

# Dynamics of Democracy

**Part I (MS100)**  
**3:10-4:50 PM**

Heather Brooks  
*University of California, Los Angeles*

Joseph Tien  
*The Ohio State University*

Susan Fennell  
*University of Limerick*

Maria D'Orsogna  
*California State University, Northridge*

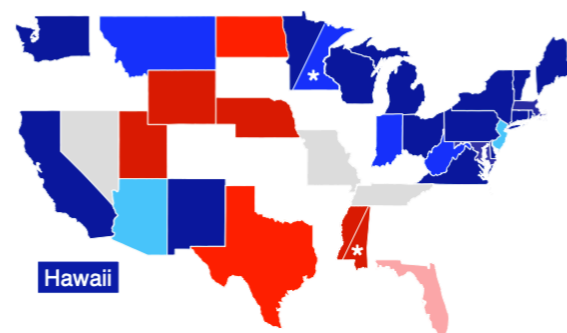
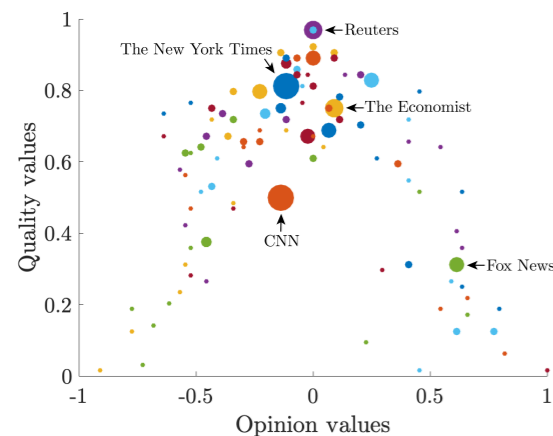
**Part II (MS112)**  
**5:00-6:40 PM**

Carlos Castillo-Chavez  
*Arizona State University*

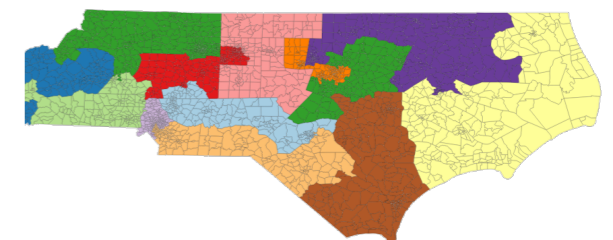
Jonathan Mattingly  
*Duke University*

Michelle Feng  
*University of California, Los Angeles*

Alexandria Volkening  
*MBI, The Ohio State University*



■ Solid Rep. ( $\geq 95\%$ )  
■ Likely Rep. ( $\geq 75\%$ )  
■ Lean Rep. ( $\geq 60\%$ )  
■ Toss-Up ( $< 60\%$ )  
■ Lean Dem. ( $\geq 60\%$ )  
■ Likely Dem. ( $\geq 75\%$ )  
■ Solid Dem. ( $\geq 95\%$ )



The background features a complex network diagram with numerous nodes and directed edges. The nodes are represented by small circles in various colors, including blue, green, and orange. The edges are thin, grey lines with arrowheads, indicating the direction of flow or influence between the nodes. The overall appearance is that of a dynamic, interconnected system.

# **A Model for the Influence of Media on the Ideology of Content in Online Social Networks**

**Heather Zinn Brooks  
CAM Assistant Professor  
Department of Mathematics, UCLA**

**Joint work with Mason A. Porter, UCLA**

# People are increasingly reliant on online social networks as sources of news and information

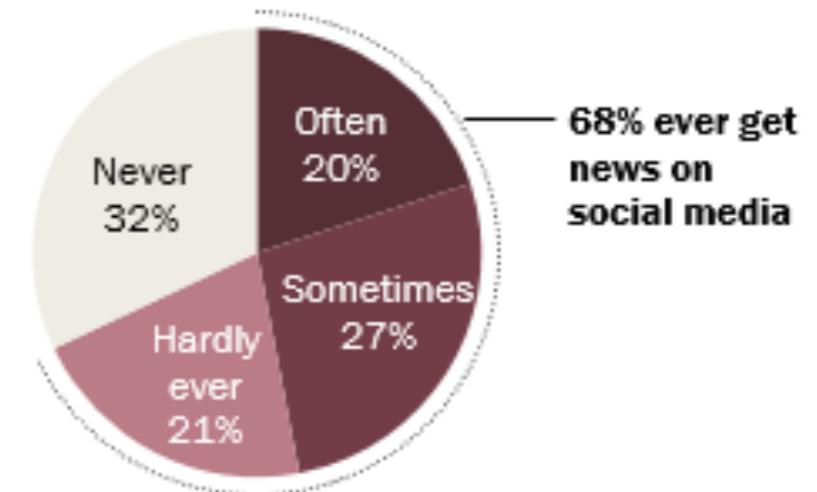


YouTube



## About two-thirds of Americans get news on social media

*% of U.S. adults who get news on social media ...*



## But most social media news consumers expect news there to be inaccurate

*% of social media news consumers who say they expect the news they see on social media to be ...*



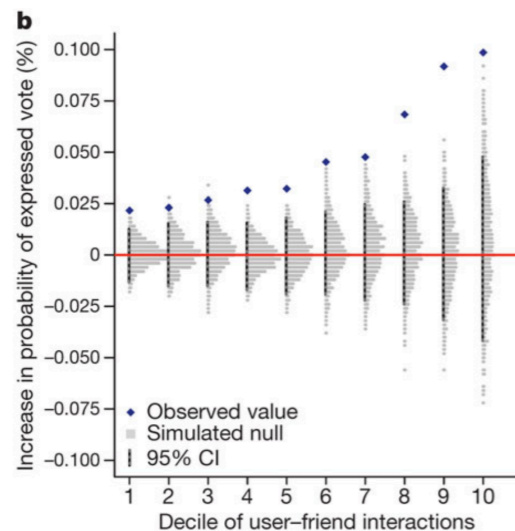
Note: No answer responses not shown.

Source: Survey conducted July 30-Aug. 12, 2018.

"News Use Across Social Media Platforms 2018"

PEW RESEARCH CENTER

# The spread of information online affects policy, opinion, and personal interactions



## LETTER

doi:10.1038/nature11421

### A 61-million-person experiment in social influence and political mobilization

Robert M. Bond<sup>1</sup>, Christopher J. Fariss<sup>1</sup>, Jason J. Jones<sup>2</sup>, Adam D. I. Kramer<sup>3</sup>, Cameron Marlow<sup>3</sup>, Jaime E. Settle<sup>1</sup> & James H. Fowler<sup>1,4</sup>

## Social Bots Distort the 2016 US Presidential Election Online Discussion

*First Monday, Volume 21, Number 11 - 7 November 2016*

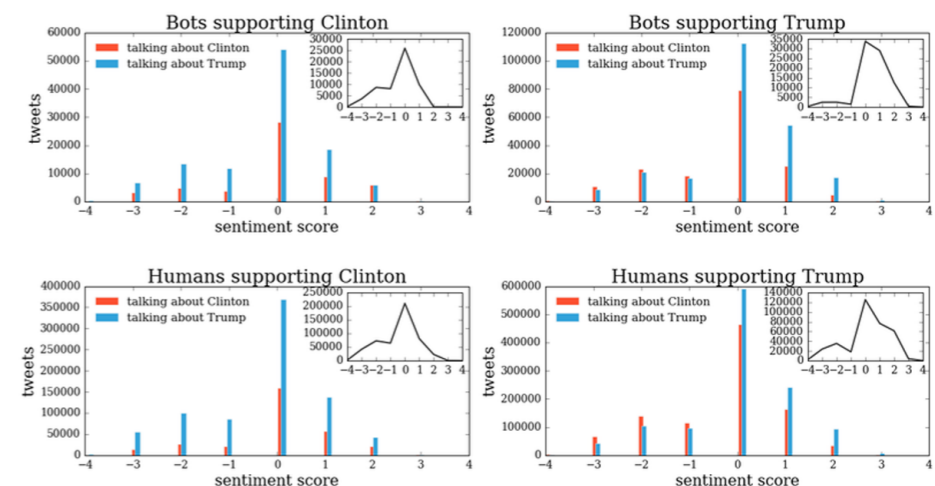
14 Pages • Posted: 8 Jun 2017

[Alessandro Bessi](#)

University of Southern California - Information Sciences Institute

[Emilio Ferrara](#)

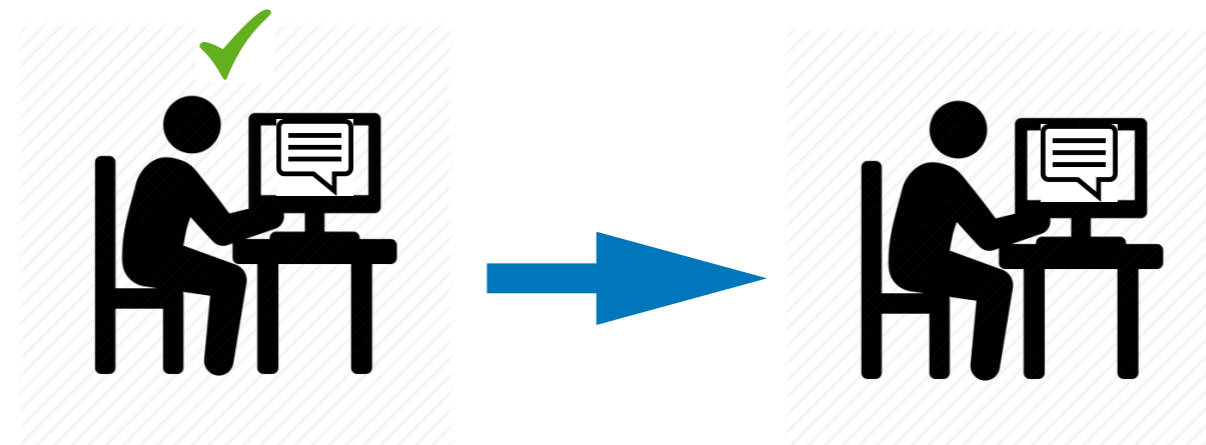
University of Southern California - Information Sciences Institute



# How is content spread?

## User preference:

Users are more likely to share a false story if it confirms or supports their biases



## Manipulation of content spread:

Bot, cyborg, and sockpuppet accounts



# Today's talk

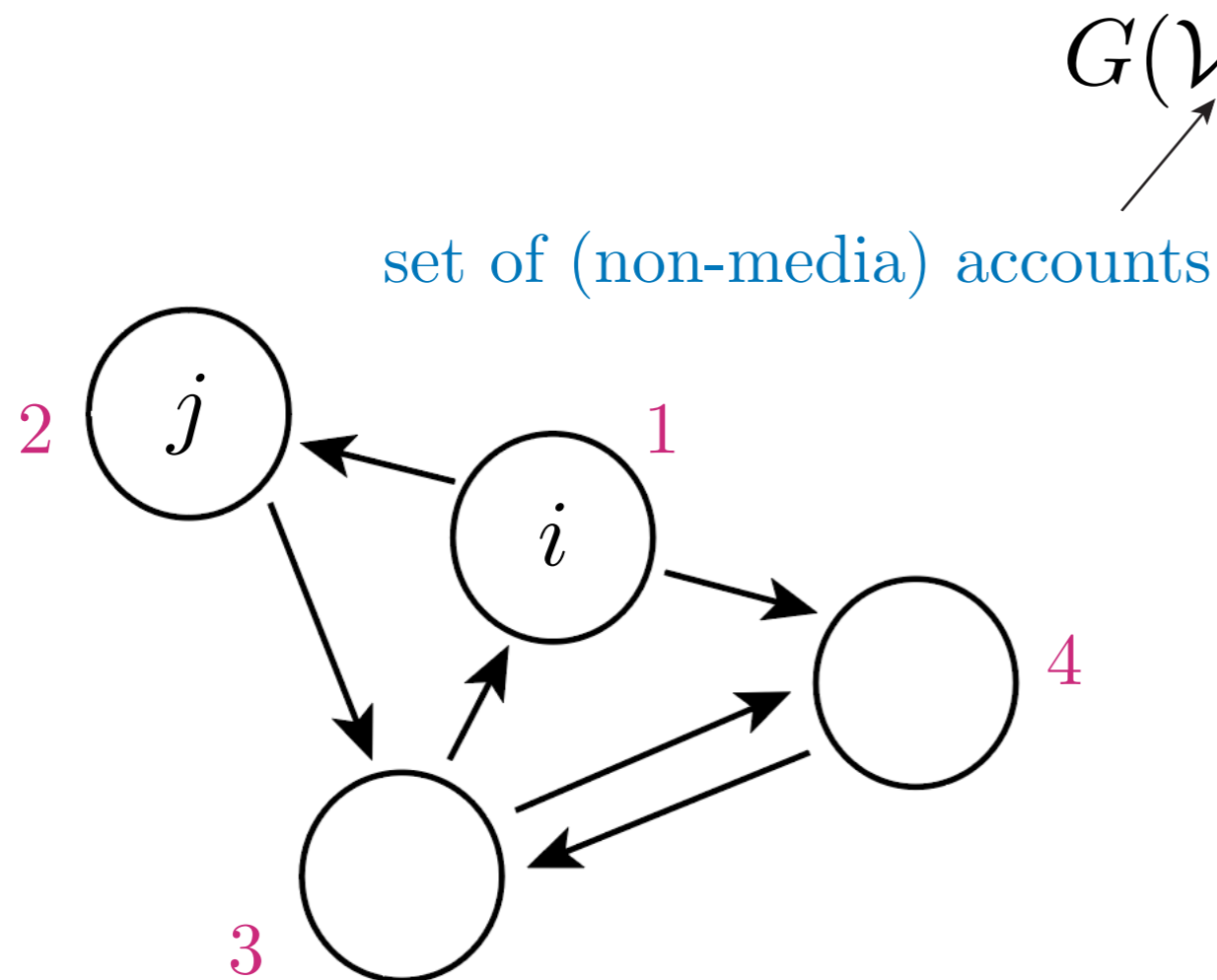
- Building our model: social network structure
- Building our model: content updating dynamics
- Quantifying media impact
- Effects of network parameters
- Model upgrade: content spread with bias and quality

## A Model for the Influence of Media on the Ideology of Content in Online Social Networks

Heather Z. Brooks, Mason A. Porter

(Submitted on 19 Apr 2019)

# We represent online social media network structure with a directed graph



set of follower relationships

Account  $i$  is a follower of account  $j$  if there is a directed edge from  $i$  to  $j$

This graph can be represented with adjacency matrix  $A$

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

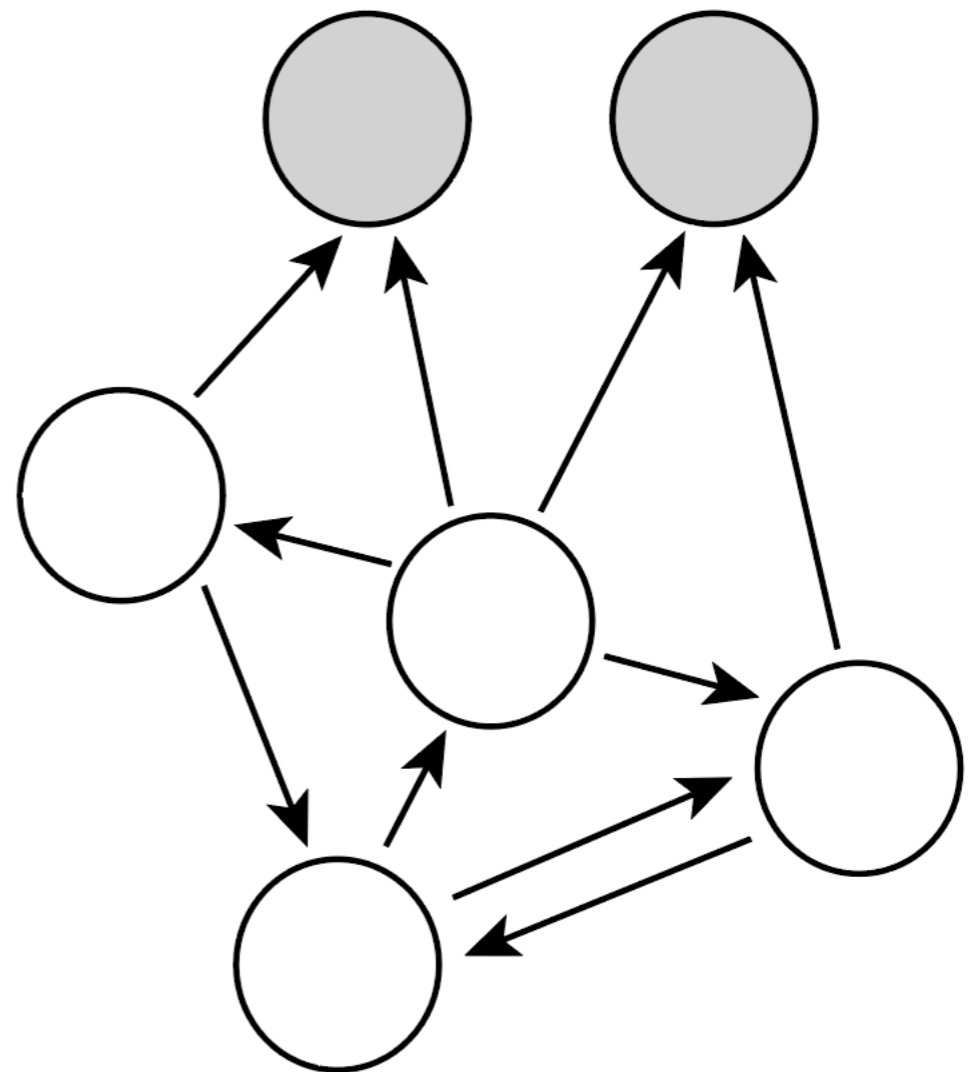
# We add media accounts as influencer nodes

Assume: media accounts are not influenced  
(they do not follow other accounts)

$M$  = number of media accounts

$n_M$  = number of followers per  
media account

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$





# Bounded-confidence mechanism for content updating

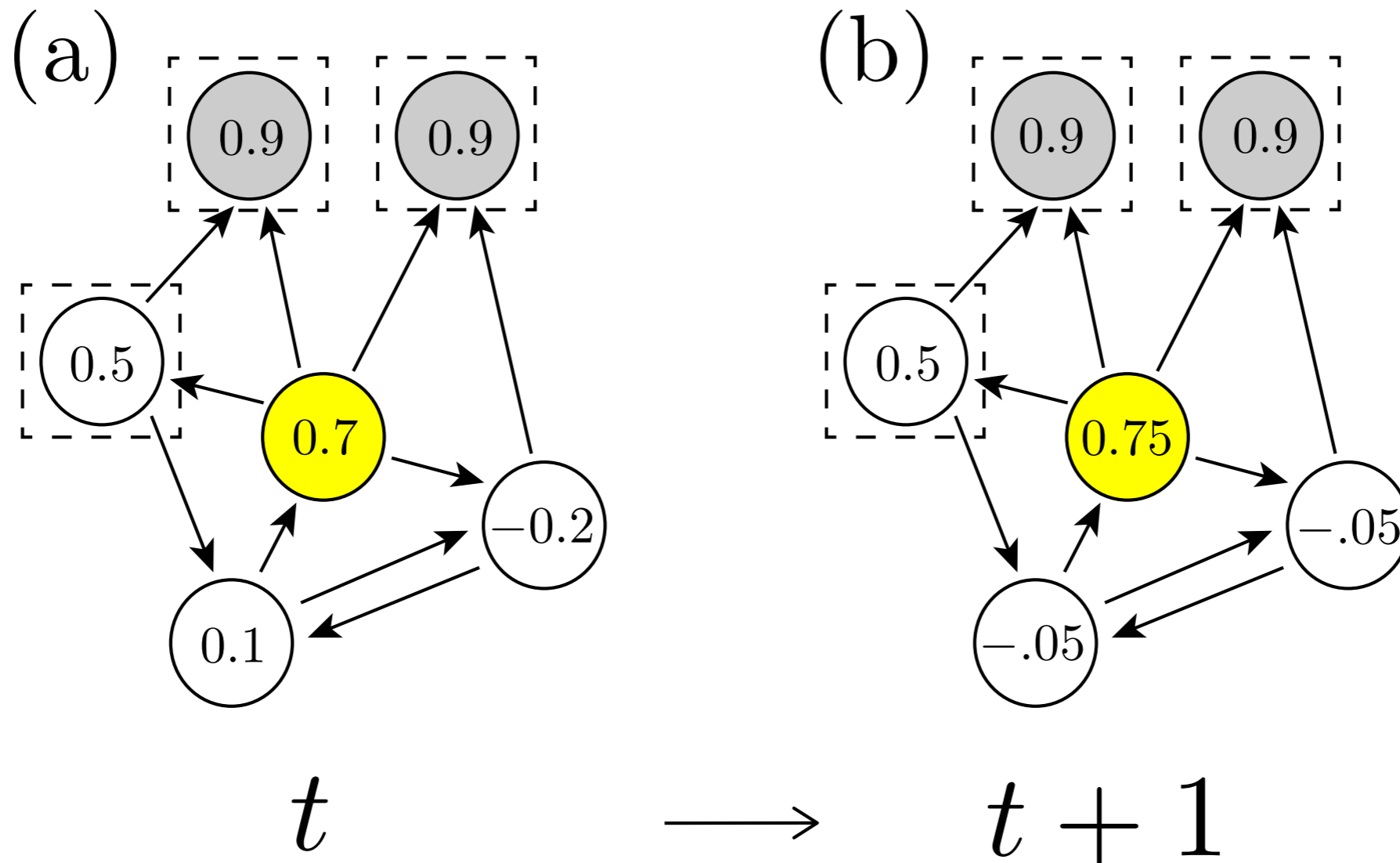
Content ideology of node  $i$  at time  $t$  is  $\mathbf{x}_i^t \in [-1, 1]^d$   
which is updated according to

$$\mathbf{x}_i^{t+1} = \frac{1}{|I_i| + 1} \left( \mathbf{x}_i^t + \sum_{j=1}^{N+M} A_{ij} \mathbf{x}_j^t f(\mathbf{x}_j^t, \mathbf{x}_i^t) \right)$$

where 
$$\begin{cases} f(\mathbf{x}_j, \mathbf{x}_i) = 1 & \text{dist}(\mathbf{x}_j, \mathbf{x}_i) < c \\ f(\mathbf{x}_j, \mathbf{x}_i) = 0 & \text{otherwise} \end{cases}$$

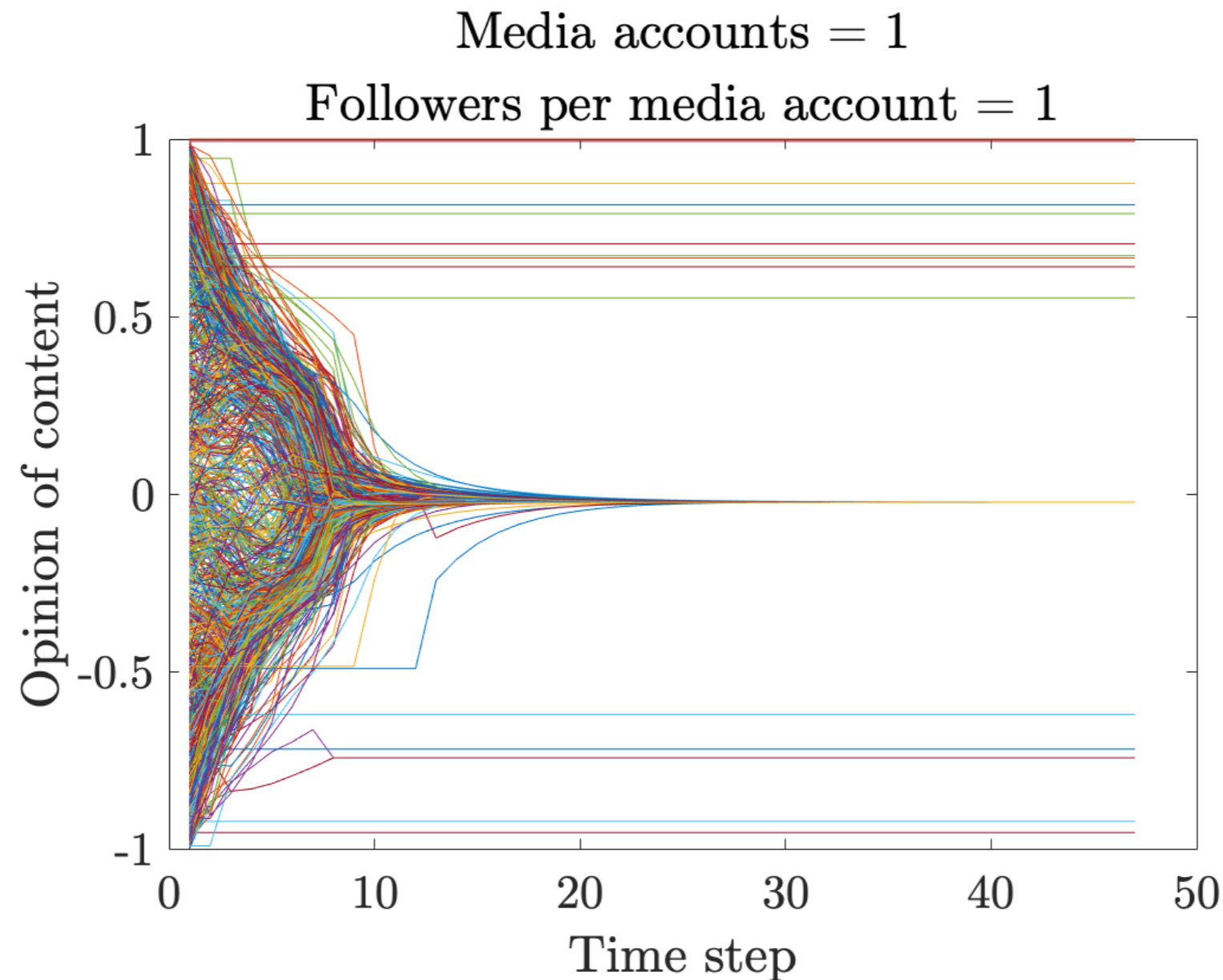
and  $I_i = \{j \in \{1, \dots, N + M\} \mid A_{ij} = 1; \text{dist}(\mathbf{x}_j, \mathbf{x}_i) < c\}$

# Schematic of content updating rule



$$x^{t+1} = \frac{1}{4} (0.7 + 0.5 + 0.9 + 0.9) = 0.75$$

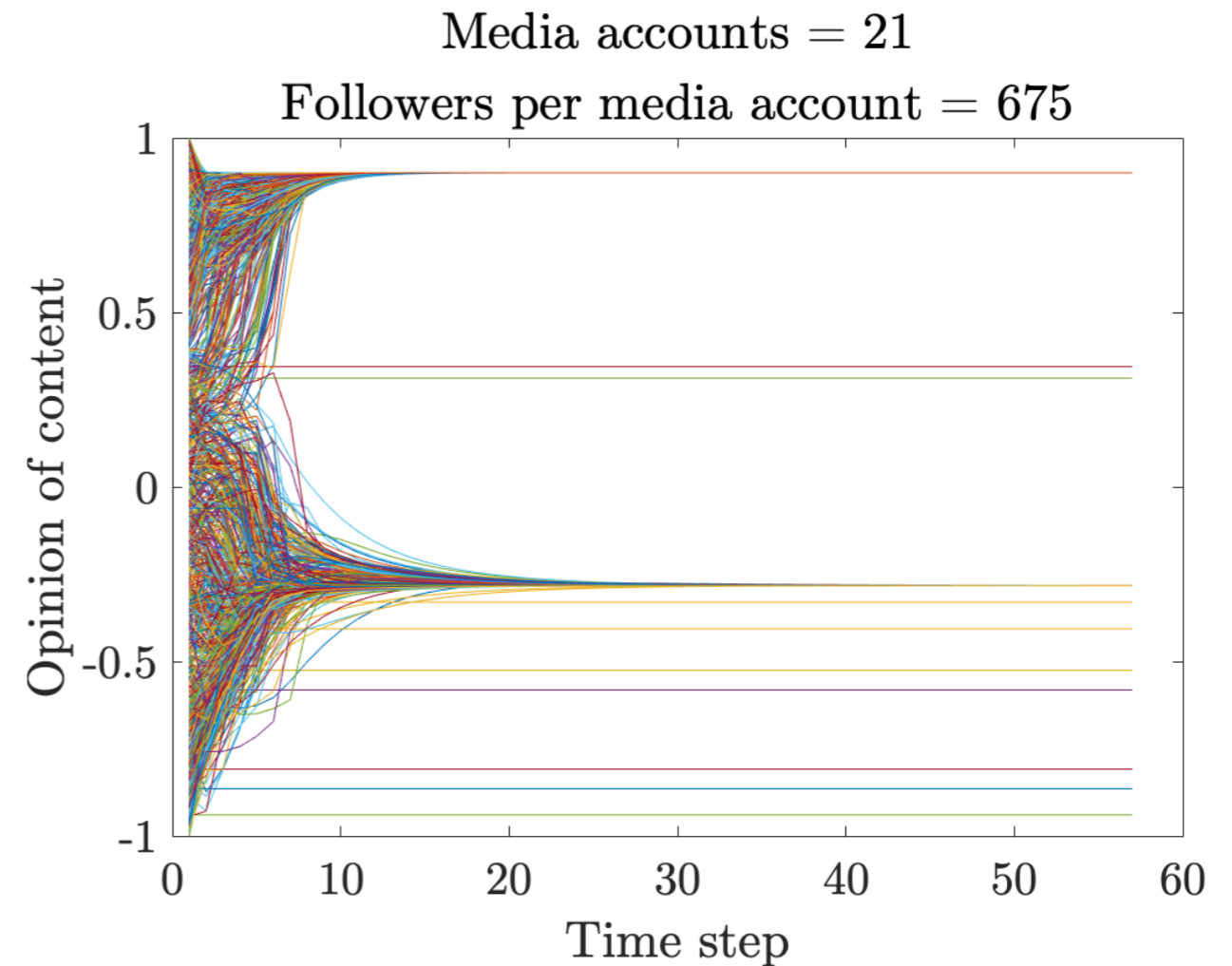
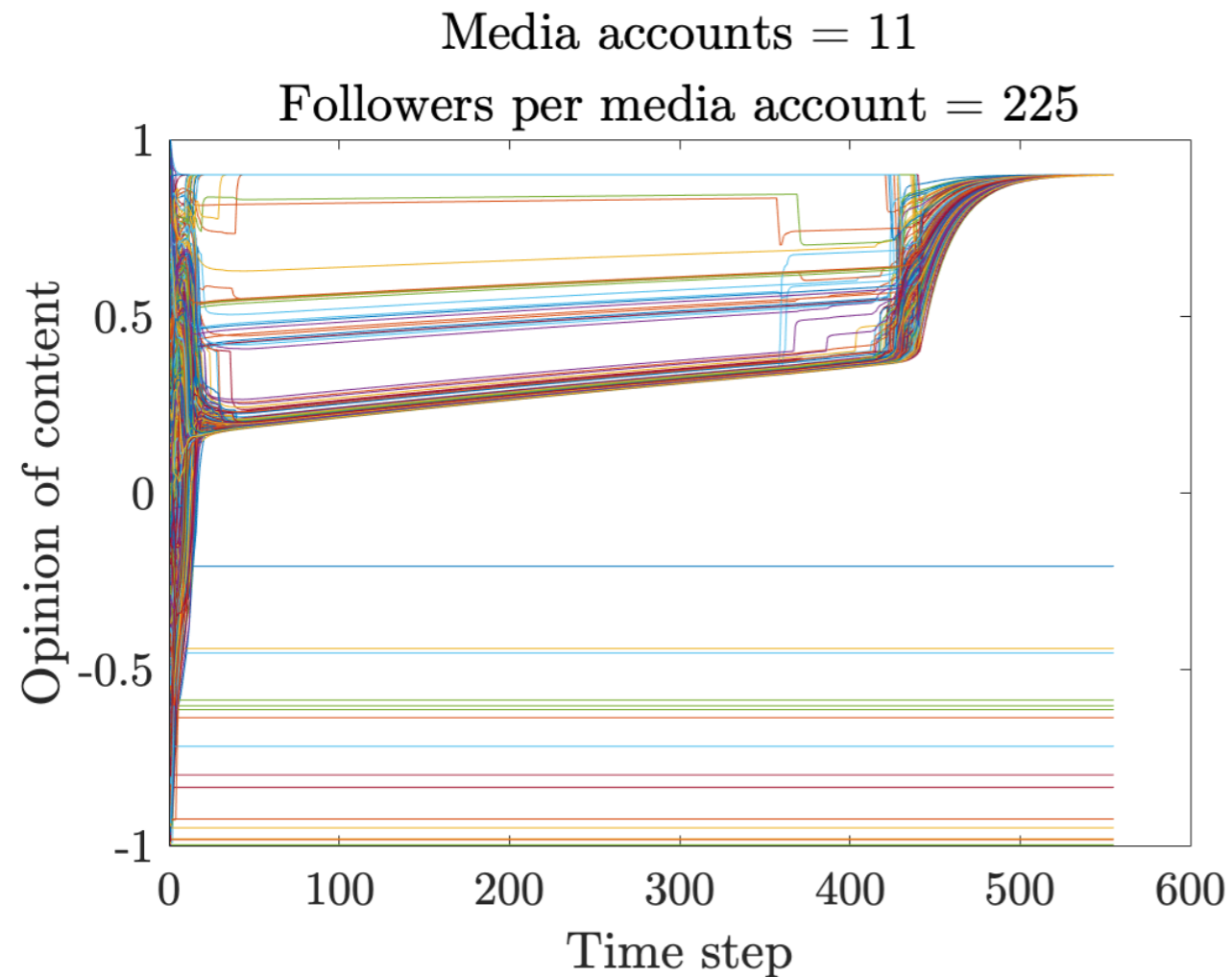
# Example: simulation of one trial with a one-dimensional ideology space



Network architecture: Facebook100 Reed College ( $N=962$ )

Media ideology:  $x_M = 0.9$

# Increasing media accounts and number of followers per account leads to different dynamics



Network architecture: Facebook100 Reed College ( $N=962$ )

Media ideology:  $x_M = 0.9$

# How to quantify impact of media accounts on the network

For each  $(n_M, M)$  pair, calculate impact summary diagnostic

$$\bar{R} = \frac{\overline{R_0}}{\overline{R_i}}$$

where the order parameter for ideology **without media** is

$$R_0 = \frac{1}{N} \sum_{i=1}^N \|x_i^b - x_M\|_2$$

and **with media** is

$$R_i = \frac{1}{N} \sum_{i=1}^N \|x_i^* - x_M\|_2$$

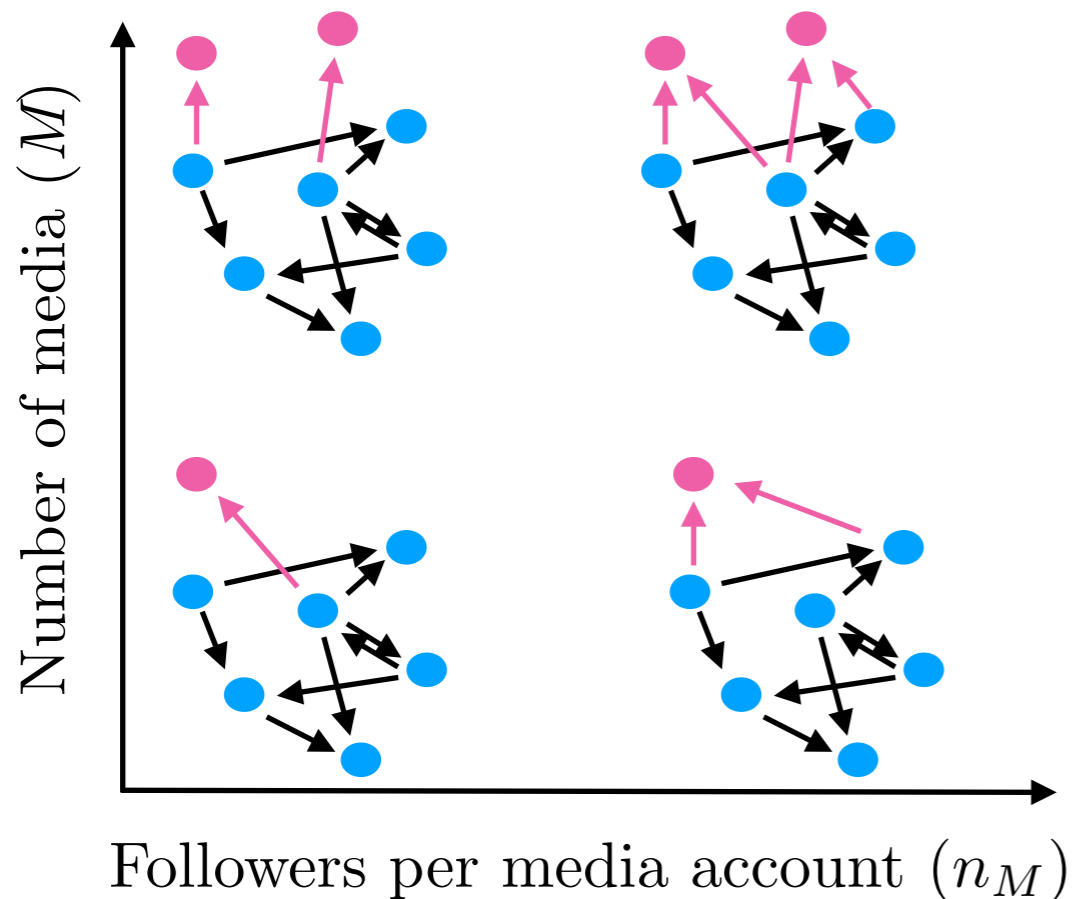


Figure 2(b)

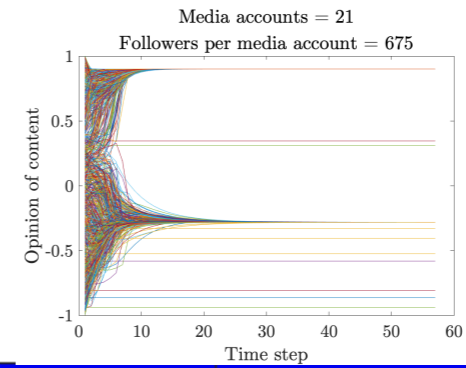
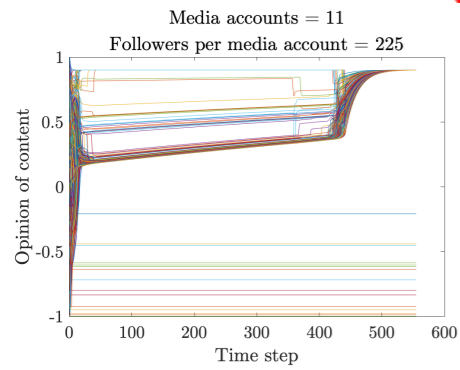


Figure 2(c)

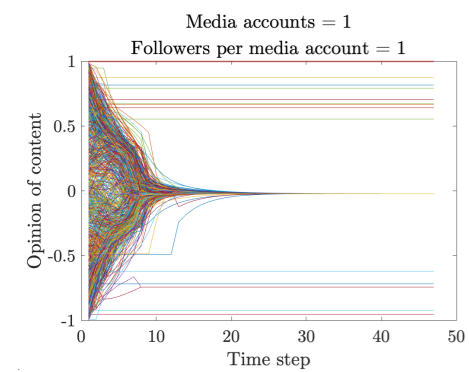
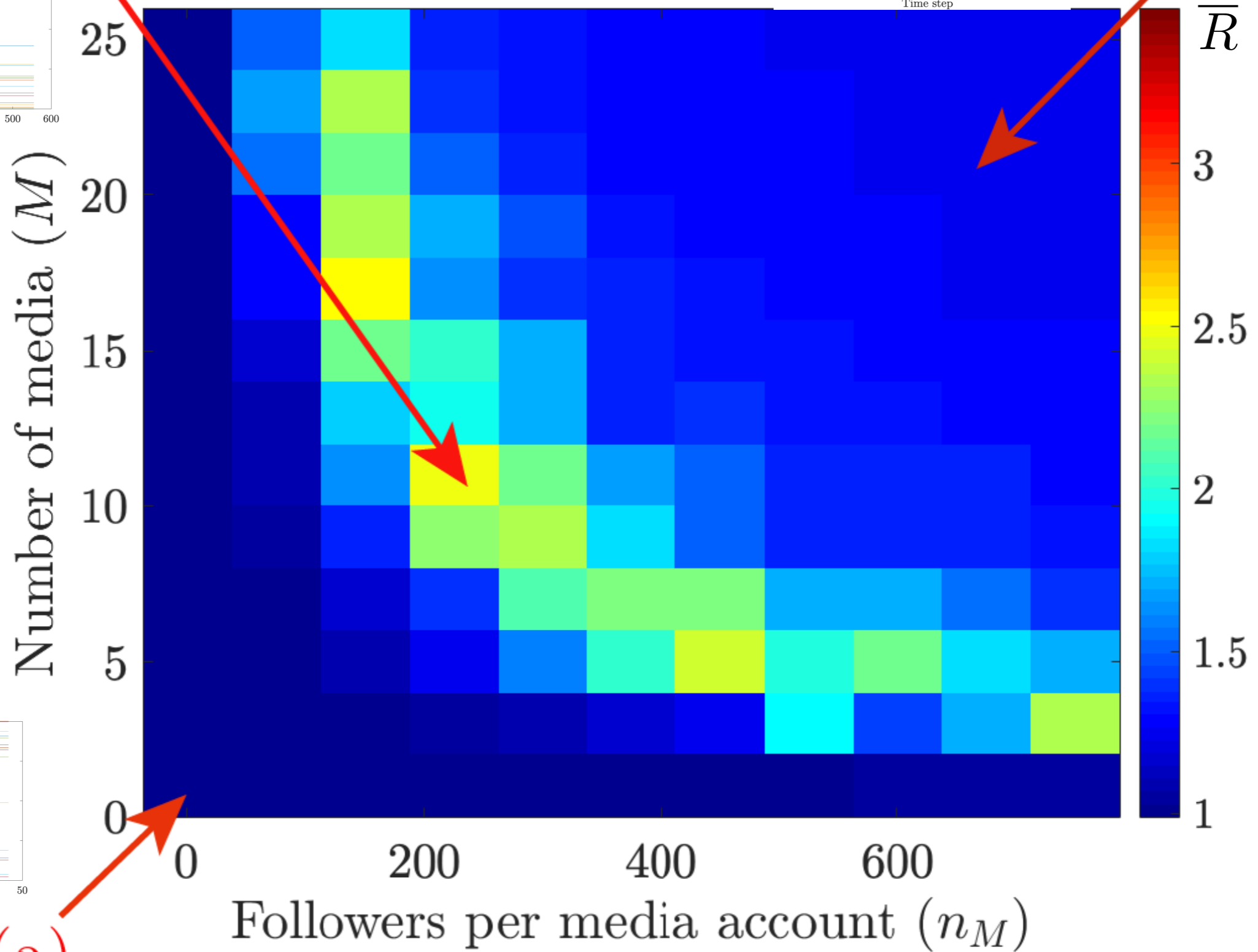
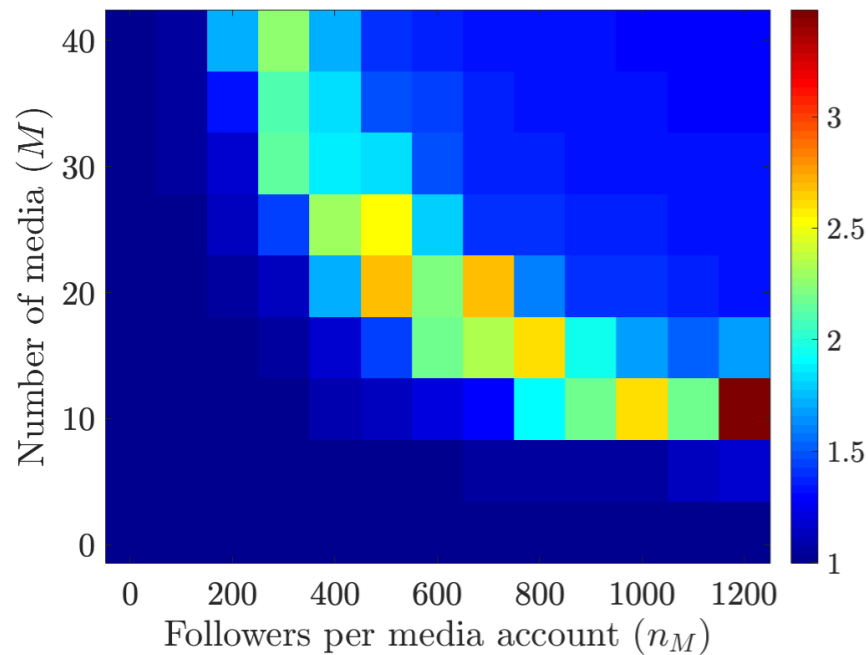


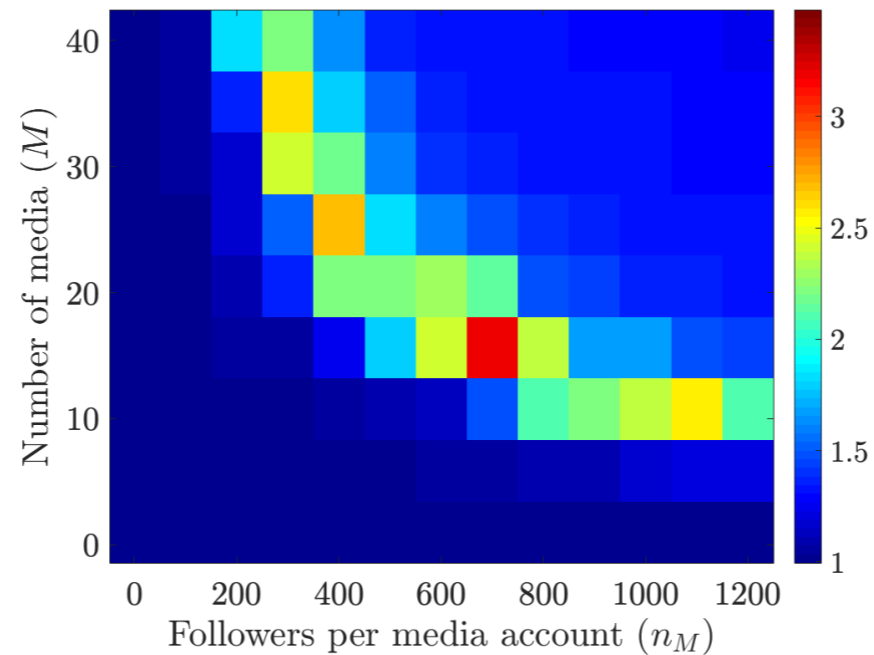
Figure 2(a)

# Media entrainment in Facebook100 networks

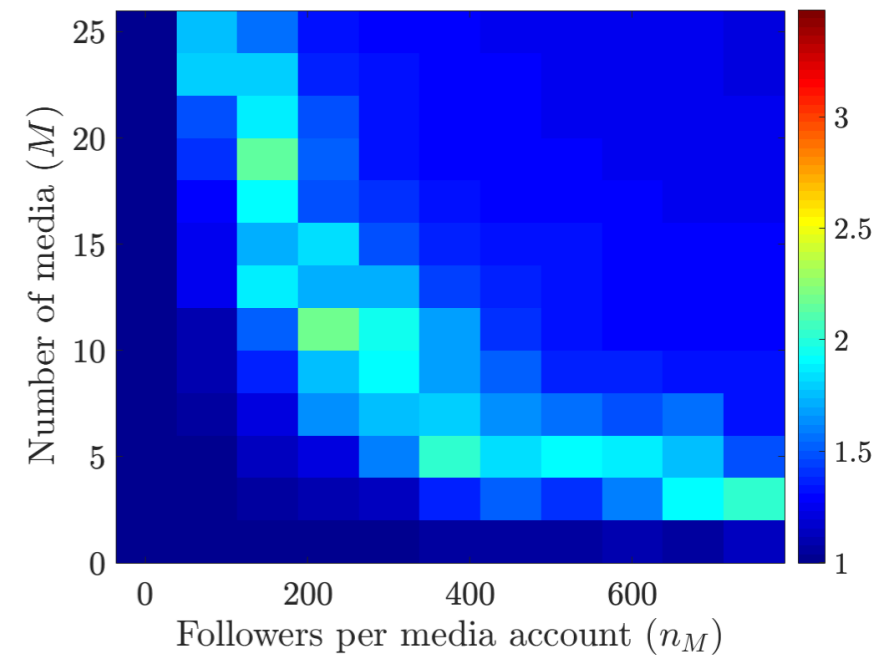
Amherst ( $N=2235$ )



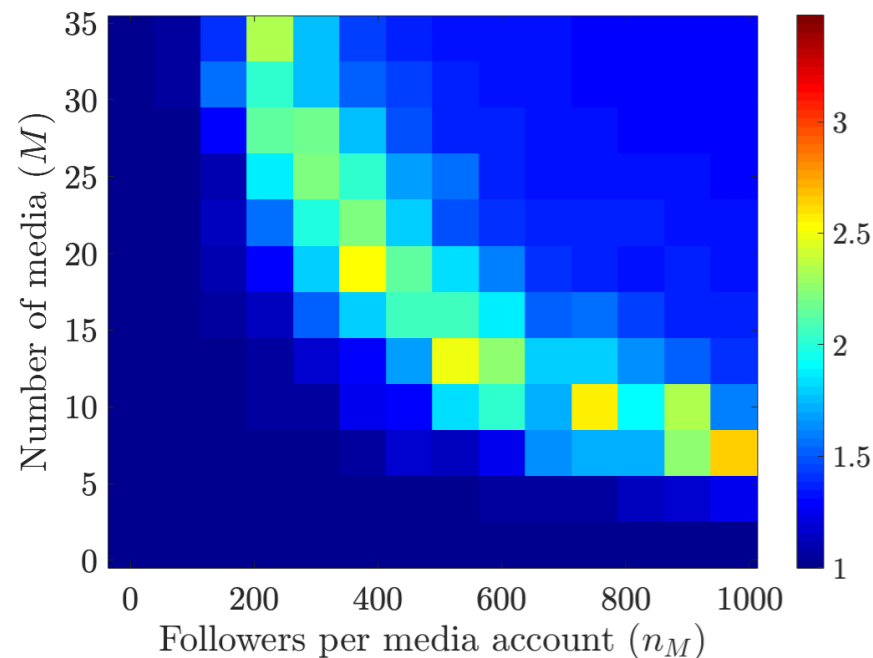
Bowdoin ( $N=2250$ )



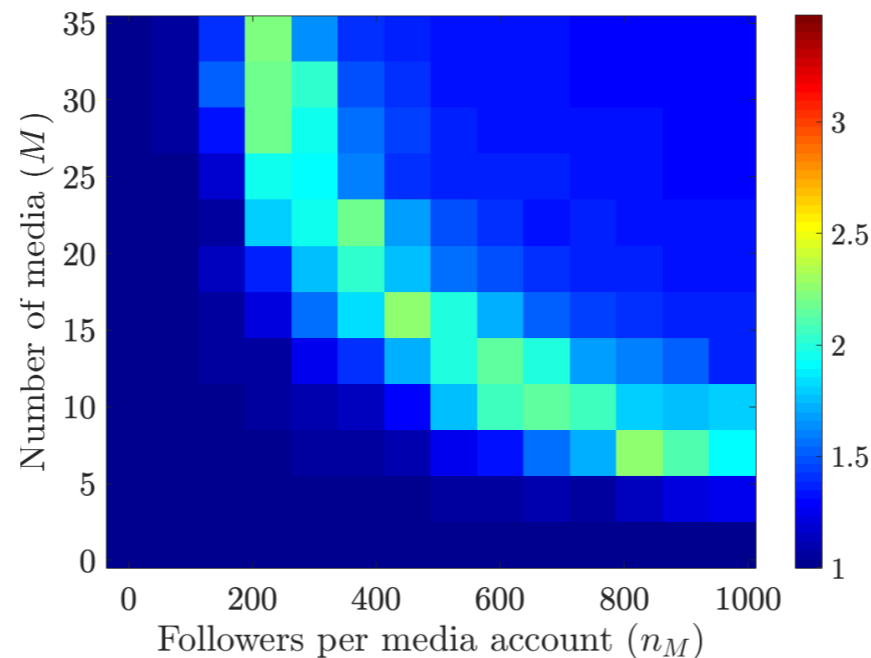
Caltech ( $N=762$ )



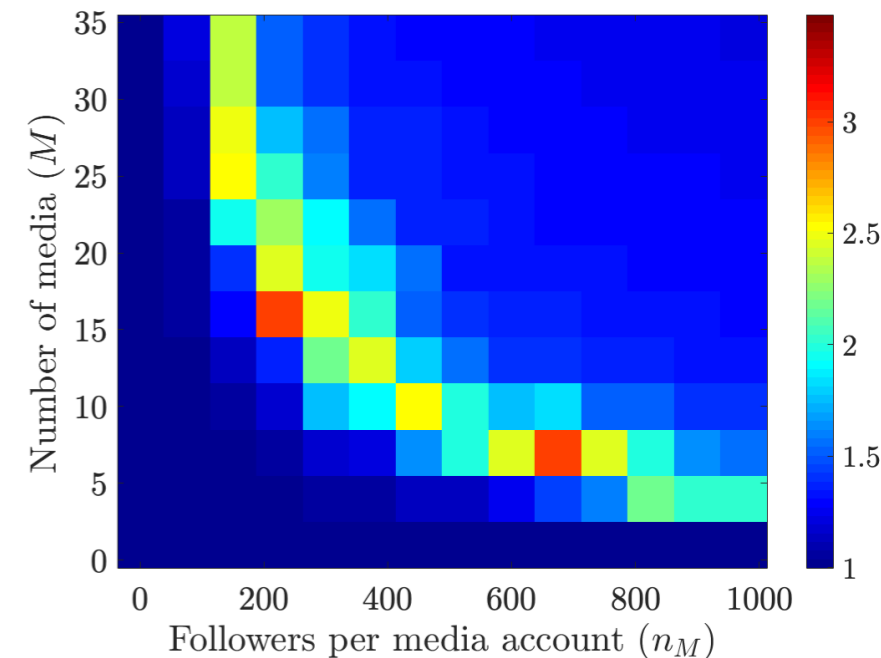
Haverford ( $N=1446$ )



Simmons ( $N=1510$ )

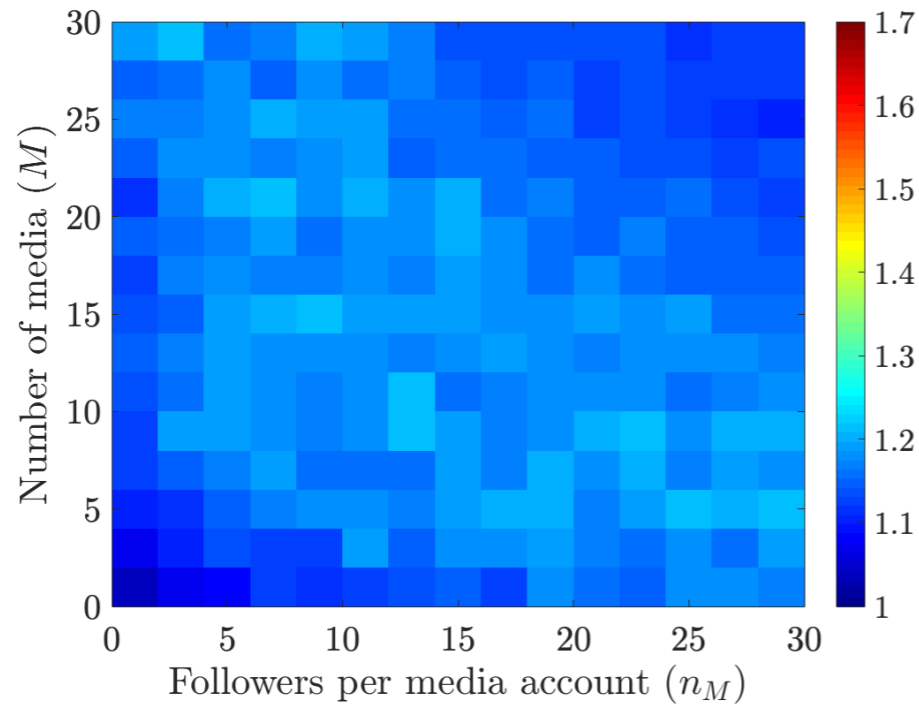


Swarthmore ( $N=1657$ )

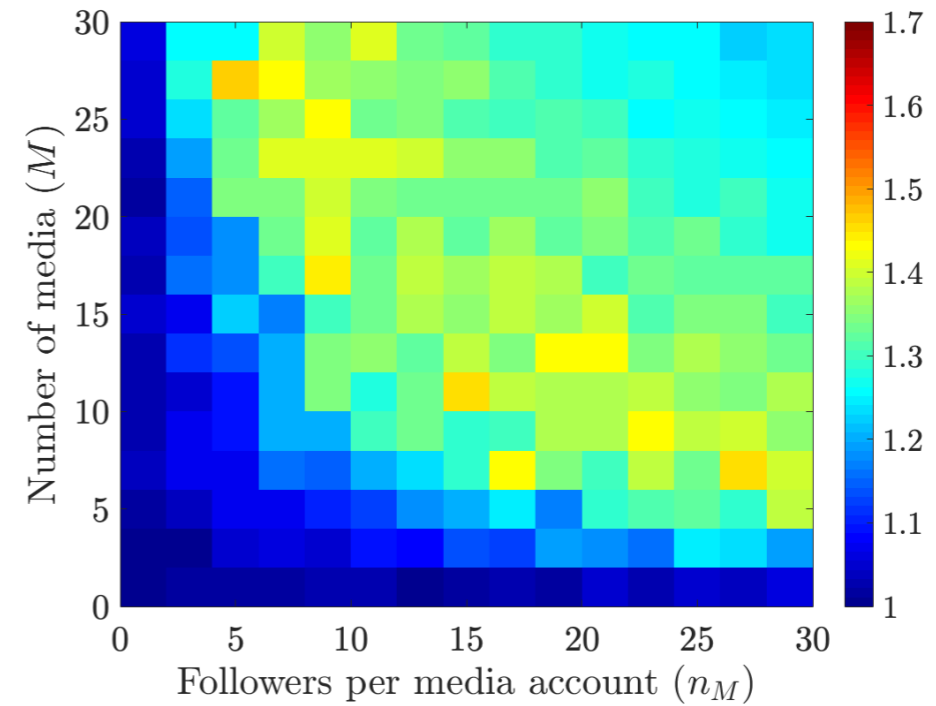


# Media entrainment in synthetic networks

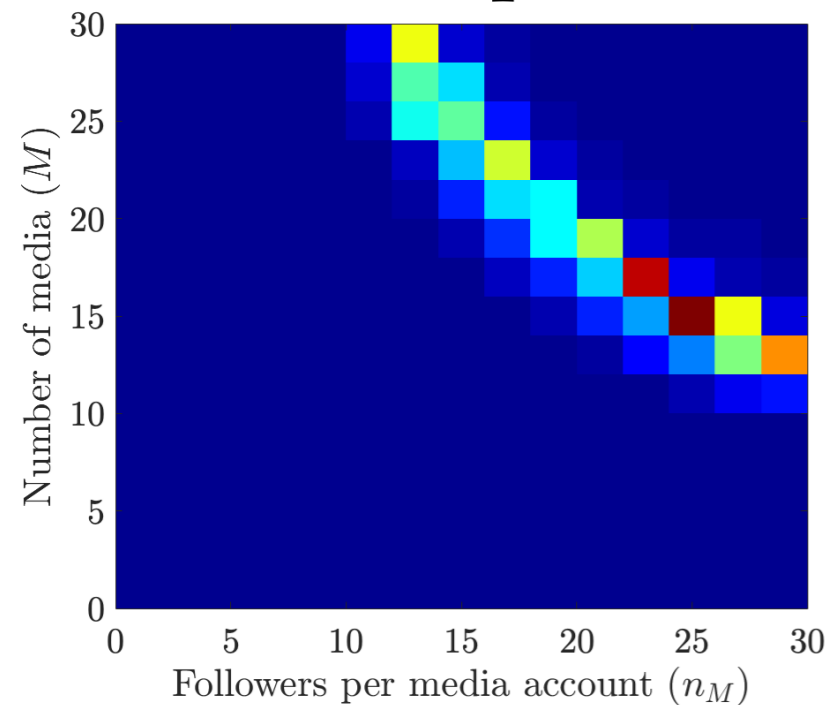
Star



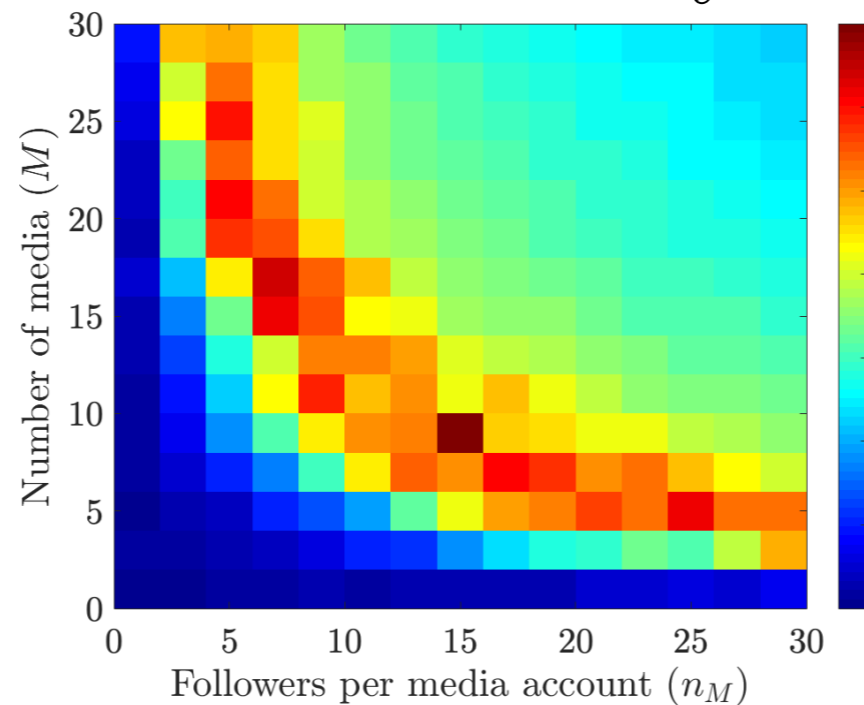
Ring Lattice



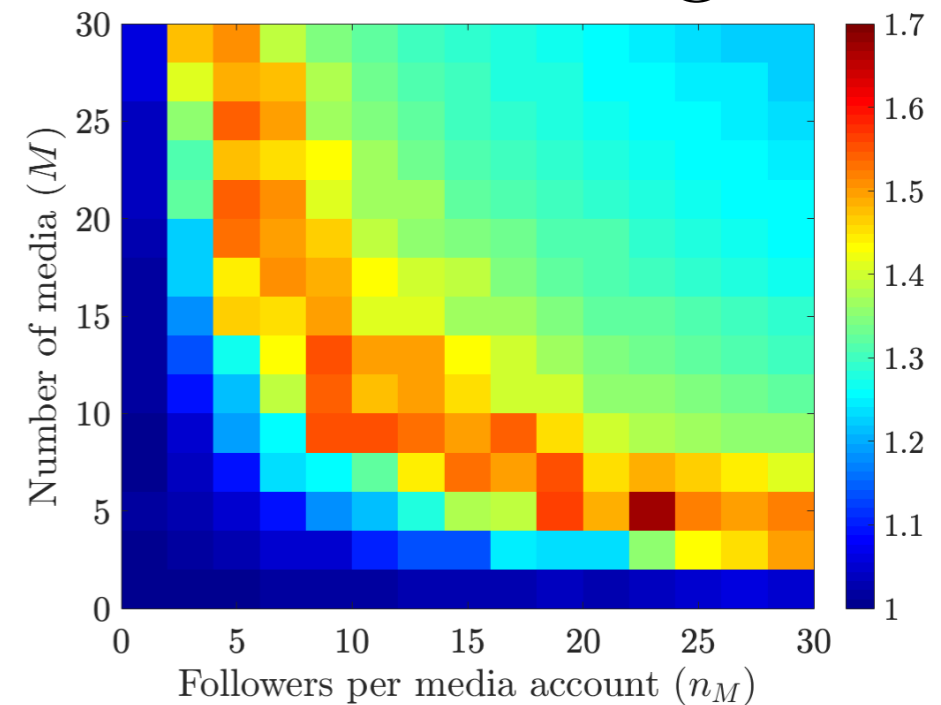
Complete



Erdős—Rényi

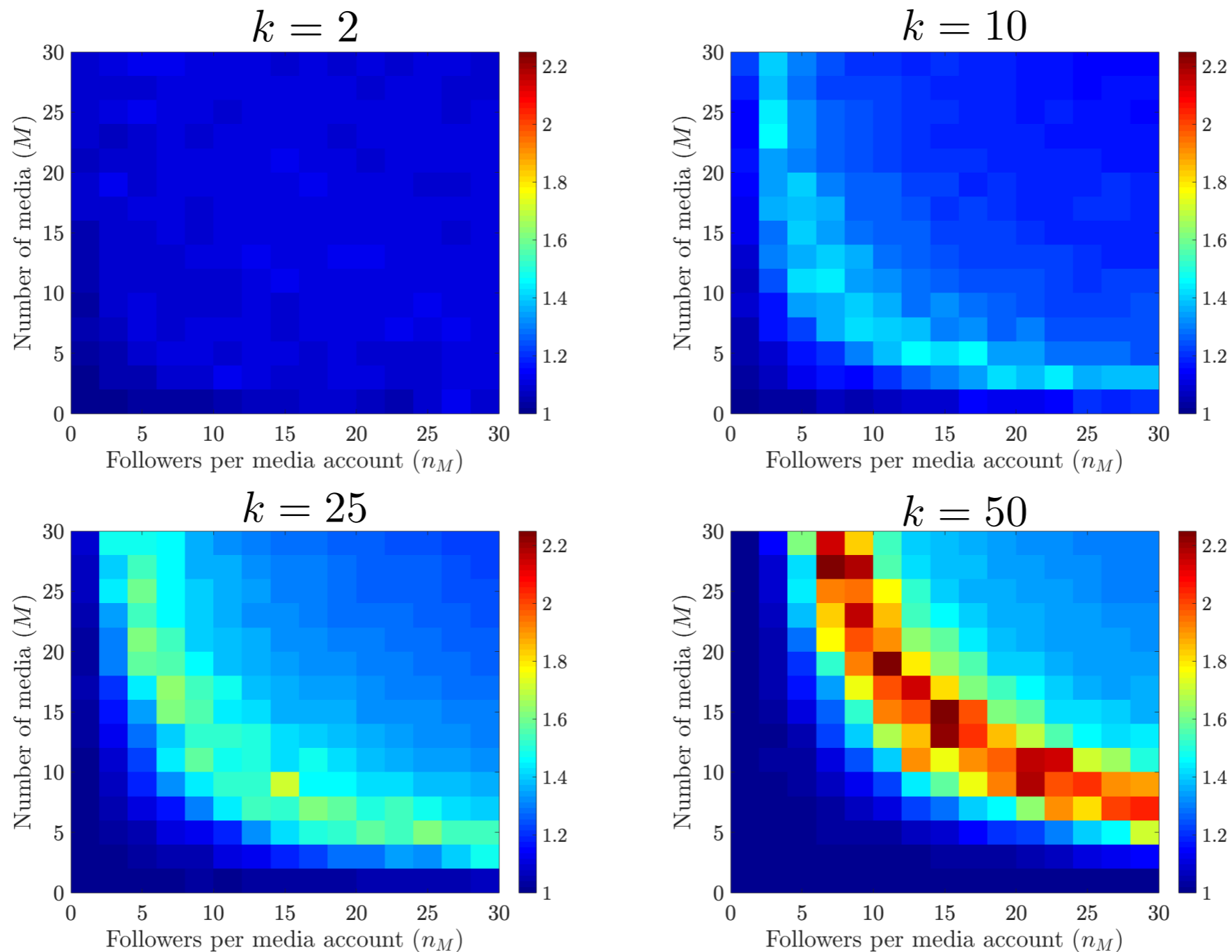


Watts—Strogatz



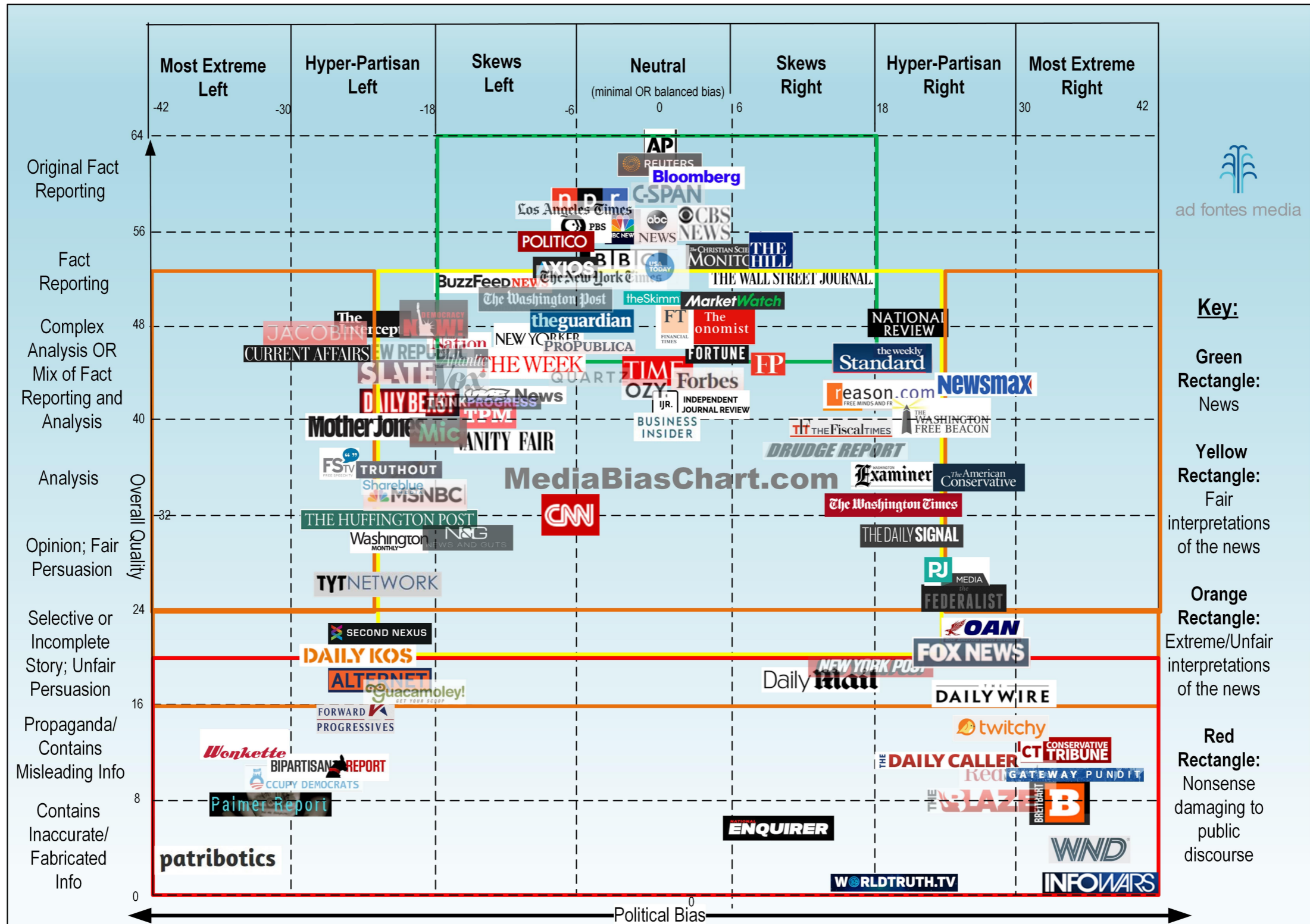


# Increasing $k$ , the average number of non-media followers, increases media entrainment



We also observe increased entrainment when increasing number of non-media accounts and increasing receptiveness parameter

# Media in real online networks: their content has a distribution of ideological bias and quality



# Our model with content ideology and quality

Content state of node  $i$  at time  $t$  is  $\mathbf{x}_i \in [-1, 1]^d \times [0, 1]$

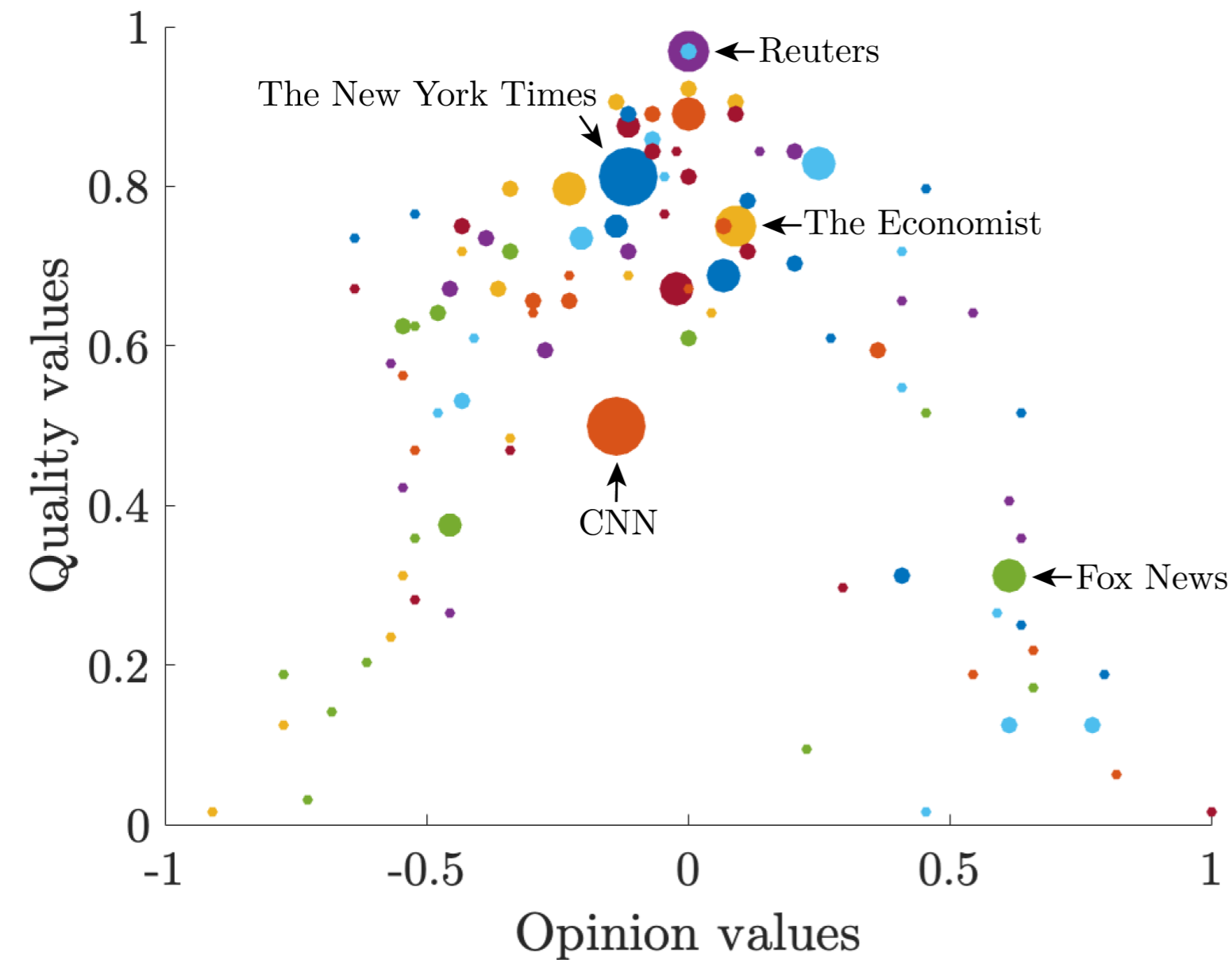
content ideology    content quality

$$\mathbf{x}_i^{t+1} = \frac{1}{|I_i| + 1} \left( \mathbf{x}_i^t + \sum_{j=1}^{N+M} A_{ij} \mathbf{x}_j^t g(\mathbf{x}_i^t, \mathbf{x}_j^t) \right)$$

$$\text{where } g(\mathbf{x}_i^t, \mathbf{x}_j^t) = \begin{cases} 1, & \text{if } x_{2,j} \geq q_{i,j} \\ 0, & \text{otherwise} \end{cases}$$

$$\text{and } q_{i,j} = \frac{1}{c} \text{dist}(x_{1,i}, x_{1,j})$$

# Create media distribution in ideology-quality space based on the Ad Fontes Media Bias Chart

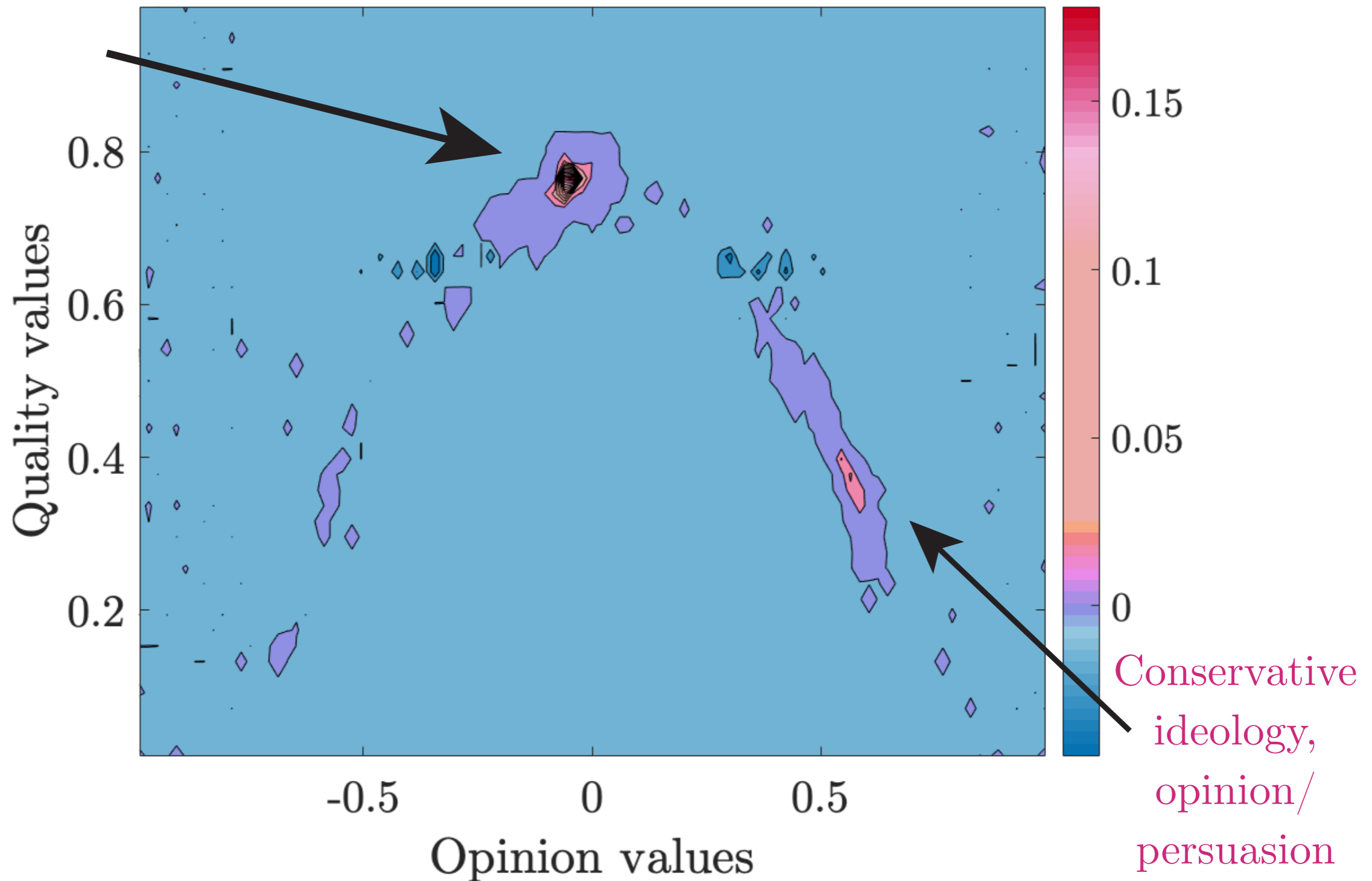


$M=103$  media accounts

We choose the number of followers per media account to be proportional to the approximate number of followers that each media source had on Twitter on 15 Feb 2019 at 17:36 PST (under the constraint that each media source in the model has at least one follower)

# We observe the emergence of two primary communities (“echo chambers”) of content

Moderate ideology, complex analysis/fact reporting



Colors of contour plot show values of media impact function over 200 trials

# There are many next steps!

- Mathematical analysis of media entrainment
- Incorporating account heterogeneity
- Structural homophily (particularly in ideology)
- Incorporating multiple types of social media: generalizing to a multilayer network
- Sentiment analysis of online content to get ideologies inferred from data
- Transient dynamics and time-dependent networks (rewiring followers)

**Our model of content spread is very generalizable, and we hope this work will build the theory of online content spread and provide a step toward the development of control strategies and novel algorithms for mitigating the spread of undesirable content.**

# Thank you!

Mason Porter (UCLA)

Franca Hoffmann (Caltech)

Andrew Stuart (Caltech)

Yi Ming Lai (Nottingham)



Find me at:

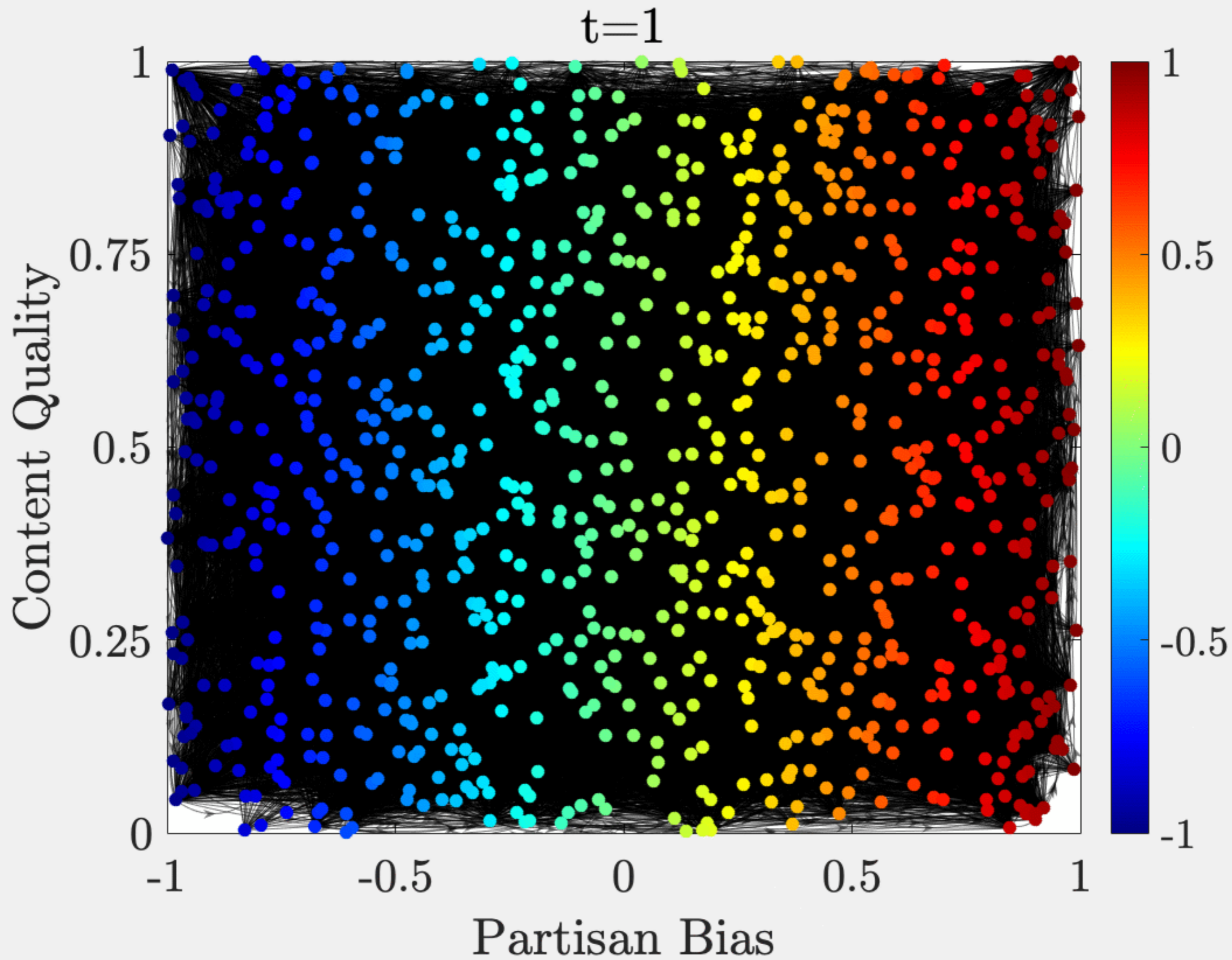
[hbrooks@math.ucla.edu](mailto:hbrooks@math.ucla.edu)

[www.math.ucla.edu/~hbrooks](http://www.math.ucla.edu/~hbrooks)

@HZinnbrooks

The paper is now on arXiv: 1904.09238

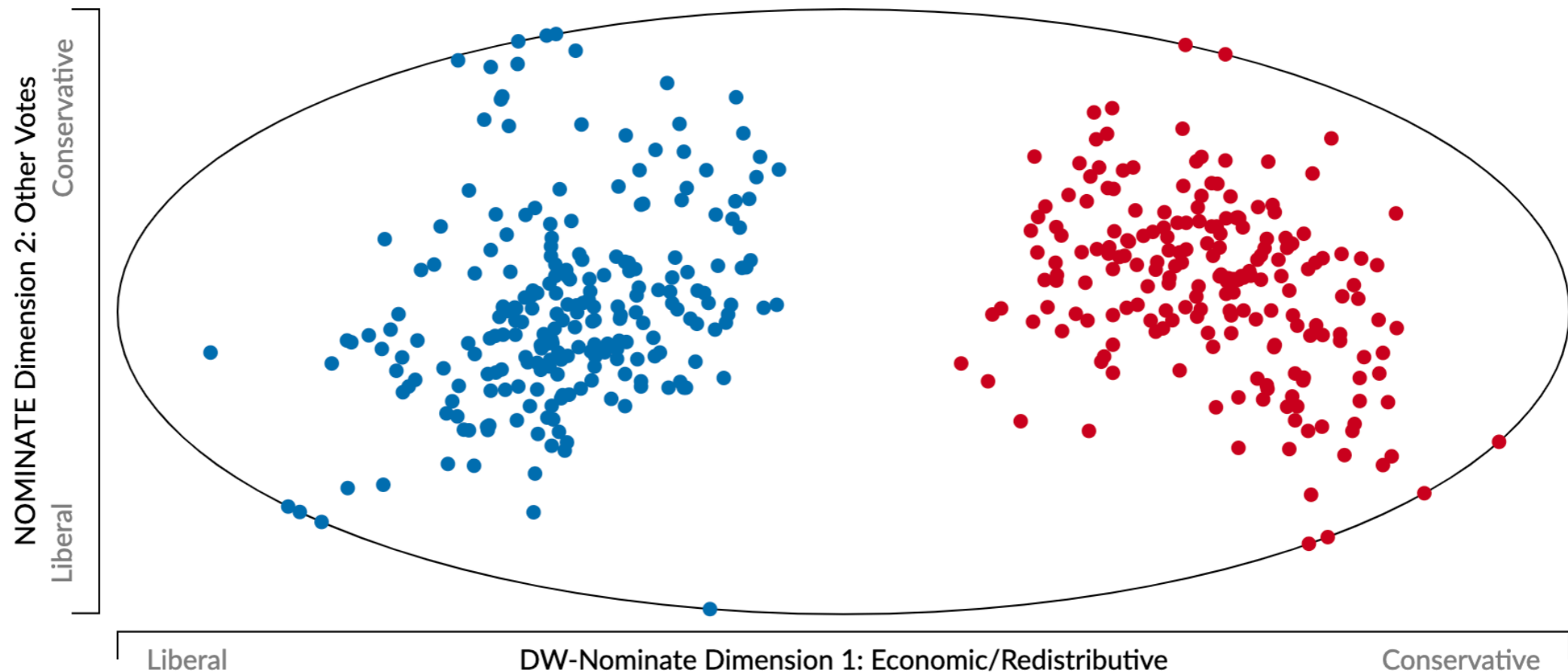




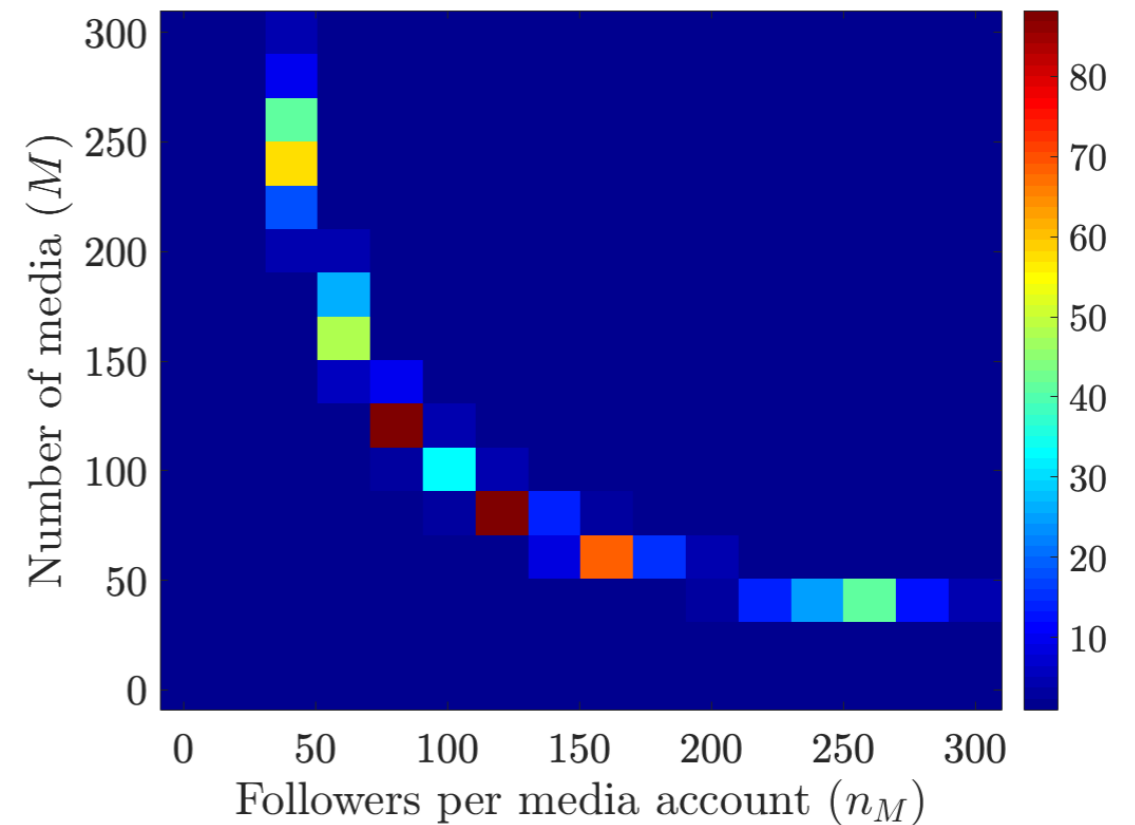
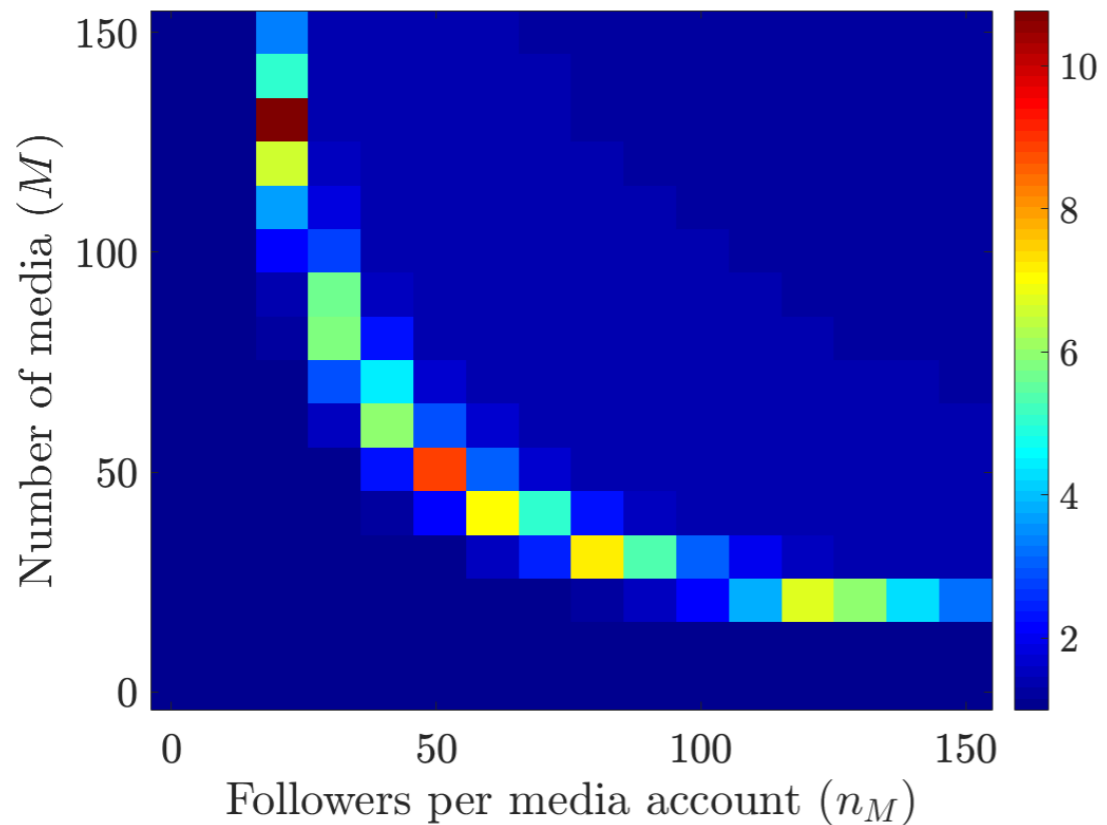
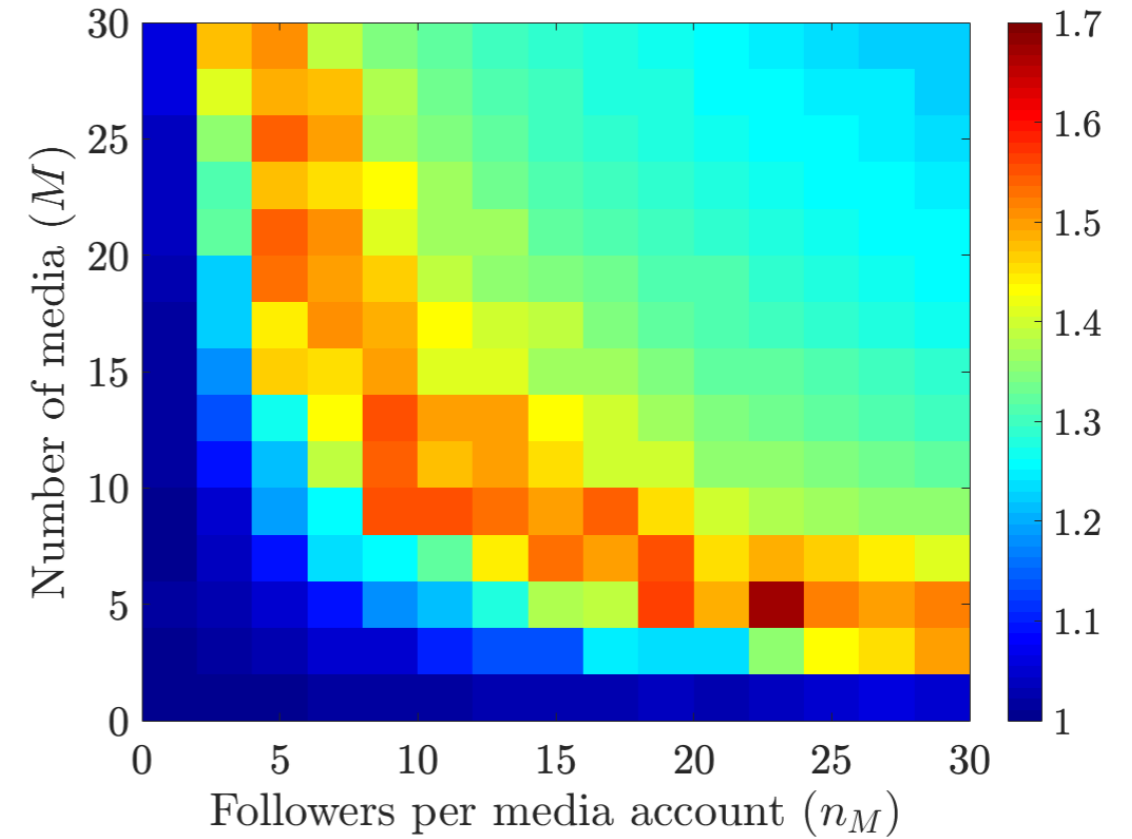
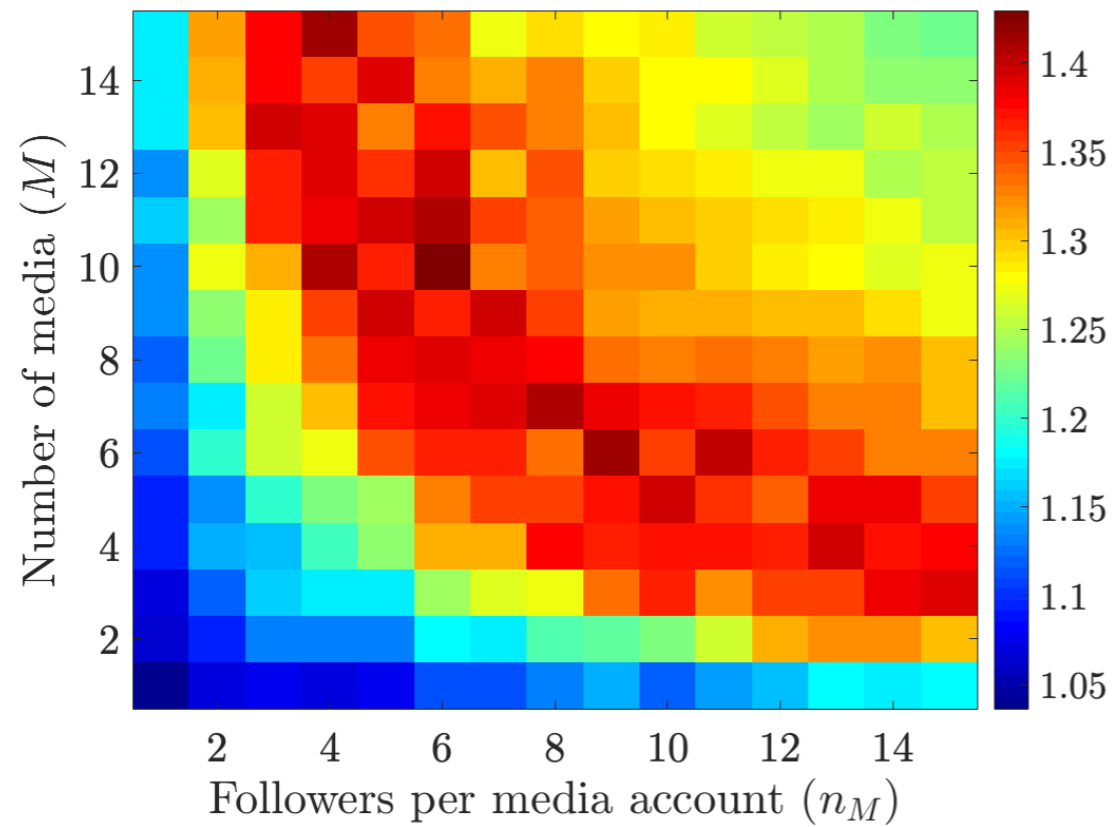
# Two key factors: ideology and quality

DW-Nominate Plot ↓

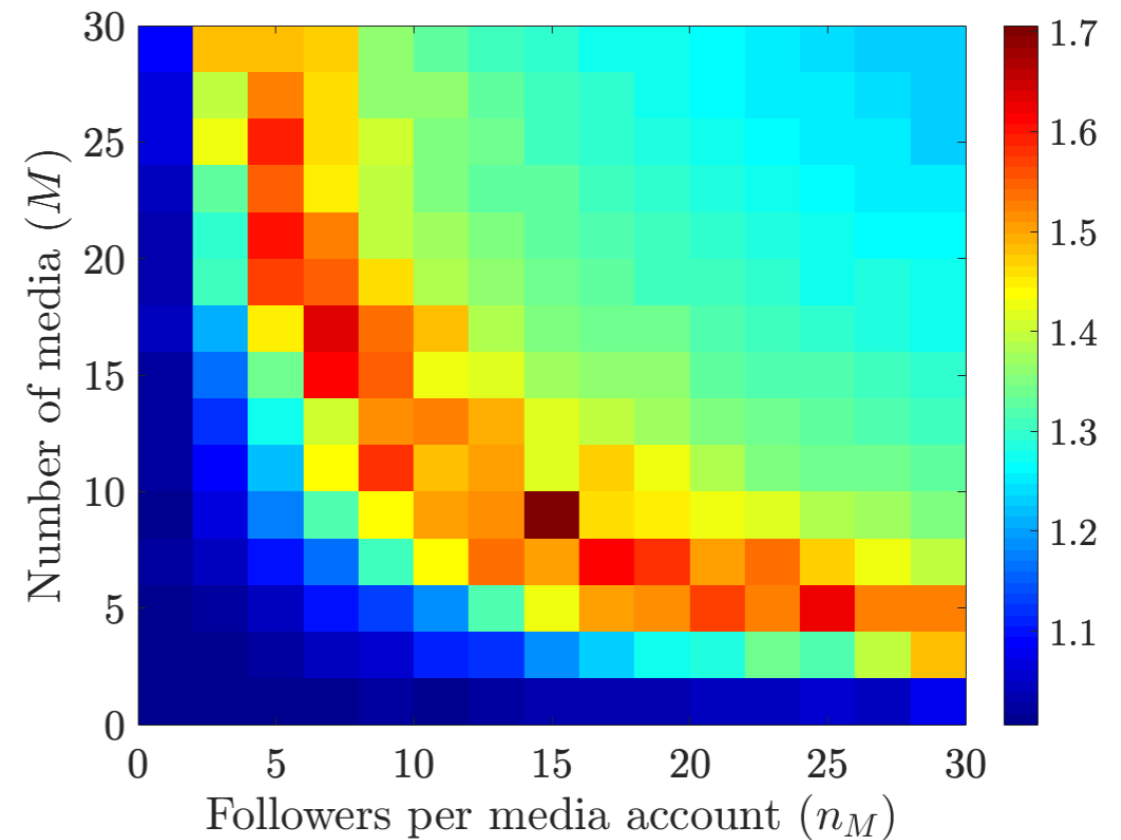
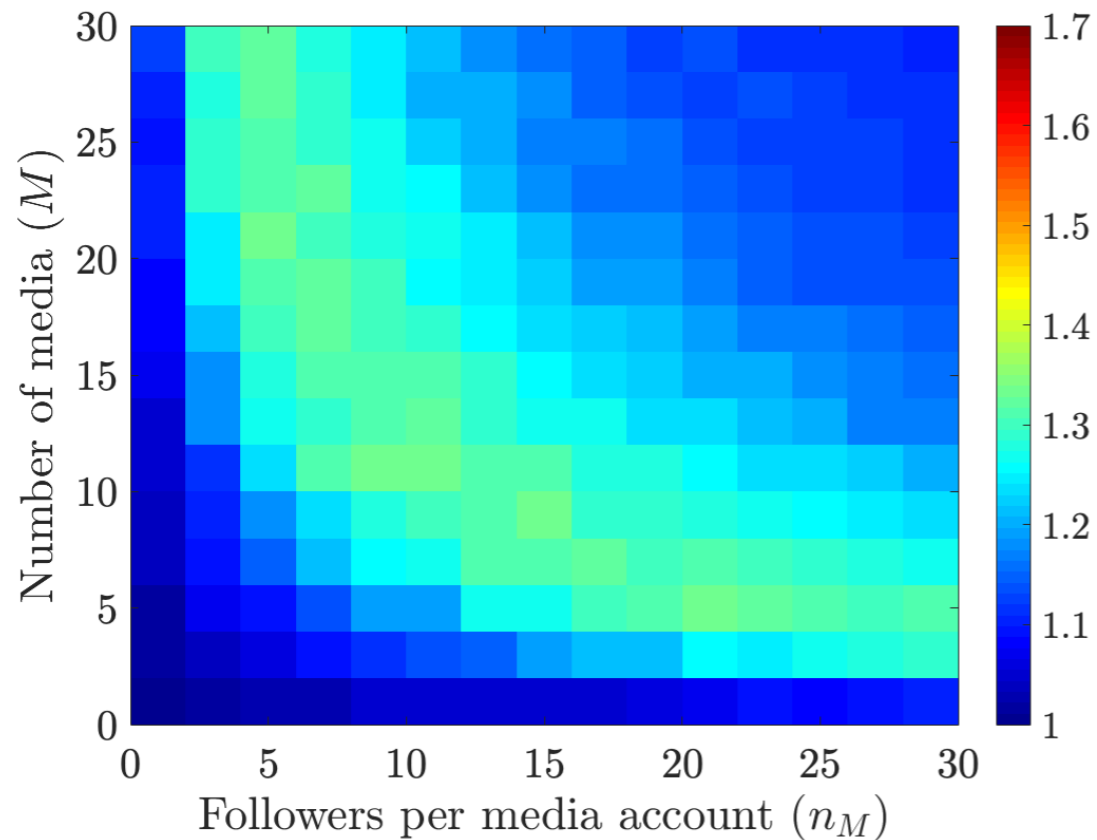
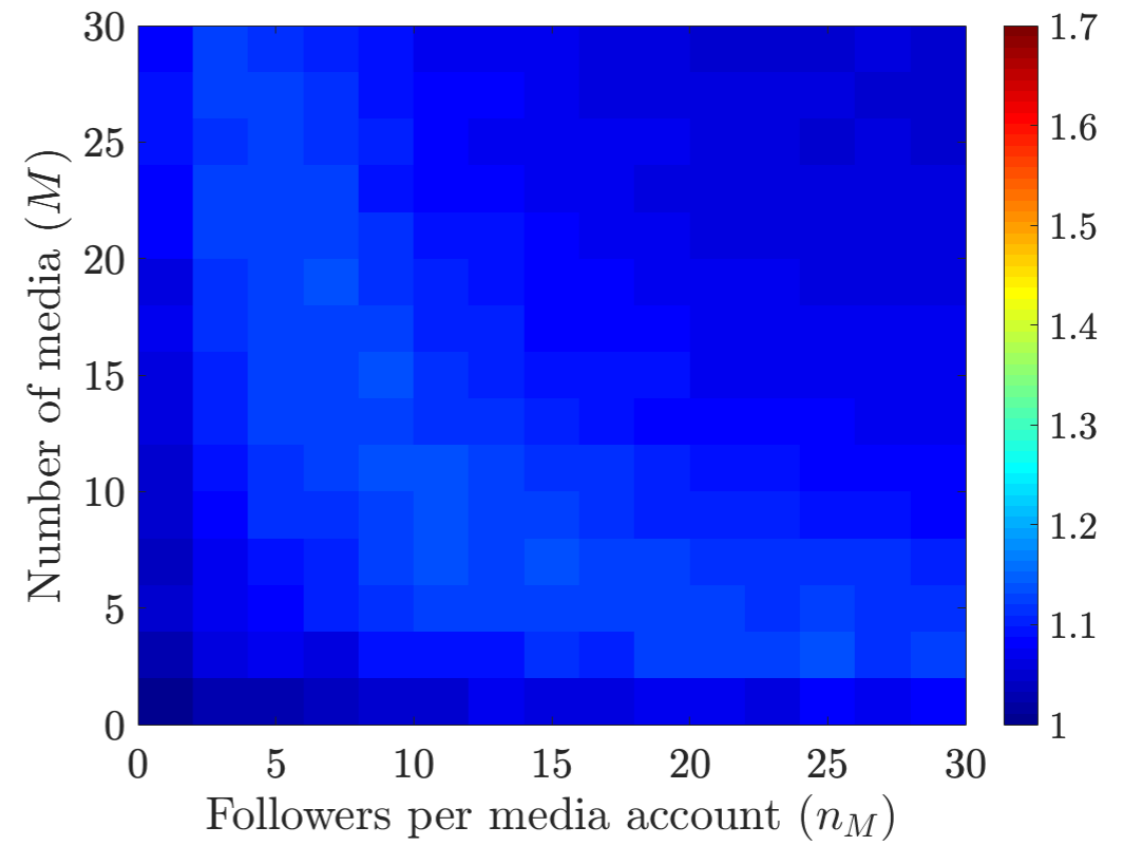
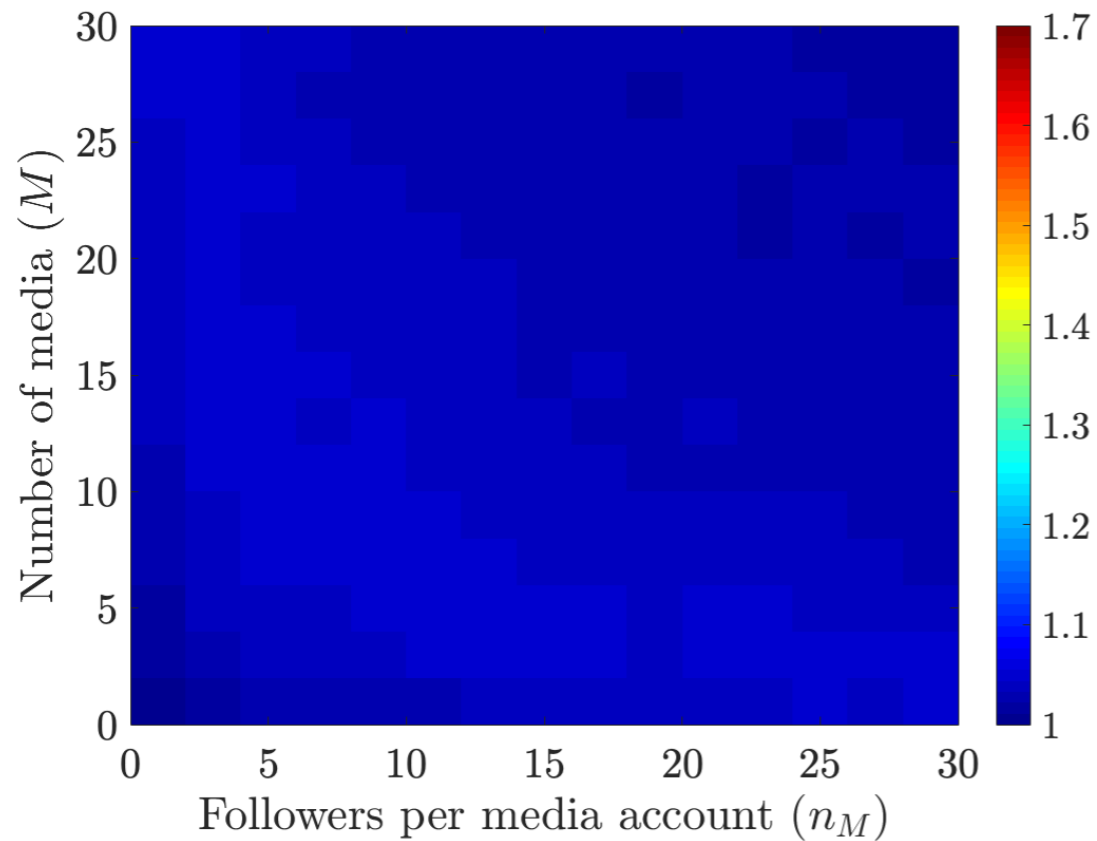
VoteView: 116th Congress, House of Representatives



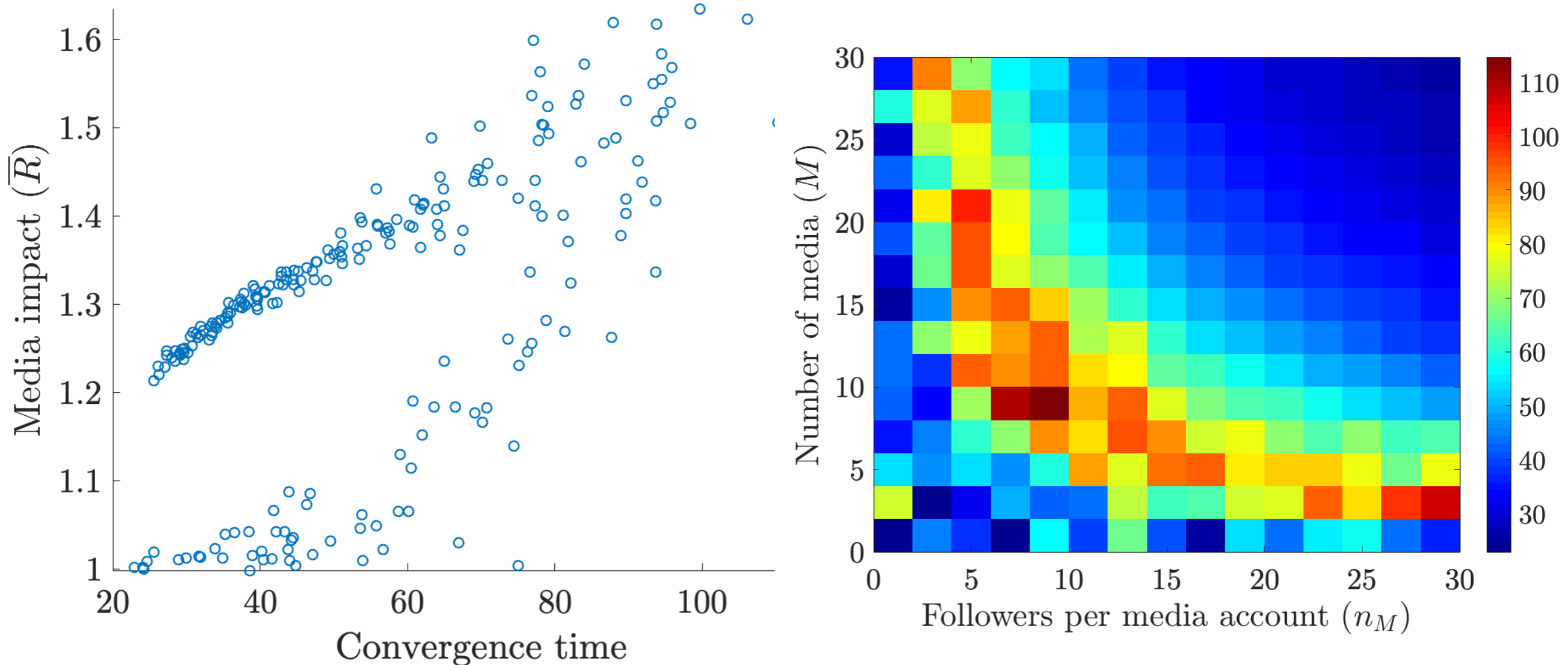
# Increasing $N$ narrows region of media entrainment



# Increasing $c$ increases media entrainment



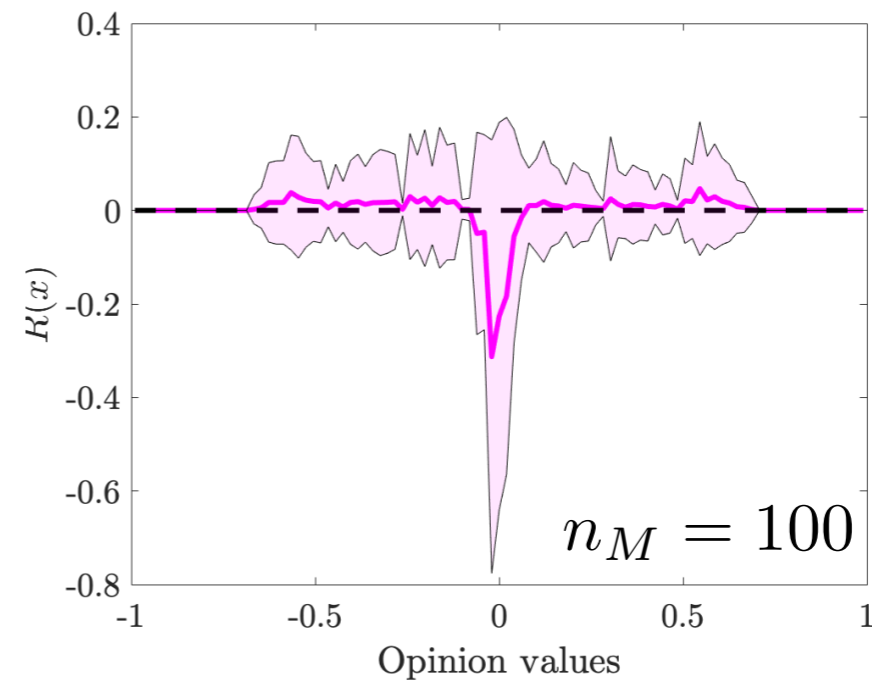
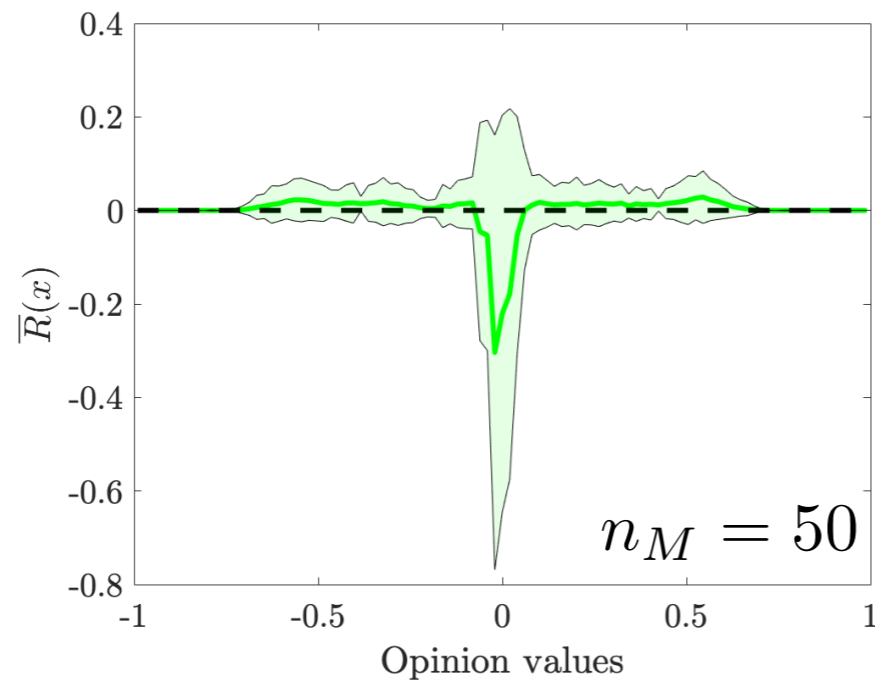
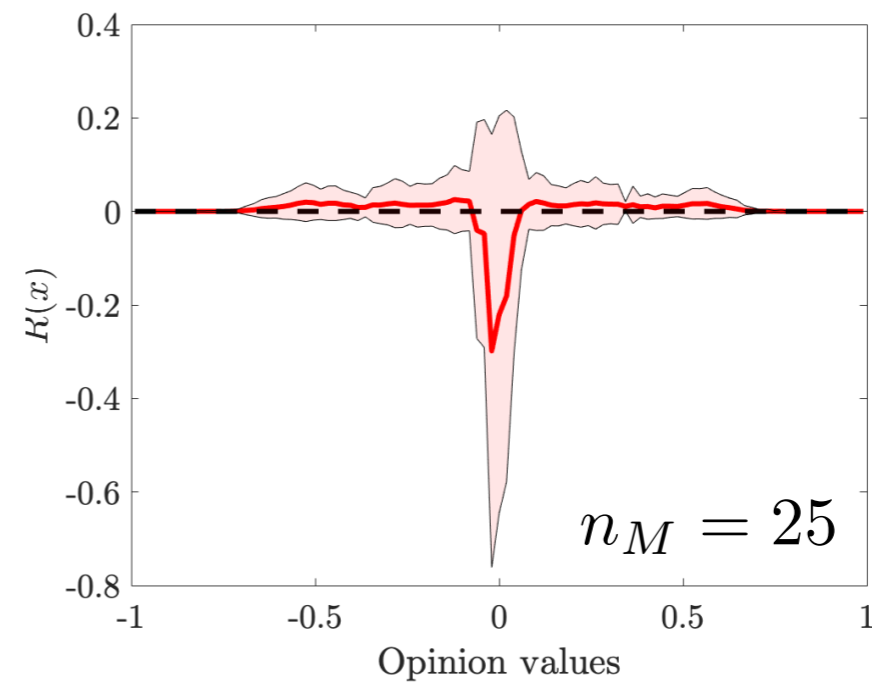
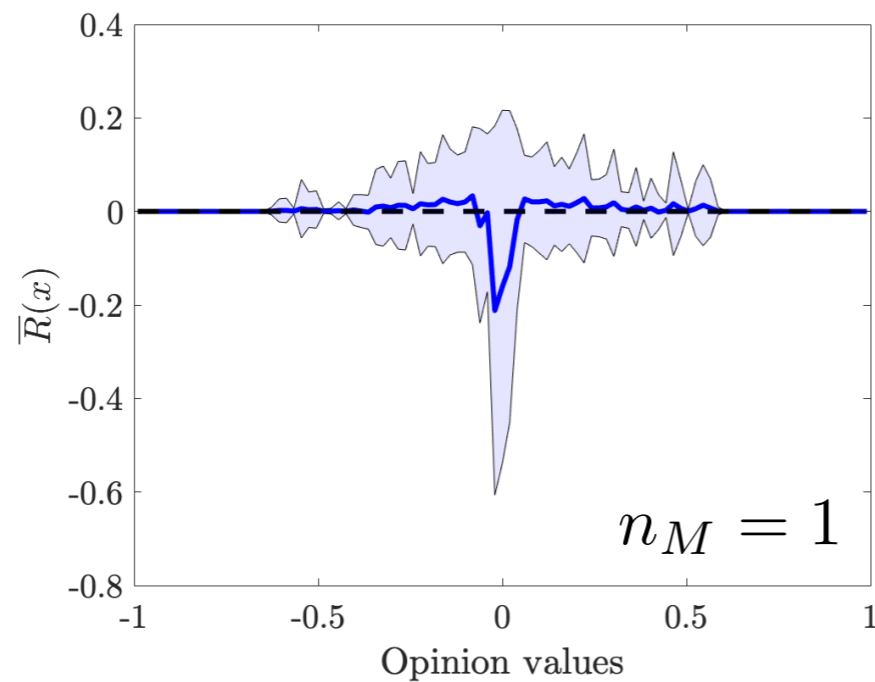
# Media impact positively correlates with convergence time



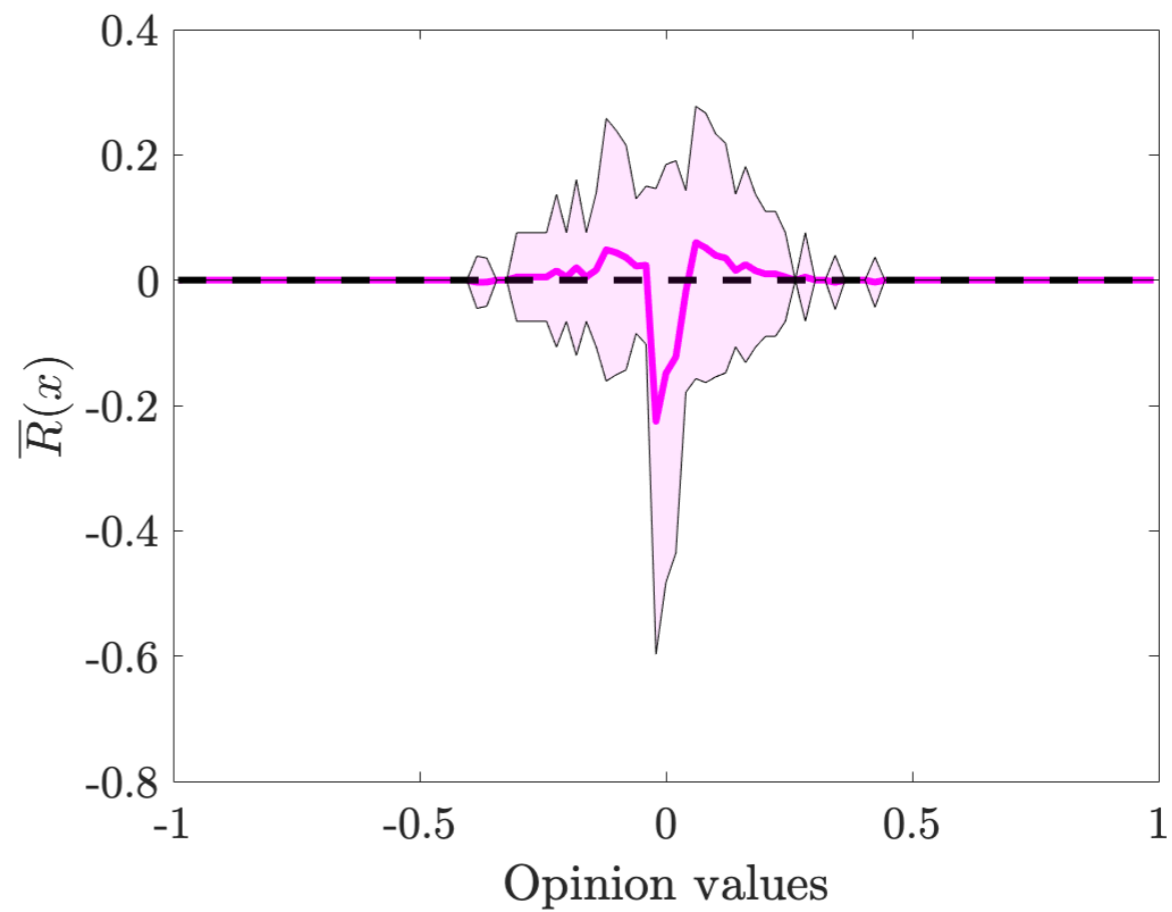
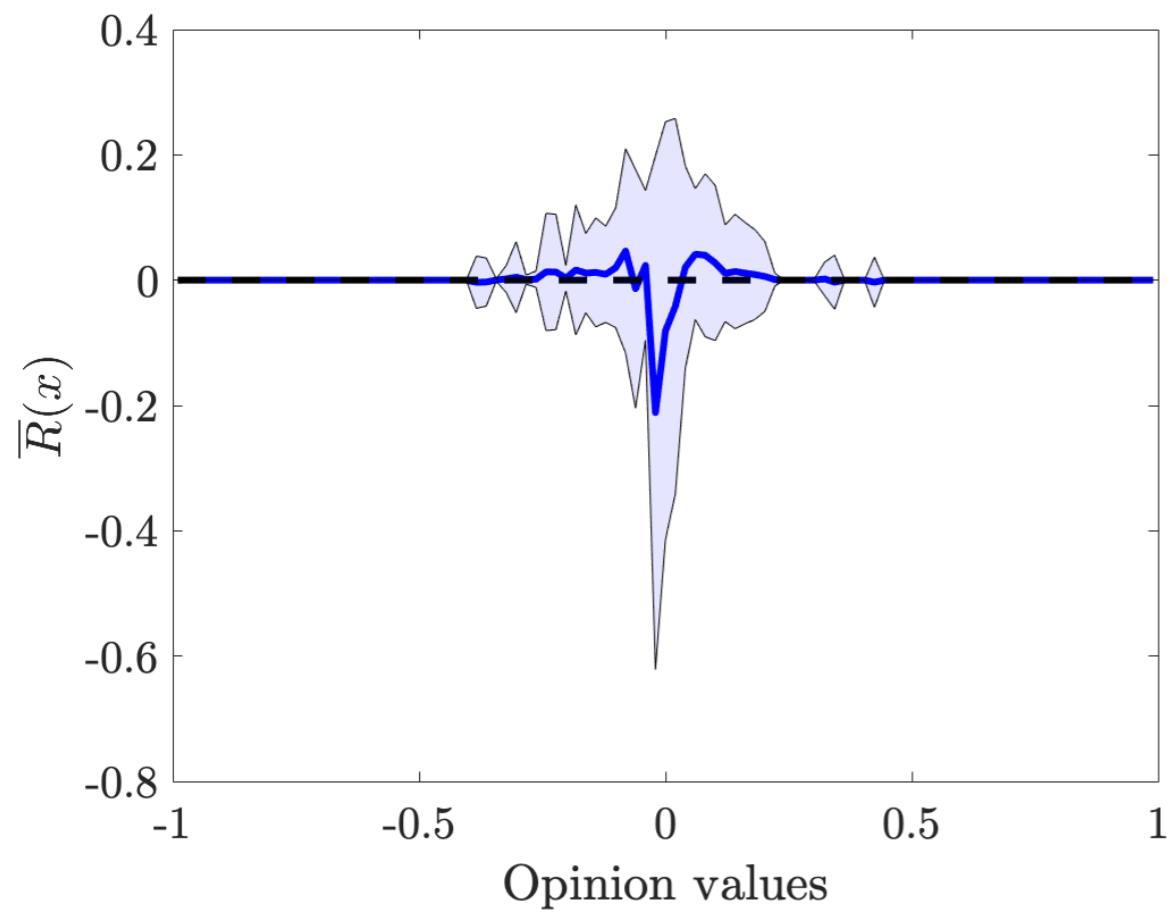
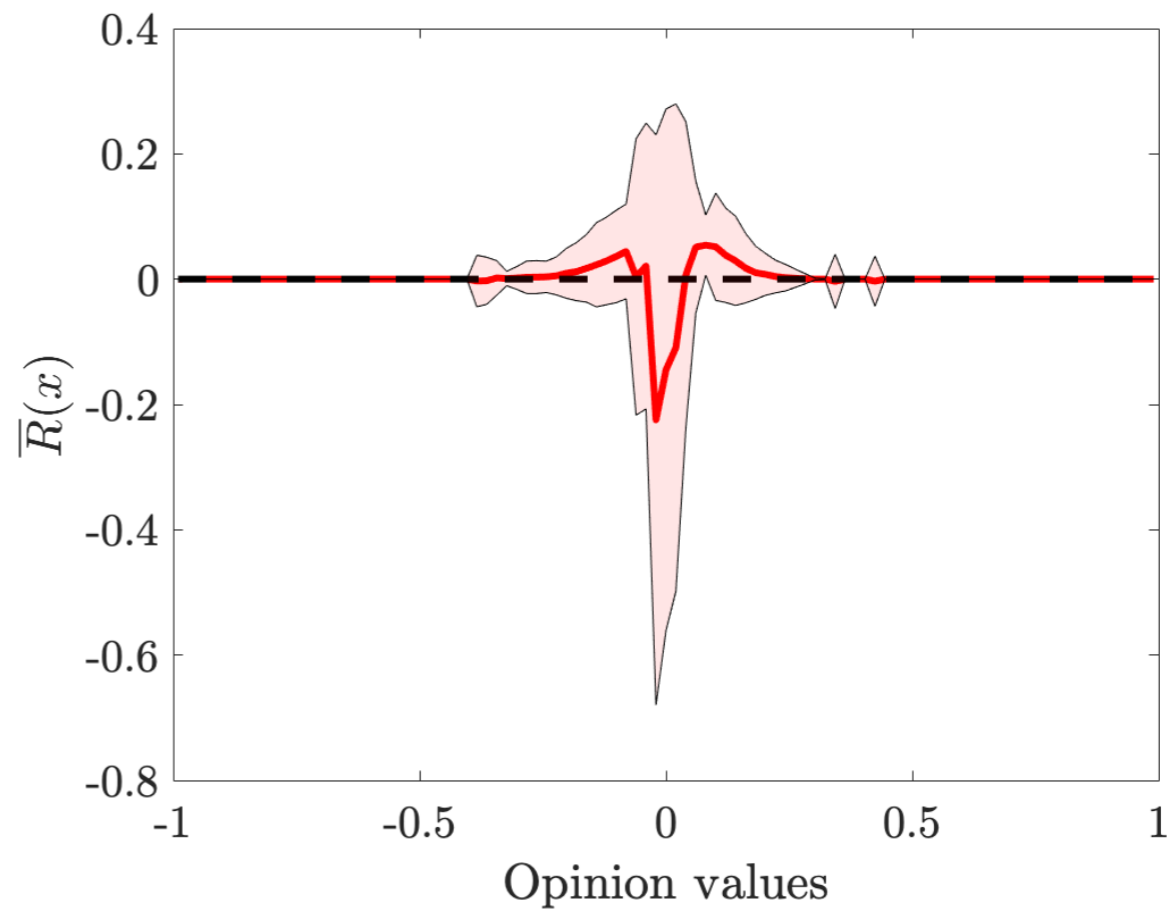
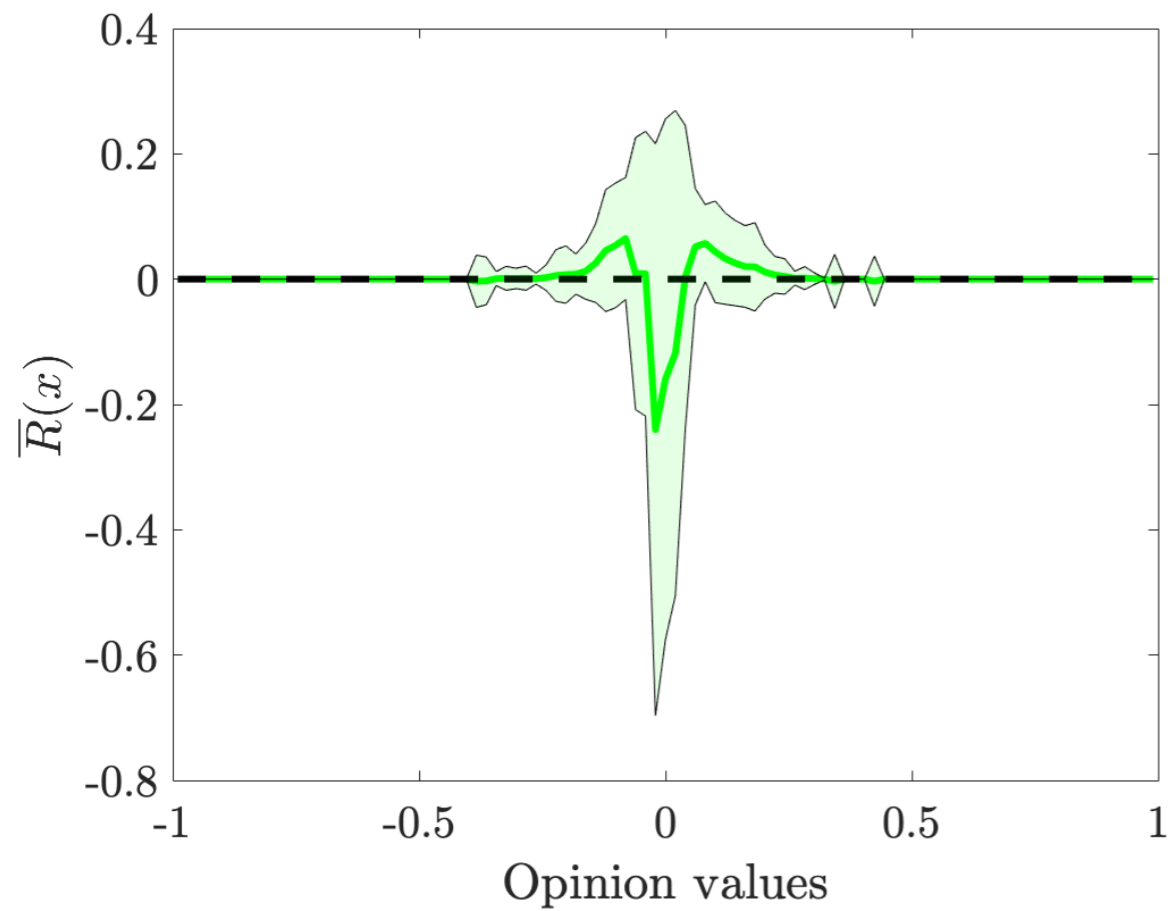
Each dot represents one trial for Erdős—Rényi graph with  $N=100$ ,  $k=25$ .

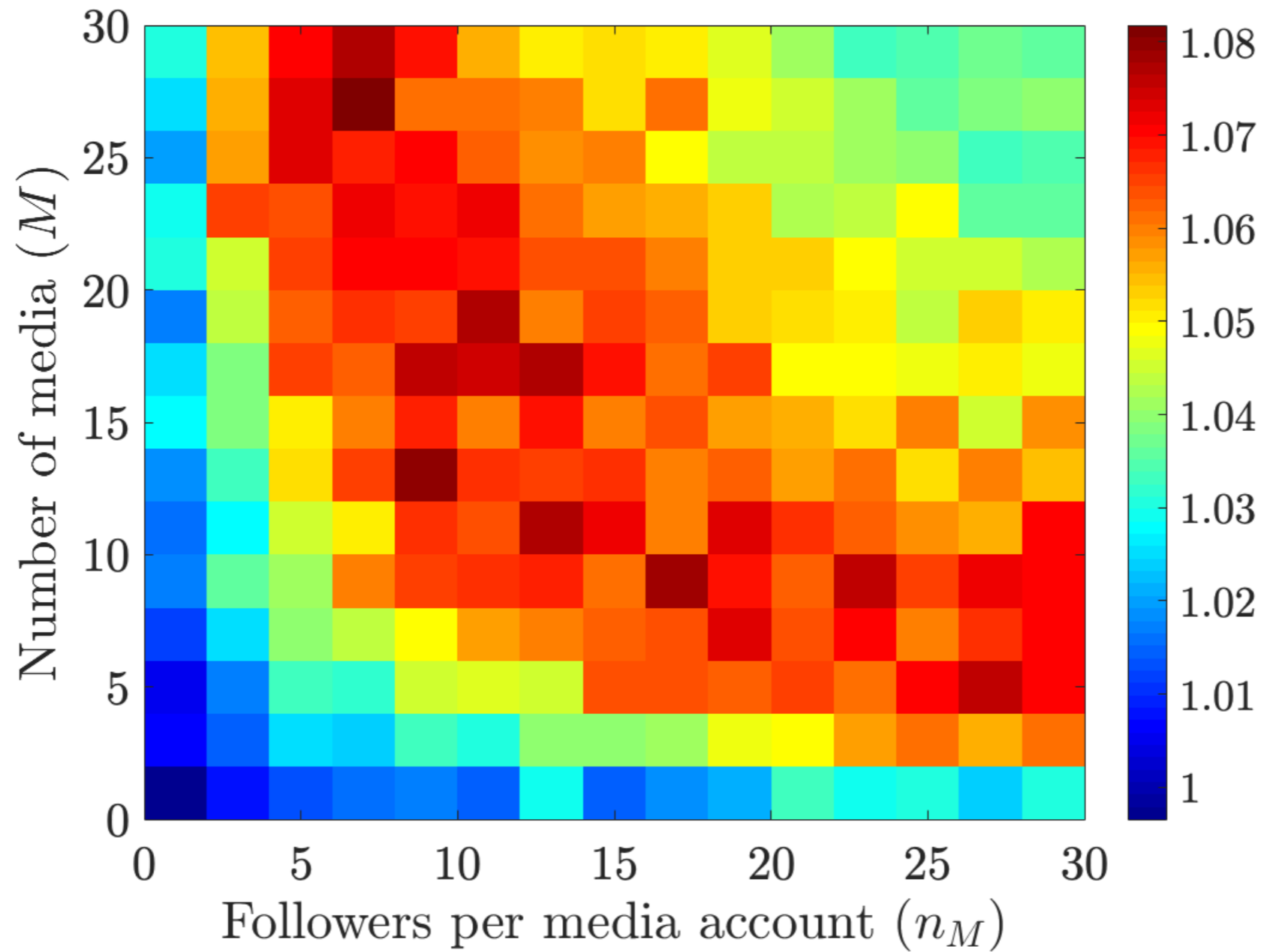
Color in heat map represents number of time steps to convergence

# We can also measure impact for media ideologies drawn from distributions



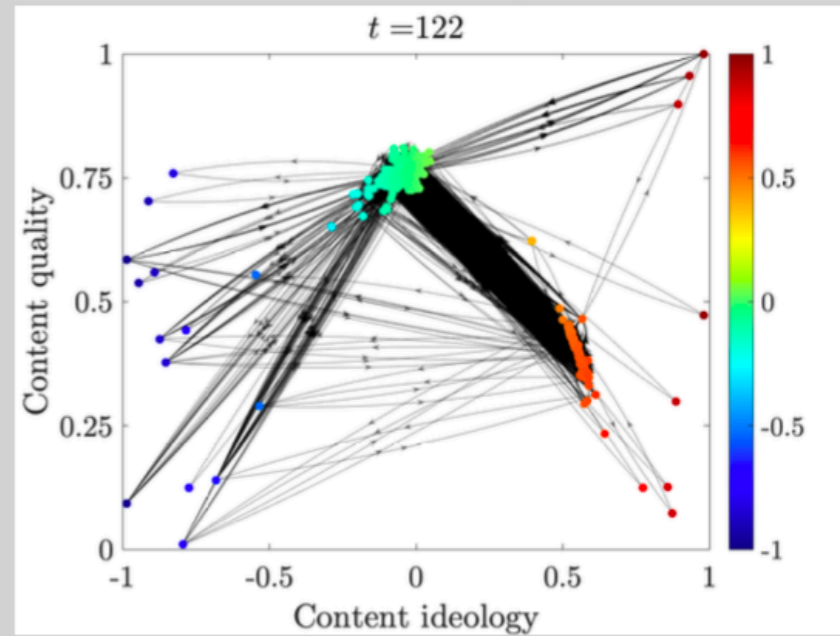
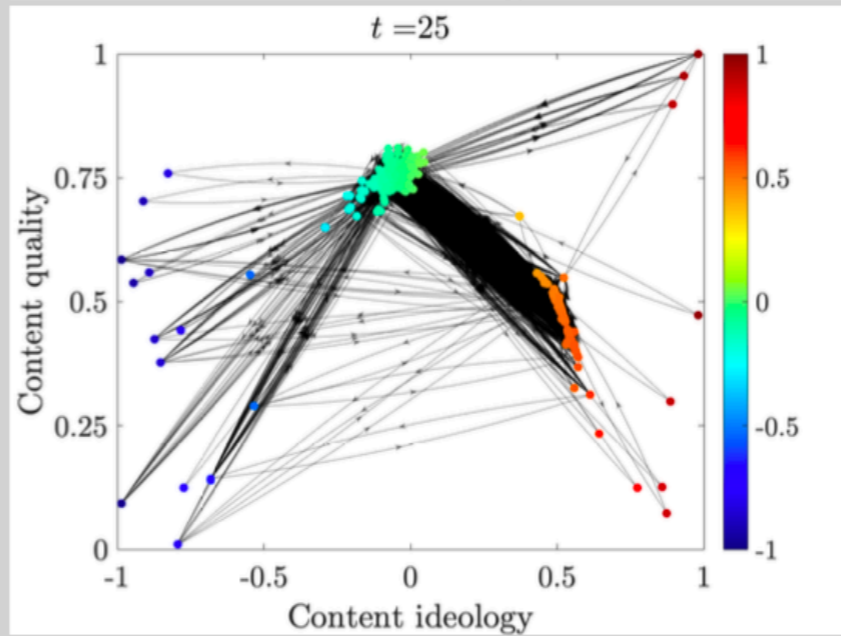
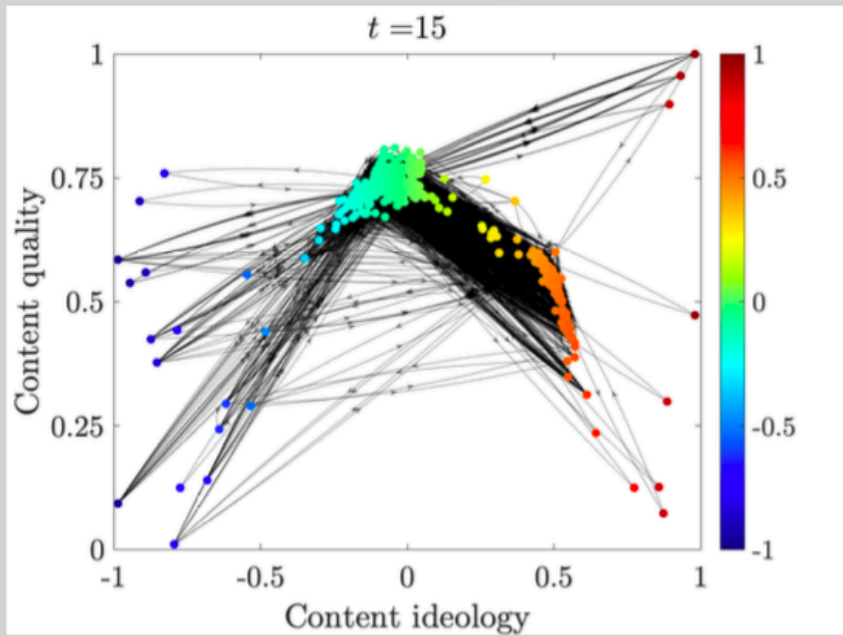
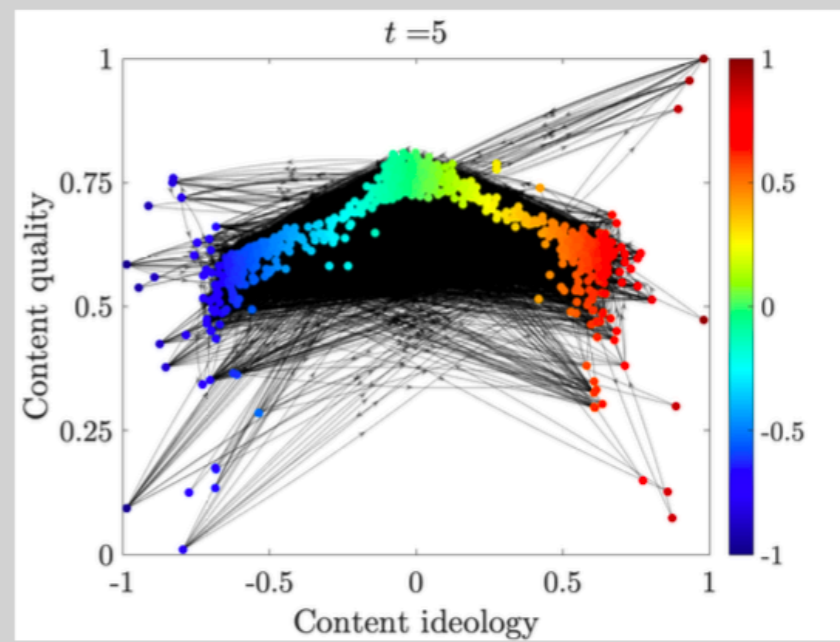
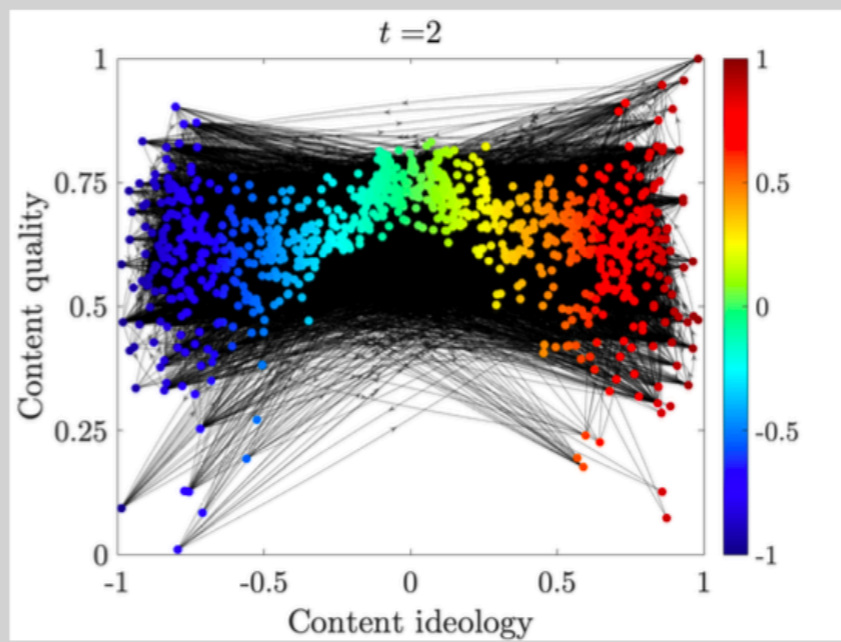
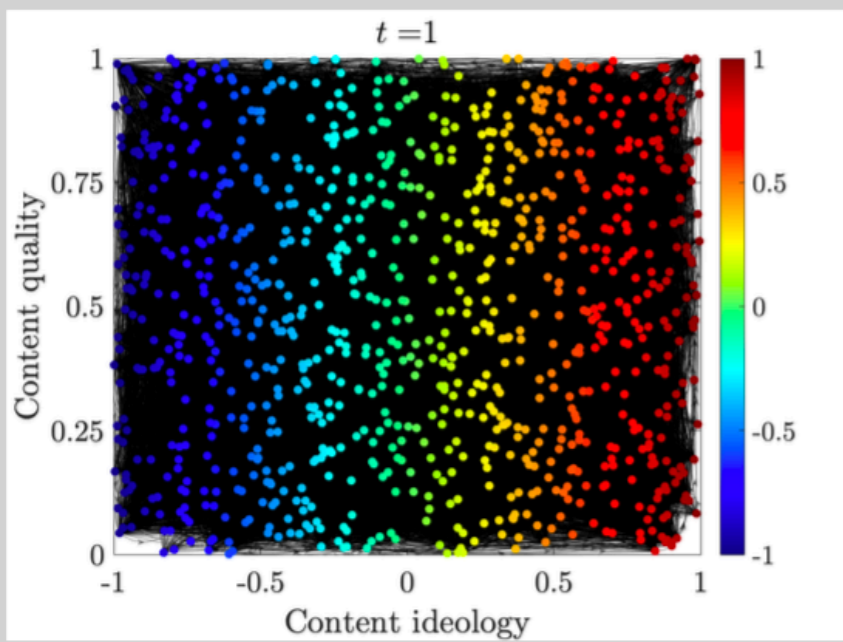
$M=100$  media drawn from a uniform random distribution on  $[-1,1]$



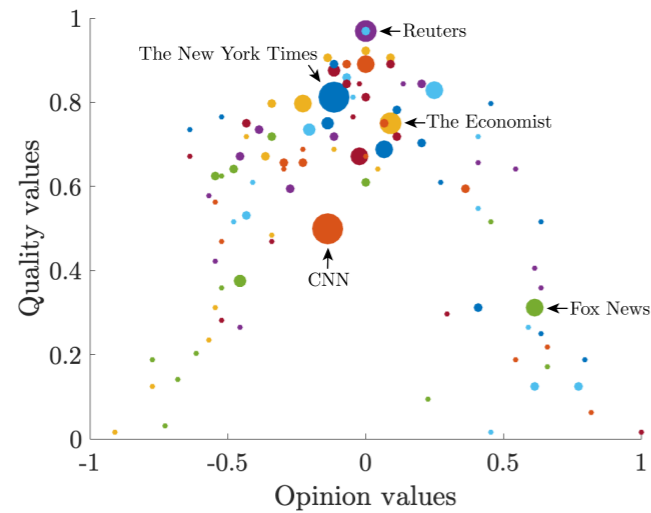


**Two ideological dimensions**





# Dynamics of democracy



Influence of Media on Opinion Dynamics in Social Networks

'Very Fine People on Both Sides' of Twitter: Analyzing the Network Structure of the Online Conversation about #Charlottesville

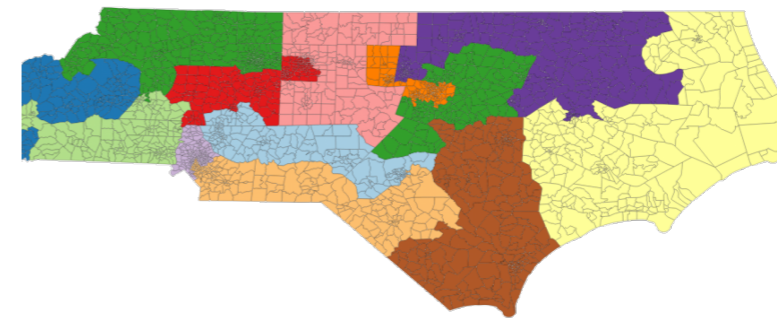
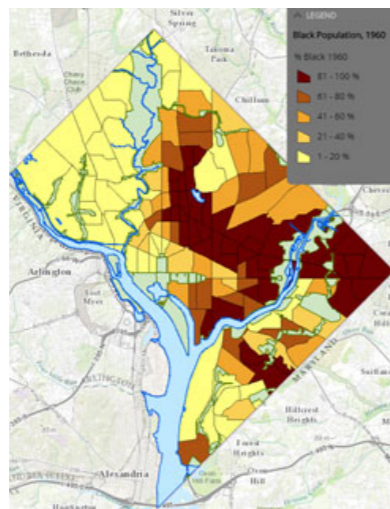
The Effect of the Convergence Parameter in the Deffuant Model of Opinion Dynamics

A Network Model of Immigration: Enclave Formation vs. Cultural Integration

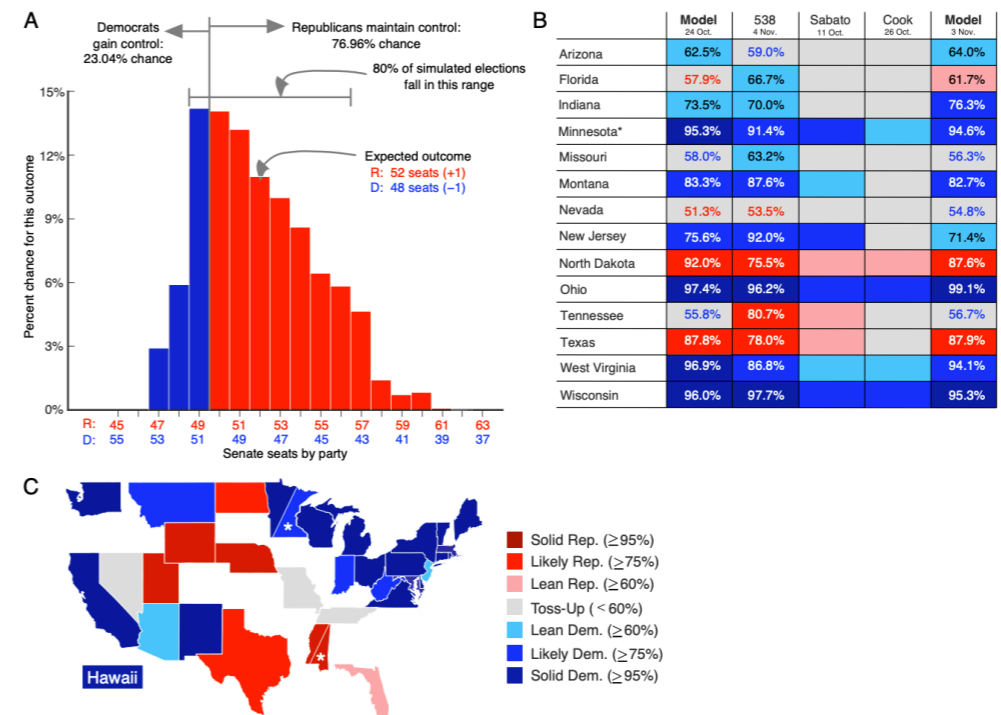
# Dynamics of democracy

Interdisciplinary Inclusive Communities of Undergraduates doing Social-Justice Inspired Research

Quantifying Gerrymandering using Random Dynamics



Forecasting U.S. Elections using Compartmental Models



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