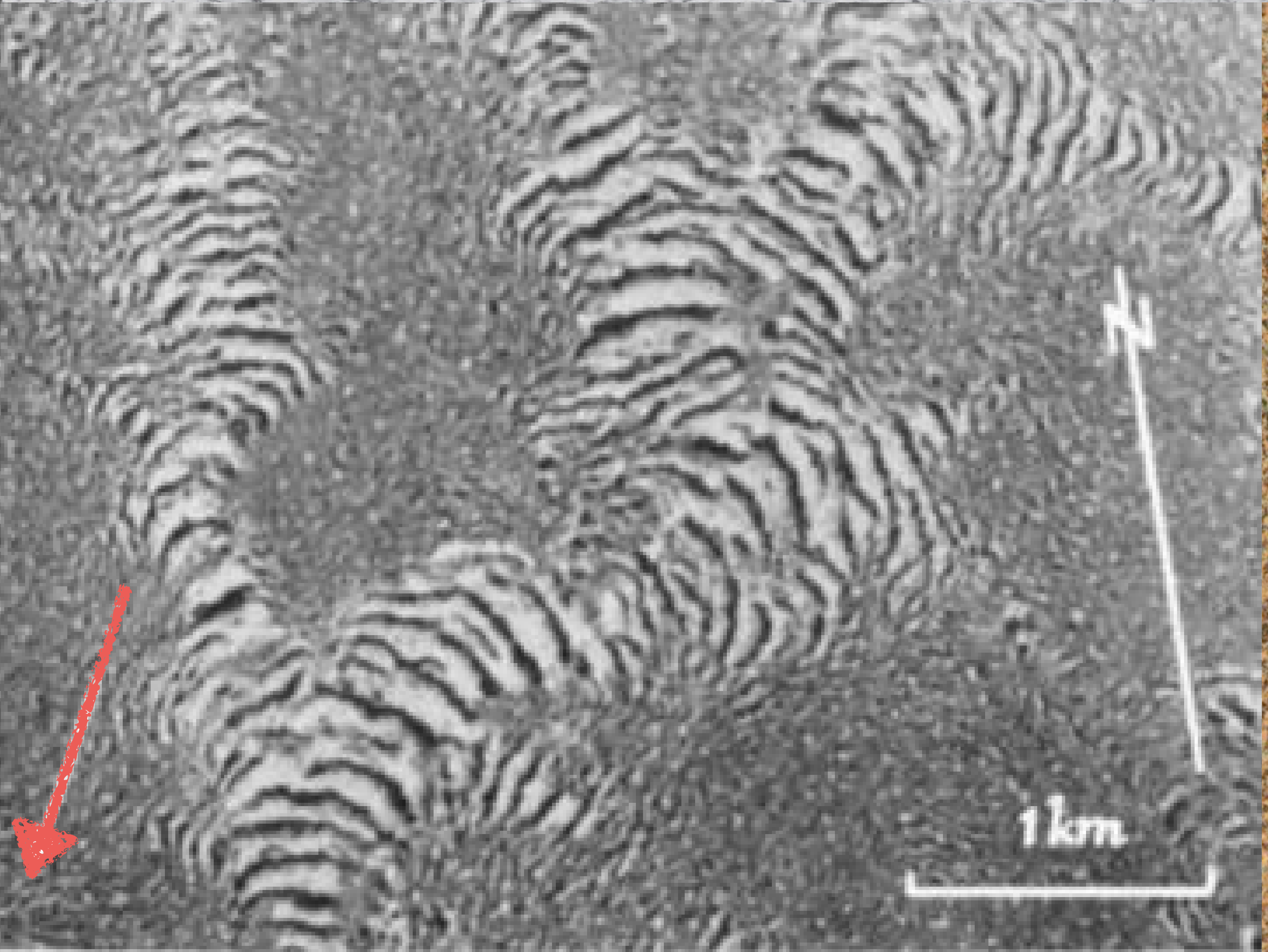
$$\frac{\partial U}{\partial t} = D_u\nabla^2 U - UV^2 + F(1 - U)$$

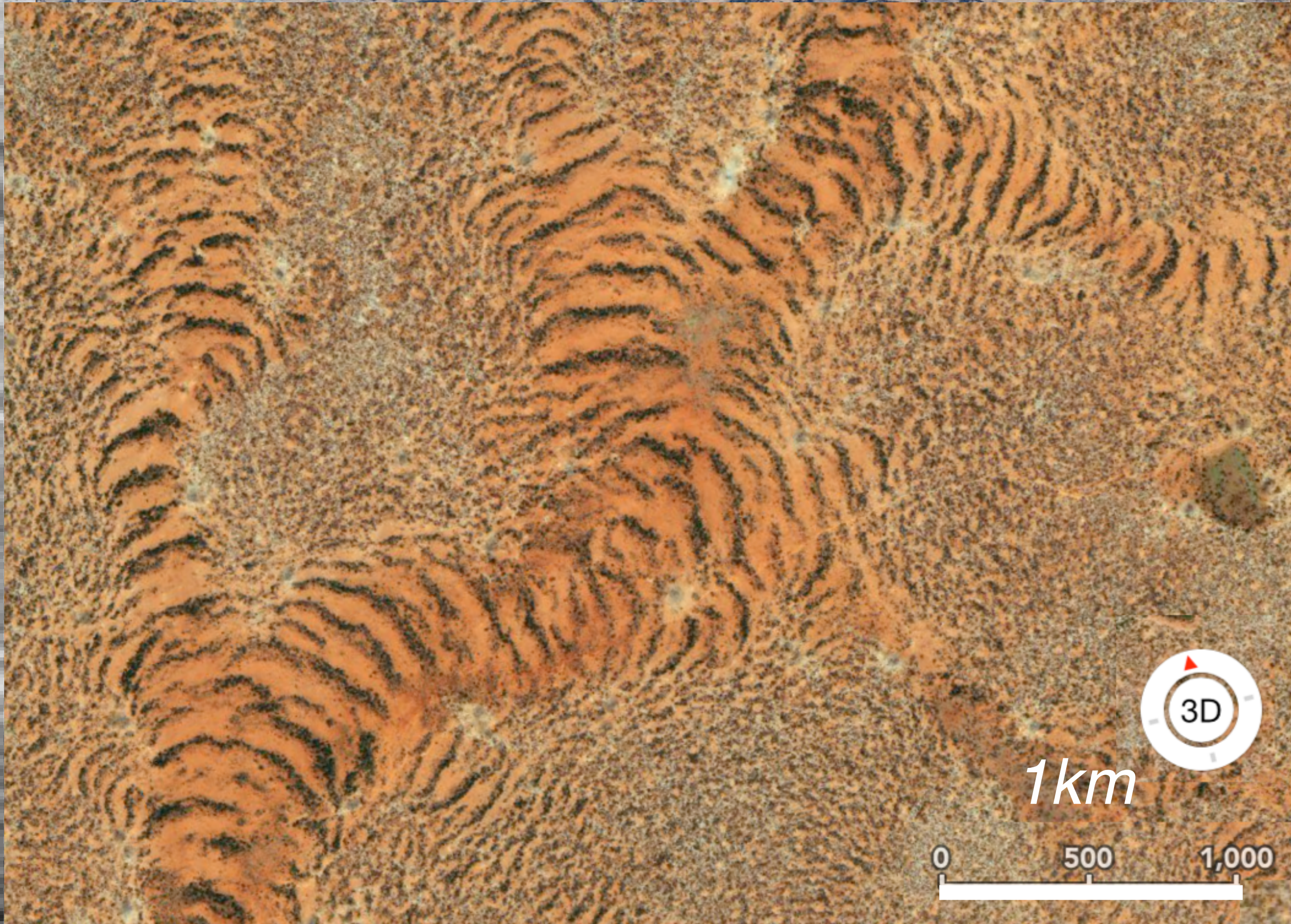
$$\frac{\partial V}{\partial t} = D_v\nabla^2 V + UV^2 - (F + k)V$$

Topographic Influence: it's about the water.





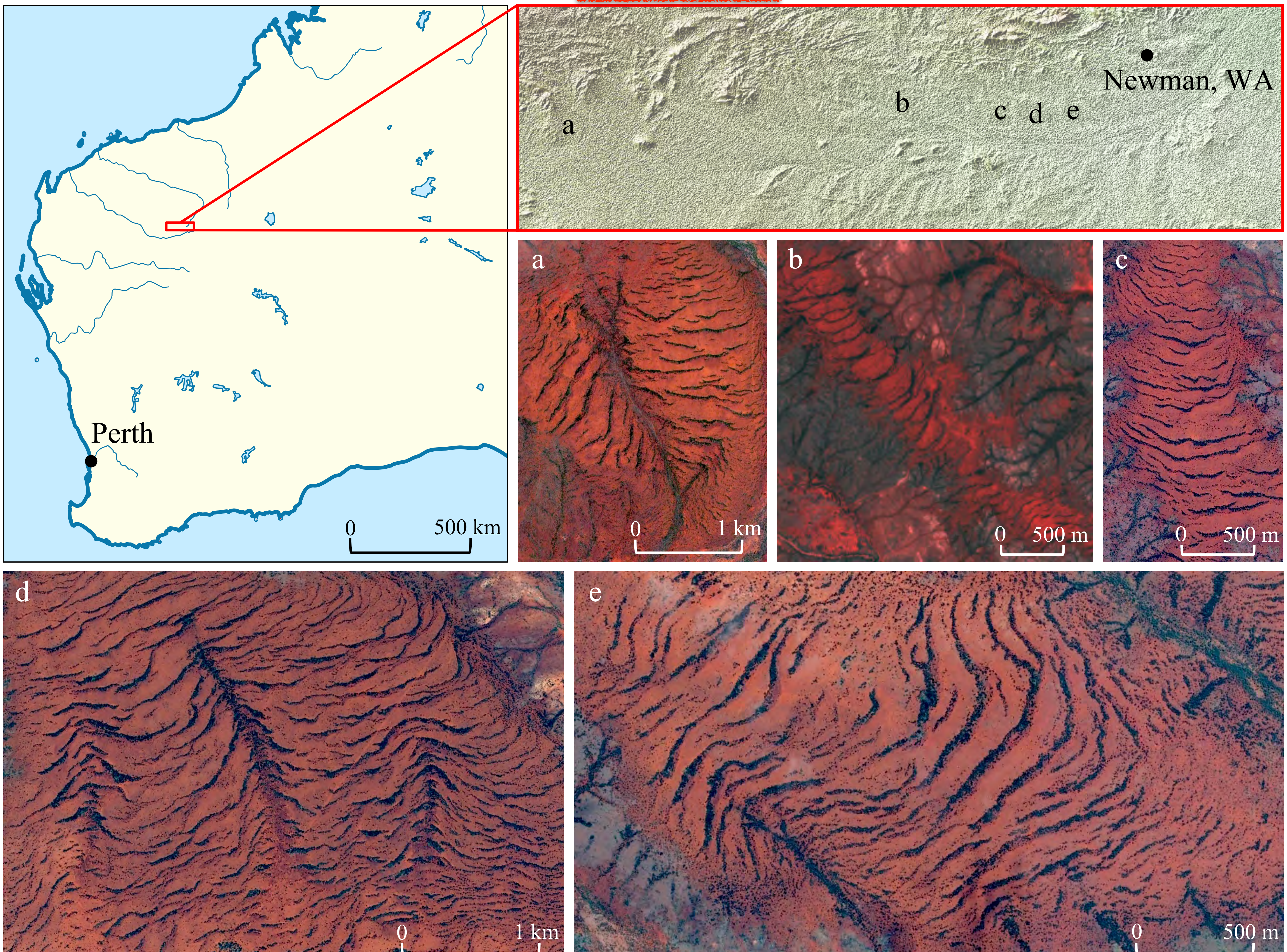




A topographic mechanism for arcing of dryland vegetation bands

Wed. 4pm MS88

Punit Gandhi,* Lucien Werner,† Sarah Iams,‡ Karna Gowda,§ Mary Silber¶



A. Regular and Irregular Patterns in Semiarid Vegetation

Christopher A. Klausmeier SCIENCE VOL 284 11 JUNE 1999

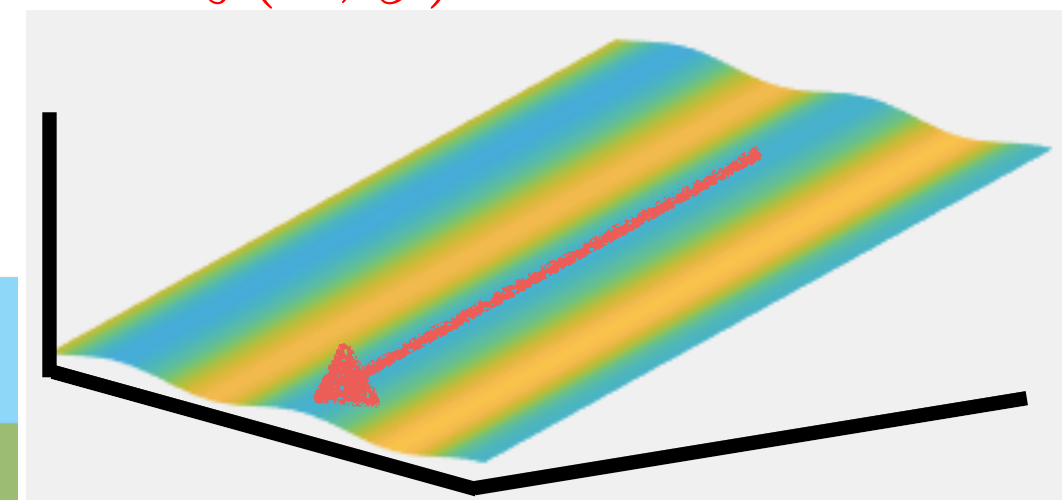
$$\partial_t B = \underbrace{J}_{\text{plant yield per unit water}} RWB^2 - \underbrace{MB}_{\text{mortality}} + \underbrace{D_B \nabla^2 B}_{\text{biomass dispersal}}$$

$$\partial_t W = \underbrace{P}_{\text{mean annual precipitation}} - \underbrace{LW}_{\text{evaporation}} - \underbrace{RWB^2}_{\text{transpiration}} + \underbrace{v\partial_x W}_{}$$

$v\partial_x W \rightarrow \nabla \cdot (W \nabla \zeta)$

"water" transport term:

$z = \zeta(x, y) = \text{elevation}$

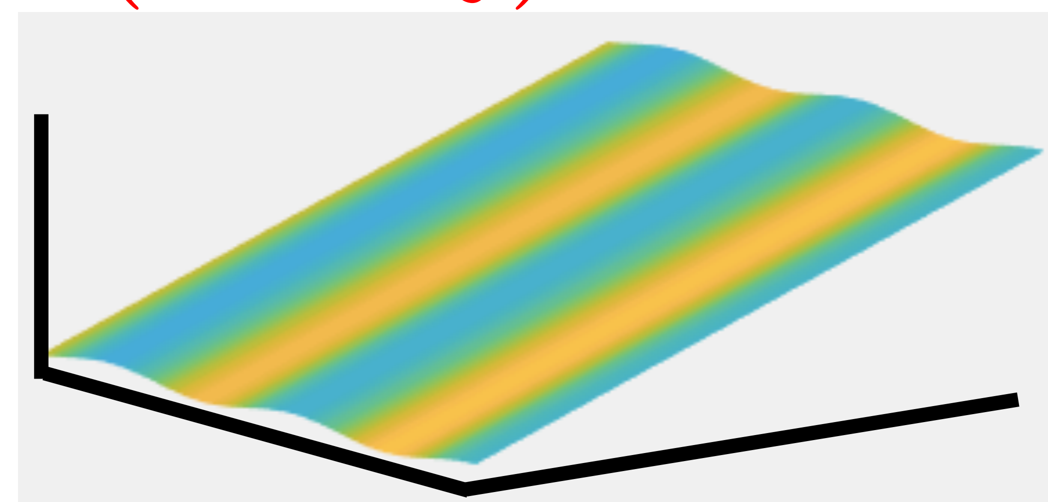


biomass density (B)

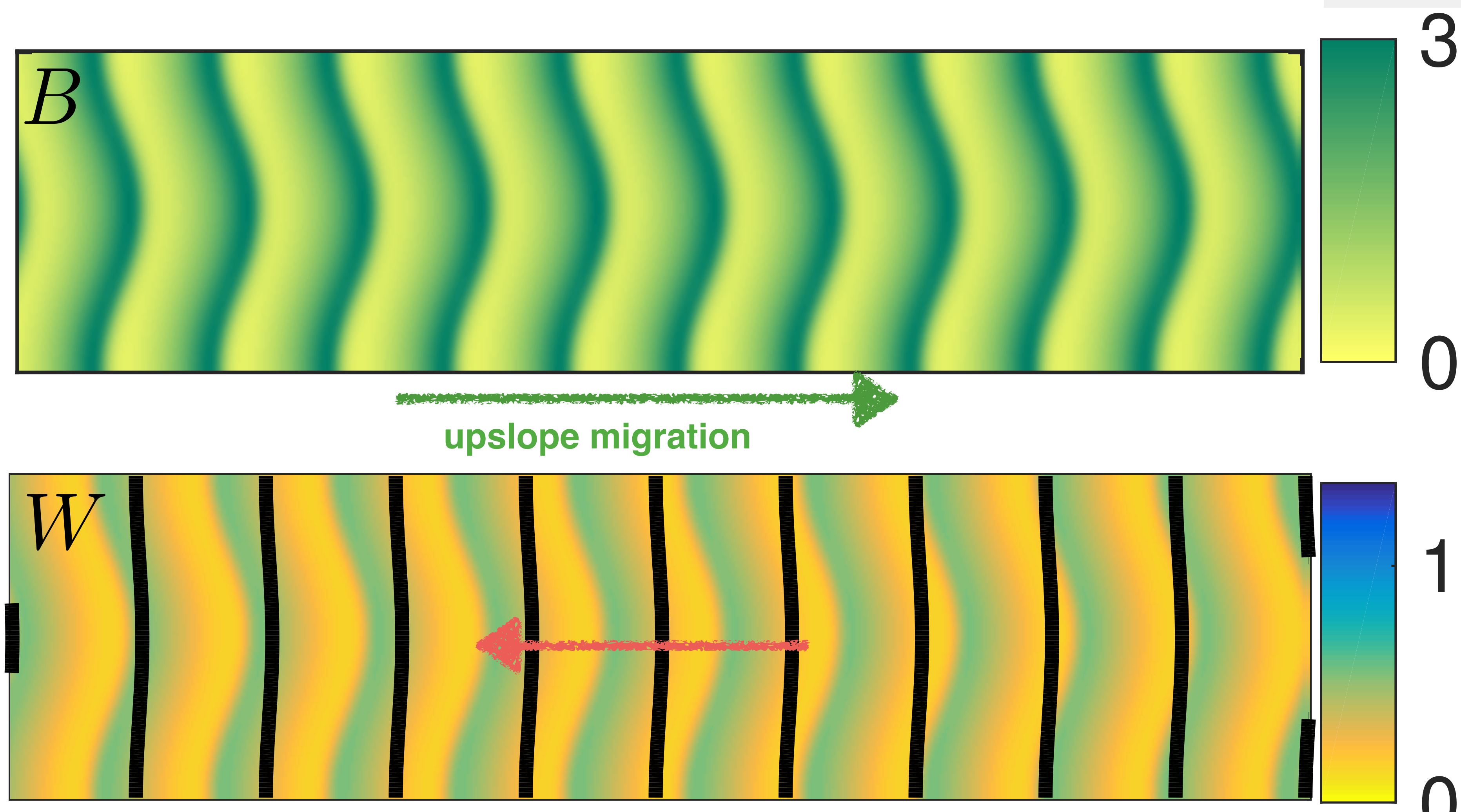
"water" (W)

$$\partial_t B = W B^2 - m B + \nabla^2 B$$

$$\partial_t W = a - W - W B^2 + \nabla \cdot (W \nabla \zeta)$$



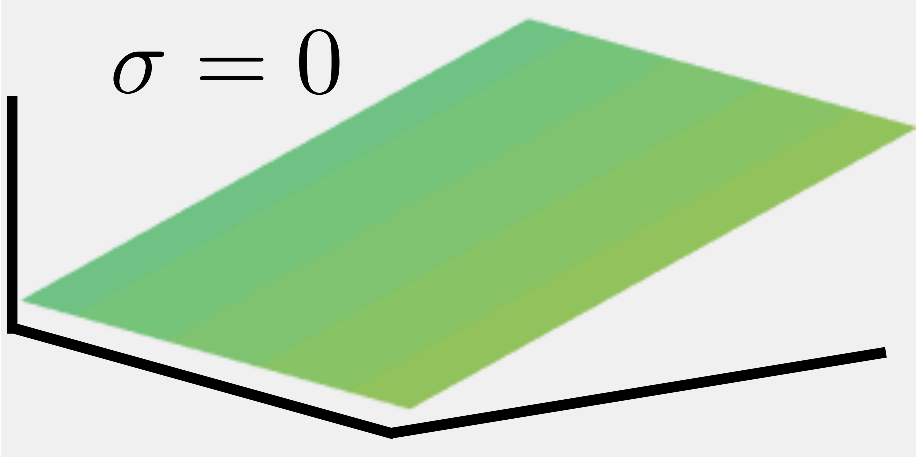
$$\nabla \cdot (W \nabla \zeta) = \nabla \zeta \cdot \nabla W + (\nabla^2 \zeta) W$$

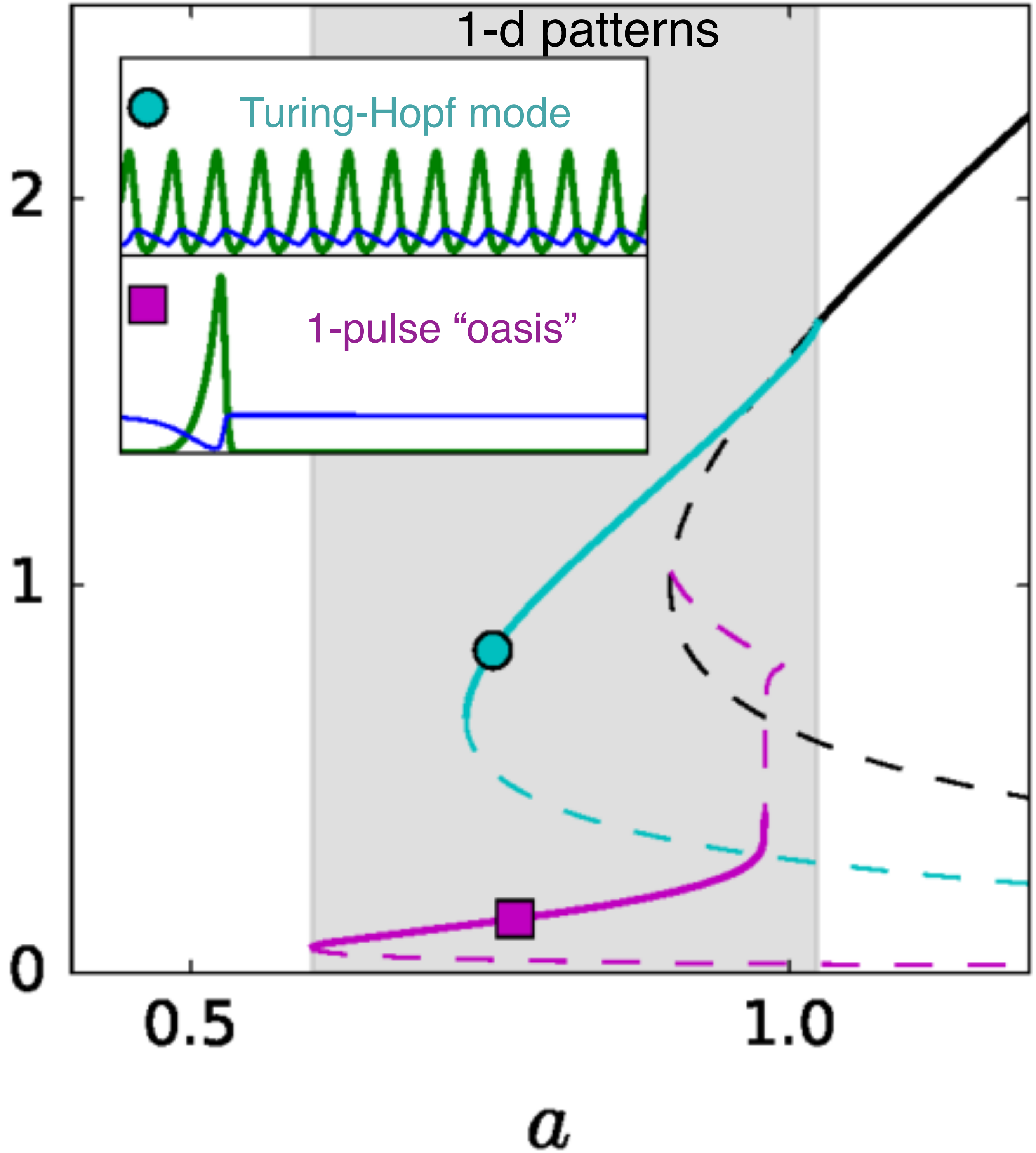


Gandhi et al. (2018)
Iams Wed. 4pm MS88

$$\partial_t B = W B^2 - mB + \partial_x^2 B$$

$$\partial_t W = a - W - W B^2 + v \partial_x W$$

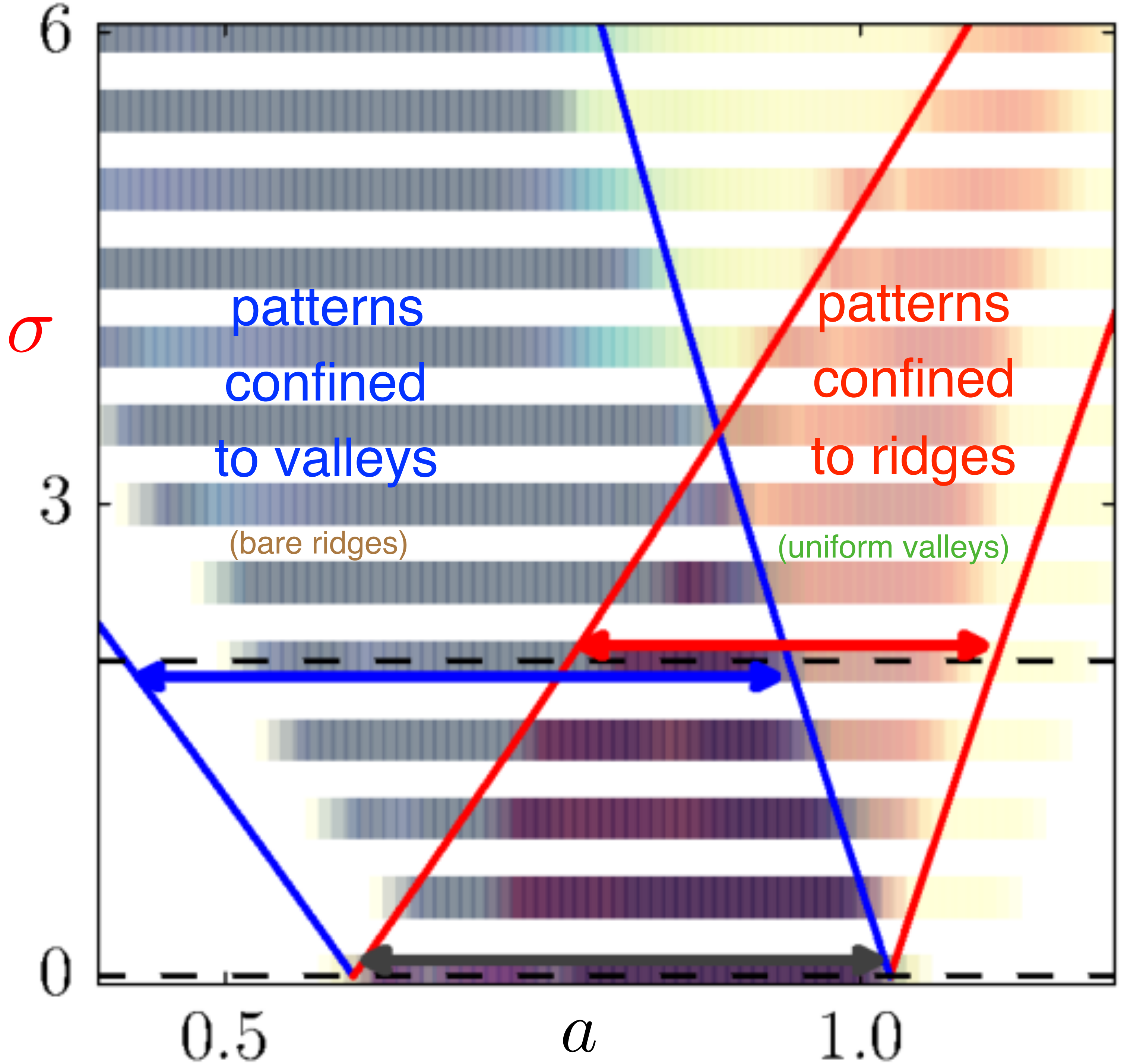


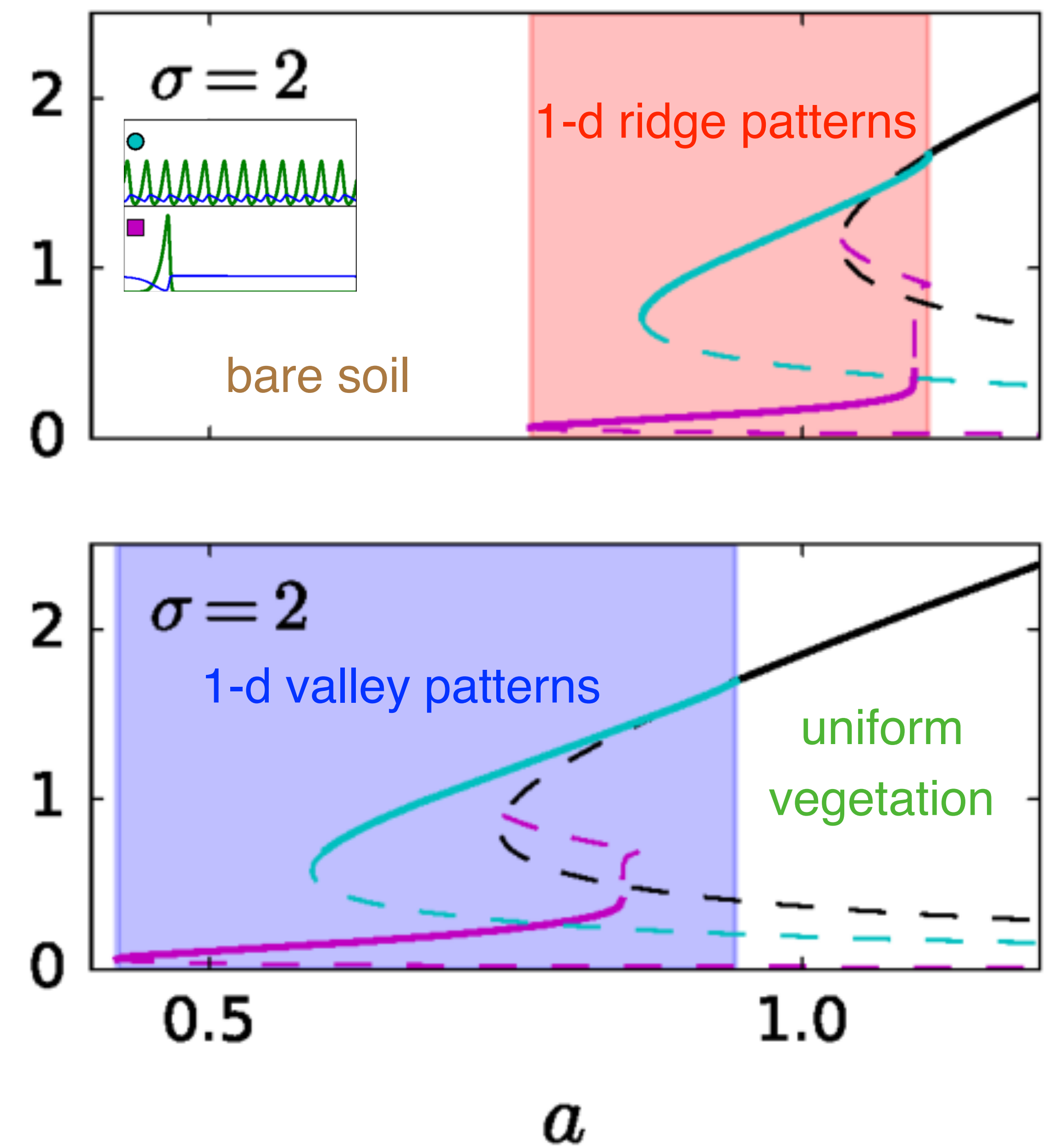


$$\partial_t B = W B^2 - m B + \nabla^2 B$$

$$\partial_t W = a - \underbrace{(1 - \nabla^2 \zeta)}_{\text{modified loss rate}} W - W B^2 + \nabla \zeta \cdot \nabla W$$

$$\zeta = v\Big(x + \sigma \cos(k_0 y)\Big)$$

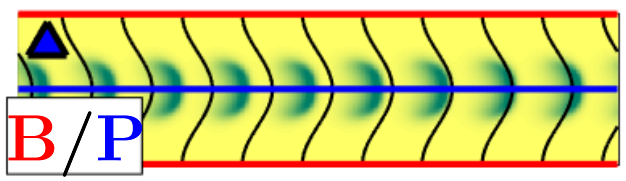


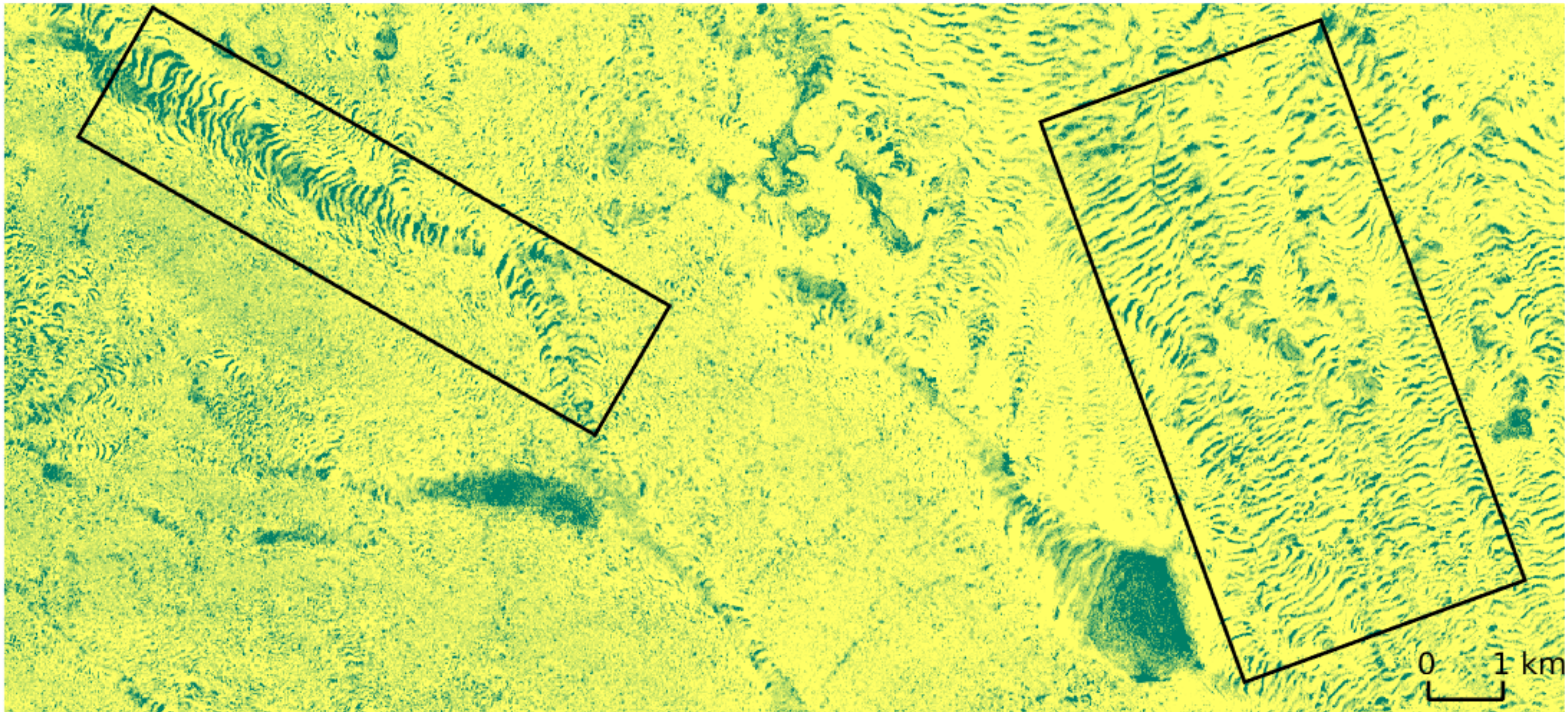


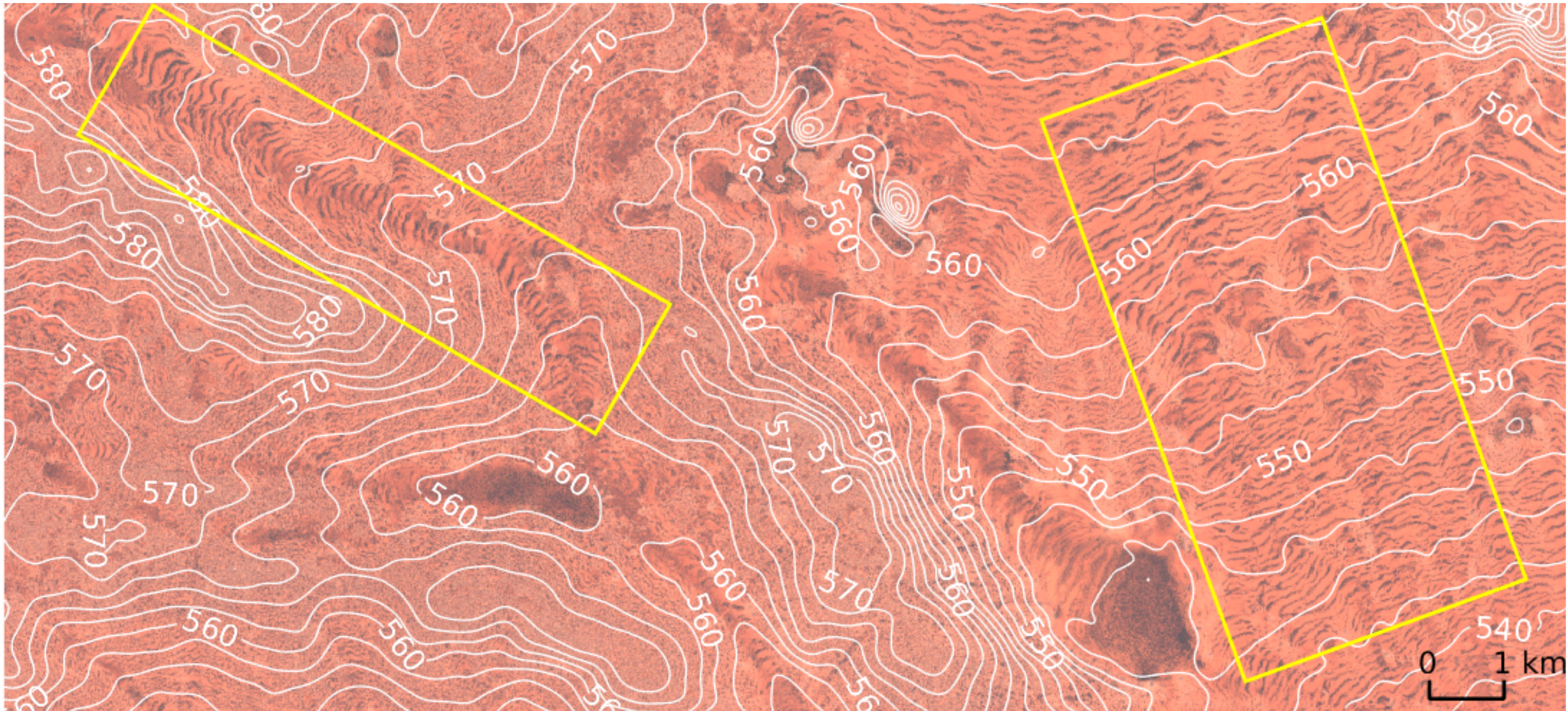
Gandhi et al. (2018)

Iams Wed. 4pm MS88

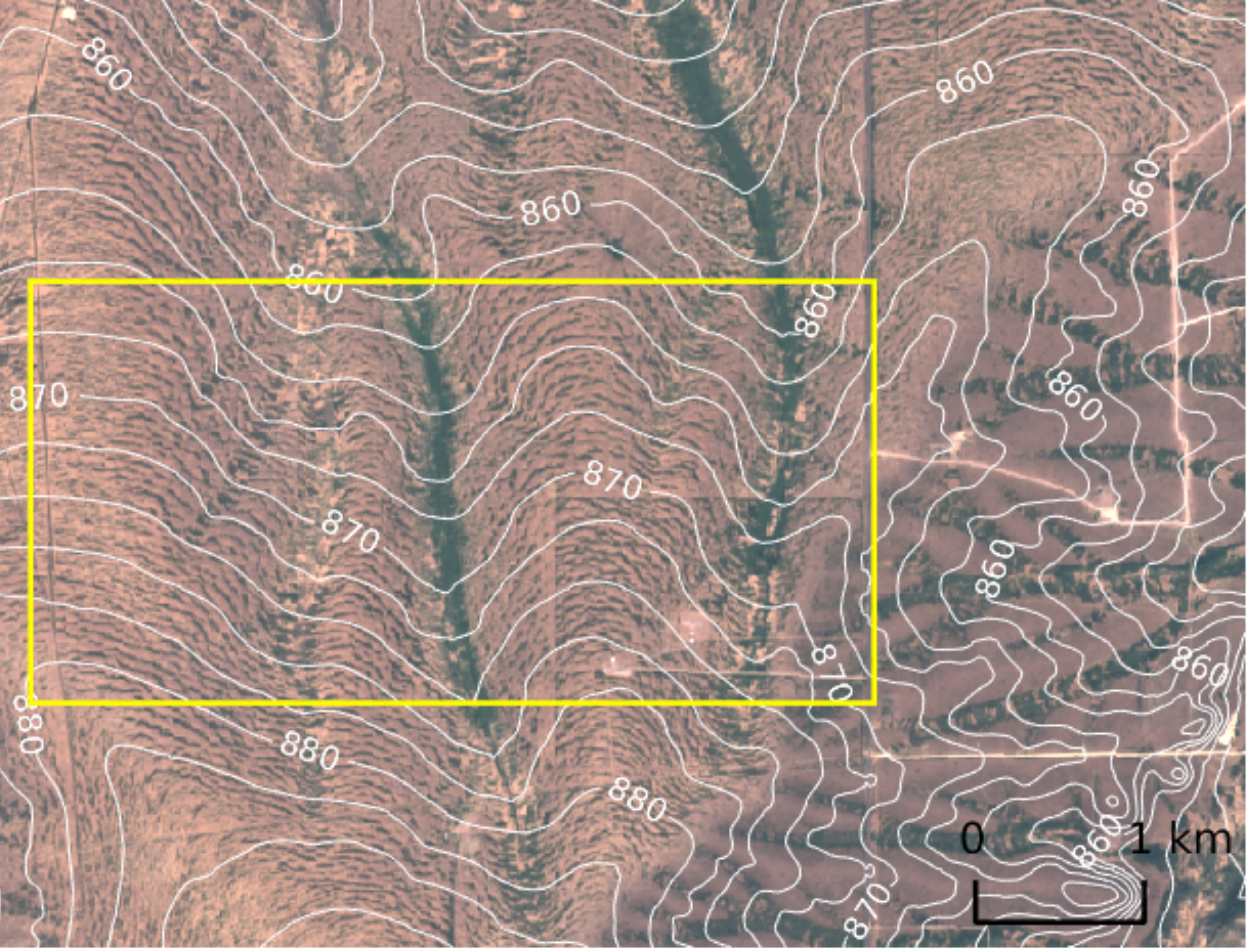
Vegetation Bands in Ethiopia
(MAP<150 mm, AI~0.08)

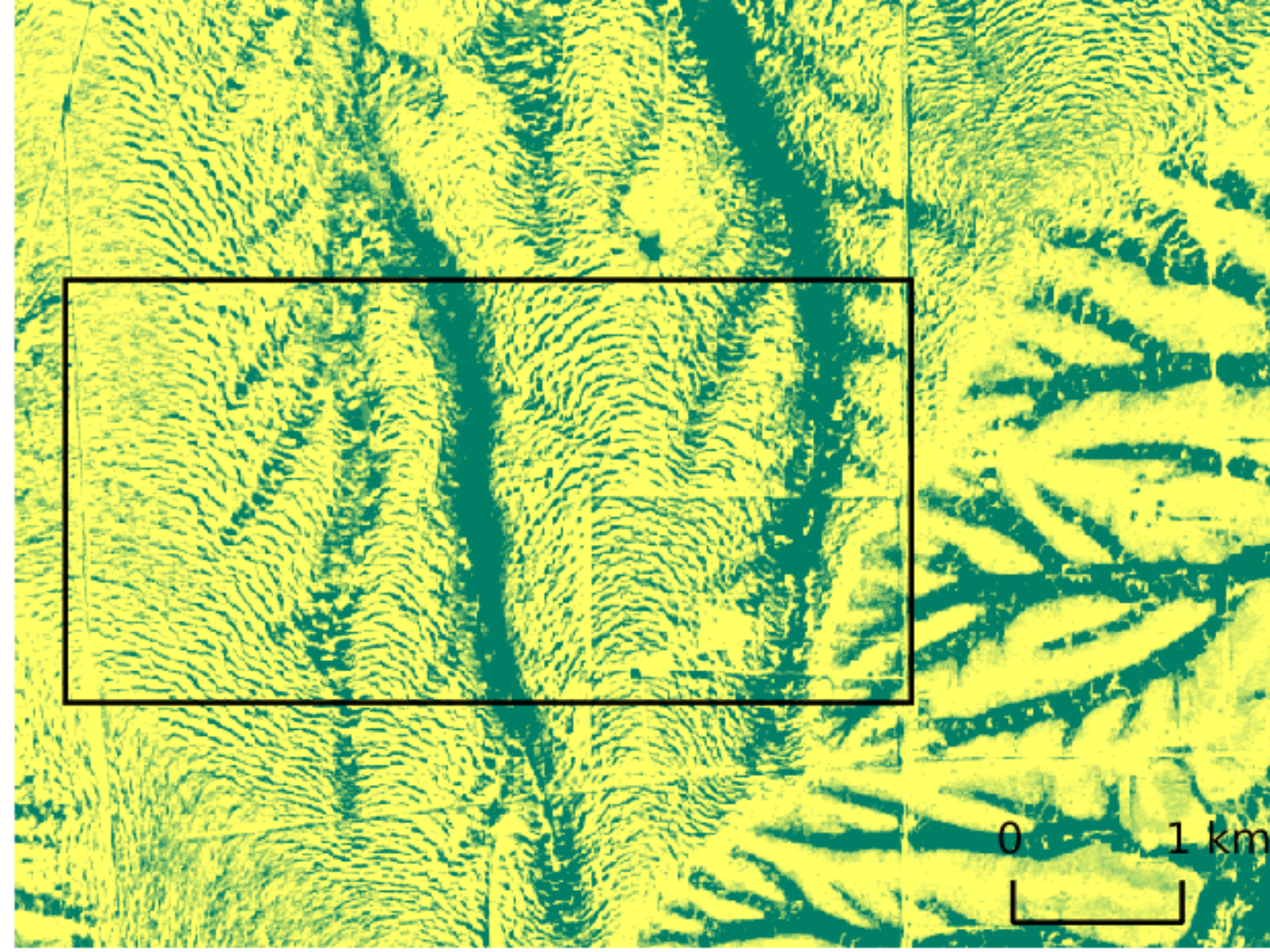




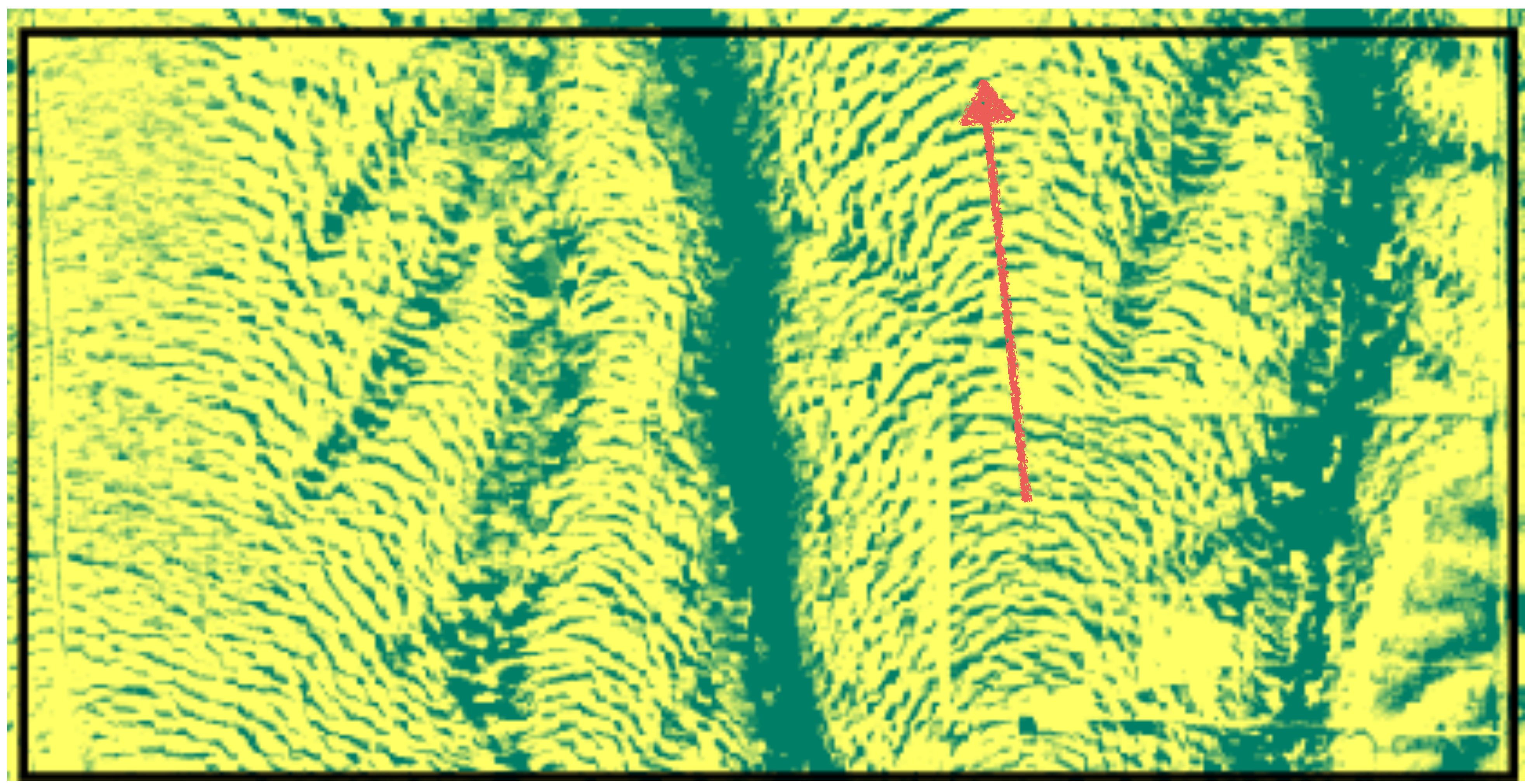


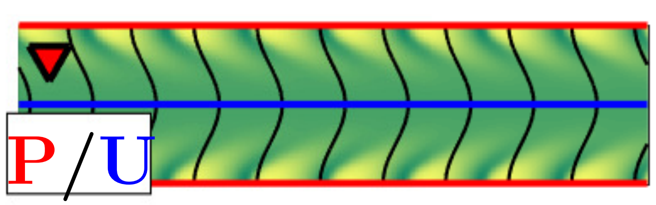
Gandhi et al. (2018)
Iams Wed. 4pm MS88





Vegetation Bands in Texas
(MAP>300 mm, AI~0.19)





Gandhi et al. (2018)
Iams Wed. 4pm MS88

B. Rietkerk model - a framework for explicitly modeling infiltration feedback, Klausmeier's "water" -> surface water+soil water

Self-Organization of Vegetation in Arid Ecosystems

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Am. Nat. (2002)

$$\partial_t h = \underbrace{p}_{\text{precip.}} - \underbrace{I(n)h}_{\text{infil.}} + \underbrace{D_h \nabla^2 h}_{\text{diffusion}}$$

surface water

$$\partial_t w = -\underbrace{\nu w}_{\text{evap.}} + \underbrace{I(n)h}_{\text{infil.}} - \underbrace{\gamma G(w)n}_{\text{transp.}} + \underbrace{D_w \nabla^2 w}_{\text{diffusion}},$$

soil water

$$\partial_t n = -\underbrace{\mu n}_{\text{mort.}} + \underbrace{G(w)n}_{\text{growth}} + \underbrace{\nabla^2 n}_{\text{dispersal}},$$

plant density

Model Nonlinearities:

$$I(n) = \alpha \frac{n+f}{n+1}$$

$$G(w) = \frac{w}{w+1}.$$

See also:

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PRL (2004)

C. And yet another, with the help of our ecohydrologist friends -

(Sara Bonetti, Amilcare Porporato)

resolving
rain events

$$\partial_T H = R(T) - K\left(\frac{B+fQ}{B+Q}\right)\left(\frac{H}{H+A}\right)\left(\frac{1-s}{1-s+\beta}\right) + \sqrt{S}\, K_w \partial_X\left(\frac{H}{1+NB}\right)$$

surface water

infiltration

$$\phi Z_r \partial_T s = K\left(\frac{B+fQ}{B+Q}\right)\left(\frac{H}{H+A}\right)\left(\frac{1-s}{1-s+\beta}\right) - rs - \Gamma s B$$

soil moisture
$s \in [0,1]$

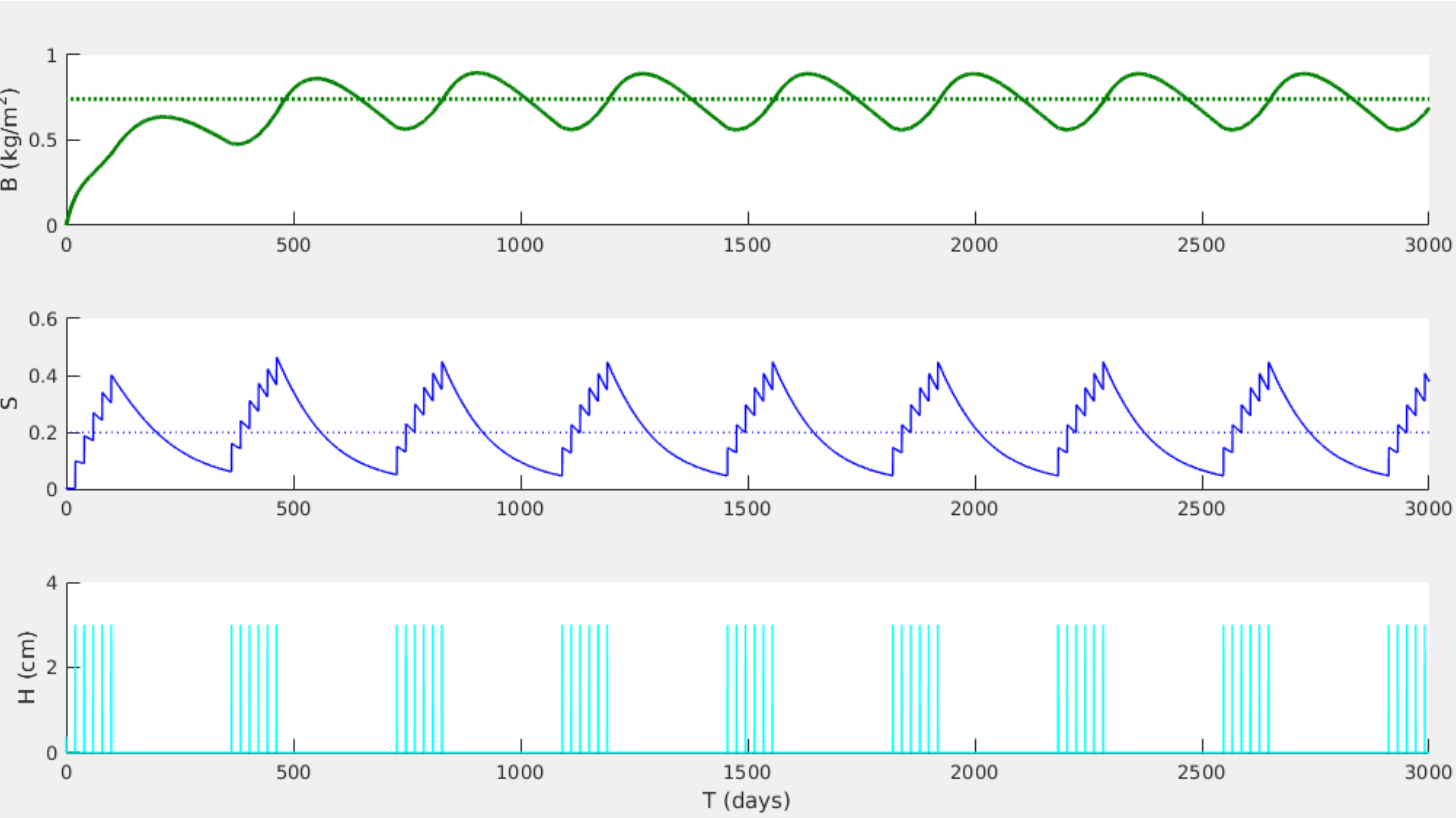
transpiration-growth

$$\partial_T B = D_b \partial_X^2 B - mB + w\Gamma s B(K_b - B)$$

biomass density

Aim 1: better constrain some model parameters.
Price: resolving fast timescales.

Challenge: 3 timescales for “reaction kinetics” years, days, hours



Water transport?

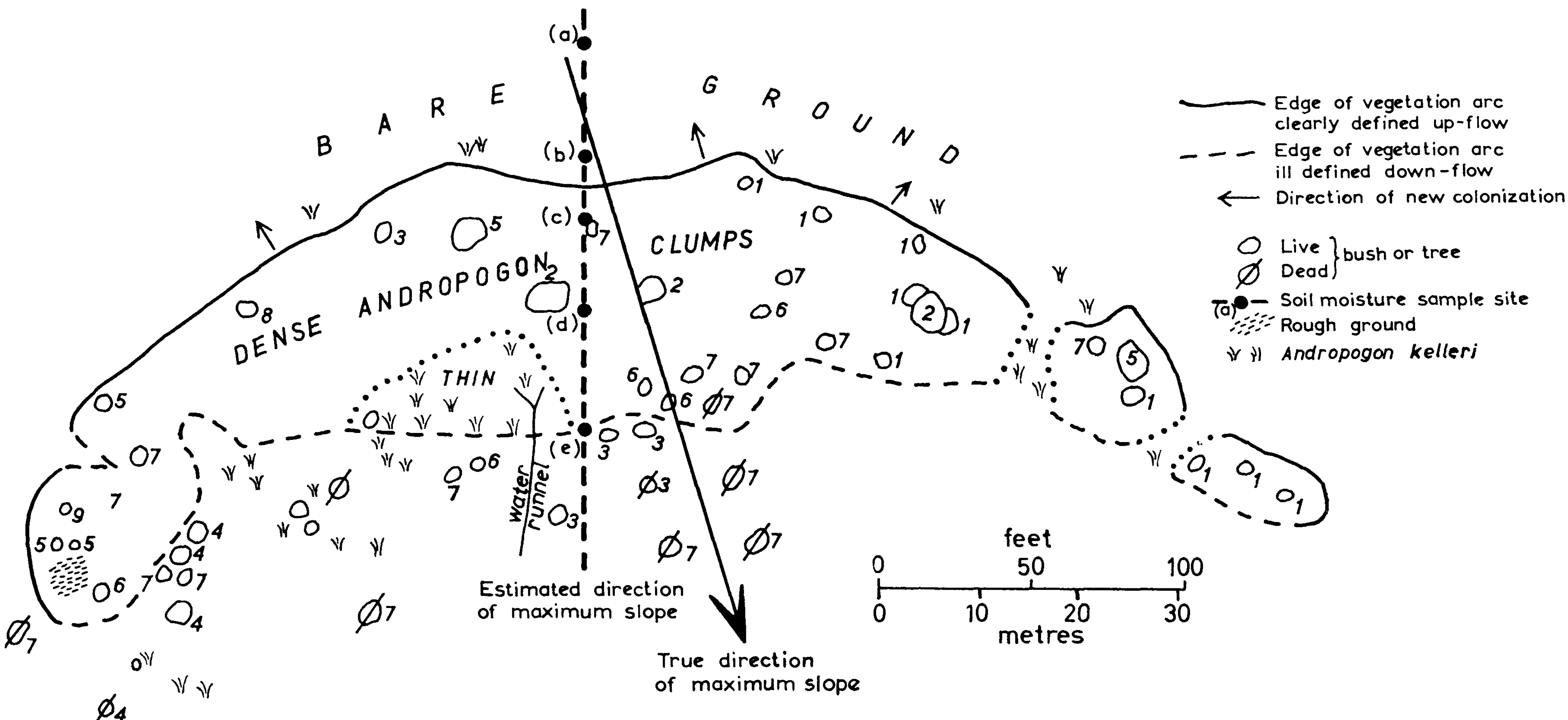
VEGETATION ARCS IN SOMALILAND

By C. F. HEMMING

Anti-Locust Research Centre, London

(1965, Journal of Ecology)

In the bare areas between the arcs much of the rain runs off as sheet-flow, and this gradually removes the fine material from the surface together with light-weight litter, such as the remains of dead grass and animal excreta. These materials are deposited where the sheet-flow is arrested by the next vegetation arc down the slope. In the dry season



Aim 2: Incorporate feedback in transport

surface roughness feedback

slope

$$\partial_T H = R(T) - K\left(\frac{B+fQ}{B+Q}\right)\left(\frac{H}{H+A}\right)\left(\frac{1-s}{1-s+\beta}\right) + \sqrt{S}\, K_w \partial_X \left(\frac{H}{1+NB}\right)$$

surface water

no soil water diffusion

$$\phi Z_r \partial_T s = K\left(\frac{B+fQ}{B+Q}\right)\left(\frac{H}{H+A}\right)\left(\frac{1-s}{1-s+\beta}\right) - rs - \Gamma s B$$

soil moisture

$s \in [0,1]$

$$\partial_T B = D_b \partial_X^2 B - mB + w\Gamma s B(K_b - B)$$

biomass density

seed dispersal (fit parameter)

hydrologist's "tomato plant term" (+ensures conservation of transport parameter #)

Some Preliminary Results on the Transport Parameters:

$$\sqrt{S} K_w \partial_X \left(\frac{H}{1+NB} \right)$$

$$D_b \partial_X^2 B$$

approach 1:

choose Turing-Hopf onset parameters: (k_c, Ω_c, p_c)
for a *constant* rain input p based on observations
(100 m wavelength, 30 cm/yr migration speed, 50-500 mm/year precipitation,
typical 0.5% grade $S = 0.005$)

solve for transport parameters: $(D_b, K_w, N) = (2\ cm^2/day,\ 3 \times 10^8\ cm/day,\ 5 \times 10^4\ cm^2/kg)$

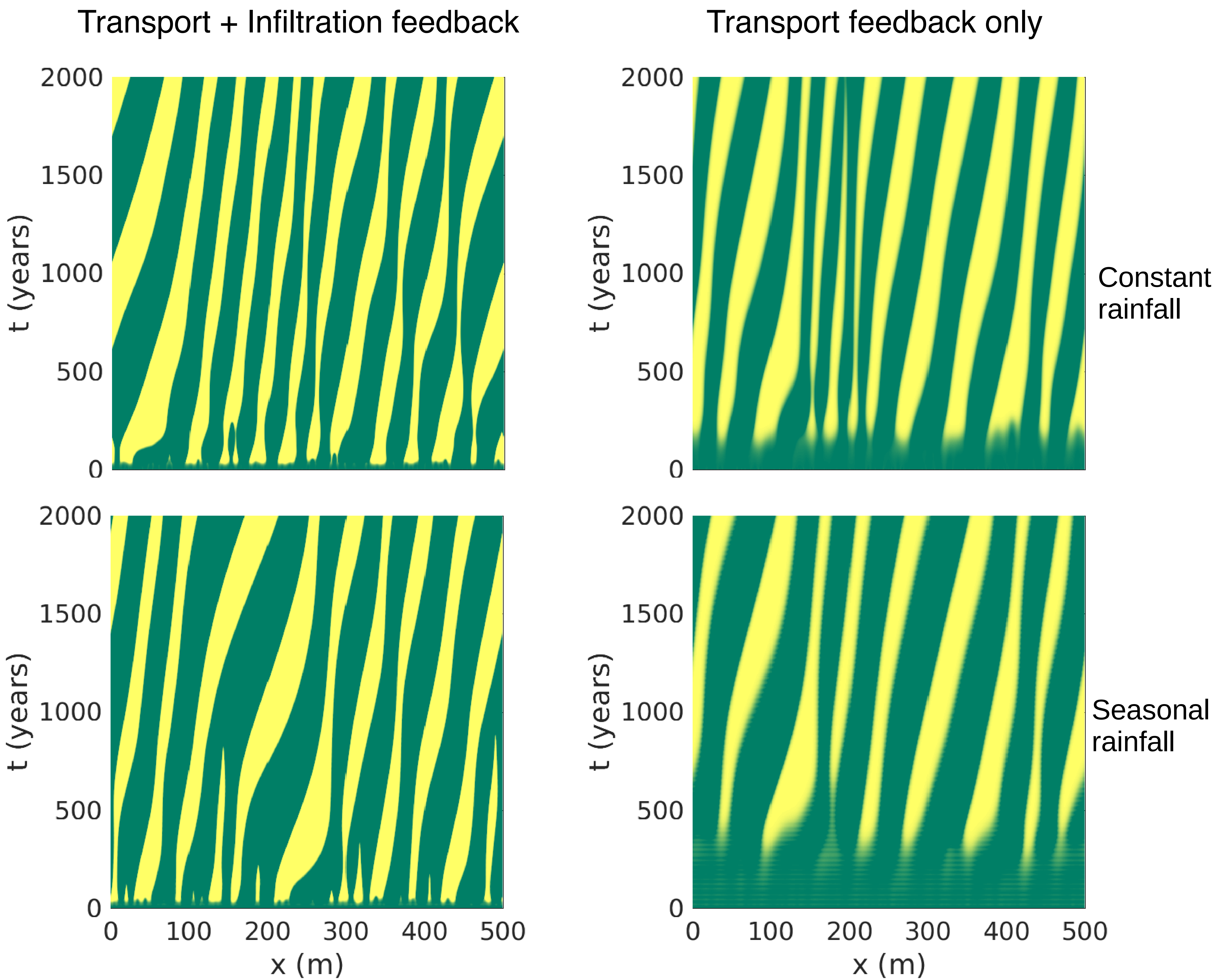
approach 2:

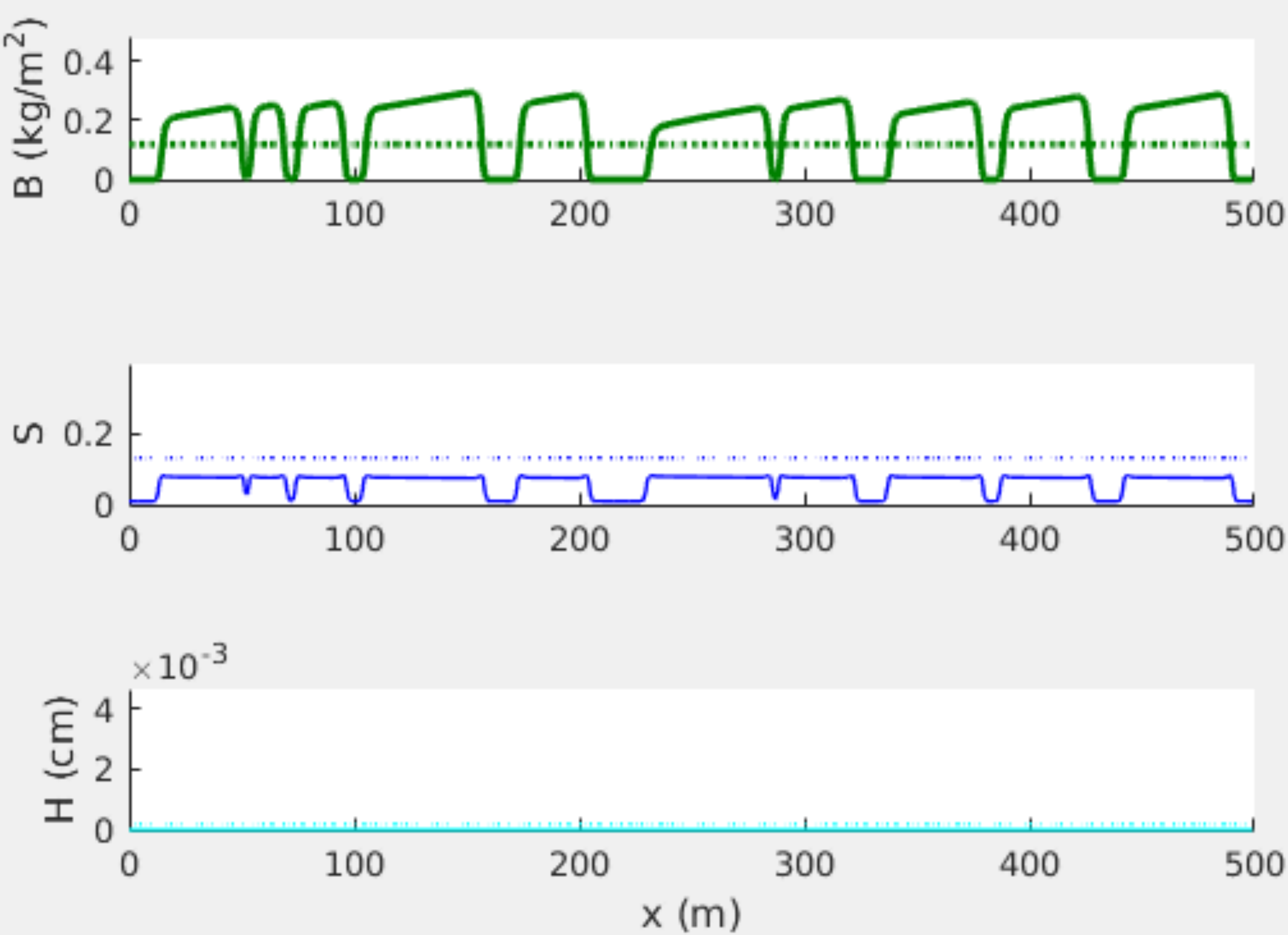
choose transport parameters (K_w, N)
based on order of magnitude estimates from Manning equation: $V = \frac{1}{n} \sqrt{S}\ H^{2/3}$

$n =$ "Manning roughness coefficient"
$n = 0.01 - 0.03\ s/m^{1/3}$ (bare ground)
$n = 0.1 - 0.8\ s/m^{1/3}$ (dense shrubs)
$H = 1 - 5\ cm$

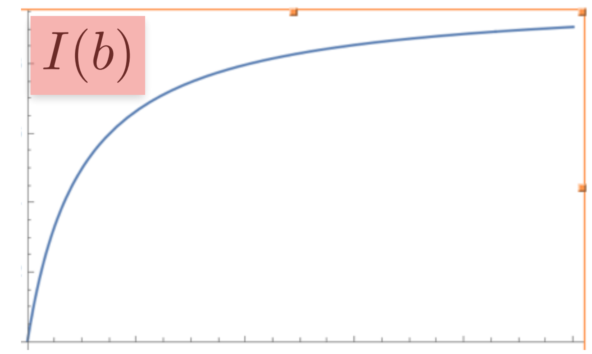
$K_w = 10^7 - 10^8\ cm/day$ ✓
$N = 10^4 - 10^5\ cm^2/kg$ ✓
$D_b = 10^{-2} - 10^3\ cm^2/day$ ✓

(Note: for these linear stability calculations, with constant precip, we can neglect the infiltration feedback nonlinearities.)


Transport + Infiltration feedback
Transport feedback only
Constant rainfall
Seasonal rainfall
t (years)
x (m)
0
500
1000
1500
2000
100
200
300
400


B (kg/m^2)
0.4
0.2
0
0
100
200
300
400
500
S
0.2
0
0
100
200
300
400
500
×10^-3
H (cm)
4
2
0
0
100
200
300
400
500
x (m)

Three Models, Three Forms to Biomass-Water Feedback



$$w_t = p - w - wb^2 + vw_x \longrightarrow \begin{cases} h_t = p - I(b)h + vh_x \\ w_t = -w + I(b)h - \gamma G(w)b + w_{xx} \end{cases}$$

$$\partial_T H = R(T) - K\left(\frac{B + fQ}{B + Q}\right)\left(\frac{H}{H + A}\right)\left(\frac{1 - s}{1 - s + \beta}\right) + \sqrt{S}\, K_w \partial_X \left(\frac{H}{1 + NB}\right)$$

Claims:
(1) Exploiting topographic heterogeneity is a good idea.
(2) To do this, we will need to better model surface water transport.

Question: Is claim (2) true?